

Holographic p-wave Superfluids

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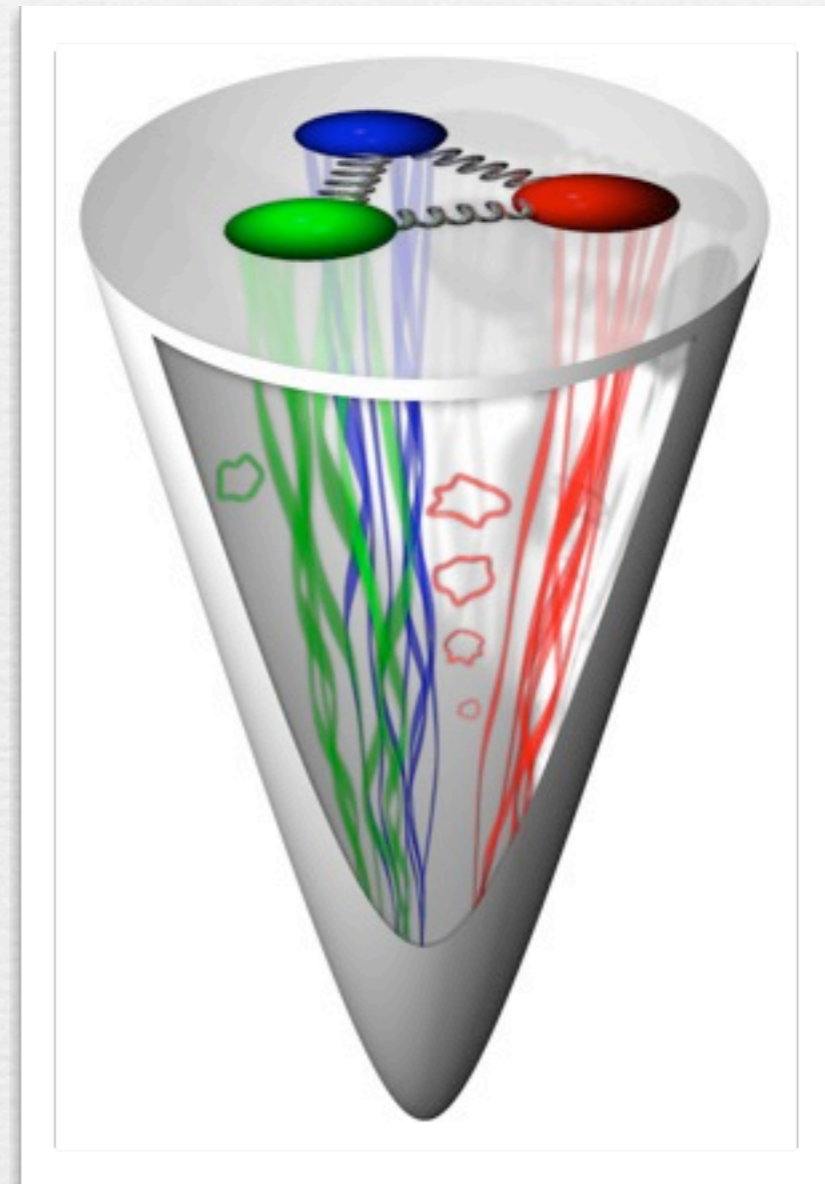
EMMI Workshop, Heidelberg

in collaboration with: M. Ammon, J. Erdmenger,
V. Grass, M. Kaminski, A. O'Bannon

based on: 0810.2316, 0903.1864, 0912.3515

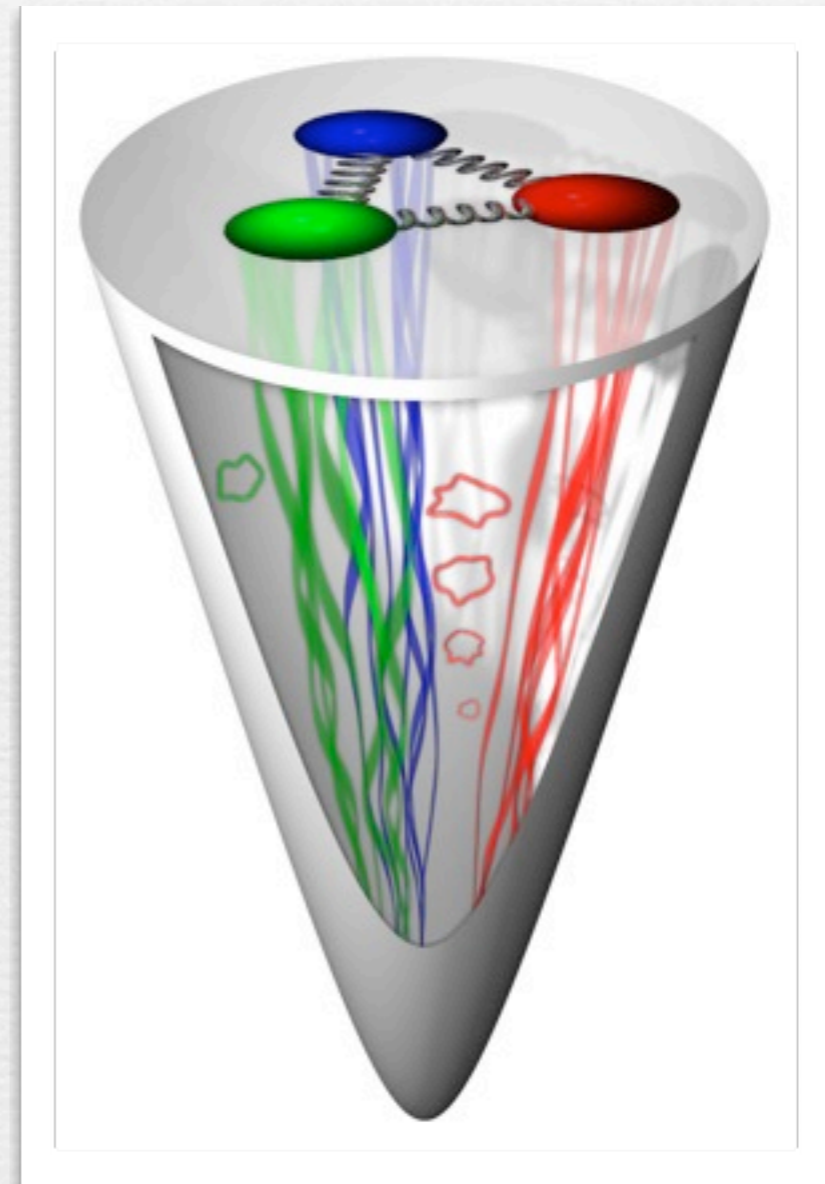
What can we learn about QCD using gauge/gravity duals?

- Inside into strong coupling regime, relevant e.g. for RHIC especially hydrodynamic expansion



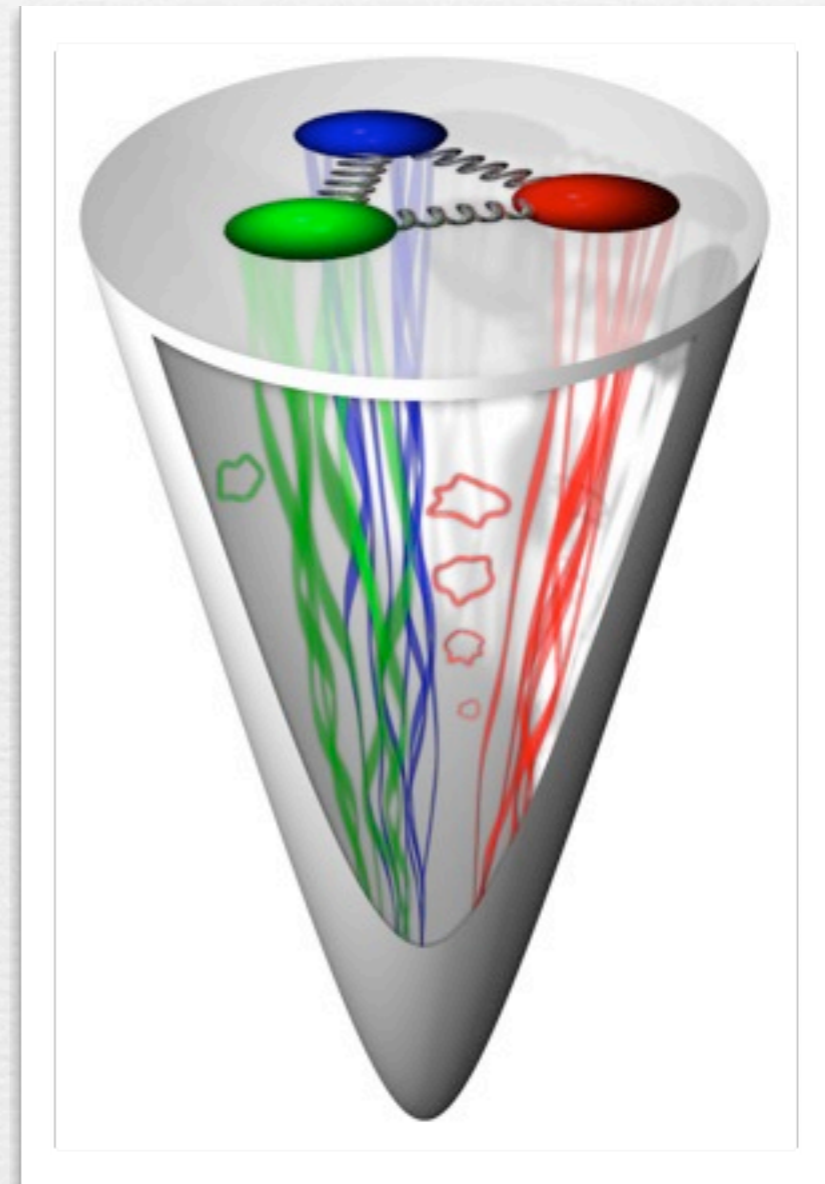
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What can we learn about QCD using gauge/gravity duals?

- Inside into strong coupling regime, relevant e.g. for RHIC especially hydrodynamic expansion
- Finite temperature and finite chemical potentials possible \Rightarrow explore the phase diagram
- What kind of matter can we find?



Meson Superfluids

- QCD has $U(N_f) = U(1) \times SU(N_f)$ global flavor symmetry. For $N_f = 2$, $SU(2)$ is isospin symmetry.

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- For $\mu > M_{\text{meson}}$, mesons condense due to BEC and break $U(1)_3$ spontaneously \Rightarrow Meson Superfluids
- If vector mesons condense \Rightarrow rotational symmetry broken \Rightarrow p-wave superfluids

p-wave Superfluids in Gauge/Gravity duals

❧ First:

0912.3515

Simple gravity model with $SU(2)$ symmetry

❧ Later:

0810.2316, 0903.1864

Embedding into string theory \Rightarrow superfluidity in explicit field theory:

$\mathcal{N} = 4$ $SU(N_c)$ SYM coupled to two $\mathcal{N} = 2$ hypermultiplets

Gauge/Gravity Duality

• Type IIB SUGRA on $AdS_5 \times X_5$

is dual to

Conformal Field Theory at large N_c and large λ

in the sense

$$Z_{\text{SUGRA}} [\phi(x, r)|_{r \rightarrow r_{\text{bdy}}} = \phi_0(x)] = \left\langle e^{\int d^4x \phi_0(x) \mathcal{O}(x)} \right\rangle$$

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• Black holes correspond to thermal field theories

• Gauge fields A_μ^a are dual to global currents J_a^μ
especially vevs A_t^0, A_t^3 induce finite chemical potentials μ_B, μ_I (source) and finite densities $\langle J_0^t \rangle, \langle J_3^t \rangle$

Gravity model


- Einstein-Yang-Mills theory with $SU(2)$ gauge group

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2\kappa_5^2} (R - \Lambda) - \frac{1}{4\hat{g}^2} F_{\mu\nu}^a F^{a\mu\nu} \right] \quad \alpha = \frac{\kappa_5}{\hat{g}}$$

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- Probe limit $\alpha = 0$ studied by Gubser and Pufu '08

Interpretation of α

- Holographic calculations of Weyl anomaly:
 $1/\kappa_5^2 \propto c$, c : number of degrees of freedom
- Correlators of $SU(2)$ currents proportional to $1/\hat{g}^2$
 $\Rightarrow 1/\hat{g}^2$ counts degrees of freedom charged under $SU(2)$.

• Intuitively,

$$\alpha^2 = \frac{\kappa_5^2}{\hat{g}^2} \propto \frac{\# \text{ charged degrees of freedom}}{\# \text{ total degrees of freedom}}$$

Behavior at finite chemical potential

• Reissner-Nordström black hole

$$ds^2 = -N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2 d\vec{x}^2$$

$$N(r) = r^2 - \frac{2m_0}{r} + \frac{2\alpha^2 q^2}{3r^4}$$

$$A = \left(\mu - \frac{q}{r} \right) \tau^3 dt$$

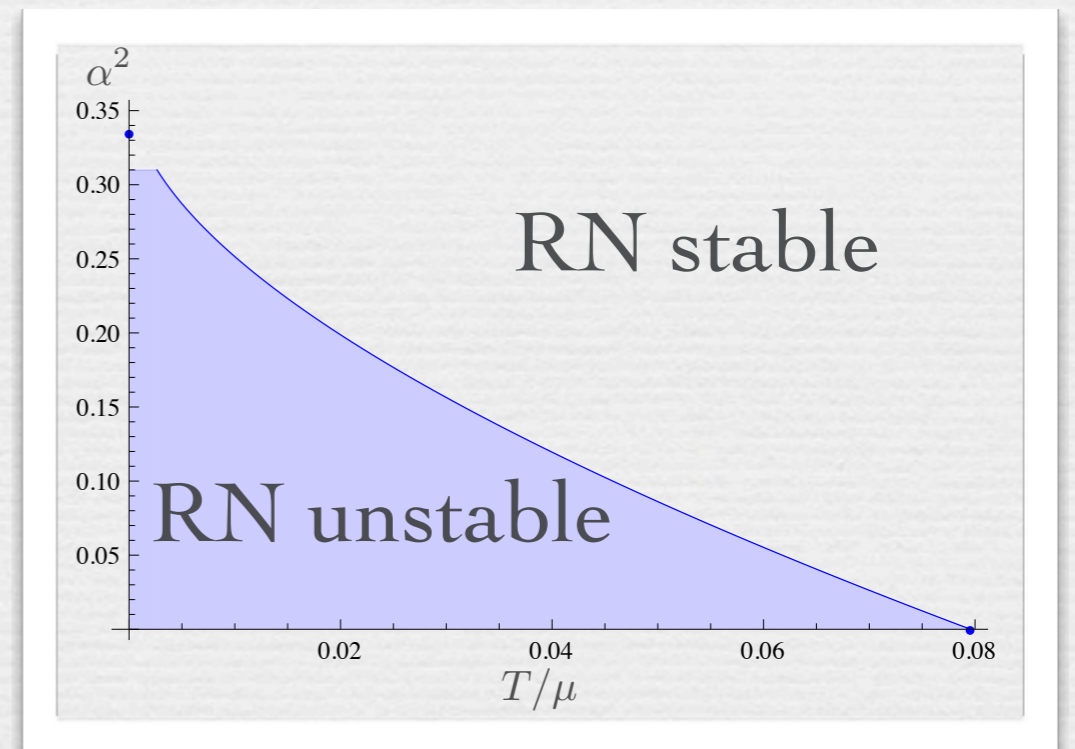
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- At low temperature the black hole unstable against fluctuations in $A_x^1 \Rightarrow$ Condensation

Condensation Process

Gubser

↪ Sketchy action for A_x^1 :

$$S \sim \partial_\mu A_x^1 \partial^\mu A_x^1 + \underbrace{2g^{tt} g^{xx} (A_t^3)^2}_{=m_{\text{eff}}} (A_x^1)^2$$

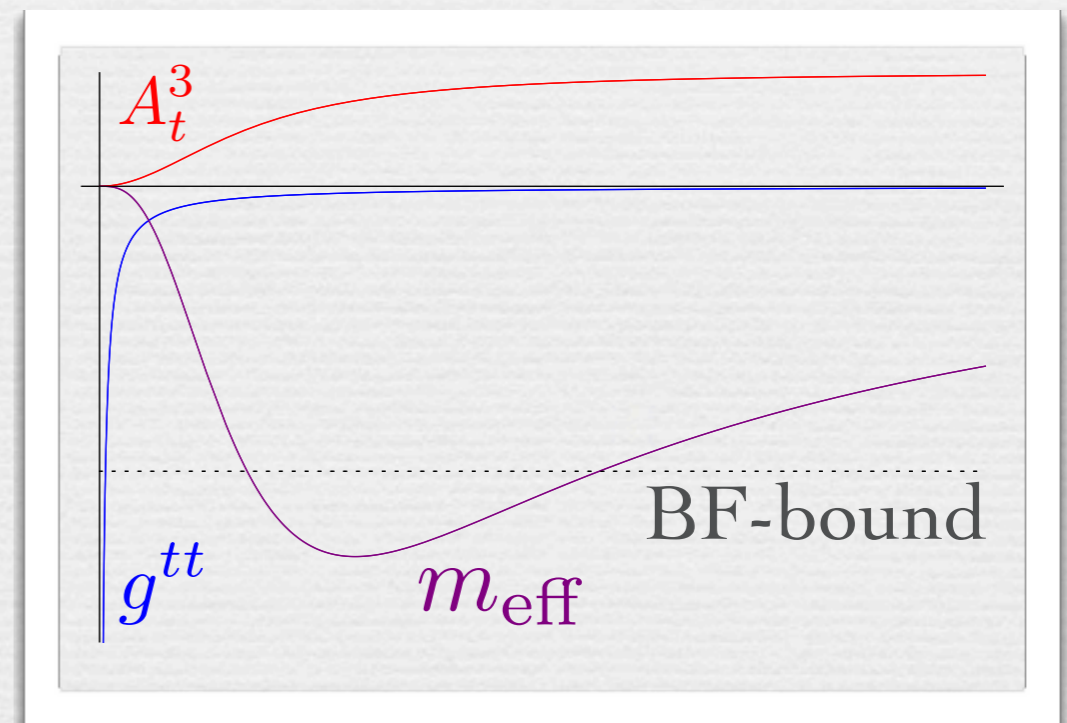
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- Due to black hole $g^{tt} \rightarrow -\infty$,
 m_{eff} can be lower than BF-bound
 \Rightarrow Instability
 \Rightarrow Condensation \Rightarrow vector hair



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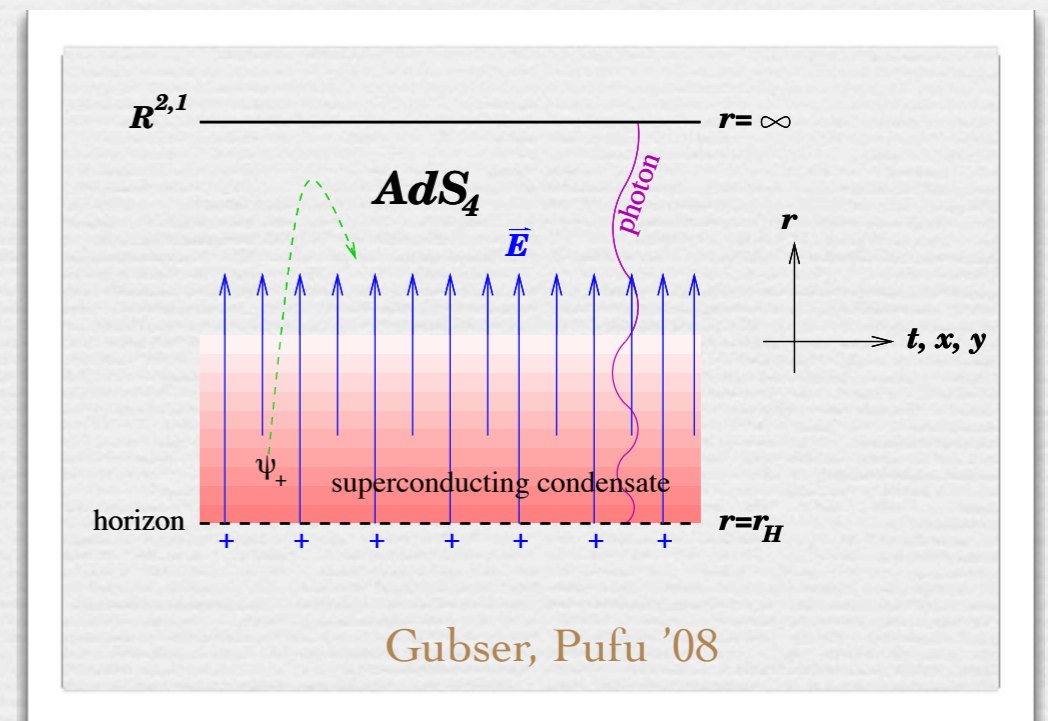
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- Hair is stabilized by the equilibrium of electric and gravitational force in AdS space.



Solutions in the broken phase

- We numerically solve the Einstein-Yang-Mills equations for the ansatz

$$ds^2 = - N(r)\sigma(r)^2 dt^2 + \frac{1}{N(r)} dr^2 \\ + r^2 f(r)^{-4} dx^2 + r^2 f(r)^2 (dy^2 + dz^2)$$

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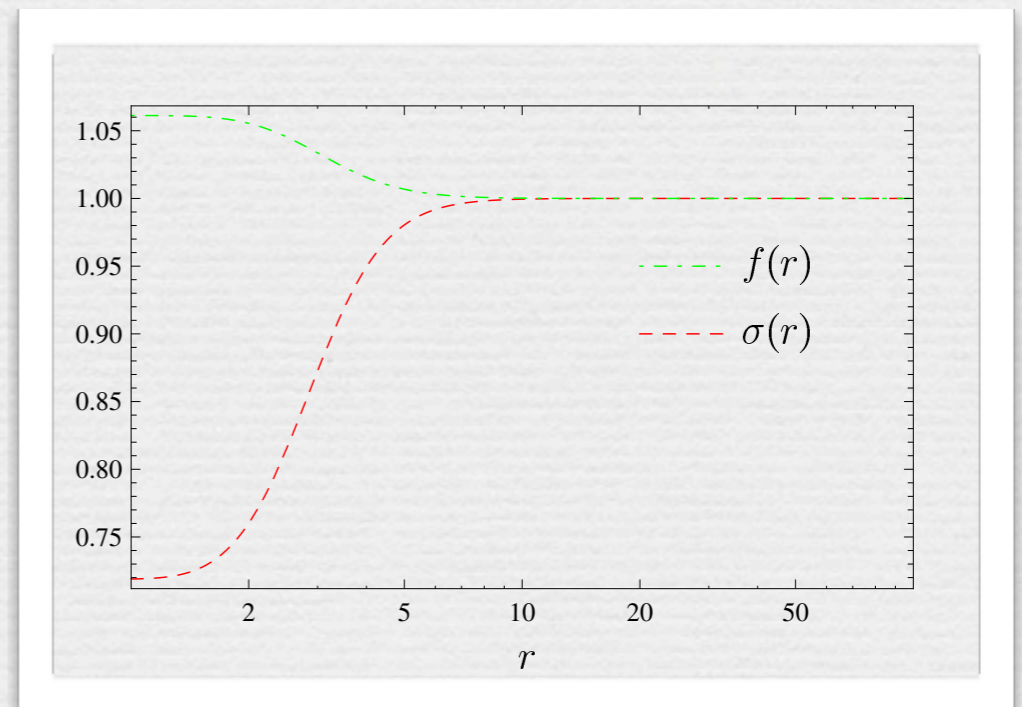
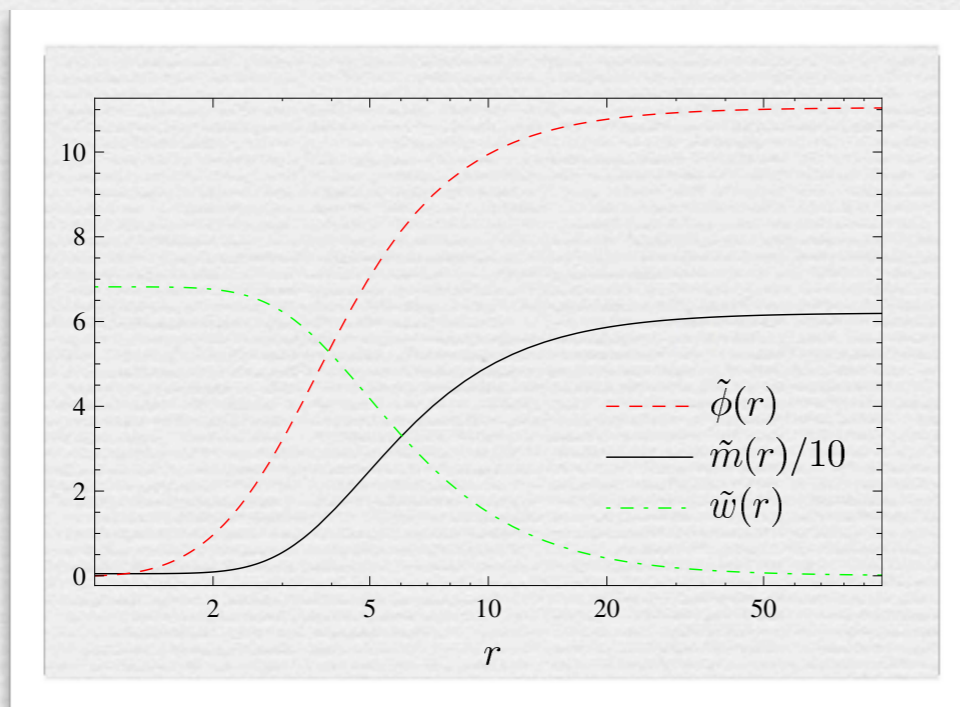
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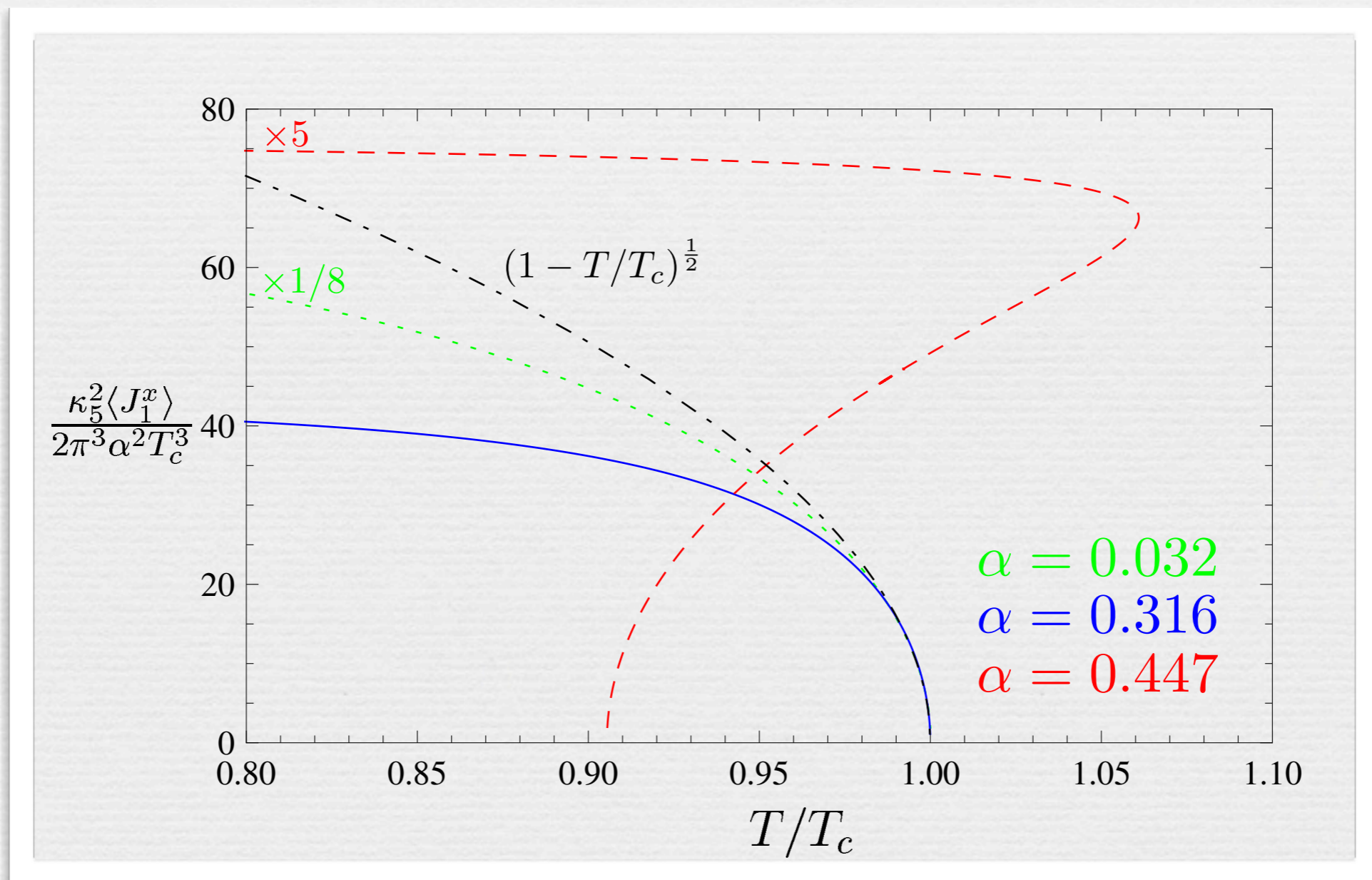
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- Typical solution for $w \neq 0$:



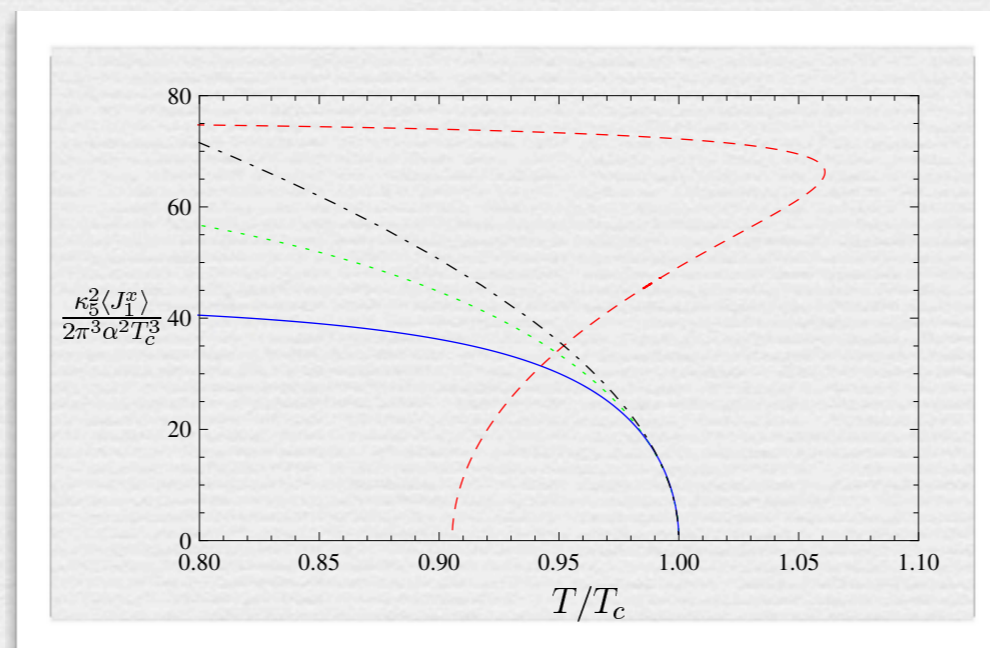
Phase transition

- Order parameter $\langle J_1^x \rangle$ determined by boundary behavior of A_x^1 .



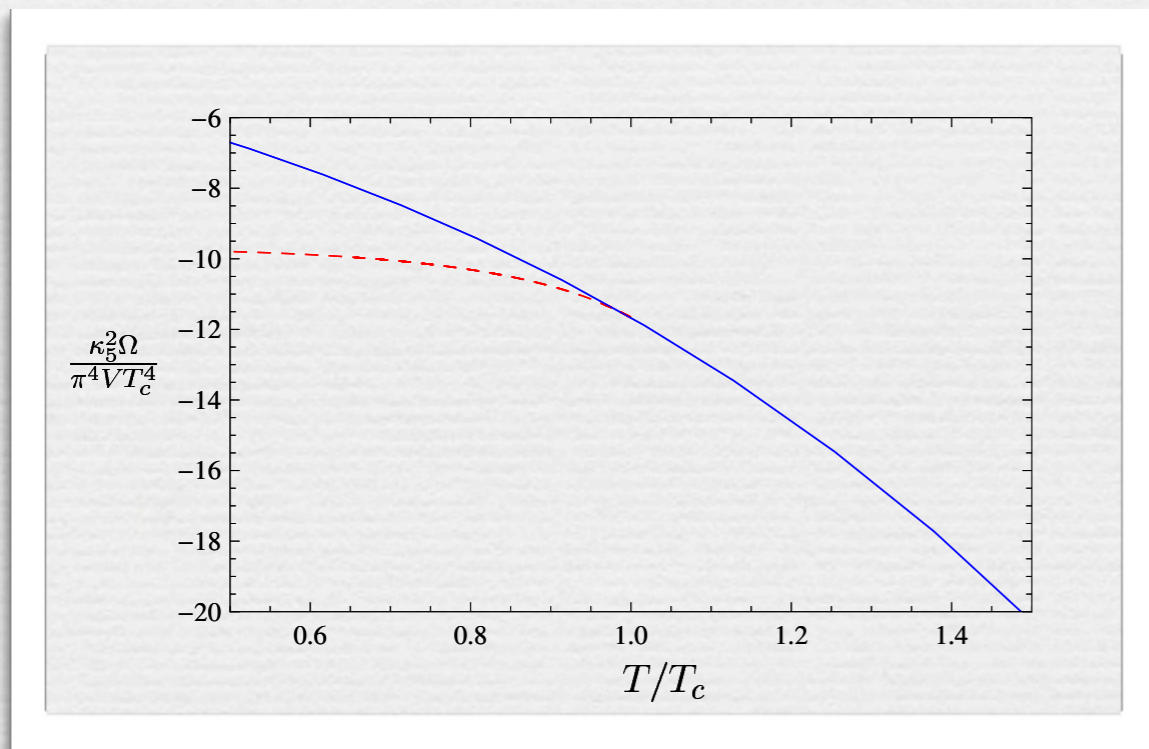
Phase transition

- Order parameter $\langle J_1^x \rangle$ determined by boundary behavior of A_x^1 .
- For $\alpha < \alpha_c$ order parameter increases monotonically \Rightarrow 2nd order transition (mean field)
- For large $\alpha > \alpha_c$ order parameter becomes multivalued \Rightarrow 1st order transition

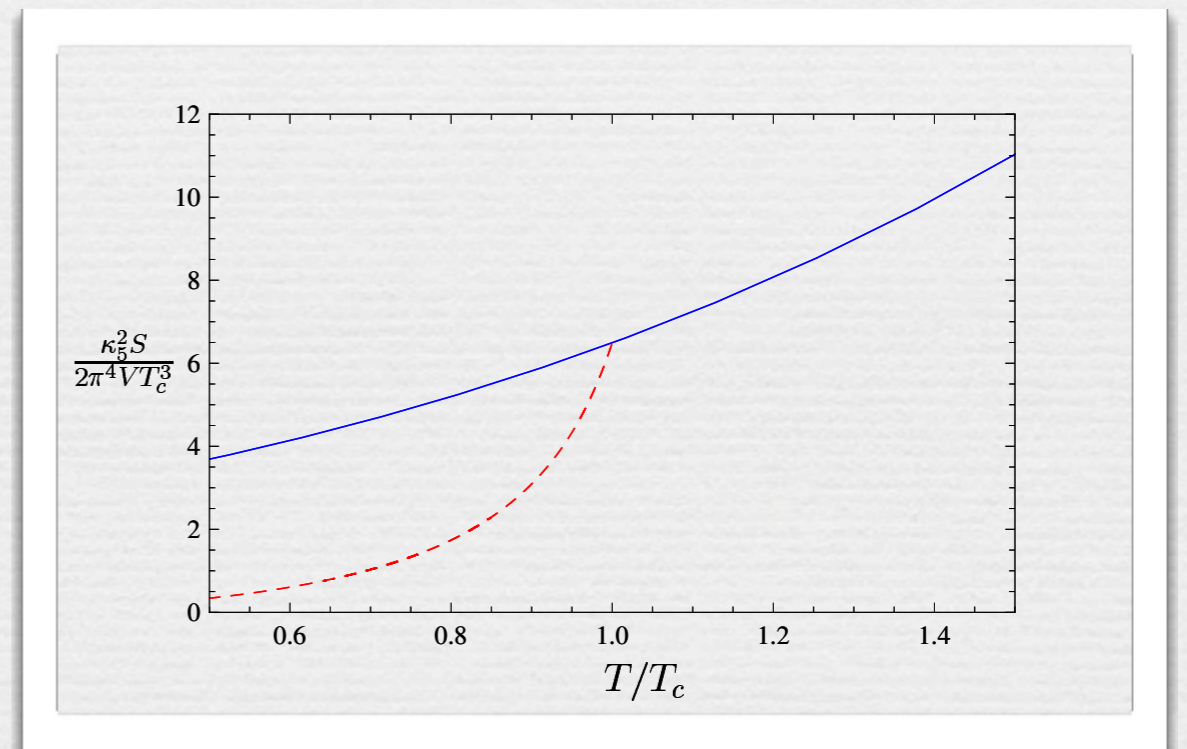


Thermodynamics

$$\alpha < \alpha_c$$



Grand potential

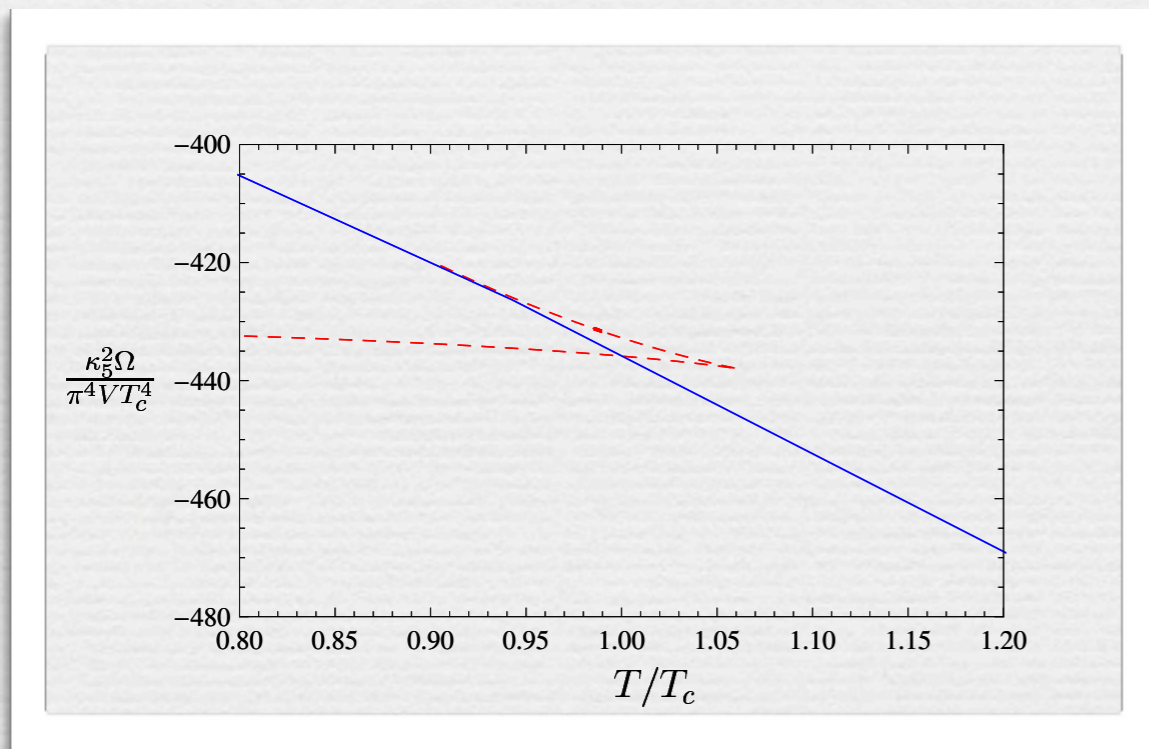


Entropy

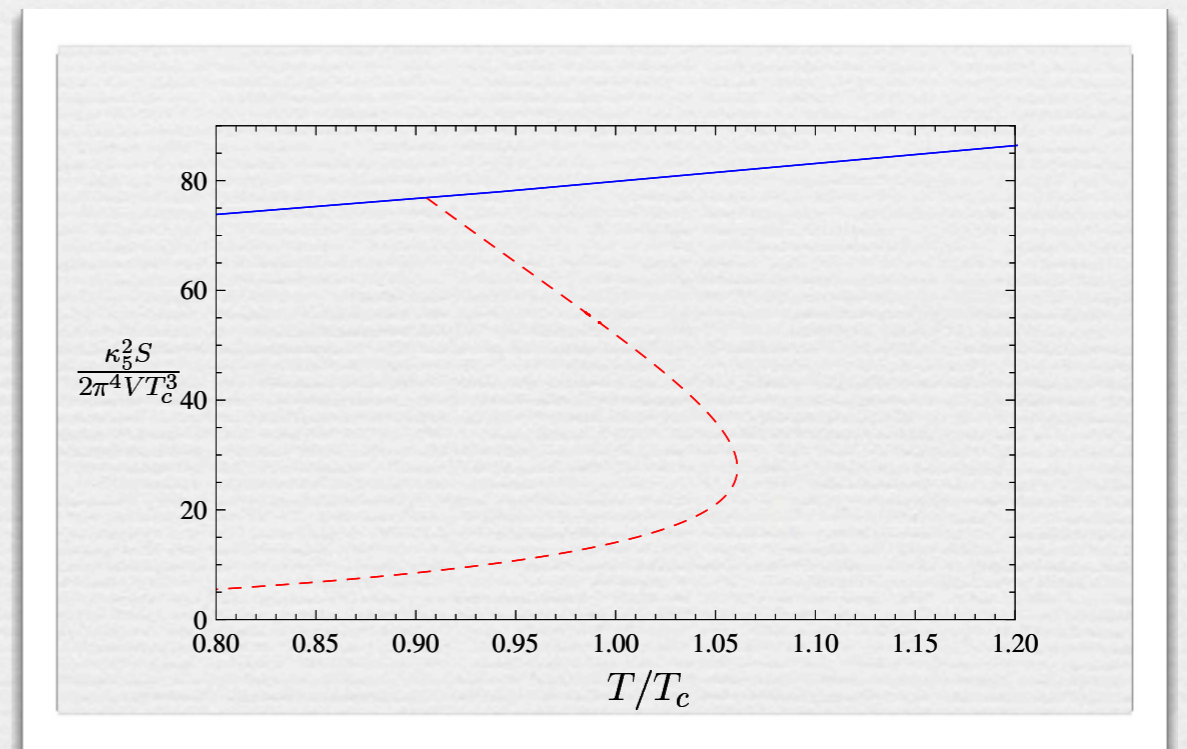
normal phase
superfluid phase

Thermodynamics

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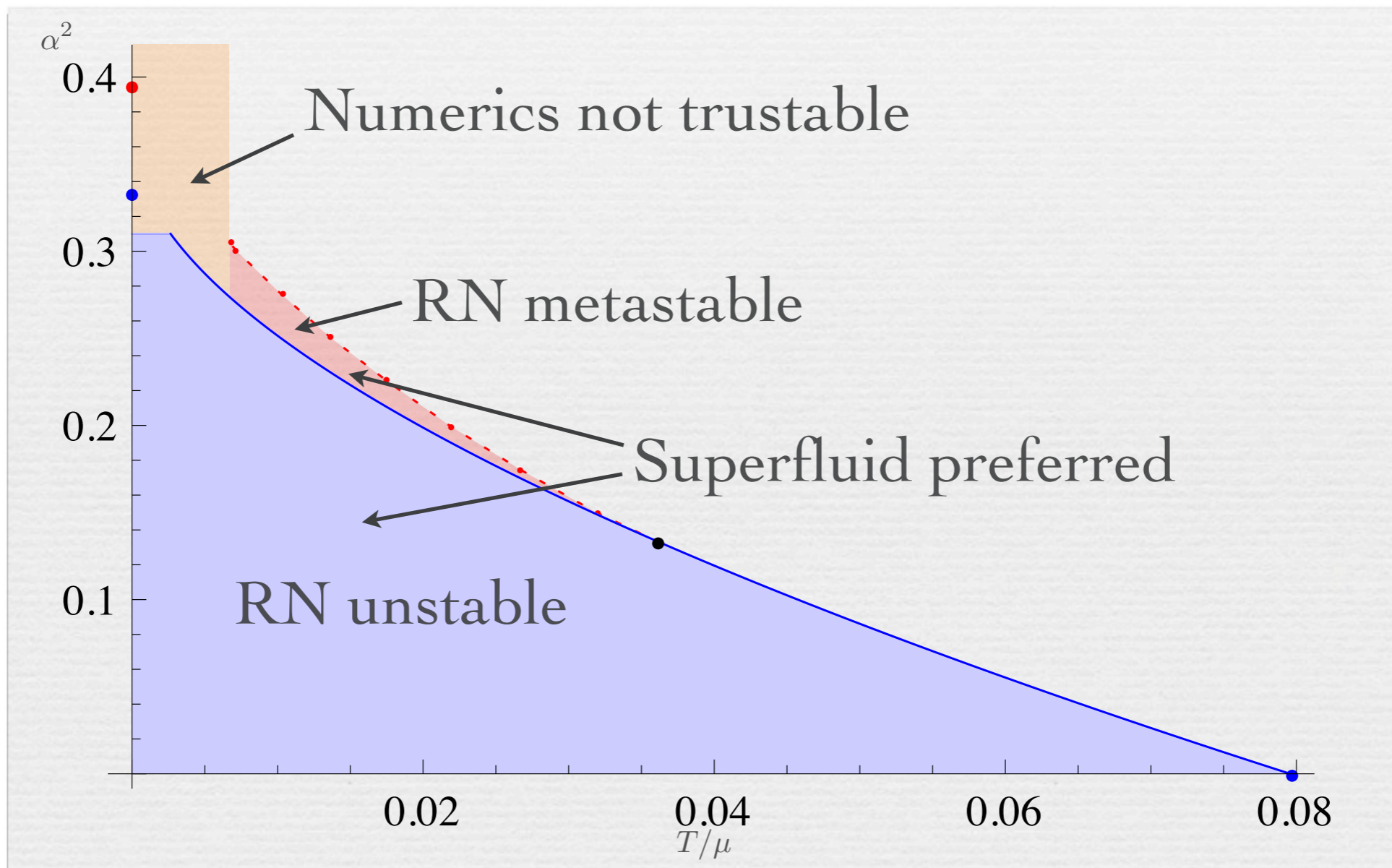
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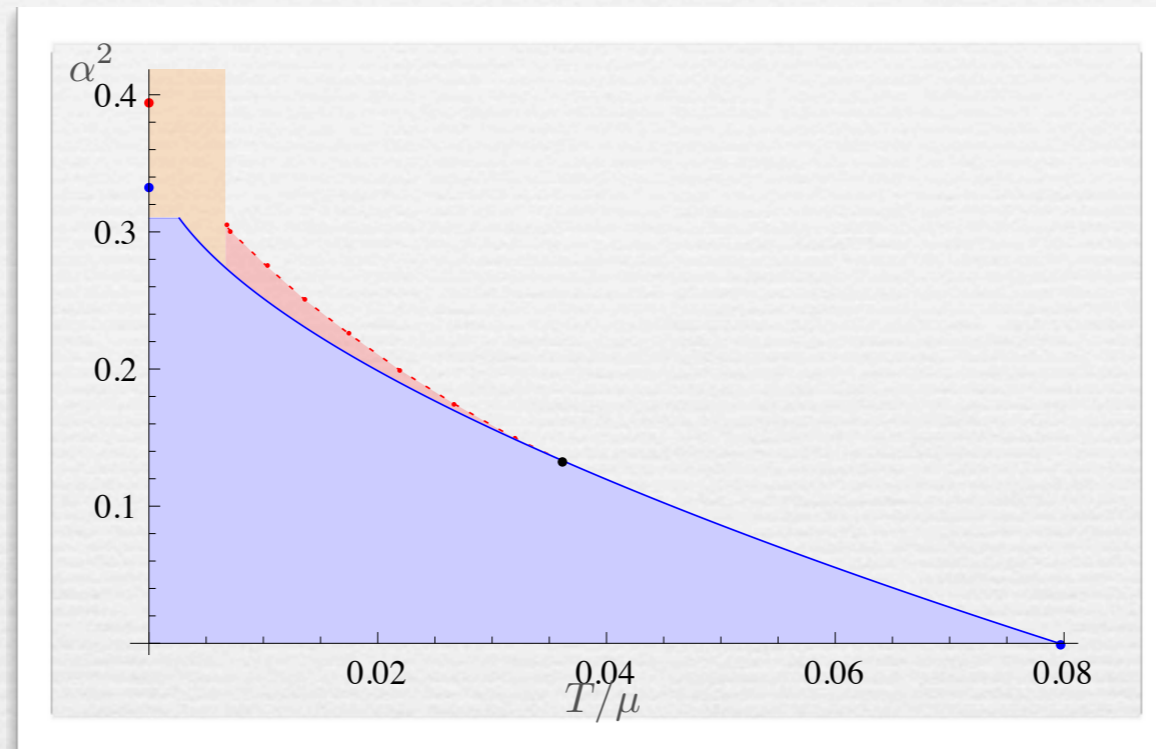
Entropy

normal phase
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“Phase diagram”



“Phase diagram”



- Superfluid thermodynamically preferred in red and blue region
- Phase transition second order for $\alpha < \alpha_c$ and first order for $\alpha > \alpha_c$ with $\alpha_c = 0.365$
- For $\alpha > 0.628$ no superfluid phase available

Embedding in String Theory Setup

- Gravity model can be embedded into D3/D7 brane setup (see Johanna's lecture)
- D3/D7 brane setup dual to $\mathcal{N} = 4$ $SU(N_c)$ SYM coupled to $\mathcal{N} = 2$ hypermultiplets
- The D7-branes provide a non-Abelian gauge field
⇒ Global flavor symmetry

Embedding in String Theory

non-Abelian DBI action

- Need non-Abelian DBI action (best guess, correct up to $(\alpha')^4$)

$$S_{\text{DBI}} = T_{D7} \text{Str} \int d^8 \xi \sqrt{\det Q} \sqrt{\det \left(P_{ab} \left[E_{MN} + E_{Mi} (Q^{-1} - \delta)^{ij} E_{jN} \right] + 2\pi\alpha' F_{ab} \right)}$$

$$E_{MN} = g_{MN} + B_{MN} \quad Q_j^i = \delta_j^i + i2\pi\alpha' [\Phi^i, \Phi^k] E_{kj} \quad \begin{array}{l} M, N = 0, \dots, 9 \\ a, b = 0, \dots, 7 \\ i, j = 8, 9 \end{array}$$

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- Use symmetries of our setup Erdmenger, Kamiski, PK, Rust 0807.2663

$$\Phi^9 = 0 \quad \Rightarrow \quad [\Phi^i, \Phi^j] = 0 \quad \Rightarrow \quad Q_j^i = \delta_j^i$$

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Looks like
Abelian action



String Theory Embedding

Evaluation of Str

- Str prescription only correct up to fourth order

- We use 2 different approaches:

- 1) expand action to fourth order

- +: include maximal number of terms we can trust

- : approximation breaks down in superfluid phase

- 2) adapt Str prescription: set $(\tau^i)^2 = 1$ inside Str

- +: can handle **all** terms

- : strictly valid only if $N_f \rightarrow \infty$ contradict probe approximation

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approximation

give the same physics!!!

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Comparison to Gravity Model

- D7-branes are probes in D3-brane background.
Background determined by Type IIB SUGRA

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Background determined by Type IIB SUGRA

$$S_{\text{IIB}} \supset N_c^2 \int d^5x \sqrt{-g} R$$

- DBI action determines embedding of the D7-branes and the gauge fields on these branes

$$S_{\text{DBI}} = -T_{D7} \int d^8\xi \sqrt{\det (P[g] + 2\pi\alpha' F)}$$
$$\supset N_c N_f \int d^5x \sqrt{-g} F^2$$

Embedding in String Theory

Comparison to Gravity Model

- D7-branes are probes in D3-brane background. Background determined by Type IIB SUGRA

$$S_{\text{IIB}} \supset N_c^2 \int d^5x \sqrt{-g} R$$

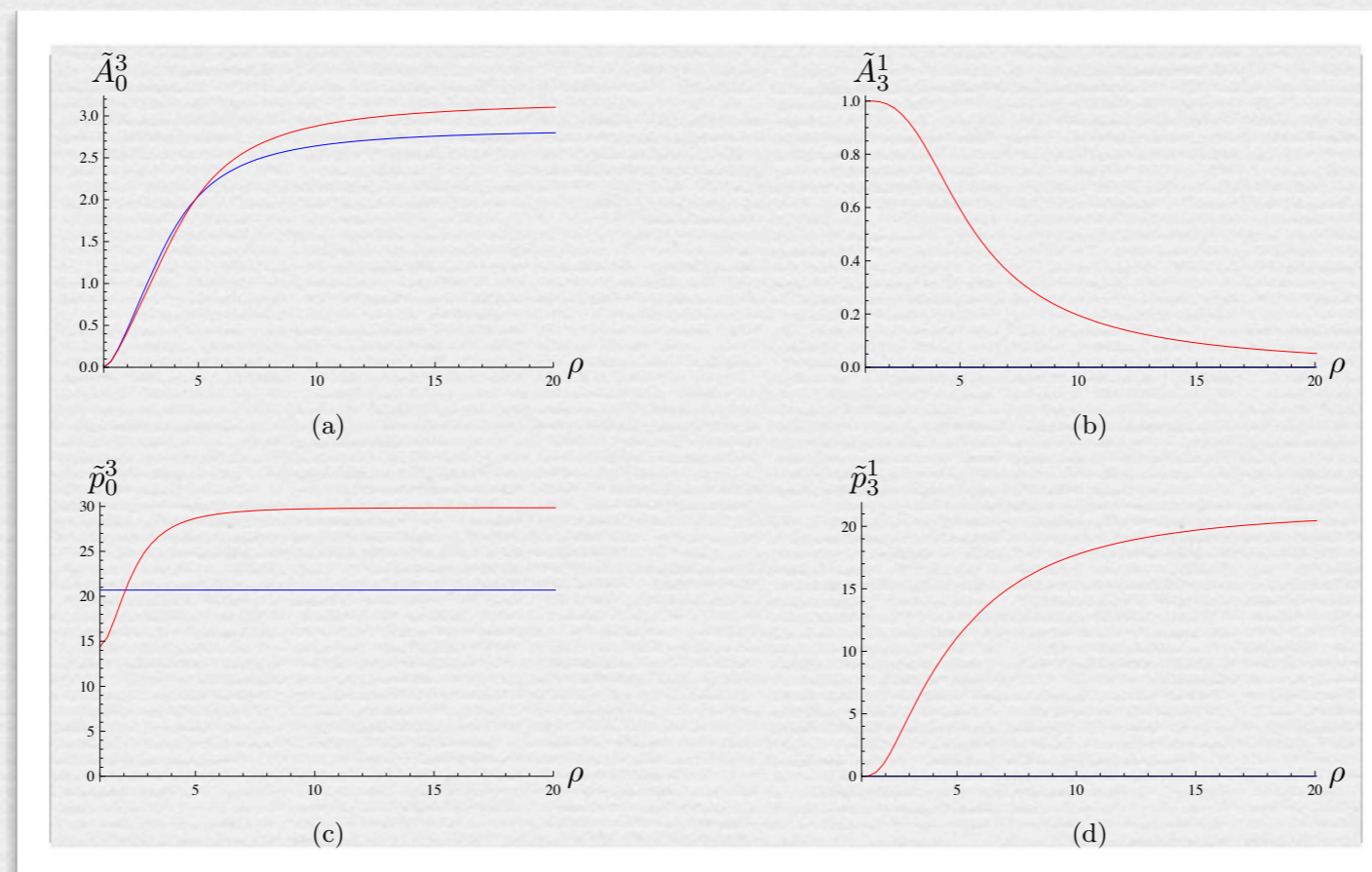
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- Action is similar to EYM action $\alpha \propto \sqrt{N_f/N_c} \stackrel{\text{here}}{=} 0$ with back-reaction dilaton have to considered, too

Solutions in normal phase

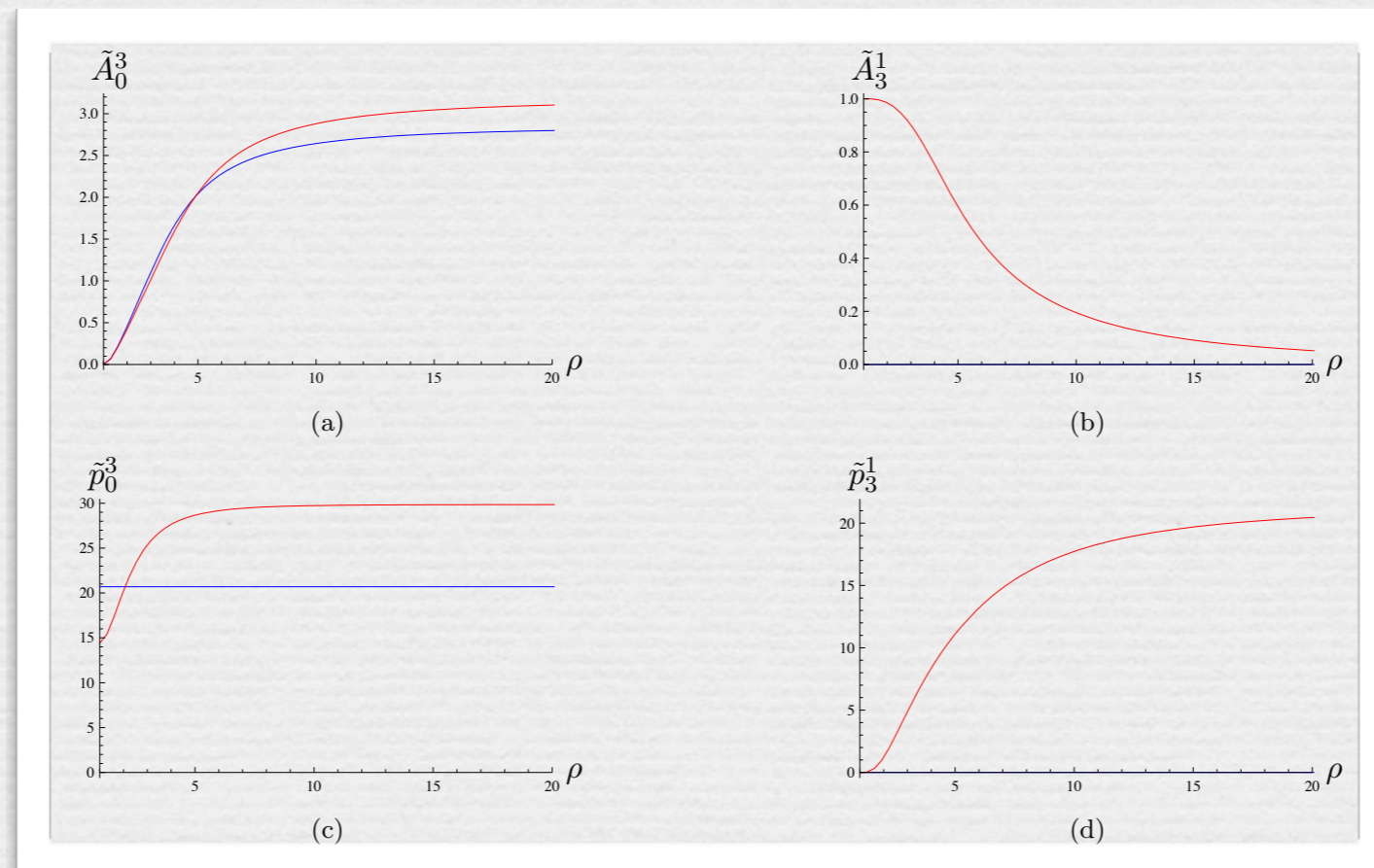
- Conjugate momenta $p = \frac{\partial S}{\partial(\partial A)}$ approach $\langle J \rangle$ at boundary
- Normal phase:
 p_t^3 constant \Rightarrow isospin density only produced at horizon



normal phase
 superfluid phase

Solutions in superfluid phase

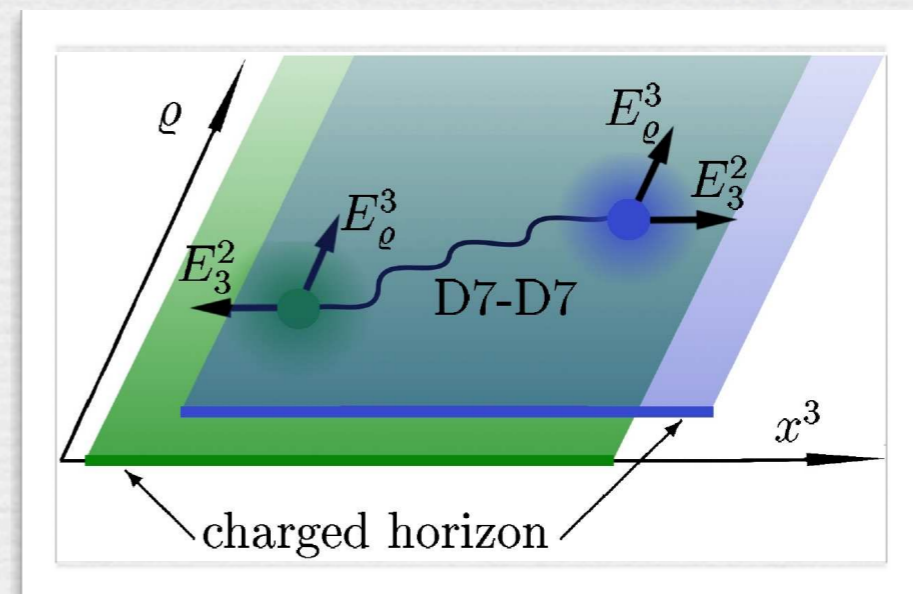
- Broken phase:
 p_t^3 not constant \Rightarrow isospin density generated in bulk
- p_x^1 increases towards boundary \Rightarrow condensate only created in bulk



normal phase
superfluid phase

Pairing Mechanism

- D3-D7 strings at horizon produce finite isospin density \Rightarrow branes have different charges.
- At large densities system unstable (cf. flashover)
- D3-D7 strings merge and form D7-D7 strings.
- D7-D7 strings propagate towards boundary \Rightarrow energy minimized



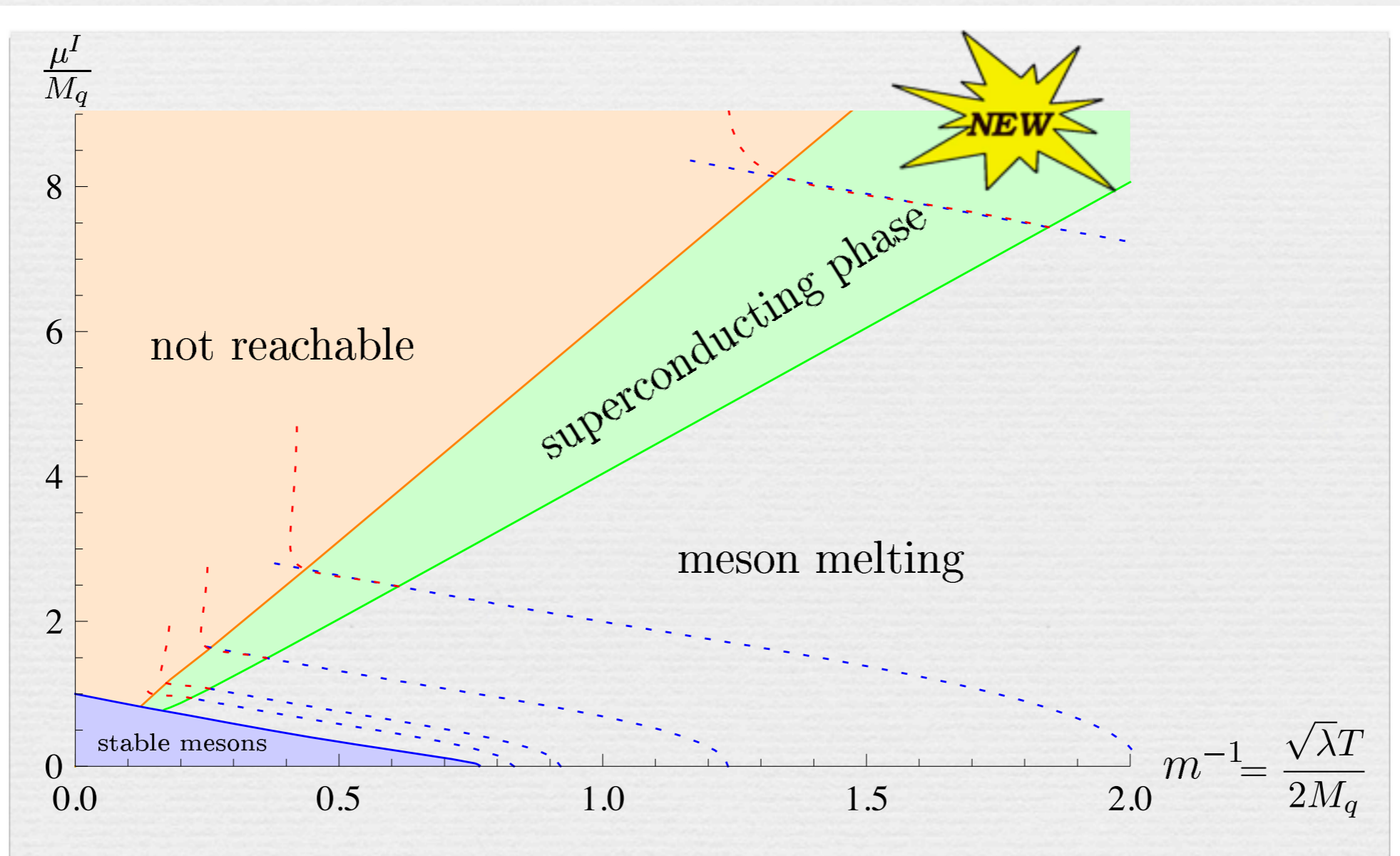
Importance of D7-D7 strings



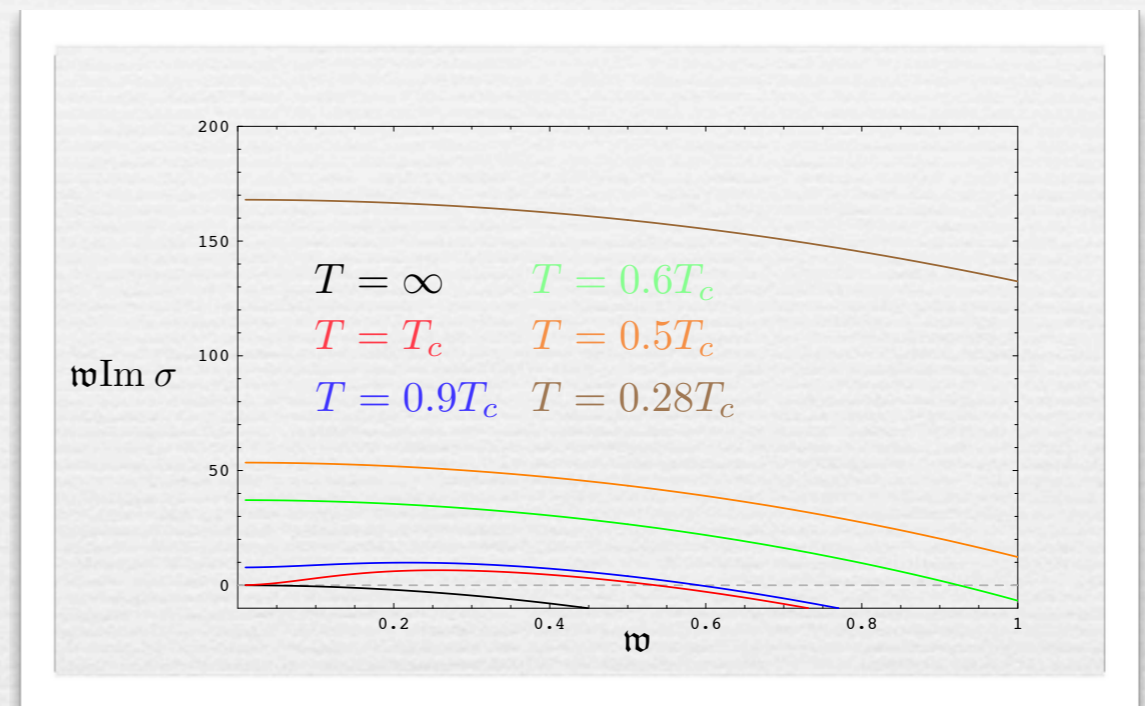
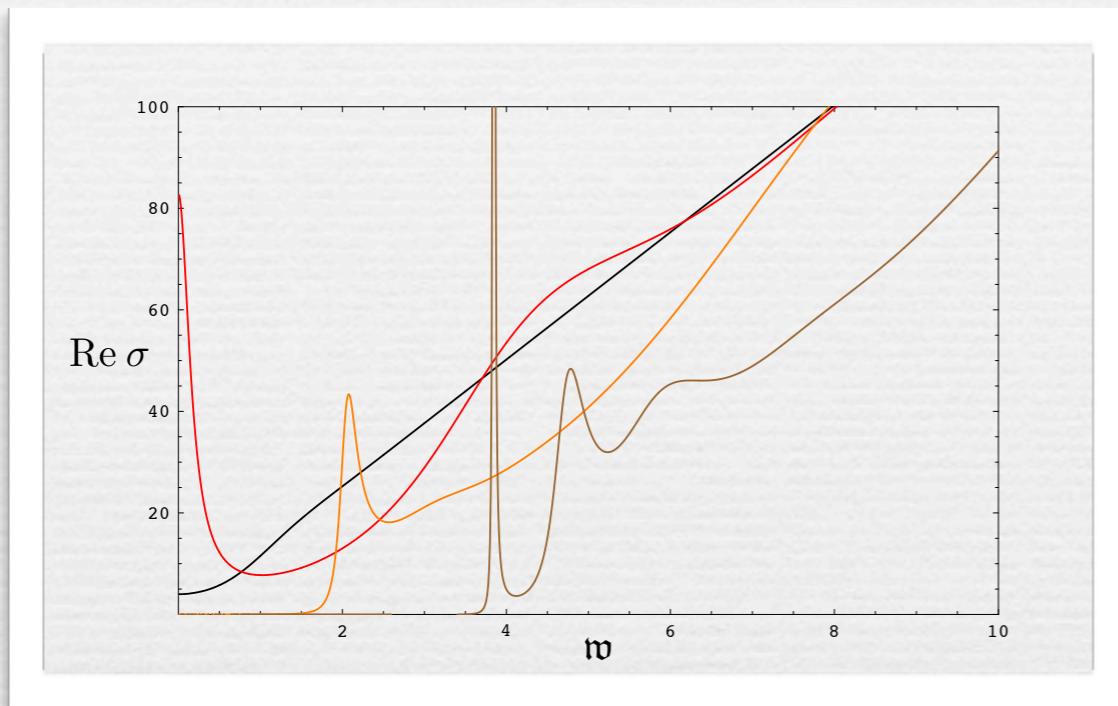
- They stabilize the system (minimal energy).
- They generate the condensate in the bulk.
⇒ They break the $U(1)_3$ symmetry.
- They are the duals of the Cooper pairs.

Phase diagram

for $\mathcal{N} = 4$ $SU(N_c)$ SYM
coupled to $\mathcal{N} = 2$ hypermultiplets



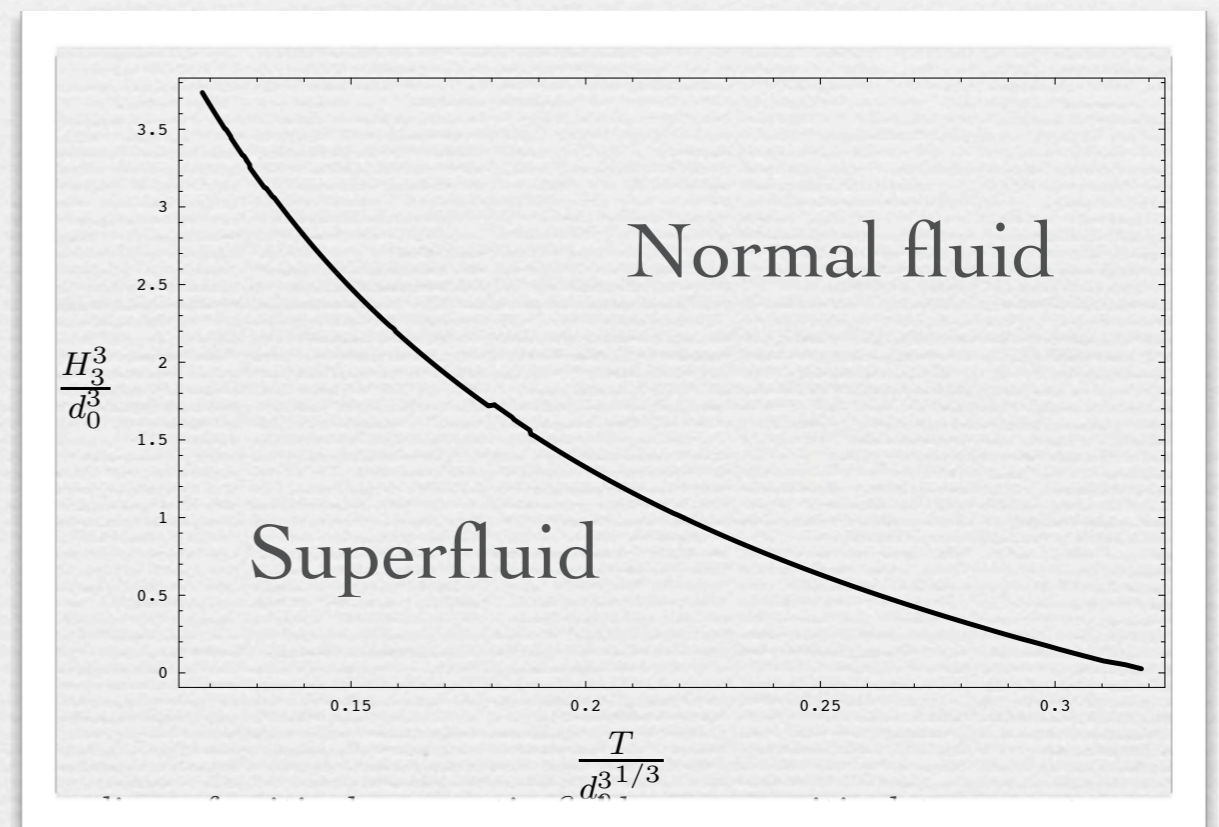
Conductivity



- Real part of conductivity develops a gap.
- Additional resonances (mesons) can appear in the gap.
- Imaginary part behaves like $n_s/\omega \Rightarrow \text{Re } \sigma \sim \pi n_s \delta(\omega)$
Superfluid density behaves as in GL $n_s \propto (1 - T/T_c)$

Remnant of Meissner-Ochsenfeld effect

- ❧ Magnetic fields can destroy the superfluid state.
- ❧ Only for small magnetic field, magnetic field and condensate can coexist.
- ❧ If symmetry was gauged, superconducting current would generate magnetic field \Rightarrow magnetic field would be expelled.



Summary

- ❧ Realization of holographic p-wave superfluids by black holes with vector hair
- ❧ Order of transition depends on number of charged degrees of freedom
- ❧ Embedding into string theory:
 - ⇒ Superfluidity in explicit field theory
 - ⇒ Pairing Mechanism in string terms

Outlook

❧ May shed light on:

meson superfluids and non-conventional superconductors due to QCP

❧ Can study additional properties:

hydrodynamic transport, response to non-zero superfluid velocities, ...

Outlook

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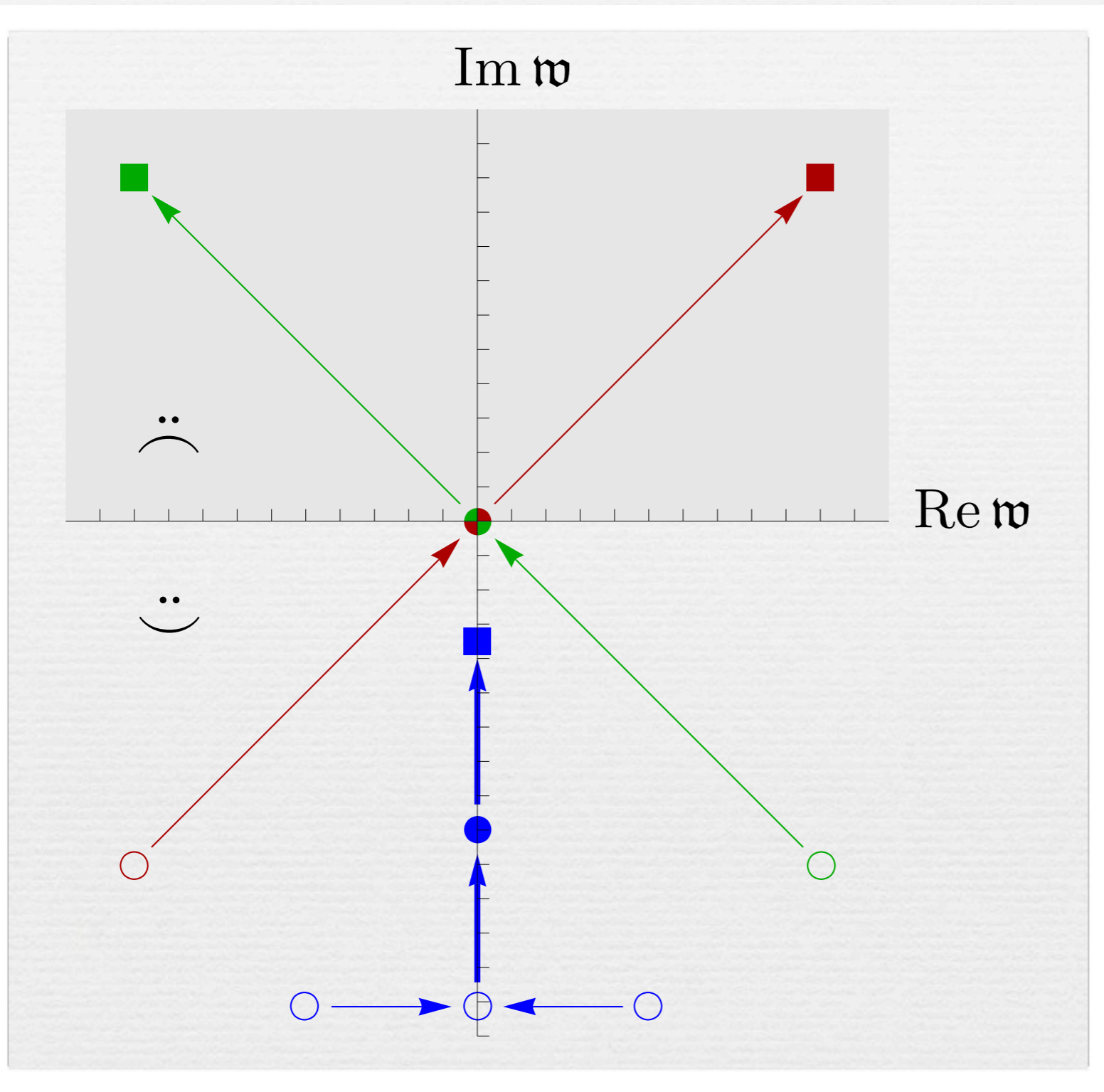
meson superfluids and non-conventional superconductors due to QCP

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Thank you!

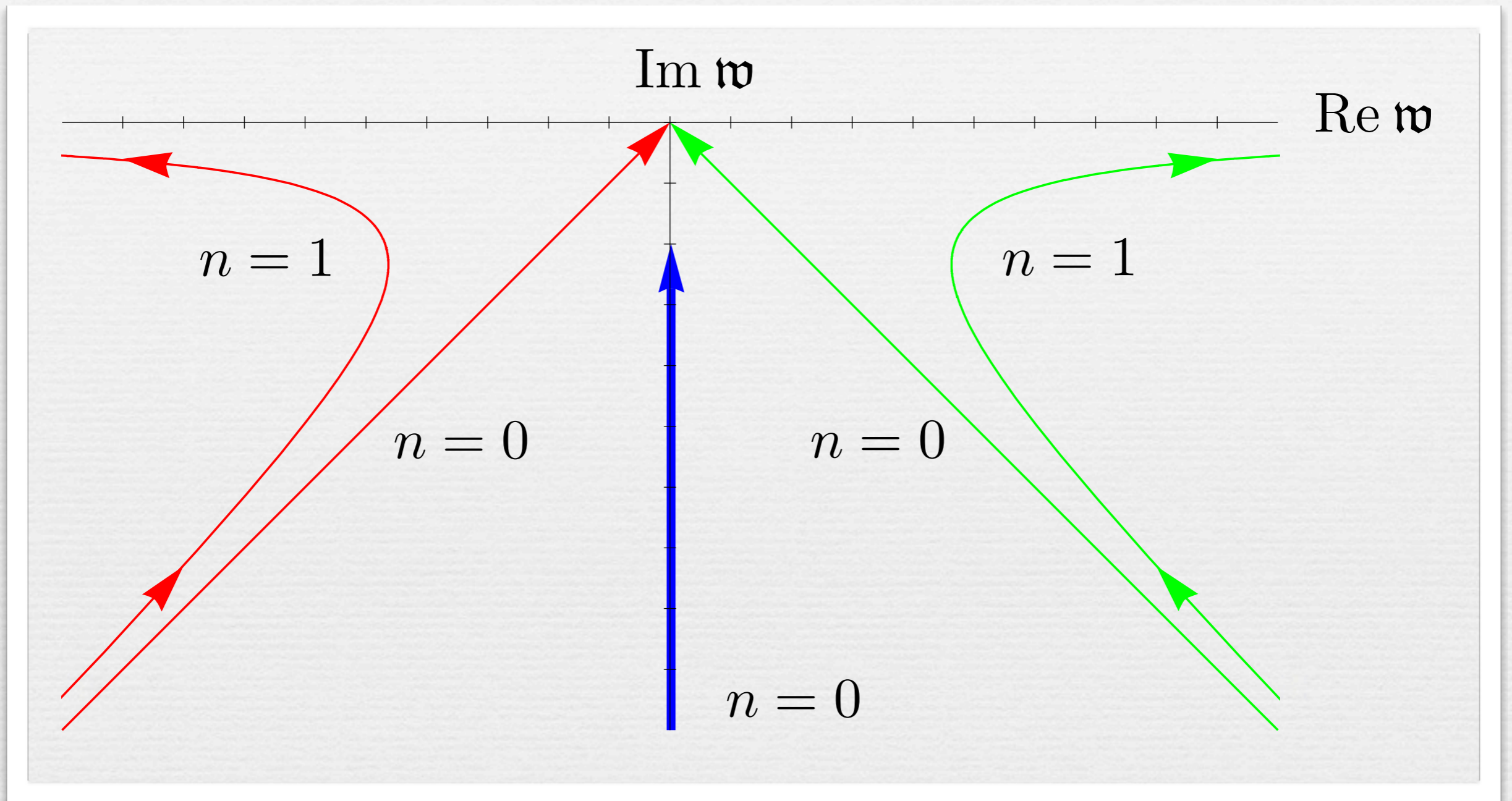
Backup slides



Fourier decomposition:

$$e^{-iw} = e^{+\text{Im}w} e^{-i\text{Re}w}$$

Quasinormal modes
in normal phase



Quasinormal modes in
superfluid phase