

Soft contributions to the shear and bulk viscosities

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- Experimental indications.
- General properties of the viscosities.
- The Kubo formulae.
- Calculation of the viscosities.
- The results.
- Concluding remarks.

Experimental indications

RHIC data exhibit strong collective phenomena in the asymmetric azimuthal distribution around the beam axis:

$$p_0 \frac{d^3 N}{dp^3} \Big|_{p_z=0} = v_0(p_\perp) [1 + 2v_2(p_\perp) \cos(2\phi) + 2v_4(p_\perp) \cos(4\phi) + \dots],$$

where $(p_x, p_y, p_z) = (p_\perp \cos \phi, p_\perp \sin \phi, p_z)$.

The large elliptic flow $v_2 \simeq 0.06$ cannot be described just by two-body interactions between partons.

Particles of different mass are emitted from the fireball with a common **fluid velocity**.

Relativistic hydrodynamics reproduces v_2 very well, up to $p_\perp \sim 1.5$ GeV (P. Huovinen, U.W. Heinz, '01).

Experimental indications

L_{mfp} of a parton, which traverses an (ideal quantum) liquid is much smaller than the thermal wavelength $\sim \beta \equiv \frac{1}{T}$, i.e.

$$\frac{L_{\text{mfp}}^{\text{liq.}}}{\beta} \ll 1.$$

Instead, in the dilute-gas model of the QGP,

$$L_{\text{mfp}}^{\text{gas}} \sim (\rho \sigma_t)^{-1},$$

where $\rho \sim T^3$ is the particle-number density, $\sigma_t \sim g_T^4 \beta^2 \ln g_T^{-1}$ is the Coulomb transport cross-section, and g_T is the perturbative finite- T QCD coupling

$$\Rightarrow \frac{L_{\text{mfp}}^{\text{gas}}}{\beta} \sim \frac{1}{g_T^4 \ln g_T^{-1}} \gg 1$$

\Rightarrow the experimental results could have only been reproduced by the dilute-gas model if σ_t were larger by an order of magnitude (D. Molnar, M. Gyulassy, '02).

General properties of the viscosities

Shear viscosity η represents the ability to transport momentum:

$$\frac{\eta}{s} \sim \frac{L_{\text{mfp}}}{\beta},$$

where s is the entropy density $\Rightarrow \frac{\eta}{s}$ is large in the dilute-gas model of the QGP, and gets smaller for a strongly interacting QGP.

E.g., for $T \sim 200$ MeV and $L_{\text{mfp}} \sim 0.1$ fm, $\frac{\eta}{s} \sim 0.1$.

When a parton propagates through the QGP over the distance L_{mfp} , its mean momentum change Δp is $\sim T$

$$\Rightarrow \frac{\eta}{s} \sim \frac{L_{\text{mfp}}}{\beta} \sim L_{\text{mfp}} \cdot \Delta p$$

is nonvanishing due to the Heisenberg uncertainty principle

$\Rightarrow \eta$ cannot vanish completely.

General properties of the viscosities

- How small can $\frac{\eta}{s}$ be ?
- What is the temperature behavior of $\frac{\eta}{s}$?

The minimal possible value of $\frac{\eta}{s}$ is conjectured to be that in $\mathcal{N} = 4$ SYM (G. Policastro, D.T. Son, A.O. Starinets, '01):

$$\left. \frac{\eta}{s} \right|_{\mathcal{N}=4\text{SYM}} = \frac{1}{4\pi} \simeq 0.08.$$

It is a temperature-independent constant (because $\mathcal{N} = 4$ SYM is a CFT).

Rather, in perturbative QCD (P. Arnold et al., '01, '03),

$$\left. \frac{\eta}{s} \right|_{\text{pQCD}} \sim \frac{1}{g_T^4 \ln g_T^{-1}} \gg 1.$$

Note that plasma instabilities can generate an anomalous viscosity η_A (M. Asakawa, S.A. Bass, B. Müller, '06):

$$\frac{\eta_A}{s} \sim \frac{1}{g_T^{3/2}} < \left. \frac{\eta}{s} \right|_{\text{pQCD}}, \quad \text{but still } \gg 1.$$

General properties of the viscosities

For known liquids, $\frac{\eta}{s} \gg 1$ for small and large T , where η is dominated by potential- and kinetic-energy contributions, respectively.

Around the liquid-gas phase transition, these two contributions are nearly equal, and $\frac{\eta}{s}$ has a minimum, corresponding to the most difficult condition to transport momentum.

This behavior is exhibited by liquids of a very different nature, such as helium, nitrogen, and water.

The empirical minima of $\frac{\eta}{s}$ are at least by one order of magnitude larger than $\frac{1}{4\pi}$.

General properties of the viscosities

The energy-momentum tensor of an ideal liquid:

$$\Theta_{\mu\nu} = -p \cdot g_{\mu\nu} + Ts \cdot u_\mu u_\nu,$$

where u_μ is the velocity of energy transport.

The principal deviation from the ideality:

$$\Delta\Theta_{\mu\nu} = \eta \cdot (\Delta_\mu u_\nu + \Delta_\nu u_\mu) + \left(\frac{2}{3}\eta - \zeta\right) H_{\mu\nu} \partial_\rho u_\rho,$$

where $H_{\mu\nu} = u_\mu u_\nu - g_{\mu\nu}$, $\Delta_\mu = \partial_\mu - u_\mu u_\nu \partial_\nu$.

The bulk viscosity ζ is the other first-order transport coefficient.

While η characterizes a change in shape of a fixed volume, ζ characterizes a change in volume of the liquid of a fixed shape.

General properties of the viscosities

Cf. the Navier–Stokes' equation in hydrodynamics:

$$\rho \cdot \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} \right] = -\text{grad } p + \eta \cdot \Delta \mathbf{v} + \left(\zeta + \frac{\eta}{3} \right) \cdot \text{grad div } \mathbf{v} \Rightarrow$$

- η enters foremost through $\eta \cdot \Delta \mathbf{v}$;
- ζ is relevant only when $\text{div } \mathbf{v} \neq 0$, i.e. for compressible liquids.

For helium, nitrogen, and water, $\frac{\zeta}{\eta}$ has a maximum near the liquid-gas phase transition.

In $\mathcal{N} = 4$ SYM, $\zeta \equiv 0$ (again because it is a CFT), unlike QCD, where the non-conformality effects are manifest in $\varepsilon - 3p$ up to $T = (2 \div 3) T_c$.

The Kubo formulae

η and ζ are defined through the spectral densities,

$$\eta = \pi \left. \frac{d\rho_T^{(s)}}{d\omega} \right|_{\omega=0} \quad \text{and} \quad \zeta = \frac{\pi}{9} \left. \frac{d\rho_T^{(b)}}{d\omega} \right|_{\omega=0},$$

which can be obtained from the Euclidean Kubo formulae (A. Hosoya et al., '84; F. Karsch & H.W. Wyld, '87)

$$\int_0^\infty d\omega \rho_T^{(s),(b)}(\omega) \frac{\cosh \left[\omega \left(x_4 - \frac{\beta}{2} \right) \right]}{\sinh(\omega\beta/2)} = \int d^3x \sum_{n=-\infty}^{+\infty} U_T^{(s),(b)}(\mathbf{x}, x_4 + \beta n),$$

where

$$U_T^{(s)}(\mathbf{x}, x_4) = \langle \Theta_{12}(0) \Theta_{12}(\mathbf{x}, x_4) \rangle_T, \quad U_T^{(b)}(\mathbf{x}, x_4) = \langle \Theta_{\mu\mu}(0) \Theta_{\nu\nu}(\mathbf{x}, x_4) \rangle_T,$$

and in the Yang–Mills theory

$$\Theta_{12} = g^2 F_{1\mu}^a F_{2\mu}^a, \quad \Theta_{\mu\mu} = \frac{\beta(g)}{2g} (F_{\mu\nu}^a)^2.$$

The Kubo formulae

$\Theta_{\mu\nu}$ of the gluon plasma receives contributions from

- stochastic background fields, characterized by $\langle g^2(F_{ij}^a)^2 \rangle_T$ and μ_T ;
- valence gluons, which are confined at large *spatial* separations.

Such a two-component model of the gluon plasma is efficient to describe

- radiative energy loss of a parton traversing the plasma (H.-J. Pirner & D.A., '08);
- pressure and interaction measure $(\varepsilon - 3p)/T^4$ of the plasma (H.-J. Pirner, M.G. Schmidt & D.A., '09).

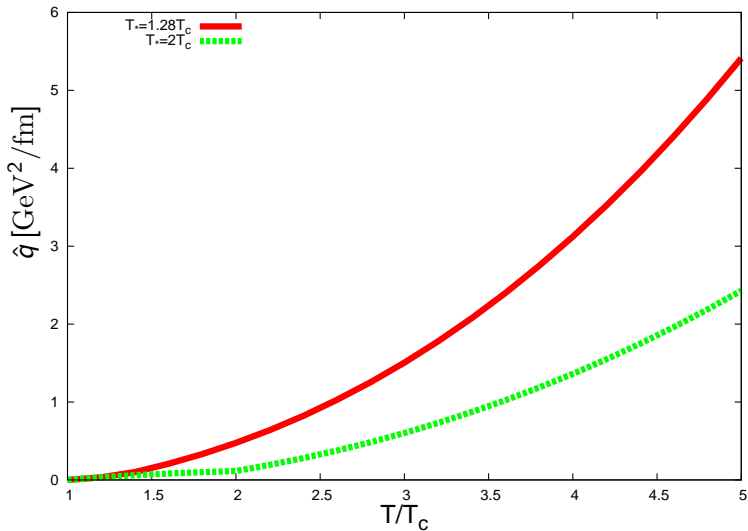


Figure: The jet quenching parameter $\hat{q}(T)$ for various values of the dimensional-reduction temperature, $T_* = 1.28T_c$ and $T_* = 2T_c$.

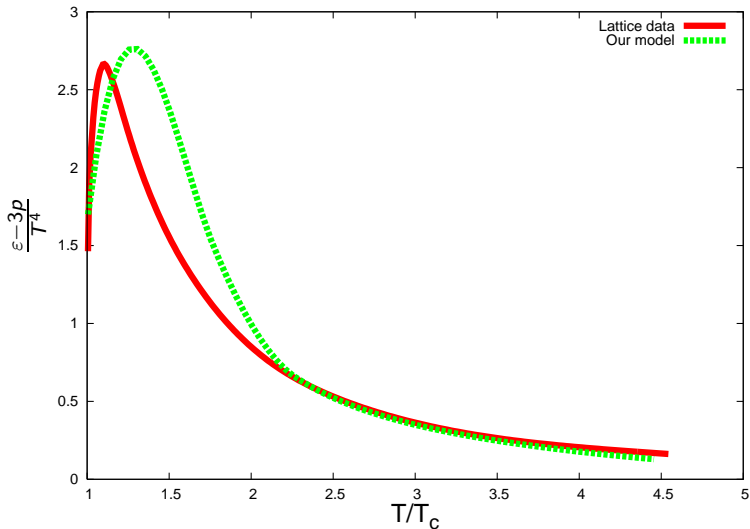


Figure: Lattice data on the interaction measure $(\epsilon - 3p)/T^4$ (courtesy of F. Karsch) compared to the prediction of the two-component model.

The Kubo formulae

At $T \gg T_c$, the two contributions become strictly additive:

$$\rho = \rho_{\text{backgr}} + \rho_{\text{pert}},$$

and $\rho_{\text{pert}} \propto g_T^4 \omega^4$ together with $\langle \Theta_{\mu\nu}(0) \Theta_{\lambda\rho}(x) \rangle_{\text{pert}} \propto g_T^4 / |x|^8$ can be isolated simultaneously.

This project (to be realized through the Kubo formulae):

– Using the stochastic vacuum model at finite temperature (Yu.A. Simonov, N.O. Agasian, '95 – '08), calculate ρ_{backgr} .

To be presented below.

– Calculate the contribution of valence gluons to ρ at $T \sim T_c$.

The Kubo formulae

A reminder on the stochastic vacuum model (SVM).

While QCD sum rules assume $\langle g^2(F_{\mu\nu}^a)^2 \rangle$, the SVM additionally assumes a finite correlation length of the vacuum, $\mu^{-1} < \infty$ (Pisa group, '86-'03):

$$\langle F_{\mu\nu}^a(x) F_{\lambda\rho}^b(0) \rangle \sim e^{-\mu|x|} \Rightarrow$$

the SVM can quantitatively describe confinement with the string tension $\sigma \propto \mu^{-2} \langle g^2(F_{\mu\nu}^a)^2 \rangle$.

The spatial string tension at $T > T_c$:

$$\sigma_s(T) \propto \mu_T^{-2} \langle g^2(F_{ij}^a)^2 \rangle_T,$$

i.e. the chromo-magnetic vacuum still confines.

The Kubo formulae

Since

$$\Theta_{\mu\nu} = \mathcal{O}(g^2(F_{\alpha\beta}^a)^2),$$

the expected contributions of the background fields to the viscosities are

$$\eta \propto \zeta \propto \frac{\langle g^2(F_{ij}^a)^2 \rangle_T^2}{\mu_T^5},$$

in agreement with

$$\sigma_{\text{total}}^{\text{SVM}} \propto \langle g^2(F_{\mu\nu}^a)^2 \rangle^2 \quad (\text{Heidelberg group, '91 - '03})$$

\Rightarrow at temperatures $T > T_*$,

$$\eta \propto \zeta \propto (g_T^2 T)^3,$$

whereas $s \propto T^3$ at $T \gtrsim 2T_c \Rightarrow$

$$\frac{\eta}{s} \propto \frac{\zeta}{s} \propto g_T^6 \quad \text{at} \quad T \gtrsim 2T_c.$$

We get the coefficients in these formulae.

Calculation of the viscosities

Notations:

$$\langle g^2(F_{\mu\nu}^a)^2 \rangle \equiv \langle G^2 \rangle, \quad \langle g^2(F_{ij}^a)^2 \rangle_T \equiv \langle G^2 \rangle_T, \quad \rho_{\text{backgr}} \equiv \rho_T, \quad \omega_k = 2\pi T k.$$

Assuming at $T = 0$ exponentially falling off Ansätze

$$U_{T=0}^{(s),(b)}(x) = N_\alpha^{(s),(b)} \langle G^2 \rangle^2 \cdot \frac{K_{2-\alpha}(M|x|)}{(M|x|)^{2-\alpha}}, \quad \text{where } \alpha > 0,$$

we get at $T > T_c$ the Fourier transformed $(\sum_k e^{i\omega_k x_4} f_k)$ Kubo formulae:

$$\int_0^\infty d\omega \rho_T^{(s),(b)}(\omega) \frac{\omega}{\omega^2 + \omega_k^2} = \pi^2 2^\alpha \Gamma(\alpha) N_\alpha^{(s),(b)} \langle G^2 \rangle_T^2 \frac{M_T^{2\alpha-4}}{(\omega_k^2 + M_T^2)^\alpha}. \quad (*)$$

Calculation of the viscosities

Lorentzian-type spectral densities

$$\rho_T^{(s),(b)}(\omega) = C_T^{(s),(b)} \cdot \frac{\omega}{(\omega^2 + M_T^2)^{\alpha + \frac{1}{2}}}$$

ensure that both sides of Eq. (*) have the same large- $|k|$ behavior.

$M_T \sim \mu_T$ is the momentum scale, below which PT breaks down.

- For $|k| \gg 1$,

$$\text{LHS of Eq. (*)} = \frac{C_T^{(s),(b)}}{\omega_k^{2\alpha}} \left[\frac{\pi}{2 \sin(\pi\alpha)} + \mathcal{O}\left(\frac{M_T^2}{\omega_k^2}\right) + \sum_{i=2}^{\infty} c_i \left(\frac{M_T}{\omega_k}\right)^{i-2\alpha} \right]$$

\Rightarrow the leading term in the brackets is k -independent only for $\alpha < 1$.

$$\text{RHS of Eq. (*)} = \pi^2 2^\alpha \Gamma(\alpha) N_\alpha^{(s),(b)} \frac{\langle G^2 \rangle_T^2}{\omega_k^{2\alpha}} M_T^{2\alpha-4} \cdot \left[1 + \mathcal{O}\left(\frac{M_T^2}{\omega_k^2}\right) \right].$$

Calculation of the viscosities

$$\Rightarrow \eta \Big|_{|k| \gg 1} = \pi^2 2^{\alpha+1} \Gamma(\alpha) N_\alpha^{(s)} \sin(\pi\alpha) \frac{\langle G^2 \rangle_T^2}{M_T^5}.$$

Note: $|k| \gg 1$ means $|k| \geq 3$, since $\frac{M_T}{\omega_3} < 0.35$ for any $T > T_c$.

• For $|k| \sim 1$ (e.g. $k = 0$ for $T > T_*$), $\mathcal{O}\left(\frac{\omega_k^2}{M_T^2}\right)$ -terms and higher can be disregarded \Rightarrow

$$\eta \Big|_{|k| \sim 1} = \pi^{5/2} 2^{\alpha+1} \Gamma\left(\alpha + \frac{1}{2}\right) N_\alpha^{(s)} \frac{\langle G^2 \rangle_T^2}{M_T^5}.$$

The ratio

$$\frac{\eta \Big|_{|k| \gg 1}}{\eta \Big|_{|k| \sim 1}} = \frac{\Gamma(\alpha) \sin(\pi\alpha)}{\sqrt{\pi} \Gamma\left(\alpha + \frac{1}{2}\right)} \quad \text{for } 0 < \alpha < 1$$

is equal to 1 at $\alpha = \frac{1}{2}$, i.e. $\eta \Big|_{\alpha=\frac{1}{2}}$ is k -independent.

Calculation of the viscosities

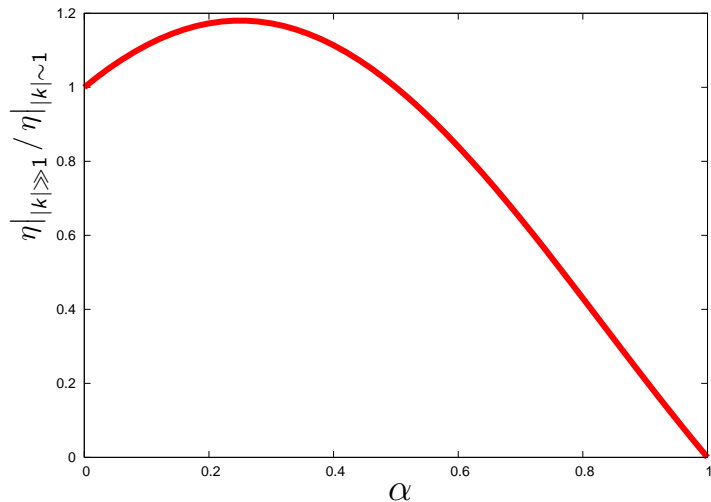


Figure: The ratio $\frac{\eta_{|k|\gg 1}}{\eta_{|k|\sim 1}}$.

Calculation of the viscosities

For $\alpha = \frac{1}{2}$, $\rho_T^{(s),(b)}(\omega)$ take the purely Lorentzian form, with

$$C_T^{(s),(b)} = (2\pi)^{3/2} N_{1/2}^{(s),(b)} \cdot \frac{\langle G^2 \rangle_T^2}{M_T^3}.$$

The coefficients $N_{1/2}^{(s),(b)}$ can be determined via the Gaussian-dominance hypothesis, which disregards the connected parts of $\langle \Theta_{\mu\nu}(0)\Theta_{\lambda\rho}(x) \rangle$.

The SVM parametrizes the remaining two-point functions.

- Retaining only confining self-interactions of the background fields:

$$\langle g^2 F_{\mu\nu}^a(x) F_{\lambda\rho}^b(0) \rangle = \frac{\langle G^2 \rangle}{12} \cdot (\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda}) \cdot \frac{\delta^{ab}}{N_c^2 - 1} \cdot D(x),$$

where $D(x) \rightarrow e^{-\mu|x|}$. The compatibility with $U_{T=0,\alpha=1/2}^{(s)}$ is achieved by

$$D(x) = \mathcal{A} \cdot \sqrt{\frac{K_{3/2}(2\mu|x|)}{(2\mu|x|)^{3/2}}} \quad \text{and} \quad M = 2\mu.$$

Calculation of the viscosities

$$\Rightarrow N_{1/2}^{(s)} = \frac{\mathcal{A}^2}{576}.$$

The constant \mathcal{A} is fixed by $\sigma_f = \frac{\langle G^2 \rangle}{144} \int d^2x D(x) \Rightarrow$

$$\mathcal{A} = \frac{4}{\int_0^\infty dz \cdot z^{1/4} \cdot \sqrt{K_{3/2}(z)}} \simeq 1.05 \Rightarrow$$

the shear viscosity

$$\eta = \frac{\pi^{5/2} \mathcal{A}^2}{4608\sqrt{2}} \cdot \frac{\langle G^2 \rangle_T^2}{\mu_T^5}.$$

Similarly, in the one-loop approximation where $\frac{\beta(g)}{2g} \simeq -\frac{11}{32\pi^2} g^2$,
the bulk viscosity

$$\zeta = \frac{\mathcal{A}^2}{1728\sqrt{2}\pi^3} \left(\frac{11}{32}\right)^2 \cdot \frac{\langle G^2 \rangle_T^2}{\mu_T^5}.$$

Calculation of the viscosities

- Accounting also for the nonconfining nonperturbative self-interactions of the background fields:

$$\langle g^2 F_{\mu\nu}^a(0) F_{\lambda\rho}^b(x) \rangle = \frac{\langle G^2 \rangle}{12} \cdot \frac{\delta^{ab}}{N_c^2 - 1} \cdot \{ \kappa (\delta_{\mu\lambda} \delta_{\nu\rho} - \delta_{\mu\rho} \delta_{\nu\lambda}) D(x) + \\ + \frac{1 - \kappa}{2} [\partial_\mu (x_\lambda \delta_{\nu\rho} - x_\rho \delta_{\nu\lambda}) + \partial_\nu (x_\rho \delta_{\mu\lambda} - x_\lambda \delta_{\mu\rho})] D_1(x) \}, \text{ where } \kappa \in [0, 1].$$

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Cf. the Wilson loop $\langle W(C) \rangle = \langle \text{tr} P \exp (ig \oint_C dx_\mu T^a A_\mu^a) \rangle$:

$$\langle W(C) \rangle = \exp \left\{ -\frac{C_2 \langle G^2 \rangle}{96(N_c^2 - 1)} \left[2\kappa \int_{\Sigma_{\min}} d\sigma_{\mu\nu}(x) \int_{\Sigma_{\min}} d\sigma_{\mu\nu}(x') D(|x-x'|) + \right. \right. \\ \left. \left. + (1-\kappa) \oint_C dx_\mu \oint_C dx'_\mu \int_{(x-x')^2}^{\infty} d\xi D_1(\sqrt{\xi}) \right] \right\}.$$

Lattice data (Pisa group) suggest that $D_1(x) = D(x)$, and $\kappa \simeq 0.83$.

Calculation of the viscosities

$U_{T=0}^{(s),(b)}$ contain terms through $\mathcal{O}((1-\kappa)^2) \Rightarrow$ seeking $D(x)$ in the form

$$D(x) = \mathcal{A}_\kappa \cdot f_\kappa(\mu|x|), \quad \text{where } f_\kappa = f_{\kappa=1} + (1-\kappa)f^{(1)} + (1-\kappa)^2f^{(2)},$$

$$\mathcal{A}_{\kappa=1} = \mathcal{A} \quad \text{and} \quad f_{\kappa=1}(z) = \sqrt{\frac{K_{3/2}(2z)}{(2z)^{3/2}}} = \frac{\pi^{1/4}}{2^{7/4}} \cdot \frac{e^{-z}}{z^{3/2}} \cdot (1+2z)^{1/2}.$$

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Then

$$f_\kappa(z) = \frac{\pi^{1/4}}{256 \cdot 2^{3/4}} \cdot \frac{e^{-z}}{[z(1+2z)]^{3/2}} \cdot \left\{ 128(1+2z)^2 + \right. \\ \left. + (1-\kappa) \cdot 16(1+2z)(3+6z+4z^2) + (1-\kappa)^2 \cdot [9 + 4z \cdot (9 + z \cdot (7 + 4z(1+z)))] \right\}$$

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The constant \mathcal{A}_κ is again fixed via the string tension:

$$\mathcal{A}_\kappa = \frac{1}{\int_0^\infty dz \cdot z \cdot f_\kappa(z)}.$$

Calculation of the viscosities

In particular, $\mathcal{A}_{\kappa=0.83} \simeq 0.97 \Rightarrow \left(\frac{\mathcal{A}_{\kappa=0.83}}{\mathcal{A}}\right)^2 \simeq 0.85 \Rightarrow$

$\eta^{\kappa=0.83}$ and $\zeta^{\kappa=0.83}$ are by 15% (that is close to 17%) smaller than, respectively, $\eta^{\kappa=1}$ and $\zeta^{\kappa=1}$.

Parameters for the numerical calculation:

- $T_c = 270 \text{ MeV}$.
- The two-loop running coupling in SU(3) YM:

$$g^{-2}(T) = 2b_0 \ln \frac{T}{\Lambda} + \frac{b_1}{b_0} \ln \left(2 \ln \frac{T}{\Lambda} \right),$$

$$b_0 = \frac{11N_c}{48\pi^2}, \quad b_1 = \frac{34}{3} \left(\frac{N_c}{16\pi^2} \right)^2, \quad N_c = 3, \quad \Lambda = 0.104 T_c.$$

Calculation of the viscosities

- Temperature dependence:

$$f(T) \equiv \begin{cases} 1 & \text{at } T_c < T < T_*, \\ \frac{g_T^2 \cdot T}{g_{T_*}^2 \cdot T_*} & \text{at } T > T_*, \end{cases}$$

$$\Rightarrow \mu_T = \mu \cdot f(T), \quad \sigma_f(T) = \sigma_f \cdot f^2(T), \quad \langle G^2 \rangle_T = \langle G^2 \rangle \cdot f^4(T).$$

where $\mu = 894 \text{ MeV}$ (Pisa group, '97), $\sigma_f = (440 \text{ MeV})^2$, $\langle G^2 \rangle = \frac{72}{\pi} \sigma_f \mu^2$.

- Determining T_* from the equation $\sigma_f(T_*) = \sigma_f$, where $\sigma_f(T) = [0.566 g^2(T) T]^2$ (Bielefeld group, '93, '96) $\Rightarrow T_* = 1.28 T_c$.
- Entropy density $s(T) = \frac{dp_{\text{stat}}}{dT} \Rightarrow s(T)/T^3$ is nearly constant at $T \gtrsim 2T_c$.

Calculation of the viscosities

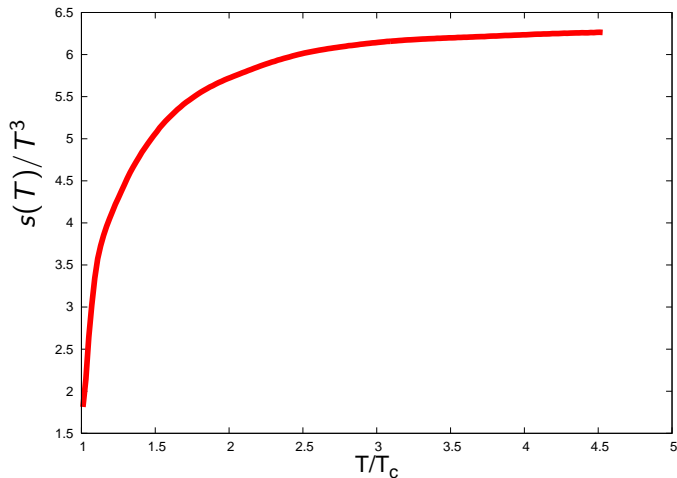


Figure: Entropy density $s(T)$, in the units of T^3 , derived from the lattice values for the pressure (G. Boyd et al., 1996; courtesy of F. Karsch).

The results

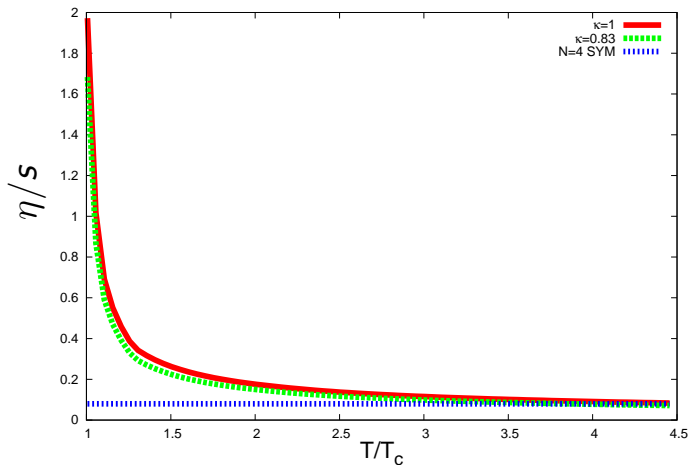


Figure: Calculated ratios of η/s . Also shown is the conjectured lower bound of $1/(4\pi)$ realized in $\mathcal{N} = 4$ SYM.

The results

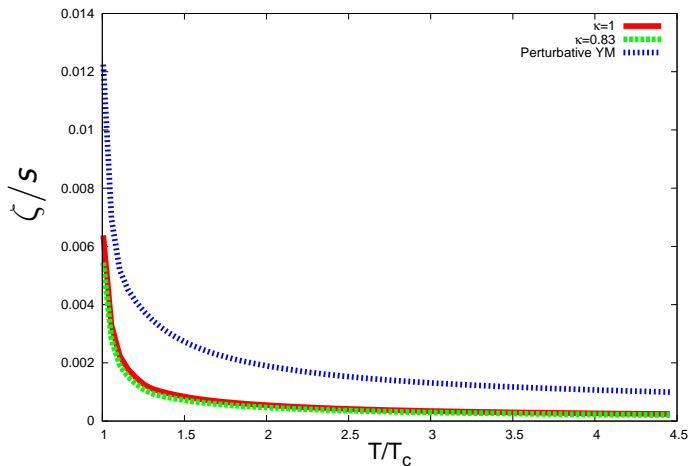


Figure: Calculated ratios of ζ/s . Also shown are the perturbative values ζ_{pert}/s , where $\zeta_{\text{pert}} = \frac{0.443\alpha_s^2 T^3}{\ln(7.14/gT)}$ (P. Arnold et al., '06) is extrapolated down to $T = T_c$.

Concluding remarks

- The calculated soft contribution to η/s falls off rapidly at $T_c < T \lesssim 2T_c$, and further as $\mathcal{O}(g_T^6)$, gradually crossing $1/(4\pi)$.
- However, at $T \gg T_c$, the perturbative result, with

$$\eta_{\text{NLL}} = \frac{T^3}{g_T^4} \cdot \frac{27.126}{\ln \frac{2.765}{g_T}}$$

(P. Arnold et al., '03), takes it over \Rightarrow A minimum of the full η/s should exist at intermediate temperatures (cf. other liquids), yielding the temperature of a **possible liquid-gas phase transition**.

- For ζ/s , perturbative contributions only enhance the $\mathcal{O}(g_T^6)$ -behavior to the $\mathcal{O}(g_T^4)$ -one.
- At $T \sim T_c$, an interference of nonperturbative contributions, produced by the background fields and by valence gluons, will be studied.

D.A., arXiv:1002.2406 (Annals Phys., in press).

Many thanks to the organizers for a very nice and interesting workshop.