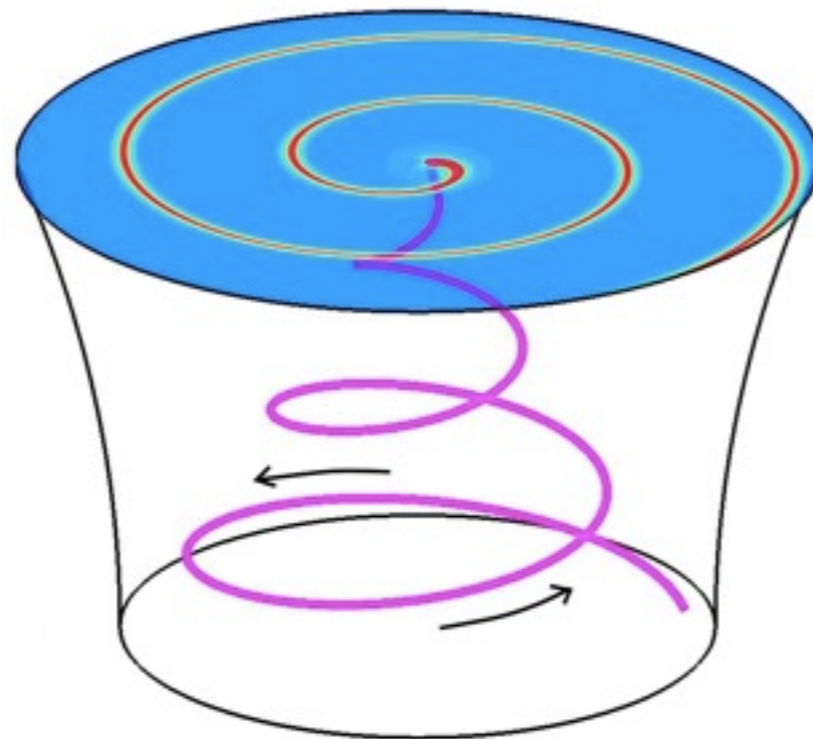


# SYNCHROTRON RADIATION IN STRONGLY COUPLED CONFORMAL FIELD THEORIES

Dominik Nickel  
Institute of Nuclear Theory, Seattle



C. Athanasiou, P. Chesler, H. Liu, DN, K. Rajagopal

[arXiv:1001.3880](https://arxiv.org/abs/1001.3880)

**W**  
UNIVERSITY *of*  
WASHINGTON

# OUTLINE

- motivation: patterns of radiation
- gauge/gravity duality
- synchrotron radiation
- outlook

# RADIATION IN CLASSICAL ELECTRODYNAMICS

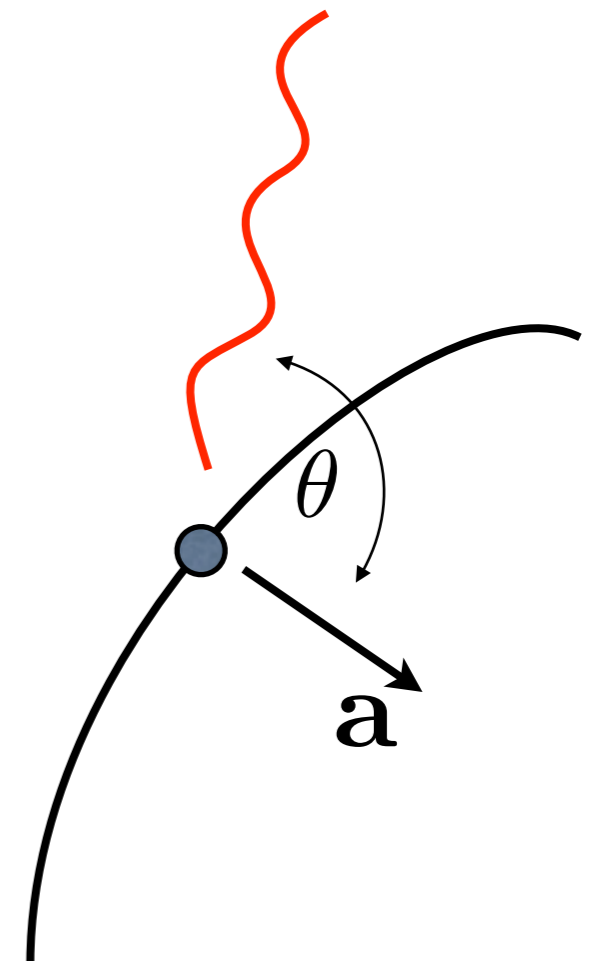
- accelerated charge radiates

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi} a^2 \sin^2 \theta$$

- ➔ total power radiated (energy loss)

$$P = \frac{2}{3} \frac{e^2}{4\pi} a^2$$

- ➔ free propagation of radiation (distribution of power)



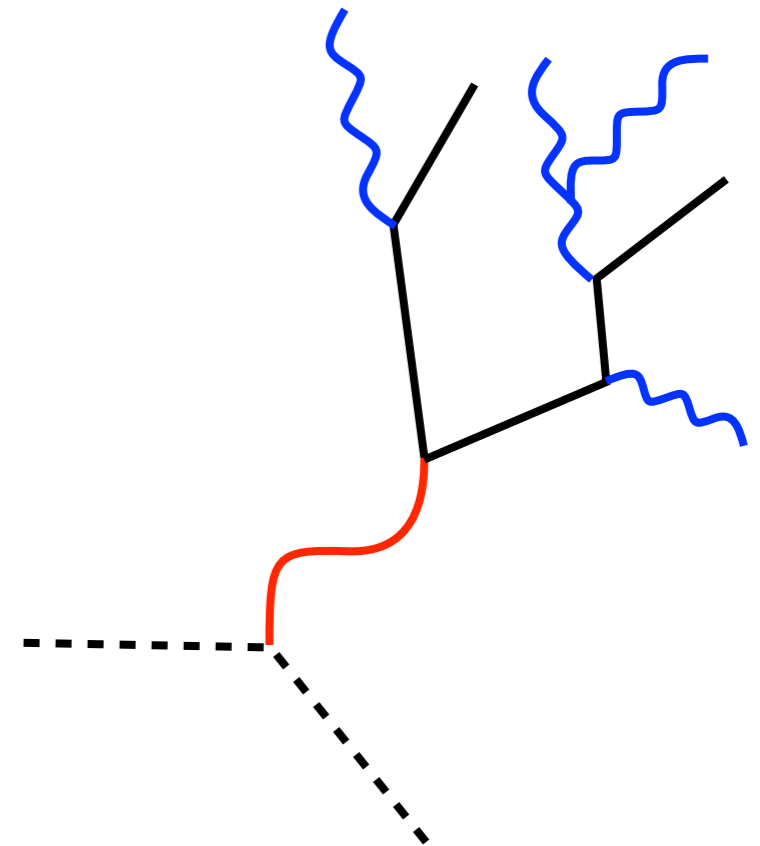
analogy in strongly coupled gauge theories?

# RADIATION IN GAUGE THEORIES? - JETS

factorization in jet production

$$e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$$

- ▶ hard production process
- ▶ showering
- ▶ hadronization



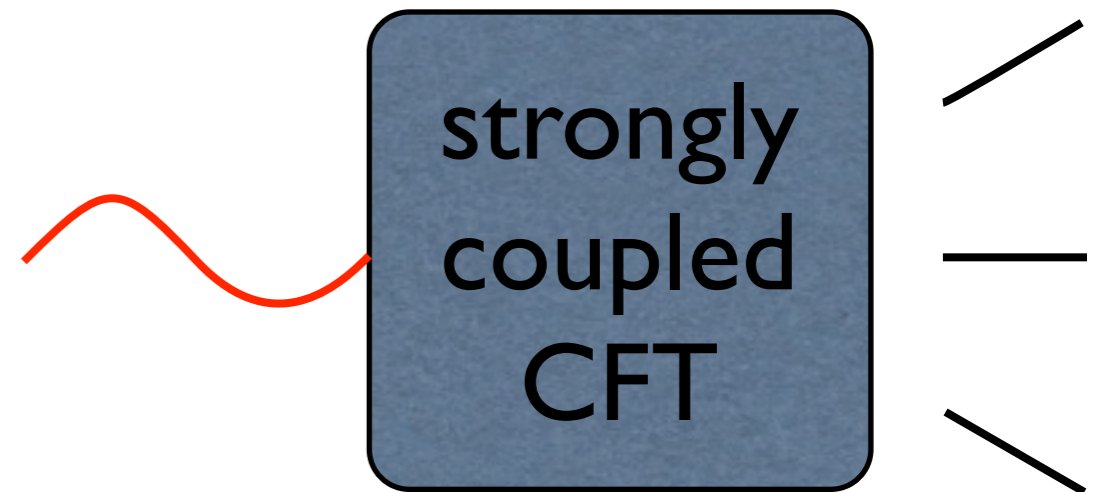
understanding from first principles challenging?

➡ guidance from other approaches?

# CONFORMAL COLLIDER PHYSICS

D. Hofman, J. Maldacena [arXiv:0803.1467]

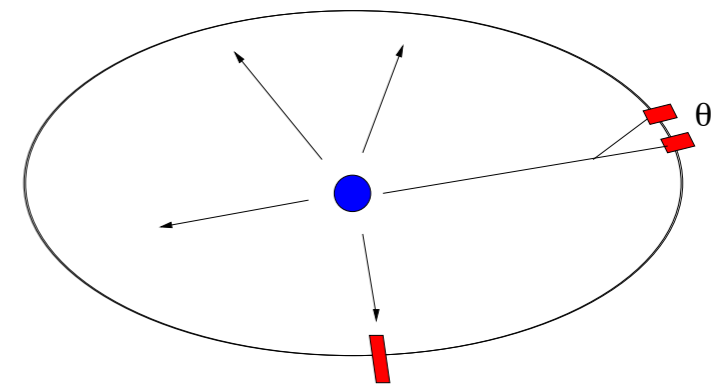
study decay of off-shell bosons in a strongly coupled CFT



quantities of interest:

$$\langle \mathcal{E}(\hat{r}) \rangle = \lim_{r \rightarrow \infty} r^2 \int dt \langle \psi(t) | \hat{r} \cdot \mathbf{S} | \psi(t) \rangle$$

$$\langle \mathcal{E}(\hat{r}) \mathcal{E}(\hat{r}') \rangle = \dots$$

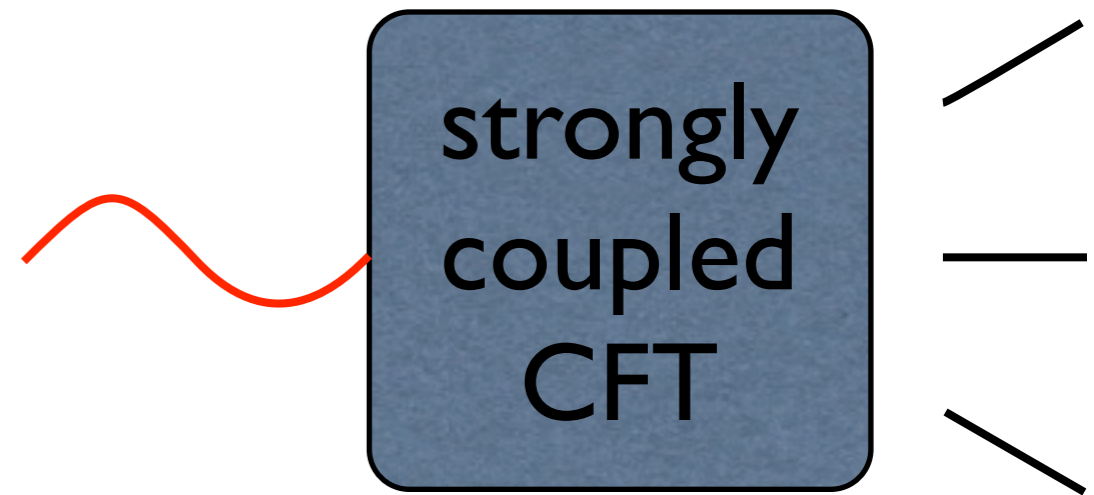


key result: all N-point functions isotropic!

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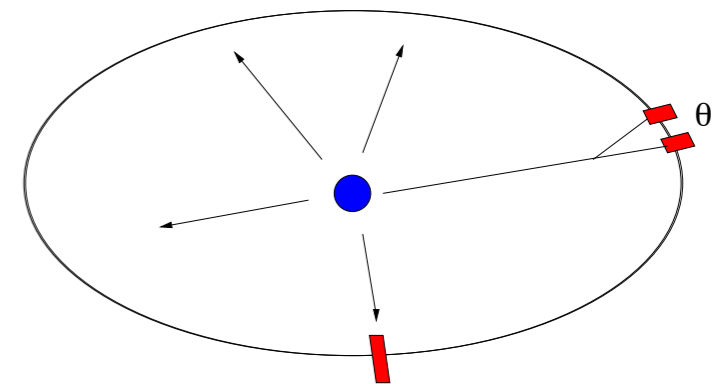
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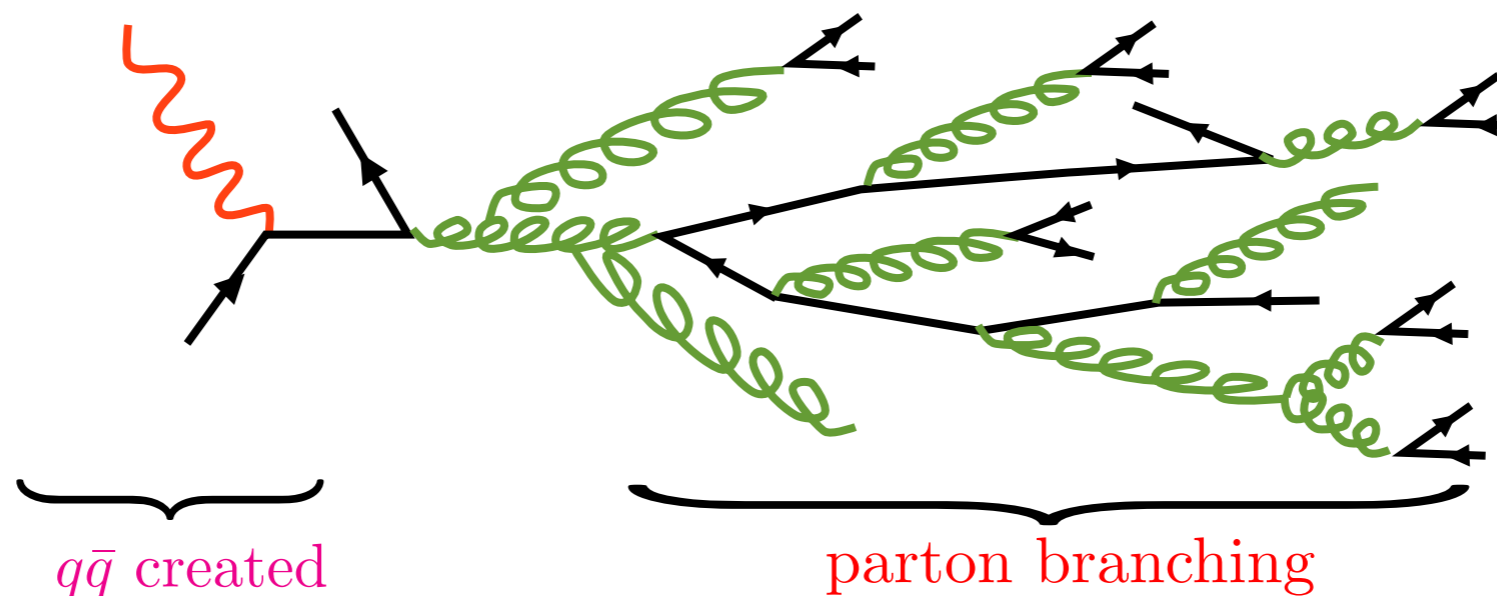
no jets at strongly coupling?

# WHERE DOES THE ISOTROPY COME FROM?

- choice of initial conditions?
- propagation through strongly coupled vacuum?

parton branching picture:

Y. Hatta, E. Iancu, A. Mueller [arXiv:0803.2481]



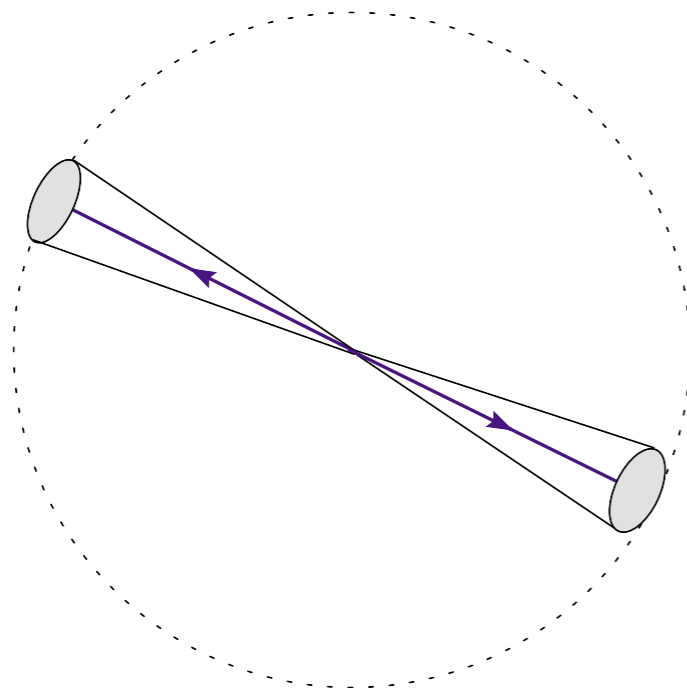
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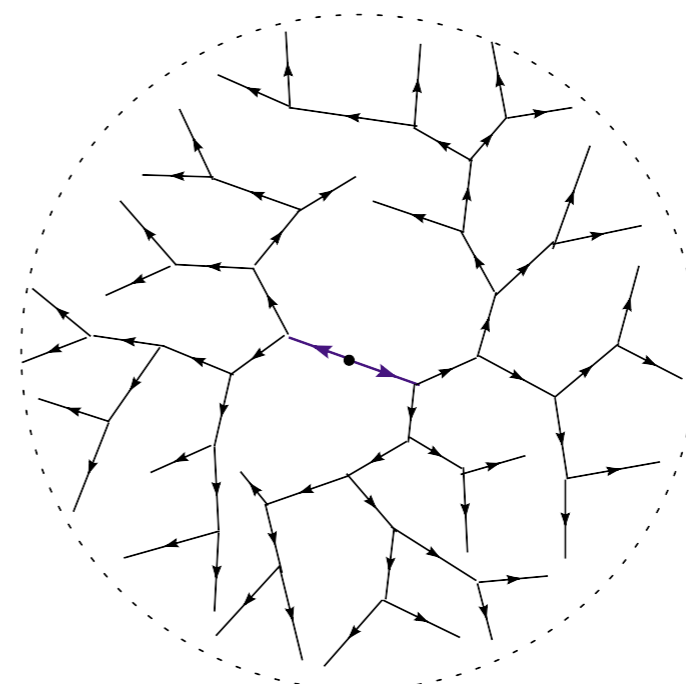
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weak coupling



strong coupling

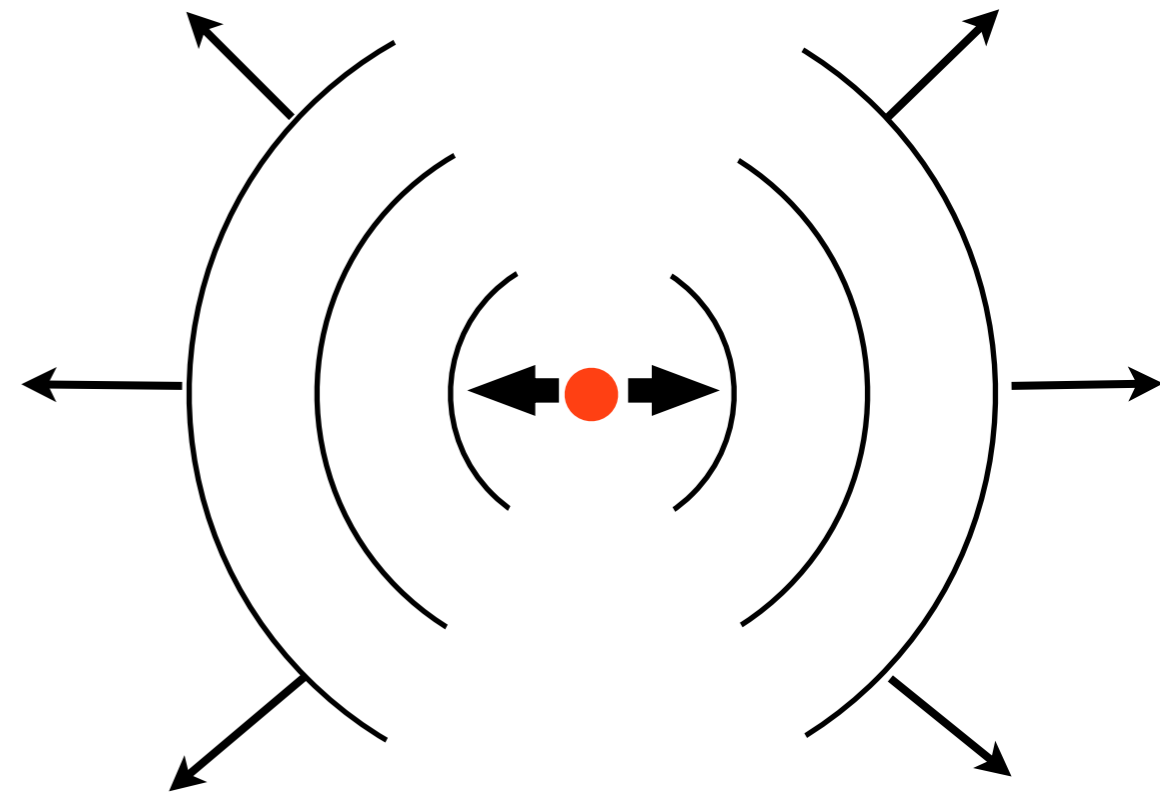
any anisotropy scrambled by successive parton branching?



# A SIMPLE EXPERIMENT

radiating source in compact domain

- ▶ real-time propagation
- ▶ power spectrum at infinity
- ▶ depletion of short-wavelength modes?



concentrate on theories with classical gravity dual  
>exact solution for synchrotron radiation<

# ADS/CFT

Maldacena's conjecture:

string theory on asymptotic  $AdS_5 \times X^5$   $\longleftrightarrow$   $4d$  (supersymmetric) CFT  
classical gravity large  $N_c, \lambda$

J. Maldacena [hep-th/9711200]

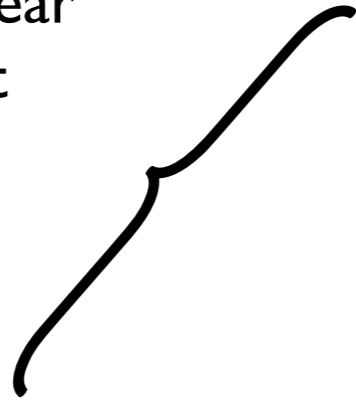
geometrical realization / 'dictionary':

gravity in 5d		4d CFT
gravitational fields		source for stress-energy tensor
gauge fields		source for global currents
open strings	$\longleftrightarrow$	quarks
black hole		finite temperature/density
$S_{\text{gravity}}[\phi] = S_{\text{gravity}}^{\text{boundary}}[\phi_0] = S_{\text{CFT}}[\phi_0]$		

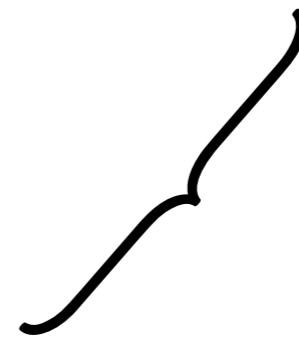
➡ classical dynamics in 5d captures dynamics of 4d QFT

# A PICTORIAL PRESENTATION OF HOLOGRAPHY

deformation of background near  
boundary encodes relevant  
information for CFT



(cartoon for illustrative purposes  
only; courtesy of P. Chesler)



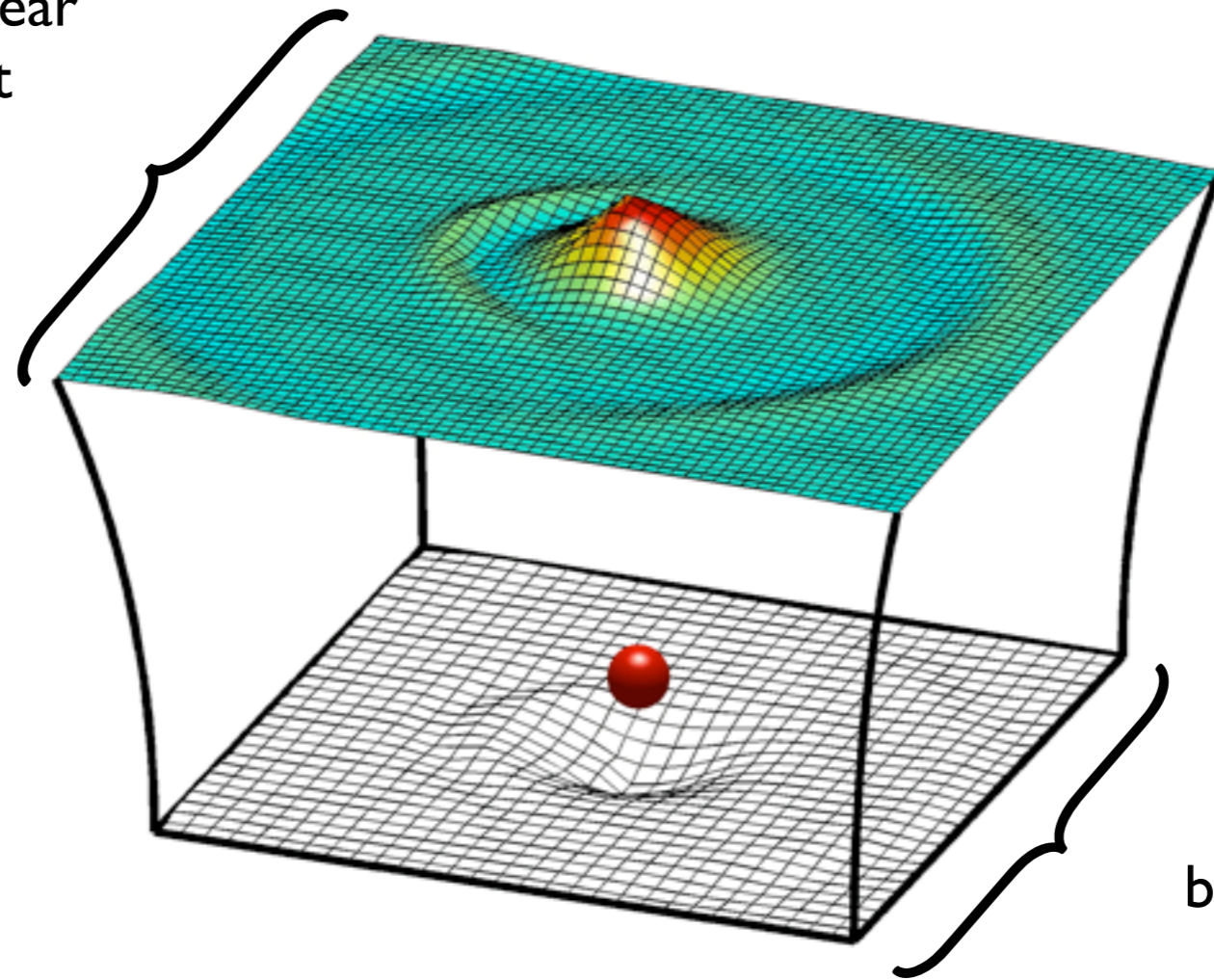
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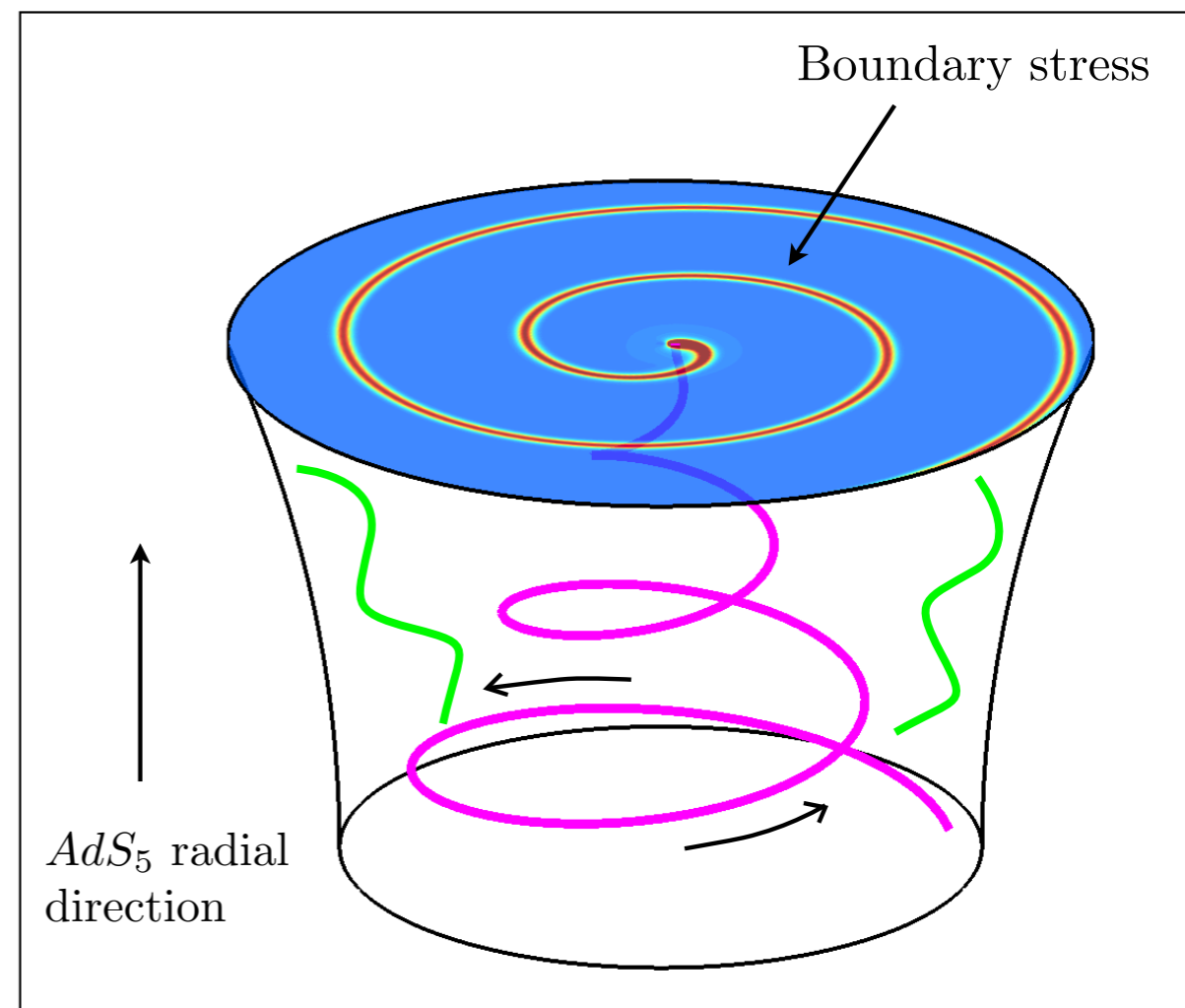
# THE GRAVITATIONAL SETUP

AdS background:

$$ds^2 = \frac{L^2}{u^2} (-dt^2 + d\mathbf{x}^2 + du^2)$$

- ➔ rotating string  $X^\mu(\sigma, \tau)$
- ➔ gravitational waves  $h_{\mu\nu}(x)$  towards boundary
- ➔ stress-energy tensor/energy density

$$T^{\mu\nu}(x) = \lim_{u \rightarrow 0} \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{gravity}}}{\delta h_{\mu\nu}(x)}$$



simplification:  $G_{\text{Newton}} \sim 1/N_c^2 \rightarrow$  gravitational response linear



# THE ROTATING STRING

- Nambu-Goto action in AdS background:

$$S_{\text{NG}} = -T_0 \int dt du \sqrt{-\det(\gamma_{ab})}, \quad \gamma_{ab} = G_{\mu\nu}^{\text{AdS}} X^\mu X^\nu$$

- parameterization of worldsheet:  $X^\mu = (t, R(u), \pi/2, \phi(u) + \omega_0 t, u)^\mu$

- equations of motion: 
$$R'' = -\frac{R(u + 2RR')(1 + R'^2)}{u(u^4\Pi^2 - R^2)} - \frac{1 + R'^2}{R(1 - \omega_0^2 R^2)},$$

$$\phi'^2 = \frac{u^4\Pi^2(1 - \omega_0^2 R^2)(1 + R'^2)}{R^2(R^2 - u^4\Pi^2)}.$$

- additional constraint:  $1 - \omega_0^2 R(u_\Lambda)^2 = 0 \Leftrightarrow R(u_\Lambda)^2 - u_\Lambda^2 \Pi^4 = 0$

- therefore only one constant of integration:  $R_0 \equiv R(u = 0)$

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- solution (**remarkable luck!**):

$$R(u) = \sqrt{R_0^2 + \frac{v^2 u^2}{1 - v^2}}, \quad \phi(u) = -\frac{u\omega_0}{\sqrt{1 - v^2}} + \arctan\left(\frac{u\omega_0}{\sqrt{1 - v^2}}\right)$$

- **bonus:** energy loss from flux on string A. Mikhailov [hep-th/0305196]

$$E = -\pi_t^u \equiv \frac{\delta S_{\text{NG}}}{\delta \partial_u X^t} = \frac{\lambda}{2\pi} \left( \frac{R_0 \omega_0^2}{1 - v^2} \right)^2$$



# GRAVITATIONAL WAVES AND GAUGE INVARIANTS

- linearized equations of motion for  $G_{MN} = G_{MN}^{\text{AdS}} + h_{MN}$  from

$$R_{MN} - \frac{1}{2}G_{MN}(R + 2\Lambda) = 8\pi G_{\text{Newton}} t_{MN}^{\text{rotating string}}$$

- decomposition into helicities:  $h_{MN} = h_a^{\text{tensor}} \mathbb{T}_{MN}^a + h_a^{\text{vector}} \mathbb{V}_{MN}^a + h_a^{\text{scalar}} \mathbb{S}_{MN}^a$

- coordinate invariance/conservation laws: only one gauge invariant per helicity required

- scalar equation

$$\partial_u^2 Z(u) - \frac{5}{u} \partial_u Z(u) + \left( -\partial_t^2 + \nabla^2 + \frac{9}{u^2} \right) Z(u) = S(u)$$

$$S(u)/L^3 = -u^2 \partial_u \left( \frac{t_{00}}{u^2} \right) + \frac{u}{6} \nabla^2 (2t_{00} - 2t_{55} + t_{ii}) - \frac{1}{2} u \nabla_i \nabla_j t_{ij} + \partial_t t_{05} + \nabla_i t_{i5}$$

- energy density:

$$\mathcal{E}(t, \mathbf{r}) = \lim_{u \rightarrow 0} \frac{Z(t, \mathbf{r}, u)}{u^3}$$

# ANALYTIC SOLUTION FOR ENERGY DENSITY

- retarded Green's function for desired boundary conditions:

$$\mathcal{G}(t, \mathbf{r}, u) = \frac{1}{\pi u^2} \delta'(-t^2 + \mathbf{r}^2 + u^2)$$

- energy density via convolution:

$$\begin{aligned} \mathcal{E}(t, \mathbf{r}) &= \int dt dr' \int_0^\infty du \mathcal{G}(t - t', \mathbf{r} - \mathbf{r}', u) S(t', \mathbf{r}', u) + \text{regulators} \\ &= \frac{\sqrt{\lambda}}{24\pi^2 \gamma^4 r^6 \Xi^6} \left[ -2r^2 \Xi^2 + 4r\gamma^2 \Xi(t_{\text{ret}} - t) + (2\gamma^2 - 4r^2 v^2 \gamma^2 \omega_0^2 \sin^2 \theta + 3r^2 \gamma^4 \omega_0^2 \Xi^2)(t_{\text{ret}} - t)^2 \right. \\ &\quad \left. + 7r\gamma^2 \omega_0^2 \Xi(t_{\text{ret}} - t)^3 + 4\gamma^2 \omega_0^2 (t_{\text{ret}} - t)^4 + 8v\gamma^2 \omega_0 r (t_{\text{ret}} - t)(t_{\text{ret}} - t + r \Xi) \sin \theta \cos(\varphi - \omega_0 t_{\text{ret}}) \right] \end{aligned}$$

where

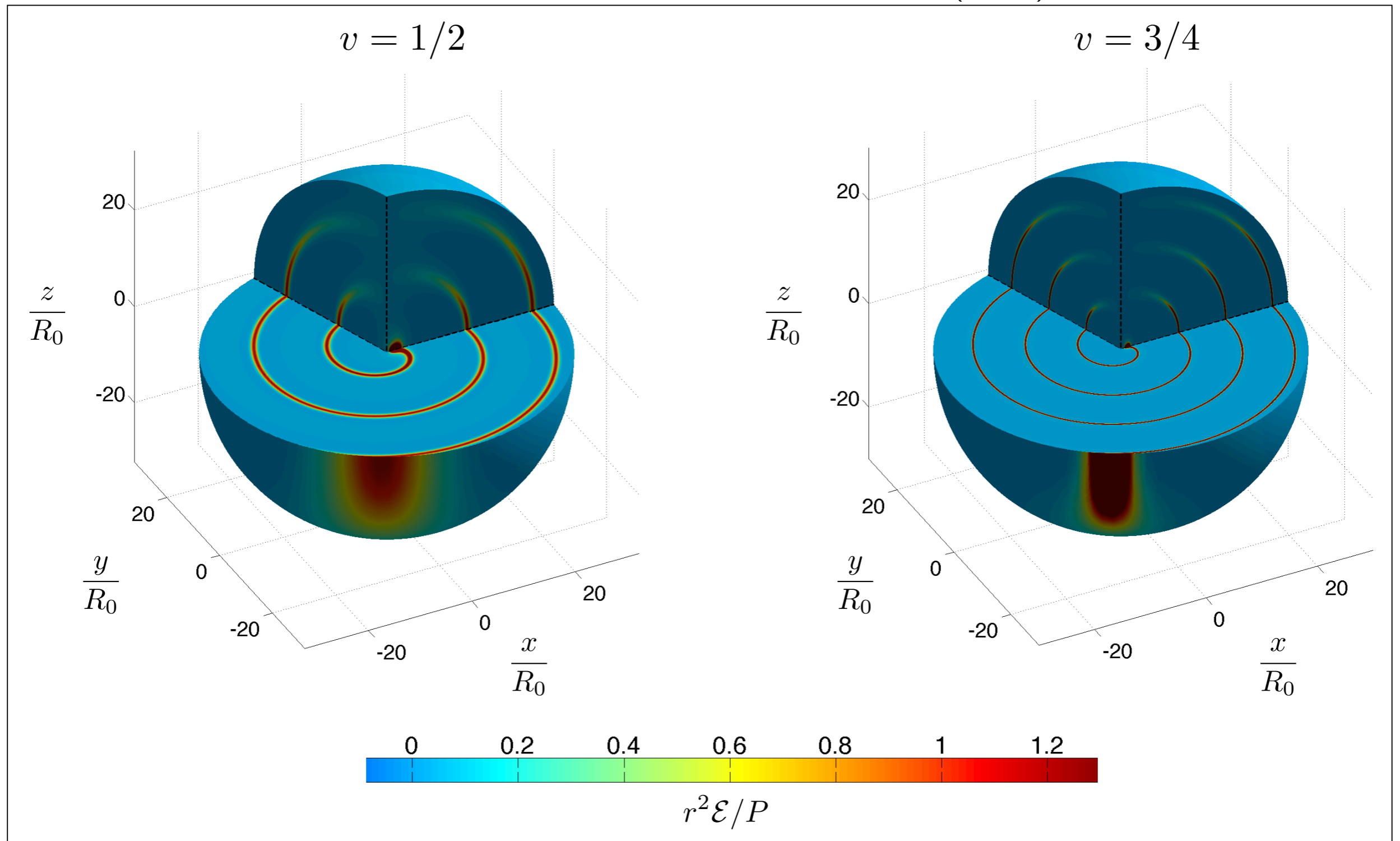
$$t - t_{\text{ret}} - |\mathbf{r} - \mathbf{r}_{\text{quark}}(t_{\text{ret}})| = 0$$

$$\Xi \equiv \frac{|\mathbf{r} - \mathbf{r}_{\text{quark}}(t_{\text{ret}})| - \mathbf{r} \cdot \dot{\mathbf{r}}_{\text{quark}}(t_{\text{ret}})}{r}$$

$$\mathbf{r}_{\text{quark}} = \text{quark's trajectory}, \quad \mathbf{r} \equiv \{r, \theta, \phi\} = \text{observer}, \quad \gamma = 1/\sqrt{1 - v^2}$$

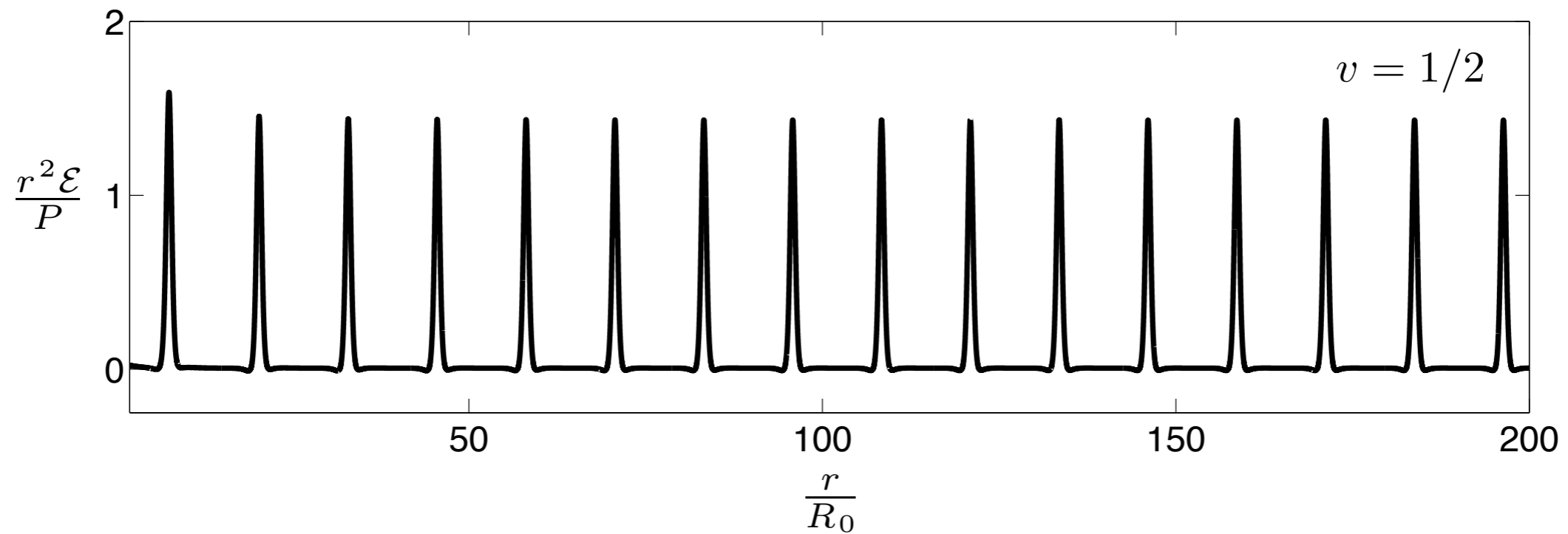
- analytic solution up to  $t_{\text{ret}}$  (as in electrodynamics)
- similar dependence on  $\Xi$  as in electrodynamics

# ILLUSTRATION OF $\mathcal{E}(t, \mathbf{r})$

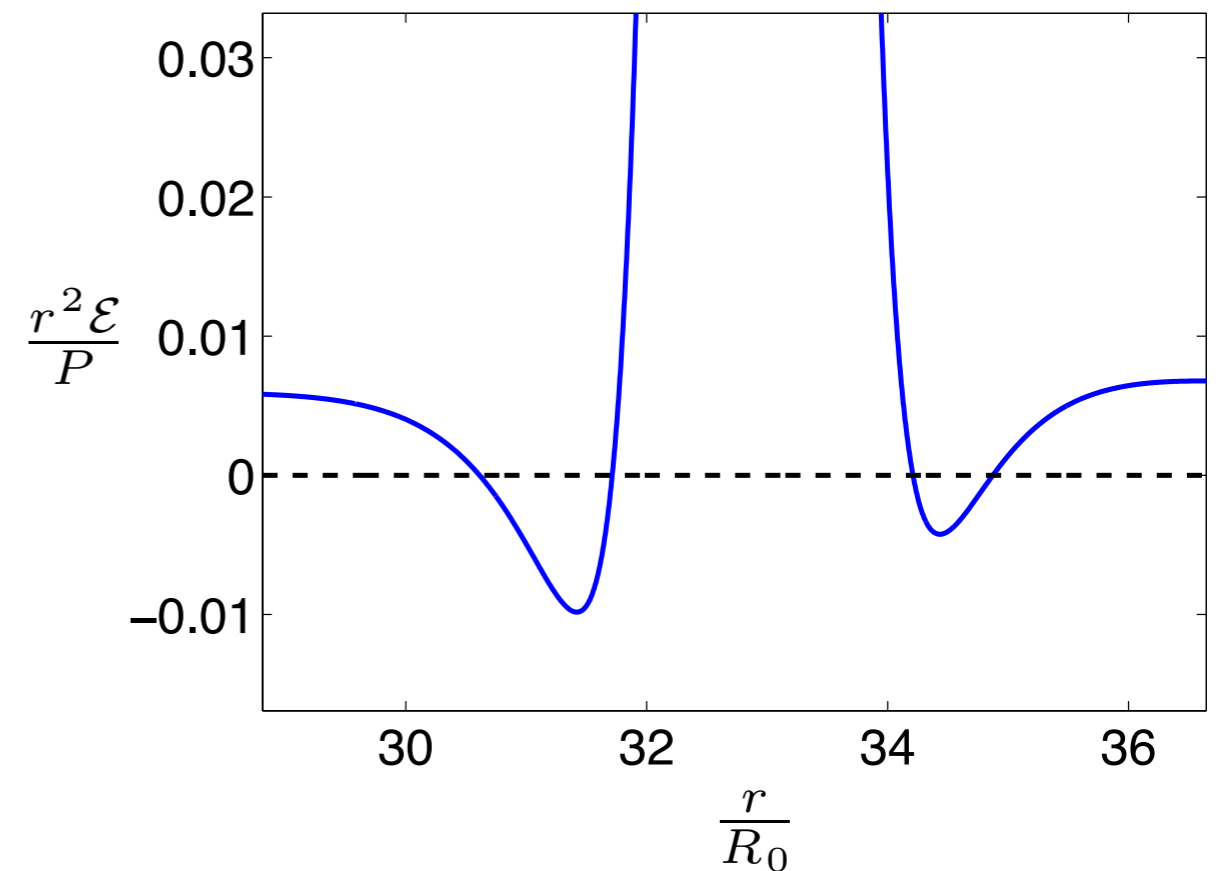


radial thickness:  $\sim 1/\gamma^3$ , azimuthal thickness:  $\sim 1/\gamma$

# BEHAVIOR IN FAR ZONE

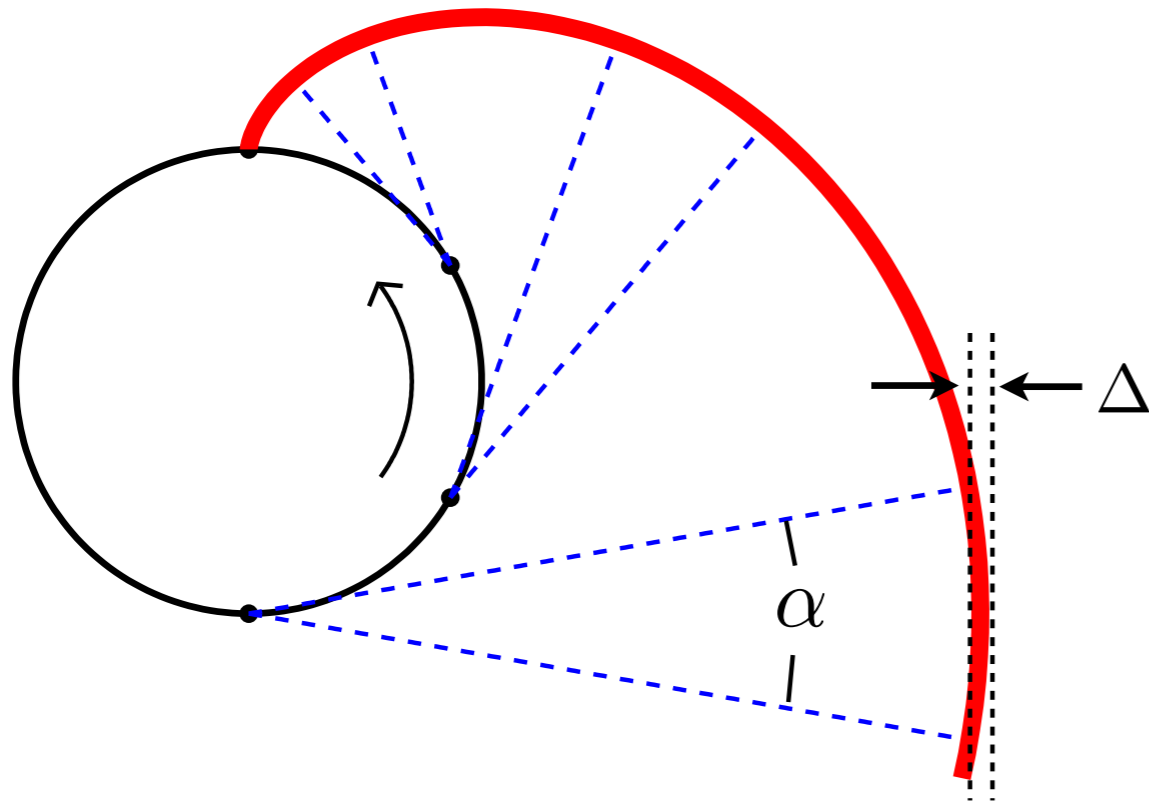


- radiation does not broaden in far zone!
- similar structure as classical synchrotron radiation
- negative energy density  $\longleftrightarrow$  quantum effects



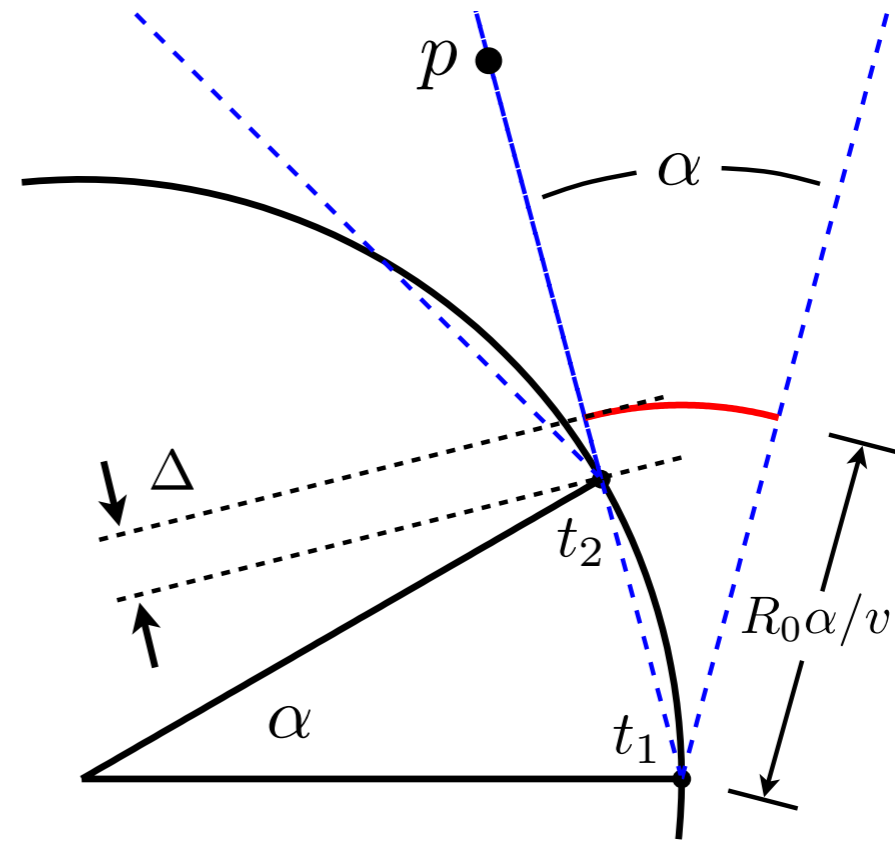
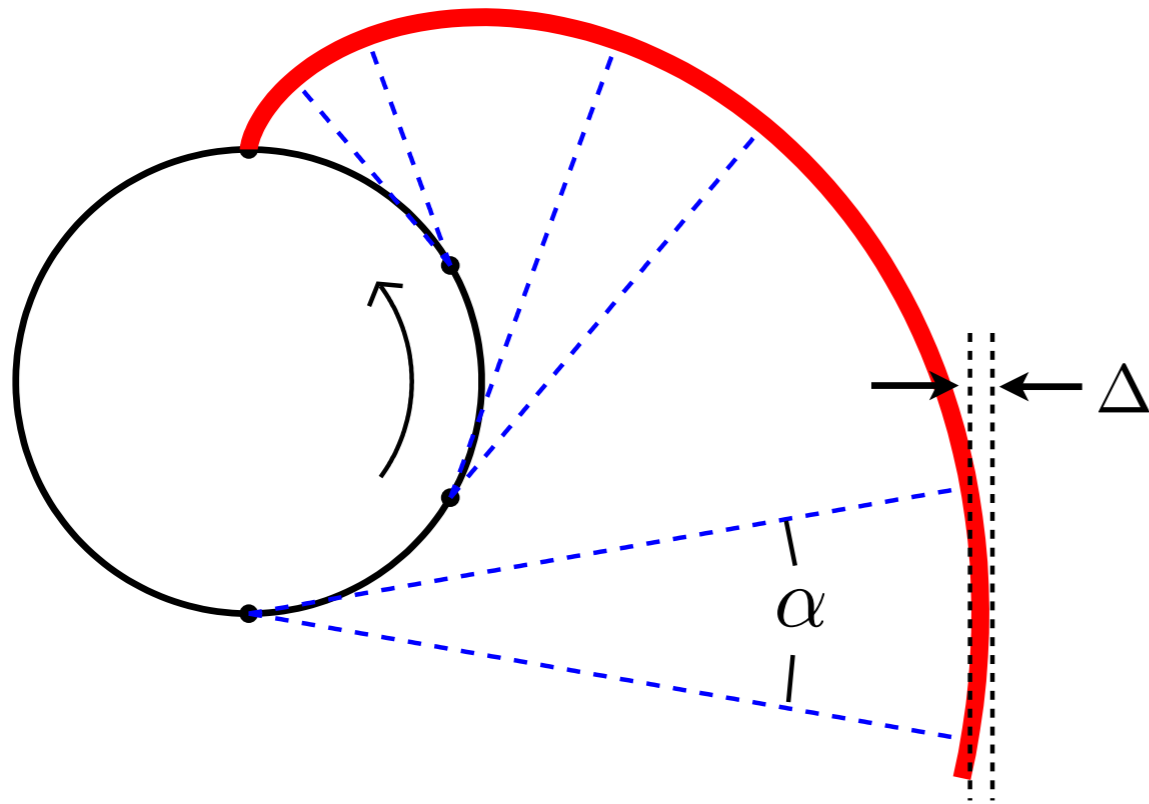
# **GEOMETRICAL INTERPRETATION**

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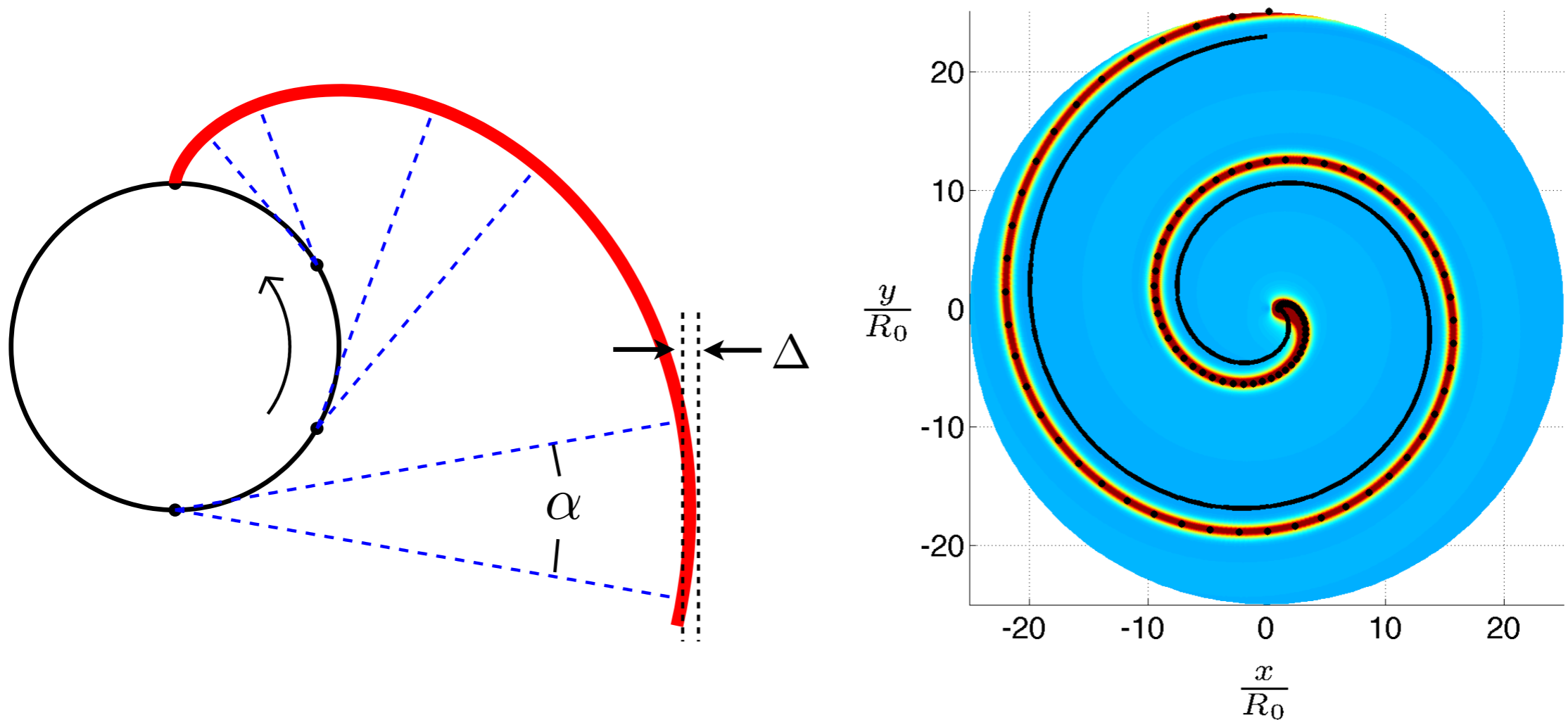
- radiation in fixed relative direction and free without broadening propagation: spiral structure

# GEOMETRICAL INTERPRETATION



- radiation in fixed relative direction and free without broadening propagation: spiral structure
- radiation in cone with width  $\alpha$  : - when radiated in moving direction  $\Delta \sim \alpha R_0/v - \alpha R_0 \sim R_0/\gamma^3$   
- otherwise  $\Delta \sim \alpha R_0$

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**Lesson:**

- radiation more or less isotropic in rest frame ( $\alpha \sim 1/\gamma$ )
- free propagation **without broadening**
- strong similarities to ED/weak coupling regime



# COMPARISON TO WEAK COUPLING AND ED

- time-averaged power distributions:

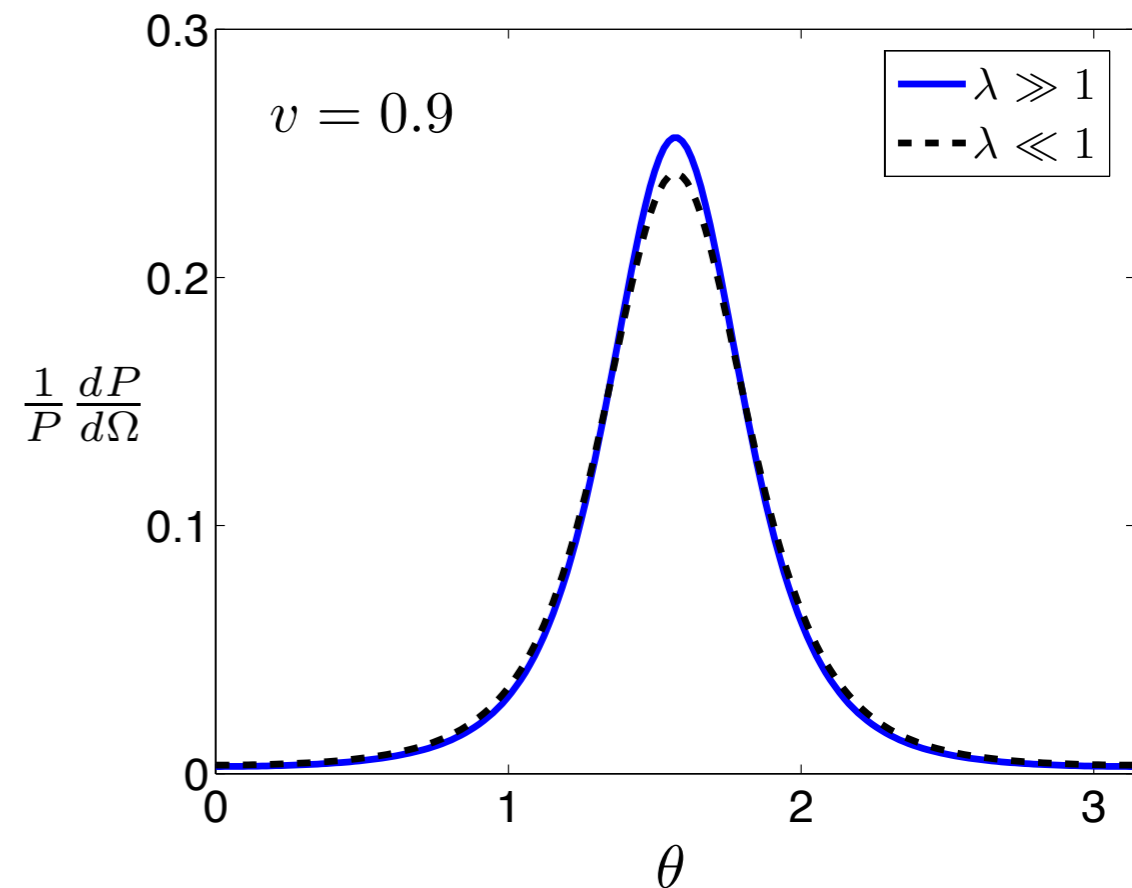
$$\frac{dP}{d\Omega} = \frac{1}{T} \int_t^{t+T} dt' \underbrace{\hat{\mathbf{n}} \cdot \mathbf{S}}_{\rightarrow r^2 \mathcal{E}}$$

- weak coupling: one vector and one scalar field coupling to  $e_{\text{eff}}^2 = \lambda/2$

$$\begin{aligned} \frac{dP^{\lambda \ll 1}}{d\Omega} &\propto \frac{dP^{\text{vector}}}{d\Omega} + \frac{dP^{\text{scalar}}}{d\Omega} \\ \frac{dP^{\lambda \gg 1}}{d\Omega} &\propto \frac{dP^{\text{vector}}}{d\Omega} + 2 \frac{dP^{\text{scalar}}}{d\Omega} \end{aligned}$$

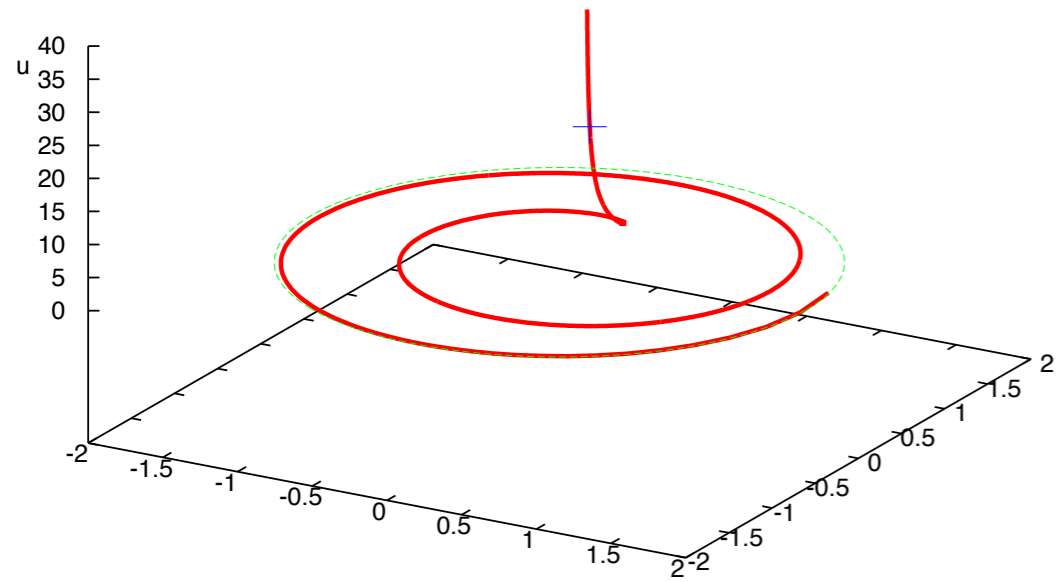
- overall power radiated:

$$P^{\lambda \ll 1} = \frac{5\lambda}{12} \frac{a^2}{2\pi} \longleftrightarrow P^{\lambda \gg 1} = \sqrt{\lambda} \frac{a^2}{2\pi}$$

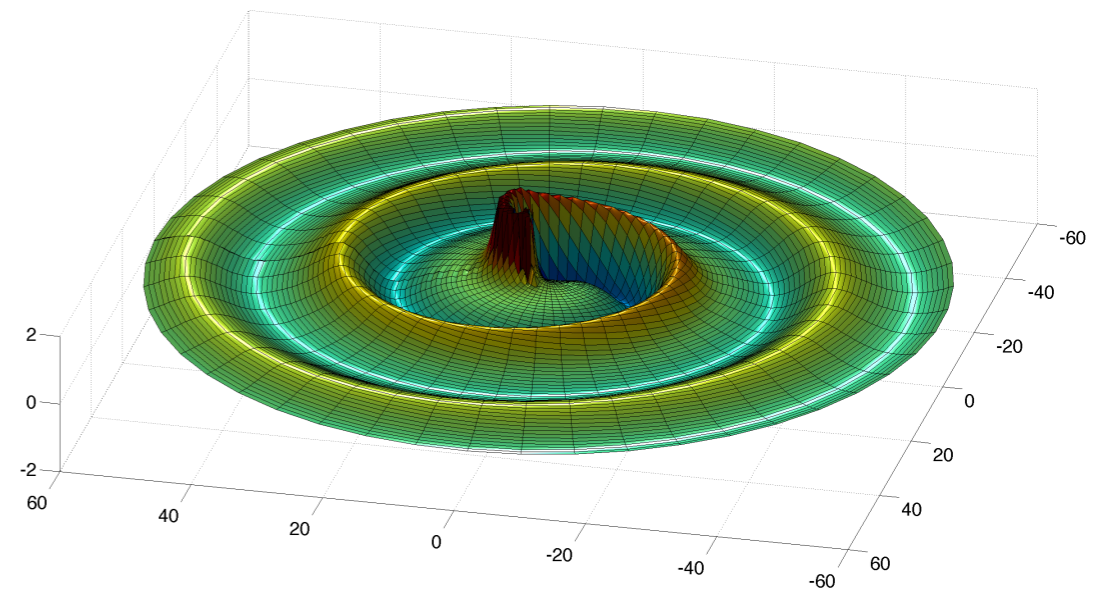


different parametric dependence on coupling, but similar profiles

# POSSIBLE EXTENSIONS



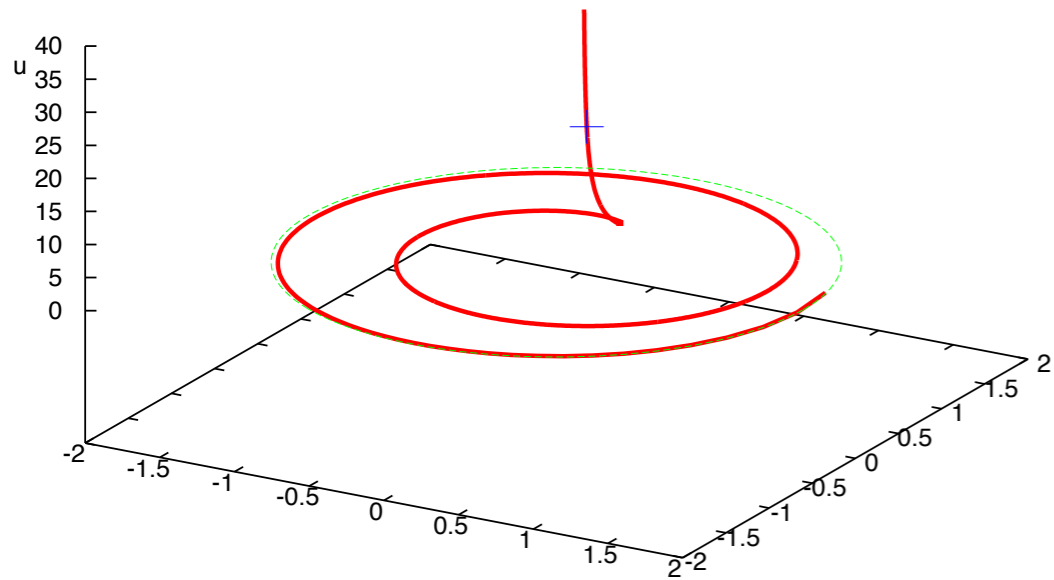
$$\omega_0^2 \gamma^3 \ll \pi^2 T^2$$



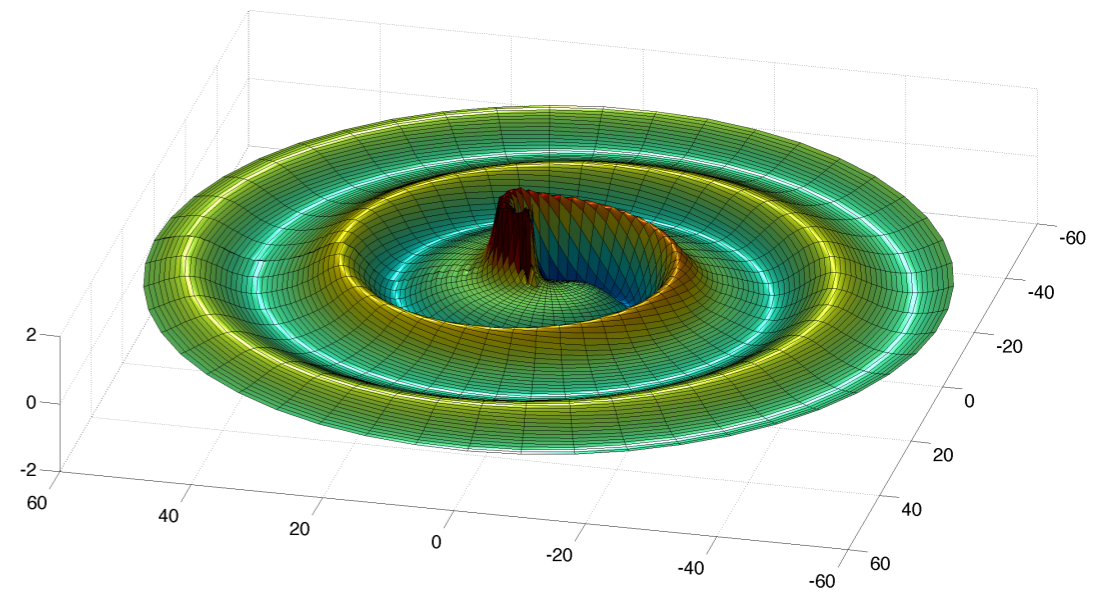
K. Fadafan, H. Liu, K. Rajagopal, U. Wiedemann [arXiv:0809.2869]

# POSSIBLE EXTENSIONS

- finite temperature: competition between radiation and thermal diffusion -  
- laboratory for jet quenching?!



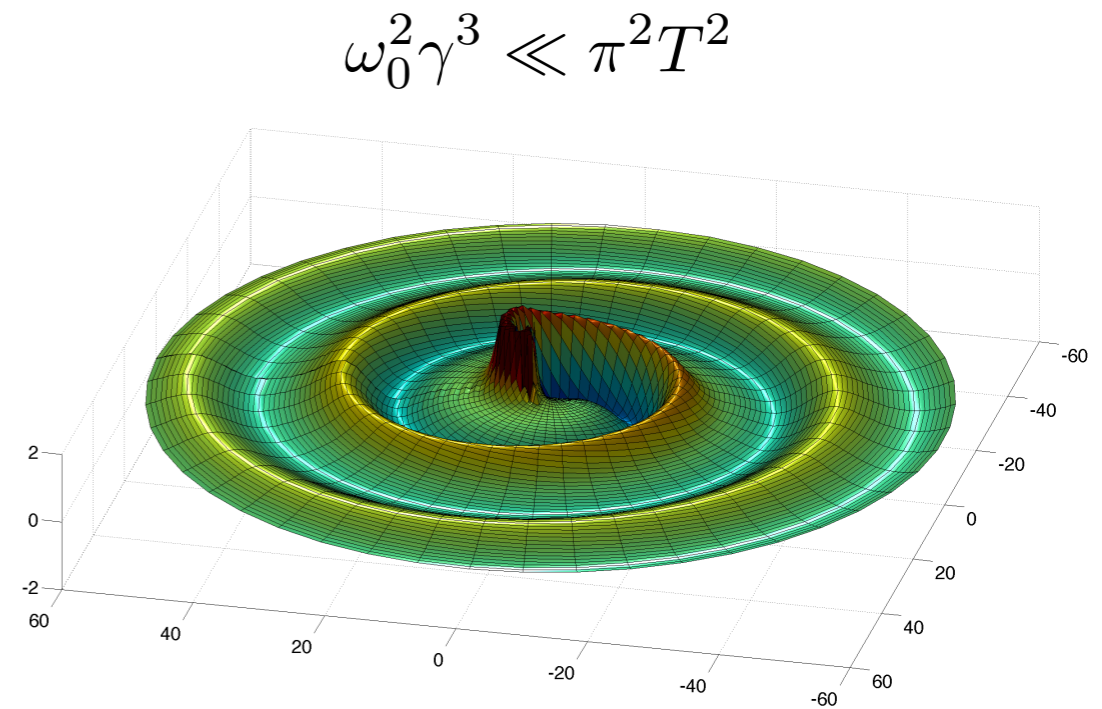
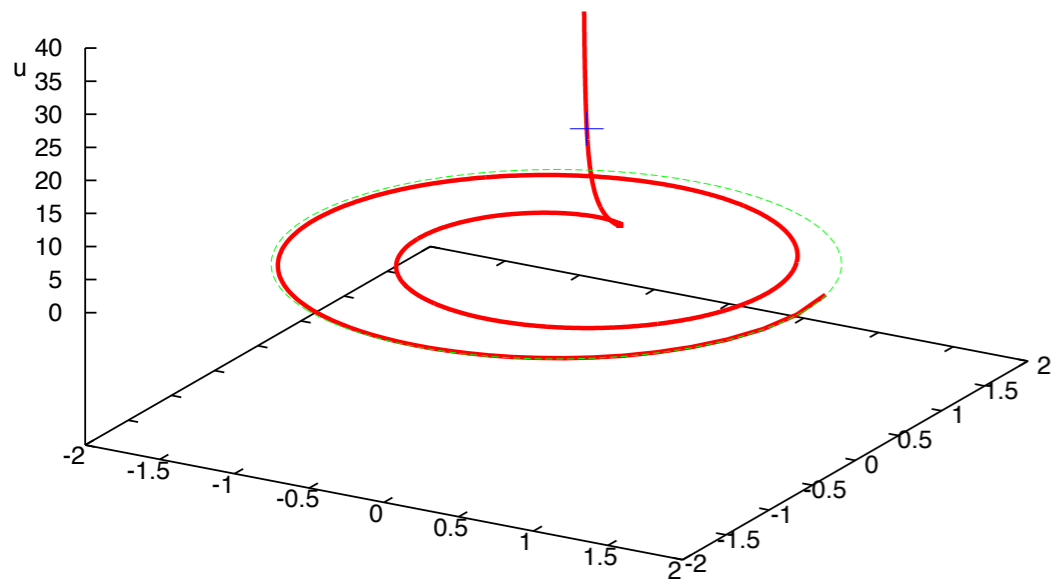
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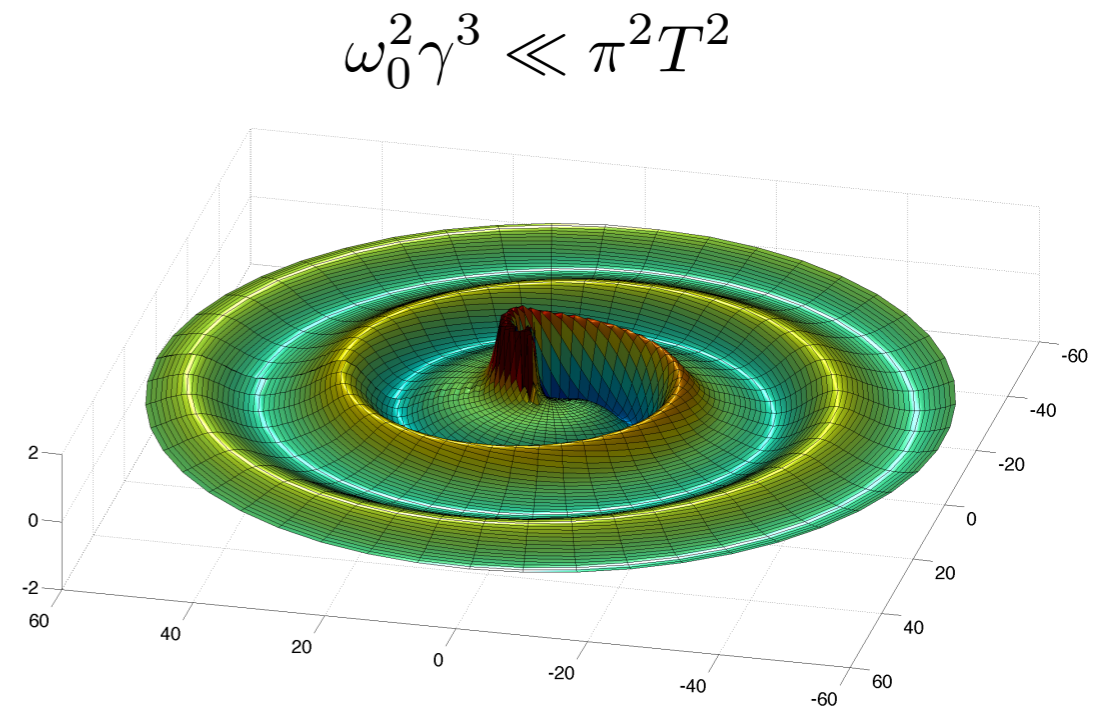
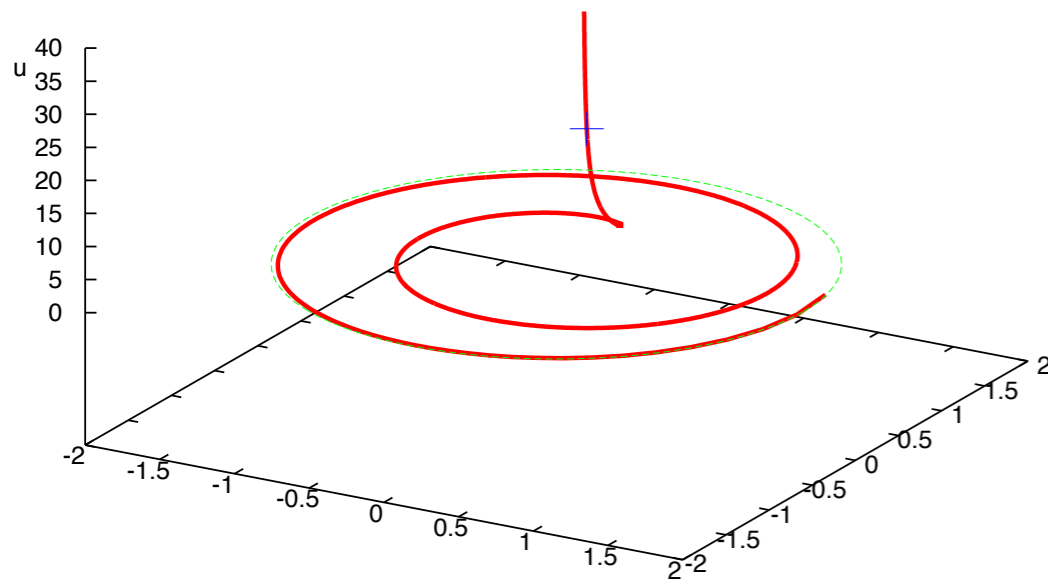
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- solution for general acceleration at  $T=0$  - unlikely (due to memory effects)

$$\mathcal{E}(t, \mathbf{r}) = \frac{L^3}{\pi} \int d^4 r' \int_0^\infty du \theta(t - t') \left[ (4ut_{00} - t_{M5} \nabla'_M \mathcal{W}) \frac{\delta''(\mathcal{W})}{u^2} + |\mathbf{r} - \mathbf{r}'|^2 (4t_{00} - 4t_{55} + 2t_{ii}) \frac{\delta'''(\mathcal{W})}{3u} - (t_{ij} \nabla'_i \mathcal{W} \nabla'_j \mathcal{W}) \frac{\delta'''(\mathcal{W})}{2u} \right]$$

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- (late-time) model for single jets - study  $\mathcal{E}(t, \mathbf{x})$  for classes of string profiles

# SUMMARY

- gauge/gravity duality nice tool to study real-time evolution
- synchrotron radiation solved analytically
- ➔ no isotropization observed
- hope/ideas for modeling jets

Thank you for your attention!