

Strings and numeric strings

Marco Panero

Institute for Theoretical Physics
ETH Zürich
Zürich, Switzerland

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ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Outline

Physical motivation

Lattice QCD

Results of this work

Conclusions and outlook

Based on:

- ▶ M.P., *Thermodynamics of the QCD plasma and the large- N limit*,
Phys. Rev. Lett. **103** 232001 (2009),
[arXiv:0907.3719 [hep-lat]]

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The physical problem

- ▶ Due to asymptotic freedom in non-Abelian gauge theories [**Gross and Wilczek, 1973; Politzer, 1973**], hadronic matter is expected to undergo a change of state to a deconfined phase at sufficiently high temperatures or densities [**Cabibbo and Parisi, 1975; Collins and Perry, 1974**].
- ▶ Extensive experimental investigation through heavy ion collisions since the Eighties: first at AGS (BNL) and SPS (CERN), then at RHIC (BNL)
- ▶ Present experimental evidence from SPS and RHIC: a 'A new state of matter' has been created [**Heinz and Jacob, 2000, Arsene et al., 2004; Back et al., 2004; Adcox et al., 2004; Adams et al., 2005**] ...
- ▶ ... which behaves as an almost ideal fluid [**Kolb and Heinz, 2003**] ('The most perfect liquid observed in Nature')
- ▶ Program to be continued with forthcoming experiments at LHC (CERN) and FAIR (GSI)
- ▶ However, the theoretical understanding of the QCD plasma [**Rischke, 2003**] is still far from complete ...

Theoretical approaches - I

- ▶ Relativistic fluidodynamics is a successful phenomenological description [**Romatschke, 2009**], but is not derived from QCD first principles
- ▶ The perturbative approach in thermal gauge theory has a non-trivial mathematical structure, involving odd powers of the coupling [**Kapusta, 1979**], as well as contributions from diagrams involving arbitrarily large numbers of loops [**Linde, 1980; Gross, Pisarski and Yaffe, 1980**] . . .
- ▶ . . . and shows poor convergence at the temperatures probed in experiments [**Kajantie, Laine, Rummukainen and Schröder, 2002**]
- ▶ Dimensional reduction [**Ginsparg, 1980; Appelquist and Pisarski, 1981**] to EQCD and MQCD [**Braaten and Nieto, 1995**], hard-thermal loop resummations [**Blaizot and Iancu, 2002**], and other effective theory approaches [**Kraemmer and Rebhan, 2004**]
- ▶ Analytical progress in strongly interacting gauge theories: the AdS/CFT conjecture [**Maldacena, 1997**] and related theories as possible models for the non-perturbative features of QCD, including spectral [**Erdmenger, Evans, Kirsch and Threlfall, 2007**] and thermal properties [**Gubser and Karch, 2009**]
- ▶ In the large- N limit, the Maldacena conjecture relates a strongly interacting gauge theory to the classical limit of a gravity model

Theoretical approaches - II

- ▶ Numerical approach: Computer simulations of QCD regularized on a lattice allow first-principle, non-perturbative studies of the finite-temperature plasma
- ▶ The lattice determination of equilibrium thermodynamic properties in $SU(3)$ gauge theory is regarded as a solved problem [**Boyd et al., 1996**]
- ▶ In recent years, finite-temperature lattice QCD has steadily progressed towards parameters corresponding to the physical point [**Karsch et al., 2000; Ali Khan et al., 2001; Aoki et al., 2005; Bernard et al., 2006; Cheng et al., 2007; Bazavov et al., 2009**—see also [**DeTar and Heller, 2009**] for a review of recent results
- ▶ Goals of this work: High-precision determination of the equilibrium thermodynamic properties in $SU(N \geq 3)$ Yang-Mills theories, comparison with holographic models, investigation of possible non-perturbative contributions to the trace anomaly—see also [**Bringoltz and Teper, 2005**] and [**Datta and Gupta, 2009**] for related works

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Motivation, definition and scope - I

- ▶ QCD is the regnant theory of strong subnuclear interactions
- ▶ Very strong experimental evidence from processes at high energies, where, thanks to asymptotic freedom, theorists can rely on perturbative calculations
- ▶ On the contrary, the main qualitative features of the low-energy domain of hadron physics (confinement and chiral symmetry breaking) are non-perturbative in nature
- ▶ Lattice QCD [Wilson, 1974] is *the* non-perturbative regularization of QCD
- ▶ Continuum fields replaced by a discrete set of variables
- ▶ Divergent integrals regularized through a finite cutoff, inversely proportional to the lattice spacing a
- ▶ Amenable to numerical simulations
- ▶ Retains invariance under gauge transformations and a discrete subgroup of rotations and translations
- ▶ Continuum physics recovered for $\lim_{a \rightarrow 0} \lim_{V \rightarrow \infty}$

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Motivation, definition and scope - II

- ▶ A systematically improvable approach—no uncontrolled approximation involved
- ▶ Intrinsically non-perturbative, allows one to extract *first principle* QCD predictions at strong coupling (e.g. hadron spectrum, running coupling α_s , quark masses, hadronic matrix elements relevant for the CKM matrix, deconfinement at high temperature ...)
- ▶ Based on the Feynman path integral formulation in Euclidean spacetime—real-time processes, non-equilibrium thermal quantities, *et c.* are typically dealt with indirectly
- ▶ Technically challenging ('sign problem') for systems at finite density

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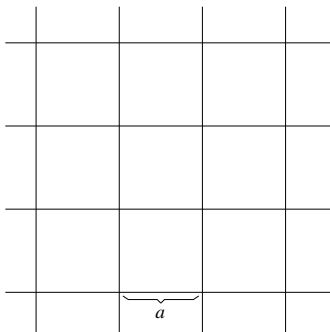
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Lattice formulation for Yang-Mills theories

- ▶ Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing a



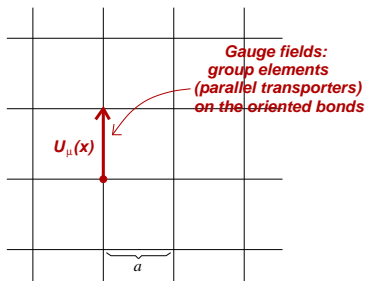
- ▶ Transcribe gauge d.o.f. to lattice elements, build lattice observables
- ▶ Isotropic lattice action for the Yang-Mills theory:

$$S = \beta \sum_{\square} \left(1 - \frac{1}{N} \text{Re Tr } U_{\square} \right), \quad \text{with: } \beta = \frac{2N}{g_0^2} a^{d-4}$$

- ▶ Invariant under gauge transformations $U_{\mu}(x) \rightarrow g(x)U_{\mu}(x)g^{\dagger}(x+a\hat{\mu})$
- ▶ Naïve continuum limit: The lattice action is equivalent to the continuum action, up to $O(a^2)$ corrections
- ▶ Integration measure: $\mathcal{D}A$ is replaced by $\prod_{x,\mu} dU_{\mu}(x)$
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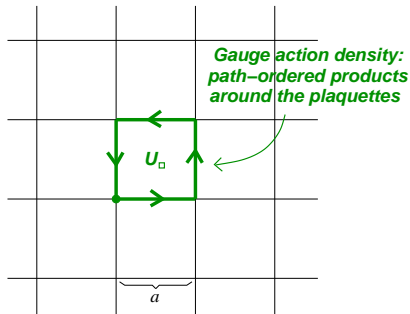
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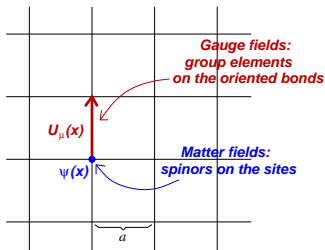
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Including fermions

- ▶ Fermionic matter fields defined on the lattice sites



- ▶ Fermionic contribution to the lattice action:

$$\sum_{q=1}^{N_f} \sum_x \left\{ m_q \bar{\psi}(x) \psi(x) + \frac{1}{2a} \sum_{\mu} \bar{\psi}(x) \gamma_{\mu} \left[U_{\mu}(x) \psi(x + a\hat{\mu}) - U_{\mu}^{\dagger}(x - a\hat{\mu}) \psi(x - a\hat{\mu}) \right] \right\}$$

- ▶ Exact analytical integration of the Grassmann variables leads to the determinant of the Dirac matrix: large computational overhead
- ▶ Naïve lattice discretization yields 2^d doublers:

$$m_q + (i/a) \sum_{\mu} \gamma_{\mu} \sin(p_{\mu} a)$$

- ▶ Wilson's fix: Include a higher-dimensional operator to remove 2^d doublers (but chiral symmetry for $m_q = 0$ is lost)

- ▶ Alternative: staggered fermions (a lattice formulation of Dirac-Kähler fermions) reduce the number of doublers to $2^{\lfloor d/2 \rfloor}$ [Kogut and Susskind, 1975]

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$$\sum_{q=1}^{N_f} \sum_x \left\{ m_q \bar{\psi}(x) \psi(x) + \frac{1}{2a} \sum_{\mu} \bar{\psi}(x) \gamma_{\mu} \left[U_{\mu}(x) \psi(x + a\hat{\mu}) - U_{\mu}^{\dagger}(x - a\hat{\mu}) \psi(x - a\hat{\mu}) \right] \right\}$$

- ▶ Exact analytical integration of the Grassmann variables leads to the determinant of the Dirac matrix: large computational overhead
- ▶ Naïve lattice discretization yields 2^d doublers:

$$m_q + (i/a) \sum_{\mu} \gamma_{\mu} \sin(p_{\mu} a)$$

- ▶ Wilson's fix: Include a higher-dimensional operator to remove $2^d - 1$ doublers (but chiral symmetry for $m_q = 0$ is lost)
- ▶ Alternative: staggered fermions (a lattice formulation of Dirac-Kähler fermions) reduce the number of doublers to $2^{\lceil d/2 \rceil}$ [Kogut and Susskind, 1975], conserving a remnant of chiral symmetry in the massless limit
- ▶ Chiral lattice fermions: domain wall fermions [Kaplan, 1992] and overlap fermions [Narayanan and Neuberger, 1993; Neuberger, 1997]—huge computational overhead

- ▶ Lattice QCD provides an *effective continuum theory* for the low-energy physics

$$S_{\text{eff}} = \int d^4x \left[\mathcal{L}_0(x) + a\mathcal{L}_1(x) + a^2\mathcal{L}_2(x) + \dots \right]$$

- ▶ The physical value of the spacing a is set using a low-energy observable (e.g., the asymptotic slope σ of the potential between infinitely heavy external sources)
- ▶ Lattice renormalization: hadronic renormalization schemes, mean-field improved perturbation theory [Parisi, 1980; Lepage and Mackenzie, 1993], recursive finite-size technique [Lüscher, Weisz and Wolff, 1991]
- ▶ Improved actions

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Thermodynamics on the lattice

- ▶ Thermal averages from simulations on a lattice with compactified Euclidean time direction, with $T = 1/(aN_\tau)$
- ▶ Pressure $p(T)$ via the 'integral method' [Engels *et al.*, 1990]:

$$\begin{aligned} p &= T \frac{\partial}{\partial V} \log \mathcal{Z} \simeq \frac{T}{V} \log \mathcal{Z} = \frac{1}{a^4 N_s^3 N_\tau} \int_{\beta_0}^{\beta} d\beta' \frac{\partial \log \mathcal{Z}}{\partial \beta'} \\ &= \frac{6}{a^4} \int_{\beta_0}^{\beta} d\beta' (\langle U_{\square} \rangle_T - \langle U_{\square} \rangle_0) \end{aligned}$$

Thermodynamics on the lattice

- ▶ Other thermodynamic observables obtained from indirect measurements
 - ▶ Trace of the stress tensor $\Delta = \epsilon - 3p$:

$$\Delta = T^5 \frac{\partial}{\partial T} \frac{p}{T^4} = \frac{6}{a^4} \frac{\partial \beta}{\partial \log a} (\langle U_{\square} \rangle_0 - \langle U_{\square} \rangle_T)$$

- ▶ Energy density:

$$\epsilon = \frac{T^2}{V} \frac{\partial}{\partial T} \log \mathcal{Z} = \Delta + 3p$$

- ▶ Entropy density:

$$s = \frac{S}{V} = \frac{\epsilon - f}{T} = \frac{\Delta + 4p}{T}$$

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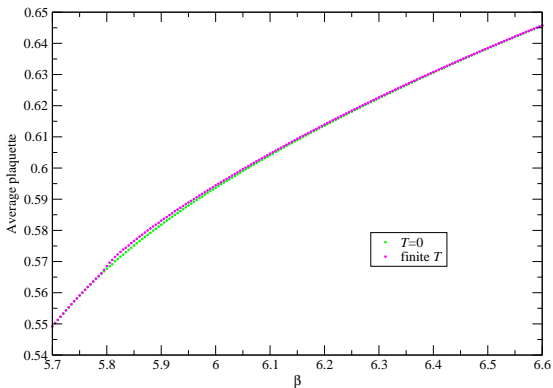
Simulation details

- ▶ Lattice sizes $N_s^3 \times N_\tau$, with $N_s = 20$ or 16 , and $N_\tau = 5$
- ▶ Simulation algorithm: heat-bath [**Kennedy and Pendleton, 1985**] for $SU(2)$ subgroups [**Cabibbo and Marinari, 1982**] and full- $SU(N)$ overrelaxation [**Kiskis, Narayanan and Neuberger, 2003; Dürr, 2004; de Forcrand and Jahn, 2005**]
- ▶ Cross-check with $T = 0$ simulations run using the Chroma suite [**Edwards and Joó, 2004**]
- ▶ Physical scale for $SU(3)$ set using r_0 [**Necco and Sommer, 2001**]
- ▶ Physical scale for $SU(N > 3)$ set using known values for the string tension σ [**Lucini, Teper and Wenger, 2004; Lucini and Teper, 2001**] in combination with the 3-loop lattice β -function [**Allés, Feo and Panagopoulos, 1997; Allton, Teper and Trivini, 2008**] in the mean-field improved lattice scheme [**Parisi, 1980; Lepage and Mackenzie, 1993**]

Measurements of the plaquette

- High precision determination of $(\langle U_{\square} \rangle_T - \langle U_{\square} \rangle_0)$ required

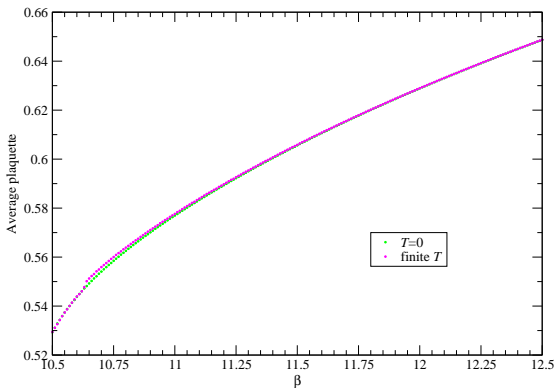
SU(3), $N_s = 20$, $N_\tau = 5$



Measurements of the plaquette

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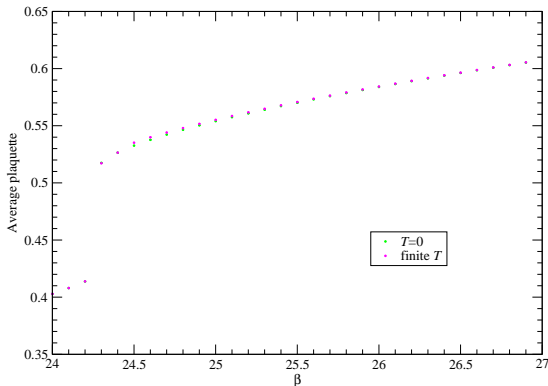
$SU(4)$, $N_s = 16$, $N_\tau = 5$



Measurements of the plaquette

- ▶ High precision determination of $(\langle U_{\square} \rangle_T - \langle U_{\square} \rangle_0)$ required
- ▶ Data reveal a strong first order bulk transition for $SU(N \geq 4)$

$$SU(6), N_s = 16, N_t = 5$$



Improved holographic QCD model vs. lattice data - I

- ▶ Kiritsis and collaborators [**Gürsoy, Kiritsis, Mazzanti and Nitti, 2008**] proposed an AdS/QCD model based on a 5D Einstein-dilaton gravity theory, with the fifth direction dual to the energy scale of the $SU(N)$ gauge theory
- ▶ Field content on the gravity side: metric (dual to the $SU(N)$ energy-momentum tensor), the dilaton (dual to the trace of F^2) and the axion (dual to the trace of $F\tilde{F}$)
- ▶ Gravity action:

$$S_{IHQCD} = -M_P^3 N^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} (\partial\Phi)^2 + V(\lambda) \right] + 2M_P^3 N^2 \int_{\partial M} d^4x \sqrt{h} K$$

- ▶ Φ is the dilaton field, $\lambda = \exp(\Phi)$ is identified with the running 't Hooft coupling of the dual $SU(N)$ YM theory
- ▶ The effective five-dimensional Newton constant $G_5 = 1 / (16\pi M_P^3 N^2)$ becomes small in the large- N limit

Improved holographic QCD model vs. lattice data - II

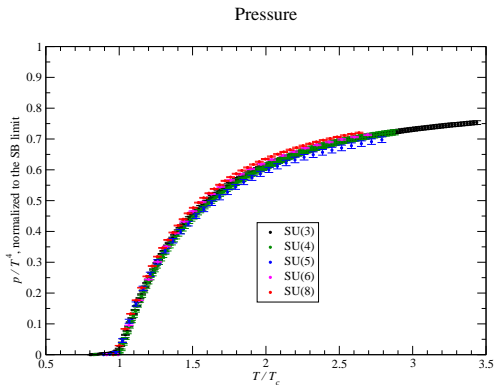
- ▶ Dilaton potential $V(\lambda)$ defined by requiring asymptotic freedom with a logarithmically running coupling in the UV and linear confinement in the IR of the gauge theory; a possible *Ansatz* is:

$$V(\lambda) = \frac{12}{\ell^2} \left[1 + V_0\lambda + V_1\lambda^{4/3} \sqrt{\log(1 + V_2\lambda^{4/3} + V_3\lambda^2)} \right], \quad (1)$$

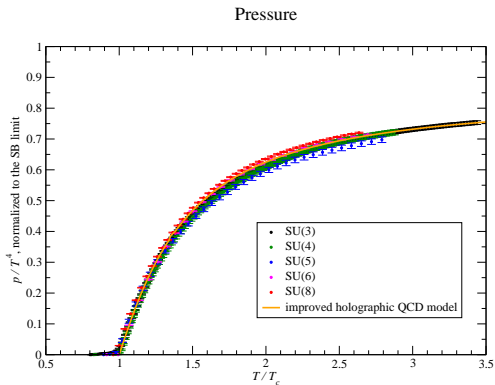
where ℓ is the AdS scale (overall normalization)

- ▶ V_0 , V_1 , V_2 and V_3 are free parameters: two of them can be fixed by imposing that the dual model reproduces the first two (scheme-independent) perturbative coefficients of the $SU(N)$ β -function, and one is left with two independent parameters

Improved holographic QCD model vs. lattice data - III

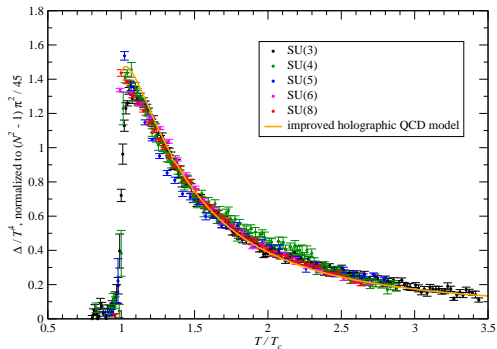


Improved holographic QCD model vs. lattice data - III

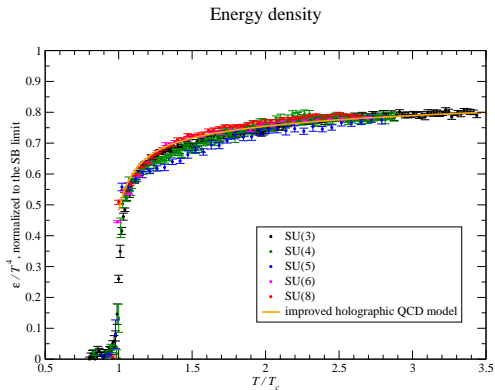


Improved holographic QCD model vs. lattice data - III

Trace of the energy-momentum tensor

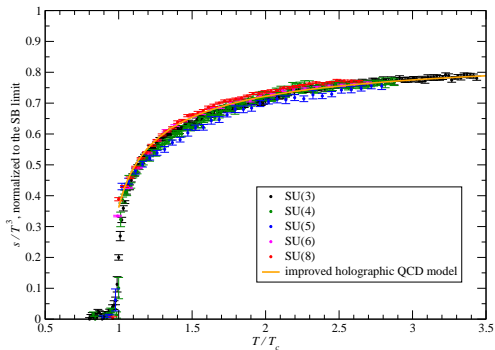


Improved holographic QCD model vs. lattice data - III



Improved holographic QCD model vs. lattice data - III

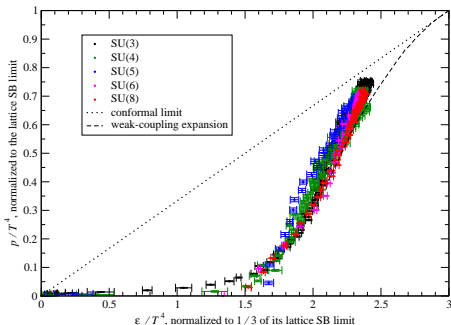
Entropy density



AdS/CFT vs. lattice data in a 'quasi-conformal' regime

For $T \simeq 3T_C$, the lattice results reveal that the deconfined plasma, while still strongly interacting and far from the Stefan-Boltzmann limit, approaches a scale-invariant regime ...

$p(\epsilon)$ equation of state and approach to conformality

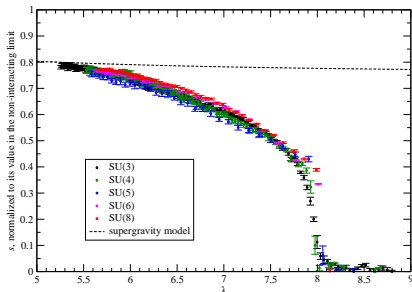


AdS/CFT vs. lattice data in a 'quasi-conformal' regime

... in which the entropy density is comparable with the supergravity prediction for $\mathcal{N} = 4$ SYM [Gubser, Klebanov and Tseytlin, 1998]

$$\frac{s}{s_0} = \frac{3}{4} + \frac{45}{32} \zeta(3) (2\lambda)^{-3/2} + \dots$$

Entropy density vs. 't Hooft coupling

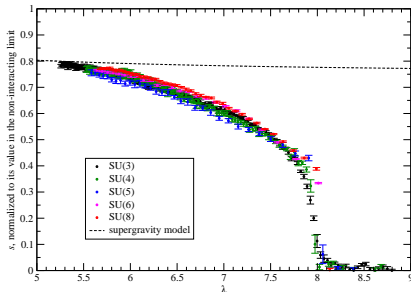


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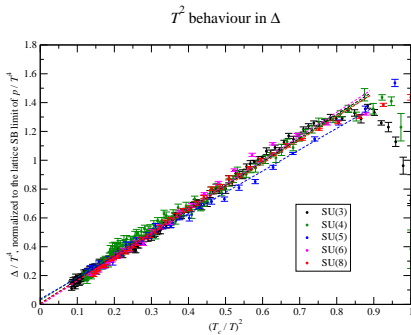
Note that a comparison of $\mathcal{N} = 4$ SYM and full-QCD lattice results for the drag force on heavy quarks also yields $\lambda \simeq 5.5$ [Gubser, 2006]

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T^2 contributions to the trace anomaly?

The trace anomaly reveals a characteristic T^2 -behavior, possibly of non-perturbative origin [Megías, Ruiz Arriola and Salcedo, 2003; Pisarski, 2006; Andreev, 2007]

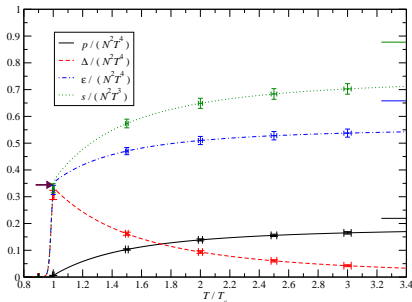


Extrapolation to $N \rightarrow \infty$

Based on the parametrization [Bazavov *et al.*, 2009]:

$$\frac{\Delta}{T^4} = \frac{\pi^2}{45} (N^2 - 1) \cdot \left(1 - \left\{ 1 + \exp \left[\frac{(T/T_c) - f_1}{f_2} \right] \right\}^{-2} \right) \left(f_3 \frac{T_c^2}{T^2} + f_4 \frac{T_c^4}{T^4} \right)$$

Extrapolation to the large- N limit



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Conclusions

- ▶ Equilibrium thermodynamic observables in $SU(N)$ YM theories at temperatures $0.8T_c \leq T \leq 3.4T_c$ show a mild dependence on N
- ▶ Successful comparison with the IHQCD model
- ▶ Quasi-conformal regime of YM and $\mathcal{N} = 4$ SYM predictions—Can lattice data help to pin down realistic parameters for AdS/CFT models of the sQGP?
[Noronha, Gyulassy and Torrieri, 2009]
- ▶ Δ seems to have a T^2 dependence also at large N
- ▶ Extrapolation to the $N \rightarrow \infty$ limit

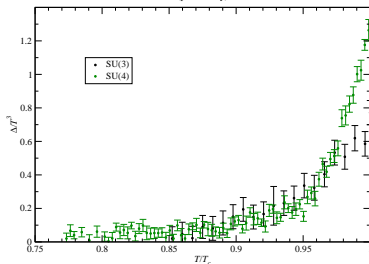
Projects for the future - I

- ▶ $SU(N)$ screening masses and spatial string tensions, comparisons with AdS/CFT [**Bak, Karch and Yaffe, 2007**] and with IHQCD [**Alanen, Kajantie and Suur-Uski, 2009**]
- ▶ $\text{Tr}F^2$ correlators and dilaton potential [**Noronha, 2009**]
- ▶ Observables related to thermodynamic fluctuations: specific heat, speed of sound *et c.* [**Gavai, Gupta and Mukherjee, 2005**]*—*relevant for the elliptic flow [**Ollitrault, 1992; Teaney, Lauret and Shuryak, 2001**]
- ▶ Renormalized Polyakov loops in various representations [**Damgaard, 1987; Damgaard and Hasenbusch, 1994; Dumitru, Hatta, Lenaghan, Orginos and Pisarski, 2004; Gupta, Hübner and Kaczmarek, 2008**]
- ▶ Transport coefficients [**Meyer, 2007**]

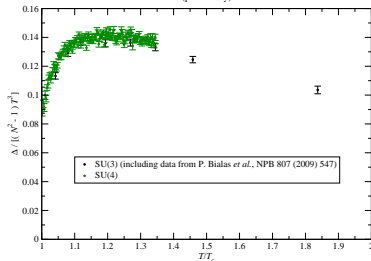
Projects for the future - II

- ▶ High-precision thermodynamics for $SU(N)$ theories in 3D (work in progress with Caselle, Castagnini, Feo and Gliozzi; see also [Bialas, Daniel, Morel and Petersson, 2008])

D=2+1 $SU(N)$ trace anomaly in the confined phase
(preliminary)



D=2+1 $SU(N)$ trace anomaly in the deconfined phase
(preliminary)



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