COMPRESSED BARYONIC MATTER EXPERIMENT

CBM-TN-2018-001
September 4, 2019

Centrality estimation with MC Glauber model

Oleksii Lubynets[1], Ilya Selyuzhenkov[2], Viktor Klochkov[3]

1. Kiev National University
2. GSI, MEPhI
3. GSI, Frankfurt University

Emails:
lubinets95@gmail.com
ilya.selyuzhenkov@gmail.com
klochkov44@gmail.com

Abstract

In this work the Glauber-model-based algorithm of centrality estimation at the CBM experiment was performed. It gives possibility to map model quantities (b, N_{part}, N_{coll}) and measurables (multiplicity, spectators energy). Centrality of *Au-Au* collisions at $12A$ GeV was estimated, the events were classified into centrality classes, average values of model quantities in centrality classes were evaluated.
Keywords: heavy-ion collisions, centrality, impact parameter, multiplicity of produced particles, Glauber model.

1 Introduction

The size and evolution of the medium produced in heavy-ion collision depend on the collision geometry, particularly on the impact parameter b, number of participants N_{part} and number of nucleon-nucleon collisions N_{coll}. These model quantities cannot be measured directly at the experiment.
Experimentally the collision is characterized with the multiplicity of produced charged particles detected with tracking system of the experiment (STS and MVD at Fig. 1) or with the projectile spectators energy detected with PSD (see Fig. 1).

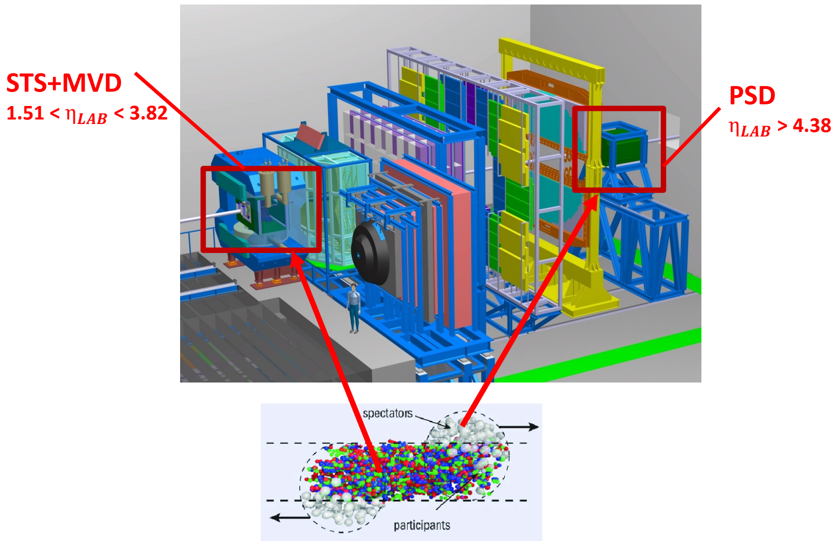

Figure 1: The CBM experiment and subdetectors for centrality determination

Events can be distributed into centrality classes according to measured quantities (multiplicity and spectators energy). Centrality is defined in the following way: events with high values of centrality have large impact parameter (corresponds to a small multiplicity and a high spectators energy), and events with low centrality have small impact parameter (corresponds to high multiplicity and low spectators energy), see Fig. 2.

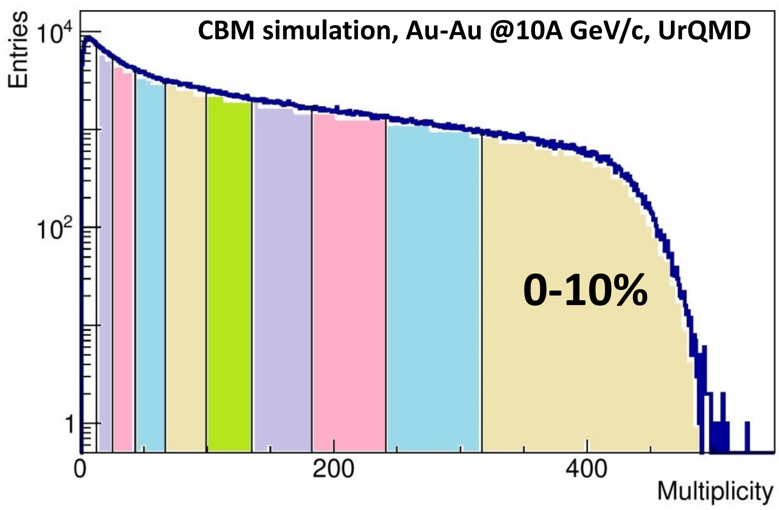

Figure 2: Multiplicity distribution and centrality classes

In terms of impact parameter centrality is defined:

$$c_b = \frac{1}{\sigma_{inel}} \int_0^b \frac{d\sigma}{db} db, \tag{1}$$

and in terms of multiplicity of charged particles:

$$c_{N_{ch}} = \frac{1}{\sigma_{inel}} \int_{N_{ch}}^{\infty} \frac{d\sigma}{dN_{ch}} dN_{ch} \tag{2}$$

These definitions are not equivalent, because the correlation between impact parameter and multiplicity has finite width. Therefore event classes selected using multiplicity can be mapped to a certain range of impact parameter. It can be done using the model which reproduces multiplicity and has impact parameter as model quantity. The model used in this work is Glauber model.

1.1 Glauber model

Glauber model considers the nuclei collision as individual interaction of nucleons. Nucleons are assumed to move independently and the size of nucleus is assumed to be much larger than nucleon-nucleon interaction scale.
Input parameters of Glauber model are nuclear density and nucleon-nucleon inelastic collision cross section. Nuclear density is usually described with Woods-Saxon potential:

$$\rho(r) = \rho_0 \frac{1 + \omega(r/R)^2}{1 + exp(\frac{r-R}{a})} \tag{3}$$

1.2 MC-Glauber fit

One of the methods to distribute events into centrality classes is phenomenological approach based on fitting the multiplicity of charged particles using Glauber model. This approach is based on assumption that number of produced charged particles in each nucleon-nucleon collision is distributed in negative binomial distribution (NBD). This assumption is motivated with measured multiplicity in proton-proton collisions, and it is well described with NBD [1].
This approach gives possibility to simulate multiplicity distribution and compare it with real or simulated data. For the event with impact parameter b the Glauber model defines number of participants N_{part} and number of nucleon-nucleon collisions N_{coll}, and the number of produced particles per one nucleon-nucleon interaction is parametrized with NBD:

$$P_{\mu,k}(n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \frac{(\mu/k)^n}{(\mu/k+1)^{n+k}}, \tag{4}$$

which gives the probability to have n particles per one ancestor, where μ is an average multiplicity and k is responsible for the distribution width.
In order to use this model to any collision with certain N_{part} and N_{coll}, the concept of independent emitting sources (ancestors) is used. We use a two-component model, which decomposes nucleon-nucleon collision into hard and soft interactions. In soft interactions the average multiplicity of produced particles is proportional to N_{part}, and in hard interactions - to N_{coll}. The number of ancestors is parametrized in the following way:

$$N_a = f N_{part} + (1-f) N_{coll}, \tag{5}$$

where $f \in [0;1]$.
For each event generated with MC-Glauber, the NBD is called N_a times in order to calculate the number

of particles produced in this event. Multiplicity distribution is simulated for different values of NBD parameters $\mu,\ k$ and N_a parameter f. Then the minimization procedure is performed in order to evaluate parameters values $\mu,\ k,\ f$ which correspond to minimal χ^2.

1.3 Framework description

The framework for centrality determination (Centrality Framework, [2]) is developed in C++(11) language with Root-6 package. The following algorithm for multiplicity distribution fitting is used:

1. NBD parameter k and N_a parameter f are set.
2. Histogram with multiplicity distribution to be fitted is given as input.
3. Glauber model root-file with N_{part} and N_{coll} is given as input.
4. For each event the number of ancestors is calculated with formula (5).
5. NBD is called N_a times, the number of produced particles is summed and written into fitting multiplicity distribution histogram.
6. Fitting histogram is normalized to have the same integral with input histogram.
7. Average multiplicity per one ancestor (μ parameter) is preliminary estimated in the following way: $\mu^* = \frac{M^{max}}{N_a^{max}}$, where M^{max} is maximal multiplicity in the input histogram and N_a^{max} is maximal number of ancestors evaluated from Glauber root-file.
8. For given $f,\ k$ and μ from $[0.7\mu^*; 1.1\mu^*]$ interval the value of χ^2 is calculated.
9. The value of μ from $[0.7\mu^*; 1.1\mu^*]$ interval which corresponds to minimal χ^2. This χ^2 is associated with $(f,\ k)$ pair.
10. Steps 1-9 are repeated for another values $(f,\ k)$ set by user.
11. In two-dimensional phase space $(f,\ k)$ the minimal χ^2 and corresponding f and k are searched.

2 Analysis

2.1 Framework self-consistency check

In order to be sure that mentioned algorithm is working correctly, the validation procedure was performed:

1. For given values of NBD and N_a parameters $(f = 0.70,\ \mu = 0.85,\ k = 9)$ the multiplicity distribution was generated (10^6 events). The values of $f,\ \mu,\ k$ were chosen in order to get the multiplicity distribution similar to typical that we have from CBM simulation.
2. This distribution was fitted with mentioned algorithm. Optimal values of $f,\ \mu,\ k$ were found.

As a result we got $\chi^2(f,k)$ dependence shown in Fig. 3.

In figures 4, 5 the $chi^2(f)$ and $\chi^2(k)$ dependencies for fixed k and f are shown.

As we can see, parameters values found with algorithm are $f = 0.68,\ \mu = 0.83,\ k = 24$ with $\chi^2 = 0.95 \pm 0.09$, and for "real" parameter values $\chi^2 = 1.04 \pm 0.09$. So, within the error, the input and found parameters sets are equivalent. Also, as we can see on figures 4, 5, there is a clear minimum of χ^2 along f, and a very shallow minimum along k.

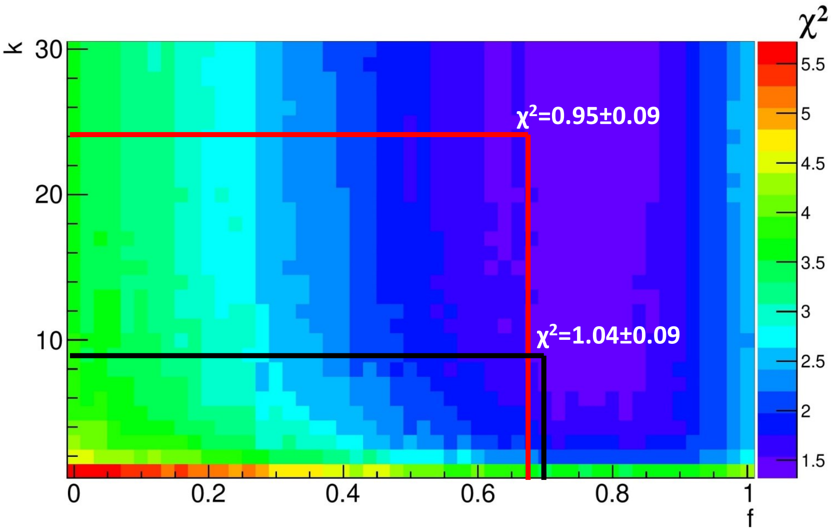

Figure 3: $\chi^2(f,k)$ dependence

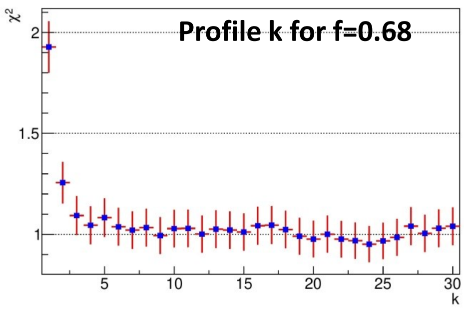

Figure 4: Dependence $\chi^2(f)$ for $k = 24$

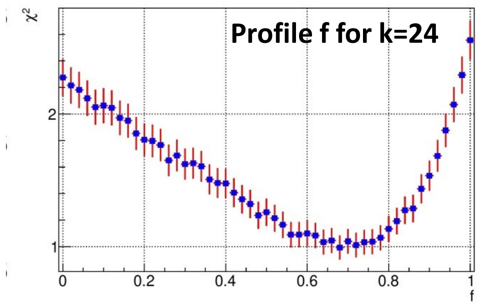

Figure 5: Dependence $\chi^2(k)$ for $f = 0.68$

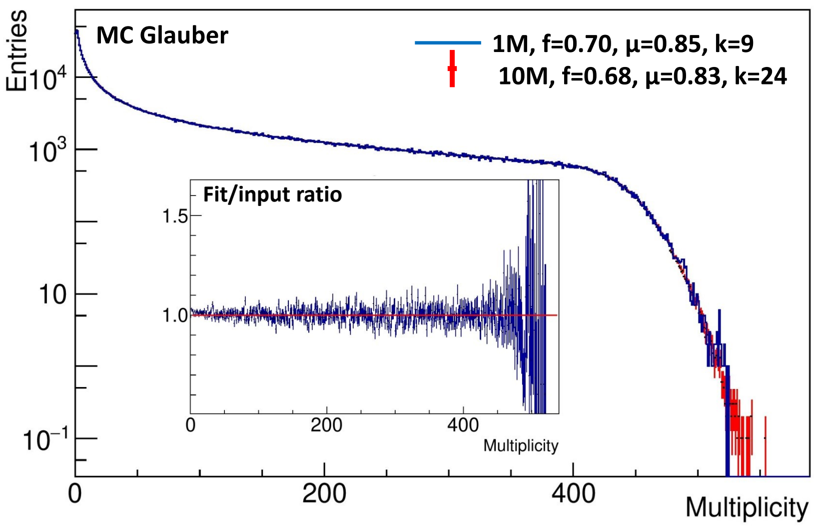

Figure 6: Input multiplicity distribution (blue line) and fitting histogram (red bars), fit-to-input ratio (inner plot)

In figure 6 we can see that fitting histogram (red bars) reproduces the input multiplicity distribution (blue line) correctly, and fit-to-input ratio is equal to 1 within the error.

So, we can conclude that fitting algorithm implemented in Centrality Framework works correctly.

2.2 Multiplicity fitting: all charged particles

Fitting procedure implemented in Centrality Framework was performed to fit multiplicity of produced *all* charged particles and to estimate centrality. To estimate centrality we used:

- 10^6*Au-Au* @ 12 *AGeV/c* in center-of-mass reference frame events generated with UrQMD [3, 4];
- to build multiplicity distribution we considered particles which have pseudorapidity $1.51 < \eta < 3.82$ in laboratory reference frame (corresponds to acceptance of STS with polar angle $2.5° < \theta < 25°$);
- for fitting this histogram 10^7 *Au-Au* simulated with Glauber model were used. The cross section of inelastic nucleon-nucleon interaction was set equal to 30 mb [5];
- fit range included region with multiplicity over 50 particles.

As a result we got fit shown in Fig. 7.

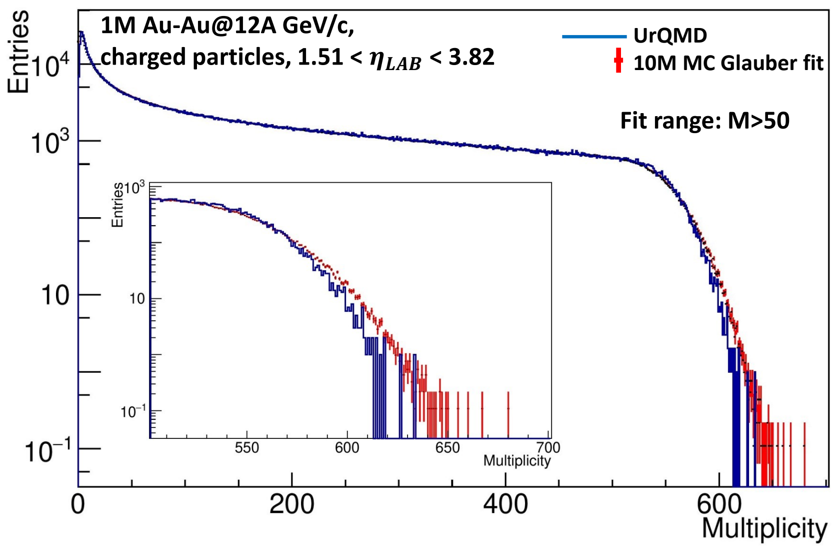

Figure 7: MC-Glauber fit (red bars) of UrQMD multiplicity distribution (blue line). All charged particles

We can see that fit does not reproduce multiplicity distribution in high multiplicity region (right tail), and $\chi^2 = 1.47 \pm 0.10$. It can be explained with that fact that *all* charged particles include protons, most of them are not produced in the collision but were the part of colliding nuclei, so their multiplicity is not described with Glauber model.

2.3 Multiplicity fitting: pions

The next step is an attempt to fit multiplicity of pions - charged particles *produced* in the collision, not exist in nuclei. Steps mentioned in previous paragraph were repeated for pions. As a result we got fit shown in Fig. 8.

We see that fit reproduces multiplicity histogram in the whole fit range, and $\chi^2 = 0.93 \pm 0.14$. The fit-to-input ratio in fit range is equal to 1 within the error. Evaluated NBD and N_a parameters ($f = 0.72,\ k = 13,\ \mu = 0.35$) can be used to distribute events into centrality classes and to evaluate average

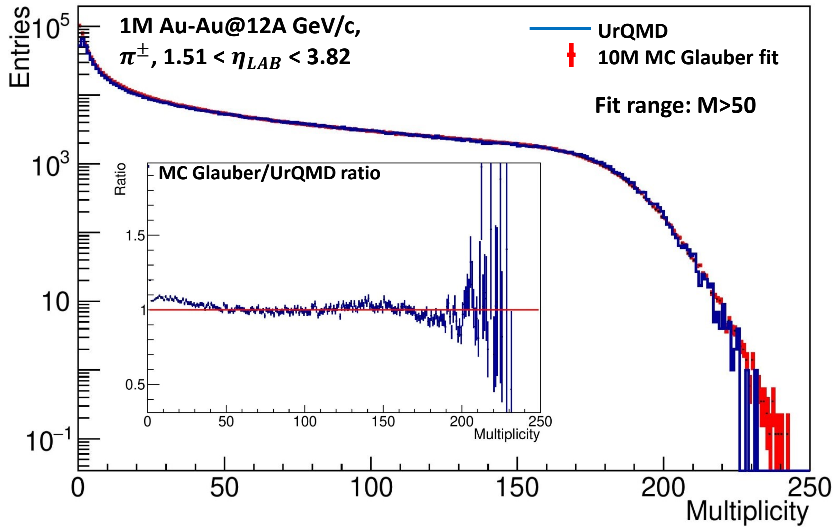

Figure 8: MC-Glauber fit (red bars) of UrQMD multiplicity distribution (blue line) and fit-to-input ratio (inner plot). Pions

values of model quantities (impact parameter b, number of participants N_{part}, number of nucleon-nucleon collisions N_{coll}) in each centrality class.
Fig. 9 shows the correlation between impact parameter and multiplicity.

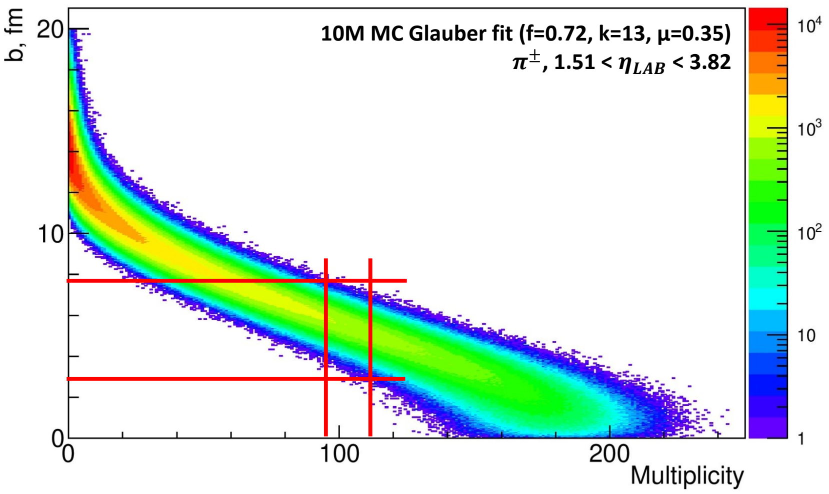

Figure 9: Correlation between particles multiplicity and impact parameter

2.4 Model values distribution in centrality classes

Figures 10, 11, 12 represent the distribution of b, N_{part}, N_{coll} in centrality classes with step 10%. Impact parameter distribution was built both with MC-Glauber fitting procedure (solid line) and directly from UrQMD simulation (dashed line).

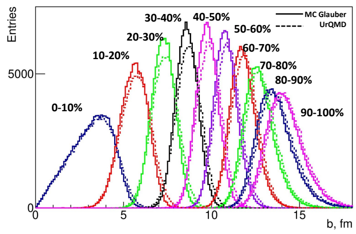

Figure 10: Impact parameter distribution in centrality classes

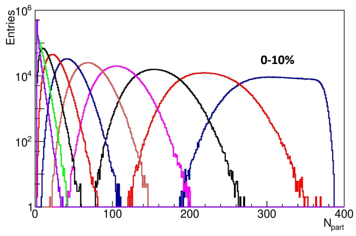

Figure 11: Participants number distribution in centrality classes. Logarithmic scale along Y-axis

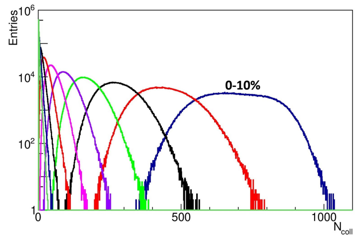

Figure 12: Nucleon-nucleon collisions number distribution in centrality classes. Logarithmic scale along Y-axis

2.5 Average values of model quantities in centrality classes

Figures 13, 14, 15 represent average values (markers) and standard deviation (bars) of b, N_{part}, N_{coll} in centrality classes.

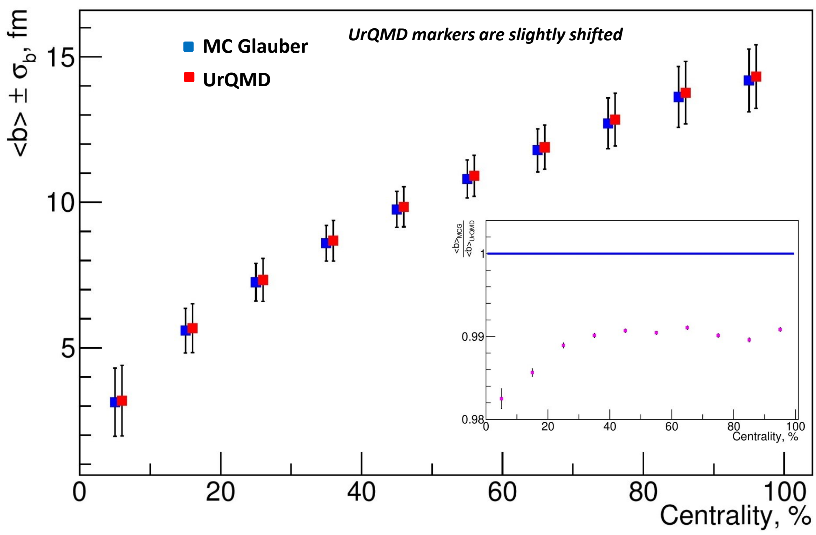

Figure 13: Impact parameter average values in centrality classes. UrQMD markers are slightly shifted

In table 1 there are average values and standard deviations of b, N_{part}, N_{coll} in centrality classes with step 10%.

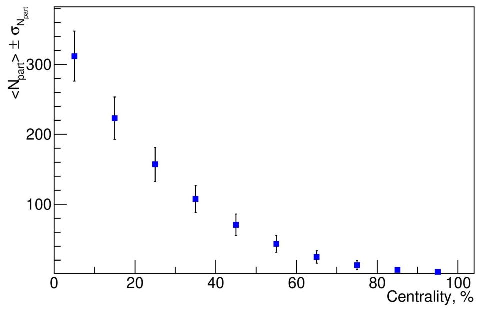

Figure 14: Participants number average values in centrality classes

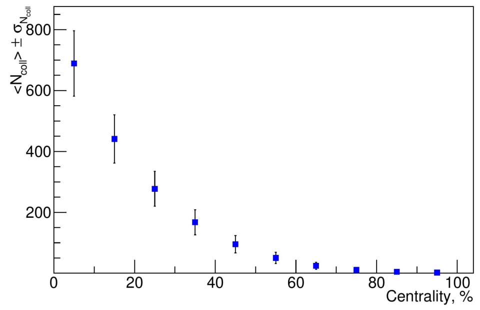

Figure 15: Nucleon-nucleon collisions number average values in centrality classes

Table 1: Model quantities in centrality classes

Centrality	$\langle b \rangle$	σ_b	$\langle N_{part} \rangle$	$\sigma_{N_{part}}$	$\langle N_{coll} \rangle$	$\sigma_{N_{coll}}$
0-10 %	3.13	1.18	311.74	35.83	688.82	107.26
10-20 %	5.59	0.77	222.96	30.41	441.16	79.50
20-30 %	7.25	0.65	157.10	24.32	277.25	57.49
30-40 %	8.60	0.62	107.63	19.57	167.40	41.27
40-50 %	9.76	0.62	70.60	15.61	95.40	28.70
50-60 %	10.81	0.66	43.44	12.15	50.30	18.89
60-70 %	11.78	0.74	24.66	9.07	24.37	11.68
70-80 %	12.72	0.88	12.79	6.32	10.86	6.77
80-90 %	13.61	1.05	6.20	4.01	4.58	3.72
90-100 %	14.19	1.08	3.43	2.28	2.20	1.97

3 Results

We observe a clear separation of centrality classes for events with low impact parameter (more central events), and overlapping of centrality classes for peripheral events. It means that for peripheral events it is advisable to make bin size larger.
We observe small (1-2%) difference between average impact parameter values evaluated with MC-Glauber procedure and taken directly from UrQMD simulation.

4 Summary

In this work the Monte-Carlo Glauber procedure was performed for fitting multiplicity of all charged particles and pions only. In case of all charged particles the fit does not reproduce high multiplicity region. For pions fit reproduces multiplicity distribution in the whole fit range.
Using evaluated NBD and N_a parameters μ, k, f, the relation between impact parameter and multiplicity centrality classes was extracted.

Outlook The following work will include:
- fitting multiplicity for full GEANT CBM simulation;
- centrality determination with energy deposed in PSD;
- using other models with fragments (DCM-QGSM) in addition to UrQMD.

References

[1] J.F. Grosse-Oetringhaus, K. Reygers: Charged-Particle Multiplicity in ProtonProton Collisions, *arXiv:0912.0023v2*

[2] V. Klochkov, I. Selyuzhenkov: Centrality Framework, *https://cbmgsi.githost.io/pwg-c2f/centrality*

[3] S. A. Bass *et al.*: Microscopic Models for Ultrarelativistic Heavy Ion Collisions, Prog. Part. Nucl. Phys. 41 (1998) 225-370

[4] M. Bleicher *et al.*: Relativistic Hadron-Hadron Collisions in the Ultra-Relativistic Quantum Molecular Dynamics Model J. Phys. G: Nucl. Part. Phys. 25 (1999) 1859-1896

[5] K.A. Olive *et al.*: Review of Particle Physics. Chinese Physics C, v.38 N9, 2014