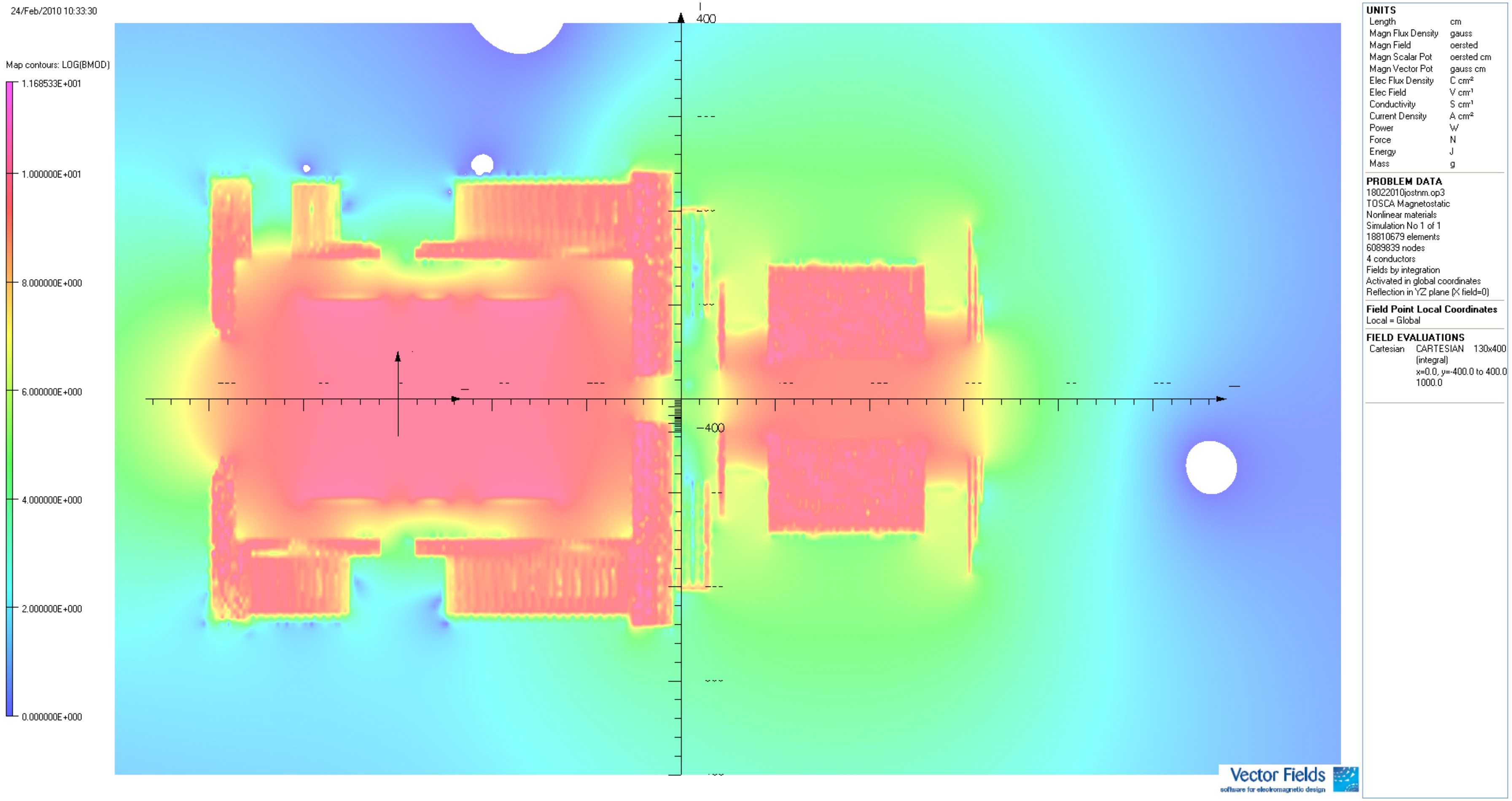
Magnetic field calculations for the solenoid and dipole magnets

Guangliang Yang
Glasgow University

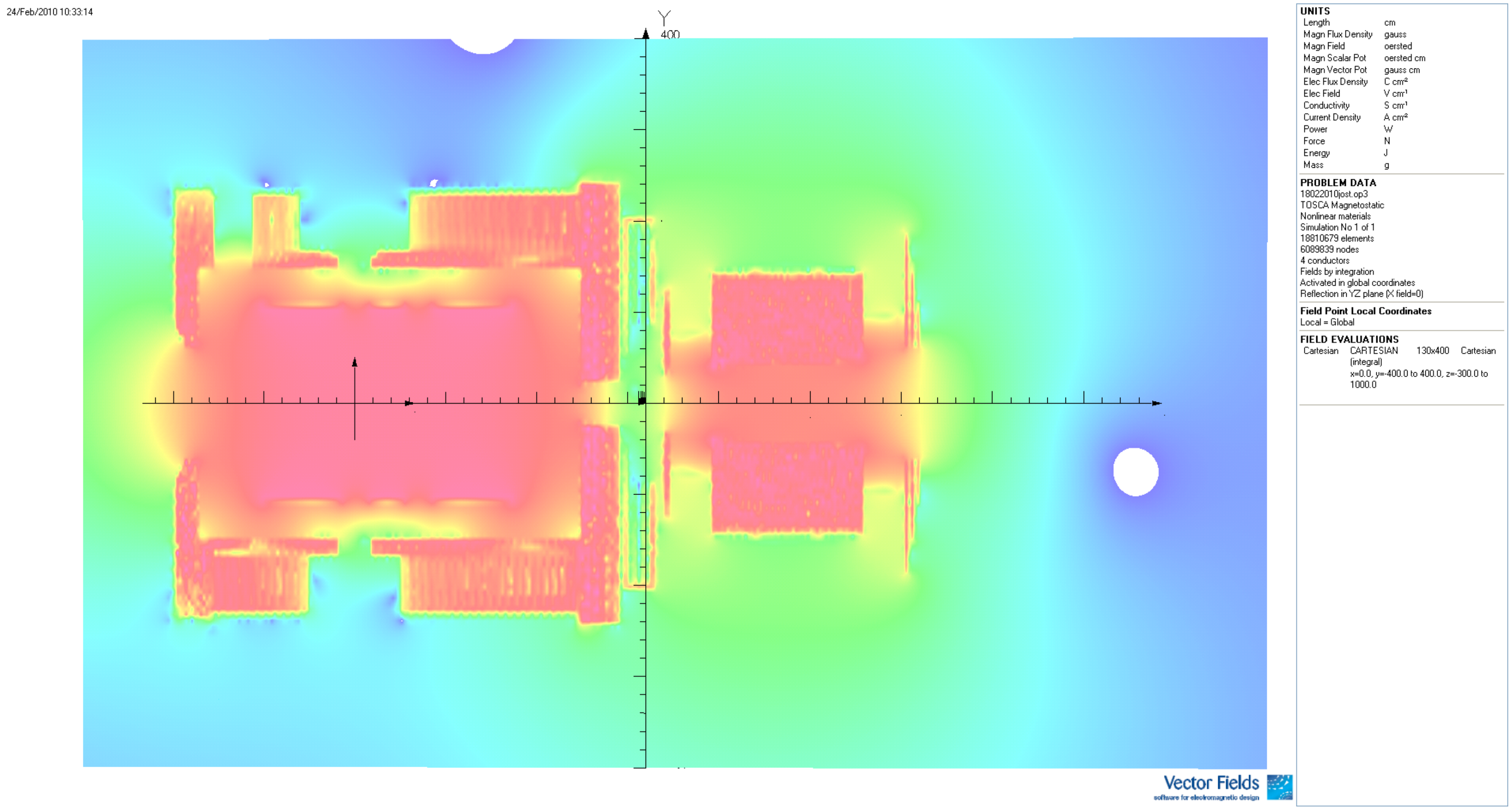
New calculations for the solenoid and the dipole magnets

- New calculations with a large background region: x -35 to 35 m, y -35 to 35 m, z -40 to 40 m.
- Previous calculations with a small background region: x -7 to 7 m, y -7 to 7 m, z -10 to 14 m.
- With a large background, field accuracy at area of interest has been improved.

Outer surface with normal magnetic boundary condition



Outer surface with tangential magnetic boundary condition



3D MULTIPOLE EXPANSION

- Motivation: help to reduce the amount of work related to field mapping.
- Theory: refer to PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 9, 012401 (2006).
- How? Using the field data on a cylindrical surface to recover 3D field inside the volume.

The field components in cylindrical coordinates

$$B_r = \sum_{m=0} \sum_{\ell=0}^{\infty} (-1)^\ell \frac{m!(2\ell + m)}{2^{2\ell} \ell!(\ell + m)!} \times C_{m,\alpha}^{[2\ell]}(z) r^{2\ell+m-1} \begin{Bmatrix} \sin(m\theta) \\ \cos(m\theta) \end{Bmatrix},$$

$$B_\theta = \sum_{m=0} \sum_{\ell=0}^{\infty} (-1)^\ell \frac{m!m}{2^{2\ell} \ell!(\ell + m)!} \times C_{m,\alpha}^{[2\ell]}(z) r^{2\ell+m-1} \begin{Bmatrix} \cos(m\theta) \\ -\sin(m\theta) \end{Bmatrix},$$

$$B_z = \sum_{m=0} \sum_{\ell=0}^{\infty} (-1)^\ell \frac{m!}{2^{2\ell} \ell!(\ell + m)!} \times C_{m,\alpha}^{[2\ell+1]}(z) r^{2\ell+m} \begin{Bmatrix} \sin(m\theta) \\ \cos(m\theta) \end{Bmatrix}.$$

Where the generalized gradient C m, α (z) ca be calculated by

$$C_{m,s}(z) = \frac{1}{2^m m!} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \exp(ikz) \frac{k^{m-1}}{I_m{}'(kR)} \tilde{\mathcal{B}}_m(R, k),$$

$$C_{m,c}(z) = \frac{1}{2^m m!} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \exp(ikz) \frac{k^{m-1}}{I_m{}'(kR)} \tilde{\mathcal{A}}_m(R, k),$$

Here $\tilde{\mathcal{B}}_m(R, k)$ and $\tilde{\mathcal{A}}_m(R, k)$ are the Fourier transforms of $\mathcal{B}_m(R, z)$ and $\mathcal{A}_m(R, z)$

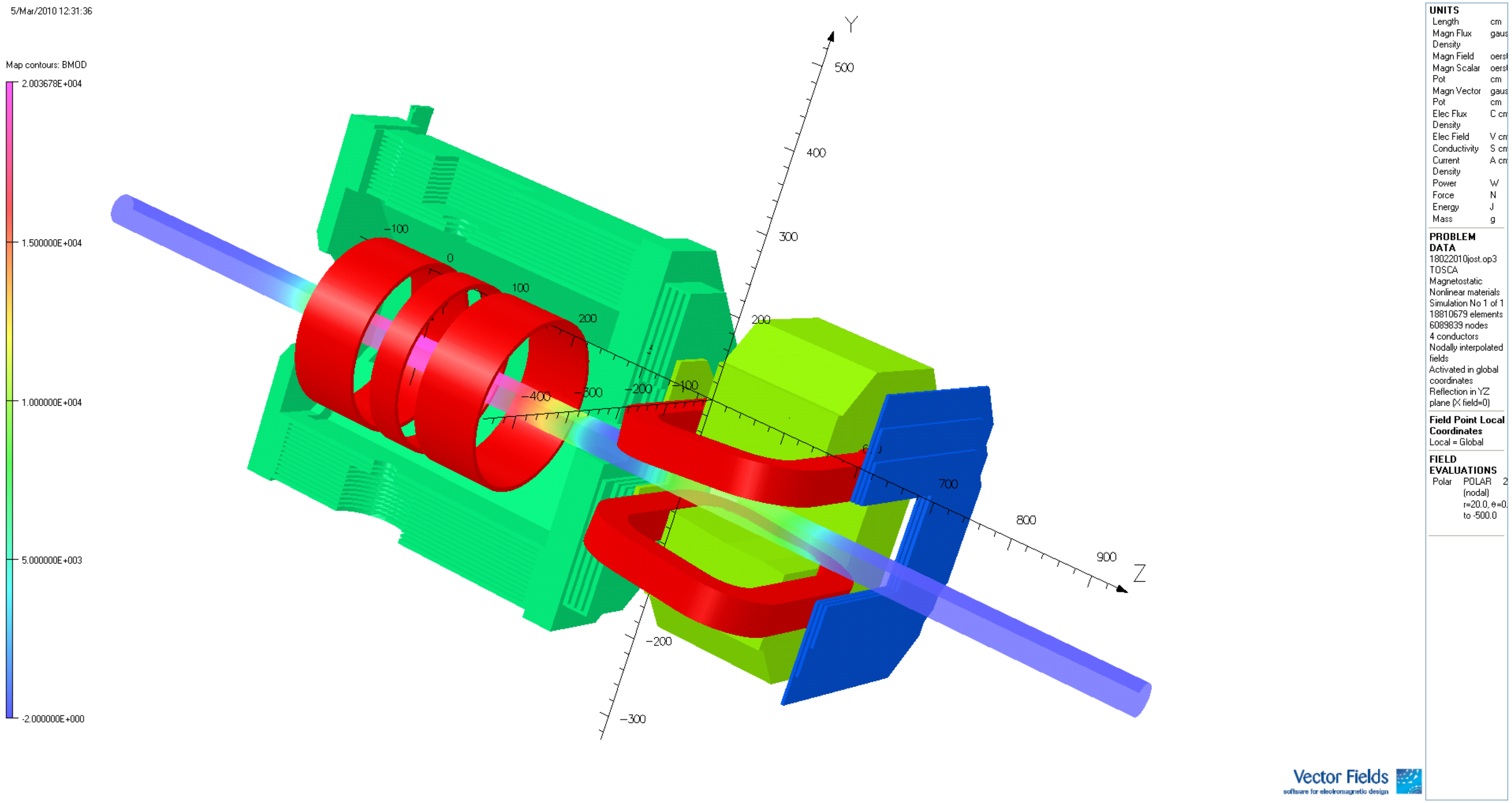
$$\tilde{\mathcal{B}}_m(R, k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dz \exp(-ikz)\mathcal{B}_m(R, z),$$

$$\tilde{\mathcal{A}}_m(R, k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dz \exp(-ikz)\mathcal{A}_m(R, z).$$

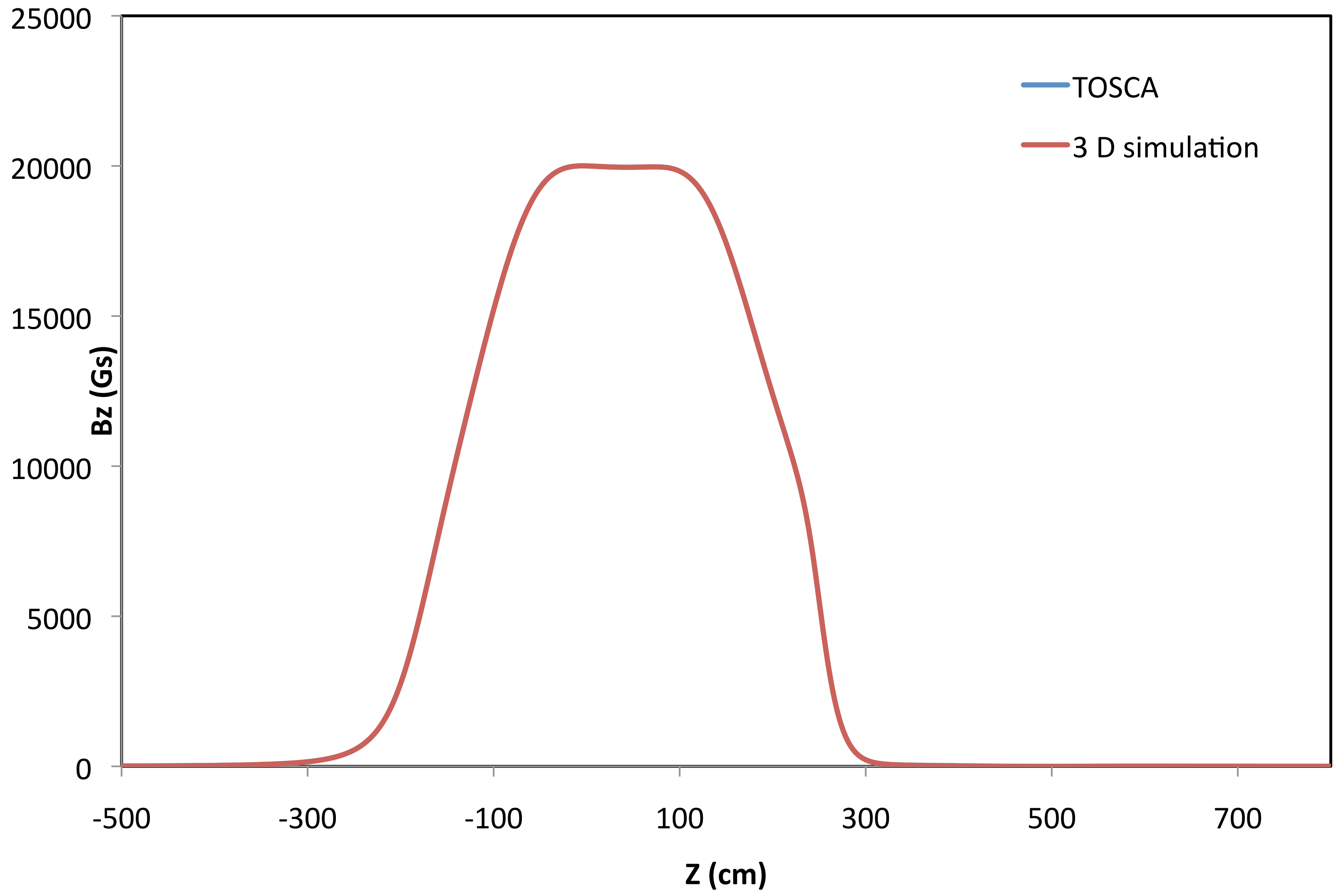
where $\mathcal{B}_m(R, z)$ and $\mathcal{A}_m(R, z)$ are the amplitudes of the normal and skew components of $B_r(R, \theta, z)$ harmonics of order m.

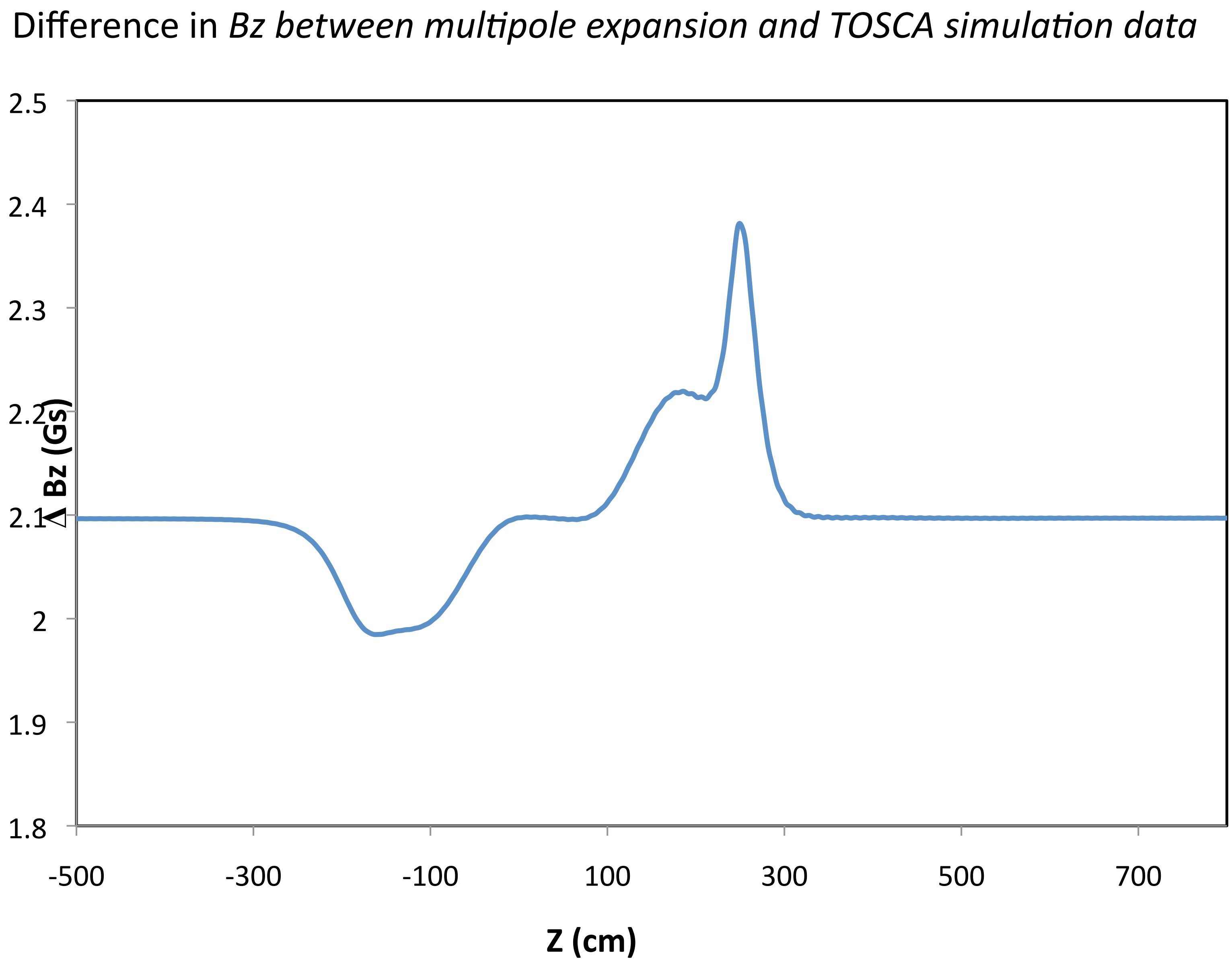
$$B_r(R, \theta, z) = \sum_{m=0}^{\infty} \mathcal{B}_m(R, z) \sin(m\theta) + \mathcal{A}_m(R, z) \cos(m\theta),$$

Magnetic field on a cylindrical surface



Bz on the z axis





Difference in Bz between multipole expansion and TOSCA simulation data
2.5
2.4
2.3
2.2
2.1
2
1.9
1.8
Δ Bz (Gs)
-500
-300
-100
100
300
500
700
Z (cm)

Conclusion and outlook

- It is possible to calculate the 3D field multiple extension from the field data on a cylindrical surface.
- If this method is not sensitive to the error raw field data on the cylindrical surface, then it can be used to help the field mapping.
- Further work need to be done, to verify how the error of raw field data will affect the calculated 3 D field.