Symmetry Energy, Neutron Star Crust & Neutron Skin

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From Neutron Stars to the r-process"
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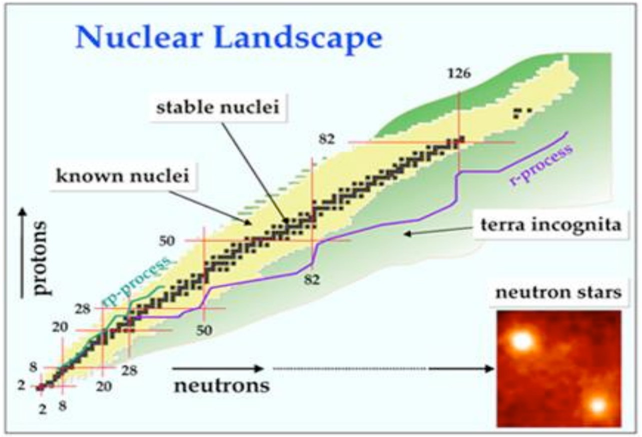
Motivation

Isospin asymmetric nuclear matter is present in:

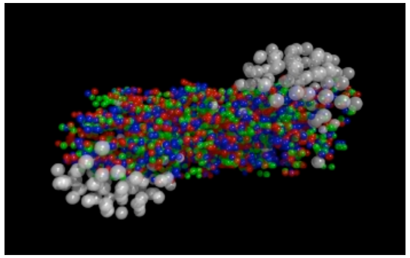
Nuclei, especially those far away from the stability line & in astrophysical systems (neutron stars)



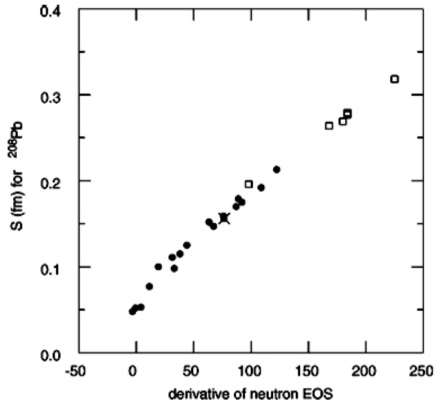
A well-grounded understanding of the properties of isospin-rich nuclear matter is necessary for both nuclear physics & astrophysics

However, some of these properties are not well constrained. In particular the density dependence of the symmetry energy is still an important source of uncertainties.

Some properties of asymmetric nuclear matter can be obtained from:



the analysis of experimental data in heavy ion collisions
(e.g., ID, GMR)



the analysis of existing correlations between different quantities in bulk matter & finite nuclei
(e.g. δR versus L)

PREX experiment @ JLAB

A major effort is being carried out to study experimentally the properties of asymmetric nuclear systems. Experiments at CSR , GSI (FAIR), RIKEN, GANIL, FRIB can probe the behavior of the symmetry energy close and above saturation density.

Astrophysical observations of compact objects
➔ window into nuclear matter at extreme isospin asymmetries

In this work ...

- We study the density dependence of the symmetry energy within the BHF approximation and compare our results with the ones obtained with several effective models (Skyrme & RMF).

- We analyze different correlations between the slope and curvature parameters of the symmetry energy and several physical quantities.

- We pay special attention to the correlations of these two parameters with the crust-core transition density in neutron stars and the neutron skin thickness.

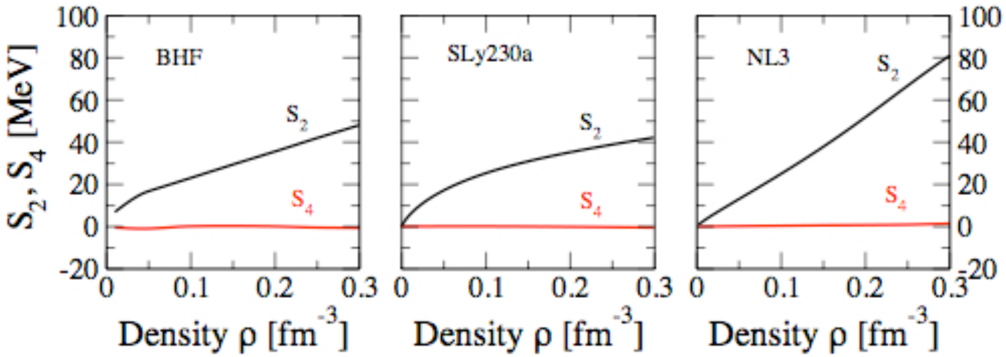
Phys. Rev. C 80, 045806 (2009)

Equation of State of Asymmetric Matter

Assuming charge symmetry for nuclear forces, the energy per particle of asymmetric matter can be expanded on the isospin asymmetry parameter $\beta=(N-Z)/(N+Z)=(\rho_n-\rho_p)/(\rho_n+\rho_p)$

$$\frac{E}{A}(\rho,\beta) = E_{SNM}(\rho) + S_2(\rho)\beta^2 + S_4(\rho)\beta^4 + O(6)$$

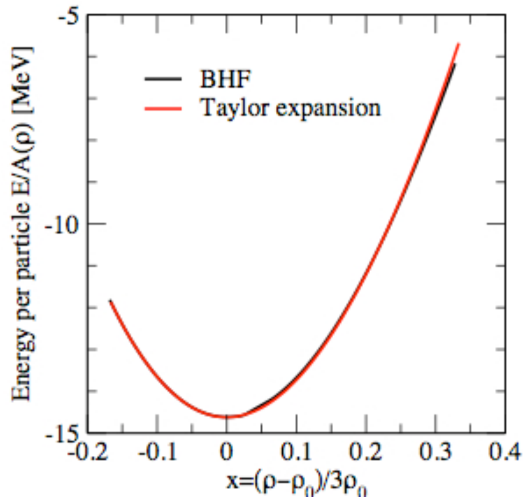
$$E_{SNM}(\rho) = \frac{E}{A}(\rho,\beta=0), \quad S_2(\rho) = \frac{1}{2}\frac{\partial^2 E/A}{\partial\beta^2}\bigg|_{\beta=0}, \quad S_4(\rho) = \frac{1}{24}\frac{\partial^4 E/A}{\partial\beta^4}\bigg|_{\beta=0}$$



$$S_2(\rho) \sim \frac{E}{A}(\rho,\beta=1) - \frac{E}{A}(\rho,\beta=0)$$

$E_{SNM}(\rho)$ it is commonly expanded around saturation density ρ_0

$$E_{SNM}(\rho) = E_0 + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{Q_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + O(4)$$

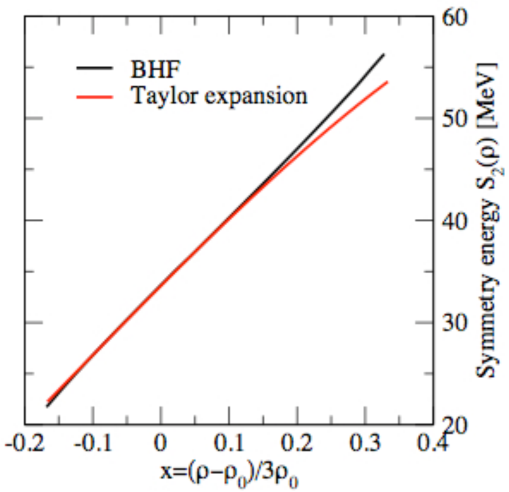


$$E_0 = E_{SNM}(\rho=\rho_0) \approx -16\, MeV$$

$$K_0 = 9\rho_0^2 \left.\frac{\partial^2 E_{SNM}(\rho)}{\partial\rho^2}\right|_{\rho=\rho_0} \approx 200 \div 300\, MeV$$

$$Q_0 = 27\rho_0^3 \left.\frac{\partial^3 E_{SNM}(\rho)}{\partial\rho^3}\right|_{\rho=\rho_0} \approx -500 \div 300\, MeV$$

Similarly the behavior of the symmetry energy $S_2(\rho)$ around ρ_0 can be also characterized in terms of a few bulk parameters



$$S_2(\rho) = E_{sym} + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{Q_{sym}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + O(4)$$

$$L = 3\rho_0 \left.\frac{\partial S_2(\rho)}{\partial\rho}\right|_{\rho=\rho_0} \quad K_{sym} = 9\rho_0^2 \left.\frac{\partial^2 S_2(\rho)}{\partial\rho^2}\right|_{\rho=\rho_0} \quad Q_{sym} = 27\rho_0^3 \left.\frac{\partial^3 S_2(\rho)}{\partial\rho^3}\right|_{\rho=\rho_0}$$

Less certain & predictions of different models vary largely

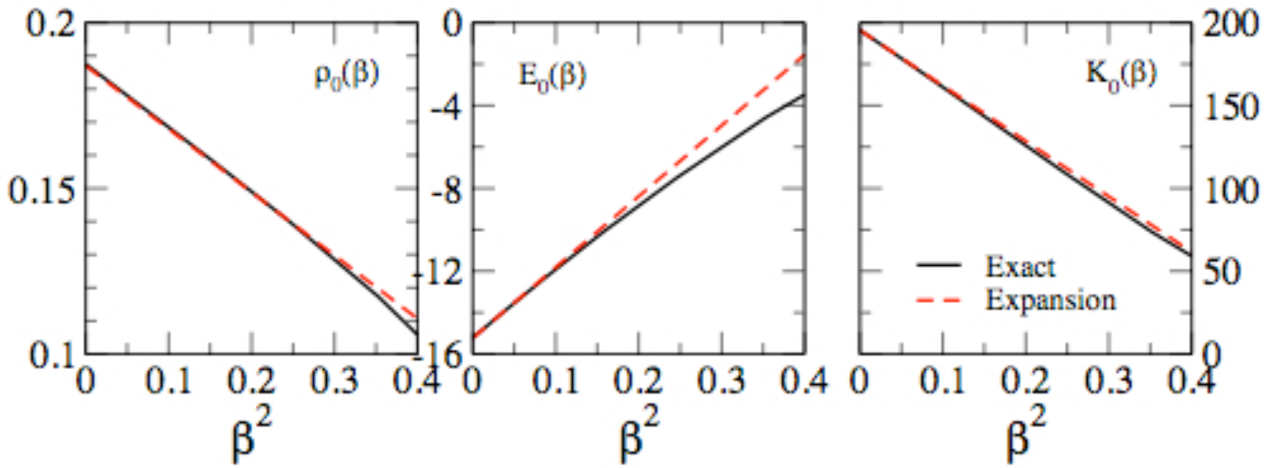
Combining the expansions of $E_{SNM}(\rho)$ and $S_2(\rho)$ one arrives at

$$\frac{E}{A}(\rho,\beta) = E_0(\beta) + \frac{K_0(\beta)}{2}\left(\frac{\rho-\rho_0(\beta)}{3\rho_0(\beta)}\right)^2 + \frac{Q_0(\beta)}{6}\left(\frac{\rho-\rho_0(\beta)}{3\rho_0(\beta)}\right)^3 + O(4)$$

where

$$\rho_0(\beta) = \rho_0 - 3\rho_0\frac{L}{K_0}\beta^2 + O(4) \qquad E_0(\beta) = E_0 + E_{sym}\beta^2 + O(4)$$

$$K_0(\beta) = K_0 + \underbrace{\left(K_{sym} - 6L - \frac{Q_0}{K_0}L\right)}_{K_\tau}\beta^2 + O(4) \qquad Q_0(\beta) = Q_0 + \left(Q_{sym} - 9L\frac{Q_0}{K_0}\right)\beta^2 + O(4)$$



Brueckner-Hartree-Fock approach of ANM

- Bethe-Goldstone Equation

$$G(\omega)_{\tau_1\tau_2;\tau_3\tau_4} = V_{\tau_1\tau_2;\tau_3\tau_4} + \sum_{ij} V_{\tau_1\tau_2;\tau_i\tau_j} \frac{Q_{\tau_i\tau_j}}{\omega - E_{\tau_i} - E_{\tau_j} + i\eta} G(\omega)_{\tau_i\tau_j;\tau_3\tau_4}$$

- Single particle energy & single particle potential

$$E_\tau(k) = \frac{\hbar^2 k^2}{2m_\tau} + \mathrm{Re}\left[U_\tau(k)\right]$$

$$U_\tau(k) = \sum_{\tau'} \sum_{k' \le k_{F_{\tau'}}} \left\langle \vec{k}\vec{k}' \middle| G(\omega = E_\tau(k) + E_{\tau'}(k')) \middle| \vec{k}\vec{k}' \right\rangle_{\mathcal{A}}$$

- Energy per particle

$$\frac{E}{A}(\rho,\beta) = \frac{1}{A} \sum_{\tau} \sum_{k \le k_{F_\tau}} \left(\frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2}\mathrm{Re}\left[U_\tau(\vec{k})\right] \right)$$

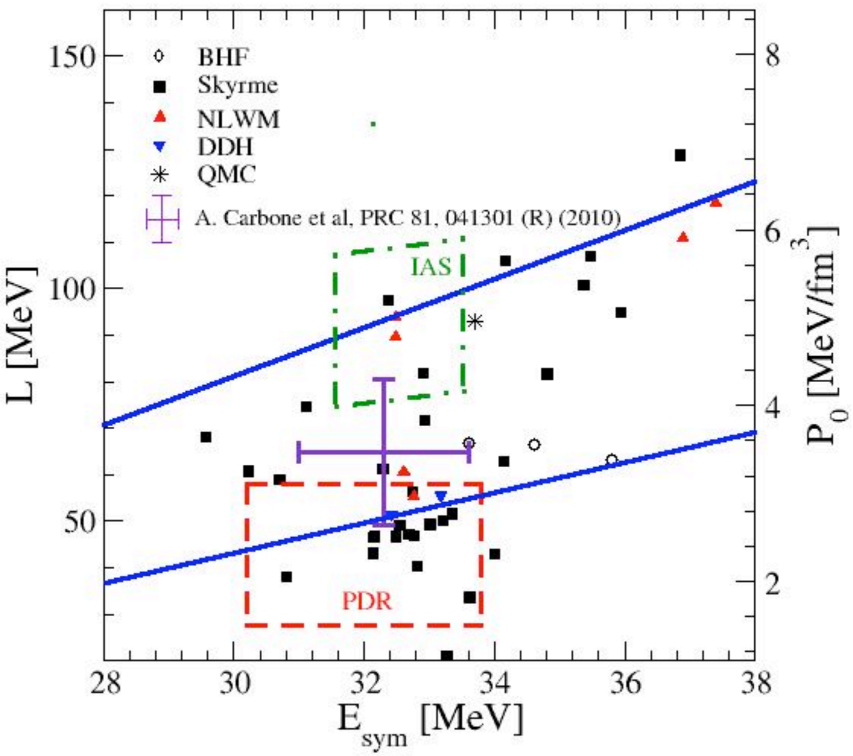
Bulk parameters of $E_{SNM}(\rho)$ & $S_2(\rho)$

Model	ρ_0	E_0	K_0	Q_0	E_{sym}	L	K_{sym}	Q_{sym}	K_τ	γ
BHF (3BFa)	0.187	-15.23	195.5	-280.9	34.3	66.5	-31.3	-112.8	-334.7	0.65
BHF (3BFb)	0.176	-14.62	185.9	-224.9	33.6	66.9	-23.4	-162.8	-343.8	0.66
BHF (2BF)	0.240	-17.30	213.6	-225.1	35.8	63.1	-27.8	-159.8	-339.6	0.59
SLy4	0.159	-15.97	229.8	-362.9	31.8	45.3	-119.8	520.8	-320.4	0.47
SLy230a	0.160	-15.98	229.9	-364.2	31.8	43.9	-98.4	602.8	-292.7	0.46
SkI4	0.162	-16.15	250.3	-335.7	29.6	59.9	-43.4	358.8	-322.5	0.67
NL3	0.148	-16.24	271.6	203.1	37.4	118.5	100.9	181.2	-698.4	1.05
TM1	0.145	-16.32	281.0	-285.2	36.8	110.8	33.6	-66.4	-518.7	1.00
FSU	0.148	-16.30	230.0	-523.4	32.6	60.5	-51.3	424.1	-276.6	0.62
TW	0.153	-16.25	240.1	-540.1	32.7	55.3	-124.7	535.2	-332.1	0.56
QMC	0.150	-15.70	291.0	-387.5	33.7	93.5	-10.0	28.0	-446.4	0.92

HIC at intermediate energies
is consistent with

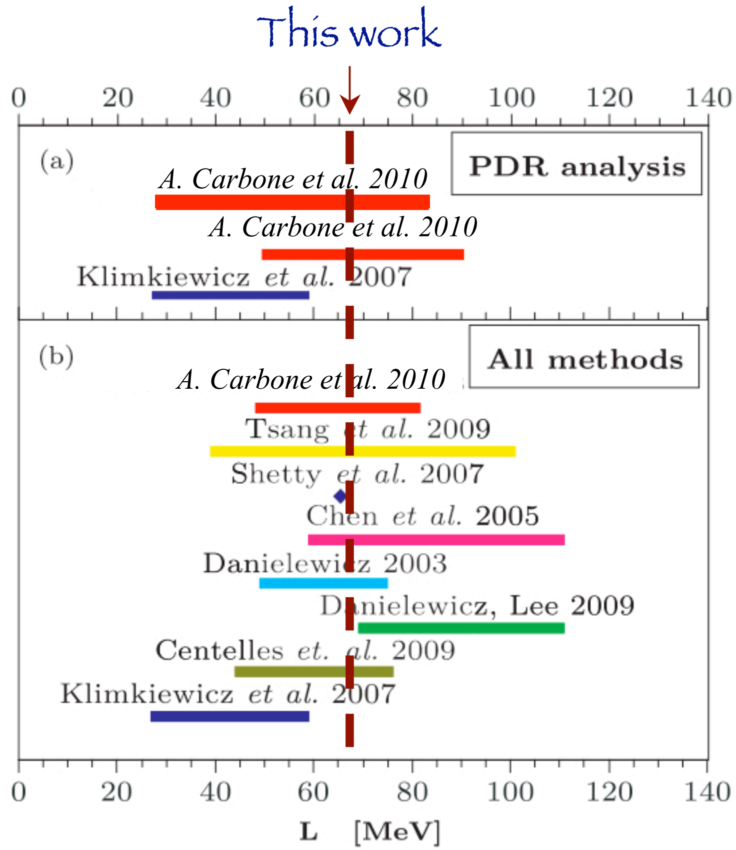
$$S_2(\rho) = E_{sym}\left(\frac{\rho}{\rho_0}\right)^{\gamma} \Rightarrow \gamma = \frac{L}{3E_{sym}}$$

Symmetry Energy versus L



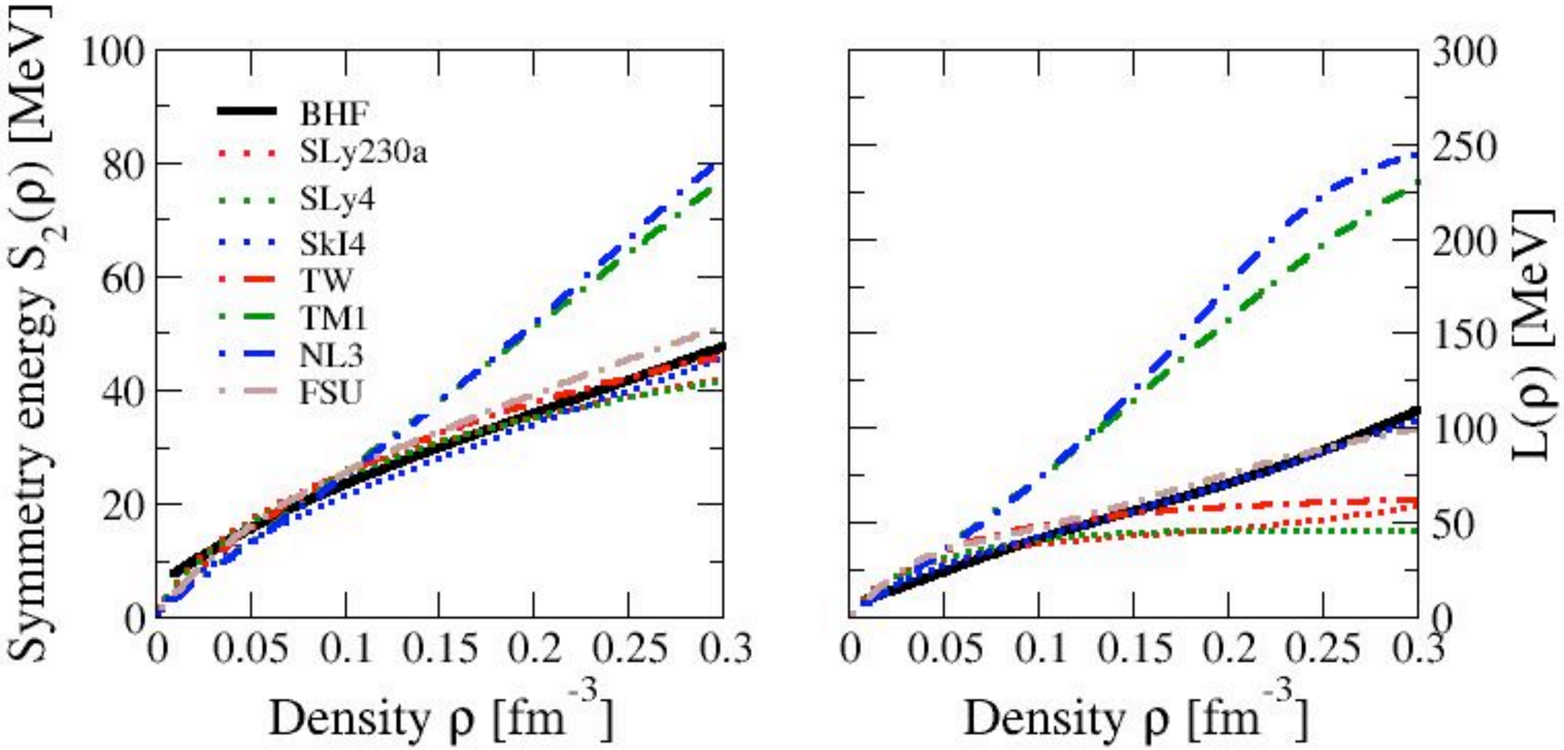
(Adapted from M. B. Tsang et al, Phys. Rev. Lett. 102, 122701 (2009))

Recent extracted values of L

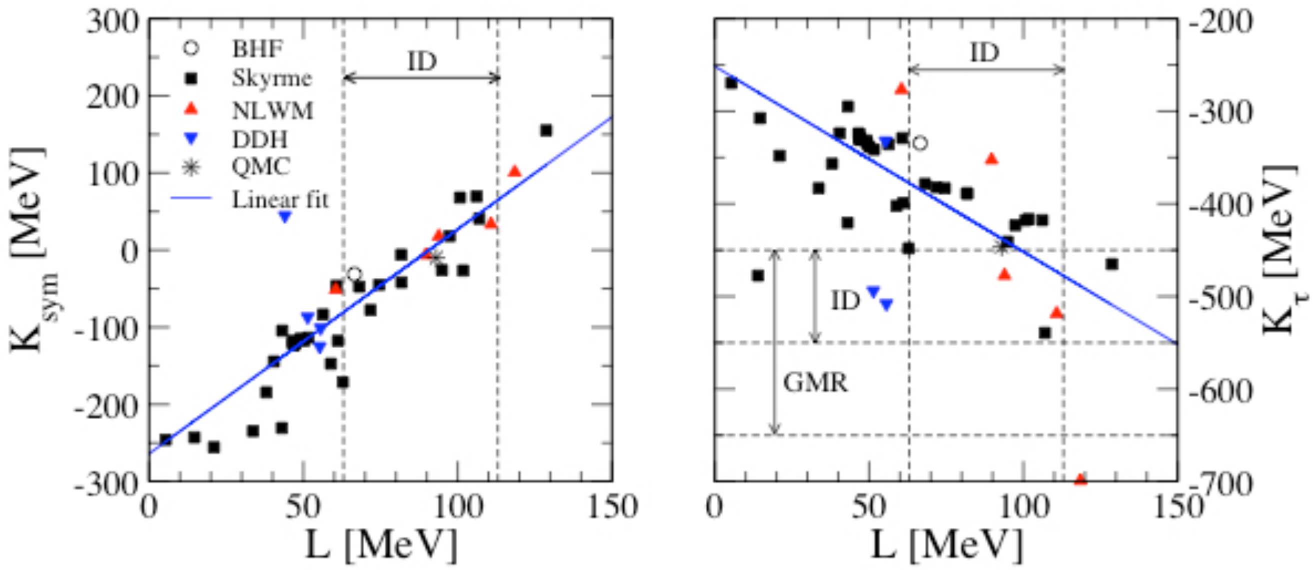


(Adapted from A. Carbone et al, Phys. Rev. C 81, 041301 (2010))

Density dependence of the Symmetry Energy

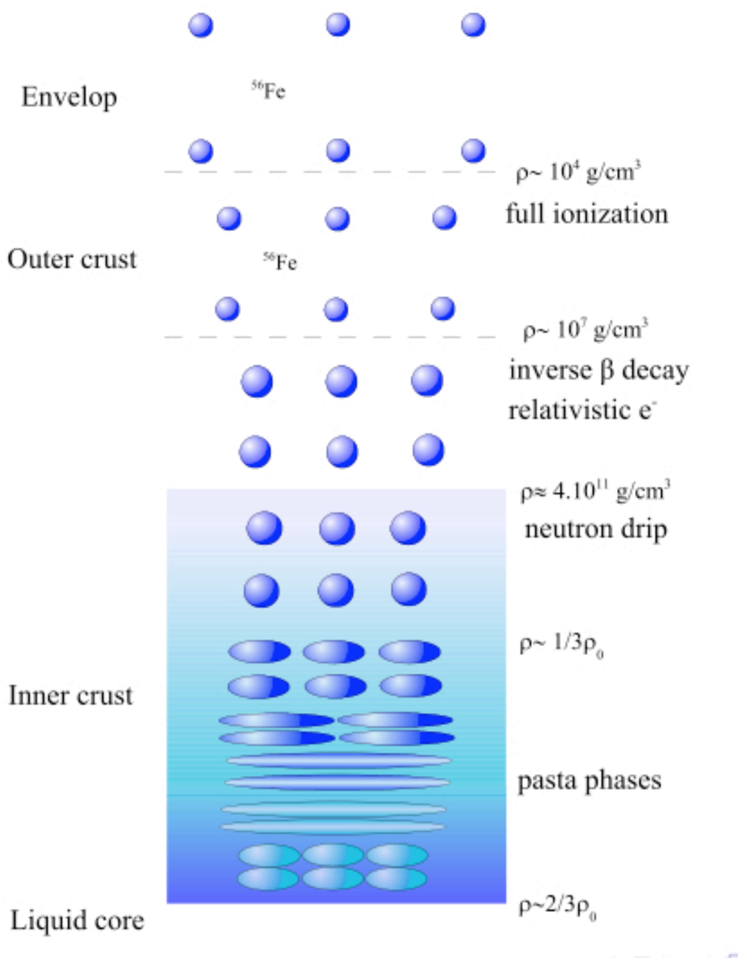


Correlations of K_{sym} & K_τ with L



Good agreement of BHF results with correlations predicted by effective models. BHF results located inside the region delimited by experimental constraints.

Neutron Stars & Symmetry Energy: Crust-Core transition density



(Picture from Nicolas Chamel)

The crust of a neutron star is very important for a number of observable properties :

- ✓ thermal evolution
- ✓ glitches
- ✓ X-ray burst
- ....

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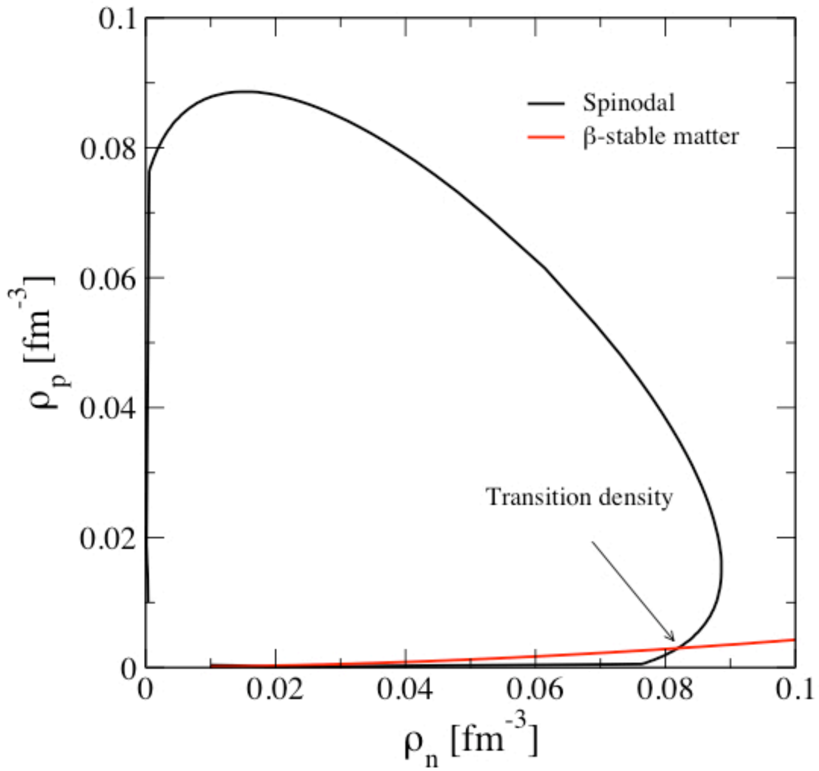
It is very important to understand well the crust-core transition region

Which constraints are set by the isospin dependence of the nuclear EoS on the transition density ?

How sensitive it is to the Symmetry Energy ?

In this work, we have estimated the crust-core transition density from the crossing of the β-equilibrium EoS and the thermodynamical spinodal instability line



Thermodynamical Stability Conditions

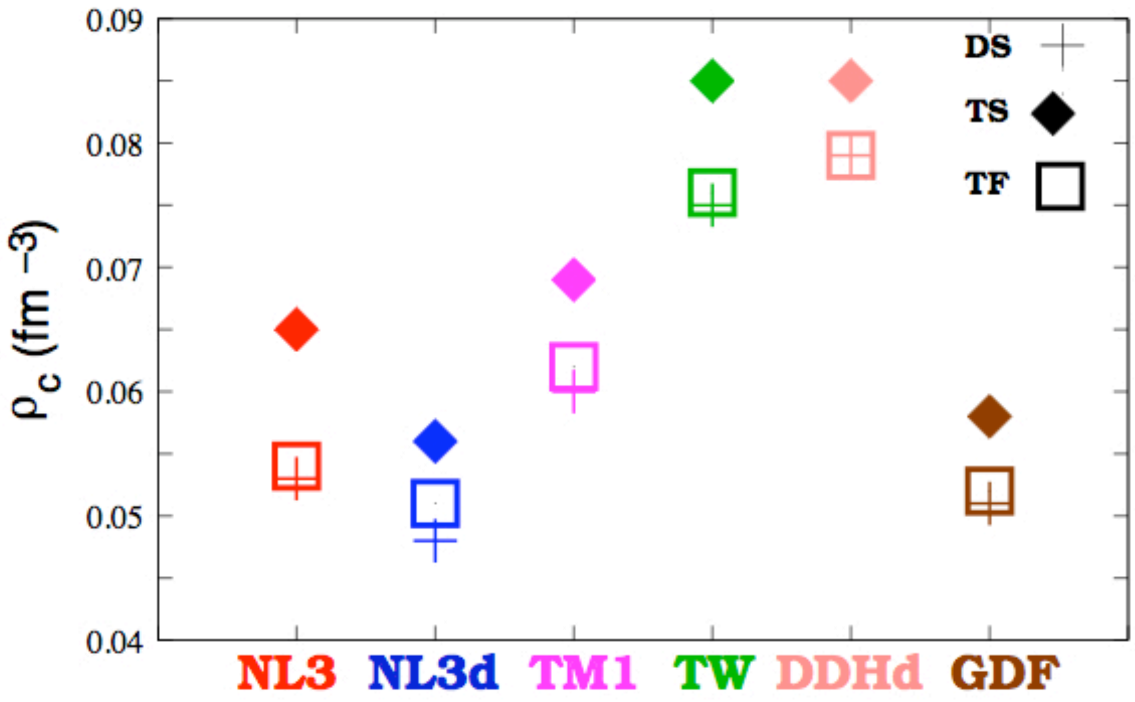
Curvature Matrix:

$$\mathcal{F}_{ij} = \left(\frac{\partial^2 \mathcal{F}}{\partial \rho_i \partial \rho_j}\bigg|_T \right) = \left(\frac{\partial \mu_i}{\partial \rho_j}\bigg|_T \right), \qquad i, j = n, p$$

positive definite:

$$Tr(\mathcal{F}_{ij}) > 0, \qquad Det(\mathcal{F}_{ij}) > 0$$

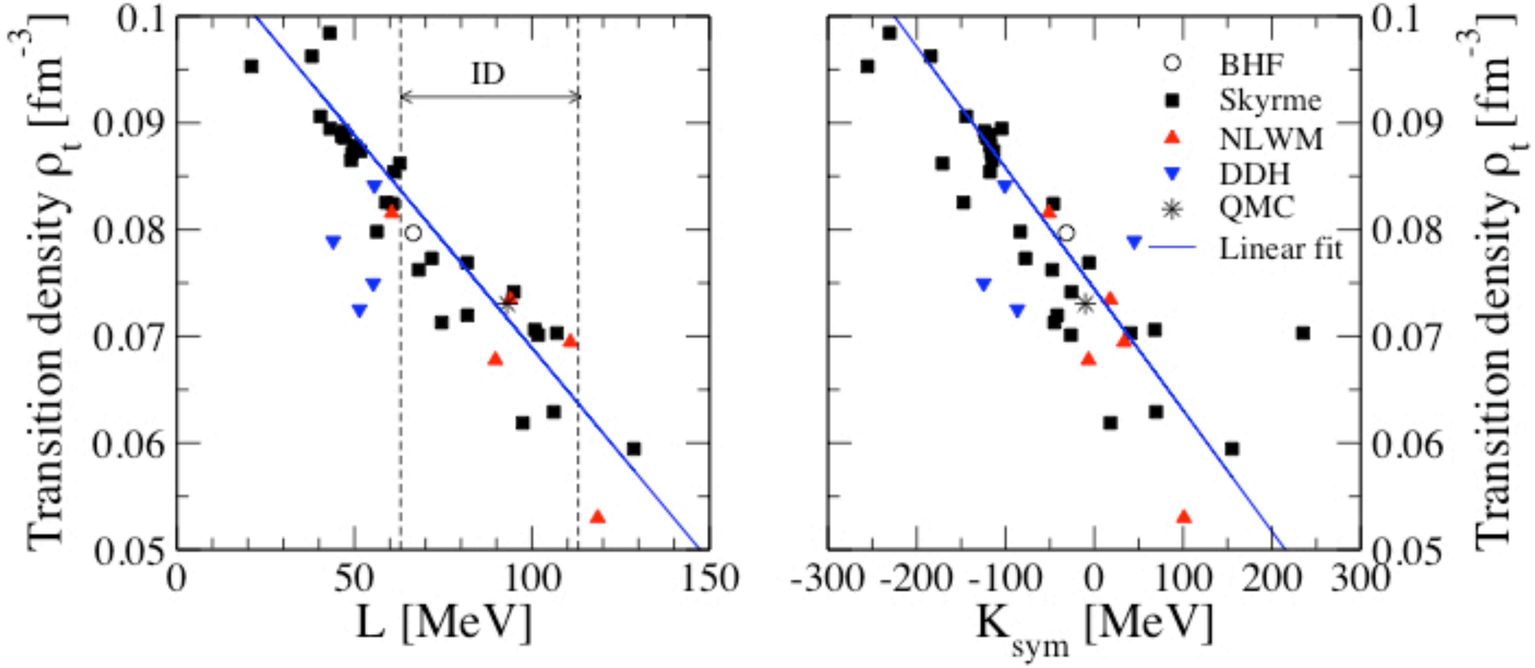
Crust-Core transition density: Thomas-Fermi calculation of Pasta Phase versus Spinodal



Results courtesy of Sidney Avacini

Predictions for ρ_t from thermodynamical spinodal ~ 15% larger than TF
➔ our estimation of ρ_t will define an upper bound of the real ρ_t.

Crust-Core transition density & Symmetry Energy Derivatives



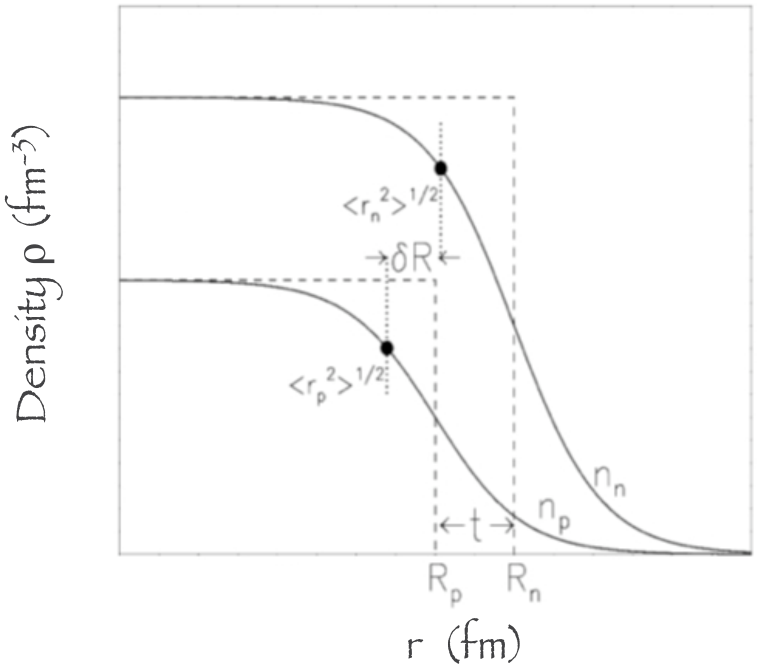
Using the experimental constraint on L

$0.063 < \rho_t < 0.083$ fm^{-3} ➔ $0.24 < P_t < 0.51$ MeV fm^{-3}

Our estimation of the crust-core transition is in reasonable agreement with calculations of other authors

Calculation	Density (fm^{-3})	Pressure (MeV fm^{-3})
Link et al. (1999)	~ 0.075	0.25 to 0.65
Xu et al. (2009)	0.04 to 0.65	0.01 to 0.26
RMF thermodynamical spinodal	0.065 to 0.079	0.26 to 0.39
RMF dynamical spinodal	0.057 to 0.072	0.26 to 0.38
This work	0.063 to 0.083	0.24 to 0.51

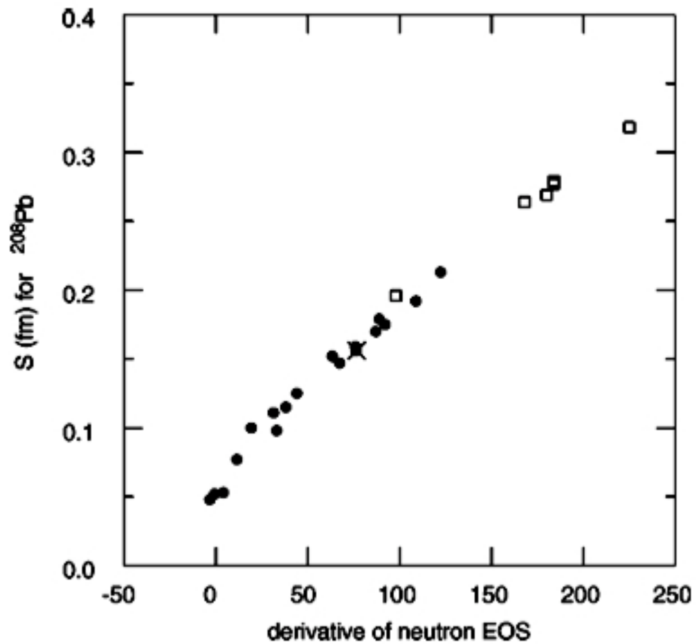
Neutron Skin Thickness & Symmetry Energy



Neutron skin thickness

$$\delta R = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$$

Typel & Brown showed that δR calculated in mean field models is very sensitive to the slope of the symmetry energy.



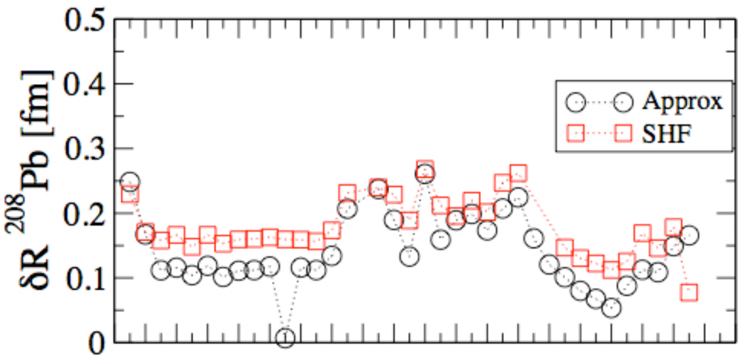
Typel-Brown correlation

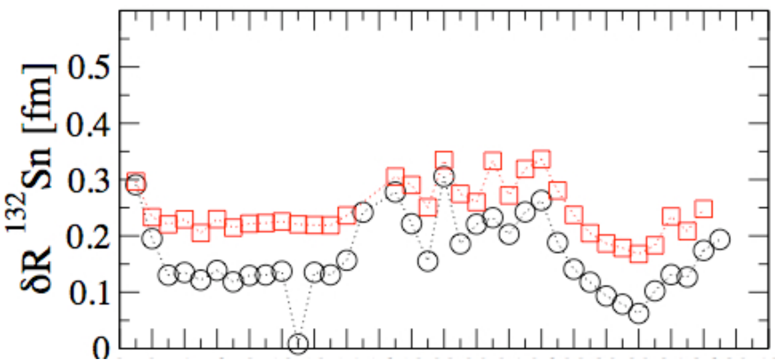
A fully self-consistent finite nuclei calculation based on the BHF approach is too difficult, therefore, following a suggestion by Steiner et al. (Phys. Rep. 411, 325 (2005)), we have estimated δR to lowest order in the diffuseness corrections

$$\delta R \approx \sqrt{\frac{3}{5}}t$$

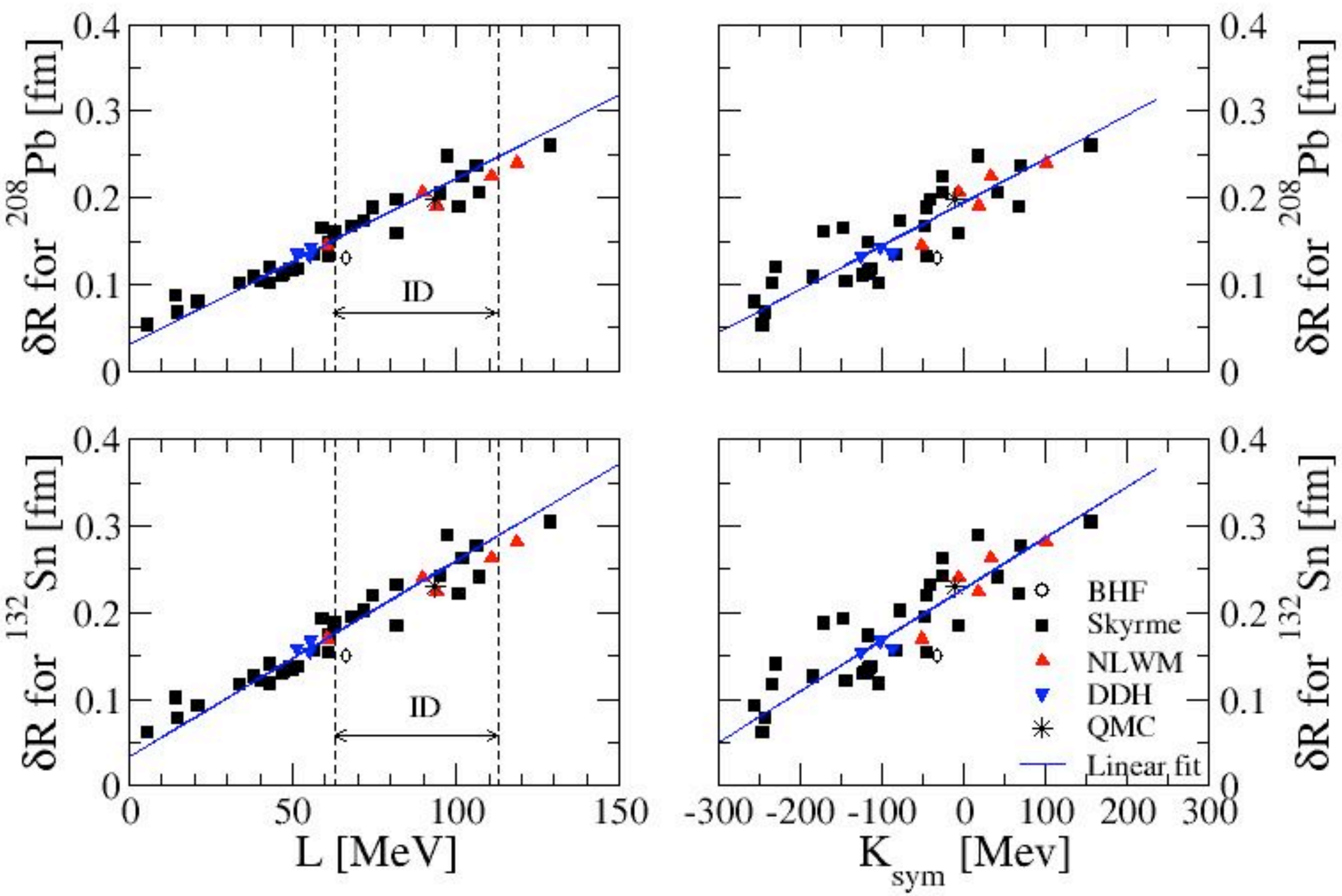
$$t = \frac{\beta_c}{\rho_0(\beta_c)(1-\beta_c^2)} \frac{E_s}{4\pi r_o^2} \frac{\int_o^{\rho_0(\beta_c)} d\rho \sqrt{\rho}\left(E_{sym}/S_2(\rho)-1\right)\left(E_{SNM}(\rho)-E_0\right)^{-1/2}}{\int_o^{\rho_0(\beta_c)} d\rho \sqrt{\rho}\left(E_{SNM}(\rho)-E_0\right)^{1/2}}$$

thickness of semi-infinite asymmetric matter





Correlation of the Neutron Skin Thickness δR with L & K_{sym}



The linear increase of δR with L & K_{sym} is not surprising since δR is determined by the pressure which pushes neutrons out.

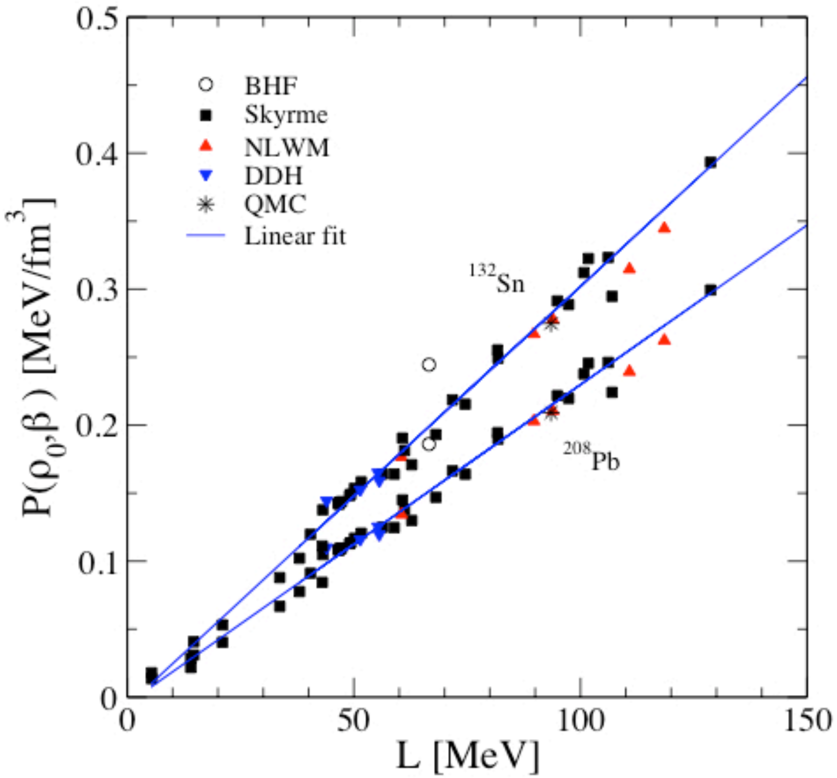
Large pressure ➔ Large neutron radius & Large δR

But the pressure is proportional to L

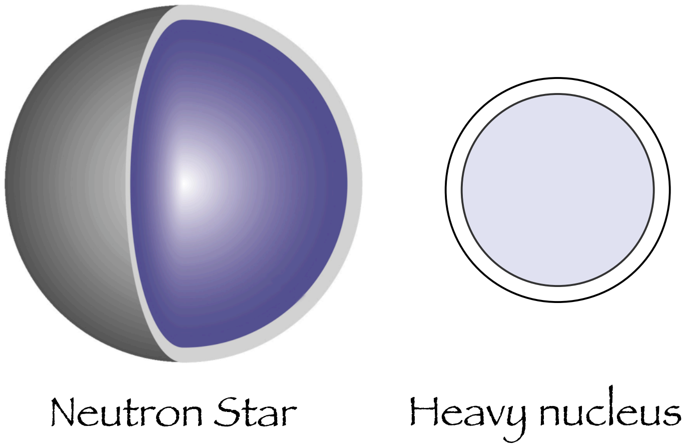
$$P(\rho,\beta) = \rho^2 \frac{dE/A}{d\rho} = \rho^2 \left(\frac{dE_{SNM}(\rho)}{d\rho} + \beta^2 \frac{dS_2(\rho)}{d\rho} \right)$$

🡇

$$P(\rho,\beta) \propto L$$



Neutron Skin Thickness & The Crust-Core Transition Density



Neutron Star Crust & Neutron Skin are made out of neutron rich matter at similar densities

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Both are governed by EoS at subnuclear densities in particular by $S_2(\rho)$ & its derivatives

Inverse correlation between δR and ρ_t (Horowiz & Piekarewicz)

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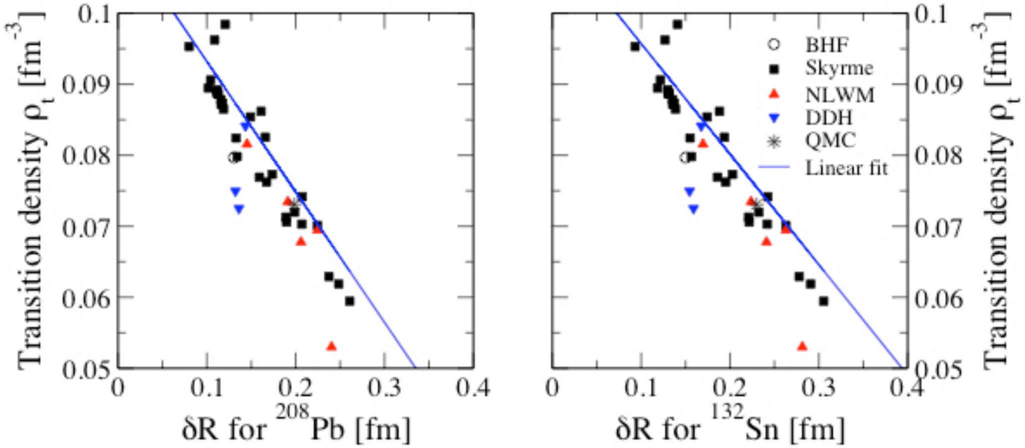
an accurate measurement of neutron skin in neutron rich nuclei can provide considerable & valuable information on the crust-core transition density.

(PREX exeriment @ JLAB)



Summary & Conclusions

- We have studied the density dependence of the symmetry energy within the BHF approximation and compared our results with the ones obtained with several effective models (Skyrme & RMF). We have found L≈66.5 MeV compatible with recent experimental constraints.

- We have studied different correlations between the slope L and curvature K_{sym} of the symmetry energy and several physical quantities such as K_τ, δR & ρ_t. Very good agreement is found with the correlations already predicted for effective models.

- Using the experimental constraint on L we have estimated a crust-core transition density between 0.063 and 0.083 fm^{-3}.

- BHF results confirm that there is an inverse correlation between δR and ρ_t ➔ an accurate measurement of neutron radius in heavy nuclei can provide considerable & valuable information on the crust-core transition density.


Thank You!