

Hadrons as Holograms

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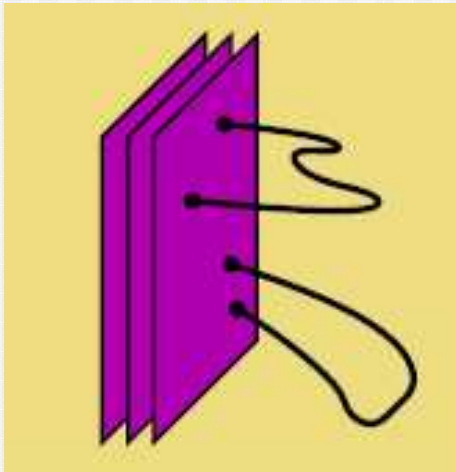
HF, T. Frederico, M. Beyer, JHEP 07 (2007) 077, IJMPE 16 (2007) 2794,
... + W. de Paula, PRD 79 (2009) 075019;
HF, PRD 78 (2008) 025071;
HF, E. Klempt, PLB 679 (2009) 77

Gauge/string correspondence and `AdS/QCD`
Meson & baryon spectra, lin. Regge traajs.
Diquark correlations in baryons
Dynamical AdS/QCD
Hadron correlators, espec. glueballs
Wrapping up...

Gauge/string correspondence in a (tiny) nutshell

2 equivalent descriptions of $E \ll 1/l_s$ excitations of $N_c \gg 1$ D3-branes:

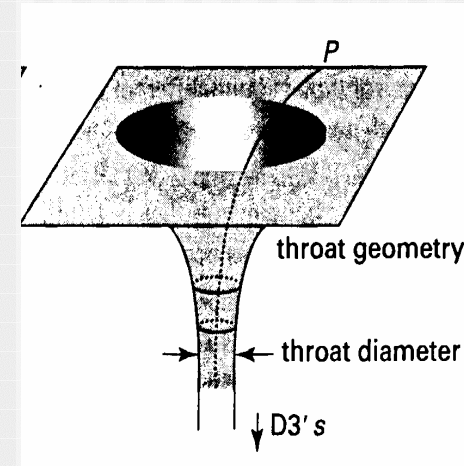
1) Open $m=0$ string modes on D3s :



$\lambda = gN_c \rightarrow 0$: $G_{(10)} \sim g^2 l_s^8 \rightarrow 0$:
 little distortion of metric: (almost) flat
 closed strings & bulk fields decouple,
 + open string modes ($m \neq 0$ inaccessible) \rightarrow

**(S)U(N_c) (S)Yang-Mills fields
 on D3s, strongly coupled (+...)**

2) SUGRA soln. sourced by branes:



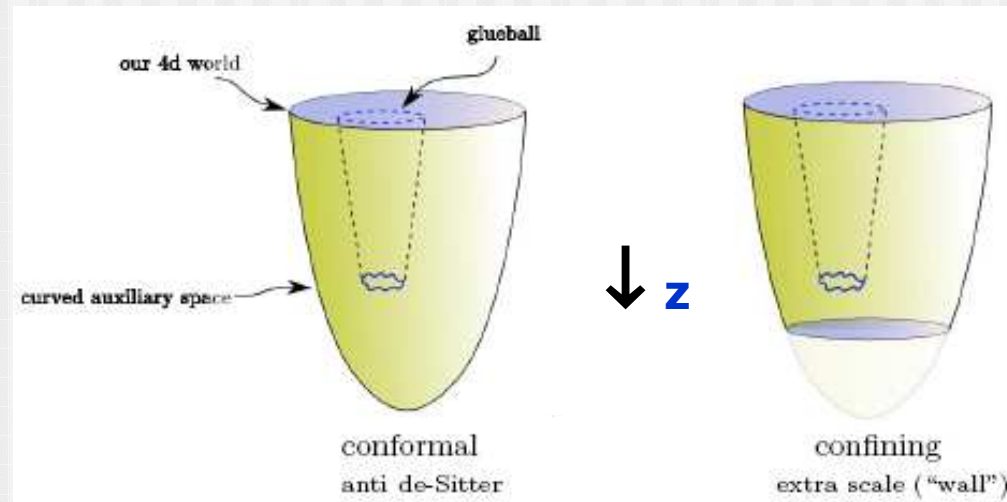
$\lambda \gg 1$: metric 'throat', ends at horizon,
 in far region: flat, gravity decoupled,
 + near horizon: fin. E excits. redshifted
 at ∞ , cannot escape from horizon

**Graviton- & other excits. in
 $AAdS_5 \times X^5$, weakly coupled (+...)**

AAdS₅ bulk backgrounds expected for QCD

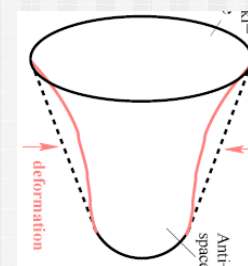
Minimal requirements on bulk geometry:

- Conf. UV, broken at Λ_{YM} : AAdS₅
- Poincaré invariant boundary
- Confinement: 'IR wall' in z
 → IR-deformed AdS₅:



$$ds^2 = g_{MN}(x) dx^M dx^N = e^{2A(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) + R^2 ds^2_{X_5}$$

IR deform. $A(z)$ with $A(z \rightarrow 0) \rightarrow 0 \Rightarrow$ in UV (ie. for $z \rightarrow 0$) metric remains conformal



+ dilaton $\Phi(z)$ in 'soft wall' + axions, tachyons, gauge fields + ...

Hadron dual-modes: rad. & orbital excitations in AAdS₅

Example: vector bulk modes:

$$V_z(x, z) = f_V(z) \varepsilon_z e^{-iPx}$$

→ **bulk vector equation:**
with (Polchinski, Strassler) bc:

$$f_i(z) \xrightarrow{z \rightarrow 0} z^{\tau_i}, \quad \tau_i = \Delta_i - \sigma_i$$

⇒ **Sturm-Liouville problem:**

$$\left[-\partial_z^2 + V_M(z) \right] \varphi(z) = M_M^2 \varphi(z)$$

Analog. for spin-0 mesons, baryons:

$$V_S(z) = \frac{3}{2} \left[A'' + \frac{3}{2} A'^2 - 3 \frac{A'}{z} + \frac{5}{2} \frac{1}{z^2} \right] + m_{5,S}^2 R^2 \frac{e^{2A}}{z^2},$$

$$V_{B,\pm}(z) = m_{5,B} R \frac{e^A}{z} \left[\pm \left(A' - \frac{1}{z} \right) + m_{5,B} R \frac{e^A}{z} \right]$$

dual to QCD meson interpolator:

$$\mathcal{O}_{M, \bar{\tau}=L+2} = \bar{q} \gamma^\mu D_{\{\ell_1 \dots D_{\ell_m}\}} q$$

$$\left[\partial_z^2 + 3 \left(A' - \frac{1}{z} \right) \partial_z + 3 \left(A'' + \frac{1}{z^2} \right) - \left(m_{5,V} R \frac{e^A}{z} \right)^2 + M^2 \right] f_V(z) = 0.$$

$$V_V(z) = \frac{3}{2} \left[-A'' + \frac{3}{2} A'^2 - 3 \frac{A'}{z} + \frac{1}{2} \frac{1}{z^2} \right] + m_{5,V}^2 R^2 \frac{e^{2A}}{z^2}$$

AdS/CFT dictionary sets bcs. ⇒

$$m_{5,S}^2 R^2 = \bar{\tau}_M (\bar{\tau}_M - 4) = L^2 - 4,$$

$$m_{5,V}^2 R^2 = \bar{\tau}_M (\bar{\tau}_M - 4) + 3 = L^2 - 1,$$

$$m_{5,B} R = \bar{\tau}_B - 2 = L + 1,$$

Trajectory structure in the light hadron spectrum

Simplest dual models, based on Polchinski-Strassler's 'hard wall':

$$e^{2A^{(hw)}(z)} = \theta(z_m - z), \quad z_m \simeq \Lambda_{\text{QCD}}^{-1},$$

predict:

$$M^2 \sim N^2, L^2, J^2$$

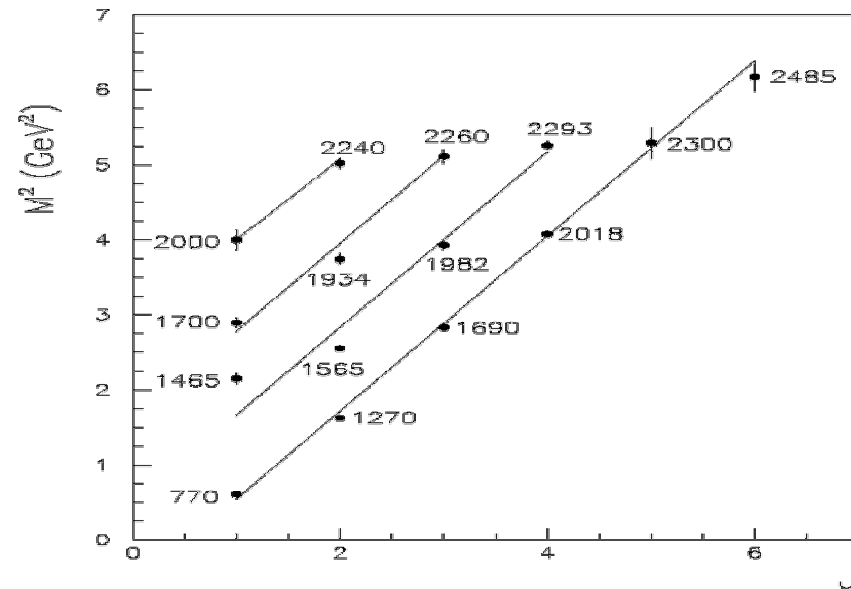
but experiment finds:

$$M^2 = M_0^2 + W(N + L)$$

$$W = 1.14 \pm 0.013 \text{ GeV}^2 \quad \text{mesons}$$

$$W = 1.081 \pm 0.035 \text{ GeV}^2 \quad \text{baryons}$$

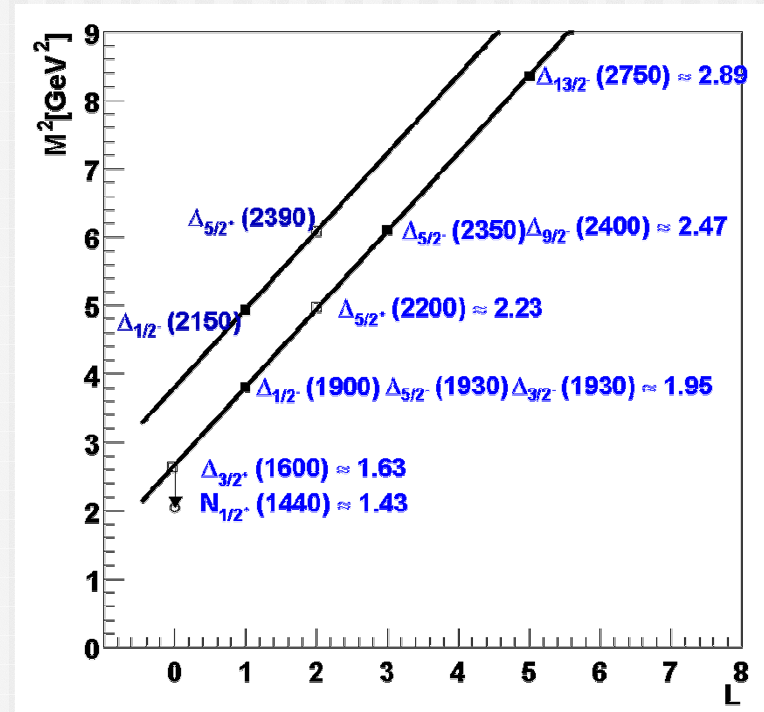
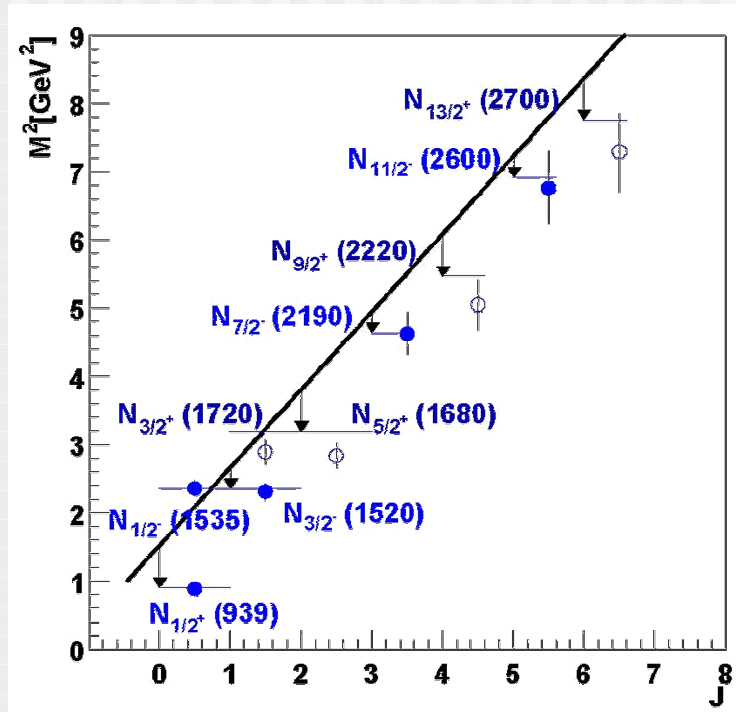
~ universal slopes!



Karch et al.: dilaton 'soft wall':
 ⇒ linear trajs. but **only for mesons!**

$$A^{(sw)}(z) \equiv 0, \quad \Phi^{(sw)}(z) = \lambda^2 z^2.$$

Linear trajectories in the baryon spectra



How to obtain observed meson & **baryon** trajs. with **universal** slope?
 Addtl. bulk fields necessary, or 5d metric background sufficient?

(Affirmative) answer constructively: assume only background metric,
 check whether **A** can be constructed such that desired spectra arise!

Construction of metric for univ. lin. trajectory spectra

Lin. Trajectories require oscillator-type Sturm-Liouville potentials

Univ. slopes can be obtained from AdS₅ 1/z² potentials via: $\bar{\tau}_i \rightarrow \bar{\tau}_i + \lambda^2 z^2.$

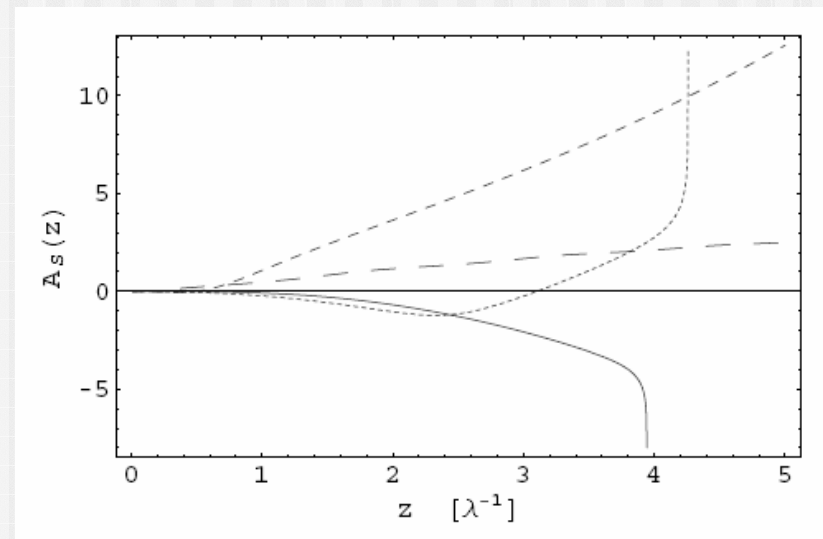
$$V_M^{(\text{LT})}(z) = \left[(\lambda^2 z^2 + L)^2 - \frac{1}{4} \right] \frac{1}{z^2}$$

$$V_{B,\pm}^{(\text{LT})}(z) = \left\{ (L+1)(L+1 \mp 1) + [2(L+1) \pm 1] \lambda^2 z^2 + \lambda^4 z^4 \right\} \frac{1}{z^2}.$$

**Derive dual geometry by equating to gen.-A potentials \Rightarrow ODEs for $A(z)$,
 \exists unique warp factor solutions $A(z)$:**

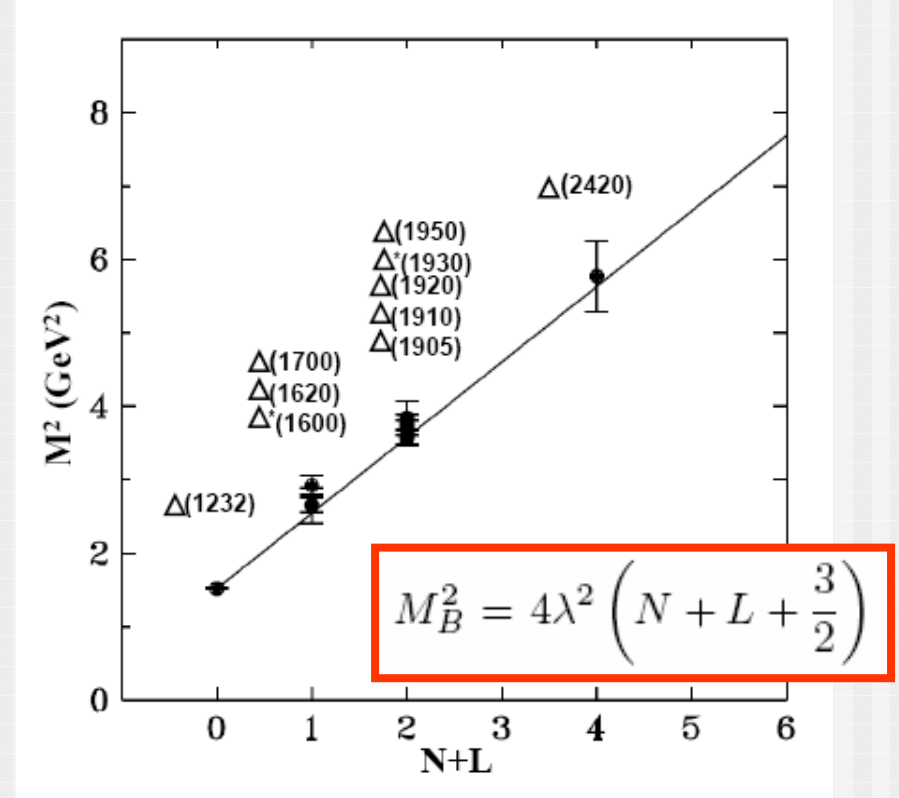
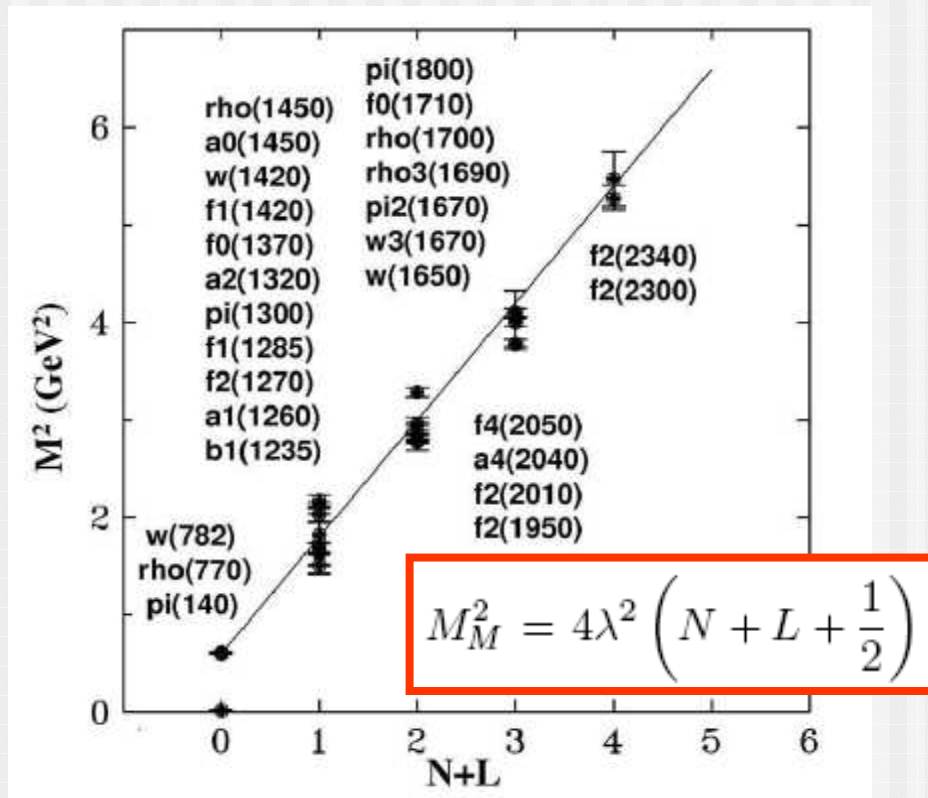
$$A_S(z) = \frac{1}{2} \ln \left[1 + \sum_{n=1}^{\infty} A_n(L) \left(\frac{\lambda^2 z^2}{2} \right)^n \right]$$

$$A_B(z) = \ln \left(1 + \frac{\lambda^2 z^2}{L+1} \right)$$



Resulting spectra and properties

Dual eigenmodes and mass spectra (obtained analytically):



- Lin. trajectories of universal slope in baryon sector, too
- Encoded via AdS_5 IR deform. (orbital fluct. backreacted?)
- Dual confinement signatures (metric singularities)
- New relations between slopes and ground state masses

Diquark correlations in light baryons holographically

Nucleon excitations improved by incl. diquark-content dependent **anomalous dims of interpolators**

$$\eta_t(x) = 2[\eta_{pd}(x) + t\eta_{sd}(x)]$$

$$\eta_{pd} = \varepsilon_{abc}(u_a^T C d_b) \gamma^5 u_c$$

$$\eta_{sd} = \varepsilon_{abc}(u_a^T C \gamma^5 d_b) u_c$$

⇒ **string-mode mass corrections:**

$$\Delta m_5^{(\kappa_{gd})} = \gamma_t^{(\kappa_{gd})} = \frac{\Delta M_{\kappa_{gd}}^2}{4\lambda^2 R}$$

dep. on “good-diquark fraction” κ !

$$M_{N,L}^2 = M_{N,L+\Delta m_5 R}^{(ms)2} = 4\lambda^2 \left(N + L + \frac{3}{2} \right) + \Delta M_{\kappa_{gd}}^2$$

$$\Delta M_{\kappa_{gd}}^2 = -2(M_{\Delta}^2 - M_N^2)\kappa_{gd}$$

Manifestly simple spectrum, reprod. all 48 observed **N res. better than quark models !**

| L, N | κ_{gd} | Resonance | | | | | Pred. |
|--------|---------------|----------------|----------------|----------------|--|--|------------------------------------|
| 0, 0 | $\frac{1}{2}$ | N(940) | | | | | input: 0.94 |
| 0, 0 | 0 | $\Delta(1232)$ | | | | | 1.27 |
| 0, 1 | $\frac{1}{2}$ | N(1440) | | | | | 1.40 |
| 1, 0 | $\frac{1}{4}$ | N(1535) | | N(1520) | | | 1.53 |
| 1, 0 | 0 | N(1650) | | N(1700) | | N(1675) | 1.64 |
| 1, 0 | 0 | $\Delta(1620)$ | $\Delta(1700)$ | | $L, N = 0, 1:$ | $\Delta(1600)$ | 1.64 |
| 2, 0 | $\frac{1}{2}$ | N(1720) | | N(1680) | | $L, N = 0, 2:$ | N(1710) 1.72 |
| 2, 0 | 0 | N(1900) | | N(1990) | | N(2000) $\Delta(1910)$ $\Delta(1920)$ 1.92 | |
| 2, 0 | 0 | $\Delta(1905)$ | $\Delta(1950)$ | | $\Delta(1900)^*$ $\Delta(1940)^*$ $\Delta(1930)^*$ | | 1.92 |
| 0, 3 | $\frac{1}{2}$ | N(2100) | | | | | 2.03 |
| 3, 0 | $\frac{1}{4}$ | N(2070) | | N(2190) | | $L, N = 1, 2:$ | N(2080) N(2090) 2.12 |
| 3, 0 | 0 | N(2200) | | N(2250) | | $\Delta(2223)$ $\Delta(2200)$ $L, N = 1, 2:$ $\Delta(2150)$ 2.20 | |
| 4, 0 | $\frac{1}{2}$ | N(2220) | | | | | 2.27 |
| 4, 0 | 0 | $\Delta(2390)$ | | $\Delta(2300)$ | | $\Delta(2420)$ $L, N = 3, 1:$ $\Delta(2400)$ $\Delta(2350)$ 2.43 | |
| 5, 0 | $\frac{1}{4}$ | N(2600) | | | | | 2.57 |
| 6, 0 | $\frac{1}{2}$ | N(2700) | | | | | 2.71 |
| 6, 0 | 0 | $\Delta(2950)$ | | | | | $L, N = 5, 1:$ $\Delta(2750)$ 2.84 |

Regge trajectories in a dynamical AdS/QCD dual

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{|g|} \left(-R + \frac{1}{2} g^{MN} \partial_M \Phi \partial_N \Phi - V(\Phi) \right),$$

$\phi = \phi(\mathbf{z})$ and \mathbf{g}_{AdS5} → Einstein-dilaton eqs.:

$$6A'^2 - \frac{1}{2}\Phi'^2 + e^{-2A}V(\Phi) = 0,$$

$$3A'' - 3A'^2 - \frac{1}{2}\Phi'^2 - e^{-2A}V(\Phi) = 0,$$

$$\Phi'' - 3A'\Phi' - e^{-2A} \frac{dV}{d\Phi} = 0$$

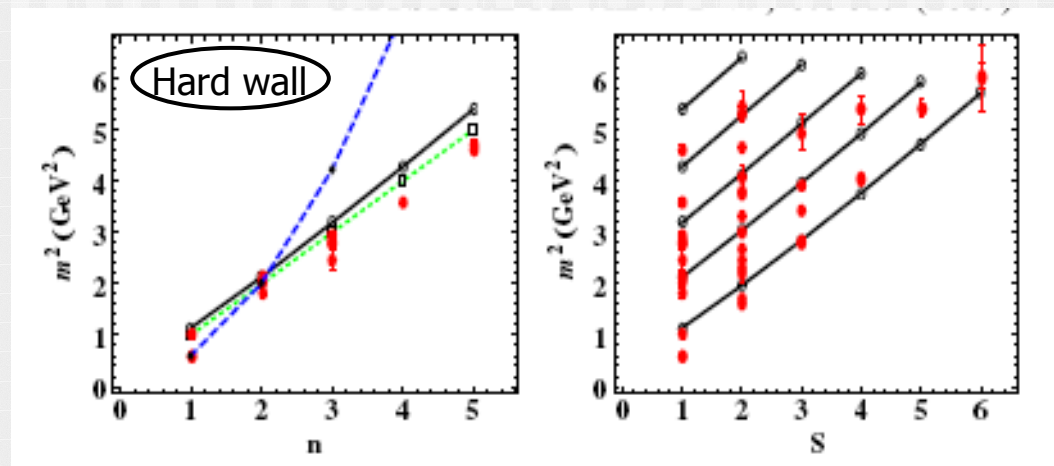
Find UV-conformal **solution** for ϕ , $V(\phi)$ with **area-law** gen. warp factor

$$A(z) = \ln z + C(z)$$

$$C(z) = \frac{1 + \sqrt{3}}{2S + \sqrt{3} - 1} \frac{(z\Lambda_{\text{QCD}})^2}{1 + e^{(1-z\Lambda_{\text{QCD}})}}$$

→ Mass spectra

$$m_{n,S}^2 \simeq \frac{1}{10}(11n + 9S - 9),$$



Pert. Log. Corrections can be included in C...

Glueball correlators: calculational strategy

On-shell action: insert solutions $\varphi(x,z)$ subject to boundary source $\varphi^{(s)}(x)$, i.e.

$$\varphi(x,z) = \int \frac{d^4 q}{(2\pi)^4} e^{-iqx} \hat{K}(q,z) \int d^4 x' e^{iqx'} \varphi^{(s)}(x')$$

where \mathbf{K} is the bulk-to-boundary propagator

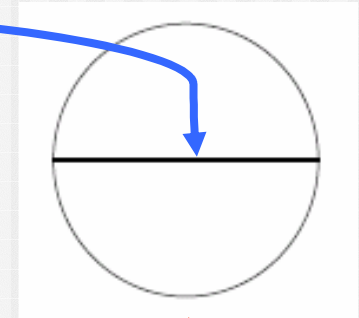
$$\hat{K}(q,z) = -\frac{R^3}{\varepsilon^3} \sum_n \frac{\psi'_n(\varepsilon) \psi_n(z)}{q^2 - m_n^2 + i\varepsilon'}$$

into the bulk action

$$S[\varphi; g, \Phi] = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{|g|} e^{-\Phi} g^{MN} \partial_M \varphi \partial_N \varphi$$

2 funct. derivatives of exp. of on-shell bulk action wrto. $\varphi^{(s)} \Rightarrow$

$$\hat{\Pi}(-q^2) = -\frac{R^3}{\kappa^2} \left[\frac{e^{-\Phi(z)}}{z^3} \hat{K}(q,z) \partial_z \hat{K}(q,z) \right]_{z=\varepsilon \rightarrow 0}$$



Glueball correlators in "hard- and soft-wall" duals

Hard-wall correlator
(IR brane at $z=z_m$):

$$\hat{\Pi}(Q^2) = \frac{R^3}{8\kappa^2} Q^4 \left[2 \frac{K_1(Qz_m)}{I_1(Qz_m)} - \ln\left(\frac{Q^2}{\mu^2}\right) \right]$$

$$\frac{R^3}{\kappa^2} = \frac{2(N_c^2 - 1)}{\pi^2}$$

Dilaton soft-wall correlator ($\phi = \lambda^2 z^2$):

$$\hat{\Pi}(Q^2) = -\frac{2R^3}{\kappa^2} \lambda^4 \left[1 + \frac{Q^2}{4\lambda^2} \left(1 + \frac{Q^2}{4\lambda^2} \right) \psi\left(\frac{Q^2}{4\lambda^2}\right) \right]$$

Spectral representation:

$$\hat{\Pi}(Q^2) = \int_{m_1^2}^{\infty} ds \frac{\rho(s)}{s + Q^2}$$

with spect. density ρ :

\Rightarrow both correlators have $\rho \geq 0$, mass gap, **zero-width Gb poles** \leftrightarrow **large N_c** :

Hard wall:

$$\rho(s) = \frac{R^3}{2\kappa^2 z_m^2} s^2 \sum_{n=1}^{\infty} \frac{\delta(s - m_n^2)}{J_0^2(j_{1,n})}$$

\Rightarrow

$$m_n^{(N)} = \frac{j_{1,n}}{z_m}$$

lin. "pomeron" trajectory!

Soft wall:

$$\rho(s) = \frac{\lambda^2 R^3}{2\kappa^2} s (s - m_0^2/2) \sum_{n=0}^{\infty} \delta(s - m_n^2)$$

\Rightarrow

$$m_n^2 = 4(n + 2) \lambda^2$$

Hard-wall content: small-instanton contributions

At large $Q \gg z_m^{-1}$ npert. Q dependence becomes exponential:

$$\hat{\Pi}^{(np)}(Q^2) \equiv \frac{R^3}{4\kappa^2} \frac{K_1(Qz_m)}{I_1(Qz_m)} Q^4$$

$$\xrightarrow{Qz_m \gg 1} \frac{4}{\pi} \left[1 + \frac{3}{4} \frac{1}{Qz_m} + O\left(\frac{1}{(Qz_m)^2}\right) \right] Q^4 e^{-2Qz_m}$$

- **No power corrections** (expected to be weak from QCD-OPE!)
- **LET** trivially satisfied (no anomaly, no gluon condensate...)

Compare with small-scale QCD instanton contribution to OPE:

$$\hat{\Pi}^{(I+\bar{I})}(Q^2) \xrightarrow{Q\bar{\rho} \gg 1} 2^4 5^2 \pi \zeta \bar{n} (Q\bar{\rho})^3 e^{-2Q\bar{\rho}}$$

⇒

$$\bar{\rho} \simeq z_m, \quad \bar{n} \simeq \frac{3}{2^4 5^2 \pi^2 \zeta} \frac{1}{z_m^4}$$

$$\rho^{(\text{hw})} \leq z_m \sim \mu^{-1}$$

Crucial **small-instanton contribs.** (semi-quant.) **reproduced!**

Soft-wall dynamcis: power corrections, UV gluon mass

Asymptotic expansion for $Q^2 \gg \lambda^2 \cong \Lambda^2$:

$$\hat{\Pi}(Q^2) = -\frac{2R^3}{\kappa^2} \lambda^4 \left[1 + \frac{Q^2}{4\lambda^2} \left(1 + \frac{Q^2}{4\lambda^2} \right) \left(\ln \frac{Q^2}{4\lambda^2} - \frac{2\lambda^2}{Q^2} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2n} \left(\frac{4\lambda^2}{Q^2} \right)^{2n} \right) \right]$$

$$= -\frac{2}{\pi^2} Q^4 \left[\ln \frac{Q^2}{\mu^2} + \frac{4\lambda^2}{Q^2} \ln \frac{Q^2}{\mu^2} + \frac{2^2 5}{3} \frac{\lambda^4}{Q^4} - \frac{2^4}{3} \frac{\lambda^6}{Q^6} + \frac{2^5}{15} \frac{\lambda^8}{Q^8} + \dots \right].$$

Contains **all OPE power corrections**: Even addtl. term expected in QCD from **UV-ren., gluon mass, 2d-nonlocal cond.** :

$$\langle G^2 \rangle \simeq -\frac{10}{3\pi^2} \lambda^4,$$

$$\langle gG^3 \rangle \simeq \frac{4}{3\pi^2} \lambda^6,$$

$$\langle G^4 \rangle \simeq -\frac{8}{15\pi^3 \alpha_s} \lambda^8.$$

$$\hat{\Pi}^{(\text{CNZ})}(Q^2) = -\frac{2}{\pi^2} Q^4 \ln \frac{Q^2}{\mu^2} \left(1 + 6 \frac{\bar{\lambda}^2}{Q^2} + \dots \right)$$

$$\bar{\lambda}^2 \simeq \frac{2}{3} \lambda^2$$

Leading coeff. has sign opposite to QCD, LET → **artefact of strong-coupl. UV!**

Glueball decay constants

Glueball observables beyond spectrum (expt. glueball signatures,...):

$$f_n := \frac{1}{m_n^2} \langle 0 | \mathcal{O}_S(0) | 0_n^{++} \rangle \quad \text{where} \quad \mathcal{O}_S(x) = G_{\mu\nu}^a(x) G^{a,\mu\nu}(x),$$

Contain crucial information on Gb structure (size!): 'WF at origin' $\mathbf{x} \rightarrow \mathbf{0}$:

$$\chi_n(x) = \left\langle 0 \left| 2\text{tr} \left\{ G_{\mu\nu} \left(-\frac{x}{2} \right) U \left(-\frac{x}{2}, \frac{x}{2} \right) G^{\mu\nu} \left(\frac{x}{2} \right) \right\} \right| 0_n^{++} \right\rangle$$

Existing QCD information:

- older lattice evidence: **except. small 0^{++} size** (via BS ampl.)
- **conflicting predictions of QCD sum rule** analyses:
 - neglect npert. Wilson coeffs. (Narison): $f_S = 0.390 \pm 0.15$ GeV
 - include npert. (small inst.) Wilson coeffs. (HF): $f_S = 1.050 \pm 0.10$ GeV
- instanton liquid model (Schäfer, Shuryak) $f_S \cong 0.8$ GeV
- (quenched) **lattice** (Chen et al. 2006): $f_S = 0.86 \pm 0.18$ GeV

Decay constants holographically

Solve bulk field eq. for norm. eigenmodes:

$$\psi_n(z) = N_n \hat{\phi}(m_n, z)$$

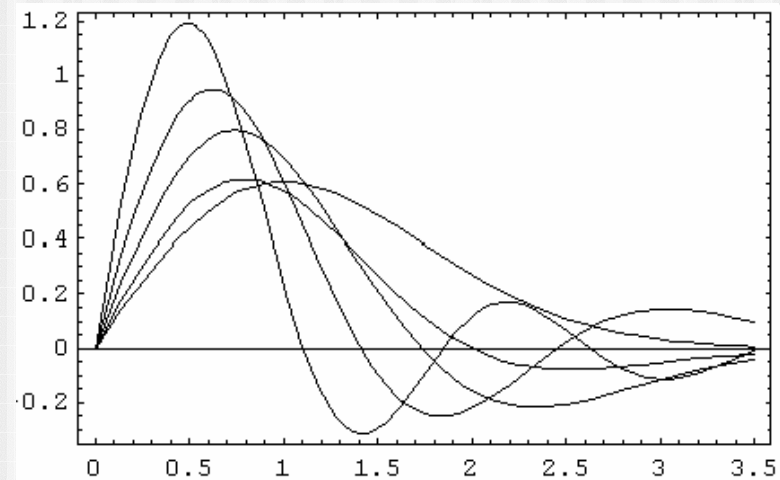
(for plot mult. by sqrt of int. measure)

For dilaton soft wall:

$$f_n^{(sw)} = 4I_n^{-1/2} \frac{\lambda^3 R^{3/2}}{m_n^2 \kappa} = \frac{1}{\sqrt{2}} \sqrt{\frac{n+1}{n+2}} \frac{\lambda R^{3/2}}{\kappa}$$

⇒

$$f_S^{(sw)} \simeq 0.3 \text{ GeV.}$$



Hard wall (expected to represent more relevant Gb physics):

$$f_n = \lim_{\varepsilon \rightarrow 0} \frac{R^3}{\kappa m_n^2} \frac{\psi'_n(\varepsilon)}{\varepsilon^3} = \frac{N_n}{2} \frac{R^3}{\kappa} m_n^2$$

⇒

$$f_S^{(hw)} \simeq 0.8 - 0.9 \text{ GeV,}$$

Consistent with ILM, IOPE sum rules & lattice results

Summary and conclusions

Hadron spectra:

- **Linear** Regge-type **trajectories** of **universal slope** for mesons AND **baryons**
 - Minimal 'metric soft wall': encoded solely via IR deformed AdS5 metric
 - New **relations between** trajectory **slopes and** ground-state **masses**
- **Diquark effects** can enter **via anom. dims.** of baryon interpolators
 - Decisively improve predictions for light baryon spectra!
- Backreacted **5d Einstein-dilaton solution**: generates area law,
 - Vector meson excitations with **linear trajectories dynamically!**

Hadron correlators:

- Detailed, quantitative testing ground → generates new insights:
- In comparison with QCD-OPE: map **impact of strongly-coupled UV**
 - Hard/soft wall duals contain complementary gauge/QCD physics
 - (beyond spectra) Predictions of **glueball decay constants**, coupls.
 - Suggests systematic bottom-up improvement strategies