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Holographic models for QCD

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Introduction

• Despite decades of progress, QCD remains a challenging theory for physics due to the strong coupling problem

• In 1974 't Hooft suggested that the large-N expansion in gauge theories may provide an alternative and controllable method to handle strong coupling, suggesting a relationship to a string theory.

• In 1997 Maldacena conjectured a precise correspondence for a more symmetric cousin of YM. There were many surprises in this duality and new intuition that developed.

• The conjecture was tested in many contexts but still remains a conjecture. Few doubt it validity.

• The duality was extended further to more theories but asymptotically-free theories remain out of (controllable) reach.

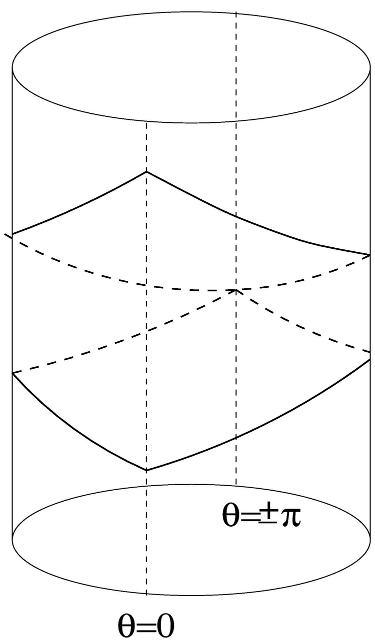
• We are still not able to solve the dual string theory even in N=4 sYM, but important progress has been done recently.

The gauge-theory/gravity duality

• The gauge-theory/gravity duality is a duality that relates a string theory with a gauge theory.

• The prime example is the AdS/CFT correspondence

Maldacena 1997



• It states that N=4 four-dimensional SU(N) gauge theory (gauge fields, 4 fermions, 6 scalars) is equivalent to ten-dimensional IIB string theory on AdS$_5 \times S^5$

$$ds^2 = \frac{\ell^2_{AdS}}{r^2}\left[dr^2 + dx^\mu dx_\mu\right] + \ell^2_{AdS}\ (d\Omega_5)^2$$

This space (AdS_5) has a single boundary, at $r = 0$.

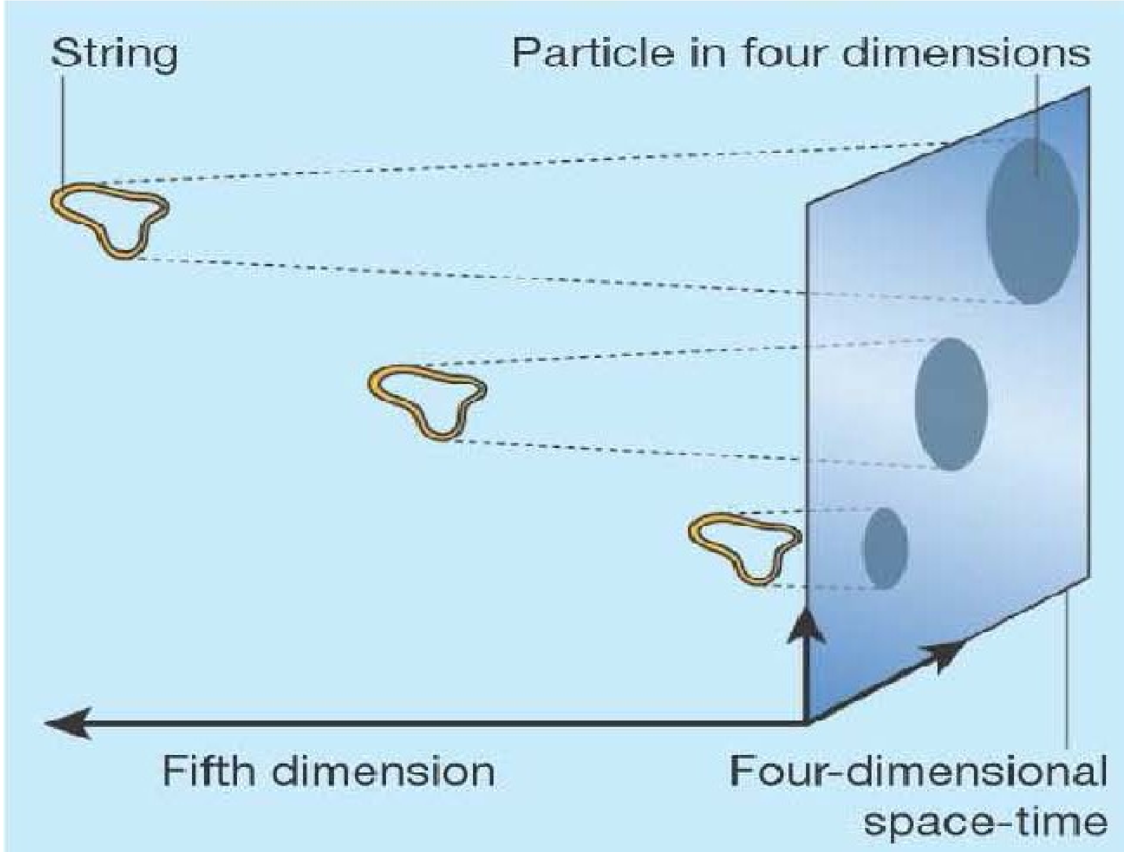
• The string theory has as parameters, g_{string}, ℓ_{string}, ℓ_{AdS}. They are related to the gauge theory parameters as

$$g^2_{YM} = 4\pi \; g_{\mathrm{string}} \quad , \quad \lambda = g^2_{YM} \; N = \frac{\ell^4_{AdS}}{\ell^4_{\mathrm{string}}}$$

• As $N \to \infty$, $g_{\mathrm{string}} \sim \frac{\lambda}{N} \to 0$.

• As $N \to \infty$, $\lambda \gg 1$ implies that $\ell_{\mathrm{string}} \ll \ell_{AdS}$ and the geometry is very weakly curved. String theory can be approximated by gravity in that regime and is weakly coupled.

• As $N \to \infty$, $\lambda \ll 1$ the gauge theory is weakly coupled, but the string theory is strongly curved.



- There is one-to-one correspondence between on-shell string states $\Phi(r, x^\mu)$ and gauge-invariant (single-trace) operators $O(x^\mu)$ in the sYM theory

- In the string theory we can compute the "S-matrix" , $S(\phi(x^\mu))$ by studying the response of the system to boundary conditions $\Phi(r = 0, x^\mu) = \phi(x^\mu)$

- This is done by doing the string path integral with sources at the boundary

$$e^{-\widehat{S}(\phi(x))} = \int_{\Phi(r=0,x^\mu)=\phi(x^\mu)} D\Phi(r,x)\ e^{-S_{\text{string}}(\Phi)}$$

• At string tree level (large N), it is enough to solve the string equations of motion with the appropriate boundary conditions.

$$\frac{\delta S}{\delta \Phi} = 0 \quad , \quad \Phi(r=0,x^\mu) = \phi(x^\mu)$$

• Substituting the solution into the string action we obtain the "S"-matrix (a functional of the sources $\phi(x)$.

• The correspondence states that this is equivalent to the generating function of c-correlators of O

$$\langle e^{\int d^4x\ \phi(x)\ O(x)} \rangle \ = \ e^{-\widehat{S}(\phi(x))}$$

Therefore the source corresponds to the "coupling constant" for the operator

$$\Phi(r,x) = \phi(x) r^{4-\Delta} + \cdots + \widehat{\phi}(x) r^{\Delta} + \cdots \quad , \quad r \to 0$$

$\widehat{\phi} \simeq \langle \phi(x) \rangle$. ϕ and $\langle \phi(x) \rangle$ ARE NOT independent: regularity of the solution determines $\langle \phi(x) \rangle$ as a function of $\phi(x)$.

The gauge-theory at finite temperature

• The finite temperature ground state of the gauge theory corresponds to a different solution in the dual string theory: the AdS-Black-hole solution

E. Witten, 1998

$$ds^2 = \frac{\ell_{AdS}^2}{r^2}\left[\frac{dr^2}{f(r)} + f(r)dt^2 + dx^i dx_i\right] + \ell_{AdS}^2\,(d\Omega_5)^2 \quad , \quad f(r) = 1 - (\pi T)^4 r^4$$

• The horizon is at $r = \frac{1}{\pi T}$

• The dynamics of low-energy gravitational fluctuations is governed by the relativistic Navier-Stokes equation.

Critical string theory holography

♠ Several "successful" holographic models of non-trivial gauge dynamics with confinement in the IR

- The non-supersymmetric D_4 solution,due to Witten, dual to $\mathcal{N} = 4_5$ sYM on a circle, whose supersymmetry is broken by the boundary conditions of the fermions. It exhibits confinement in the IR.

- Flavor has been successfully incorporated by Sakai+Sugimoto by adding D_8 (dipole) branes.

- The Chamseddine-Volkov solution interpreted by Maldacena and Nuñes as the dual of a confining compactified gauge theory (emerging by wrapping NS_5 branes on a two-cycle).

- The Klebanov-Strassler solution corresponding to a cascade of quiver gauge theories, that confine in the IR.

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♠ In all of the above, confinement related quantities (string tension, gluebal masses, finite temperature effects etc) can be calculated controllably and analytically.

♠ The same applies to the Sakai-Sugimoto model for flavor, except two major drawbacks:
The absence of bare quark masses and the chiral-symmetry-breaking condensate.

♠ In all the above solutions, the scale of KK excitations is of the same order as Λ of the confining gauge theory.

♠ None so far has managed to overcome this obstacle in critical string theory models.

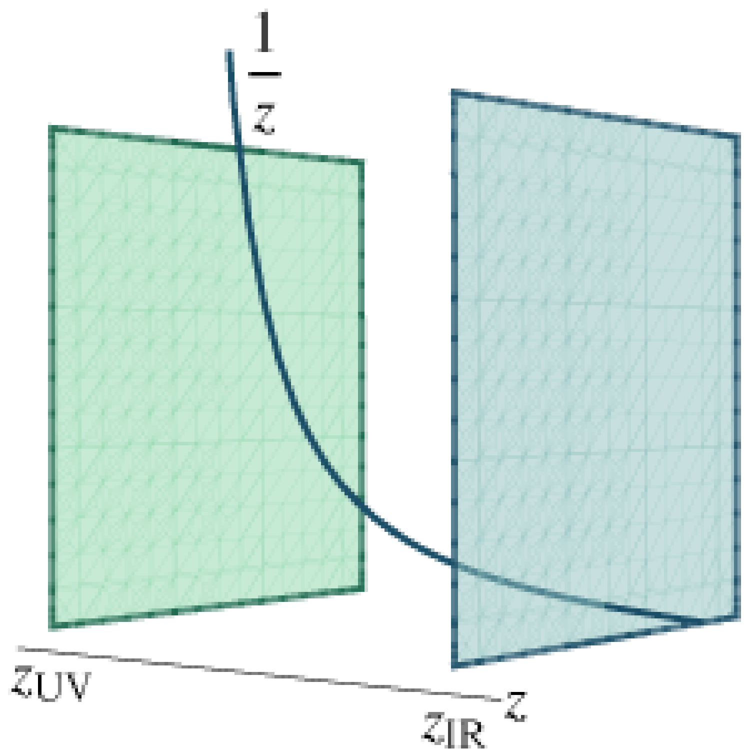
AdS/QCD

♠ A basic phenomenological approach: use a slice of AdS_5, with a UV cutoff, and an IR cutoff.
Polchinski+Strassler

♠ It successfully exhibits confinement (trivially via IR cutoff), and power-like behavior in hard scattering amplitudes

♠ It may be equipped with a bifundamental scalar, T, and $U(N_f)_L \times U(N_f)_R$, gauge fields to describe mesons.
Erlich+Katz+Son+Stepanov, DaRold+Pomarol

Chiral symmetry is broken by hand, via IR boundary conditions. The low-lying meson spectrum looks "reasonable".



♠ Shortcomings:

- The glueball spectrum does not fit very well the lattice calculations. It has the wrong asymptotic behavior $m_n^2 \sim n^2$ at large n.

- Magnetic quarks are confined instead of screened.

- Chiral symmetry breaking is input by hand.

- The meson spectrum has also the wrong UV asymptotics $m_n^2 \sim n^2$.

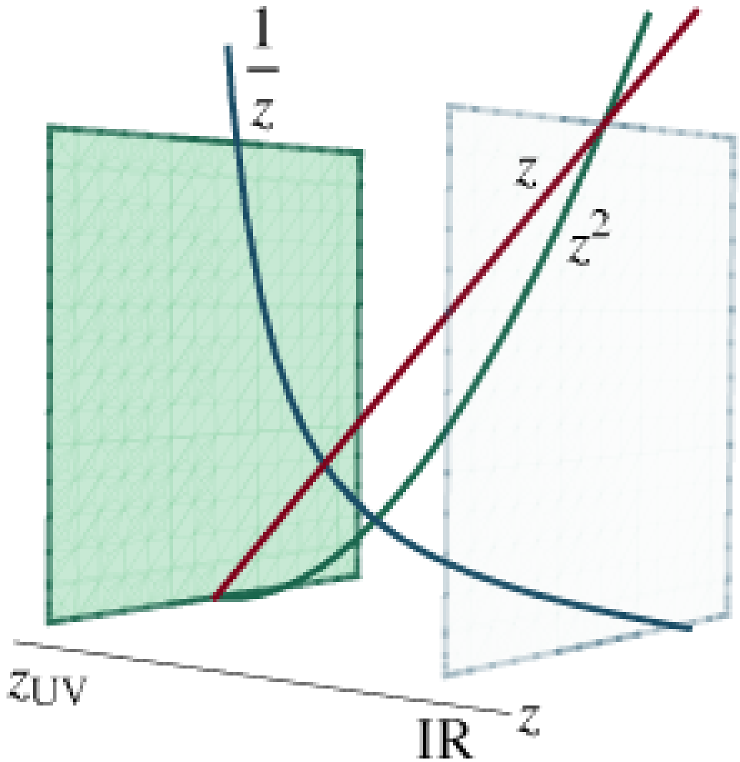
- at finite temperature there is a deconfining transition but the equation of state is trivial (conformal) ($e - 3p = 0$) and the speed of sound is $c_s^2 = \frac{1}{3}$.

The "soft wall"

♠ The asymptotic spectrum can be fixed by introducing a **non-dynamical** dilaton profile $\Phi \sim r^2$ (soft wall)

Karch+Katz+Son+Stephanov

• It is not a solution of equations of motion: the metric is still AdS: Neither $g_{\mu\nu}$ nor Φ solves the eq



• This is really an "inconsistent" phenomenological model.

A string theory for QCD:(Very) basic expectations

• Pure SU(N_c) d=4 YM at large N_c is expected to be dual to a string theory in 5 dimensions only. Essentially a single adjoint field $\rightarrow$ a single extra dimension.

♠ The four vector components are related by the expected Lorentz invariance of the vacuum.

♠ Therefore: a single eigenvalue distribution $\rightarrow$ an extra dimension

♠ Intuition well tested in several matrix models including the "old-ones".

♠ The counting of dimensions can become complicated by the presence of several fields, "evanescent dimensions" and the knowledge/structure of RG topography.

• The theory becomes asymptotically free and conformal at high energy

• Following on N=4 intuition we might expect that $\ell_{AdS} \to 0 \quad \to \quad$ singularity.

• There are several possibilities for such singularities:

(a) They are "mirage": the geometry stabilizes at $\ell \sim \ell_s$. (different examples from WZW models and DBI actions).

(b) The singularity is resolved by the stringy or higher dimensional physics. The true string metric is regular (some examples from higher dimensional resolutions)

(c) The singularity remains (not our case we think)

• The N=4 relation $\ell^4 \sim \lambda \sim \frac{1}{\log r}$. seems to indicate a naked singularity.

• Another possibility is that the classical saddle point solution should asymptote to a regular but stringy ($\ell = \ell_s$) AdS_5. This option has several advantages and provides a lot of mileage:

♠ It allows in principle the machinery of holography to be applied

♠ It realizes the geometrical implementation of the asymptotic conformal symmetry of YM theory in the UV.

The low energy spectrum

♠ In YM only $Tr[FF]$ and maybe $Tr[F \wedge F]$ have a source. However many operators can have a vev. We expect $\langle O_\Delta \rangle \sim (\Lambda_{QCD})^\Delta$.

♠ If that is the case this implies that many stringy states will have non-trivial profiles in the vacuum solution.

♠ Operators of higher dimension are not important in the UV (that's why we can truncate the RG flow). In the bulk, they have positive m^2, that suppresses their solutions.

These are scalar YM operators with $\Delta_{UV} > 4 \to m^2 > 0$ or higher spin fields.

• But higher dimension operators may become important in the IR.

♠ Indications from SVZ sum rules plus data suggest that the coefficients of higher dimension operators are "unnaturally" small.

• It seems a reasonable assumption to neglect all $\Delta > 4$ fields when looking for the vacuum solution.

• What are all gauge invariant YM operators of dimension 4 or less?

• They are given by $Tr[F_{\mu\nu}F_{\rho\sigma}]$.
Decomposing the lowest ones (in spin) are, the stress tensor, the scalar and the pseudoscalar

♠ Therefore we will consider

$$T_{\mu\nu} \leftrightarrow g_{\mu\nu},\ tr[F^2] \leftrightarrow \phi,\ tr[F \wedge F] \leftrightarrow a$$

• The "axion" action will be suppressed by $1/N_c^2$ since the axion is a RR field.

general expectations

• In the UV (near the boundary) the coupling is small and stringy behavior is important. We expect an AdS space to emerge from the asymptotic conformal invariance and it will be of stringy size.

• The rest of the asymptotics are perturbative around the AdS space, and we obtain an expansion in powers of $(1/\log r)^n$

• We do expect that $\lambda \to \infty$ (or becomes large) at the IR bottom.

• Intuition from N=4 and other 10d strongly coupled theories suggests that in this regime there should be an (approximate) two-derivative description of the physics.

• The simplest solution with this property is the linear dilaton solution with

$$\lambda \sim e^{Qr} \quad , \quad V(\lambda) \sim \delta c = 10 - D \quad \to \quad \text{constant} \quad , \quad R = 0$$

• Self-consistency of this assumption implies that the string frame curvature should vanish in the IR.

• This property persists with potentials $V(\lambda) \sim (\log \lambda)^P$. Moreover all such cases have confinement, a mass gap and a discrete spectrum (except the P=0 case).

• At the IR bottom (in the string frame) the scale factor vanishes, and 5D space becomes (asymptotically) flat.

Improved Holographic QCD: a model

• We would like to write down a model that captures the holographic behavior of YM:

• The basic fields will be $g_{\mu\nu}, \phi, a$. We can neglect a when studying the basic vacuum solution (down by N_c^{-2}).

• In the IR the action should have two derivatives and admit solutions with weak curvature (in the string frame)

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right] \quad , \quad \lambda = N_c \, e^{\phi}$$

• Although in the UV we expect higher derivatives to be important we will extend this by demanding that the solution is asymptotically AdS_5 and the 't Hooft coupling will run logarithmically.

• Although we do not expect this simple model to capture all aspects of YM dynamics we will see that it goes a long way.

The UV solution

• In order to obtain an AdS_5 solution V should become a constant when $\lambda \to 0$.

• We therefore write an expansion for the potential in the UV as

$$\lim_{\lambda\to 0} V(\lambda) = \frac{12}{\ell^2}\left(1 + \sum_{n=1}^{\infty} c_n \lambda^n\right)$$

• The potential should be strictly monotonic to drive the theory to strong coupling without IR fixed points.

• In particular, the UV fixed point $\lambda = 0$ satisfies $V^{(n)}(0) = 0$.

• The vacuum solution ansatz is

$$ds^2 = e^{2A(r)}(dr^2 + dx^\mu dx_\mu) \quad , \quad \lambda(r)$$

and is the most general one that preserves 4d Poincaré invariance.

• The classical solution represents the YM "vacuum" at large N_c.

• We may choose the holographic "energy" scale as the scale factor in the Einstein frame

$$E = e^{A_E}$$

This asymptotes properly in the UV, $E \sim 1/r$, is everywhere monotonic and becomes zero in the IR. This is a choice (scheme). Physical quantities do not depend on it. This translates into RG invariance in QFT.

• We may now solve the equations perturbatively in λ around $\lambda = 0$ and $r = 0$ (this is a weak coupling expansion) to find

$$\frac{1}{\lambda} = L - \frac{b_1}{b_0}\log L + \frac{b_1^2}{b_0^2}\frac{\log L}{L} + \left(\frac{b_1^2}{b_0^2} + \frac{b_2}{b_0}\right)\frac{1}{L} + \frac{b_1^3}{2b_0^3}\frac{\log^2 L}{L^2} + \cdots$$

$$L \equiv -b_0 \log(r\Lambda)$$

$$e^A = \frac{\ell}{r}\left[1 + \frac{4}{9\log r\Lambda} + \mathcal{O}\left(\frac{\log\log r\Lambda}{\log^2 r\Lambda}\right)\right]$$

The identification is

$$c_1 = \frac{8}{9} b_0 \quad , \quad c_2 = \frac{23\ b_0^2 - 36\ b_1}{3^4} \quad , \quad c_3 = -2\frac{324\ b_2 + 124\ b_0^3 + 189\ b_1 b_0}{3^7}$$

with

$$V = \frac{12}{\ell^2}\left[1 + c_1\lambda + c_2\lambda^2 + c_3\lambda^3 + \cdots\right]$$

$$\frac{d\lambda}{d\log E} \equiv \beta(\lambda) = -b_0\lambda^2 + b_1\lambda^3 + b_2\lambda^4 + \cdots$$

♠ The asymptotic expansion of the potential is in one-to-one correspondence with the perturbative β-function.

Organizing the vacuum solutions

- The β-function can be mapped uniquely to the dilaton potential $V(\lambda)$.

- A useful variable is the phase variable

$$X \equiv \frac{\lambda'}{3\lambda A'} = \frac{\beta(\lambda)}{3\lambda}$$

- We can introduce a (pseudo)superpotential

$$V(\lambda) = \left(\frac{4}{3}\right)^3 \left[W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \Phi}\right)^2\right]$$

and write the equations in a first order form:

$$A' = -\frac{4}{9}W \quad , \quad \Phi' = \frac{dW}{d\Phi}$$

$$\beta(\lambda) = -\frac{9}{4}\lambda \frac{d\log W}{d\log \lambda}$$

♠ The equations have three integration constants: (two for Φ and one for A) One is fixed by $\lambda \to 0$ in the UV. The other is Λ. The one in A is the choice of energy scale.

The IR regime

For any asymptotically AdS$_5$ solution ($e^A \sim \frac{\ell}{r}$):

• The scale factor $e^{A(r)}$ is monotonically decreasing

Girardelo+Petrini+Porrati+Zaffaroni
Freedman+Gubser+Pilch+Warner

• Moreover, there are only three possible, mutually exclusive IR asymptotics:

♠ *there is another asymptotic AdS_5 region, at $r \to \infty$, where* $\exp A(r) \sim \ell'/r$, *and* $\ell' \leq \ell$ (equality holds if and only if the space is exactly AdS_5 everywhere);

♠ there is a curvature singularity at some finite value of the radial coordinate, $r = r_0$;

♠ there is a curvature singularity at $r \to \infty$, where the scale factor vanishes and the space-time shrinks to zero size.

On naked holographic singularities

• In this case all Poincaré invariant solutions end up in a naked IR singularity.

• In GR we abhor naked singularities.

• In holographic gravity some many be acceptable. The reason is that they do not signal a breakdown of predictability as is the case in GR. They could be resolved by stringy or KK physics, or they could be shielded for finite energy configurations.

Something similar happens in the "Liouville wall" of 2d gravity: all finite energy physics is not affected by the $e^{\phi} \to \infty$ singularity.

• An important task in EHT is to therefore ascertain when such naked singularities are acceptable (alias "good")

♠ Gubser gave the first criterion for good singularities: They should be limits of solutions with a regular horizon.

Gubser

• The second criterion amounts to having a well-defined spectral problem for fluctuations around the solution: The second order equations describing all fluctuations are Sturm-Liouville problems (no extra boundary conditions needed at the singularity).

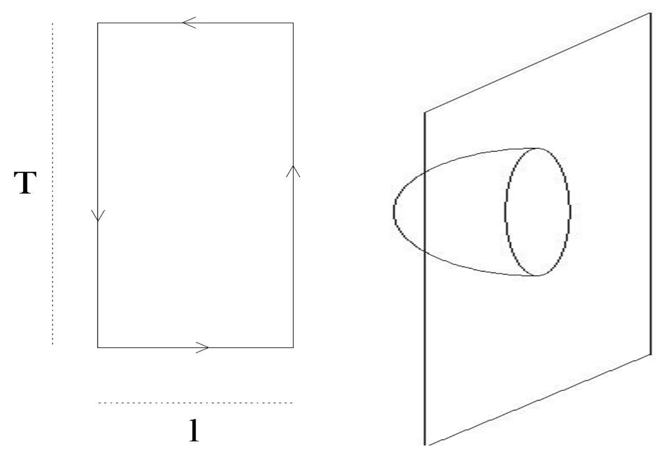
Gursoy+E.K.+Nitti

• The singularity is "repulsive" (like the Liouville wall). It has an overlap with the previous criterion. It involves the calculation of "Wilson loops"

Gursoy+E.K.+Nitti

• It is not known whether the list is complete. The 1st and 2-3rd criteria are non-overlapping.

Wilson-Loops and confinement



- Calculation of the static quark potential using the vev of the Wilson loop calculated via an F-string world-sheet.

Rey+Yee, Maldacena

$$T\ E(L) = S_{minimal}(X)$$

We calculate

$$L = 2\int_0^{r_0} dr \frac{1}{\sqrt{e^{4A_S(r)-4A_S(r_0)} - 1}}.$$

It diverges when e^{A_s} has a minimum (at $r = r_*$). Then

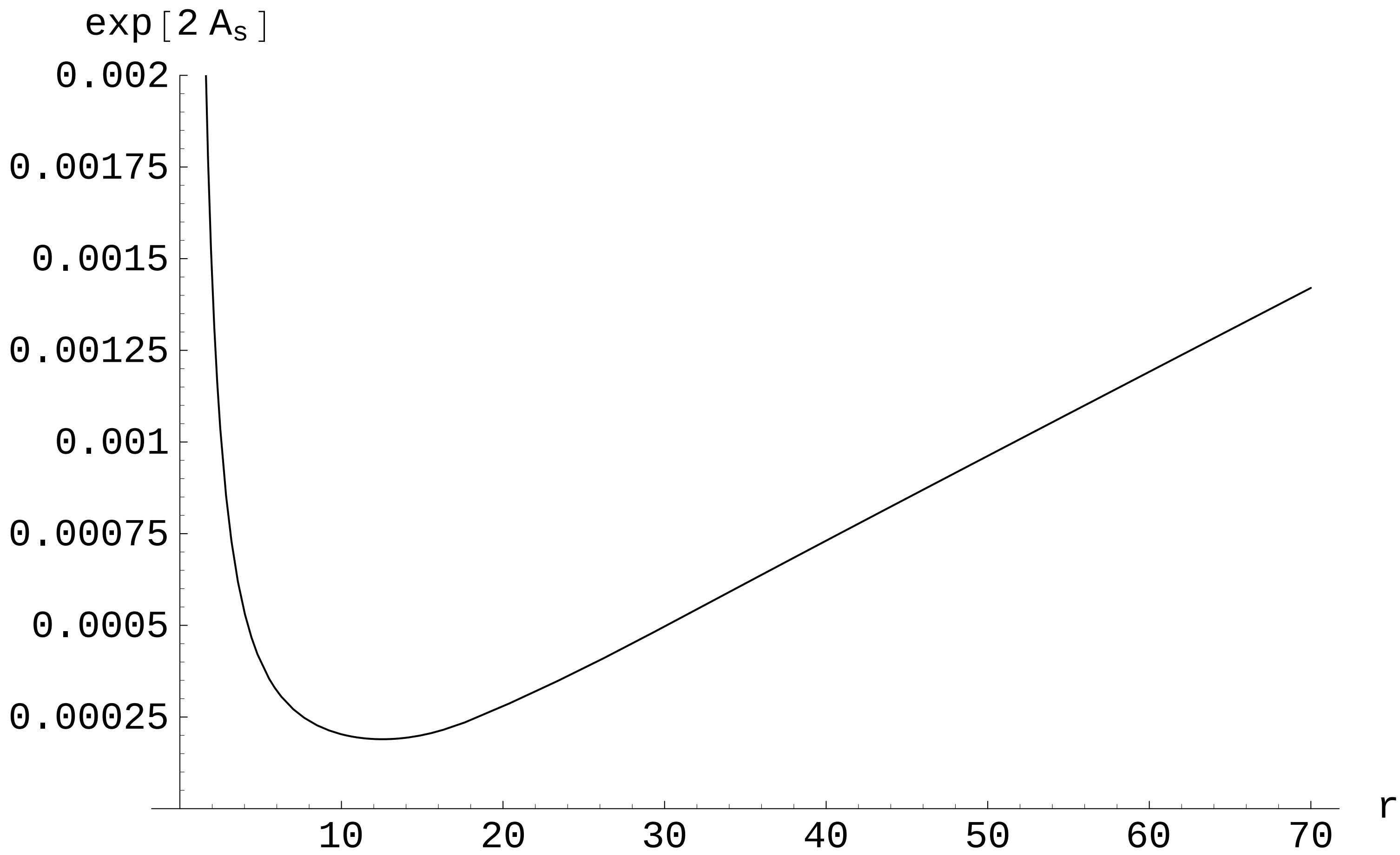
$$E(L) \sim T_f\ e^{2A_S(r_*)}\ L$$

- Confinement $\to$ $A_s(r_*)$ is finite. This is a more general condition that considered before as A_S `is not monotonic in general`. $A_S = A_E + \frac{2}{3}\Phi$

- Effective string tension

$$T_{\rm string} = T_f\ e^{2A_S(r_*)}$$

• In simple cases like AdS/QCD, Φ is constant, but r is bounded below.



The string frame scale factor in a background that confines non-trivially.

An assessment of IR asymptotics

$$V(\lambda) \sim V_0 \lambda^{2Q} \quad , \quad \lambda \equiv e^{\phi} \to \infty$$

- The solutions can be parameterized in terms of a fake superpotential

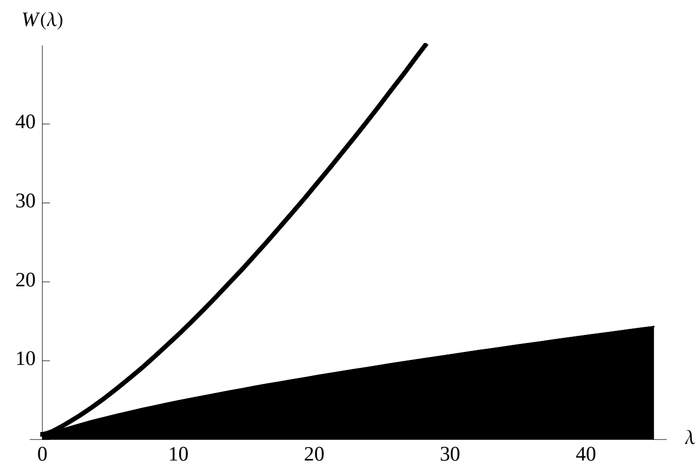
$$V = \frac{64}{27} W^2 - \frac{4}{3} \lambda^2 \, W'^2 \quad , \quad W \geq \frac{3}{8} \sqrt{3V}$$

The crucial parameter resides in the solution to the diff. equation above. There are three types of solutions for $W(\lambda)$:

Gursoy+E.K.+Mazzanti+Nitti

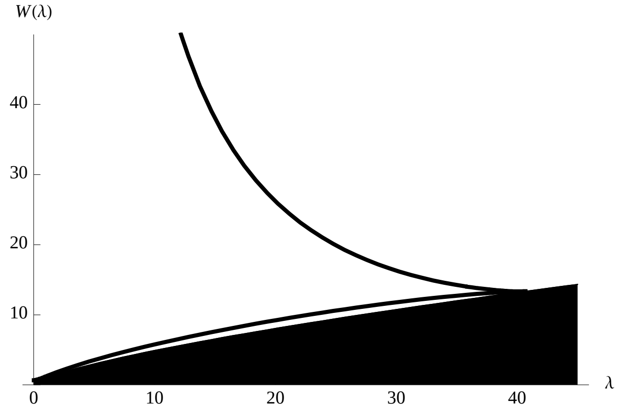
1. Generic Solutions (bad IR singularity)

$$W(\lambda) \sim \lambda^{\frac{4}{3}} \quad , \quad \lambda \to \infty$$



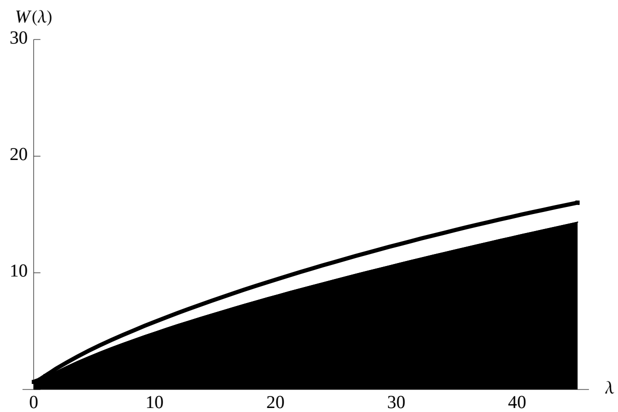
2. Bouncing Solutions (bad IR singularity)

$$W(\lambda) \sim \lambda^{-\frac{4}{3}} \quad , \quad \lambda \to \infty$$



3. The "special" solution.

$$W(\lambda) \sim W_\infty \lambda^Q \quad , \quad \lambda \to \infty \quad , \quad W_\infty = \sqrt{\frac{27V_0}{4(16-9Q^2)}}$$



Good+repulsive IR singularity if $Q < \frac{4\sqrt{2}}{3}$

- For $Q > \frac{4}{3}$ all solutions are of the bouncing type (therefore bad).

- There is another special asymptotics in the potential namely $Q = \frac{2}{3}$. Below $Q = \frac{2}{3}$ the spectrum changes to continuous without mass gap.

In that region a finer parametrization of asymptotics is necessary

$$V(\lambda) \sim V_0 \, \lambda^{\frac{4}{3}} \, (\log \lambda)^P$$

- For $P > 0$ there is a mass gap, discrete spectrum and confinement of charges. There is also a first order deconfining phase transition at finite temperature.

- For $P < 0$, the spectrum is continuous, without mas gap, and there is a transition at T=0 (as in N=4 sYM).

- At $P = 0$ we have the linear dilaton vacuum. The theory has a mass gap but continuous spectrum. The order of the deconfining transition depends on the subleading terms of the potential and can be of any order larger than two.

Gurdogan+Gursoy+E.K.

Comments on confining backgrounds

• For all confining backgrounds with $r_0 = \infty$, although the space-time is singular in the Einstein frame, the string frame geometry is asymptotically flat for large r. Therefore only λ grows indefinitely.

• String world-sheets do not probe the strong coupling region, at least classically. The string stays away from the strong coupling region.

• Therefore: singular confining backgrounds have generically the property that the singularity is *repulsive*, i.e. only highly excited states can probe it. This will also be reflected in the analysis of the particle spectrum (to be presented later)

• The confining backgrounds must also screen magnetic color charges. This can be checked by calculating 't Hooft loops using D_1 probes:

♠ All confining backgrounds with $r_0 = \infty$ and most at finite r_0 screen properly

♠ In particular "hard-wall" AdS/QCD confines also the magnetic quarks.

Selecting the IR asymptotics

The $Q = 4/3$, $0 \leq P < 1$ solutions have a singularity at $r = \infty$. They are compatible with

- Confinement (it happens non-trivially: a minimum in the string frame scale factor)
- Mass gap+discrete spectrum (except P=0)
- "good+repulsive" singularity
- $R \to 0$ justifying the original assumption. More precisely: the string frame metric becomes flat at the IR .

♠ It is interesting that the lower endpoint: P=0 corresponds to linear dilaton and flat space (string frame). It is confining with a mass gap but continuous spectrum.

- For linear asymptotic trajectories for fluctuations (glueballs) we must choose $P = 1/2$

$$V(\lambda) =\sim \lambda^{\frac{4}{3}}\sqrt{\log\lambda} + \text{subleading} \qquad \text{as} \qquad \lambda \to \infty$$

Particle Spectra: generalities

- Linearized equation:

$$\ddot{\xi} + 2\dot{B}\dot{\xi} + \Box_4 \xi = 0 \quad , \quad \xi(r,x) = \xi(r)\xi^{(4)}(x), \qquad \Box \xi^{(4)}(x) = m^2 \xi^{(4)}(x)$$

- Can be mapped to Schrodinger problem

$$-\frac{d^2}{dr^2}\psi + V(r)\psi = m^2\psi \quad , \quad V(r) = \frac{d^2 B}{dr^2} + \left(\frac{dB}{dr}\right)^2 \quad , \quad \xi(r) = e^{-B(r)}\psi(r)$$

- Mass gap and discrete spectrum visible from the asymptotics of the potential.

- Large n asymptotics of masses obtained from WKB

$$n\pi = \int_{r_1}^{r_2} \sqrt{m^2 - V(r)}\; dr$$

- Spectrum depends only on initial condition for λ ($\sim \Lambda_{QCD}$).

- scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log\frac{\beta(\lambda)^2}{9\lambda^2}$$

- tensor glueballs

$$B(r) = \frac{3}{2}A(r)$$

- pseudo-scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log Z(\lambda)$$

- Universality of asymptotics

$$\frac{m^2_{n\to\infty}(0^{++})}{m^2_{n\to\infty}(2^{++})} \to 1 \quad , \quad \frac{m^2_{n\to\infty}(0^{+-})}{m^2_{n\to\infty}(0^{++})} = \frac{1}{4}(d-2)^2$$

predicts $d = 4$ via

$$\frac{m^2}{2\pi\sigma_a} = 2n + J + c,$$

Summary

• We argued that an Einstein dilaton system with a potential can capture some important properties of YM: asymptotic freedom in the UV and confinement in the IR

$$S \sim \int \left[R - \frac{4}{3}(\partial\phi)^2 + V(\phi) \right]$$

• The potential is regular in the UV

$$V \to \frac{12}{\ell^2}\left[1 + c_1\lambda + c_2\lambda^2 + \cdots\right]$$

• In the IR it should behave as

$$V \sim \lambda^{\frac{4}{3}}(\log\lambda)^P$$

for linear trajectories $P = 1/2$.

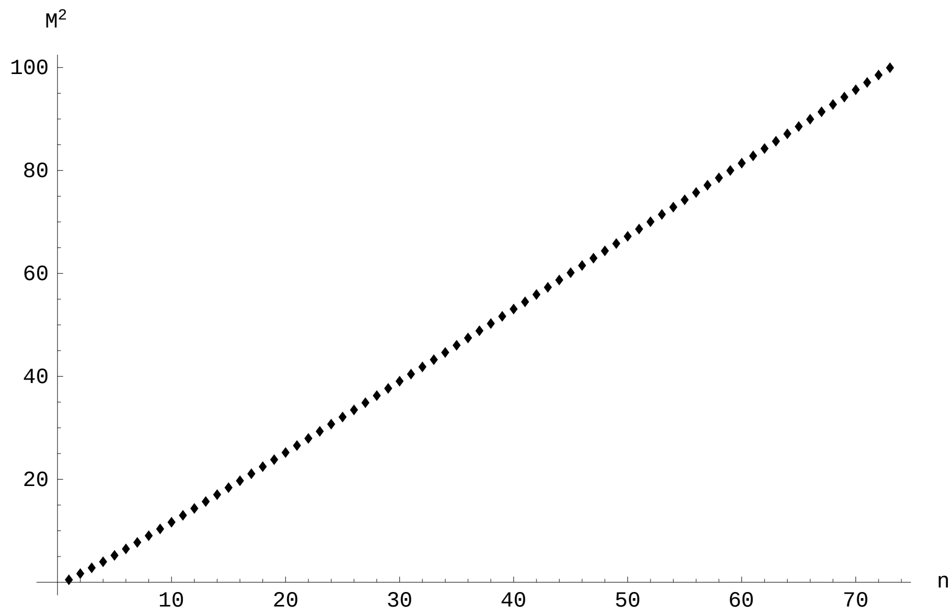
• We can solve the equations of motion with $\lambda \to 0$ in the UV.

• The solutions have only one parameter: Λ_{QCD}

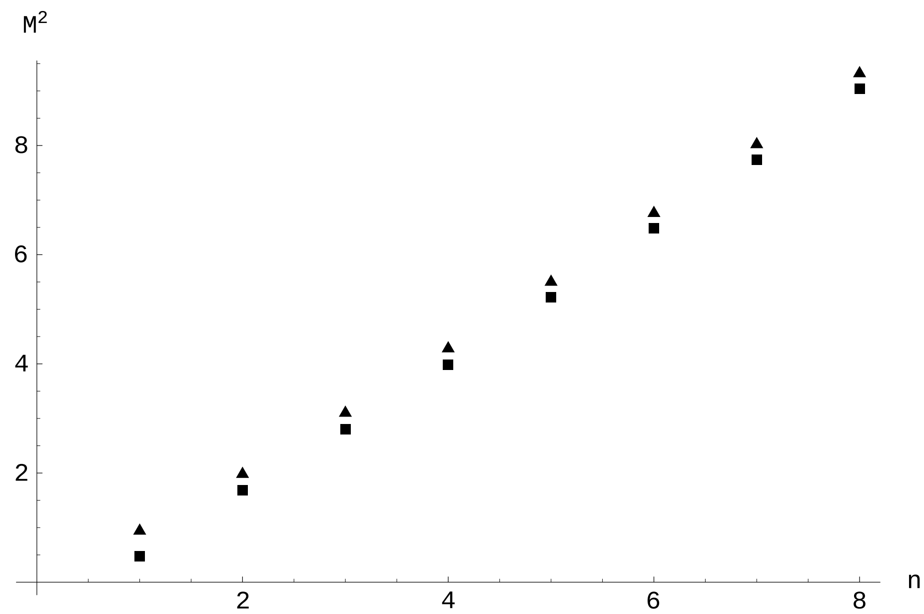
• The intermediate behavior of the potential is not fixed (phenomenological parameters).

• The axion solution is non-trivial, non-perturbative and it asymptotes to zero in the IR.

Linearity of the glueball spectrum



(a)

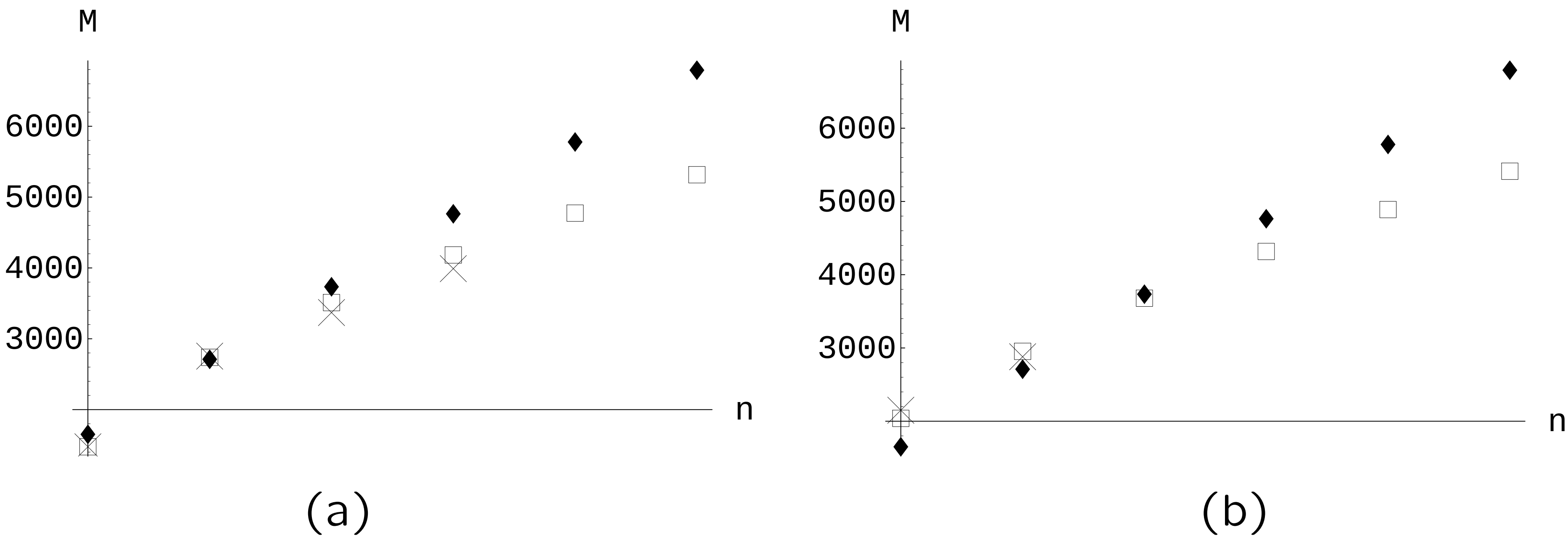


(b)

(a) Linear pattern in the spectrum for the first 40 0^{++} glueball states. M^2 is shown units of $0.015\ell^{-2}$.

(b) The first 8 0^{++} (squares) and the 2^{++} (triangles) glueballs. These spectra are obtained in the background I with $b_0 = 4.2, \lambda_0 = 0.05$.

Comparison with lattice data (Meyer)



Comparison of glueball spectra from our model with $b_0 = 4.2, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. I (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. I.

The fit to glueball lattice data

J^{PC}	Ref I (MeV)	Our model (MeV)	Mismatch	$N_c \to \infty$	Mismat
0^{++}	**1475 (4%)**	**1475**	0	**1475**	0
2^{++}	2150 (5%)	2055	4%	2153 (10%)	5%
0^{-+}	**2250 (4%)**	**2243**	0		
0^{++*}	**2755 (4%)**	**2753**	0	2814 (12%)	2%
2^{++*}	2880 (5%)	2991	4%		
0^{-+*}	3370 (4%)	3288	2%		
0^{++**}	3370 (4%)	3561	5%		
0^{++***}	3990 (5%)	4253	6%		

Comparison between the glueball spectra in Ref. I and in our model. The states we use as input in our fit are marked in red. The parenthesis in the lattice data indicate the percent accuracy.

Finite temperature

The theory at finite temperature can be described by:
(1) The "thermal vacuum solution". This is the zero-temperature solution we described so far with time periodically identified with period β.

(2) "black-hole" solutions

$$ds^2 = b(r)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx^i\right], \qquad \lambda = \lambda(r)$$

♠ We need VERY UNUSUAL boundary conditions: The dilaton (scalar) is diverging at the boundary$\phi \to -\infty$, so that $\lambda \sim e^{\phi} \to \frac{1}{\log r} \to 0$

♠ The boundary AdS is a very stiff minimum of the potential.

• Such type of solutions have not been analyzed so far in the literature.

• BH solutions where the scale factor is the same as at T=0 exist ONLY for V =constant, or $V \sim e^{a\Phi}$.

General phase structure

- For a general potential (with no minimum) the following can be shown :

i. *There exists a phase transition at finite $T = T_c$, if and only if the zero-T theory confines.*

ii. *This transition is of the first order for all of the confining geometries, with a single exception described in iii:*

iii. *In the limit confining geometry $b_0(r) \to e^{-Cr}$, $\lambda_0 \to e^{\frac{3}{2}Cr}$, (as $r \to \infty$), the phase transition is of the second or higher order and happens at $T = 3C/4\pi$.*
This is the linear dilaton vacuum solution in the IR.

iv. *All of the non-confining geometries at zero T are always in the black hole phase at finite T. They exhibit a second order phase transition at $T = 0^+$.*

Finite-T Confining Theories

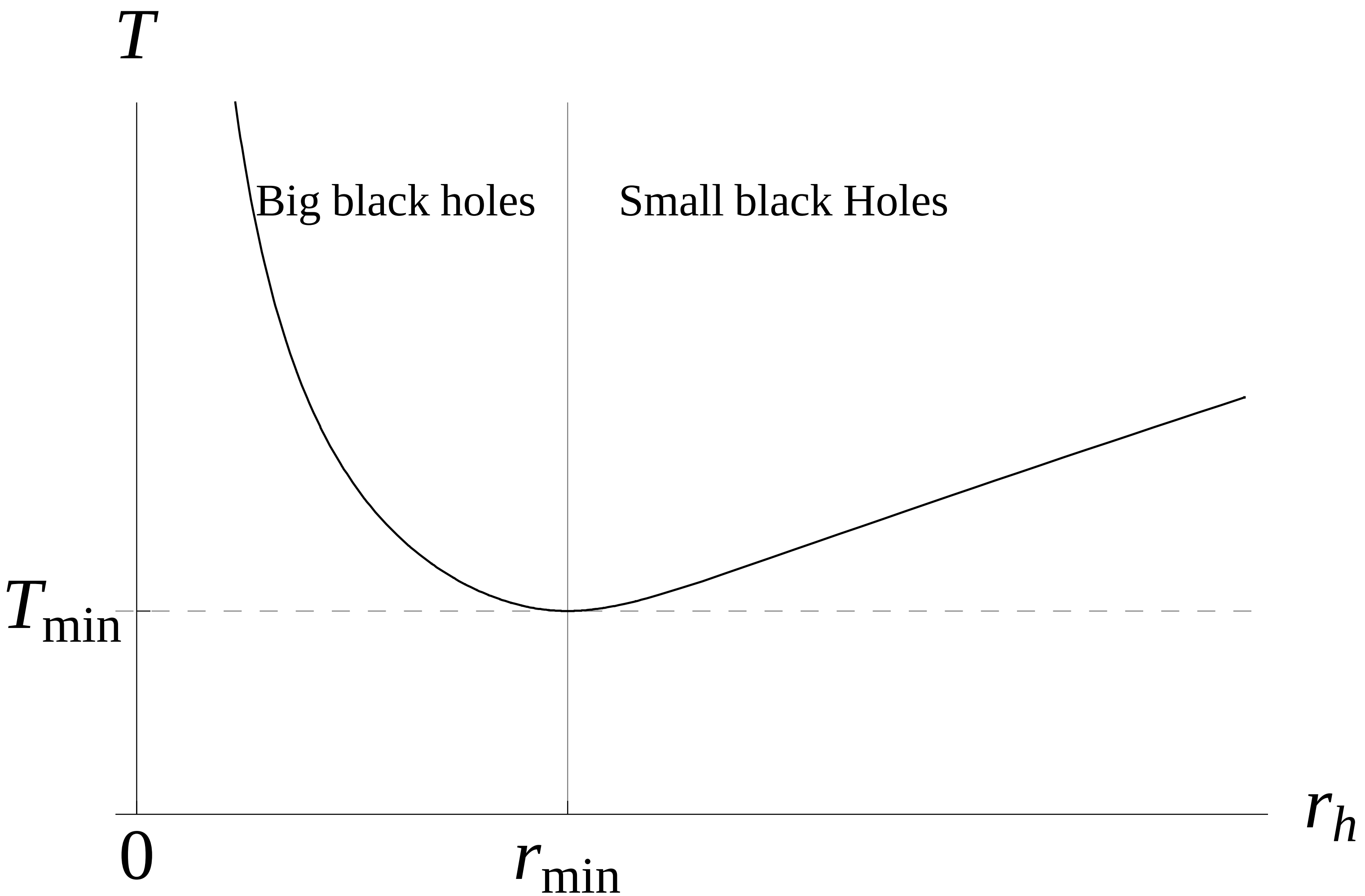
• There is a minimal temperature T_{min} for the existence of Black-hole solutions

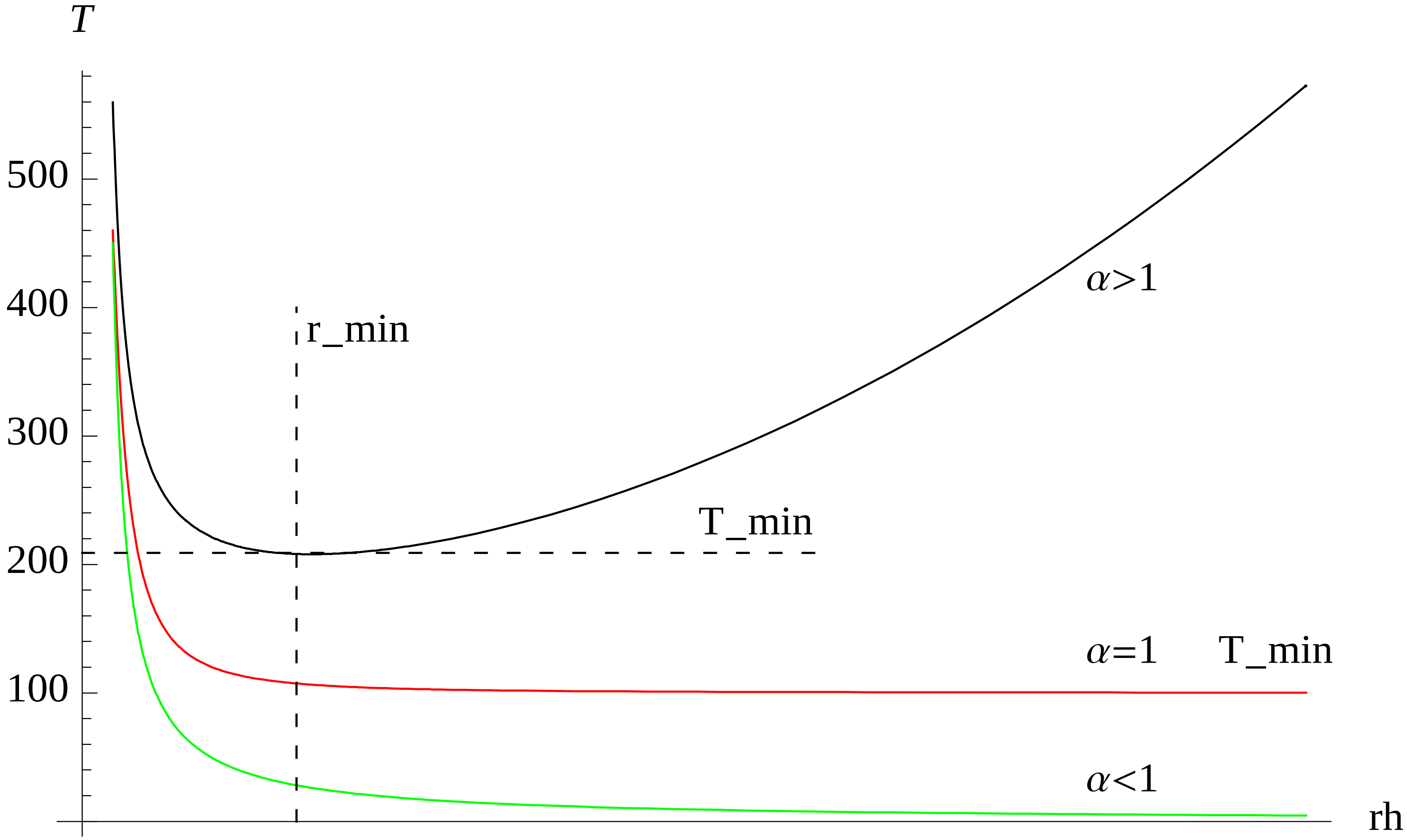
• When $T < T_{\mathrm{min}}$ only the "thermal vacuum solution" exists: it describes the confined phase at small temperatures.

• For $T > T_{\mathrm{min}}$ there are two black-hole solutions with the same temperature but different horizon positions. One is a "large" BH the other is "small".

• When $T > T_{\mathrm{min}}$ three competing solutions exist. The large BH has the lowest free energy for $T > T_c > T_{min}$. It describes the deconfined "Gluon-Glass" phase.

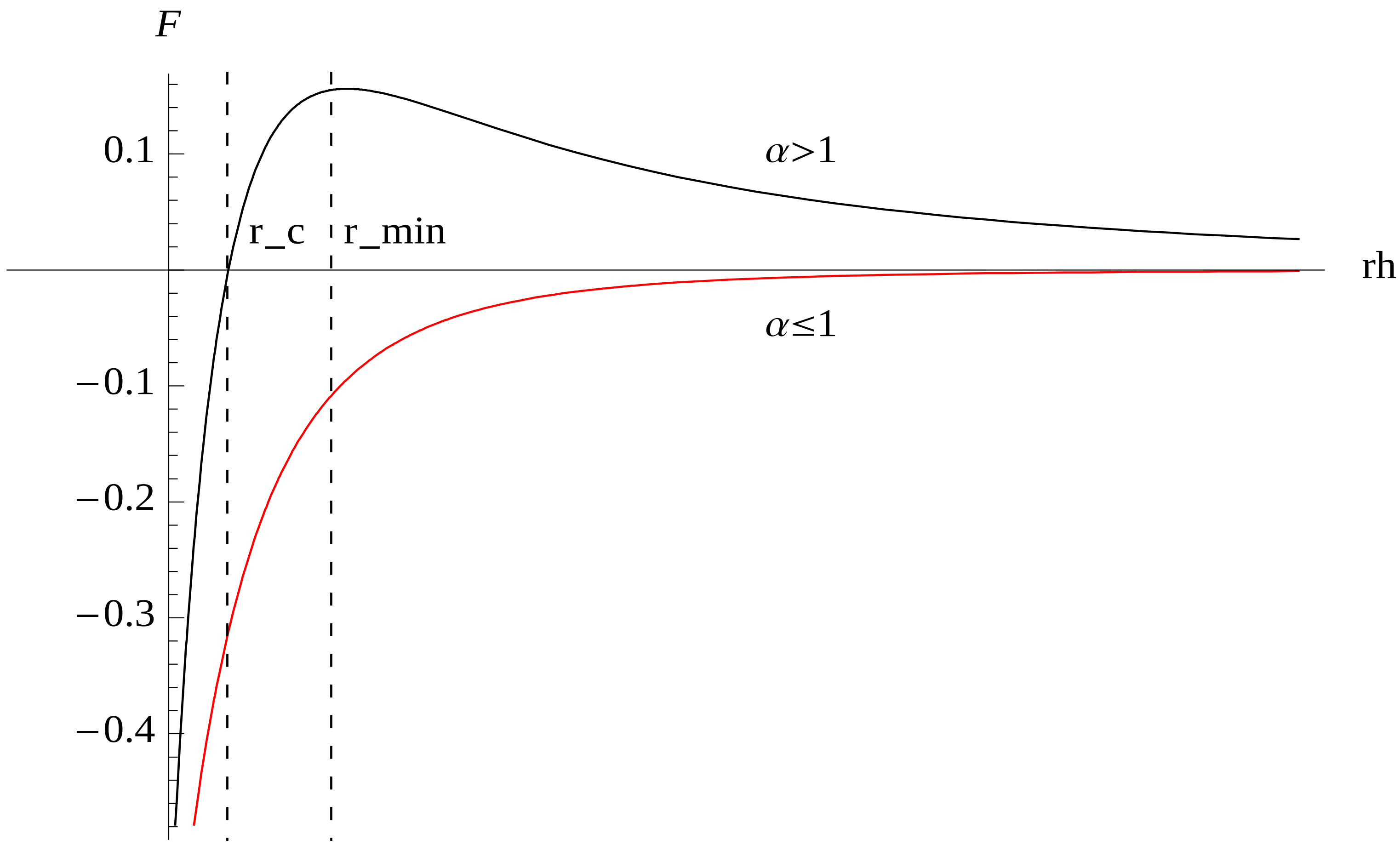
Temperature versus horizon position





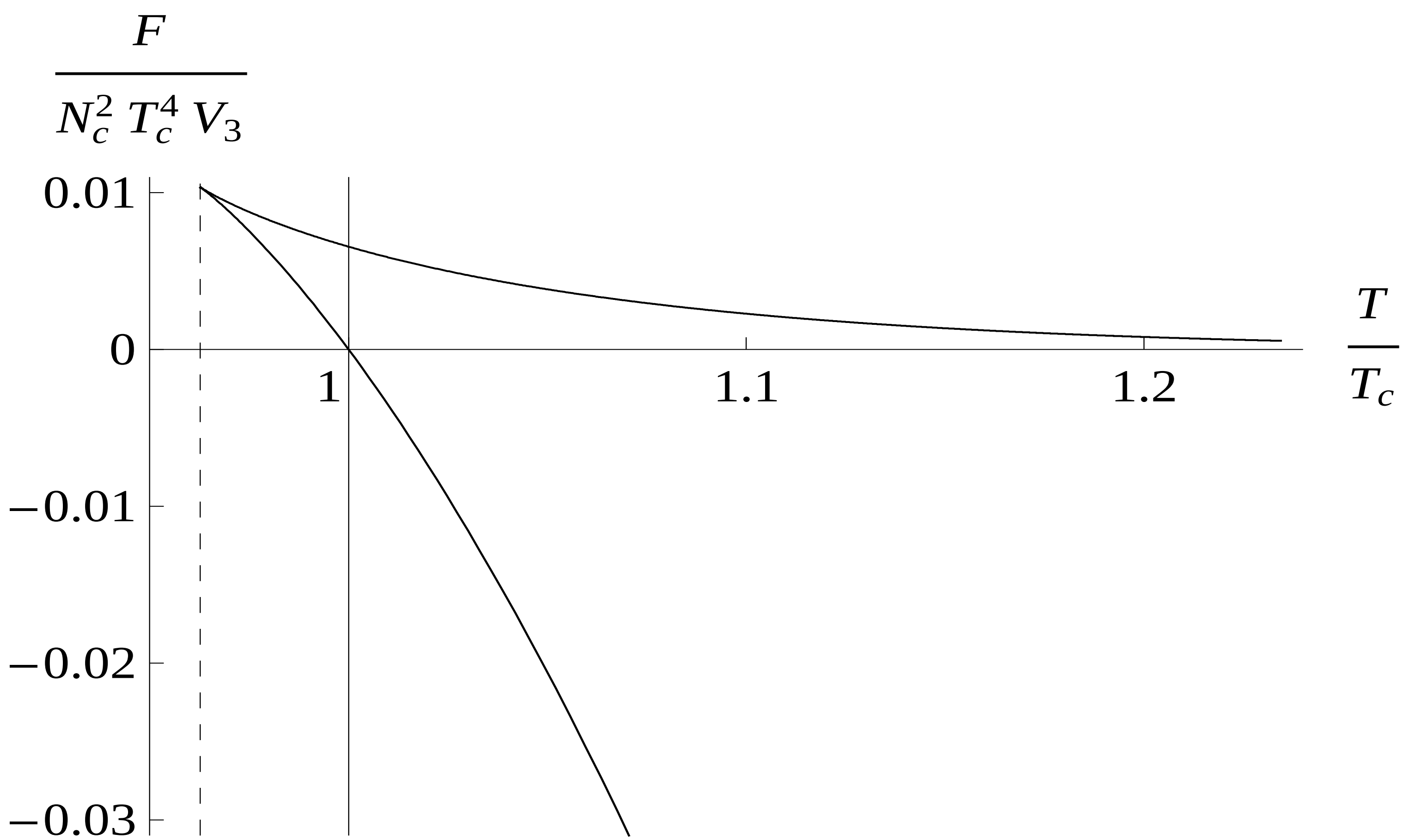
We plot the relation $T(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is only one branch of black-hole solutions.

Free energy versus horizon position



We plot the relation $\mathcal{F}(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is no first order phase transition .

The transition in the free energy



The free energy

• The free energy is calculated from the action as a boundary term for both the black-holes and the thermal vacuum solution. They are all UV divergent but their differences are finite.

$$\frac{\mathcal{F}}{M_p^3 V_3} = 12\mathcal{G}(T) - T\ S(T)$$

• $\mathcal{G}$ is the temperature-depended gluon condensate $\langle Tr[F^2]\rangle_T - \langle Tr[F^2]\rangle_{T=0}$ defined as

$$\lim_{r\to 0} \quad \lambda_T(r) - \lambda_{T=0}(r) = \mathcal{G}(T)\ r^4 + \cdots$$

• It is $\mathcal{G}$ the breaks conformal invariance essentially and leads to a non-trivial deconfining transition (as $S > 0$ always)

• The axion solution must be constant above the phase transition (black-hole). This is the only regular solution. (the would be normalizable solution diverges at the BH horizon). Therefore $\langle F \wedge F\rangle$ vanishes in agreement with indications from lattice data.

The conformal anomaly in flat space

- In YM we have the following anomaly equation in flat space:

$$T^\mu_\mu = \frac{\beta(\lambda_t)}{4\lambda_t^2} Tr[F^2],$$

- Defining the pressure p and energy density ρ,

$$p = -\frac{\mathcal{F}}{V_3}, \qquad \rho = \frac{\mathcal{F} + TS}{V_3},$$

the trace is

$$\langle T^\mu_\mu \rangle_R = \rho - 3p = 60 M_p^3 N_c^2 \mathcal{G}(T) = \frac{\beta(\lambda_t)}{4\lambda_t^2} (\langle Tr[F^2] \rangle_T - \langle Tr[F^2] \rangle_o),$$

- The left hand side is the trace of the renormalized thermal stress tensor, $\langle T^\mu_\mu \rangle_R = \langle T^\mu_\mu \rangle - \langle T^\mu_\mu \rangle_o$, and it is proportional to $\mathcal{G} \sim \langle Tr[F^2] \rangle$,

Parameters

- We have 3 initial conditions in the system of graviton-dilaton equations:

♠ One is fixed by picking the branch that corresponds asymptotically to $\lambda \sim \frac{1}{\log(r\Lambda)}$

♠ The other fixes $\Lambda \to \Lambda_{QCD}$.

♠ The third is a gauge artifact as it corresponds to a choice of the origin of the radial coordinate.

- We parameterize the potential as

$$V(\lambda) = \frac{12}{\ell^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\left[\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)\right]^{1/2}\right\},$$

- We fix the one and two loop β-function coefficients:

$$V_0 = \frac{8}{9}b_0 \quad , \quad V_2 = b_0^4\left(\frac{23 + 36b_1/b_0^2}{81V_1^2}\right)^2, \quad \frac{b_1}{b_0^2} = \frac{51}{121}.$$

and remain with two leftover arbitrary (phenomenological) coefficients.

• We also have the Planck scale M_p
Asking for correct $T \to \infty$ thermodynamics (free gas) fixes

$$(M_p \ell)^3 = \frac{1}{45\pi^2} \quad , \quad M_{\text{physical}} = M_p N_c^{\frac{2}{3}} = \left(\frac{8}{45\pi^2 \ell^3}\right)^{\frac{1}{3}} \simeq 4.6 \text{ GeV}$$

• The fundamental string scale. It can be fixed by comparing with lattice string tension

$$\sigma = \frac{b^2(r_*)\lambda^{4/3}(r_*)}{2\pi \ell_s^2},$$

$\ell/\ell_s \sim \mathcal{O}(1)$.

• ℓ is not a parameter for bulk calculations due to a special "scaling" pseudosymmetry:

$$e^\phi \to \kappa\, e^\phi \quad , \quad g_{\mu\nu} \to \kappa^{\frac{4}{3}}\, g_{\mu\nu} \quad , \quad \ell \to \kappa^{\frac{2}{3}}\, \ell \quad , \quad \ell_s \to \kappa^{\frac{2}{3}}\, \ell_s \quad , \quad V(e^\phi) \to V(\kappa\, e^\phi)$$

• It is a parameter when using the Nambu-Goto action.

Fit and comparison

	HQCD	lattice $N_c = 3$	lattice $N_c \to \infty$	Parameter
$[p/(N_c^2 T^4)]_{T=2T_c}$	**1.2**	**1.2**	-	$V1 = 14$
$L_h/(N_c^2 T_c^4)$	**0.31**	0.28 (Karsch)	**0.31** (Teper+Lucini)	$V3 = 170$
$[p/(N_c^2 T^4)]_{T\to+\infty}$	$\pi^2/45$	$\pi^2/45$	$\pi^2/45$	$M_p \ell = [45\pi^2]^{-1/3}$
$m_{0^{++}}/\sqrt{\sigma}$	**3.37**	3.56 (Chen)	**3.37** (Teper+Lucini)	$\ell_s/\ell = 0.92$
$m_{0^{-+}}/m_{0^{++}}$	**1.49**	**1.49** (Chen)	-	$c_a = 0.26$
χ	$(191 MeV)^4$	$(191 MeV)^4$ (DelDebbio)	-	$Z_0 = 133$
$T_c/m_{0^{++}}$	0.167	-	0.177(7)	
$m_{0^{*++}}/m_{0^{++}}$	1.61	1.56(11)	1.90(17)	
$m_{2^{++}}/m_{0^{++}}$	1.36	1.40(4)	1.46(11)	
$m_{0^{*-+}}/m_{0^{++}}$	2.10	2.12(10)	-	

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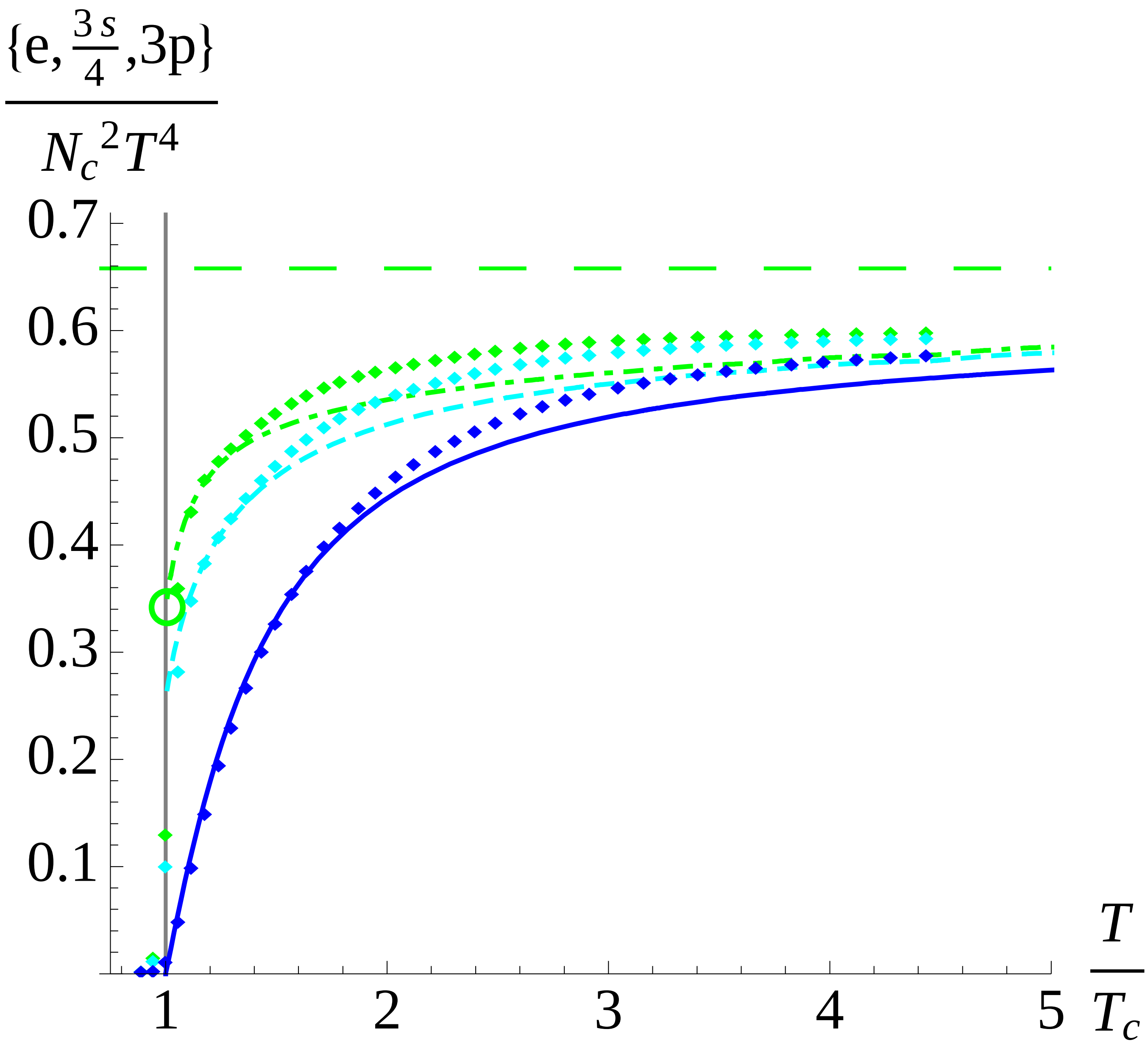
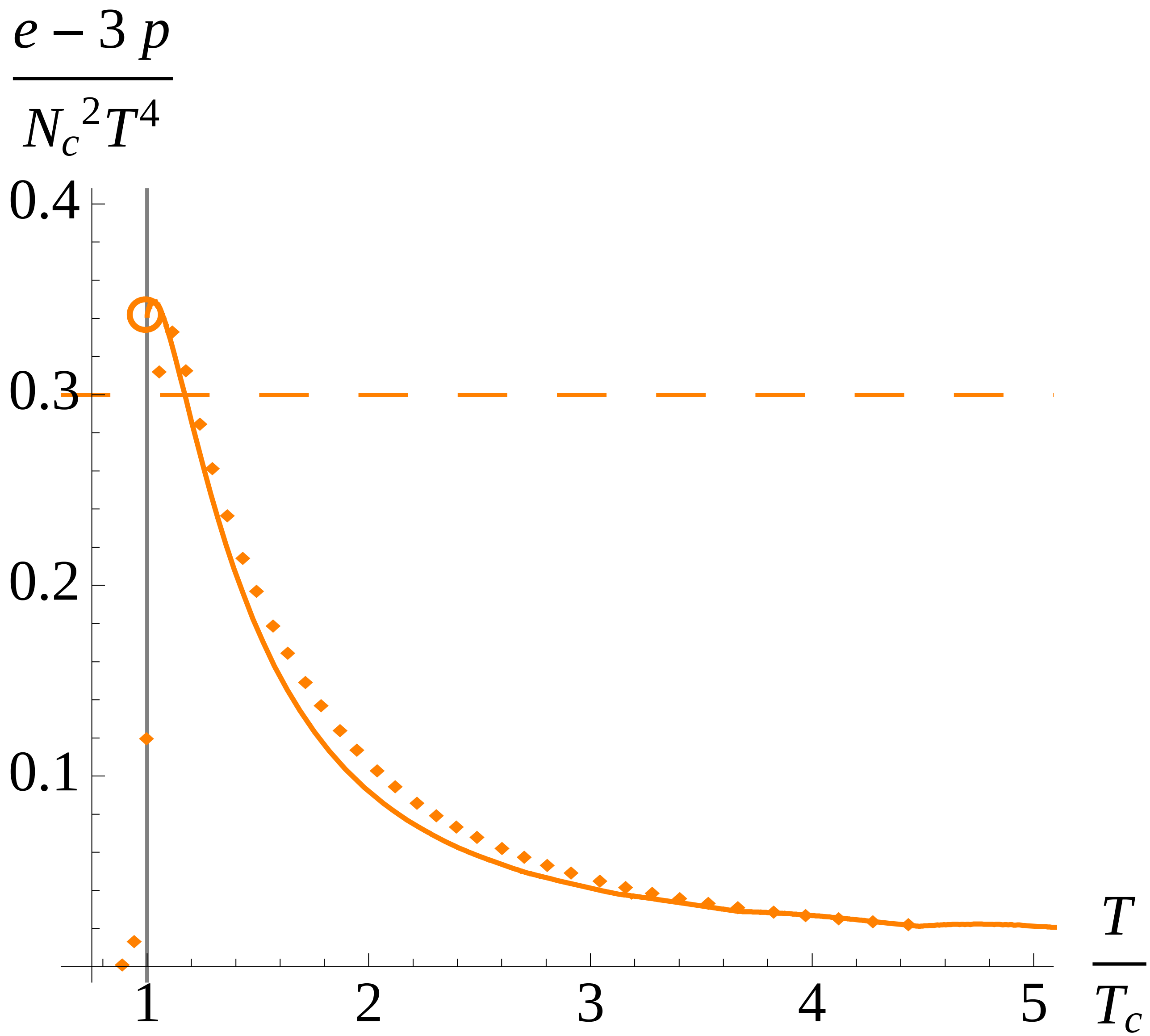
Thermodynamic variables



Equation of state



The presure from the lattice at different N

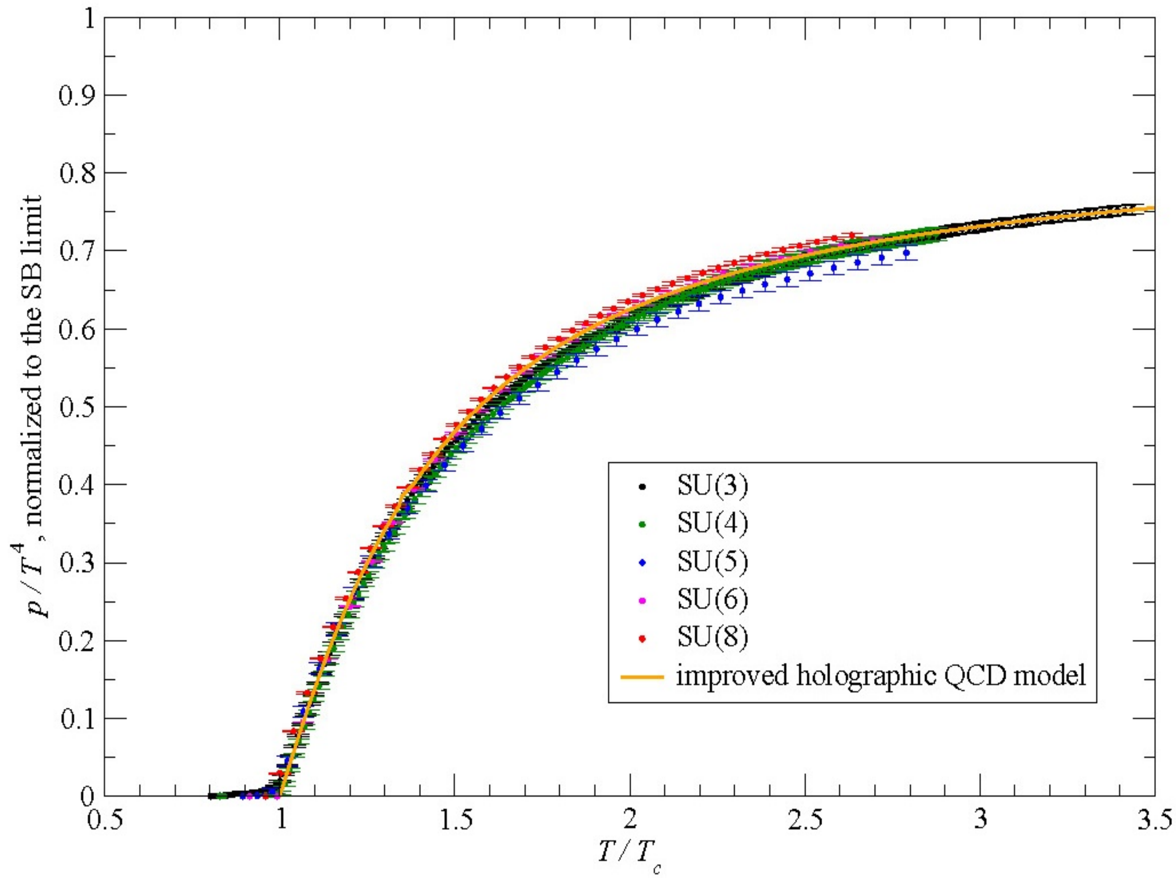


Figure 1: (Color online) The dimensionless ratio p/T^4, normalized to the lattice SB limit $\pi^2(N^2-1)R_I(N_t)/45$, *versus* T/T_c, as obtained from simulations of SU(N) lattice gauge theories on $N_t = 5$ lattices. Errorbars denote statistical uncertainties only. The results corresponding to different gauge groups are denoted by different colors, according to the legend. The yellow solid line denotes the prediction from the improved holographic QCD model from ref. [75] (with a trivial, parameter-free rescaling to our normalization).

Marco Panero arXiv: 0907.3719

The entropy from the lattice at different N

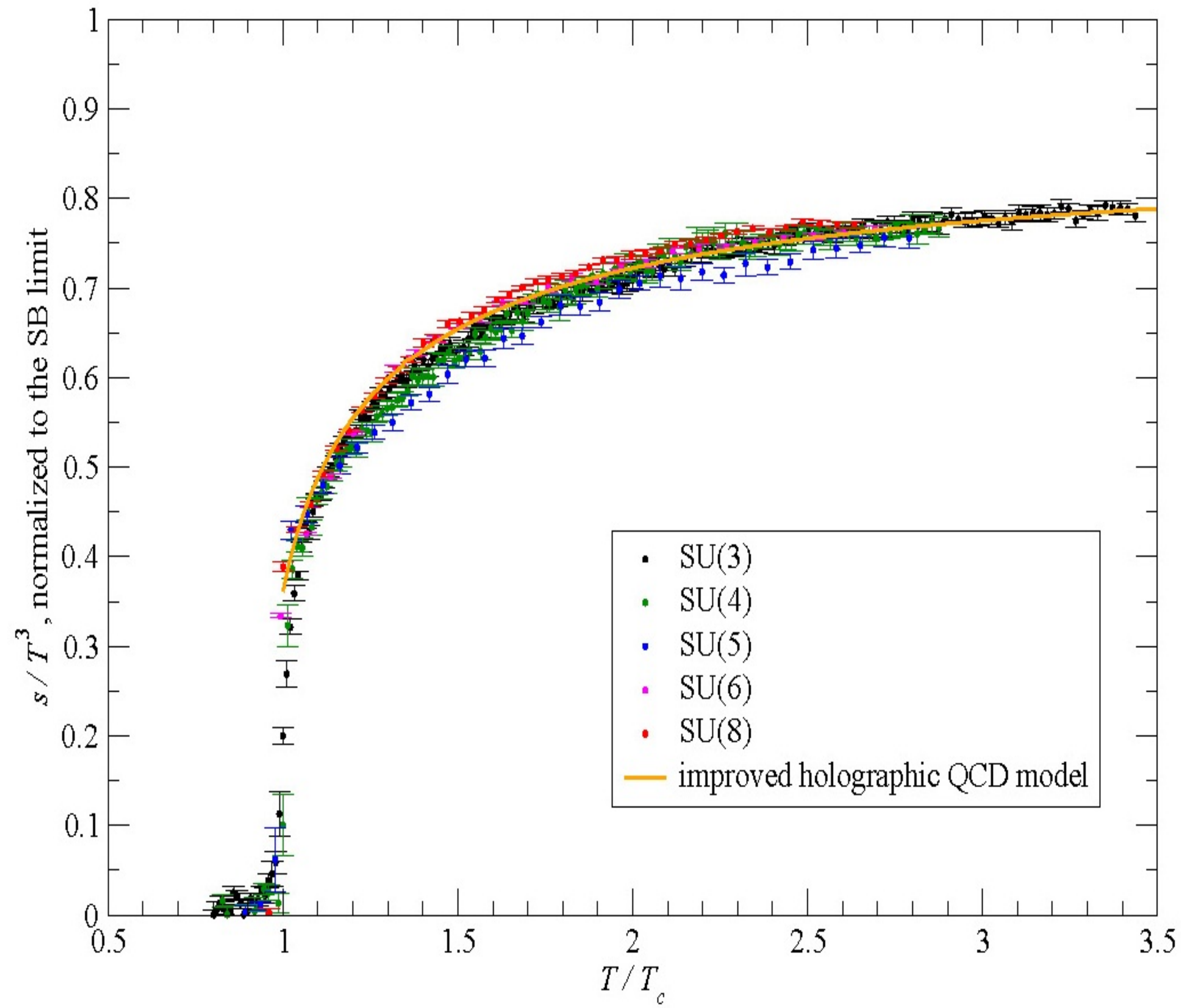


Figure 4: (Color online) Same as in fig. 1, but for the s/T^3 ratio, normalized to the SB limit.

Marco Panero arXiv: 0907.3719

The trace from the lattice at different N

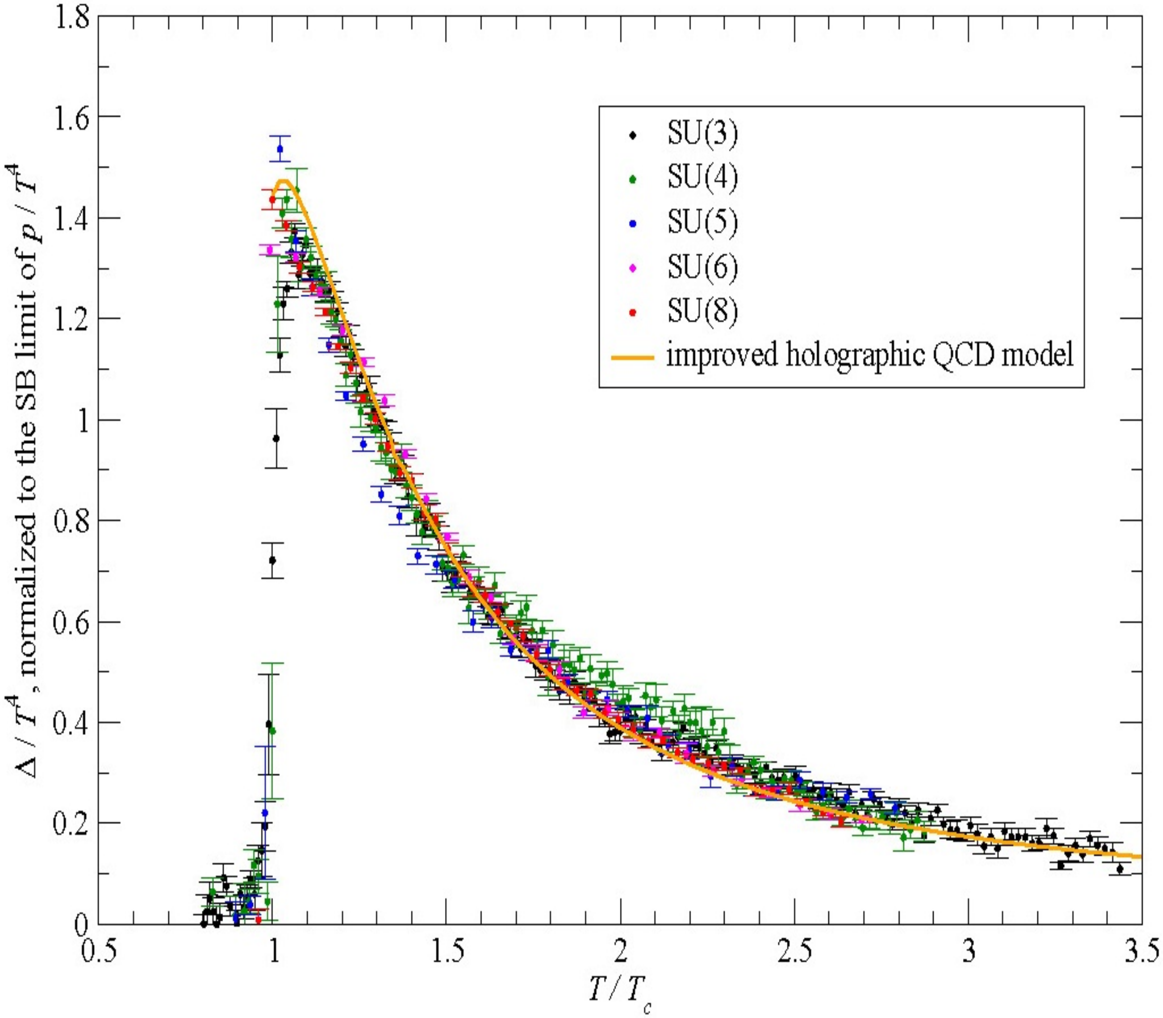
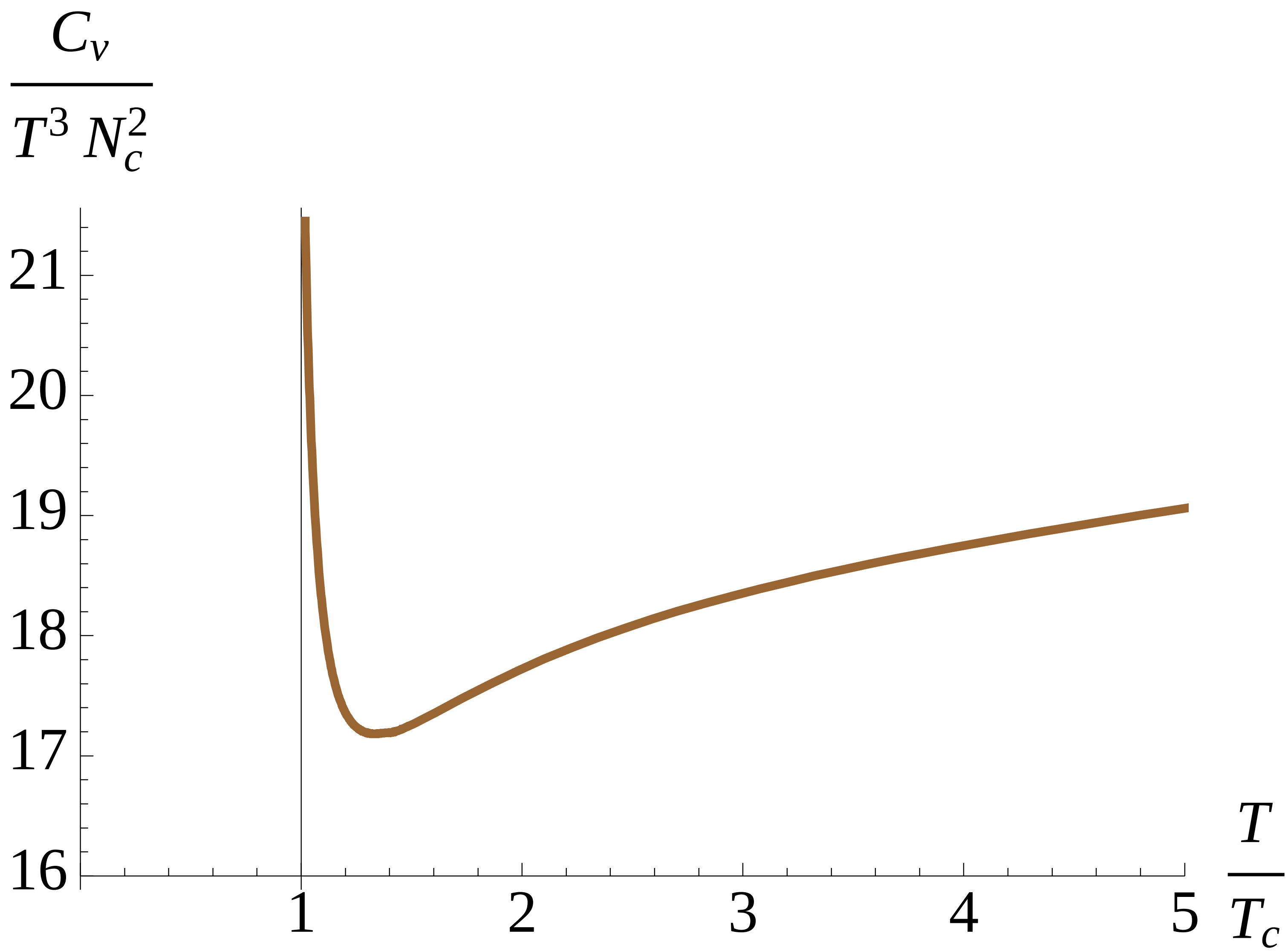


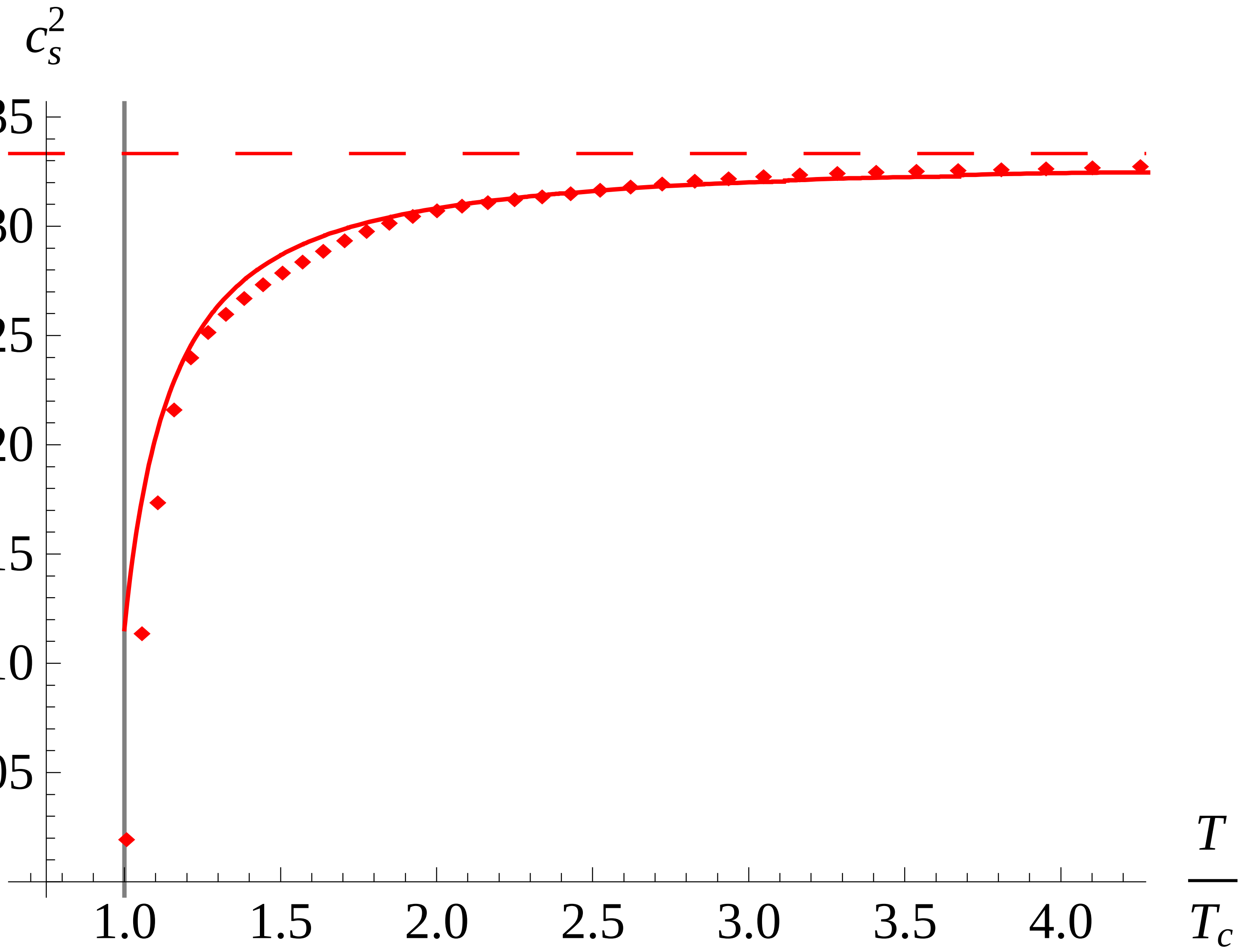
Figure 2: (Color online) Same as in fig. 1, but for the Δ/T^4 ratio, normalized to the SB limit of p/T^4.

Marco Panero arXiv: 0907.3719

The specific heat



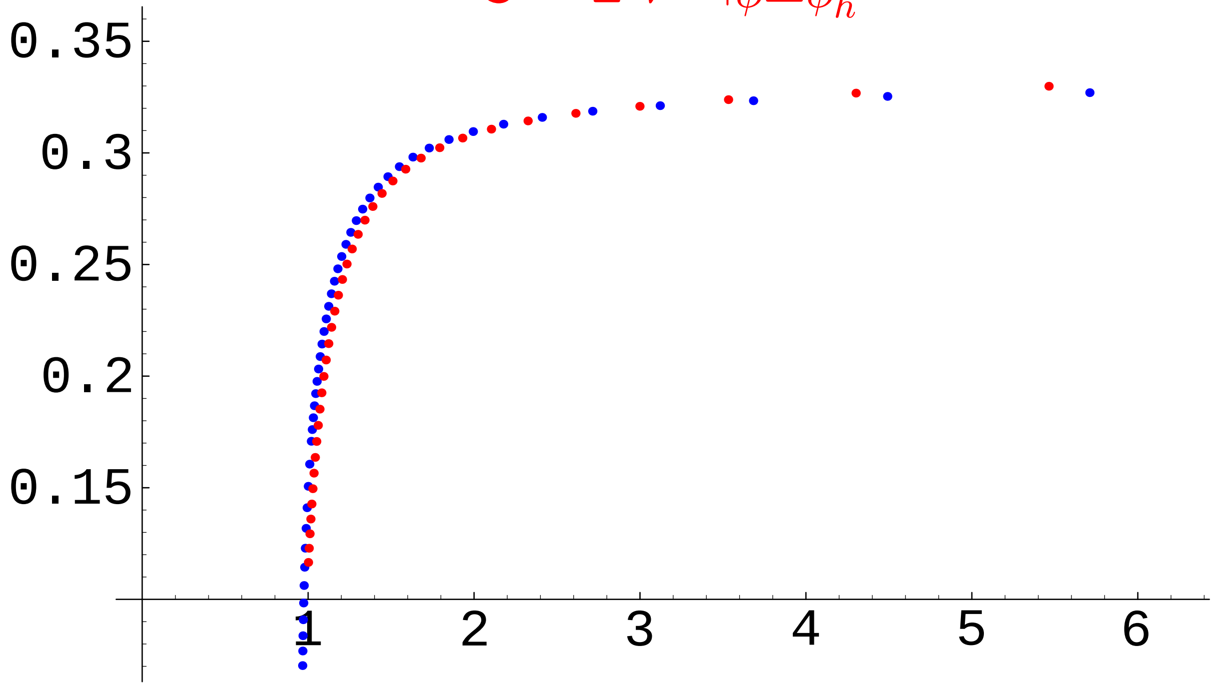
The speed of sound



Comparing to Gubser+Nelore's formula

• Gubser+Nelore proposed the following approximate formula for the speed of sound

$$c_s^2 \simeq \frac{1}{3} - \frac{1}{2}\frac{V'^2}{V^2}\bigg|_{\phi=\phi_h}$$



Gursoy (unpublished) 2009

• Red curve=numerical calculation, Blue curve=Gubser's adiabatic/approxima formula.

Adding flavor

• To add N_f quarks q_L^I and antiquarks $q_R^{\bar{I}}$ we must add (in 5d) space-filling N_f D_4 and N_f $\bar{D}_4$ branes.
(tadpole cancellation=gauge anomaly cancellation)

• The q_L^I should be the "zero modes" of the $D_3 - D_4$ strings while $q_R^{\bar{I}}$ are the "zero modes" of the $D_3 - \bar{D}_4$

• The low-lying fields on the D_4 branes ($D_4 - D_4$ strings) are U$(N_f)_L$ gauge fields A_μ^L. The low-lying fields on the $\bar{D}_4$ branes ($\bar{D}_4 - \bar{D}_4$ strings) are U$(N_f)_R$ gauge fields A_μ^R. They are dual to the J_L^μ and J_μ^R

$$\delta S_A \sim \bar{q}_L^I \ \gamma^\mu \ (A_\mu^L)^{IJ} \ q_L^J + \bar{q}_R^{\bar{I}} \ \gamma^\mu \ (A_\mu^R)^{\bar{I}\bar{J}} \ q_R^{\bar{J}} = Tr[J_L^\mu \ A_\mu^L + J_R^\mu \ A_\mu^R]$$

• There are also the low lying fields of the ($D_4 - \bar{D}_4$ strings), essentially the string-theory "tachyon" $T_{I\bar{J}}$ transforming as $(N_f, \bar{N}_f)$ under the chiral symmetry $U(N_f)_L \times U(N_f)_R$. It is dual to the quark mass terms

$$\delta S_T \sim \bar{q}_L^I \ T_{I\bar{J}} \ q_R^{\bar{J}} + \text{complex} \quad \text{congugate}$$

• The interactions on the flavor branes are weak, so that $A_\mu^{L,R}, T$ are as sources for the quarks.

• Integrating out the quarks, generates an effective action $S_{flavor}(A_\mu^{L,R}, T)$, so that $A_\mu^{L,R}, T$ can be thought as effective $q\bar{q}$ composites, that is : mesons

• On the string theory side: integrating out $D_3 - D_4$ and $D_3 - \bar{D}_4$ strings gives rise to the DBI action for the $D_4 - \bar{D}_4$ branes in the D_3 background:

$$S_{flavor}(A_\mu^{L,R}, T) \quad \longleftrightarrow \quad S_{DBI}(A_\mu^{L,R}, T) \quad \text{holographically}$$

• In the "vacuum" only T can have a non-trivial profile: $T^{I\bar{J}}(r)$. Near the AdS_5 boundary ($r \to 0$)

$$T^{I\bar{J}}(r) = M_{I\bar{J}}\, r + \cdots + \langle \bar{q}_L^I \, q_R^{\bar{J}} \rangle r^3 + \cdots$$

Casero+Kiritsis+Paredes

• A typical solution is T vanishing in the UV and $T \to \infty$ in the IR. At the point $r = r_*$ where $T = \infty$, the D_4 and $\bar{D}_4$ branes "fuse". The true vacuum is a brane that enters folds on itself and goes back to the boundary. A non-zero T breaks chiral symmetry.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• T=0 is always a solution. However it is excluded from the absence of IR boundary for the flavor branes: Holographic Coleman-Witten theorem.

• Fluctuations around the T solution for $T, A_\mu^{L,R}$ give the spectra (and interactions) of various meson trajectories.

• A GOR relation is satisfied (for an asymptotic AdS$_5$ space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\,\langle\bar{q}q\rangle \quad , \quad m_q \to 0$$

The tachyon DBI action

- The flavor action is the $D_4 - \bar{D}_4$ action: $S[T, A^L, A^R] = S_{DBI} + S_{WZ}$

$$S_{DBI} = \int drd^4x \, \frac{N_c}{\lambda} \, \mathbf{Str} \left[V(T) \left(\sqrt{-\det \left(g_{\mu\nu} + D_{\{\mu} T^\dagger D_{\nu\}} T + F^L_{\mu\nu} \right)} + \right.\right.$$

$$\left.\left. + \sqrt{-\det \left(g_{\mu\nu} + D_{\{\mu} T^\dagger D_{\nu\}} T + F^R_{\mu\nu} \right)} \right) \right]$$

$$D_\mu T \equiv \partial_\mu T - iTA^L_\mu + iA^R_\mu T \quad , \quad D_\mu T^\dagger \equiv \partial_\mu T^\dagger - iA^L_\mu T^\dagger + iT^\dagger A^R_\mu$$

transforming covariantly under flavor gauge transformations

$$T \to V_R T V_L^\dagger \quad , \quad A^L \to V_L (A^L - iV_L^\dagger dV_L) V_L^\dagger \quad , \quad A^R \to V_R (A^R - iV_R^\dagger dV_R) V_R^\dagger$$

- For the vacuum structure and spectrum $Str = Tr$.

- The tachyon potential in flat space can be computed from boundary CFT.

Kutasov+Marino+Moore

$$V(T) = K_0 \, e^{-\mu^2 TT^\dagger}$$

- Two extrema: $T = 0$ (unbroken chiral symmetry) and $T = \infty$ (broken chiral symmetry).

A "warmup" model

Take a simple confining background: AdS_6 soliton, a solution of non-critical string theory

$$ds_6^2 = \frac{R^2}{z^2}\left[dx_{1,3}^2 + f_\Lambda^{-1} dz^2 + f_\Lambda\, d\eta^2\right] \quad , \quad f_\Lambda = 1 - \frac{z^5}{z_\Lambda^5} \quad , \quad z \in [0, z_\Lambda]$$

with η periodic, $\Phi \to$ constant.

• We consider N_f $D4 + \bar{D}4$ branes at a fixed η, and we will will neglect the coordinate of the branes transverse to the η circle.

$$S = -\int d^4x dz V(|T|) \left(\sqrt{-\det \mathbf{A}_L} + \sqrt{-\det \mathbf{A}_R}\right)$$

$$\mathbf{A}_{(i)MN} = g_{MN} + 2\pi\alpha' F^{(i)}_{MN} + \pi\alpha' \left((D_M T)^*(D_N T) + (D_N T)^*(D_M T)\right)$$

$$D_M T = (\partial_M + iA_M^L - iA_M^R)T.$$

• The active fields are two 5-d gauge fields and a complex scalar $T = \tau\, e^{i\theta}$, which are dual to the low-lying quark bilinear operators which correspond to states with $J^{PC} = 1^{--}, 1^{++}, 0^{-+}, 0^{++}$,

- We will take $T = \tau \; 1$

$$V = \mathcal{K}\, e^{-\frac{\pi}{2}\tau^2} \quad , \quad R^2 = 6\alpha'$$

- Tachyon equation:

$$\tau'' - \frac{4\pi\, z\, f_\Lambda}{3}\tau'^3 + \left(-\frac{3}{z} + \frac{f'_\Lambda}{2 f_\Lambda}\right)\tau' + \left(\frac{3}{z^2 f_\Lambda} + \pi\, \tau'^2\right)\tau = 0$$

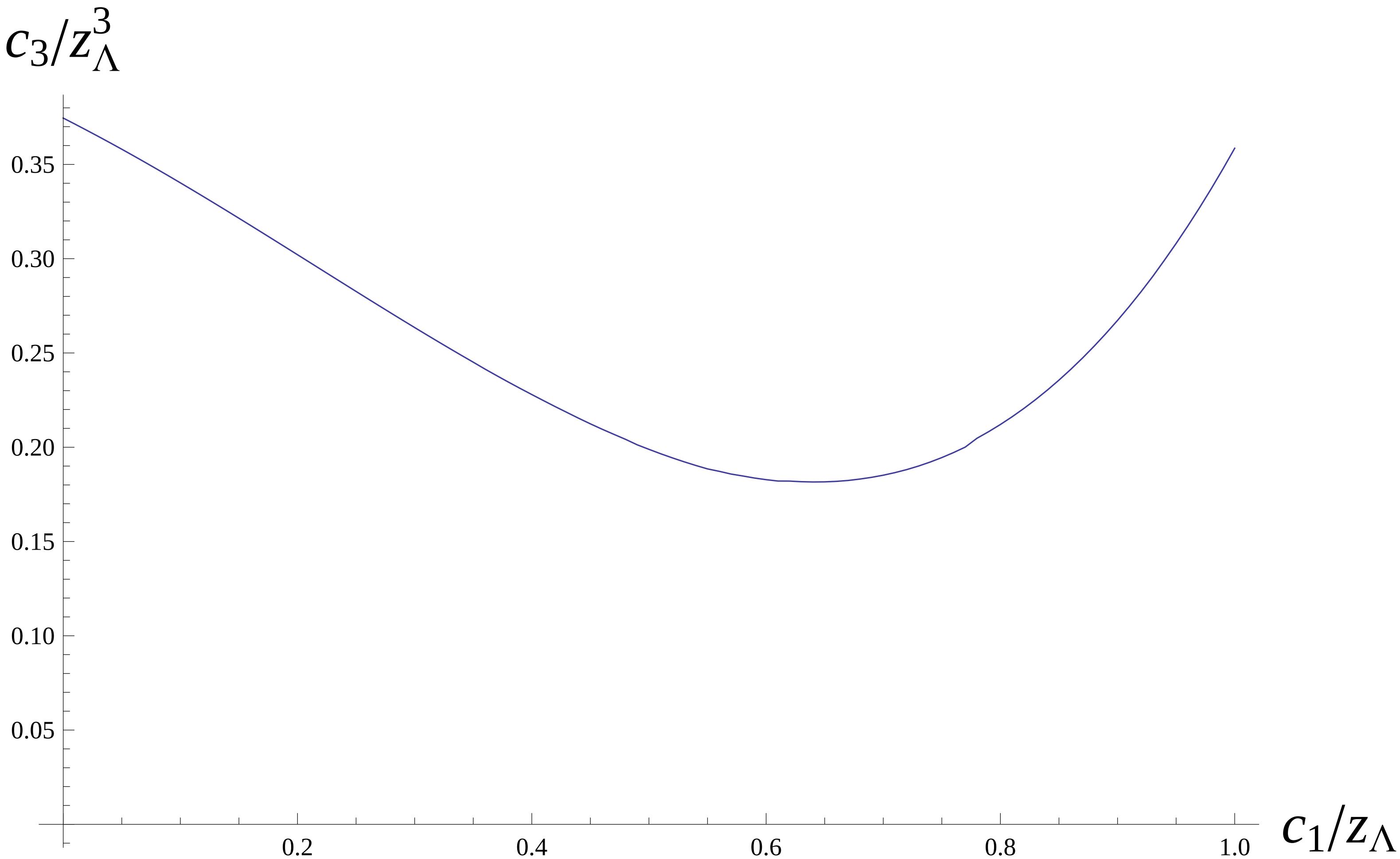
- Near the boundary $z = 0$, the solution can be expanded in terms of two integration constants as:

$$\tau = c_1 z + \frac{\pi}{6} c_1^3 z^3 \log z + c_3 z^3 + \mathcal{O}(z^5)$$

- c_1, c_3 are related to the quark mass and condensate

- There is a one-parameter family of diverging solutions in the IR:

$$\tau = \frac{C}{(z_\Lambda - z)^{\frac{3}{20}}} - \frac{13}{6\pi C}(z_\Lambda - z)^{\frac{3}{20}} + \dots$$



- Chiral symmetry breaking is manifest.

For the vectors

$$z_\Lambda m_V^{(1)} = 1.45 + 0.718 c_1 , \qquad z_\Lambda m_V^{(2)} = 2.64 + 0.594 c_1 , \qquad z_\Lambda m_V^{(3)} = 3.45 + 0.581 c_1 ,$$
$$z_\Lambda m_V^{(4)} = 4.13 + 0.578 c_1 , \qquad z_\Lambda m_V^{(5)} = 4.72 + 0.577 c_1 , \qquad z_\Lambda m_V^{(6)} = 5.25 + 0.576 c_1 .$$

For the axial vectors:

$$z_\Lambda m_A^{(1)} = 1.93 + 1.23 c_1 , \qquad z_\Lambda m_A^{(2)} = 3.28 + 1.04 c_1 , \qquad z_\Lambda m_A^{(3)} = 4.29 + 0.997 c_1$$
$$z_\Lambda m_A^{(4)} = 5.13 + 0.975 c_1 , \qquad z_\Lambda m_A^{(5)} = 5.88 + 0.962 c_1 , \qquad z_\Lambda m_A^{(6)} = 6.55 + 0.954 c_1$$

For the pseudoscalars:

$$z_\Lambda m_P^{(1)} = \sqrt{2.47 c_1^2 + 5.32 c_1} , \qquad z_\Lambda m_P^{(2)} = 2.79 + 1.16 c_1 , \qquad z_\Lambda m_P^{(3)} = 3.87 + 1.08 c_1 ,$$
$$z_\Lambda m_P^{(4)} = 4.77 + 1.04 c_1 , \qquad z_\Lambda m_P^{(5)} = 5.54 + 1.01 c_1 , \qquad z_\Lambda m_P^{(6)} = 6.24 + 0.997 c_1 .$$

For the scalars:

$$z_\Lambda m_S^{(1)} = 2.47 + 0.683 c_1 , \qquad z_\Lambda m_S^{(2)} = 3.73 + 0.488 c_1 , \qquad z_\Lambda m_S^{(3)} = 4.41 + 0.507 c_1 ,$$
$$z_\Lambda m_S^{(4)} = 4.99 + 0.519 c_1 , \qquad z_\Lambda m_S^{(5)} = 5.50 + 0.536 c_1 , \qquad z_\Lambda m_S^{(6)} = 5.98 + 0.543 c_1 .$$

- Valid up to $c_1 = 1$
- In qualitative agreement with lattice results

Laerman+Schmidt., Del Debbio+Lucini+Patela+Pica, Bali+Bursa

We fit the two parameters to the "confirmed" isospin 1 mesons

$$\frac{1}{z_\Lambda} = 503 MeV \quad , \quad c_1^{\text{light}} = 0.0135$$

J^{PC}	Meson	Measured (MeV)	Model (MeV)
1^{--}	$\rho(770)$	775	735
	$\rho(1450)$	1465	1331
	$\rho(1700)$	1720	1742
	$\rho(1900)$	1900	2083
	$\rho(2150)$	2150	2380
1^{++}	$a_1(1260)$	1230	980
	$a_1(1640)$	1647	1661
0^{-+}	π_0	135.0	135.3
	$\pi(1300)$	1300	1411
	$\pi(1800)$	1816	1955
0^{++}	$a_0(1450)$	1474	1249

- The RMS error defined as $100 \times \frac{1}{\sqrt{n}} \sqrt{\sum_O \frac{\delta O^2}{O^2}}$ with n=11-2 is 11%

- "less confirmed mesons"

J^{PC}	Meson	Measured (MeV)	Model (MeV)
1^{--}	$\rho(2270)$	2270	2649
1^{++}	$a_1(1930)$	1930	2166
	$a_1(2096)$	2096	2591
	$a_1(2270)$	2270	2965
	$a_1(2340)$	2340	3303
0^{-+}	$\pi(2070)$	2070	2406
	$\pi(2360)$	2360	2798
0^{++}	$a_0(2020)$	2025	1883

- The RMS error here is 23%

- Axial vector mesons are consistently overestimated.

"$s\bar{s}$" states

They can be "estimated" using

$$m(\text{"}\eta\text{"}) = \sqrt{2m_K^2 - m_\pi^2} \quad , \quad m(\text{"}\phi(1020)\text{"}) = 2m(K^*(892)) - m(\rho(770)) \quad , \quad .$$

Allton+Gimenez+Giusti+Rapuano

J^{PC}	Meson	Measured (MeV)	Model (MeV)
1^{--}	"$\phi(1020)$"	1009	857
	"$\phi(1680)$"	1363	1432
1^{++}	"$f_1(1420)$"	1440	1188
0^{-+}	"η"	691	740
	"$\eta(1475)$"	1620	1608
0^{++}	"$f_0(1710)$"	1386	1365

The "mass" of the s-quark is $c_{1,s} = 0.350$. The rms error for this set of observables ($n = 6 - 1$) is $\varepsilon_{rms} = 11\%$.

- $\frac{2m_s}{m_u + m_d} \simeq \frac{c_{1,s}}{c_{1,l}} \simeq 26$

- $T_{deconf} = \frac{5}{45\pi z_\Lambda} \simeq 200 MeV$.

Advantages of this simple model

• Compared to the SS model it contains all trajectories corresponding to $1^{--}, 1^{++}, 0^{-+}, 0^{++}$ and can accommodate a mass of the quarks. The asymptotic masses of mesons are $m_n^2 \sim n$ are they should.

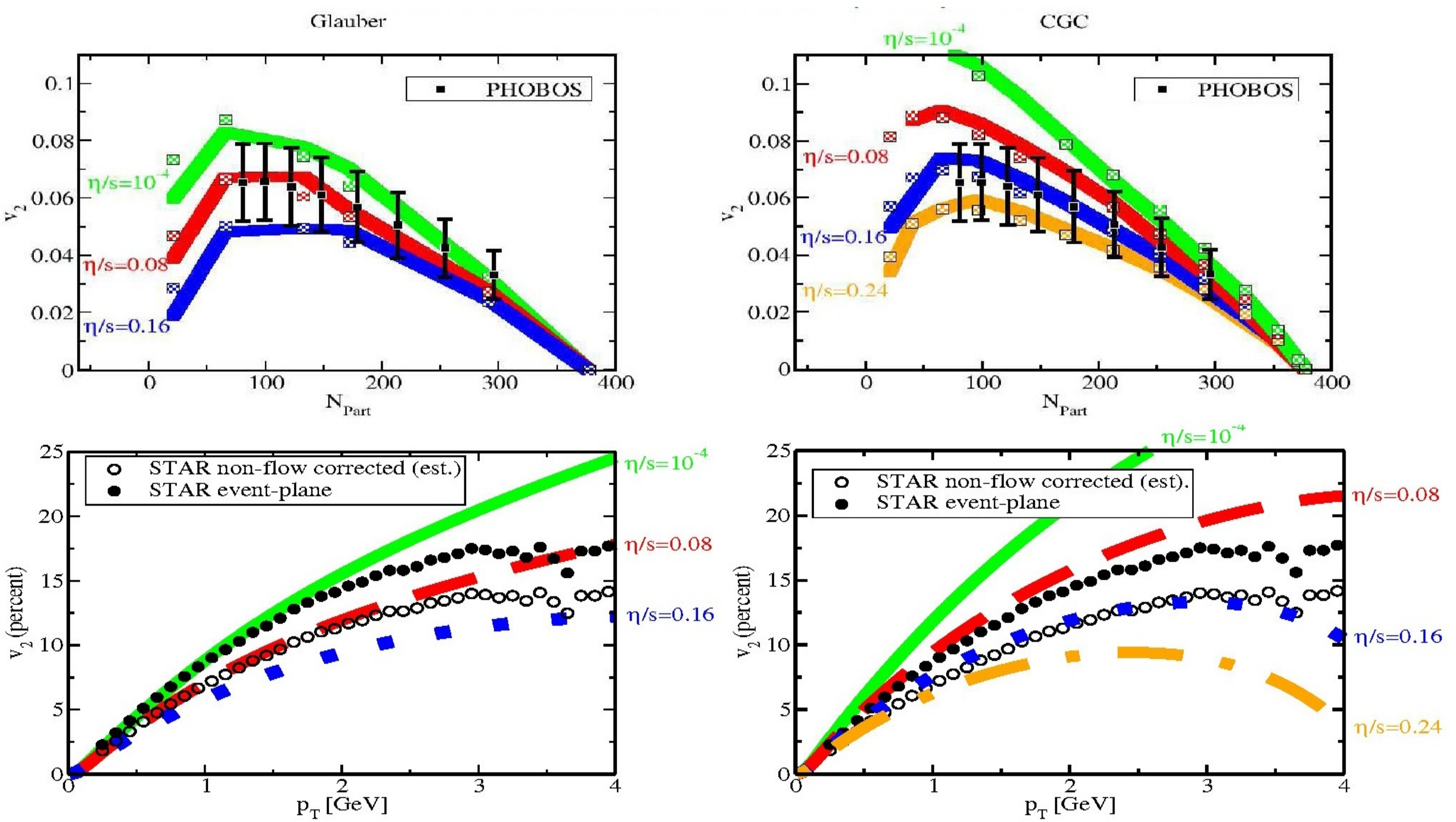
• Compared to the hard wall AdS/QCD model chiral symmetry breaking is dynamical and not input by hand. Asymptotic masses behave as $m_n^2 \sim n^2$.

• In the soft wall model, chiral symmetry breaking is not dynamical and different aspects of that model are inconsistent.

• It needs to be improved along the lines of the glue sector+add the non-abelian structure.

shear viscosity data

- V_2 is the elliptic flow coefficient



Luzum+Romatchke 2008

Viscosity

- Viscosity (shear and bulk) is related to dissipation and entropy production

$$\frac{\partial s}{\partial t} = \frac{\eta}{T}\left[\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial\cdot v\right]^2 + \frac{\zeta}{T}(\partial\cdot v)^2$$

- Hydrodynamics is valid as an effective description when relevant length scales $\gg$ mean-free-path:
- Conformal invariance imposes that $\zeta = 0$.
- Viscosity can be calculated from a Kubo-like formula (fluctuation-dissipation)

$$\eta\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right) + \zeta\delta_{ij}\delta_{kl} = -\lim_{\omega\to 0}\frac{Im\ G^R_{ij;kl}(\omega)}{\omega}$$

$$G^R_{ij;kl}(\omega) = -i\int d^3x \int dt\ e^{i\omega t}\theta(t)\ \langle 0|[T_{ij}(\vec{x},t), T_{kl}(\vec{0},0)]|0\rangle$$

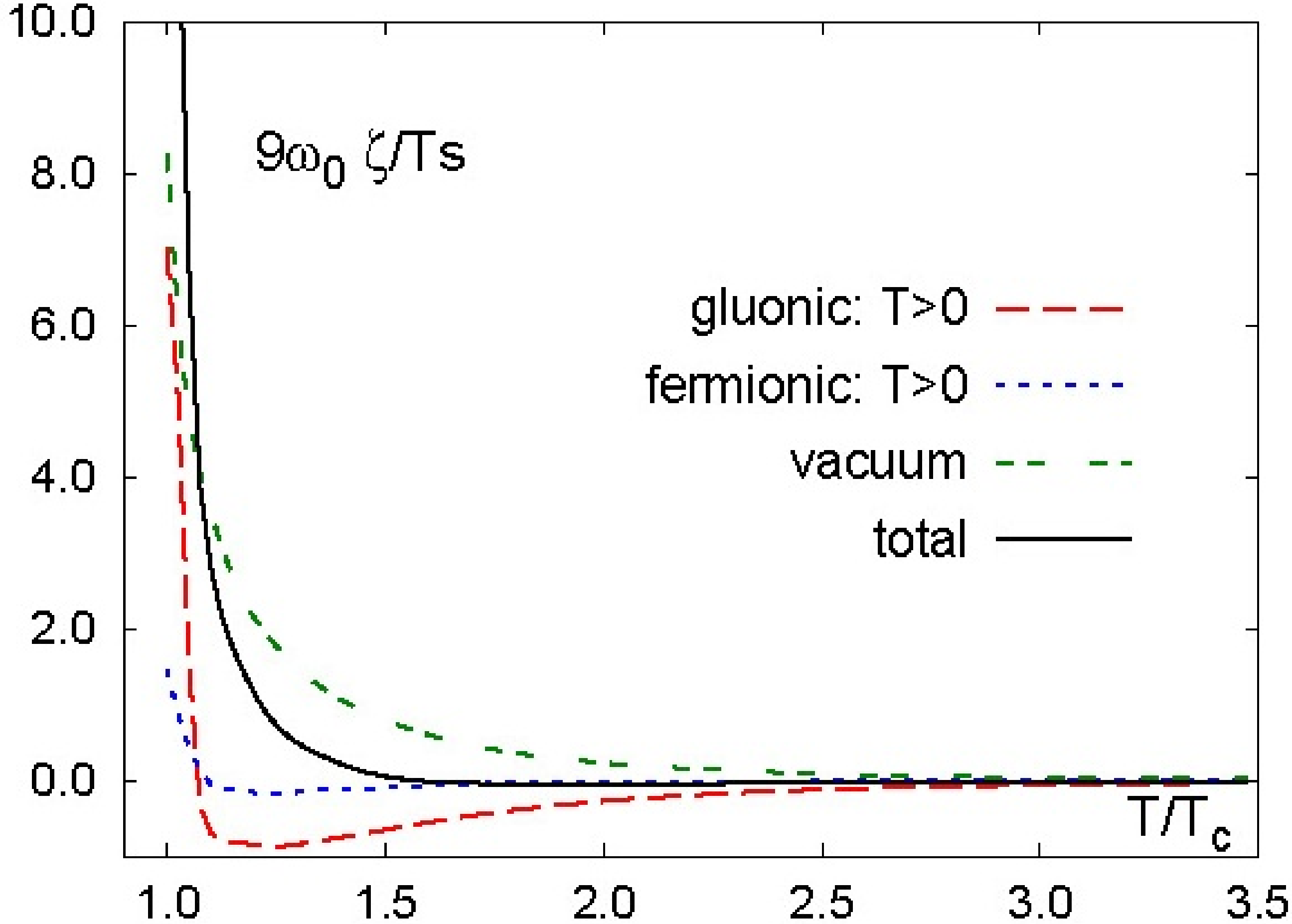
- In all theories with gravity duals ($\lambda\to\infty$) at two-derivative level

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Policastro+Starinets+Son 2001, Kovtun+Son+Starinets 2003, Buchel+Liu 2003

- In Einstein-dilaton gravity shear viscosity is equal to the universal value.

The sum rule method

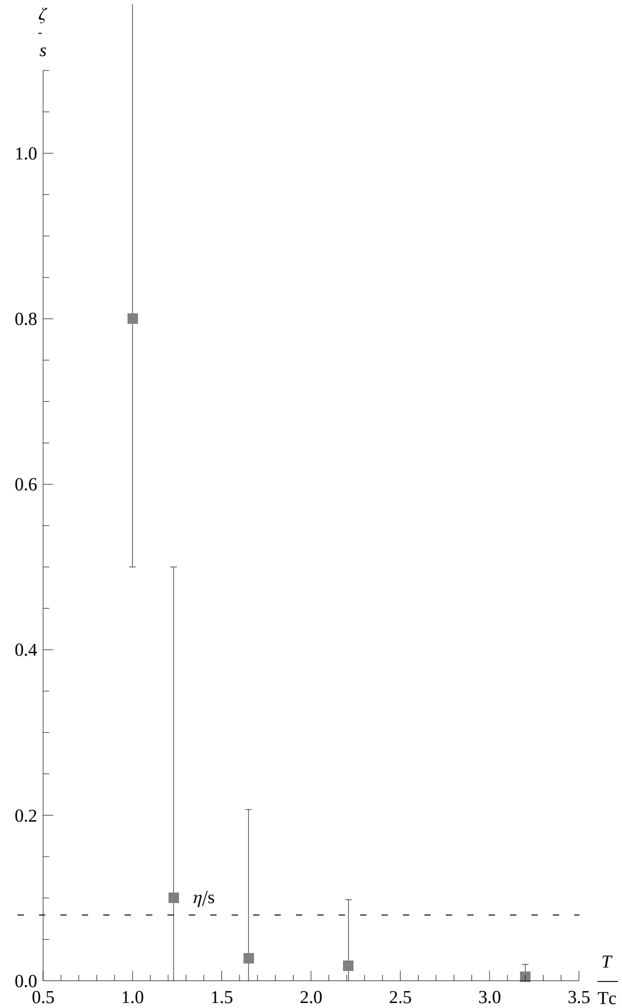


Karsch+Kharzeev+Tuchin, 2008

- A rise near the phase transition but the scale cannot be fixed.

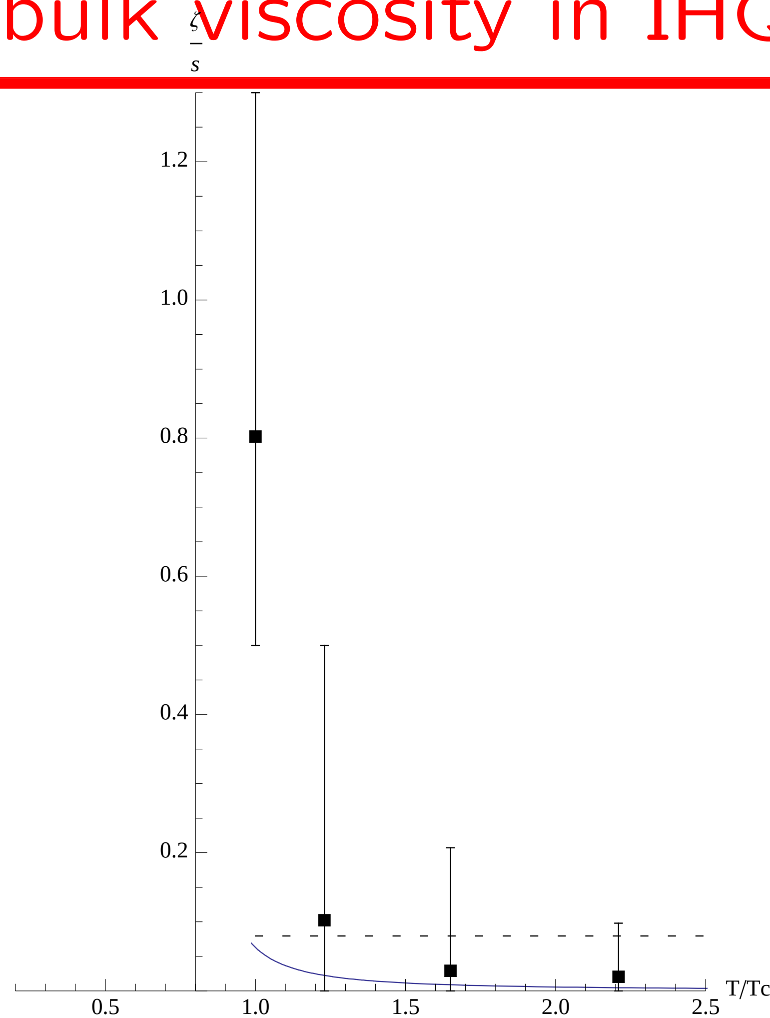
The bulk viscosity in lattice

H. Meyer 2007



Pure YM only. Error bar are statistical only.

The bulk viscosity in IHQCD



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

- Pure glue only.

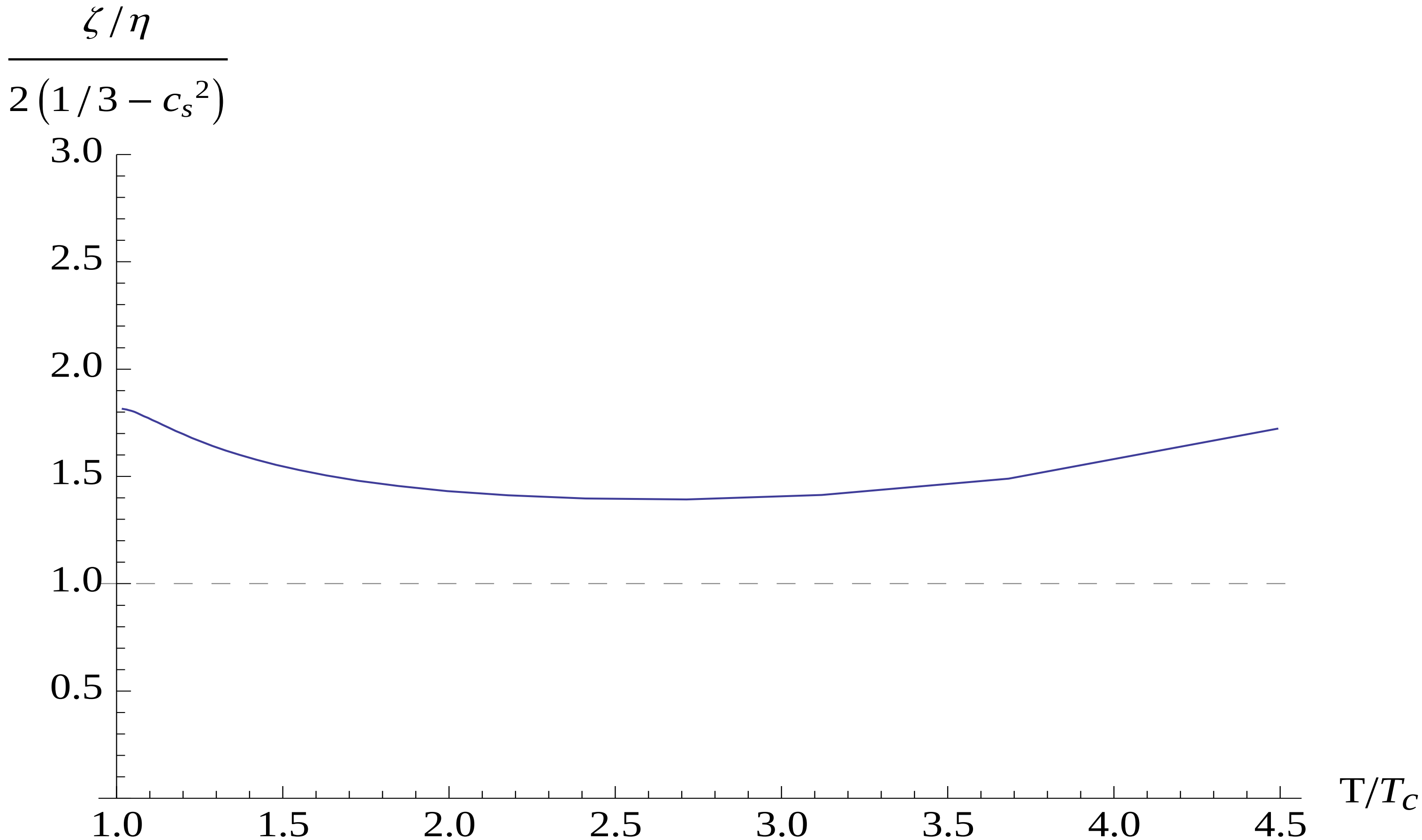
- Calculations with other potentials show robustness.

Gubser

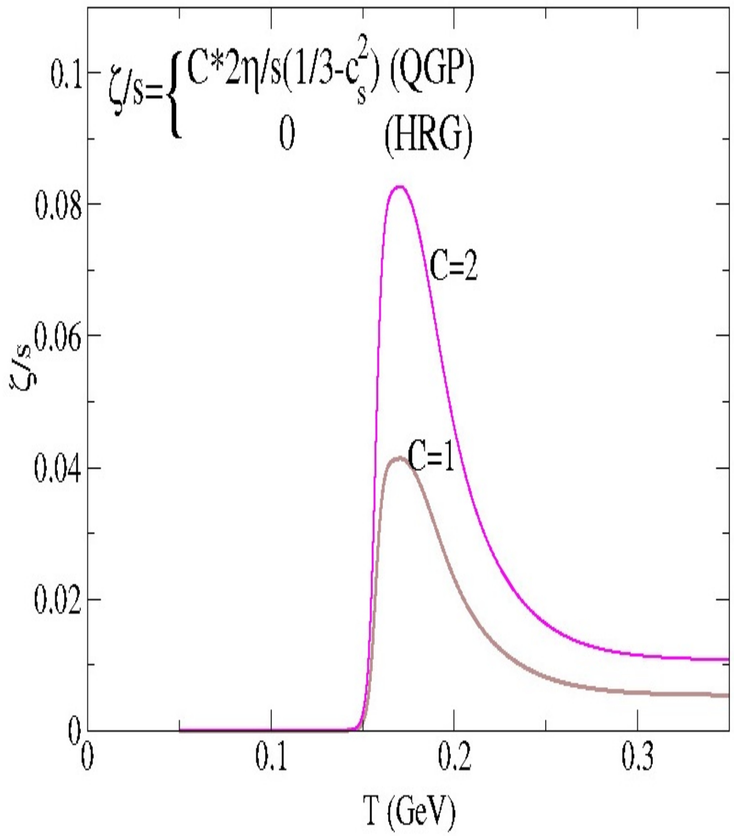
The Buchel parametrization (bound)

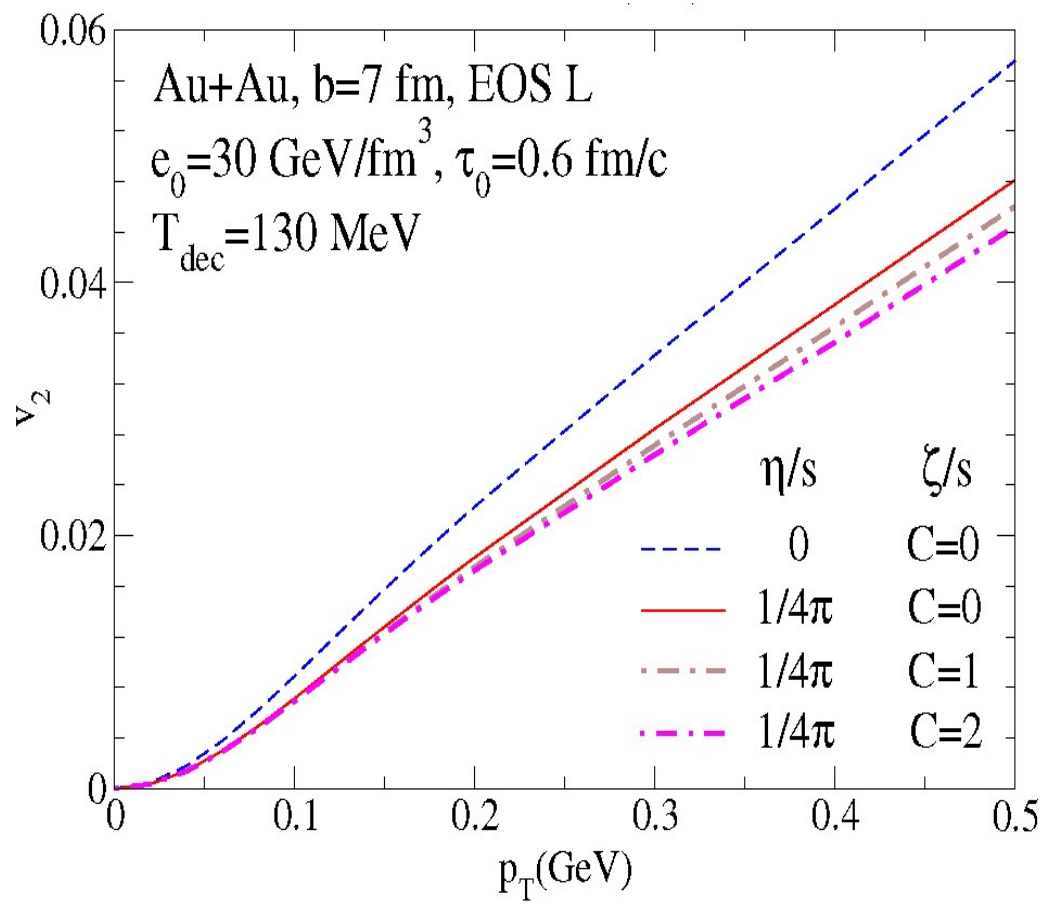
$$\frac{\zeta}{\eta} \geq 2\left(\frac{1}{3} - c_s^2\right)$$

Buchel 2007



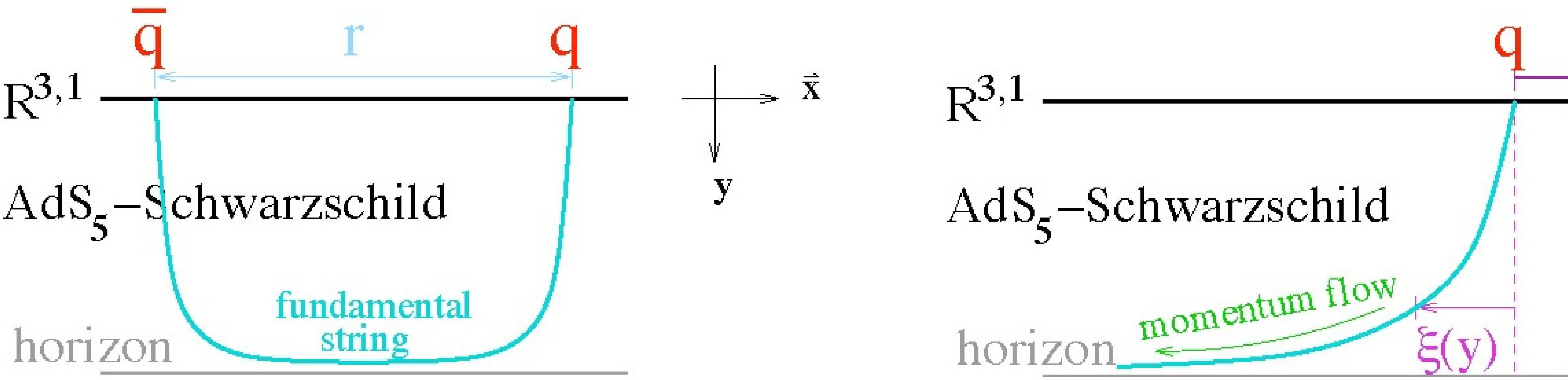
Elliptic Flow vs bulk viscosity





U Heinz+H.Song 2008

Heavy quarks and the drag force



- The dynamics is determined by the Nambu-Goto action.

$$S_{NG} = -\frac{1}{2\pi\ell_s^2}\int d\sigma d\tau \sqrt{det\left(-g_{MN}\partial_\alpha X^M \partial_\beta X^N\right)} \ ,$$

- We must find a solution to the string equations with

$$x^1 = vt + \xi(r) \quad , \quad x^{2,3} = 0 \quad , \quad \sigma^1 = t \quad , \quad \sigma^2 = r$$

The spacetime metric is a black-hole metric (in string frame)

$$ds^2 = b(r)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x}\cdot d\vec{x}\right]$$

- The "momentum" conjugate to ξ is conserved

$$\pi_\xi = -\frac{1}{2\pi\ell_s^2}\frac{g_{00}g_{11}\xi'}{\sqrt{-g_{00}g_{rr} - g_{00}g_{11}\xi'^2 - g_{11}g_{rr}v^2}}\ .$$

We solve for ξ' to obtain

$$\xi' = \frac{\sqrt{-g_{00}g_{rr} - g_{11}g_{rr}v^2}}{\sqrt{g_{00}g_{11}\left(1 + g_{00}g_{11}/(2\pi\ell_s^2\pi_\xi)^2\right)}}\ .$$

- The solution profile is

$$\xi'(r) = \frac{C}{f(r)}\sqrt{\frac{f(r) - v^2}{b^4(r)f(r) - C^2}} \quad , \quad C = -(2\pi\ell_s^2)\ \pi_\xi = vb(r_s)^2 \quad , \quad f(r_s) = v^2$$

with r_s the turning point.

• The induced metric on the world-sheet is a 2d black-hole with horizon at the turning point $r = r_s$ $(t = \tau + \zeta(r))$.

$$ds^2 = b^2(r)\left[-(f(r) - v^2)d\tau^2 + \frac{1}{(f(r) - \frac{b^4(r_s)}{b^4(r)}v^2)}dr^2\right]$$

• The associated Hawking temperature is different from the plasma temperature

$$4\pi T_s \equiv \sqrt{f(r_s)f'(r_s)\left[\frac{4b'(r_s)}{b(r_s)} + \frac{f'(r_s)}{f(r_s)}\right]}.$$

• We can calculate the drag force:

$$F_{\text{drag}} = \pi_\xi = -\frac{b^2(r_s)\sqrt{f(r_s)}}{2\pi\ell_s^2}$$

• In $\mathcal{N} = 4$ sYM it is given by

$$F_{\text{drag}} = -\frac{\pi}{2}\sqrt{\lambda}\;T^2\frac{v}{\sqrt{1-v^2}} = -\frac{1}{\tau}\frac{p}{M} \quad , \quad \tau = \frac{2M}{\pi\sqrt{\lambda}\;T^2}$$

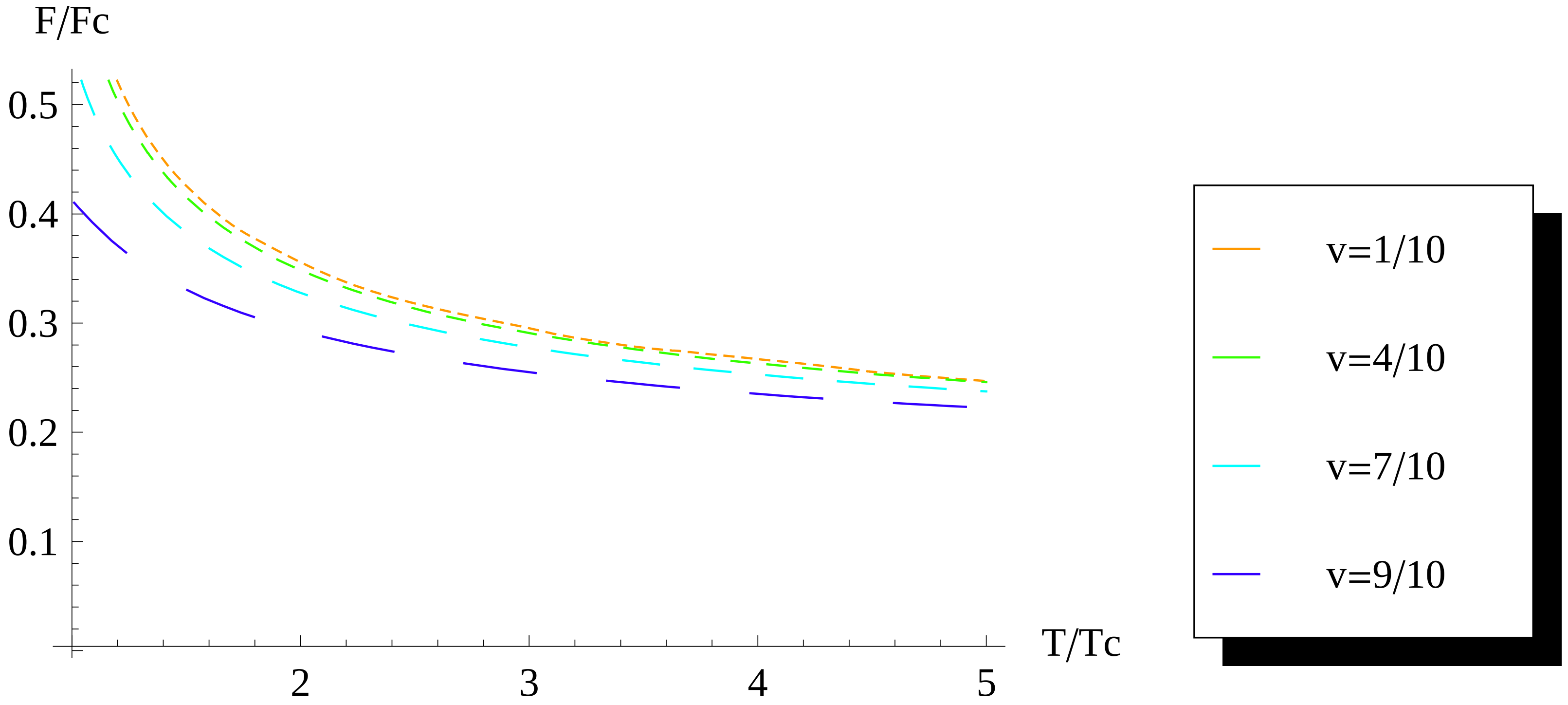
with τ the diffusion time.For non-conformal theories it is a more complicated function of momentum and temperature.

The drag force in IhQCD

Systematic errors:

(a) Flavor description (heavy quark)

(b) Ignore light fermionic degrees of freedom in plasma



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

- F_{conf} calculated with $\lambda = 5.5$

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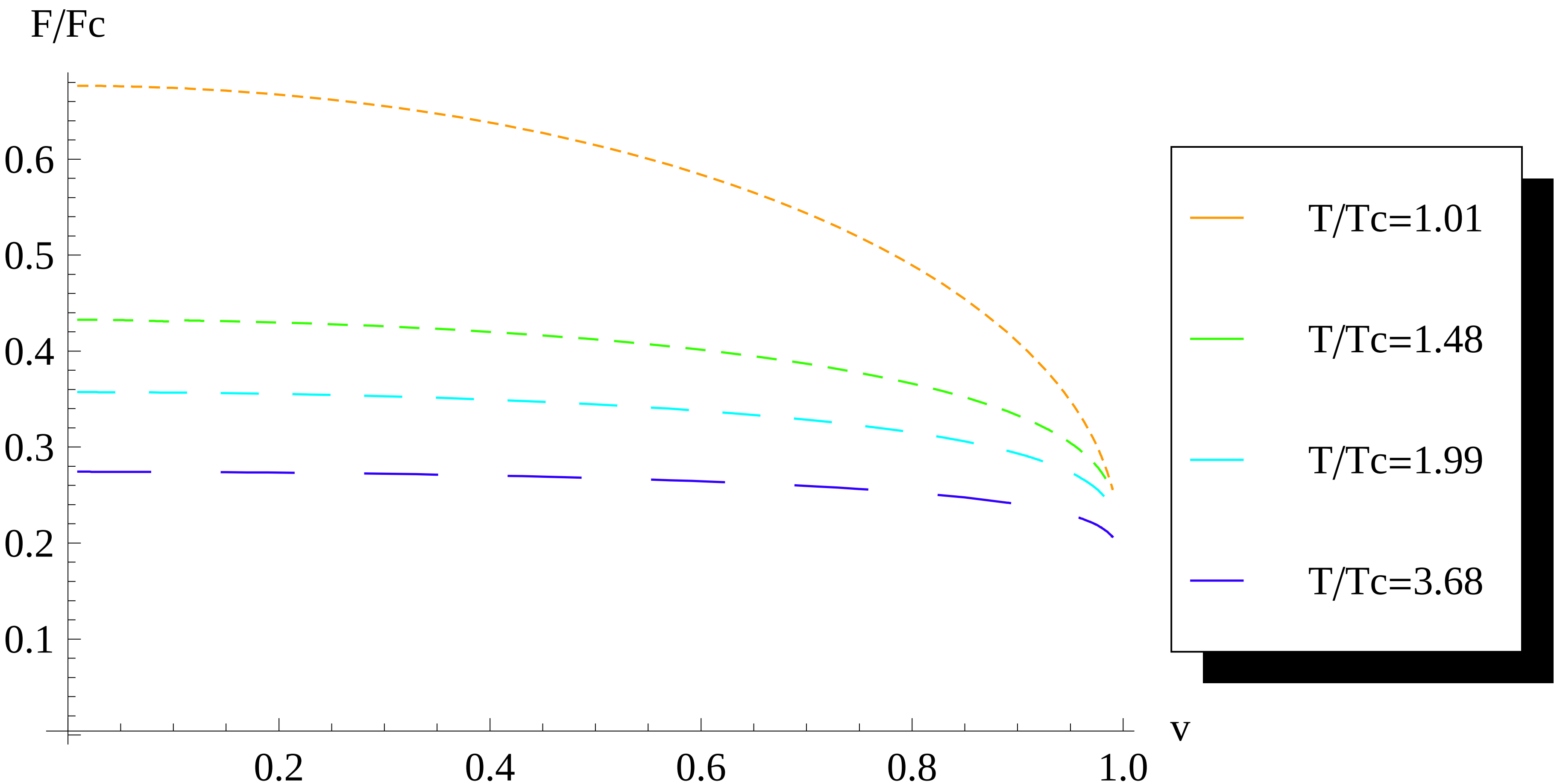
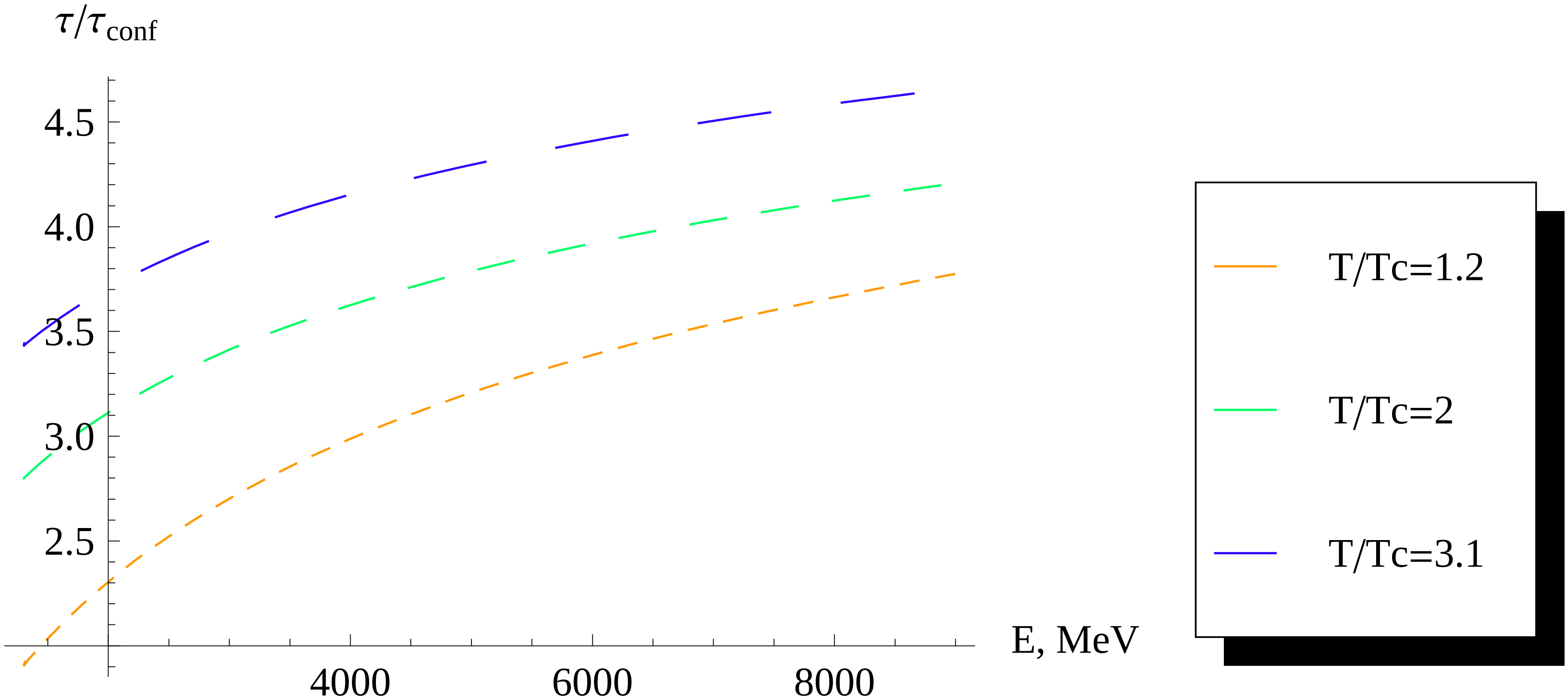


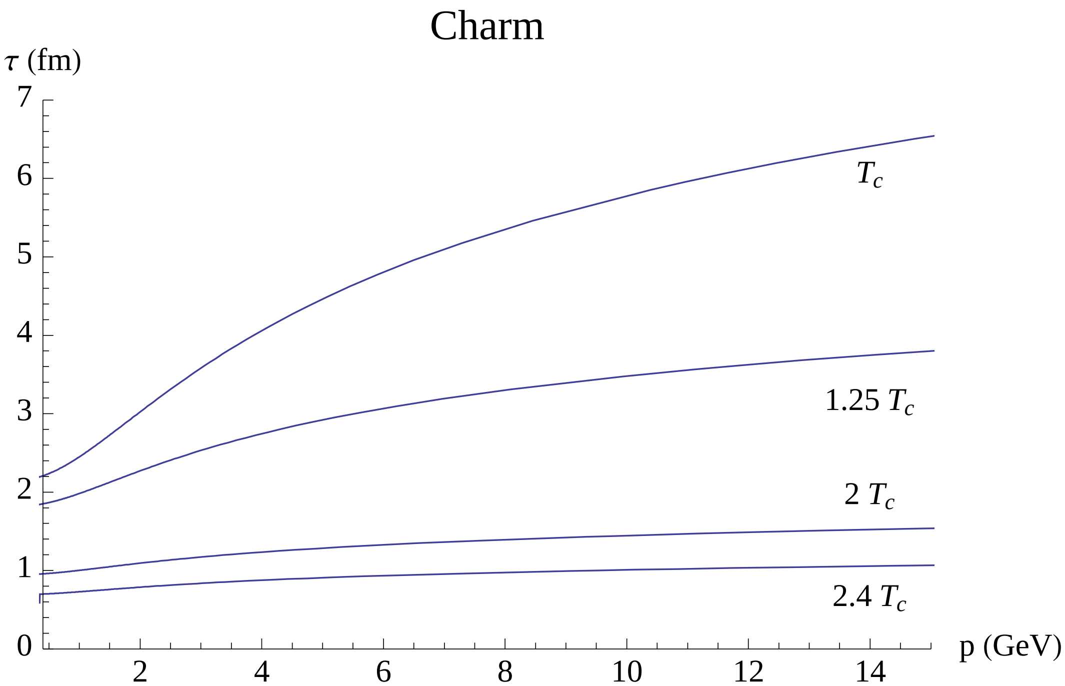
Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

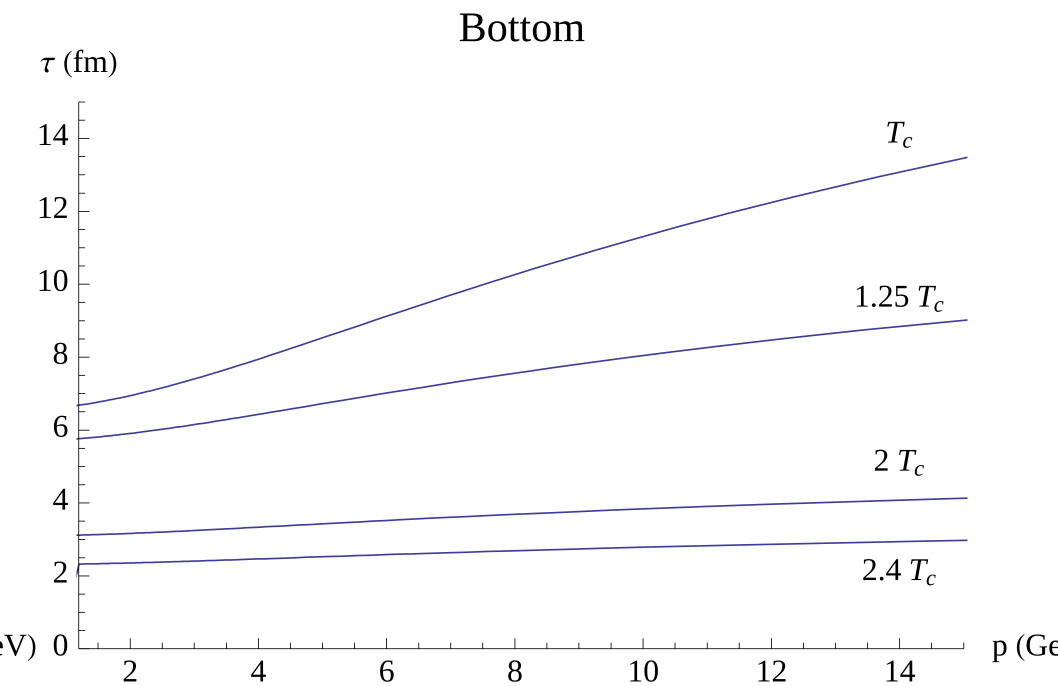
The diffusion time



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

$$\frac{dp}{dt} = -\frac{p}{\tau(p)}$$





Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

	$\gamma = 0.3$	$\gamma = 1$	$\gamma = 3$
τ_c[fm]	22	6.7	2.2
τ_b[fm]	72	21	7.2

thermalized

not thermalized

Akamatsu+Hatsuda+Hirano, 2008

Fluctuations

- We now allow the string to fluctuate

$$X^1 = vt + \xi(r) + \delta X^1 \quad , \quad X^{2,3} = \delta X^{2,3} \quad , \quad \delta X^i(r,\tau) = e^{i\omega\tau}\delta X^i(r,\omega)$$

- At the quadratic level

$$\partial_r \left[\sqrt{(f-v^2)(b^4 f - C^2)}\;\partial_r\left(\delta X^\perp\right)\right] + \frac{\omega^2 b^4}{\sqrt{(f-v^2)(b^4 f - C^2)}}\delta X^\perp = 0$$

$$\partial_r \left[\frac{1}{Z^2}\sqrt{(f-v^2)(b^4 f - C^2)}\;\partial_r\left(\delta X^\parallel\right)\right] + \frac{\omega^2 b^4}{Z^2\sqrt{(f-v^2)(b^4 f - C^2)}}\delta X^\parallel = 0$$

with

$$Z \equiv b(r)^2\sqrt{\frac{f(r)-v^2}{b(r)^4 f(r) - C^2}} \quad , \quad C = b^2(r_s)\; v^2$$

determine the frequency dependent correlators.

- As standard, the retarded correlator is determined with incoming boundary conditions at the ws BH horizon.

The diffusion constant is given by

$$\kappa = \lim_{\omega\to 0} G_{sym}(\omega) = -\lim_{\omega\to 0} \coth\left(\frac{\omega}{2T_s}\right) \text{Im}\ G_R(\omega)\ .$$

- For general backgrounds we obtain

$$\kappa_\perp = \frac{1}{\pi\ell_s^2} b^2(r_s) T_s \quad , \quad \kappa_\parallel = \frac{16\pi}{\ell_s^2} \frac{b^2(r_s)}{f'^2(r_s)} T_s^3$$

$$\hat{q}_\perp = \frac{2}{v}\kappa_\perp = \frac{2\pi}{\ell_s^2}\frac{b^2(r_s)}{v} T_s \quad , \quad \hat{q}_\parallel = \frac{2}{v}\kappa_\parallel = \frac{32\pi}{\ell_s^2}\frac{b^2(r_s)}{v\dot{f}^2(r_s)} T_s^3$$

- Universal inequality

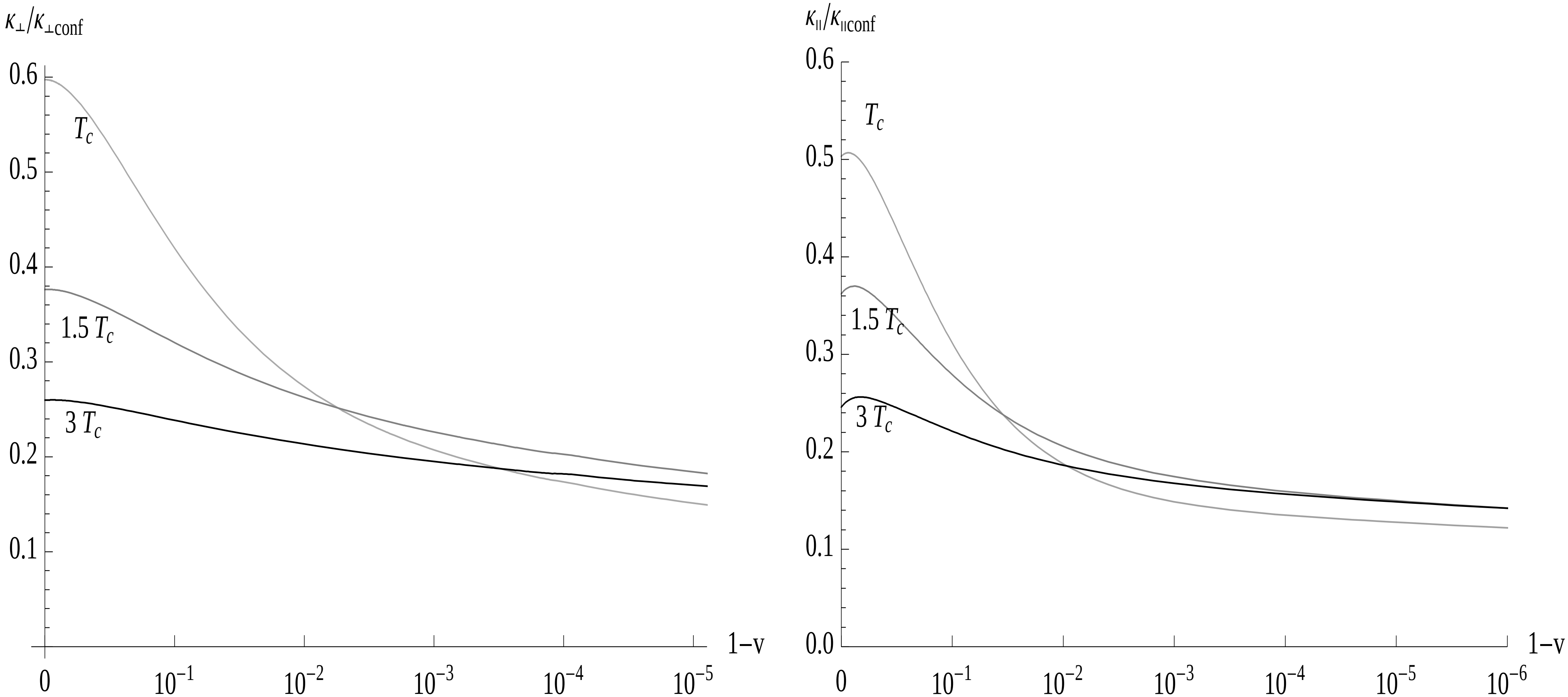
$$\kappa_\parallel \geq \kappa_\perp$$

- For CFT backgrounds the formulae simplify:

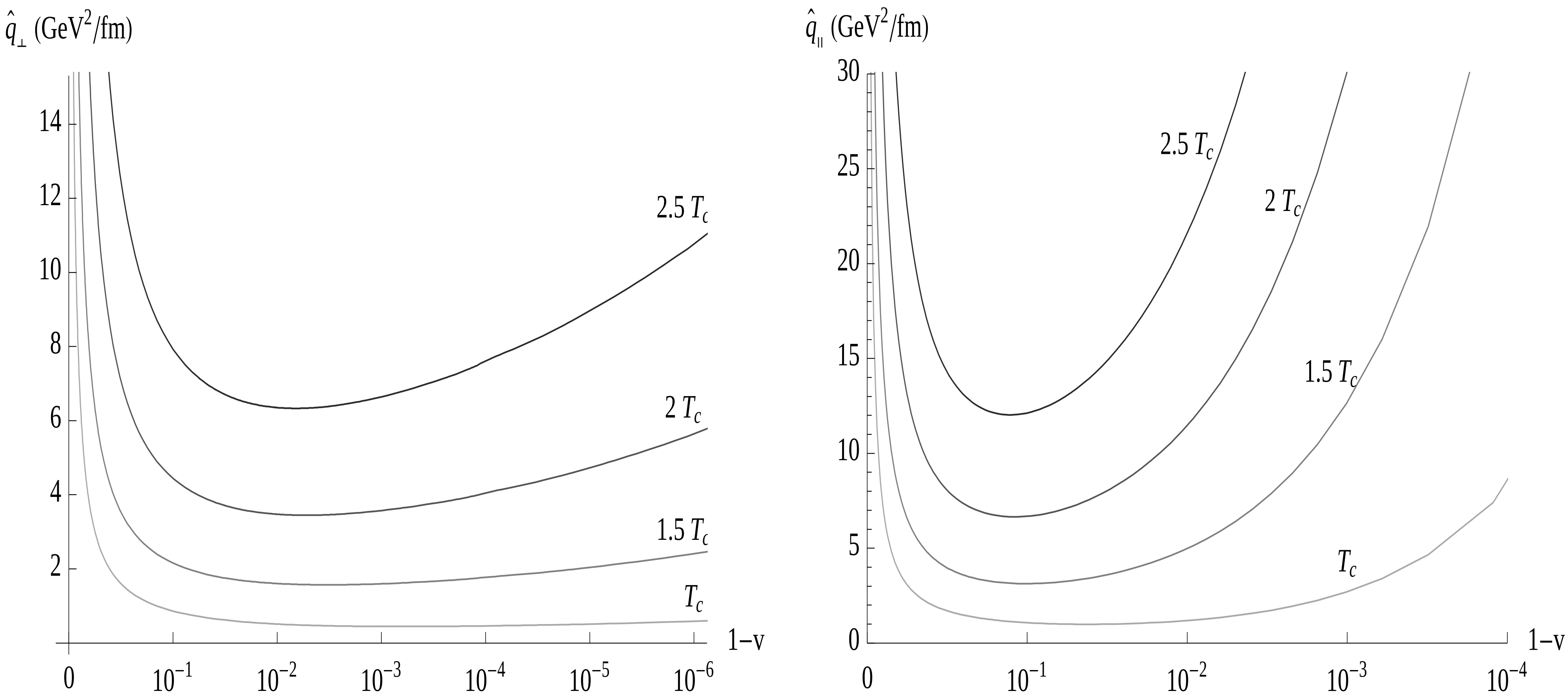
$$\kappa_\perp = \pi\sqrt{\lambda}\gamma^{1/2}T^3 \quad , \quad \kappa_\parallel = \pi\sqrt{\lambda}\gamma^{5/2}T^3$$

- In the non-relativistic limit

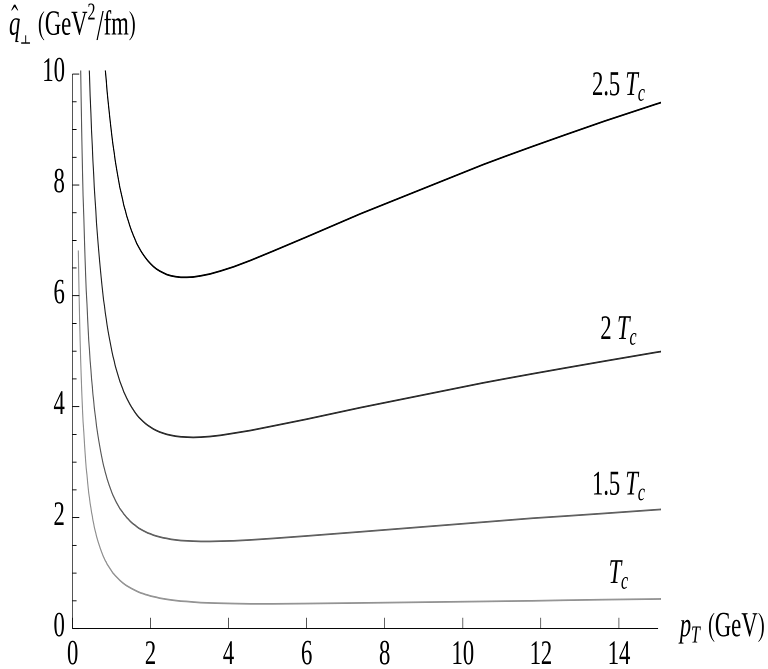
$$\kappa_\perp = \kappa_\parallel$$



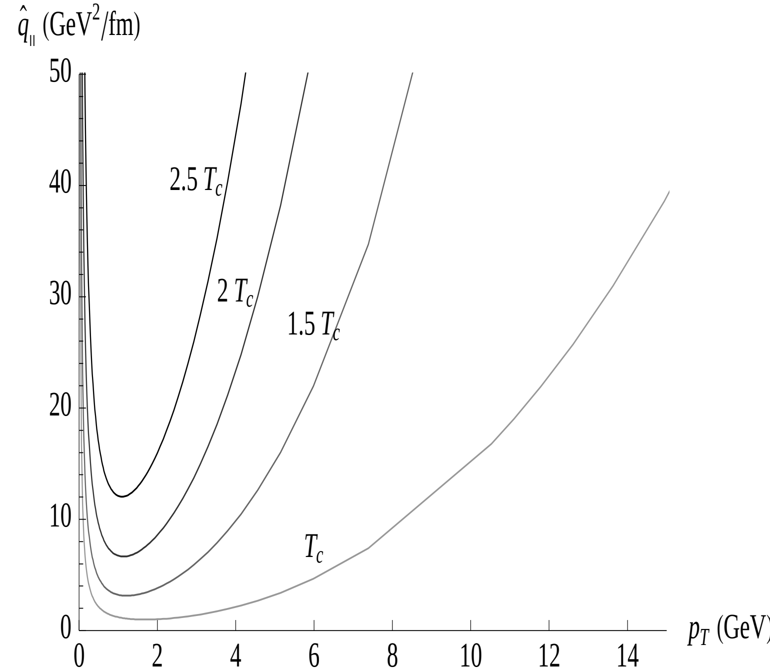
- The ratio of the diffusion coefficients $\kappa_\perp$ and $\kappa_\parallel$ to the corresponding value in the holographic conformal $\mathcal{N} = 4$ theory (with $\lambda_{\mathcal{N}=4} = 5.5$) are plotted as a function of the velocity v (in logarithmic horizontal scale).



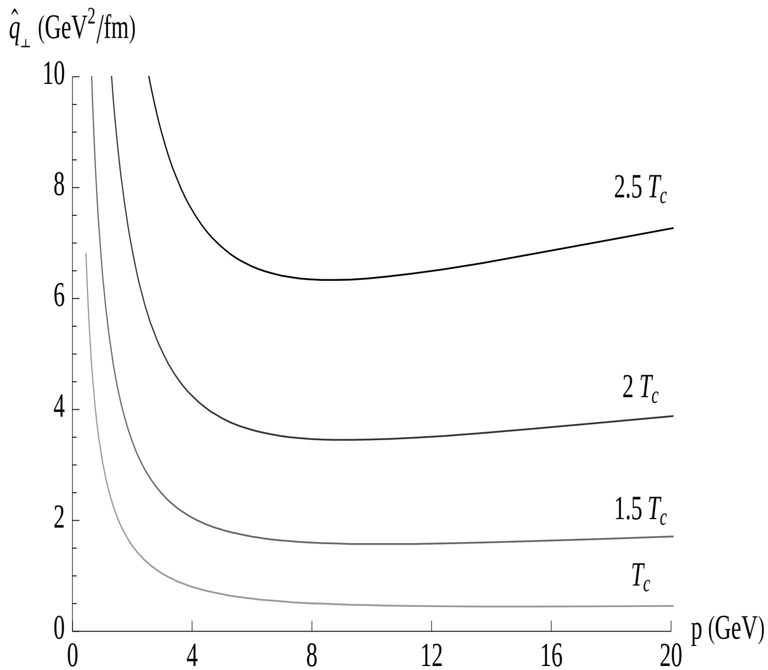
• The jet quenching parameters $\widehat{q}_\perp$ and $\widehat{q}_{||}$ are plotted as a function of the velocity v (in a logarithmic horizontal scale). The results are evaluated at different temperatures.



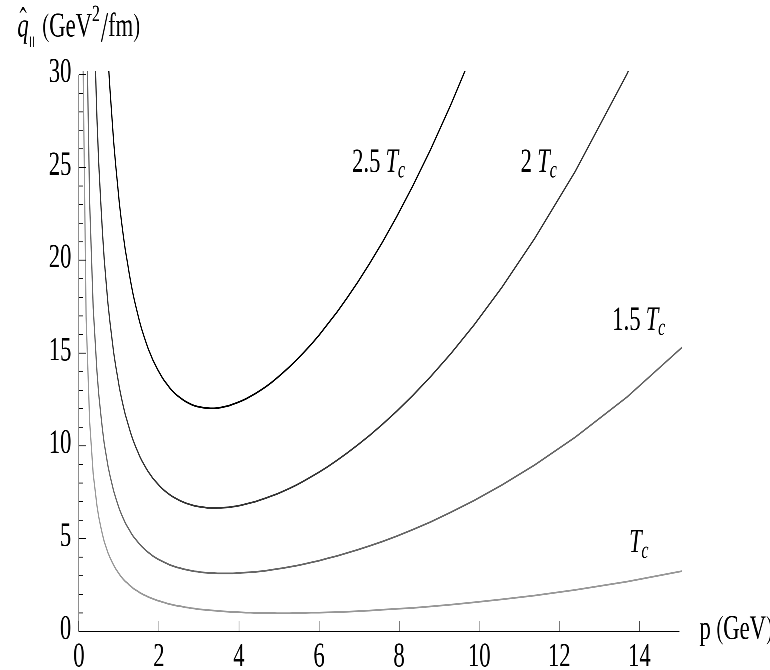
$\widehat{q}_\perp$ charm



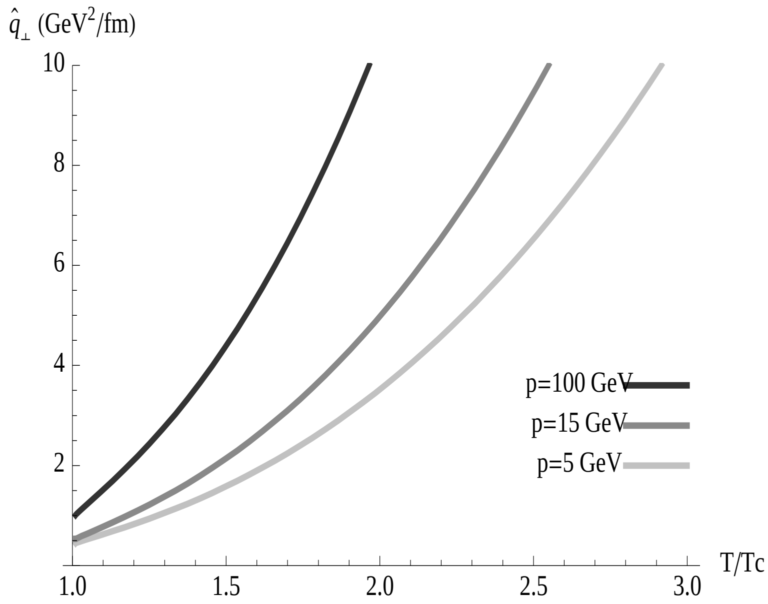
$\widehat{q}_{||}$ charm



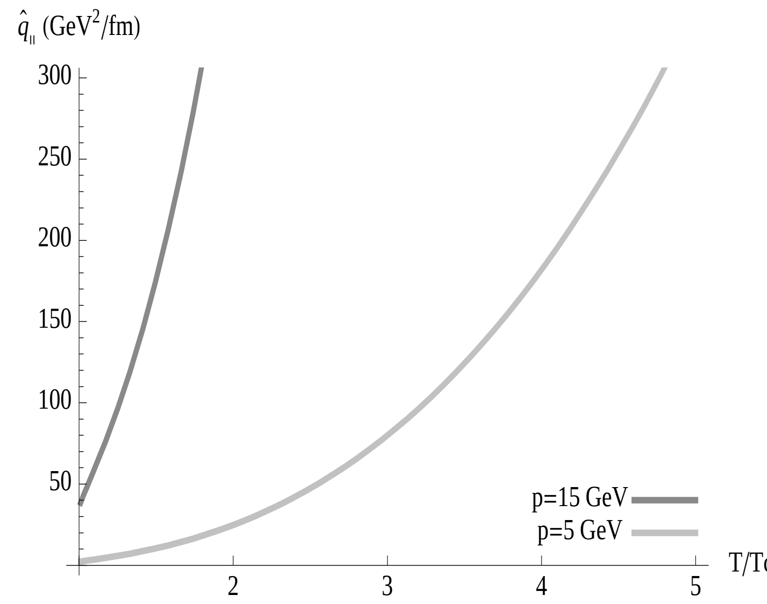
$\widehat{q}_\perp$ bottom



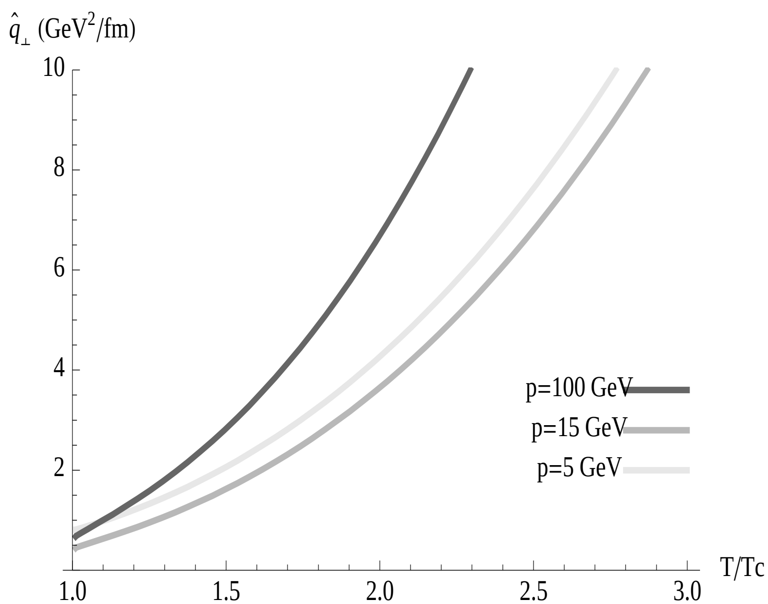
$\widehat{q}_{||}$ bottom



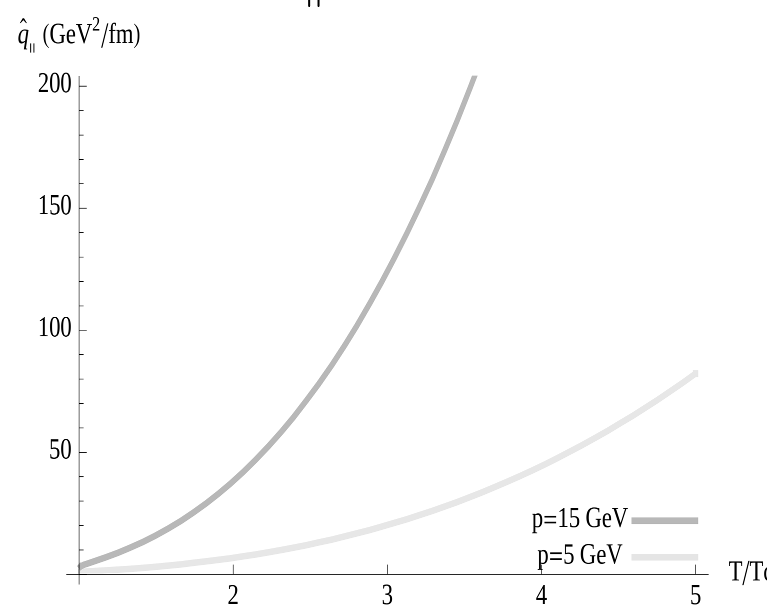
$\widehat{q}_\perp$ charm



$\widehat{q}_{||}$ charm



$\widehat{q}_\perp$ bottom



$\widehat{q}_{||}$ bottom

Shortcomings

Not everything is perfect: There are some shortcomings localized at the UV

- The conformal anomaly (proportional to the curvature) is incorrect.

- Shear viscosity ratio is constant and equal to that of N=4 sYM.

(This is not expected to be a serious error in the experimentally interesting $T_c \leq T \leq 4T_c$ range.)

Both of the above need Riemann curvature corrections.

- Many other observables come out very well both at T=0 and finite T

Open problems

- Explore further the applicability of such a model to various YM observables: Wilson+Polyakov Loops, quark potentials, Debye screening lengths in various symmetry channels, etc

- Investigate quantitatively the meson sector: spectra, interactions, finite temperature effects

- Calculate the phase diagram in the presence of baryon number.

- Find the Baryons as instantons on the flavor branes and calculate their properties.

- Proceed beyond the quenched approximation for flavor.

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Thank you for your Patience

General criterion for confinement

• **the geometric version**:
A geometry that shrinks to zero size in the IR is dual to a confining 4D theory if and only if the Einstein metric in conformal coordinates vanishes as (or faster than) e^{-Cr} as $r \to \infty$, for some $C > 0$.

• It is understood here that a metric vanishing at finite $r = r_0$ also satisfies the above condition.

♠ **the superpotential**
A 5D background is dual to a confining theory if the superpotential grows as (or faster than)

$$W \;\sim\; (\log\lambda)^{P/2}\lambda^{2/3} \quad \text{as} \quad \lambda \to \infty \quad , \quad P \geq 0$$

♠ **the β-function** A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda\to\infty}\left(\frac{\beta(\lambda)}{3\lambda}+\frac{1}{2}\right)\log\lambda = K, \qquad -\infty \leq K \leq 0$$

(No explicit reference to any coordinate system) Linear trajectories correspond to $K = -\frac{3}{16}$

Classification of confining superpotentials

Classification of confining superpotentials $W(\lambda)$ as $\lambda \to \infty$ in IR:

$$W(\lambda) \sim (\log \lambda)^{\frac{P}{2}} \lambda^{Q} \quad , \quad \lambda \sim E^{-\frac{9}{4}Q}\left(\log \frac{1}{E}\right)^{\frac{P}{2Q}}, \qquad E \to 0.$$

- $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

$$e^{A}(r) \sim \begin{cases} (r_0 - r)^{\frac{4}{9Q^2-4}} & Q > \frac{2}{3} \\ \exp\left[-\frac{C}{(r_0-r)^{1/(P-1)}}\right] & Q = \frac{2}{3} \end{cases}$$

- $Q = 2/3$, and $0 \leq P < 1$ leads to confinement and a singularity at $r = \infty$ The scale factor e^A vanishes there as

$$e^{A}(r) \sim \exp[-Cr^{1/(1-P)}].$$

- $Q = 2/3, P = 1$ leads to confinement but the singularity may be at a finite or infinite value of r depending on subleading asymptotics of the superpotential.

♠ If $Q < 2\sqrt{2}/3$, no *ad hoc* boundary conditions are needed to determine the glueball spectrum $\to$ One-to-one correspondence with the β-function This is unlike standard AdS/QCD and other approaches.

- when $Q > 2\sqrt{2}/3$, the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

Confining β-functions

A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda\to\infty}\left(\frac{\beta(\lambda)}{3\lambda}+\frac{1}{2}\right)\log\lambda = K, \qquad -\infty \le K \le 0$$

(No explicit reference to any coordinate system). Linear trajectories correspond to $K = -\frac{3}{16}$

- We can determine the geometry if we specify K:

- $K = -\infty$: the scale factor goes to zero at some finite r_0, not faster than a power-law.

- $-\infty < K < -3/8$: the scale factor goes to zero at some finite r_0 faster than any power-law.

- $-3/8 < K < 0$: the scale factor goes to zero as $r \to \infty$ faster than $e^{-Cr^{1+\epsilon}}$ for some $\epsilon > 0$.

- $K = 0$: the scale factor goes to zero as $r \to \infty$ as e^{-Cr} (or faster), but slower than $e^{-Cr^{1+\epsilon}}$ for any $\epsilon > 0$.

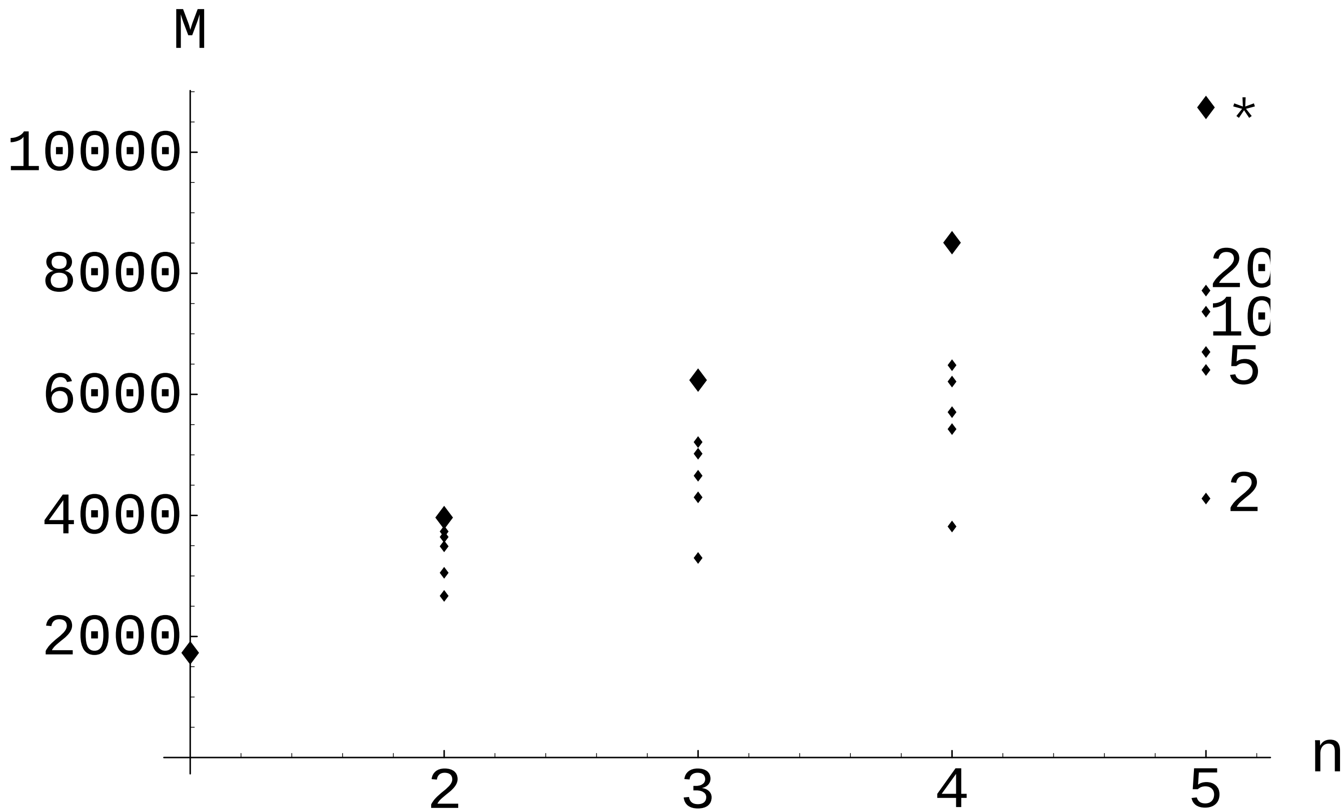
The borderline case, $K = -3/8$, is certainly confining (by continuity), but whether or not the singularity is at finite r depends on the subleading terms.

The lattice glueball data

J^{++}	Ref. I $(m/\sqrt{\sigma})$	Ref. I (MeV)	Ref. II (mr_0)	Ref. II (MeV)	$N_c \to \infty (m/\sqrt{}$
0	3.347(68)	1475(30)(65)	4.16(11)(4)	1710(50)(80)	3.37(15)
0^*	6.26(16)	2755(70)(120)	6.50(44)(7)	2670(180)(130)	6.43(50)
0^{**}	7.65(23)	3370(100)(150)	NA	NA	NA
0^{***}	9.06(49)	3990(210)(180)	NA	NA	NA
2	4.916(91)	2150(30)(100)	5.83(5)(6)	2390(30)(120)	4.93(30)
2^*	6.48(22)	2880(100)(130)	NA	NA	NA
R_{20}	1.46(5)	1.46(5)	1.40(5)	1.40(5)	1.46(11)
R_{00}	1.87(8)	1.87(8)	1.56(15)	1.56(15)	1.90(17)

Available lattice data for the scalar and the tensor glueballs. Ref. I =H. B. Meyer, [arXiv:hep-lat/0508002]. and Ref. II = C. J. Morningstar and M. J. Peardon, [arXiv:hep-lat/9901004] + Y. Chen *et al.*, [arXiv:hep-lat/0510074]. The first error corresponds to the statistical error from the the continuum extrapolation. The second error in Ref.I is due to the uncertainty in the string tension $\sqrt{\sigma}$. (Note that this does not affect the mass ratios). The second error in the Ref. II is the estimated uncertainty from the anisotropy. In the last column we present the available large N_c estimates according to B. Lucini and M. Teper, [arXiv:hep-lat/0103027]. The parenthesis in this column shows the total possible error followed by the estimations in the same reference.

α-dependence of scalar spectrum



The 0^{++} spectra for varying values of α that are shown at the right end of the plot. The symbol * denotes the AdS/QCD result.

$B_2 - C_2$ mixing

- B_2 and C_2 are typically massless.
- In the presence of C_4 flux, this is not the case:

$$S=-M^3\int d^5x\sqrt{g}\left[\frac{e^{-2\phi}}{2\cdot 3!}H_3^2+\frac{1}{2\cdot 3!}F_3^2+\frac{1}{2\cdot 5!}F_5^2\right]\ ,\ F_3=dC_2\ ,\ H_3=dB_2\ ,\ F_5=dC_4-C_2\wedge H_3$$

The equations of motion that stem from this action are*

$$\nabla^\mu(e^{-2\phi}H_{3,\mu\nu\rho})+\frac{1}{4}F_{5,\nu\rho\alpha\beta\gamma}{F_3}^{\alpha\beta\gamma}=0\quad ,\quad \nabla^\mu F_{3,\mu\nu\rho}+\frac{1}{4}F_{5,\nu\rho\alpha\beta\gamma}{H_3}^{\alpha\beta\gamma}=0$$

$$\nabla^\mu F_{5,\mu\nu\rho\sigma\tau}=0\quad\to\quad F_{5,\mu\nu\rho\sigma\tau}=\frac{\epsilon_{\mu\nu\rho\sigma\tau}}{\sqrt{g}}\frac{2N_c}{3\ell_s}$$

Substituting

$$\nabla^\mu(e^{-2\phi}H_{3,\mu\nu\rho})+\frac{N_c}{6\ell_s}\frac{\epsilon_{\nu\rho\alpha\beta\gamma}}{\sqrt{g}}{F_3}^{\alpha\beta\gamma}=0\quad ,\quad \nabla^\mu F_{3,\mu\nu\rho}+\frac{N_c}{6\ell_s}\frac{\epsilon_{\nu\rho\alpha\beta\gamma}}{\sqrt{g}}{H_3}^{\alpha\beta\gamma}=0$$

We finally decouple the equations:

$$\nabla^\mu\left[\nabla^\nu(e^{-2\phi}H_{3,\mu\rho\sigma}+\text{cyclic}\right]+\frac{N_c^2}{12\cdot 5!\ell_s^2}H_{3,\nu\rho\sigma}=0$$

and a similar one for F_3. This equation has uniform N_c scaling for $e^\phi\sim\frac{\lambda}{N_c}$

- Both B_2 and C_2 combine to a massive two-tensor, that is dual to the $C-odd$ non-conserved operator $Tr[F_{[\mu a}F^{ab}F_{b\nu]}+\frac{1}{4}F_{ab}F^{ab}F_{\mu\nu}]$ with UV dimension 6.

RETURN

$D_0 - F_1$ charges

We may dualize $C_2 \to C_1$

$$(F_3)_{\mu\nu\rho} = \frac{\epsilon_{\mu\nu\rho\sigma\tau}}{2\sqrt{g}}\left(F^{\sigma\tau} + \frac{N_c}{\ell_s}B^{\sigma\tau}\right) \quad , \quad F = dC_1$$

The equations become

$$\nabla^\mu\left(e^{-2\phi}H_{\mu\nu\rho}\right) + \left(\frac{N_c}{2\ell_s}\right)^2 B_{\nu\rho} + \frac{N_c}{4\ell_s}F_{\nu\rho} = 0 \quad , \quad \nabla^\sigma\left(F_{\sigma\tau} + \frac{N_c}{\ell_s}B_{\sigma\tau}\right) = 0$$

and stem from a Stuckelberg-type action

$$S = -M^3\int d^5x\sqrt{g}\left[\frac{e^{-2\phi}}{2\cdot 3!}H_3^2 + \frac{1}{4}\left(F_{\mu\nu} + \frac{N_c}{\ell_s}B_{\mu\nu}\right)^2 + \frac{2N_c^2}{9\ell_s^2}\right]$$

Under B_2 gauge transformations C_1 transforms

$$\delta B_2 = d\Lambda_1 \quad , \quad \delta C_1 = -\frac{N_c}{\ell_s}\Lambda_1$$

- This implies that N_c units of fundamental string charge can cancel one unit of C_1 charge.

RETURN

$D_1 - NS_0$ charges

We now dualize $B_2 \to \tilde{B}_1$

$$e^{-2\phi}(H_3)_{\mu\nu\rho} = \frac{\epsilon_{\mu\nu\rho\sigma\tau}}{2\sqrt{g}}\left(\tilde{F}^{\sigma\tau} + \frac{N_c}{\ell_s}C^{\sigma\tau}\right) \quad , \quad \tilde{F} = d\tilde{B}_1$$

The equations become

$$\nabla^\mu\left((F_3)_{\mu\nu\rho}\right) + e^{2\phi}\left(\frac{N_c}{2\ell_s}\right)^2 C_{\nu\rho} + e^{2\phi}\frac{N_c}{4\ell_s}\tilde{F}_{\nu\rho} = 0 \quad , \quad \nabla^\sigma\left[e^{2\phi}\left(F_{\sigma\tau} + \frac{N_c}{\ell_s}B_{\sigma\tau}\right)\right] = 0$$

and stem from a Stuckelberg-type action

$$S = -M^3\int d^5x\sqrt{g}\left[\frac{1}{2\cdot 3!}F_3^2 + \frac{e^{2\phi}}{4}\left(\tilde{F}_{\mu\nu} + \frac{N_c}{\ell_s}C_{\mu\nu}\right)^2 + \frac{2N_c^2}{9\ell_s^2}\right]$$

Under C_2 gauge transformations C_1 transforms

$$\delta C_2 = d\Lambda_1 \quad , \quad \delta\tilde{B}_1 = -\frac{N_c}{\ell_s}\Lambda_1$$

- This implies that N_c units of fundamental D-string charge can cancel one unit of $\tilde{B}_1$ charge.

RETURN

Bosonic string or superstring? II

• Consider the axion a dual to $Tr[F\wedge F]$. We can show that it must come from a RR sector.

In large-N$_c$ YM, the proper scaling of couplings is obtained from

$$\mathcal{L}_{YM} = N_c\ Tr\left[\frac{1}{\lambda}F^2 + \frac{\theta}{N_c}F\wedge F\right] \quad , \quad \zeta \equiv \frac{\theta}{N_c} \sim \mathcal{O}(1)$$

It can be shown

$$E_{YM}(\theta) = N_c^2\ E_{YM}(\zeta) = N_c^2\ E_{YM}(-\zeta) \simeq C_0\ N_c^2 + C_1\theta^2 + C_2\frac{\theta^4}{N_c^2} + \cdots$$

Witten

In the string theory action

$$S \sim \int e^{-2\phi}[R+\cdots] + (\partial a)^2 + e^{2\phi}(\partial a)^4 + \cdots \quad , \quad e^{\phi} \sim g_{YM}^2 \quad , \quad \lambda \sim N_c e^{\phi}$$

$$\sim \int \frac{N_c^2}{\lambda^2}[R+\cdots] + (\partial a)^2 + \frac{\lambda^2}{N_c^2}(\partial a)^4 + \cdots \quad , \quad a = \theta[1+\cdots]$$

RETURN

bosonic string or superstring?

• The string theory must have no on-shell fermionic states at all because there are no gauge invariant fermionic operators in pure YM. (even in the presence of quarks and modulo baryons that are expected to be solitonic).

♠ We do expect a superstring however since there should be RR fields.

♠ A RR field we expect to have is the RR 4-form, as it is necessary to "seed" the D_3 branes responsible for the gauge group.

• It is non-propagating in 5D

• We will see later however that it is responsible for the non-trivial IR structure of the gauge theory vacuum.

• The most solid indication: There is a direct argument that the axion, dual to the instanton density $F \wedge F$ must be a RR field (as in $\mathcal{N} = 4$).

• Therefore the string theory must be a 5d-superstring theory resembling the II-0 class.

The minimal effective string theory spectrum

- NS-NS $\quad \rightarrow \quad g_{\mu\nu} \leftrightarrow T_{\mu\nu} \quad , \quad B_{\mu\nu} \leftrightarrow Tr[F]^3 \quad , \quad \phi \leftrightarrow Tr[F^2]$

- RR $\quad \rightarrow \quad$ Spinor$_5 \times$Spinor$_5 = F_0 + F_1 + F_2 + (F_3 + F_4 + F_5)$

♠ $F_0 \leftrightarrow F_5 \rightarrow \quad C_4$, background flux $\rightarrow$ no propagating degrees of freedom.

♠ $F_1 \leftrightarrow F_4 \rightarrow \quad C_3 \leftrightarrow C_0$: C_0 is the axion, C_3 its 5d dual that couples to domain walls separating oblique confinement vacua.

♠ $F_2 \leftrightarrow F_3 \rightarrow \quad C_1 \leftrightarrow C_2$: C_2 mixes with B_2 because of the C_4 flux, and is massive. C_1 is associated with baryon number (as we will also see later when we add flavor).

- In an ISO(3,1) invariant vacuum solution, only $g_{\mu\nu}, \phi, C_0 = a$ can be non-trivial.

$$ds^2 = e^{2A(r)}(dr^2 + dx_4^2) \quad , \quad a(r), \phi(r)$$

The relevant "defects"

- $B_{\mu\nu} \to$ Fundamental string (F_1). This is the YM (glue) string: fundamental tension $\ell_s^2 \sim \mathcal{O}(1)$

- Its dual $\tilde{B}_\mu \to NS_0$: Tension is $\mathcal{O}(N_c^2)$. It is an effective magnetic baryon vertex binding N_c magnetic quarks.

- $C_5 \quad \to D_4$: Space filling flavor branes. They must be introduced in pairs: $D_4 + \bar{D}_4$ for charge neutrality/tadpole cancelation $\quad \to \quad$ gauge anomaly cancelation in QCD.

- $C_4 \to D_3$ branes generating the gauge symmetry.

.

• $C_3 \to D_2$ branes : domain walls separating different oblique confinement vacua (where $\theta_{k+1} = \theta_k + 2\pi$). Its tension is $\mathcal{O}(N_c)$

• $C_2 \to D_1$ branes: These are the magnetic strings:
(strings attached to magnetic quarks) with tension $\mathcal{O}(N_c)$

• $C_1 \to D_0$ branes. These are the baryon vertices: they bind N_c quarks, and their tension is $\mathcal{O}(N_c)$.
Its instantonic source when we add flavor is the (solitonic) baryon in the string theory.

• $C_0 \to D_{-1}$ branes: These are the Yang-Mills instantons.

The string effective action

• as $N_c \to \infty$, only string tree-level is dominant.

• Relevant field for the vacuum solution: $g_{\mu\nu}, a, \phi, F_5$.

• The vev of $F_5 \sim N_c\ \epsilon_5$. It appears always in the combination $e^{2\phi}F_5^2 \sim \lambda^2$, with $\lambda \sim N_c\ e^{\phi}$ All higher derivative corrections $(e^{2\phi}F_5^2)^n$ are $\mathcal{O}(1)$.
A non-trivial potential for the dilaton will be generated already at string tree-level.

• This is not the case for all other RR fields: in particular for the axion as $a \sim \mathcal{O}(1)$

$$(\partial a)^2 \sim \mathcal{O}(1) \quad , \quad e^{2\phi}(\partial a)^4 = \frac{\lambda^2}{N_c^2}(\partial a)^4 \sim \mathcal{O}\left(N_c^{-2}\right)$$

Therefore to leading order $\mathcal{O}(N_c^2)$ we can neglect the axion.

The UV regime

- In the far UV, the space should asymptote to AdS$_5$.

- The 't Hooft coupling should behave as ($r \to 0$)

$$\lambda \sim \frac{1}{\log(r\Lambda)} + \cdots \quad \to \quad 0 \quad , \quad r \sim \frac{1}{E}$$

- The effective action to leading order in N_c is

$$S_{eff} \sim \int d^5x \sqrt{g}\; e^{-2\phi} \left(F(R\;,\;\xi\;) + \;4(\partial\phi)^2\;\right) \quad , \quad \xi \equiv -e^{2\phi}\frac{F_5^2}{5!}$$

- For weak background fields

$$F = \frac{2}{3}\frac{\delta c}{\ell_s^2} + R + \frac{1}{2}\xi + \mathcal{O}(R^2, R\xi, \xi^2) \quad , \quad \delta c = 10 - 5 = 5$$

The equation for the four form is

$$\nabla^\mu \left(F_\xi\; F_{\mu\nu\rho\sigma\tau}\right) = 0 \quad , \quad F_\xi\; F_{\mu\nu\rho\sigma\tau} = \frac{N_c}{\ell_{AdS}}\frac{\epsilon_{\mu\nu\rho\sigma\tau}}{\sqrt{g}} \to \xi\; F_\xi(\xi, R)^2 = \frac{\lambda^2}{\ell_{AdS}^2}$$

We may use the alternative action where the 4-form is "integrated-out"

$$S_{\rm tree} = M^3 N_c^2 \int d^5x\sqrt{g}\ \frac{1}{\lambda^2}\left[4\frac{\partial\lambda^2}{\lambda^2} + F(R,\xi) - 2\xi F_\xi(R,\xi)\right] \quad , \quad \xi\ F_\xi^2 = \frac{\lambda^2}{\ell_{AdS}^2}$$

To continue further we must solve $\xi\ F_\xi^2 = \frac{\lambda^2}{\ell_{AdS}^2}$. There are several possibilities:

(a) $\xi \to 0$ as $\lambda \to 0$ (turns out to be inconsistent with equations of motion).

(b) $\xi \to \xi_*(R)$ as $\lambda \to 0$.

$$F \simeq c_0(R) + \frac{c_1(R)}{2}(\xi - \xi_*(R))^2 + \mathcal{O}\left[(\xi - \xi_*(R))^3\right]$$

$$\xi \equiv \xi_*(R) + \delta\xi \simeq \xi_*(R) - \frac{\lambda}{c_1(R)\ \ell_{AdS}\ \sqrt{\xi_*(R)}} + \mathcal{O}(\lambda^2)$$

The gravitational equation implies that for AdS to be the leading solution (at $\lambda = 0$) we must have

$$c_0(R_*) = 0 \quad , \quad \left.\frac{\partial c_0(R)}{\partial R}\right|_{R=R_*} = 0$$

F is therefore zero to next order and the first non-trivial contribution is at quadratic order

$$F(R,\xi) = \frac{\lambda^2}{2c_1(R_*)\ \ell_{AdS}^2\ \xi_*(R_*)} + \frac{1}{2}\left.\frac{\partial^2 c_0(R)}{\partial R^2}\right|_{R=R_*}(R-R_*)^2 + \cdots$$

Solving the equations we find the one-loop β-function coefficients as

$$b_0 = \frac{\ell_{AdS}\sqrt{\xi_*(R_*)}}{16}$$

and the correction subleading correction to the AdS_5 metric

$$e^A = \frac{\ell}{r}\left[1 + \frac{w}{\log(\Lambda r)} + \cdots\right] \quad , \quad \delta R = \frac{40w}{\ell^2\log(\Lambda r)} + \cdots$$

$$w = \frac{-5 + \frac{\frac{\delta\xi_*}{\delta R}(R*)}{\xi_*(R_*)}R_*}{c_0''(R_*)}\ \frac{\xi_*(R_*)}{80R_*}$$

• This turns out to be a regular expansion of the solution in powers of

$$\frac{P_n(\log\log(r\Lambda))}{(log(r\Lambda))^n}$$

• Effectively this can be rearranged as a "perturbative" expansion in $\lambda(r)$. In the case of running coupling, the radial coordinate can be substituted by $\lambda(r)$.

• Using λ as a radial coordinate the solution for the metric can be written

$$E \equiv e^A = \frac{\ell}{r(\lambda)}\left[1 + c_1\lambda + c_2\lambda^2 + \cdots\right] = \ell\,(e^{-\frac{b_0}{\lambda}})\left[1 + c'_1\lambda + c'_2\lambda^2 + \cdots\right] \quad , \quad \lambda -$$

The axion

Similar arguments lead to an action of the form

$$S = N_c^2 \; S_{g,\phi} + S_{axion} + \cdots$$

$$S_{axion} \sim \int d^5x \sqrt{g}\; G(R,\lambda)\; (\partial a)^2$$

- Higher powers of $(\partial a)^2$ are subleading in N_c.

- We may therefore find the solution using the solution of the metric-dilaton system.

UV conclusions

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Conclusion 1: The asymptotic AdS_5 is stringy, but the rest of the geometry is "perturbative around the asymptotics". We cannot however do computations even if we know the structure.

Conclusion 2: It has been a mystery how can one get free field theory at the boundary. This is automatic here since all non-trivial connected correlators are proportional to positive powers of λ that vanishes in the UV.

The axion background

- The axion solution can be interpreted as a "running" θ-angle

- This is in accordance with the absence of UV divergences (all correlators $\langle Tr[F\wedge F]^n\rangle$ are UV finite), and Seiberg-Witten type solutions.

- The axion action is down by $1/N_c^2$

$$S_{axion} = -\frac{M_p^3}{2}\int d^5x\sqrt{g}\ Z(\lambda)\ (\partial a)^2$$

$$\lim_{\lambda\to 0} Z(\lambda) = Z_0\left[1+c_1\lambda+c_2\lambda^2+\cdots\right]\quad , \quad \lim_{\lambda\to\infty} Z(\lambda) = c_a\lambda^d+\cdots\quad , \quad d=4$$

- The equation of motion is

$$\ddot{a}+\left(3\dot{A}+\frac{\dot{Z}(\lambda)}{Z(\lambda)}\right)\dot{a}=0\quad\to\quad \dot{a}=\frac{C\ e^{-3A}}{Z(\lambda)}$$

- The full solution is

$$a(r)=\theta_{UV}+2\pi k+C\ \int_0^r dr\frac{e^{-3A}}{Z(\lambda)}\quad , \quad C=\langle Tr[F\wedge F]\rangle$$

- $a(r)$ is a running effective θ-angle. Its running is non-perturbative,

$$a(r) \sim r^4 \sim e^{-\frac{4}{b_0\lambda}}$$

- The vacuum energy is

$$E(\theta_{UV}) = -\frac{M^3}{2}\int d^5x\sqrt{g}\ Z(\lambda)\ (\partial a)^2 = -\frac{M^3}{2}Ca(r)\ \Big|_{r=0}^{r=r_0}$$

- Consistency requires to impose that $a(r_0) = 0$. This determines C and

$$E(\theta_{UV}) = \frac{M^3}{2}\mathrm{Min}_k\ \frac{(\theta_{UV} + 2\pi k)^2}{\int_0^{r_0} \frac{dr}{e^{3A}Z(\lambda)}}$$

$$\frac{a(r)}{\theta_{UV} + 2\pi k} = \frac{\int_r^{r_0} \frac{dr}{e^{3A}Z(\lambda)}}{\int_0^{r_0} \frac{dr}{e^{3A}Z(\lambda)}}$$

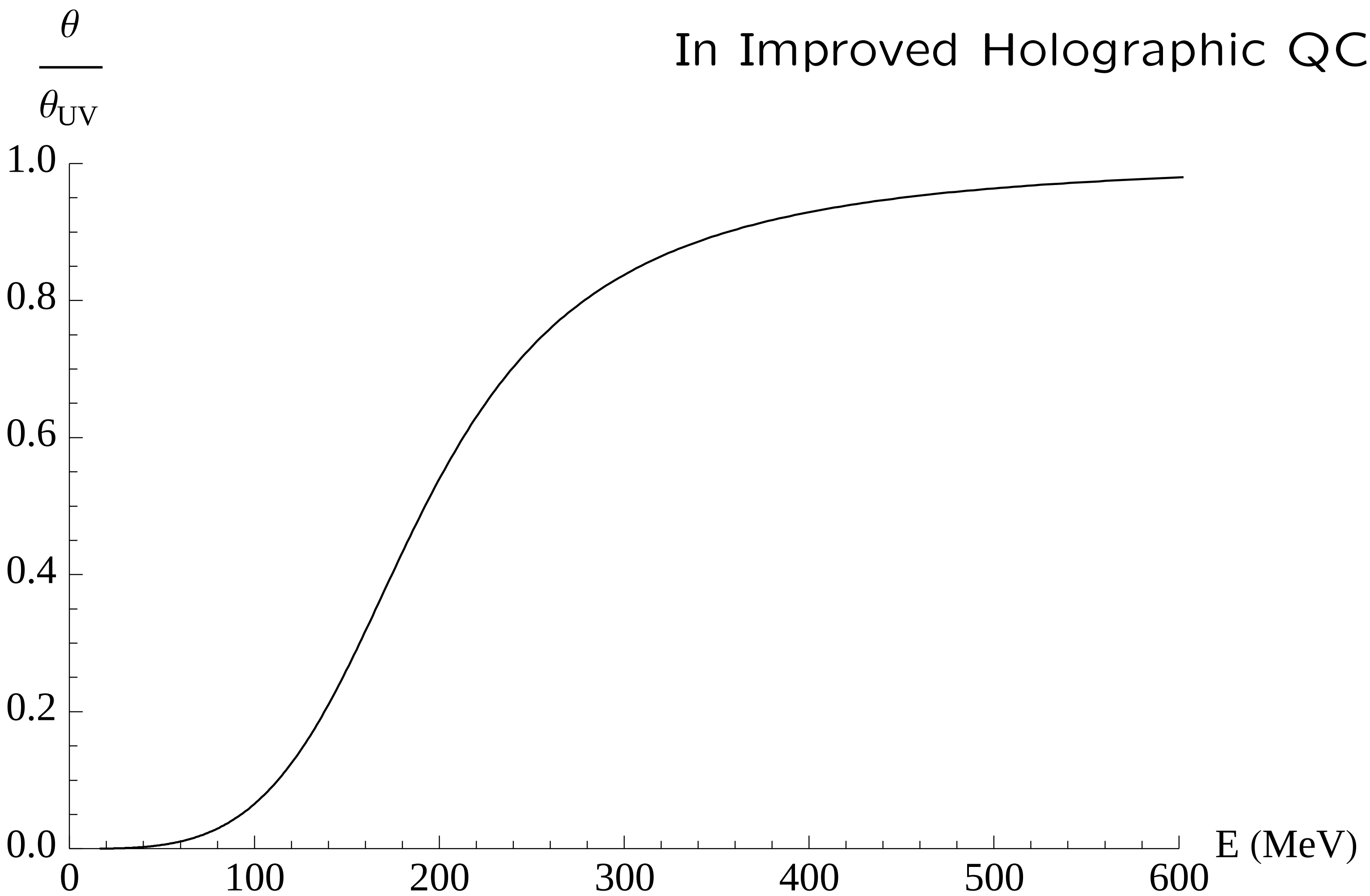
- The topological susceptibility is given by

$$E(\theta) = \frac{1}{2}\chi\ \theta^2 + \mathcal{O}(\theta^4) \quad , \quad \chi = \frac{M^3}{\int_0^{r_0} \frac{dr}{e^{3A}Z(\lambda)}}$$

• The effective θ-angle "runs" also in the D4 model for QCD, and also vanishes in the IR

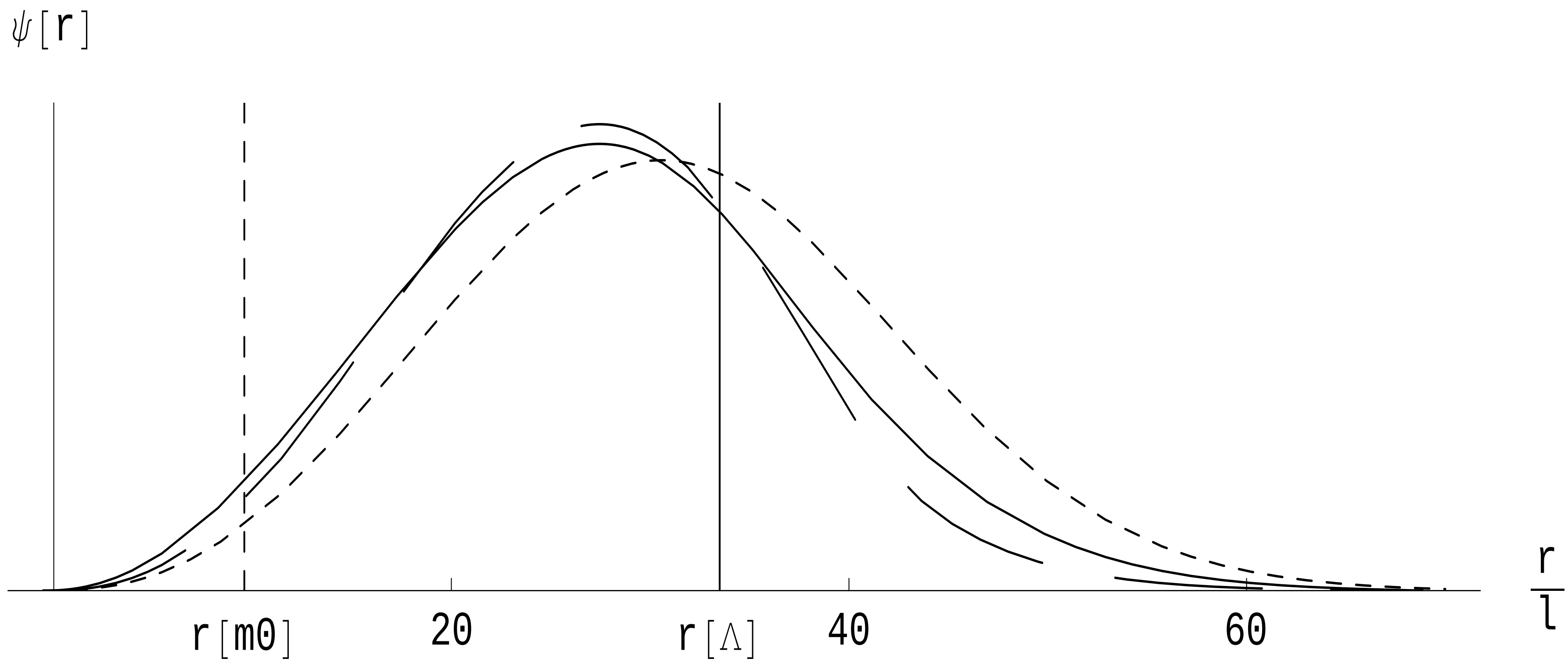
$$\theta(U) = \theta(1 - U_0^3/U^3)$$

• In Improved Holographic QCD:



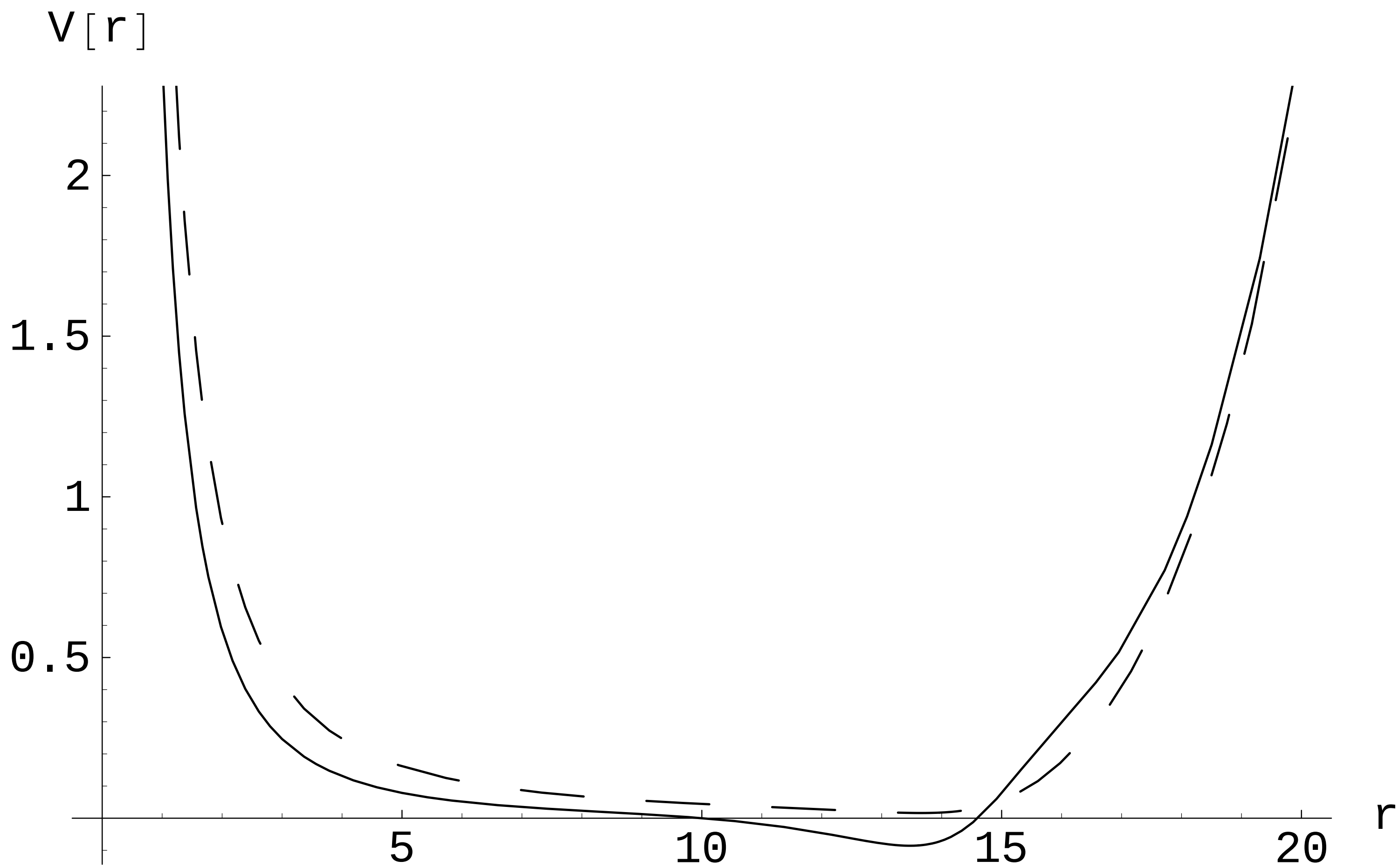
We have taken: $Z(\lambda) = Z_0(1 + c_a\lambda^4) \simeq 133(1 + 0.26\lambda^4)$

The glueball wavefunctions



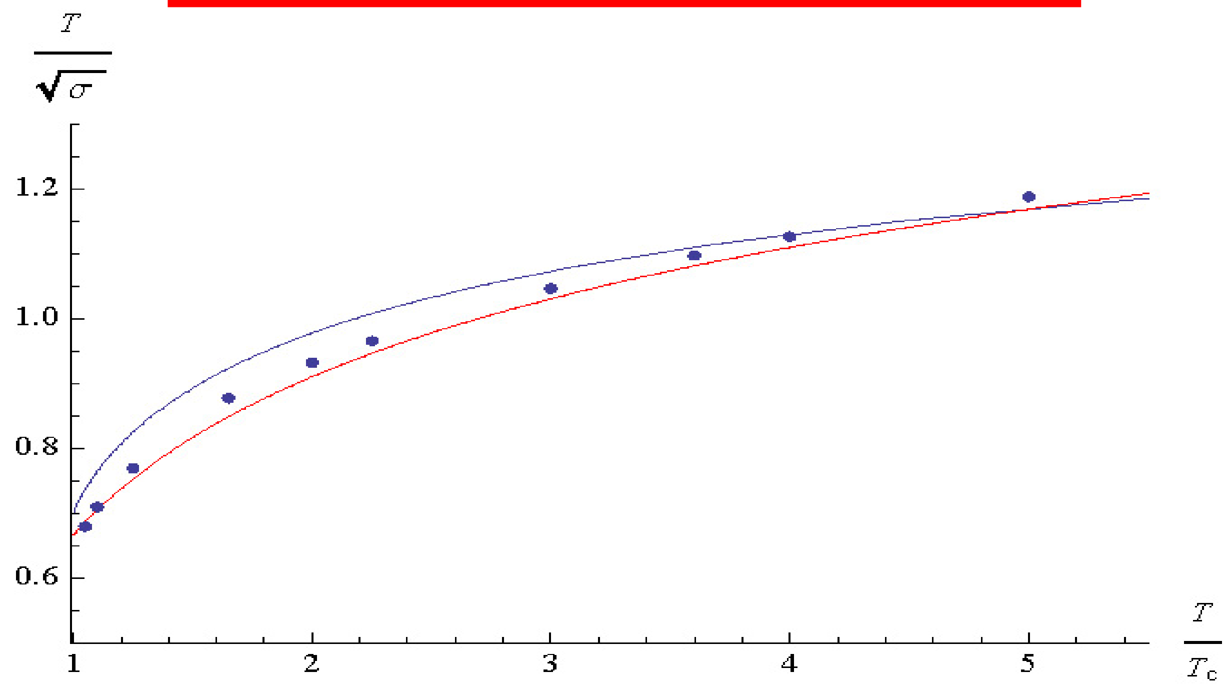
Normalized wave-function profiles for the ground states of the 0^{++} (solid line) ,0^{-+} (dashed line), and 2^{++} (dotted line) towers, as a function of the radial conformal coordinate. The vertical lines represent the position corresponding to $E = m_{0^{++}}$ and $E = \Lambda_p$.

Comparison of scalar and tensor potential



Effective Schrödinger potentials for scalar (solid line) and tensor (dashed line) glueballs. The units are chosen such that $\ell = 0.5$.

Spatial string tension



G. Boyd et al. 1996

- The blue line is the spatial string tension as calculated in Improved hQCD, with no additional fits.

Nitti (unpublished) 2009

- The red line is a semi-phenomenological fit using

$$\frac{T}{\sqrt{\sigma_s}} = 0.51 \left[\log\frac{\pi T}{T_c} + \frac{51}{121}\log\left(2\log\frac{\pi T}{T_c}\right)\right]^{\frac{2}{3}}$$

Alanen+Kajantie+Suur-Uski, 2009

The tachyon WZ action

- The WZ action is given by

Kennedy+Wilkins, Kraus+Larsen, Takayanagi+Terashima+Uesugi

$$S_{WZ} = T_4 \int_{M_5} C \wedge \mathrm{Str}\ \exp\left[i2\pi\alpha'\mathcal{F}\right]$$

- M_5 is the world-volume of the D4-$\overline{\text{D4}}$ branes that coincides with the full space-time.

- C is a formal sum of the RR potentials $C = \sum_n (-i)^{\frac{5-n}{2}} C_n$,

- $\mathcal{F}$ is the curvature of a superconnection $\mathcal{A}$:

$$i\mathcal{A} = \begin{pmatrix} iA_L & T^\dagger \\ T & iA_R \end{pmatrix}, \qquad i\mathcal{F} = \begin{pmatrix} iF_L - T^\dagger T & DT^\dagger \\ DT & iF_R - TT^\dagger \end{pmatrix}$$

$$\mathcal{F} = d\mathcal{A} - i\mathcal{A}\wedge\mathcal{A} \quad , \quad d\mathcal{F} - i\mathcal{A}\wedge\mathcal{F} + i\mathcal{F}\wedge\mathcal{A} = 0$$

• Under (flavor) gauge transformation it transforms homogeneously

$$\mathcal{F} \to \begin{pmatrix} V_L & 0 \\ 0 & V_R \end{pmatrix} \mathcal{F} \begin{pmatrix} V_L^\dagger & 0 \\ 0 & V_R^\dagger \end{pmatrix}$$

• Expanding:

$$S_{WZ} = T_4 \int C_5 \wedge Z_0 + C_3 \wedge Z_2 + C_1 \wedge Z_4 + C_{-1} \wedge Z_6$$

where Z_{2n} are appropriate forms coming from the expansion of the exponential of the superconnection.

• $Z_0 = 0$, signaling the global cancelation of 4-brane charge, which is equivalent to the cancelation of the gauge anomaly in QCD.

$$Z_2 = d\Omega_1 \quad , \quad \Omega_1 = iSTr(V(T^\dagger T))Tr(A_L - A_R) - \log\det(T)d(StrV(T^\dagger T))$$

Casero+Kiritsis+Paredes

• This term provides the Stuckelberg mixing between $Tr[A^L_\mu - A^R_\mu]$ and the QCD axion that is dual to C_3. Dualizing the full action we obtain

$$S_{CP-odd} = \frac{M^3}{2N_c^2} \int d^5x \sqrt{g} Z(\lambda) \left(\partial a + i\Omega_1\right)^2$$

$$= \frac{M^3}{2}\int d^5x\sqrt{g}Z(\lambda)\left(\partial_\mu a + \zeta\partial_\mu V(\tau) - \sqrt{\frac{N_f}{2}}V(\tau)A^A_\mu\right)^2$$

$$\zeta = \Im \log\det T \quad , \quad A_L - A_R \equiv \frac{1}{2N_f}A^A\mathbf{I} + (A^a_L - A^a_R)\lambda^a$$

• This term is invariant under the $U(1)_A$ transformations, reflecting the QCD $U(1)_A$ anomaly.

$$\zeta \to \zeta + \epsilon \quad , \quad A^A_\mu \to A^A_\mu - \sqrt{\frac{2}{N_f}}\partial_\mu\epsilon \quad , \quad a \to a - N_f\epsilon V(\tau)$$

• This is responsible for the mixing between the QCD axion and the η' $\to$ we have two scalars a, ζ and an (axial) vector, A^A_μ. Then an appropriate linear combination of the two scalars will become the 0^{-+} glueball field while the other will be the η'. The transverse (5d) vector will provide the tower of $U(1)_A$ vector mesons.

• The term $C_1 \times Z_4 \sim V\ C_1\,[F_L\wedge F_L + F_R\wedge F_R] + \cdots$ couples the flavor instanton density to the baryon vertex.

• Using $Z_6 = d\Omega_5$ we may rewrite the last term as

$$\int F_0\wedge\Omega_5 \quad , \quad F_0 = dC_{-1}$$

$F_0 \sim N_c$ is nothing else but the dual of the five-form field strength. This term then provides the correct Chern-Simons form that reproduces the flavor anomalies of QCD. It contains the tachyon non-trivially

Casero+Kiritsis+Paredes

• To proceed further and analyze the vacuum solution we set $T = \tau\ \mathbf{1}$ and set the vectors to zero. Then the DBI action collapses to

$$S[\tau, A_M] = N_c N_f \int dr d^4x\ e^{-\Phi} V(\tau) \sqrt{-\det\left(g_{\mu\nu} + \partial_\mu \tau \partial_\nu \tau\right)}$$

We assume the following tachyon potential, motivated/calculated in studies of tachyon condensation:

$$V(\tau) = V_0 e^{-\frac{\mu^2}{2}\tau^2}$$

where μ has dimension of mass. It is fixed by the requirement that τ has the correct bulk mass to couple to the quark bilinear operator on the boundary.

Tachyon dynamics

- In the vacuum the gauge fields vanish and $T \sim 1$. Only DBI survives

$$S[\tau] = T_{D_4} \int dr d^4x \, \frac{e^{4A_s(r)}}{\lambda} \, V(\tau) \, \sqrt{e^{2A_s(r)} + \dot{\tau}(r)^2} \quad , \quad V(\tau) = e^{-\frac{\mu^2}{2}\tau^2}$$

- We obtain the nonlinear field equation:

$$\ddot{\tau} + \left(3\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right)\dot{\tau} + e^{2A_S}\mu^2\tau + e^{-2A_S}\left[4\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right]\dot{\tau}^3 + \mu^2\tau \, \dot{\tau}^2 = 0.$$

- In the UV we expect

$$\tau = m_q \, r + \sigma \, r^3 + \cdots \quad , \quad \mu^2\ell^2 = 3$$

- We expect that the tachyon must diverge before or at $r = r_0$. We find that indeed it does `at the (dilaton) singularity`. For the $r_0 = \infty$

backgrounds

$$\tau \sim \exp\left[\frac{2}{a}\ \frac{R}{\ell^2}\ r\right] \qquad \text{as} \qquad r \to \infty$$

• Generically the solutions have spurious singularities: $\tau(r_*)$ stays finite but its derivatives diverges because:

$$\tau \sim \tau_* + \gamma\sqrt{r_* - r}.$$

The condition that they are absent determines σ as a function of m_q.

• The easiest spectrum to analyze is that of vector mesons. We find $(r_0 = \infty)$

$$\Lambda_{glueballs} = \frac{1}{R}, \qquad \Lambda_{mesons} = \frac{3}{\ell}\left(\frac{\alpha\ell^2}{2R^2}\right)^{(\alpha-1)/2} \propto \frac{1}{R}\left(\frac{\ell}{R}\right)^{\alpha-2}.$$

This suggests that $\alpha = 2$ preferred also from the glue sector (linear trajectories).

Detailed plan of the presentation

- Title page 1 minutes
- Collaborators 2 minutes
- Introduction 4 minutes
- The gauge theory/string theory duality 10 minutes
- The gauge theory at finite temperature 12 minutes
- Critical string theory holography 14 minutes
- AdS/QCD 17 minutes
- The "soft wall" 18 minutes
- A string theory for QCD:(Very) basic expectations 23 minutes
- The low energy spectrum 27 minutes
- general expectations 30 minutes
- Improved Holographic QCD: a model 33 minutes
- The UV solution 40 minutes

- General criterion for confinement 180 minutes
- Classification of confining superpotentials 183 minutes
- Confining β-functions 186 minutes
- The lattice glueball data 187 minutes
- α-dependence of scalar spectrum 188 minutes
- $B_2 - C_2$ mixing 191 minutes
- $D_0 - F_1$ charges 193 minutes
- $D_1 - NS_0$ charges 195 minutes
- Bosonic string or superstring? 200 minutes
- The minimal string theory spectrum 205 minutes
- The relevant "defects" 208 minutes
- The string effective action 210 minutes
- The UV regime 222 minutes
- The axion 223 minutes
- UV conclusions 225 minutes