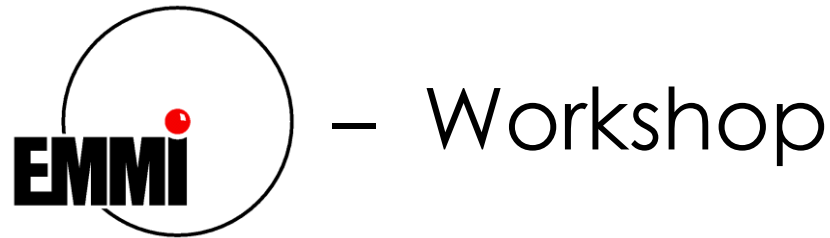


Meson and Baryon Screening in Hot Moving Plasmas



– String Theory and Extreme Matter –

in collaboration with Carlo Ewerz

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Motivation

- Understand the dynamics of the quark-gluon plasma (QGP)
 - Method needed for strongly coupled gauge theory
 - Computation of dynamical quantities
- AdS/CFT correspondence is promising candidate:
- η/s Bound
 - Jet Quenching Parameter \hat{q}

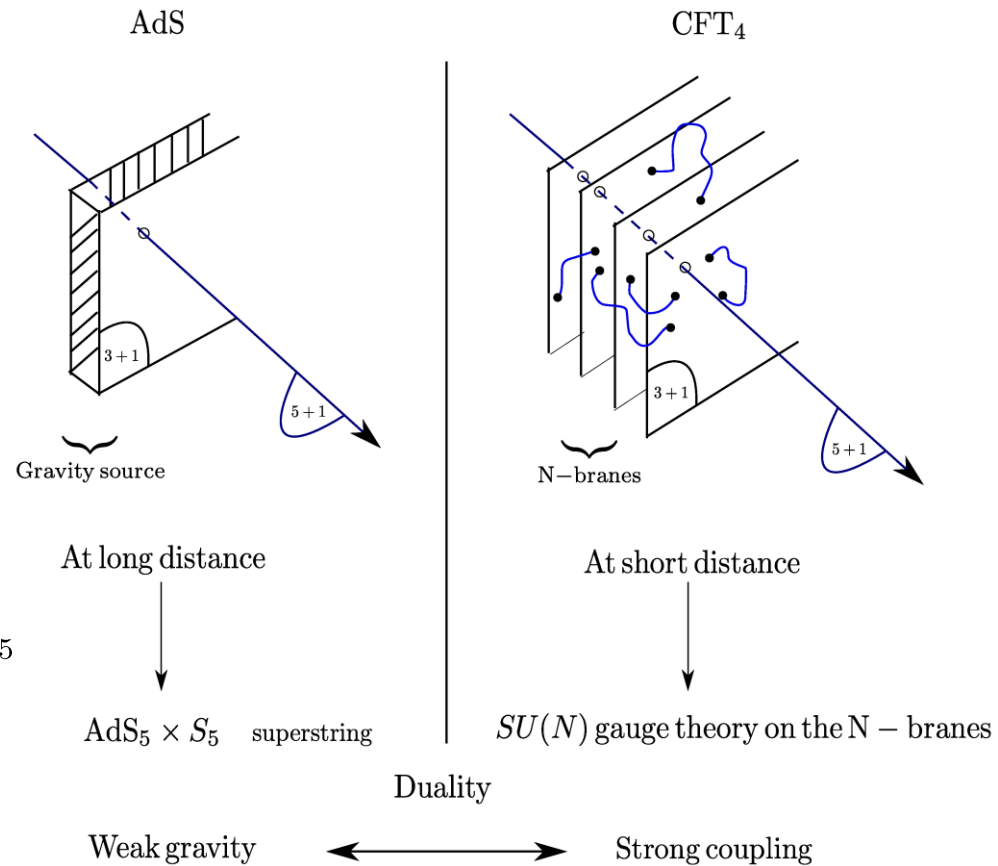
AdS/CFT correspondence

Gauge theory side:

$\mathcal{N} = 4$ supersymmetric Yang-Mills theory in 3+1 dimensions in the limit of $N_c \rightarrow \infty$

String theory side:

Type IIB string theory on a $AdS_5 \times S^5$ spacetime



$$ds^2 = \frac{r^2}{R^2} \left(-dt^2 + \sum_{i=1}^3 dx_i^2 \right) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5 \quad \text{with curvature } R = 4\pi g_{\text{YM}}^2 \alpha'^2 N_c$$

$\mathcal{N} = 4$ and QCD

Properties of $\mathcal{N} = 4$ SYM:

- Conformal theory, coupling is constant
- Maximally supersymmetric
- No confinement
- No chiral symmetry breaking
- $N_c \rightarrow \infty$ for duality

At finite temperature T :

- No confinement in QCD
- Chiral symmetry restoration in QCD
- Finite T breaks supersymmetry

→ $\mathcal{N} = 4$ SYM is closer to QCD at finite T

How to extract information

1. Search for gravity dual to QCD (not yet known)
2. Deformation of existing dualities to come closer to QCD
 - KTY model
 - 2 – parameter models which solve equations of motion
3. How do observables depend on deformations?
 - Robustness: small changes
 - Universality: no change, or systematically in one direction

-
- η/s Bound is famous example
 - $c_s^2 < \frac{1}{3}$ in a wide class of theories

Kovtun, Starinets, Son
Cherman, Cohen,
Nellore; Hohler,
Stephanov

Metric models at finite temperature

AdS_5 metric at finite temperature:

$$ds^2 = -f dt^2 + \frac{r^2}{R^2} (dx_1^2 + dx_2^2 + dx_3^2) + \frac{1}{f} dr^2 = G_{\mu\nu} dx^\mu dx^\nu,$$

$$f \equiv \frac{r^2}{R^2} \left(1 - \frac{r_0^4}{r^4} \right) \quad \text{where } r_0 \text{ is the black hole horizon}$$

- T is Hawking temperature of the BH, with horizon r_0 (z_0):

$$T = \frac{r_0}{\pi R^2}, \quad T = \frac{1}{\pi z_0} \quad \text{with } z = \frac{R^2}{r}$$

Metric models at finite temperature

KTY Model:

Kajantie, Tahkokallio, Yee

$$ds^2 = e^{\frac{29}{20}c\frac{R^4}{r^2}} \left[-\frac{r^2}{R^2} \left(1 - \frac{r_0^4}{r^4}\right) dt^2 + \frac{r^2}{R^2} d\vec{x}^2 + \frac{R^2}{r^2} \frac{dr^2}{1 - \frac{r_0^4}{r^4}} \right]$$
$$= \frac{R^2 e^{\frac{29cz^2}{20}}}{z^2} \left[-\left(1 - \frac{z^4}{z_0^4}\right) dt^2 + d\vec{x}^2 + \frac{dz^2}{1 - \frac{z^4}{z_0^4}} \right] \quad \text{with parameter } \frac{c}{T^2}$$

- Temperature is the same as in the conformal case

$$T = \frac{r_0}{\pi R^2}$$

- Relevant parameter range is $0 < \frac{c}{T^2} < 4$
- Realistic thermodynamics for $c = 0.127 \text{ GeV}^2$
- KTY is not a solution to supergravity equations of motion

Metric models at finite temperature

Deformed 2-parameter metric:

DeWolfe, Rosen; Gubser

- Supergravity action with dilaton potential V

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{1}{2} (\partial_\mu \Phi)^2 - V(\Phi) \right)$$

- General ansatz for metric

$$ds^2 = e^{2A(\Phi)} (-h(\Phi) dt^2 + d\vec{x}^2) + \frac{e^{2B(\Phi)}}{h(\Phi)} d\Phi^2 \quad \text{with } \frac{c}{T^2}, \alpha \equiv \frac{c}{\phi} \quad \text{and } \Phi = \sqrt{\frac{3}{2}} \phi z^2$$

- Translation invariance, $SO(3)$ symmetry but not $SO(3,1)$
- Temperature in this case

$$T = \frac{e^{A(\Phi_h) - B(\Phi_h)} |h'(\Phi_h)|}{4\pi}$$

Metric models at finite temperature

Deformed 2-parameter metric:

- Definition of A, B :

$$A(\Phi) = \frac{1}{2} \ln \left(\sqrt{\frac{3}{2}} c \frac{R^2}{\alpha} \right) - \frac{1}{2} \ln \Phi - \frac{\alpha}{\sqrt{6}} \Phi$$

$$B(\Phi) = \ln \left(\frac{R}{2} \right) + \frac{1 + 2\alpha^2}{2\alpha^2} \ln \left(1 + \alpha \sqrt{\frac{2}{3}} \Phi \right) - \ln \Phi - \frac{1}{\alpha\sqrt{6}} \Phi$$

- Exponential factor of KTY metric is obtained for $\alpha_{\text{KTY}} = \frac{20}{49}$

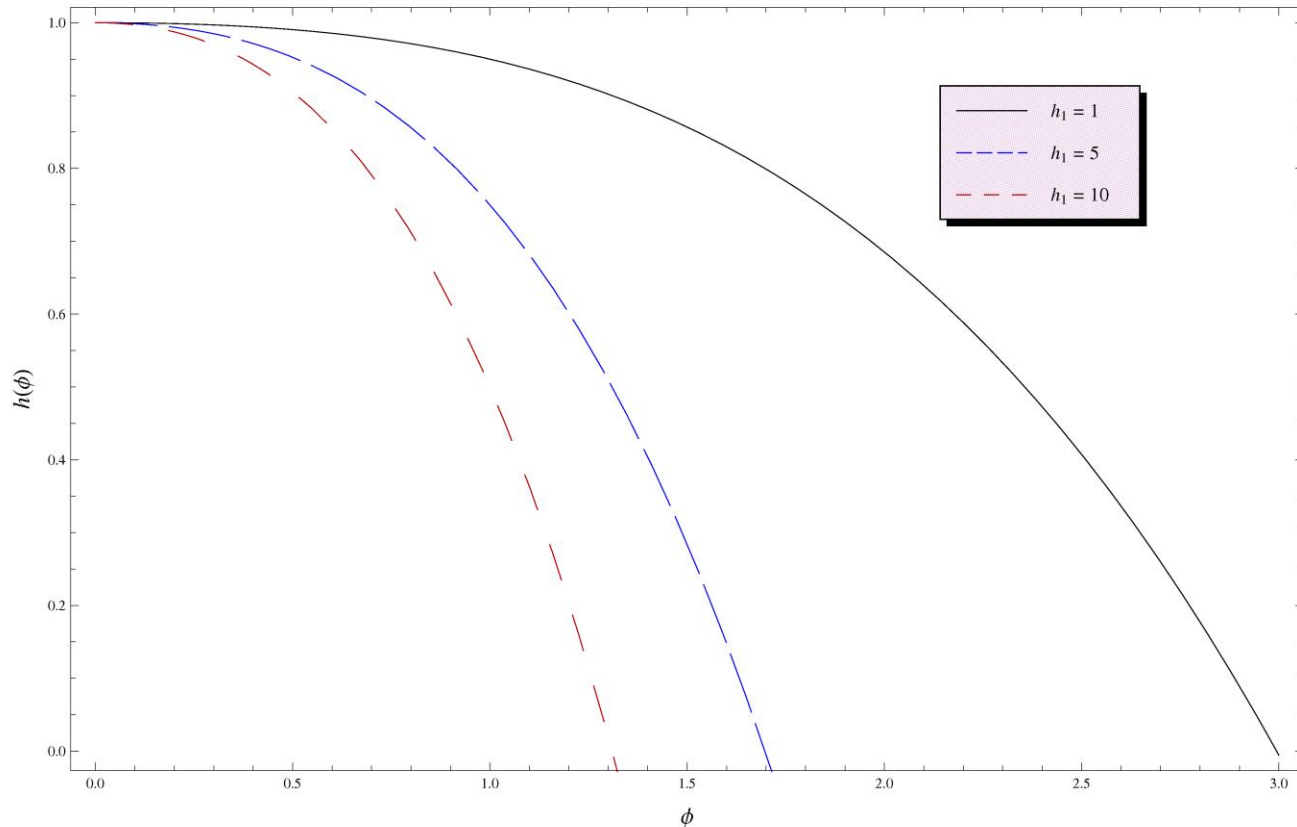
- Two versions of this metric:

- “Einstein frame“: scalar Φ is not dilaton
- “String frame“: scalar Φ is dilaton

$$g_{\mu\nu}^E e^{\frac{4}{3}\Phi} = g_{\mu\nu}^S$$

- 2-parameter model solves supergravity equations of motion

Horizon function



- AdS limit is obtained for $\phi \rightarrow 0$ in a $z = \phi$ gauge
- The horizon ϕ_0 is defined as the zero of h

$$h(\Phi) = h_0 - c^2 R^3 h_1 \int_0^{\Phi} d\Phi' e^{-4A(\Phi') + B(\Phi')}$$

Screening Length in a hot moving plasma

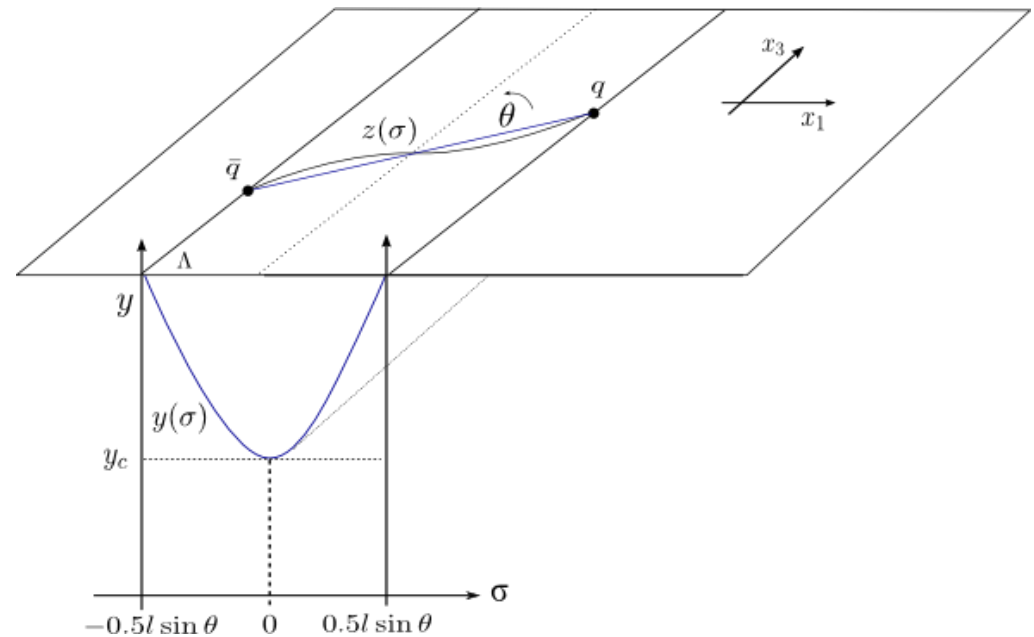
- Static quark-antiquark pair in a hot moving plasma „wind“ blowing in $-x_3$ – direction
- velocity $v = \tanh \eta$
- orientation angle θ

Nambu-Goto action:

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\det g_{\alpha\beta}}$$

with $g_{\alpha\beta} = G_{\mu\nu} \partial_\alpha x^\mu \partial_\beta x^\nu$

Quark-antiquark configuration



by Liu, Rajagopal, Wiedemann, '06

Parametrisation

- Boost into the rest frame of the quark-antiquark pair (t', x'_3) :

$$\begin{aligned} dt &= dt' \cosh \eta - dx'_3 \sinh \eta \\ dx_3 &= -dt' \sinh \eta + dx'_3 \cosh \eta \end{aligned}$$

- The metric has then the special form given by (for $\mathcal{N} = 4$):

$$ds^2 = -A dt^2 - 2B dt dx_3 + C dx_3^2 + \frac{r^2}{R^2} (dx_1^2 + dx_2^2) + \frac{1}{f} dr^2$$

$$\text{with } A = \frac{r^2}{R^2} \left(1 - \frac{r_1^4}{r^4} \right), \quad B = \frac{r_1^2 r_2^2}{r^2 R^2}, \quad C = \frac{r^2}{R^2} \left(1 + \frac{r_2^4}{r^4} \right)$$

$$\text{and } r_1^4 = r_0^4 \cosh^2 \eta, \quad r_2^4 = r_0^4 \sinh^2 \eta$$

- Use appropriate parametrisation for the different orientations:

For $\theta = \frac{\pi}{2}$:

$$X^\mu = \left(t, \sigma = x_1 \in \left[-\frac{L}{2}, \frac{L}{2} \right], x_2, x_3, r(\sigma) \right)$$

For $\theta = 0$:

$$X^\mu = \left(t, x_1, x_2, x_3 = \sigma \in \left[-\frac{L}{2}, \frac{L}{2} \right], r(\sigma) \right)$$

Calculation in $\mathcal{N} = 4$

- Calculate determinant of the induced metric and Nambu-Goto action (shown for $\mathcal{N} = 4$ case):

$$\text{For } \theta = \frac{\pi}{2} \quad S = \frac{T}{2\pi\alpha'} \int_{-\frac{L}{2}}^{\frac{L}{2}} d\sigma \sqrt{A \left(\frac{(\partial_\sigma r)^2}{f} + \frac{r^2}{R^2} \right)}$$

- Calculate conserved Hamiltonian and solve for the coordinate functions (with $r \equiv r_0 y$):

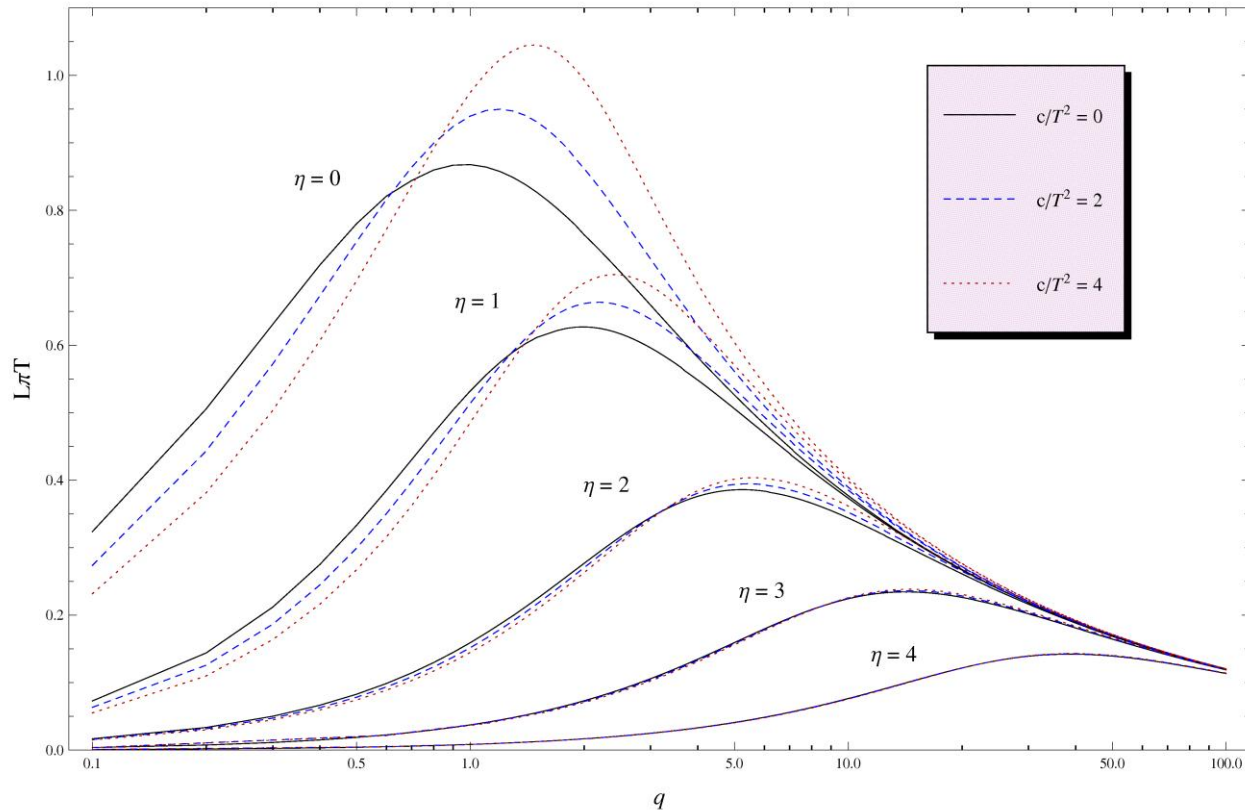
$$\mathcal{H} \equiv \mathcal{L} - y' \frac{\partial \mathcal{L}}{\partial y'} = \frac{y^4 - \cosh^2 \eta}{\mathcal{L}} = q$$

$$y' = \frac{1}{q} \sqrt{(y^4 - 1)(y^4 - y_c^4)} \quad \text{with} \quad y_c^4 \equiv \cosh^2 \eta + q^2$$

- Use boundary conditions to obtain the distance:

$$L\pi T = 2 \int_0^{\frac{L\pi T}{2}} d\sigma = 2q \int_{y_c}^{\infty} \frac{dy}{y'} = 2q \int_{y_c}^{\infty} dy \frac{1}{\sqrt{(y^4 - 1)(y^4 - y_c^4)}}$$

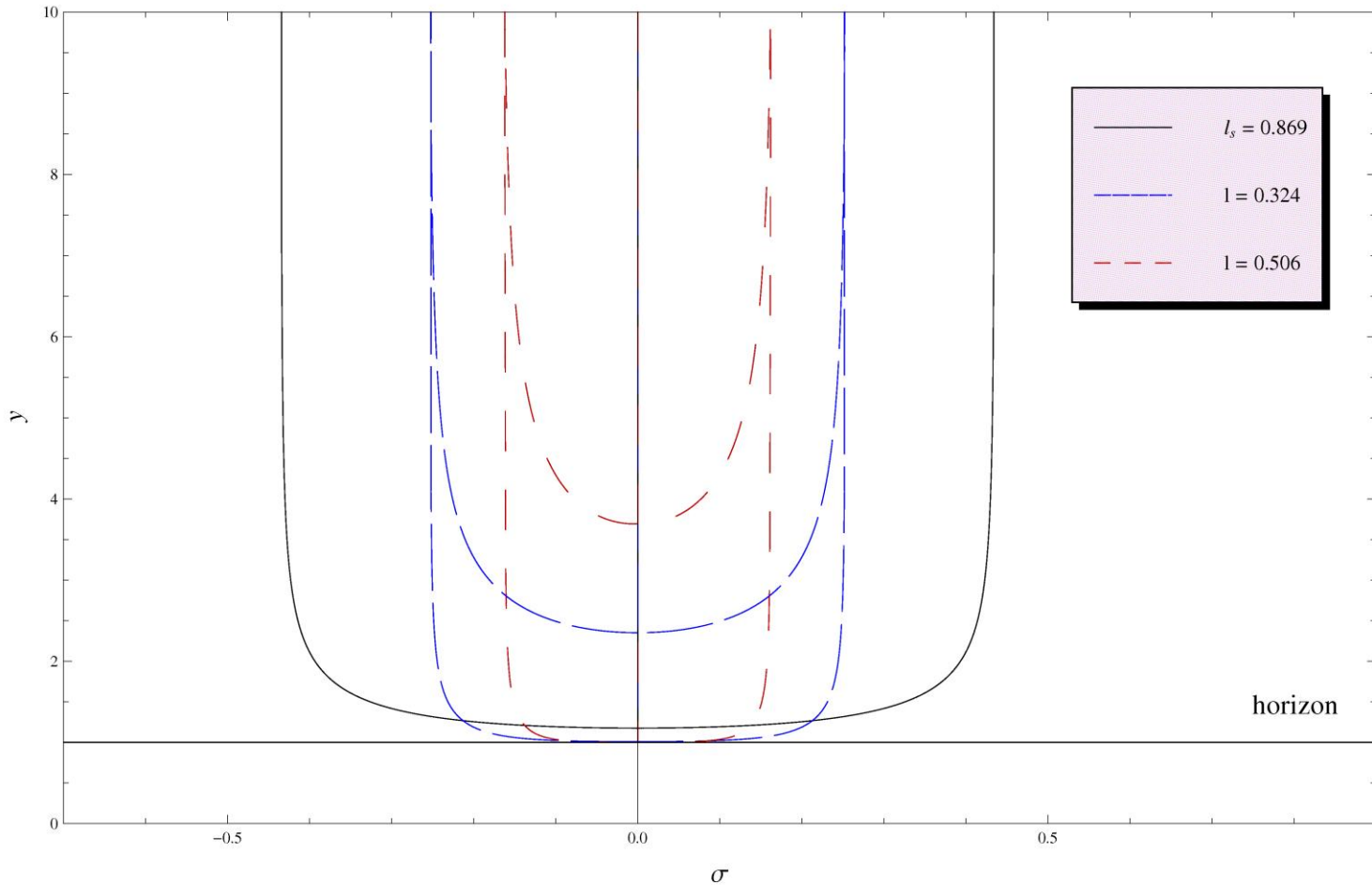
Quark-antiquark distance (KTY)



- Two solutions for each distance L up to a maximal L_{\max}

→ L_{\max} is the screening length

Configuration of the strings



- The string coming closer to the horizon is unstable

Free energy

- General formulation of the quark-antiquark free energy:

$$E(L, \eta, \theta)T = S(L, \eta, \theta) - S_0$$

- The string action reads:

$$S = \sqrt{\lambda}TT \int_{y_c}^{\infty} \frac{d\sigma}{y'} \sqrt{(y^4 - \cosh^2 \eta) \left(1 + \frac{y'^2}{y^4 - 1}\right)} \quad \Rightarrow \quad \frac{S}{\sqrt{\lambda}TT} = \int_{y_c}^{\infty} dy \frac{y^4 - \cosh^2 \eta}{\sqrt{(y^4 - y_c)(y^4 - 1)}}$$

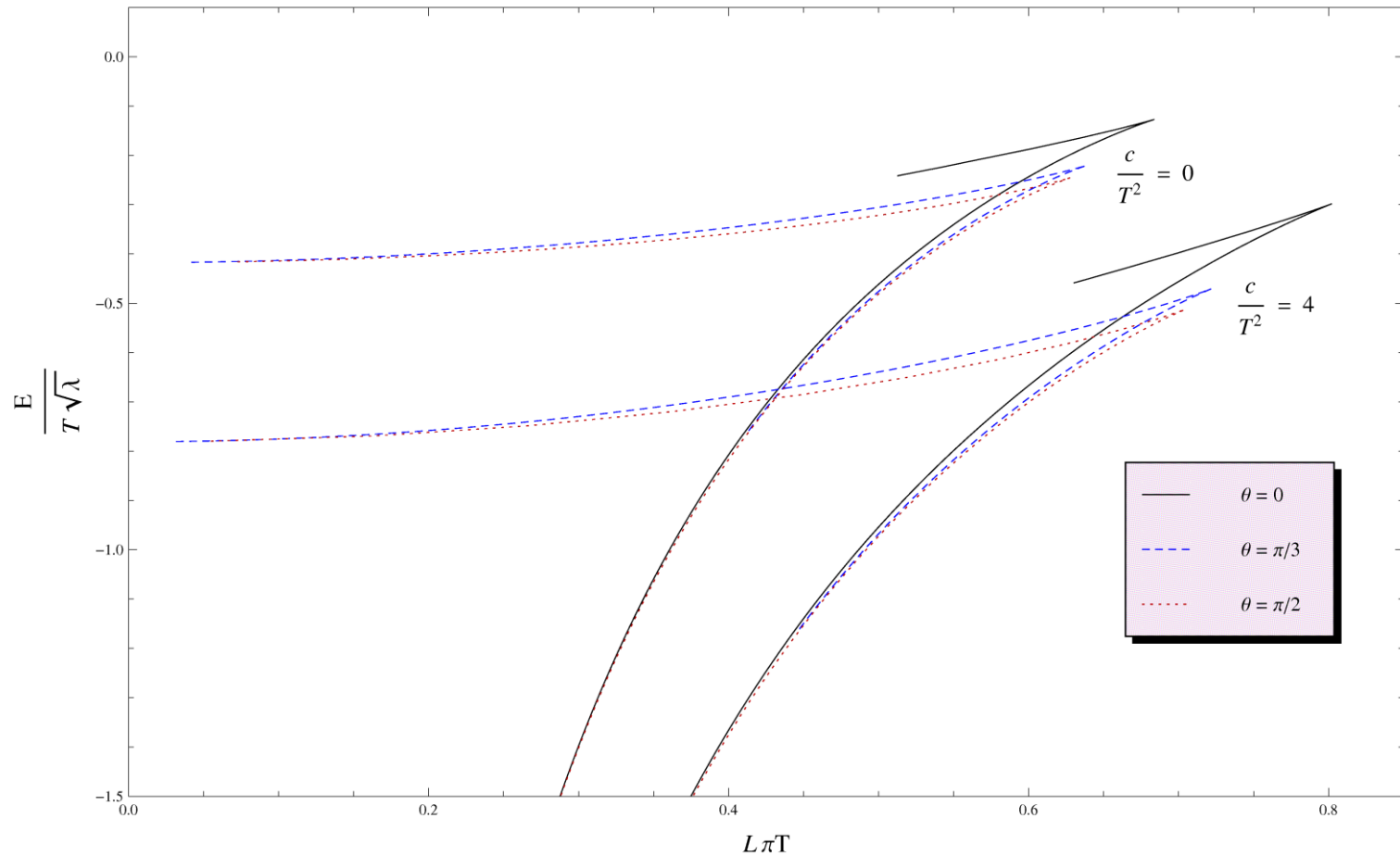
- Drag solution for a moving heavy quark:

$$S_0 = \sqrt{\lambda}TT \int_1^{\infty} dy$$

- Equation for the free energy:

$$E\left(q(L), \eta, \frac{\pi}{2}\right)T = \sqrt{\lambda}TT \int_{y_c}^{\infty} \left[\frac{y^4 - \cosh^2 \eta}{\sqrt{(y^4 - y_c)(y^4 - 1)}} - 1 \right] dy - \sqrt{\lambda}TT(y_c - 1)$$

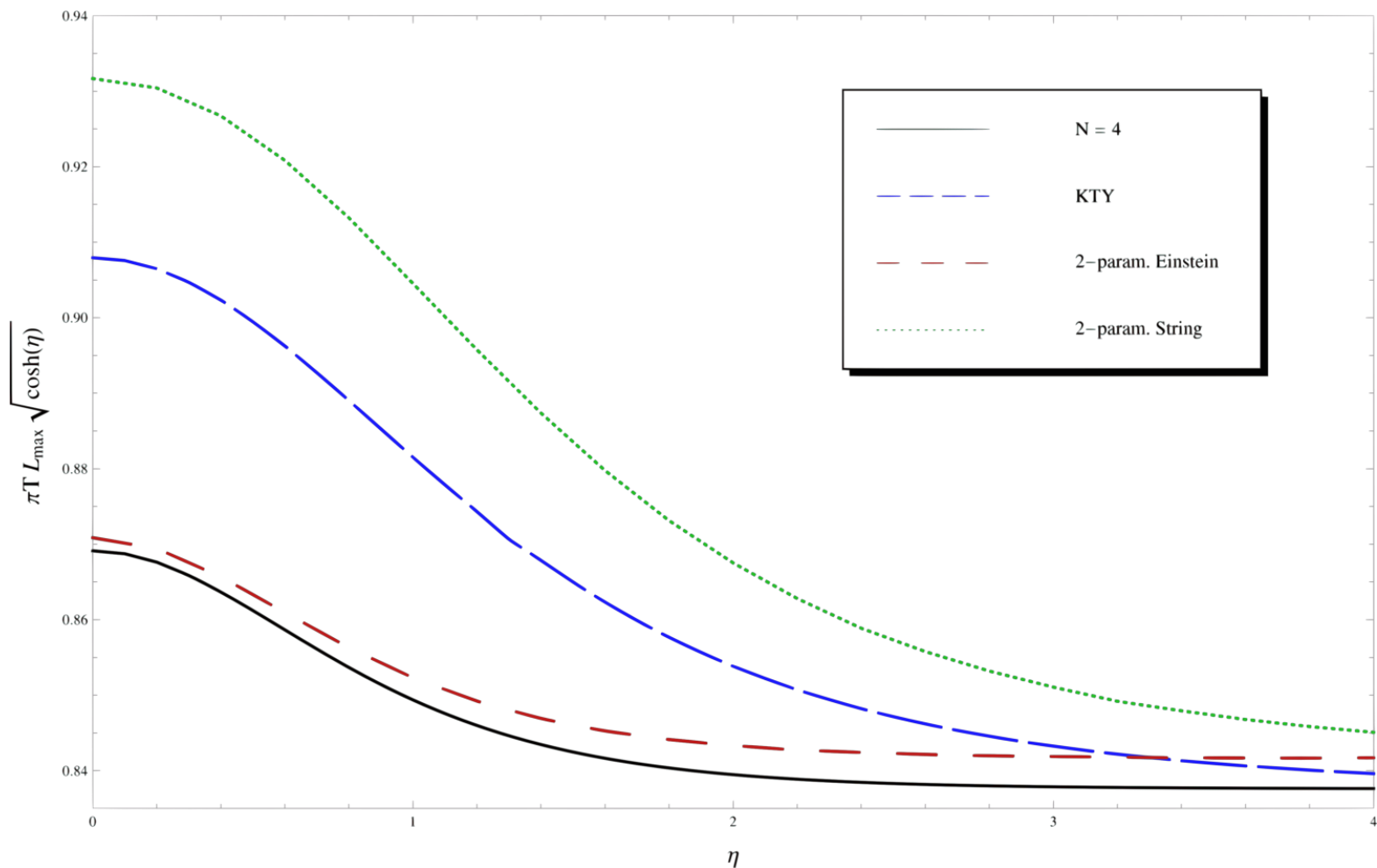
Free energy of a quark-antiquark pair



- Free energy for different orientation angles and $\eta = 1$

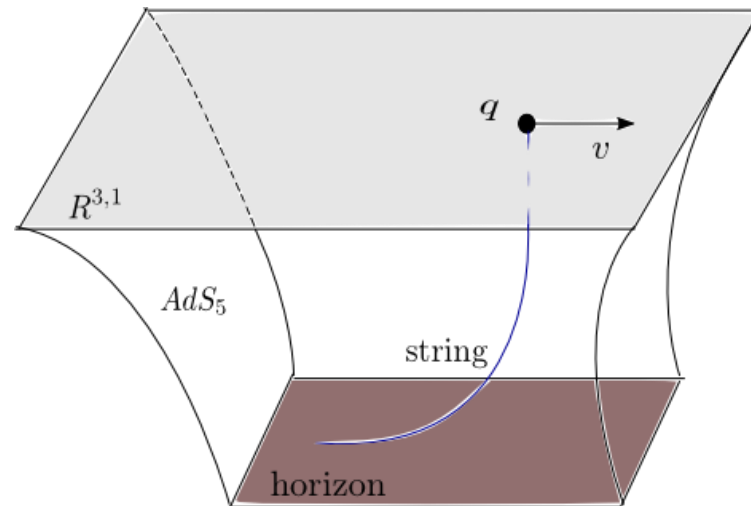
Screening length bound

- L_{\max} is minimal for $\mathcal{N} = 4$



Single quark drag force

- Consider single quark being pulled through medium with velocity v



- Drag force is defined as:

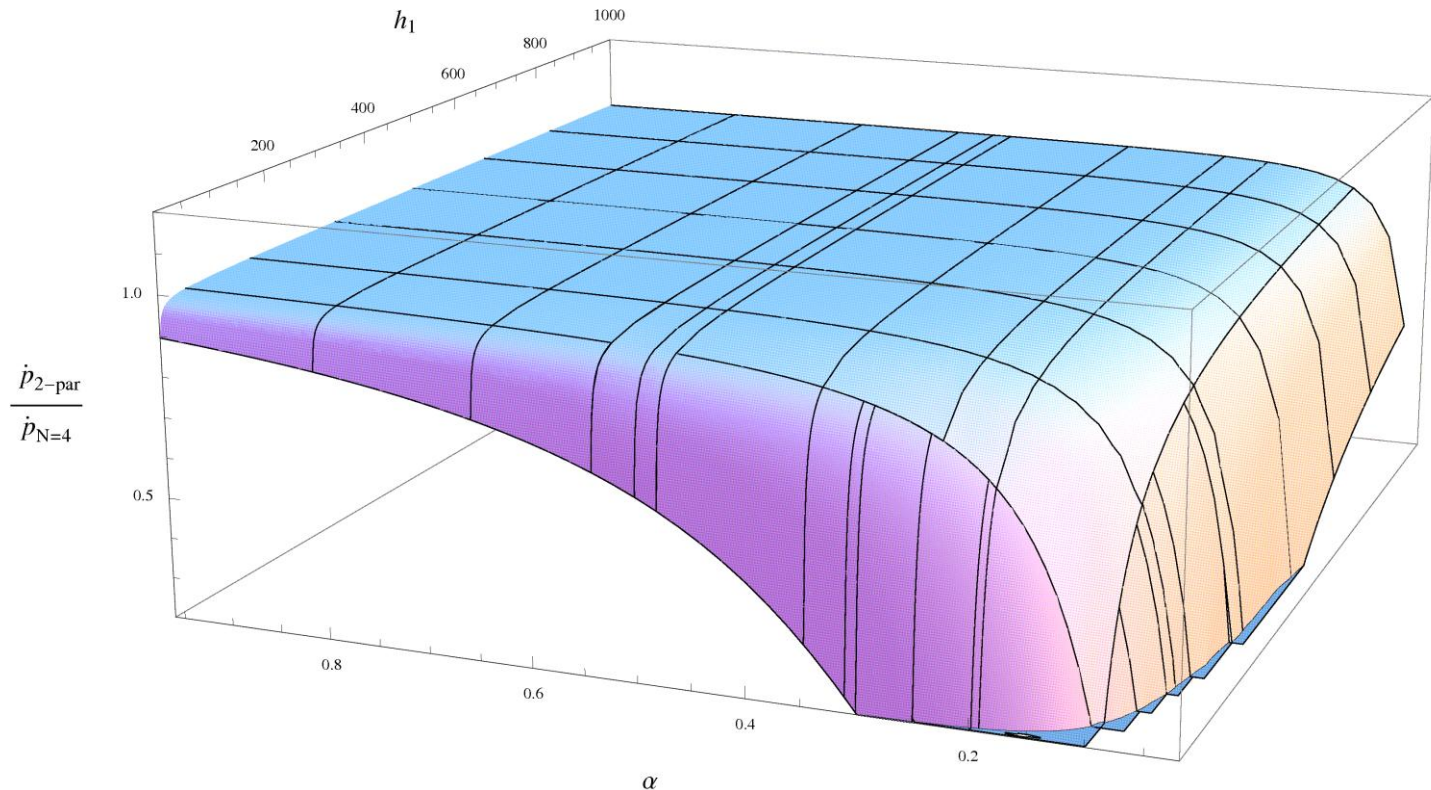
$$\frac{dp_1}{dt} = -\frac{\pi_z}{2\pi\alpha'} \cdot \frac{r_0^2}{R^2} = -\frac{q}{2\pi\alpha'} \cdot \frac{r_0^2}{R^2}$$

- For $\mathcal{N} = 4$ we have

$$\frac{dp_1}{dt} = -\frac{\pi\sqrt{\lambda}T^2}{2} \cdot \frac{v}{\sqrt{1-v^2}}$$

Drag Force bound

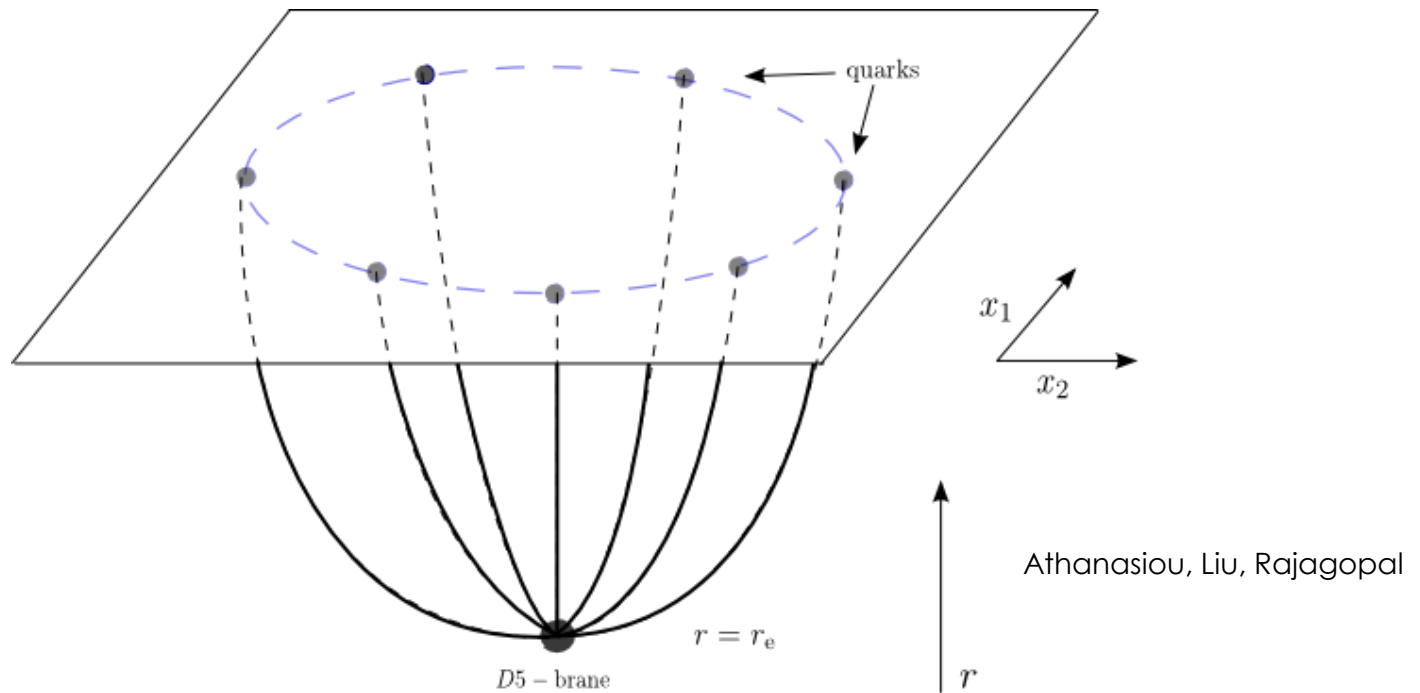
- Among 2-parameter models drag force has a maximum in $\mathcal{N} = 4$



- α is the conformal breaking parameter
- h_1 is a integration constant related to the temperature

Heavy baryon screening

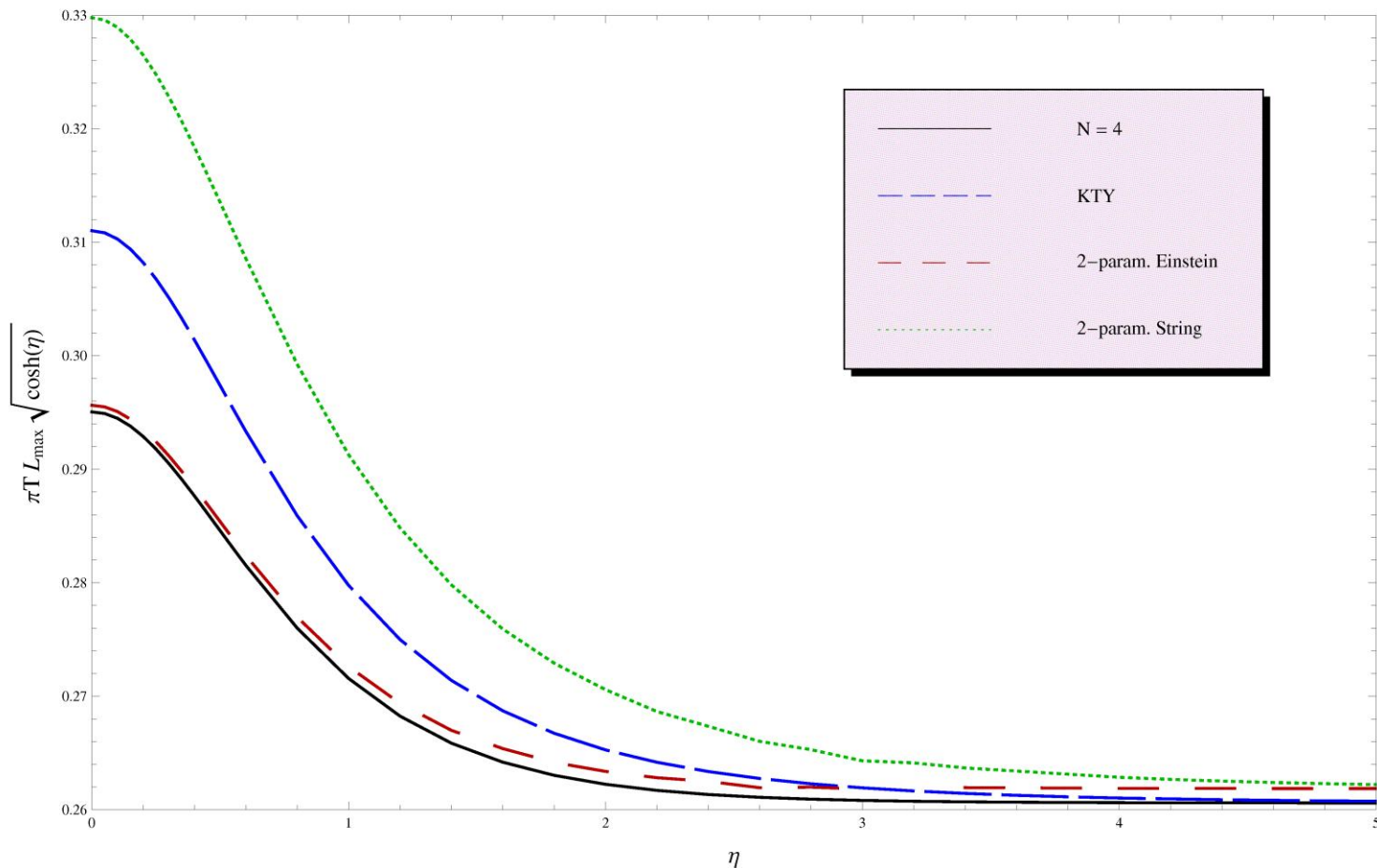
- Consider baryon configuration with N_c quarks arranged on a circle



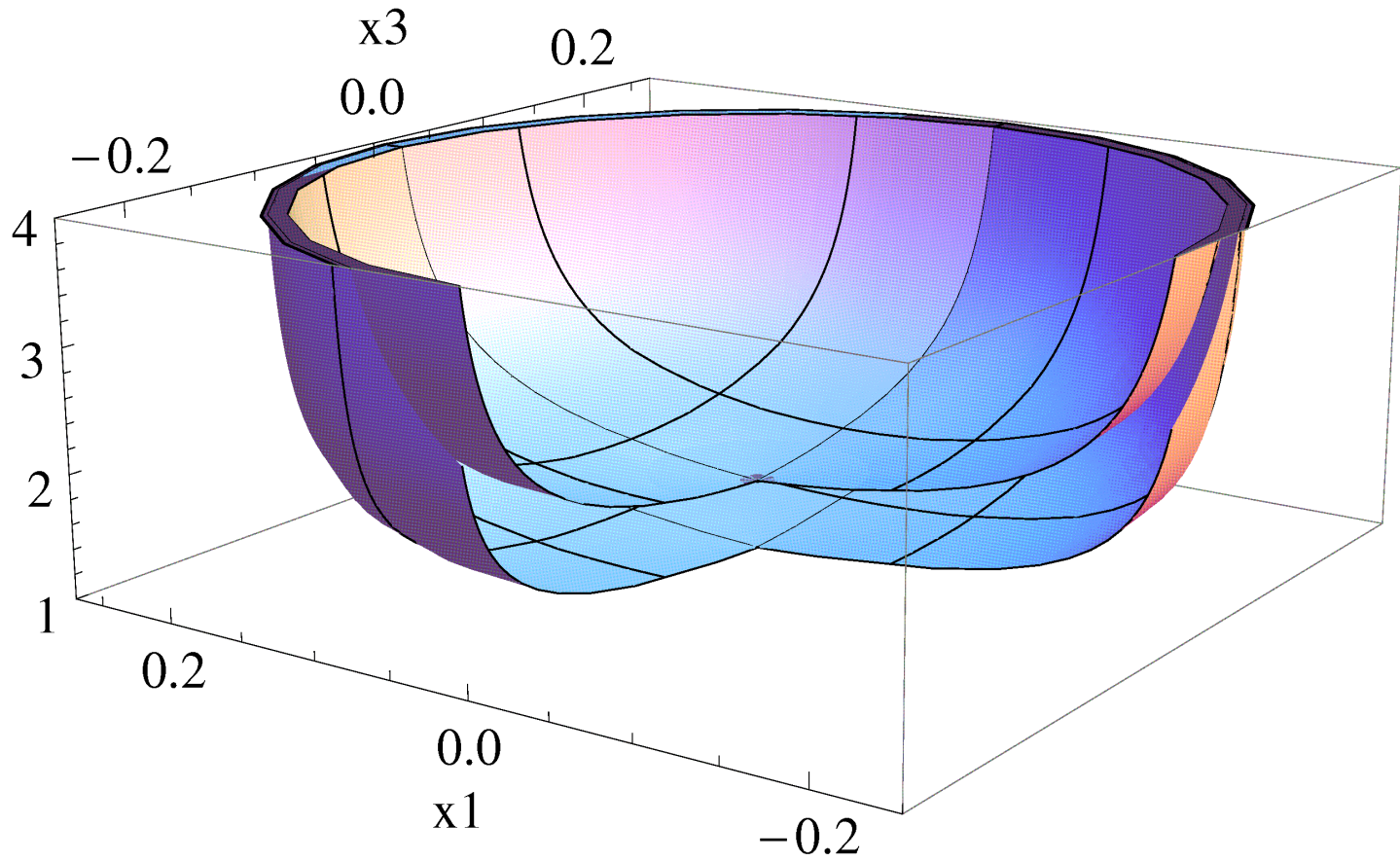
- Introduce density of quarks along the circle

Screening length bound for Baryons

- Again L_{\max} is minimal for $\mathcal{N} = 4$



Baryon string configuration



Summary

- We have calculated heavy meson and baryon screening in the wind of hot strongly coupled plasmas for two kinematic parameters (velocity $v = \tanh \eta$ and orientation angle θ)
- The screening length is a robust quantity
- The screening length in $\mathcal{N} = 4$ is minimal for all kinematic parameters in large class of theories
- We conjecture that it is a universal lower bound for an even wider range of theories.

Outlook

- General analytic study of small perturbations of the $AdS_5 \times S_5$ metric

- For KTY model a first order correction has the form:

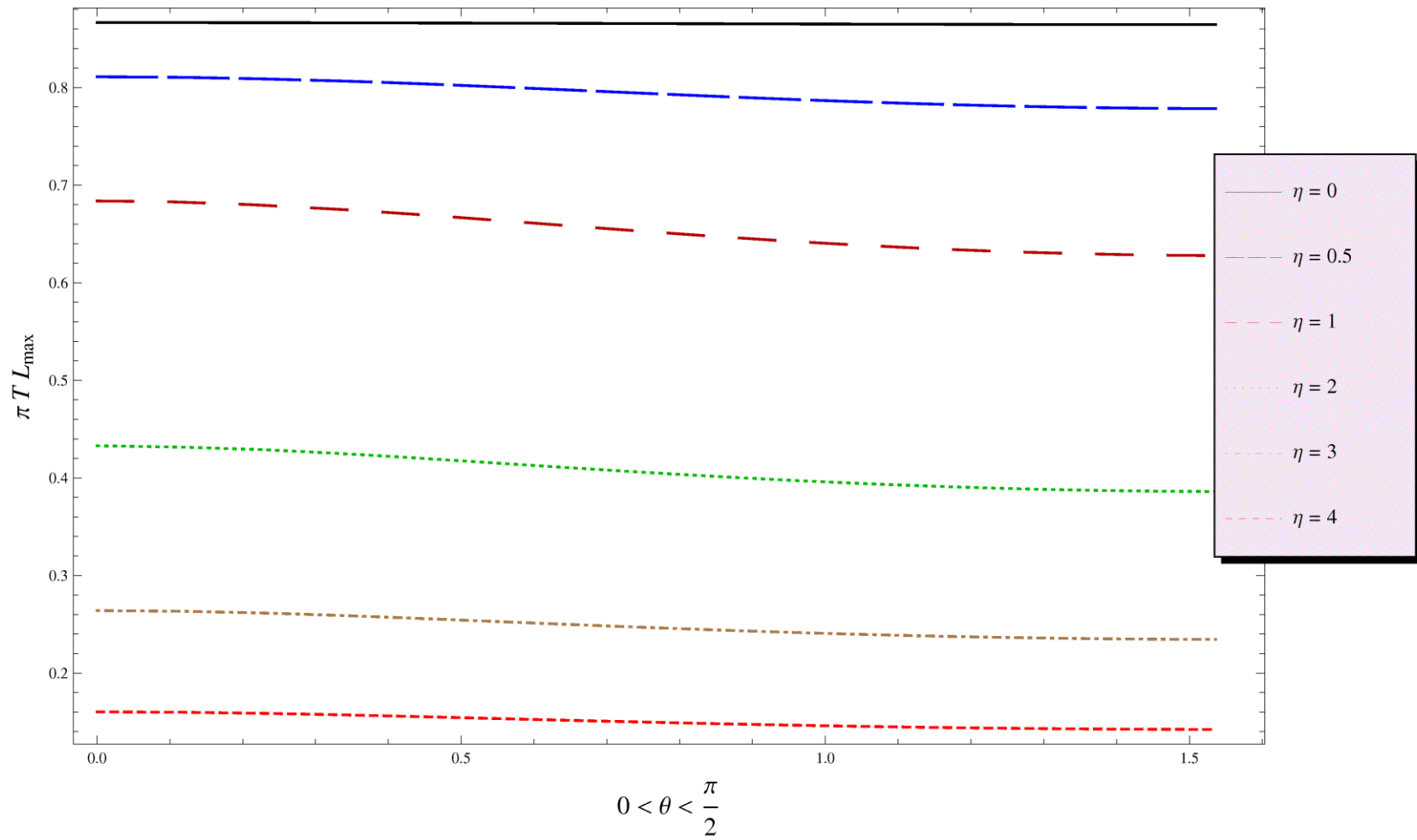
$$L\pi T = L\pi T|_{\mathcal{N}=4} + c \cdot \frac{\sqrt{y_c^4 - 1}}{\pi^2 y_c^9} \left((3 + 4y_c^4) \frac{\sqrt{\pi} \Gamma(\frac{5}{4})}{\Gamma(\frac{3}{4})} \cdot {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{1}{y_c^4} \right) + \dots \right) + \mathcal{O}(c^2)$$

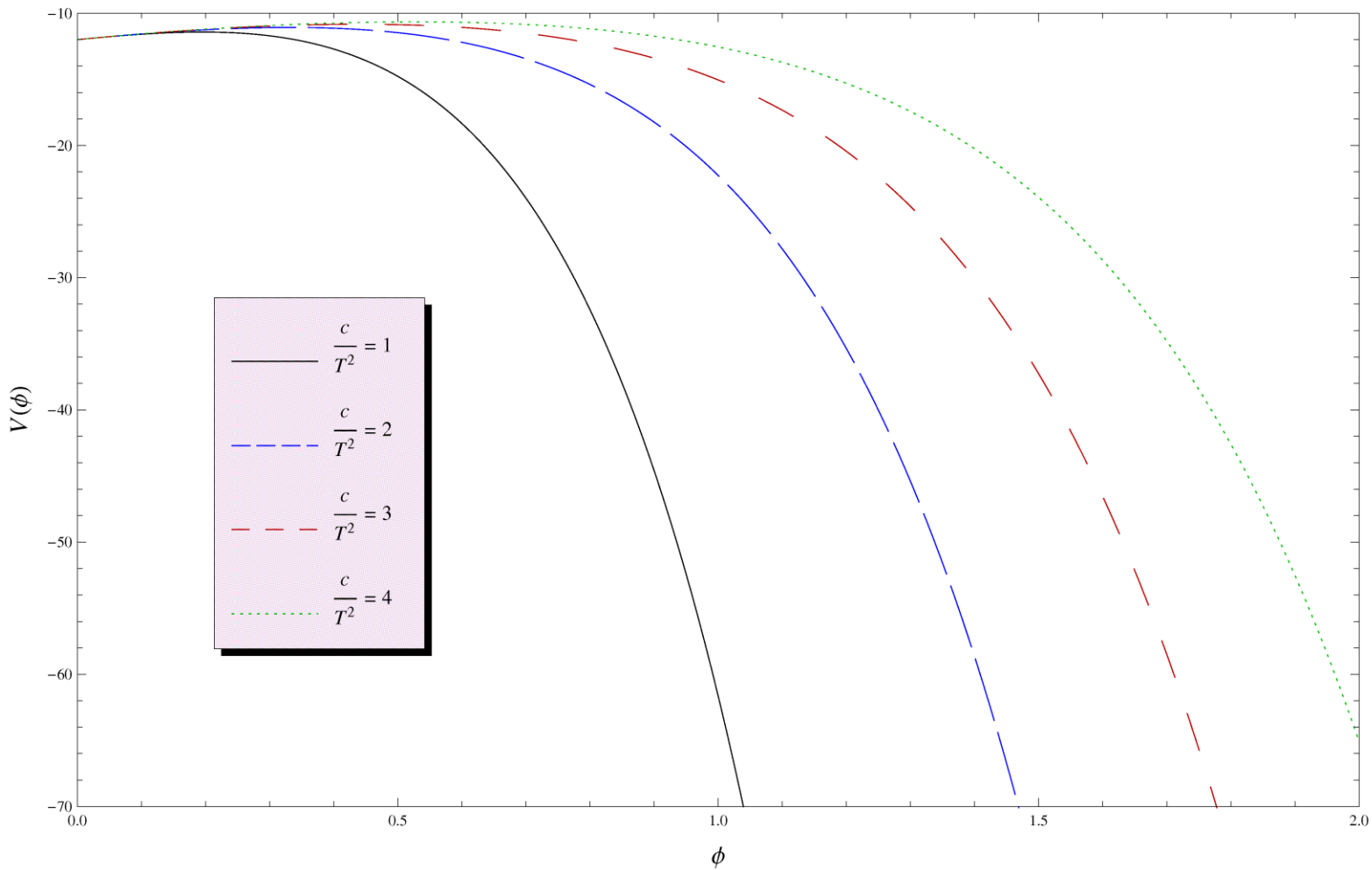
- For the 2-parameter model an analytic computation is more difficult



Thank you for
your attention!

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Analytic verification: Computation

- Easy for KTY. Use first order expansion and analytic computation of the screening length in $\mathcal{N} = 4$

$$L\pi T = L\pi T|_{\mathcal{N}=4} + c \cdot \frac{\sqrt{y_c^4 - 1}}{\pi^2 y_c^9} \left((3 + 4y_c^4) \frac{\sqrt{\pi} \Gamma(\frac{5}{4})}{\Gamma(\frac{3}{4})} \cdot {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{1}{y_c^4} \right) + \dots \right) + \mathcal{O}(c^2)$$

- Not possible for 2-parameter model (intractable equations)

→ Consider small perturbations of the $AdS_5 \times S_5$ metric

$$g_{\mu\nu} = g_{\mu\nu}^0 + h_{\mu\nu} \quad \text{with} \quad h_{\mu\nu} \ll g_{\mu\nu}^0$$

1. Derivation of the linearized Einstein equations
2. Derivation of the equations of motion for general metric ansatz

$$\delta A(z), \quad \delta B(z), \quad \delta h(z)$$

Analytic verification: Results

3. Use polynomial ansatz for $\delta A(z)$ and calculate $\delta B(z)$ and $\delta h(z)$:

$$\delta A = \phi z^k$$

- For $k = \frac{26}{9}$ an analytical calculation is possible:
 - The Drag Force is maximal in $\mathcal{N} = 4$

$$\frac{dp_{\text{pert}}/dt}{dp_{\mathcal{N}=4}/dt} = 1 + \left(\text{Sech}(\eta)^{22/9} - \frac{7}{3} \right) \phi + \mathcal{O}(\phi^2)$$

- For the Screening length in $\mathcal{N} = 4$ an analogue result can be obtained

$$L\pi T = L\pi T|_{\mathcal{N}=4} + \phi \left(\frac{\sqrt{y_c^4 - 1}}{18\sqrt{\pi}y_c^{71/9}} \cdot (7y_c^{44/9} - 3) \frac{\Gamma(\frac{7}{4})}{\Gamma(\frac{5}{4})} \cdot {}_2F_1 \left(\frac{1}{2}, \frac{3}{4}, \frac{5}{4}, \frac{1}{y_c^4} \right) + \dots \right) + \mathcal{O}(c^2)$$