Neutron Star Masses and Radii

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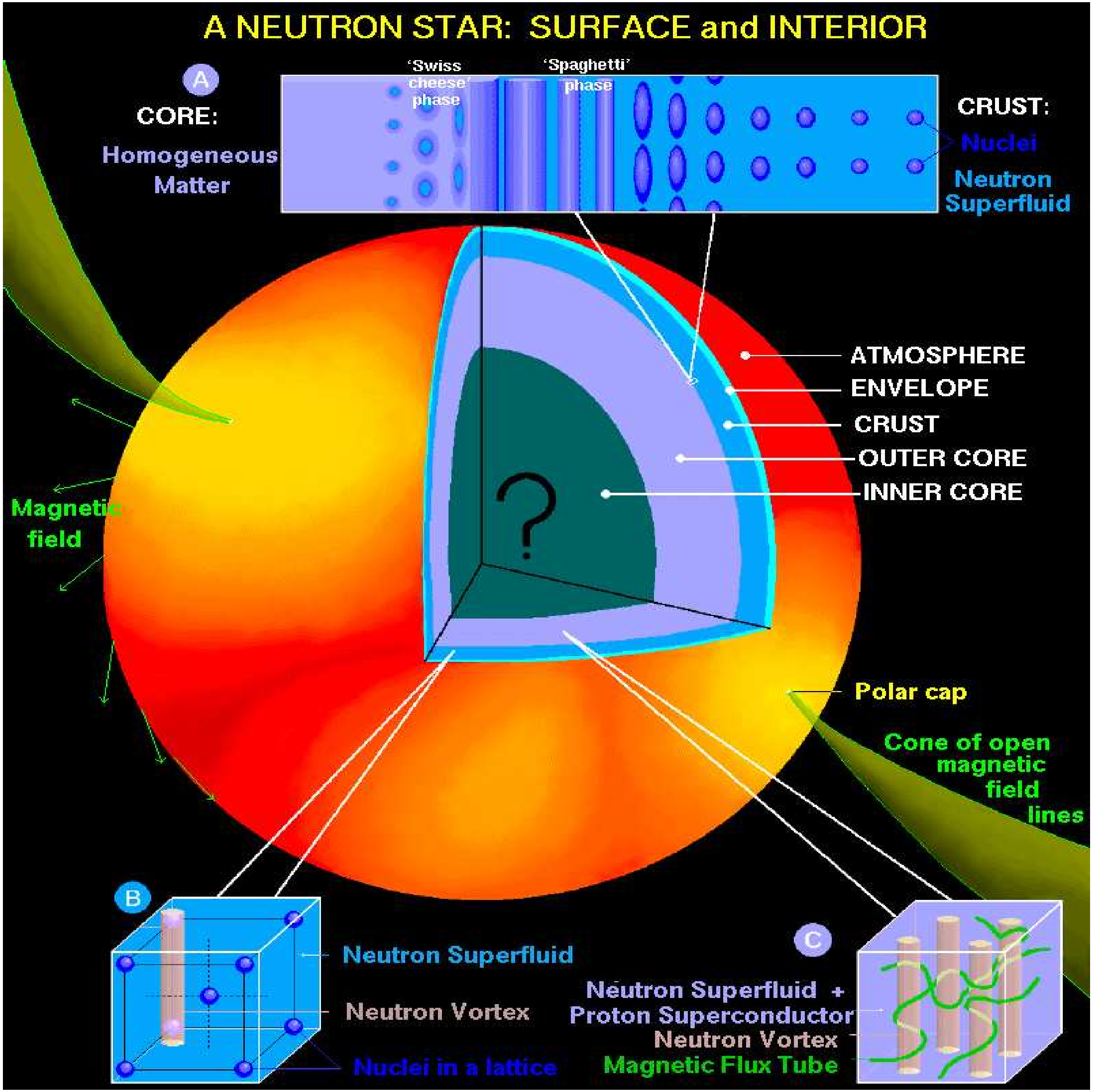


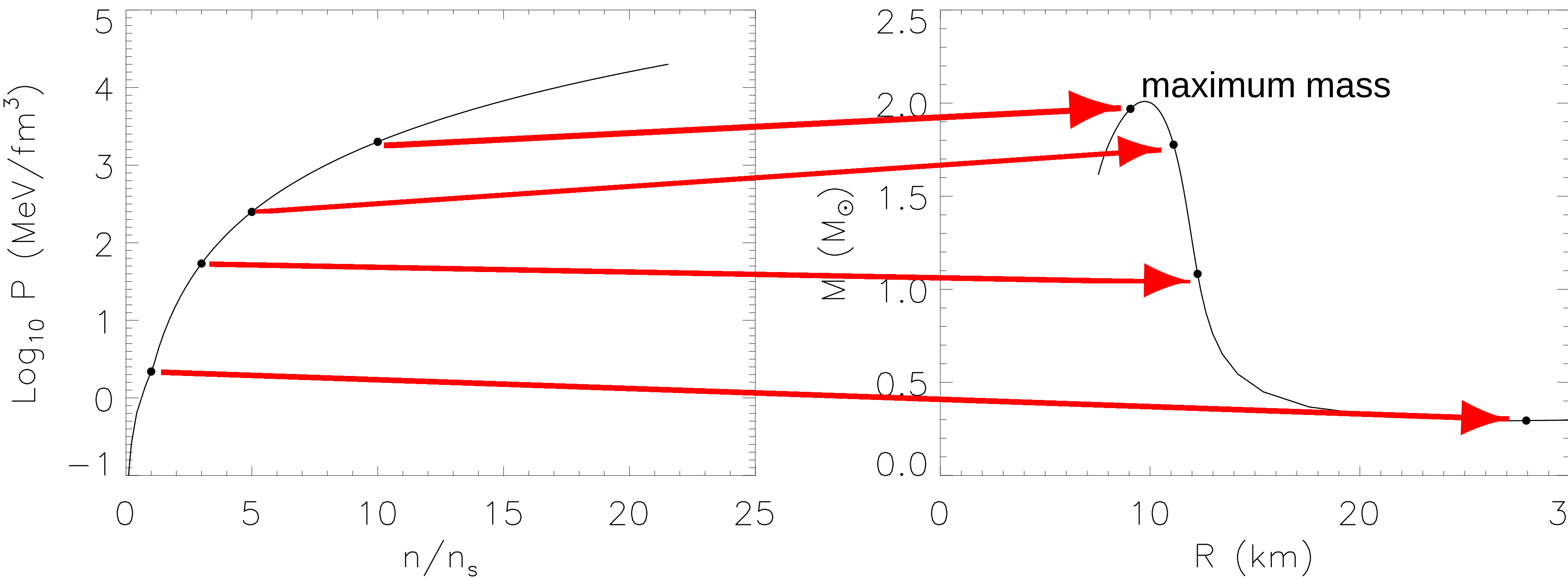
Credit: Dany Page, UNAM

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

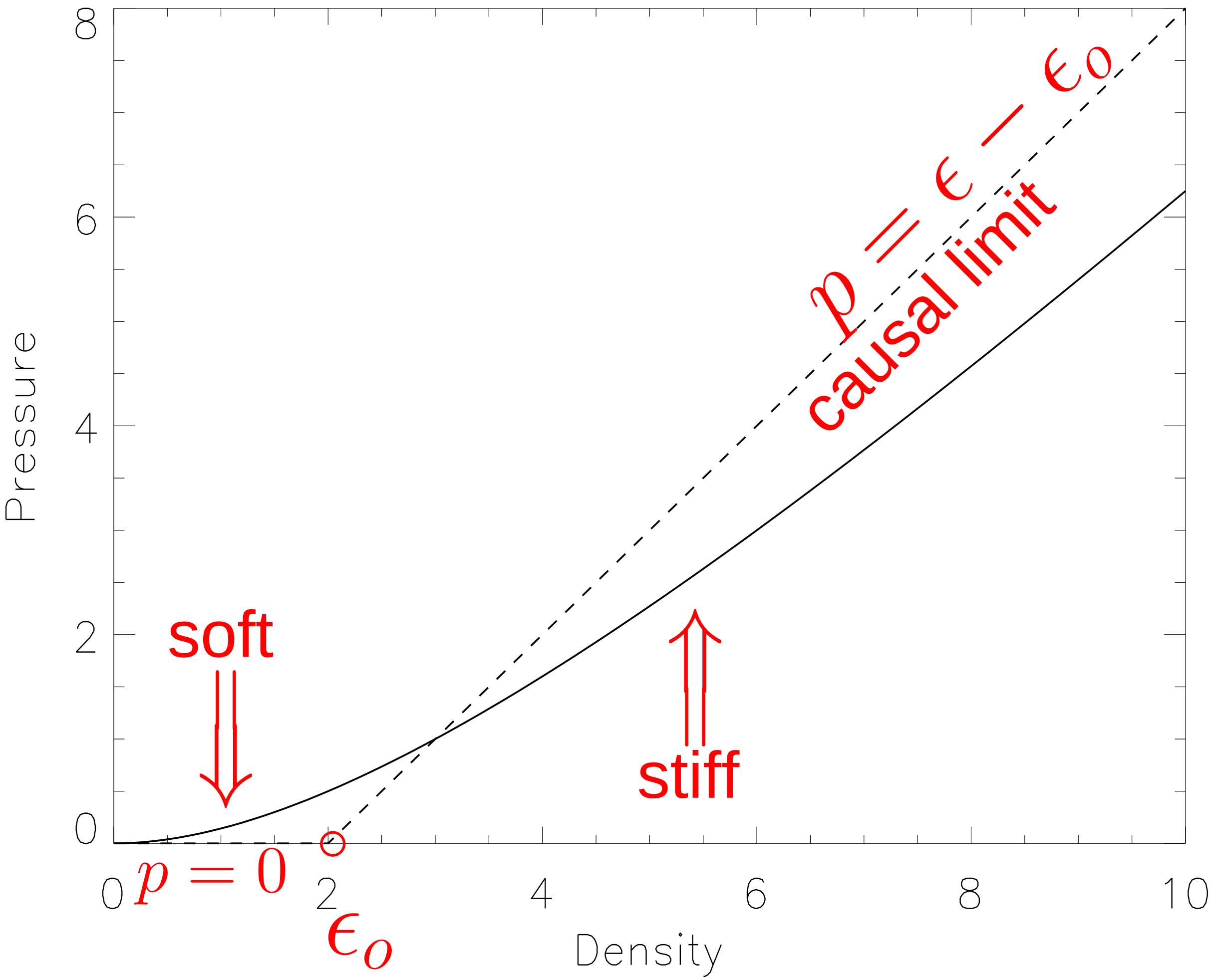
$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi pr^3)(\epsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\epsilon}{c^2}r^2$$



Extreme Properties of Neutron Stars

- The most compact configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff".



ϵ_0 is the only EOS parameter

The TOV solutions scale with ϵ_0

Maximum Mass, Minimum Period

Theoretical limits from GR and causality

- $M_{max} = 4.2(\epsilon_s/\epsilon_f)^{1/2}\ \mathrm{M}_\odot$ **Rhoades & Ruffini (1974), Hartle (1978)**
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)\ \mathrm{km}$

  Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)
- $\epsilon_c < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2\ \mathrm{g\ cm}^{-3}$ **Lattimer & Prakash (2005)**
- $P_{min} \simeq 0.74(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}\ \mathrm{ms}$

  Koranda, Stergioulas & Friedman (1997)
- $P_{min} \simeq 0.96 \pm 0.03(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}\ \mathrm{ms}$

  (empirical) **Lattimer & Prakash (2004)**
- $\epsilon_c > 0.91 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2\ \mathrm{g\ cm}^{-3}$ (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

Mass-Radius Diagram and Theoretical Constraints

GR:

$R > 2GM/c^2$

$P < \infty$:

$R > (9/4)GM/c^2$

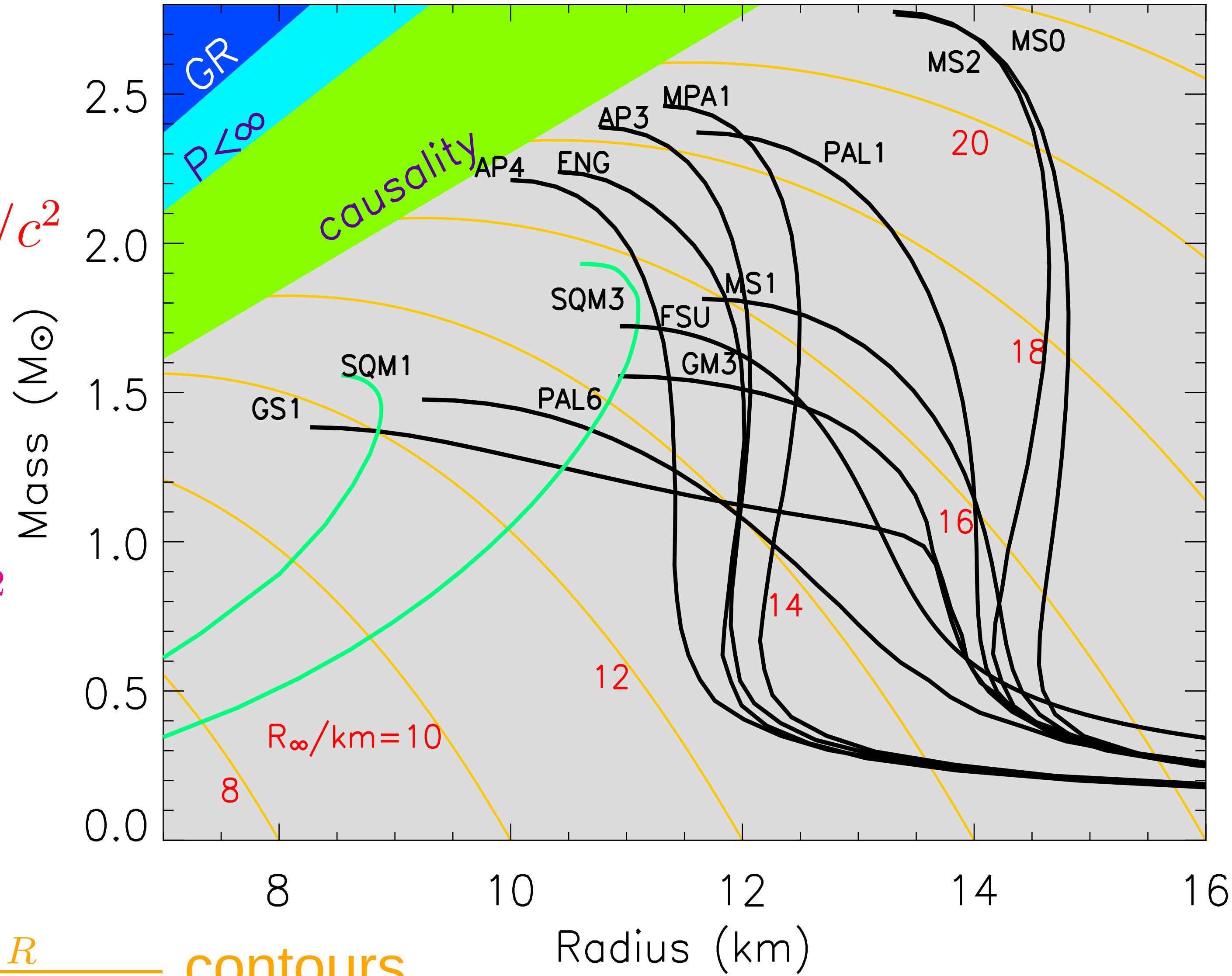
causality:

$R \gtrsim 2.9GM/c^2$

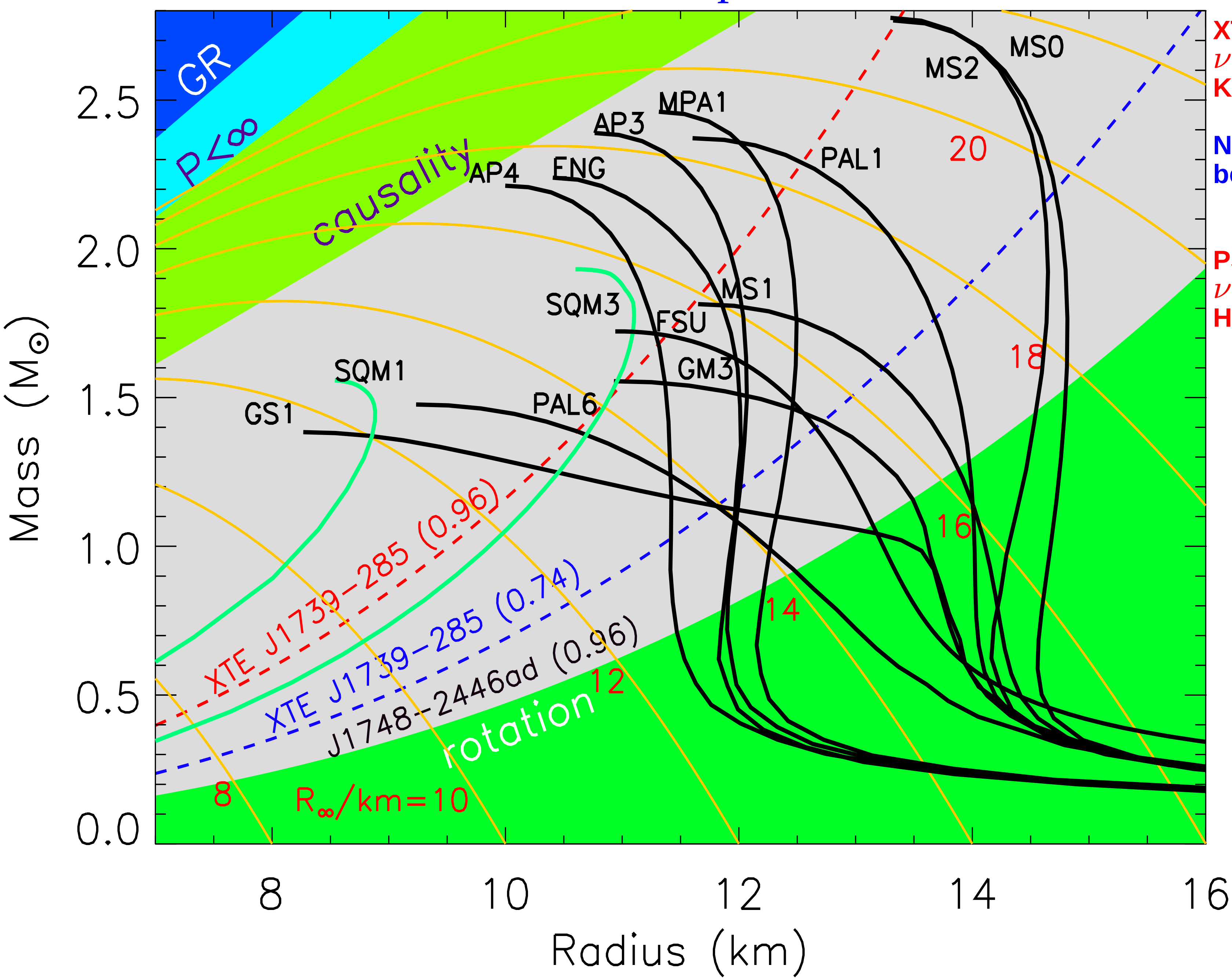
— normal NS

— SQS

— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$ contours



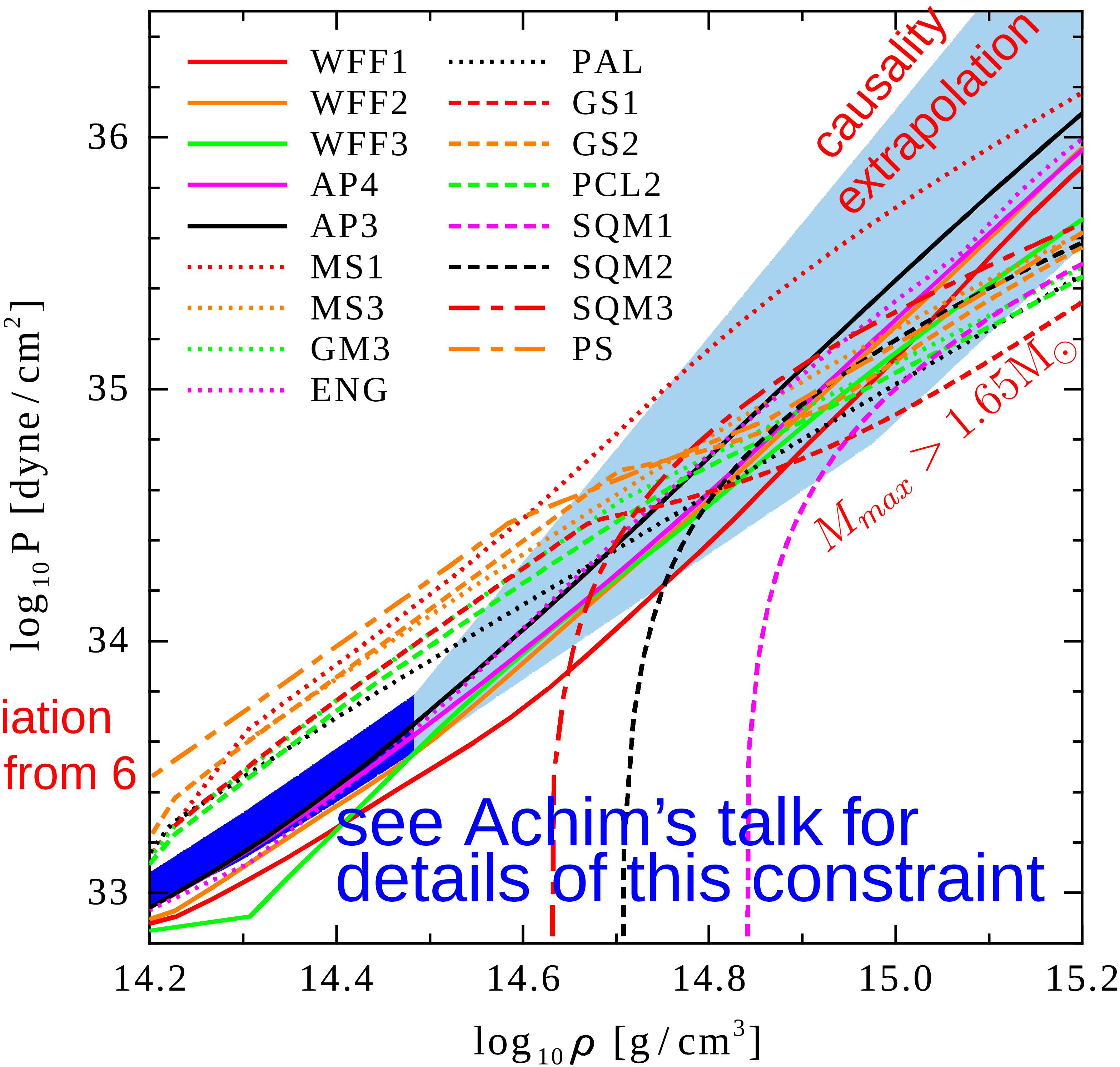
Constraints from Pulsar Spins



XTE J1739-285
$\nu = 1122$ **Hz**
Kaaret et al. 2006

Not confirmed to be a rotation rate

PSR J1748-2446ad
$\nu = 716$ **Hz**
Hessels et al. 200

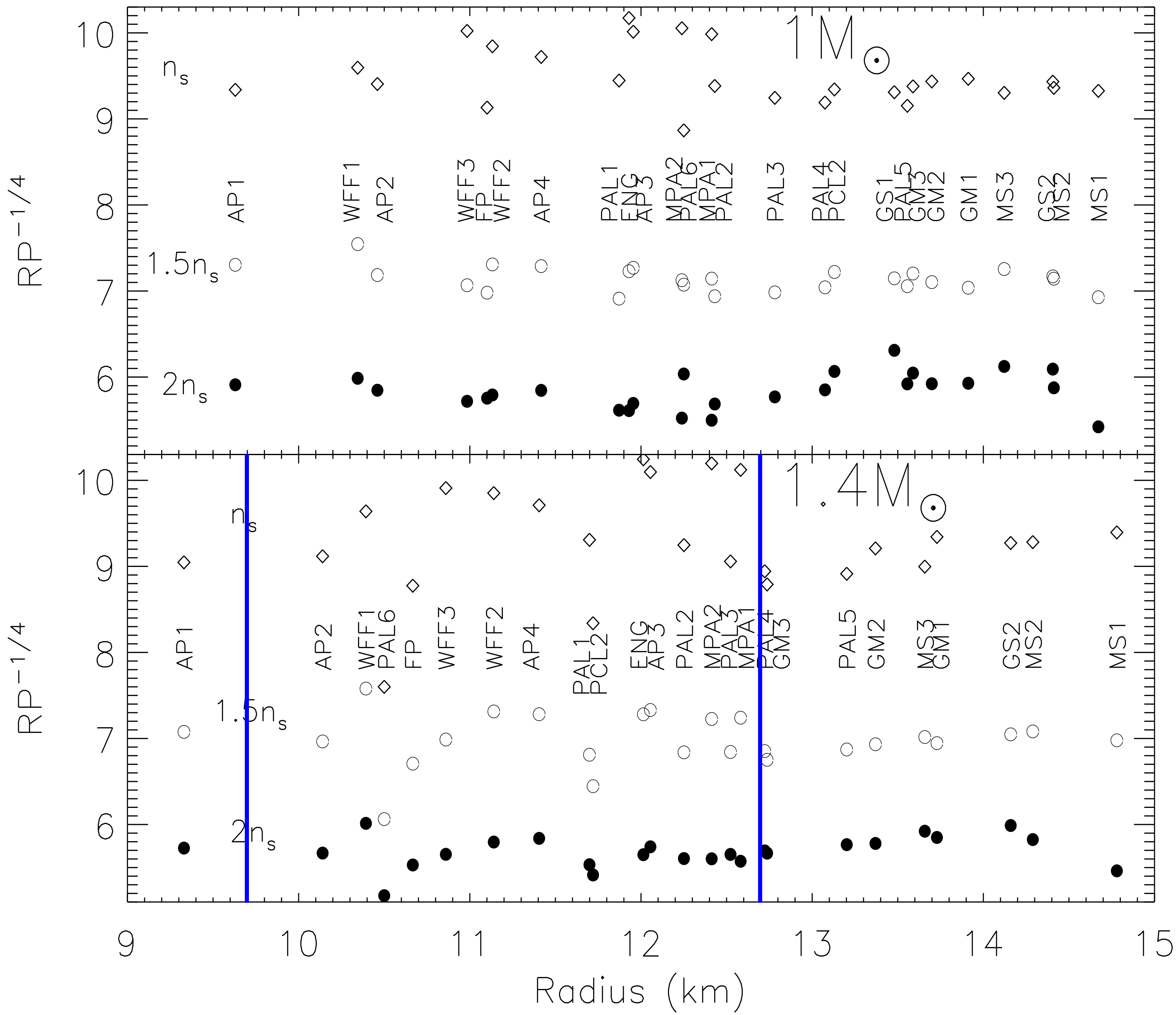


$p(\rho_0)$ variation reduced from 6 to 1.5

see Achim's talk for details of this constraint

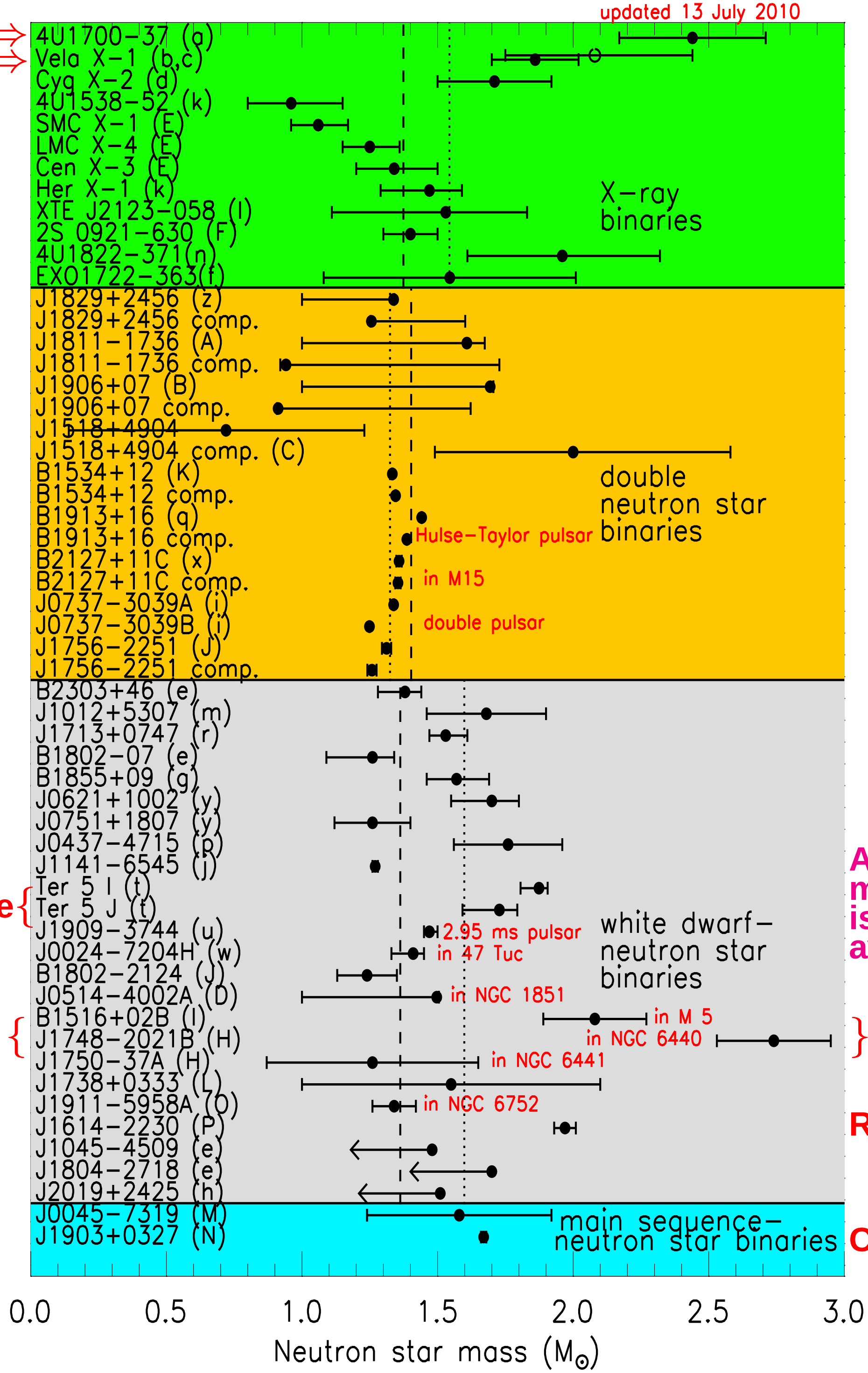
The Radius – Pressure Correlation

$R \propto p^{1/4}$



Lattimer & Prakash (2001)

Black hole? ⇒
Firm lower mass limit? ⇒



$M > 1.68$ M$_\odot$, 95% confidence {

Freire et al. 2007 {

Although simple average mass of w.d. companions is 0.27 M$_\odot$ larger, weighted average is 0.08 M$_\odot$ smaller

} **w.d. companion? statistics?**

Ransom et al. 2010

Champion et al. 2008

Possible Kinds of Observations

- Maximum and Minimum Mass (binary pulsars)
- Minimum Rotational Period*
- Radiation Radii or Redshifts from X-ray Thermal Emission*
- Crustal Cooling Timescale from X-ray Transients*
- X-ray Bursts from Accreting Neutron Stars*
- Seismology from Giant Flares in SGR's*
- Neutron Star Thermal Evolution (URCA or not)*
- Moments of Inertia from Spin-Orbit Coupling*
- Neutrinos from Proto-Neutron Stars (Binding Energies, Neutrino Opacities, Radii)*
- Pulse Shape Modulations*
- Gravitational Radiation from Neutron Star Mergers* (Masses, Radii from tidal Love numbers)

* Significant dependence on symmetry energy

Potentially Observable Quantities

- Apparent angular diameter from flux and temperature measurements
  $\beta \equiv GM/Rc^2$

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2\beta}} = \sqrt{\frac{F_\infty}{\sigma}\frac{1}{f_\infty^2 T_\infty^2}}$$

- Redshift

$$z = (1-2\beta)^{-1/2} - 1$$

- Eddington flux

$$F_{Edd} = \frac{GMc}{\kappa c^2 D^2}(1-2\beta)^{1/2}$$

- Crust thickness

$$\frac{m_b c^2}{2}\ln\mathcal{H} \equiv \int_0^{p_t}\frac{dp}{n} = \mu_{n,t} - \mu_{n,t}(p=0)$$

$$\frac{\Delta}{R} \equiv \frac{R-R_t}{R} \simeq \frac{(\mathcal{H}-1)(1-2\beta)}{\mathcal{H}+2\beta-1} \simeq (\mathcal{H}-1)\left(\frac{1}{2\beta}-1\right).$$

- Moment of Inertia

$$I \simeq (0.237 \pm 0.008) MR^2(1 + 2.84\beta + 18.9\beta^4)\ \mathrm{M}_\odot\ \mathrm{km}^2$$

- Crustal Moment of Inertia

$$\frac{\Delta I}{I} \simeq \frac{8\pi}{3}\frac{R^6 p_t}{IMc^2}$$

- Binding Energy

$$\mathrm{B.E.} \simeq (0.60 \pm 0.05)\frac{\beta}{1-\beta/2}$$

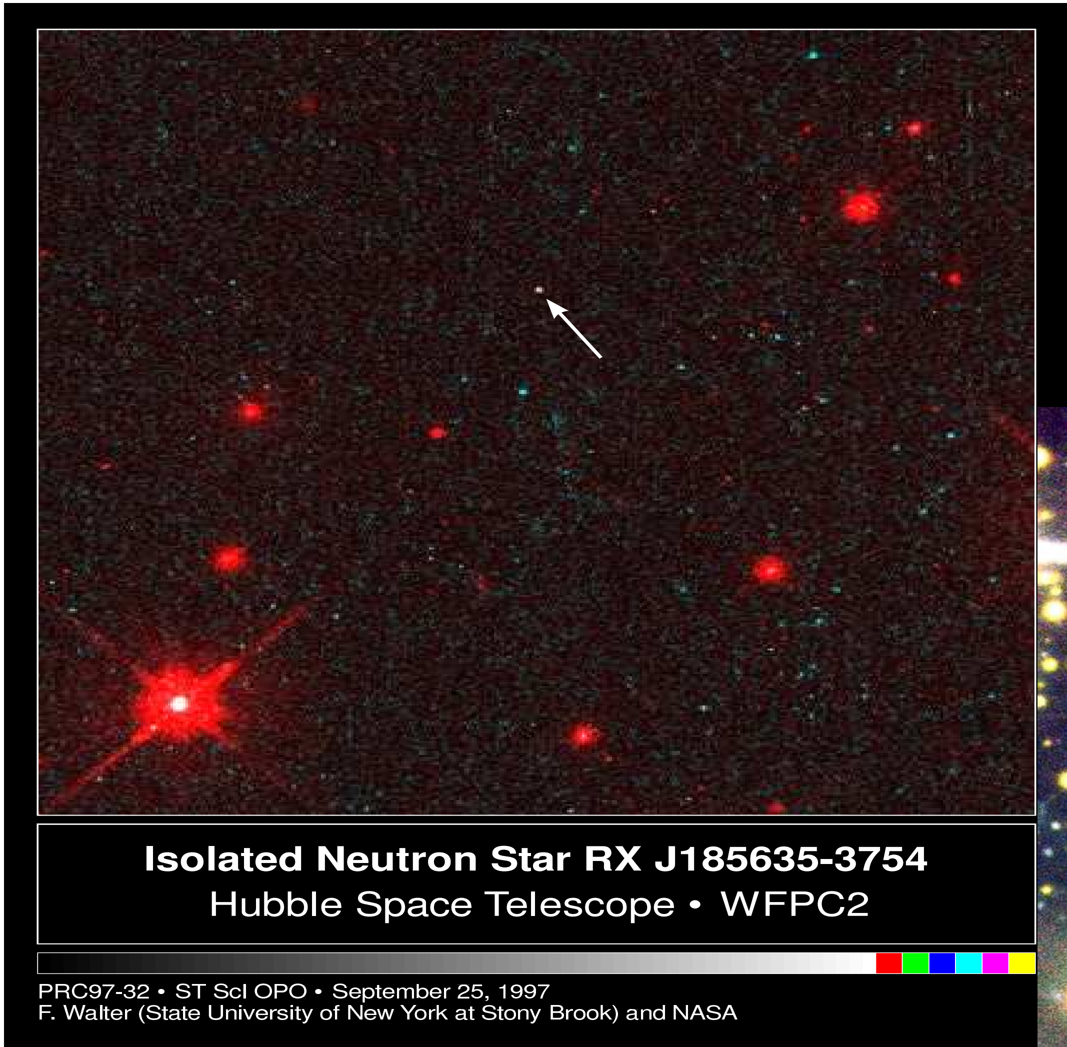
Radiation Radius

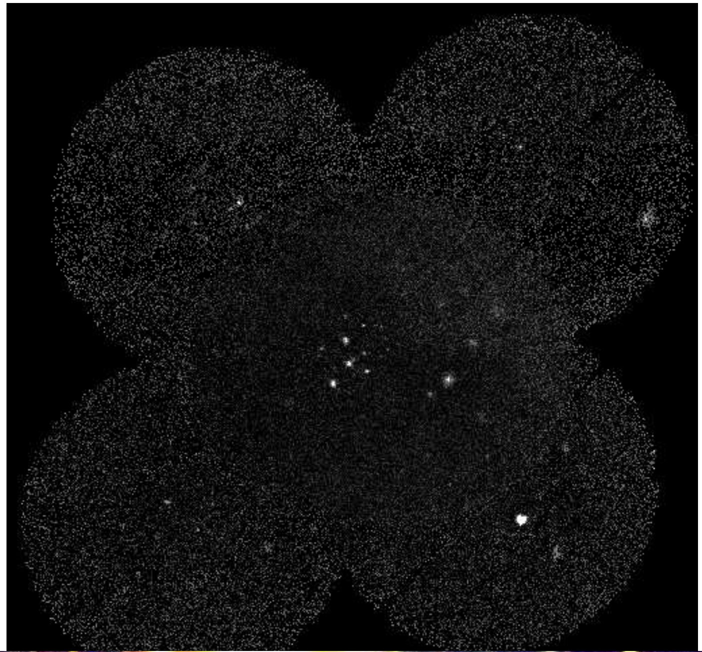
- Combination of flux and temperature measurements yields apparent angular diameter (pseudo-BB):

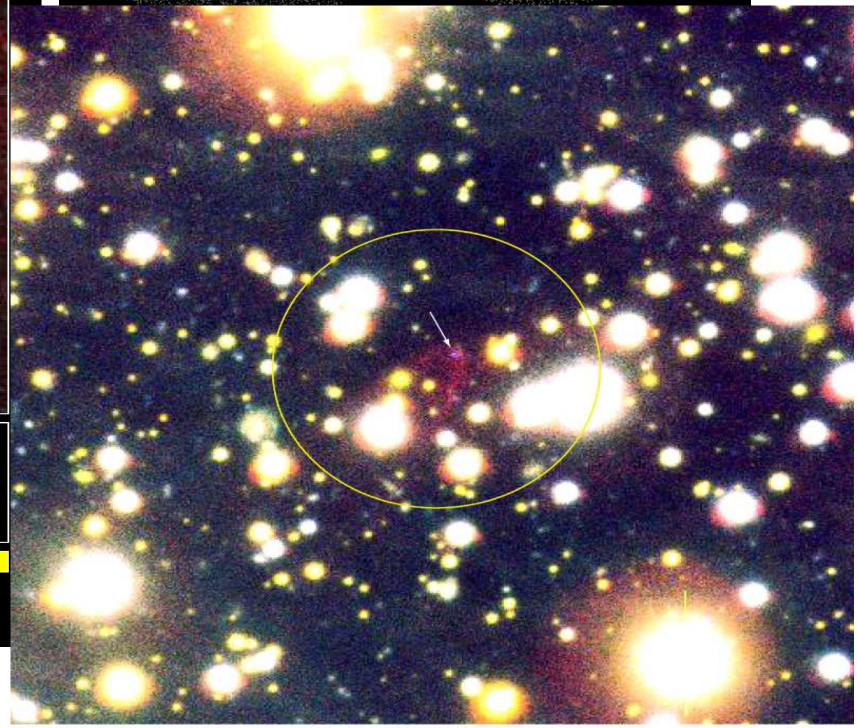
$$\frac{R_\infty}{d} = \frac{R}{d}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

- Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- Best chances for accurate radii are from
  - Nearby isolated neutron stars (parallax measurable)
  - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmosperes)

RX J1856-3754





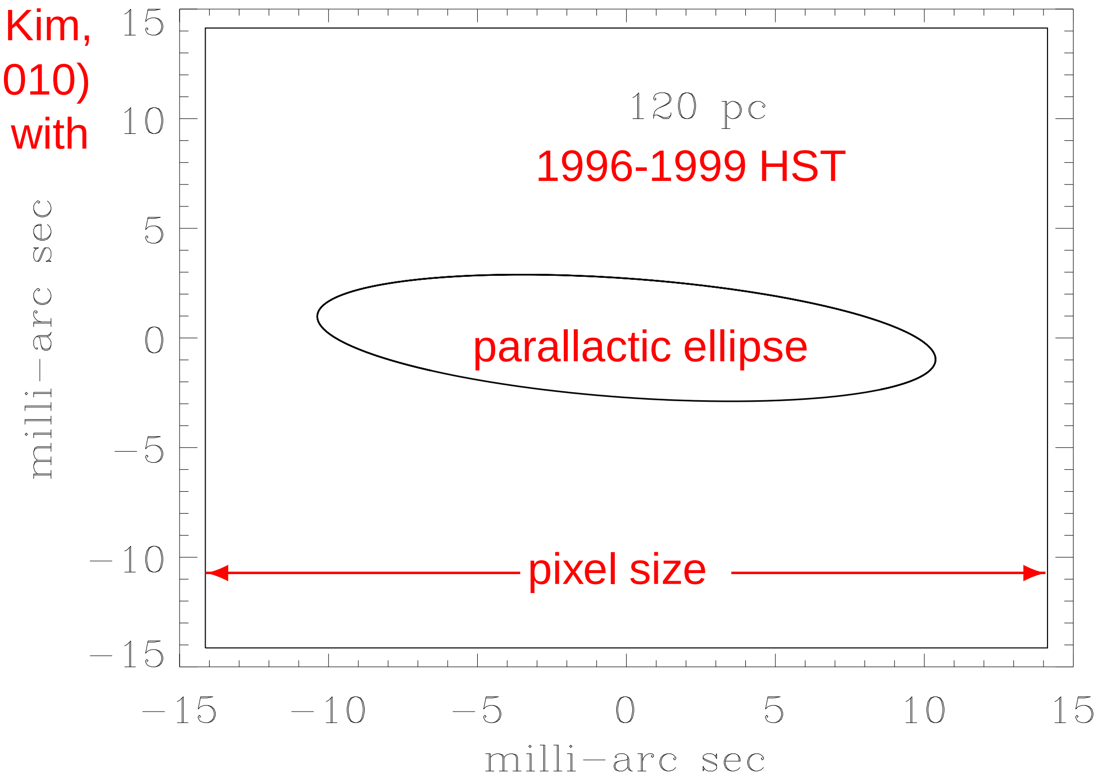


A Bowshock Nebula Near the Neutron Star RX J1856.5-3754 (Detail)
(VLT KUEYEN + FORS2)

ESO PR Photo 23b/00 (11 September 2000) ©European Southern Observatory

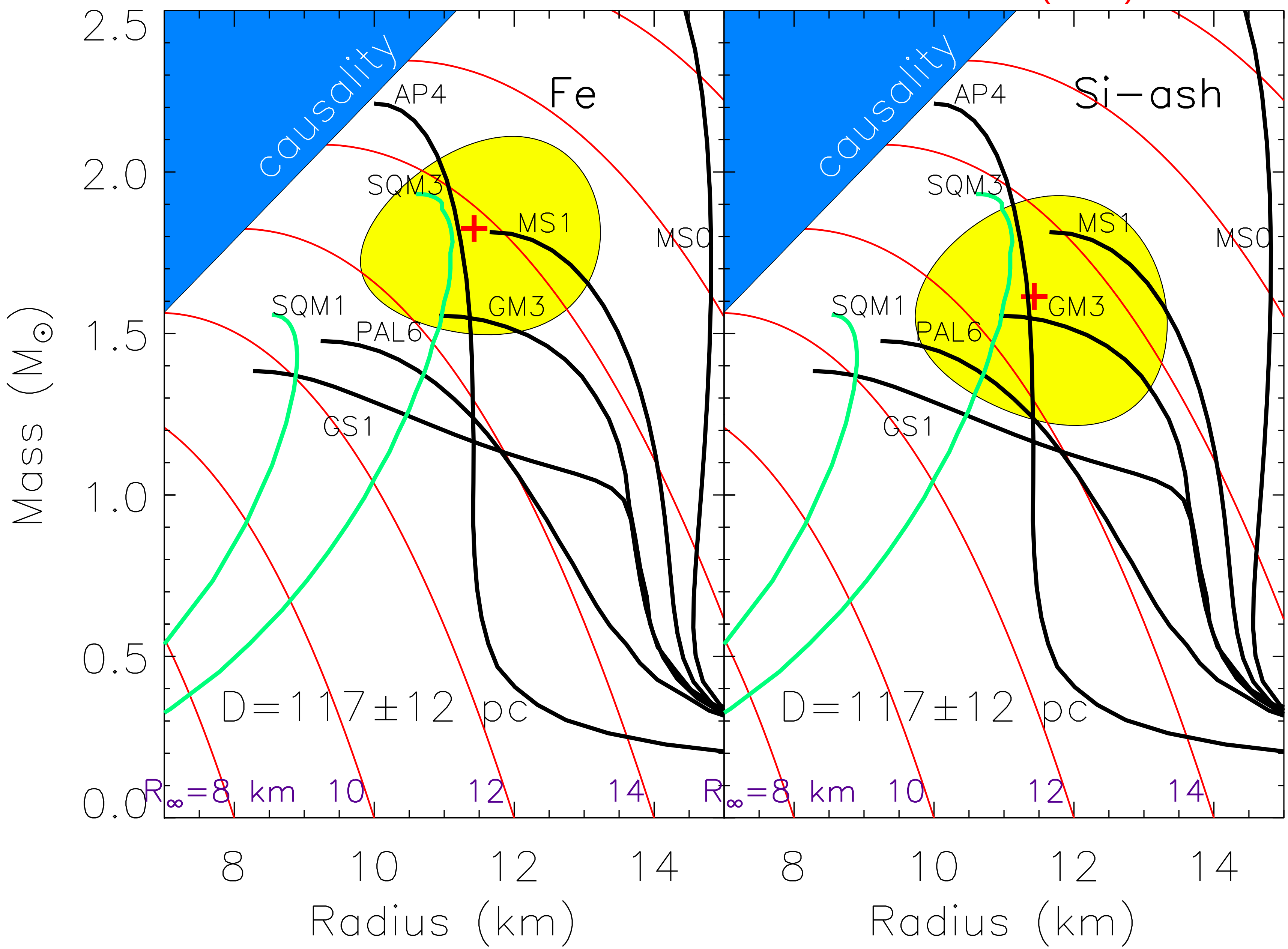
Astrometry of RXJ 1856-3754

- Walter & Lattimer (2002) determined $D = 117 \pm 12$ pc and $v \simeq 190$ km/s from 1996-1999 HST Planetary Camera observations
- Star's age is probably 0.5 million years
- Kaplan, van Kerkwijk & Anderson (2002): $D = 140 \pm 40$ pc using same data
- van Kerkwijk & Kaplan (2007, conference proceeding) revised this to $D = 161 \pm 16$ pc based on 2002-2004 High-Resolution Camera of the Advanced Camera for Surveys HST observations (double the resolution)
- Walter, Eisenbeiβ, Lattimer, Kim, Hambaryan & Neuhäuser (2010) determined $D \simeq 115 \pm 8$ pc with 2002-2004 HST data

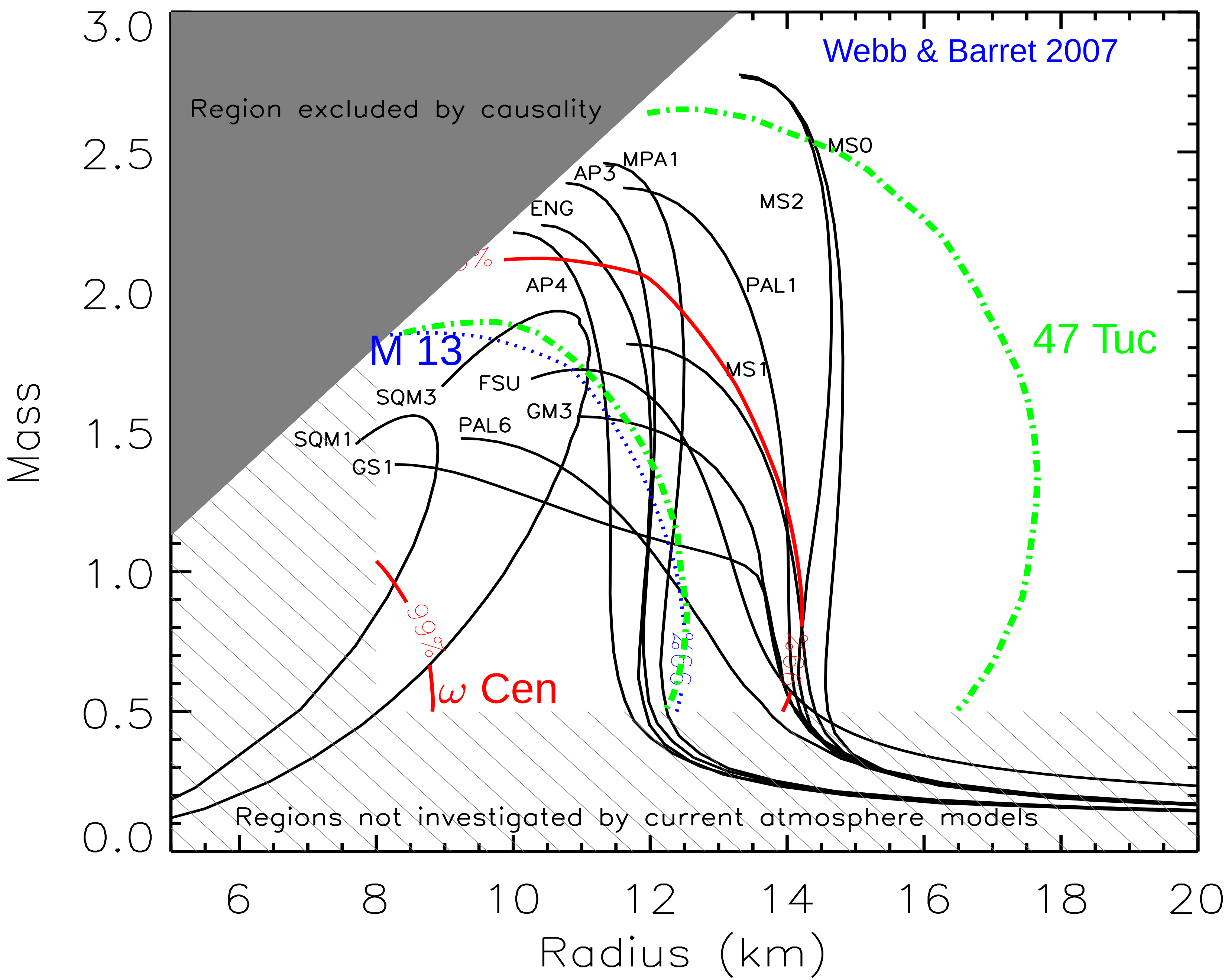


RX J1856-3754

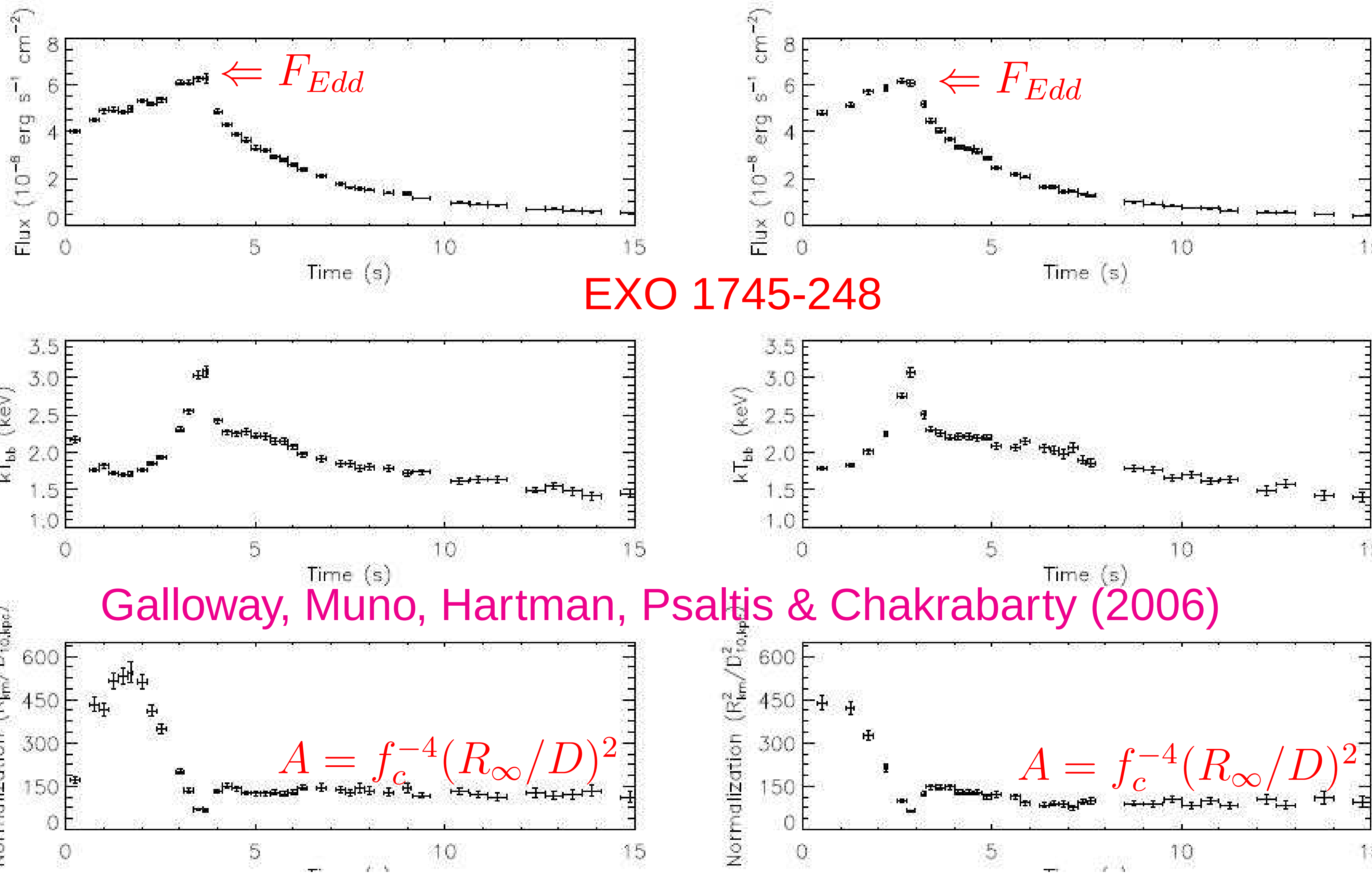
Walter & Lattimer (2002)



Radiation Radius: Globular Cluster Sources



Photospheric Radius Expansion X-Ray Bursts



EXO 1745-248

Galloway, Muno, Hartman, Psaltis & Chakrabarty (2006)

Systematics with $R_{ph} = R$

$$F_{Edd} = \frac{GMc}{\kappa D^2}\sqrt{1 - 2\frac{GM}{R_{ph}c^2}} = \frac{GMc}{\kappa D^2}\sqrt{1-2\beta}$$

$$\kappa \simeq 0.2(1+X)\ \mathrm{cm^2 g^{-1}}$$

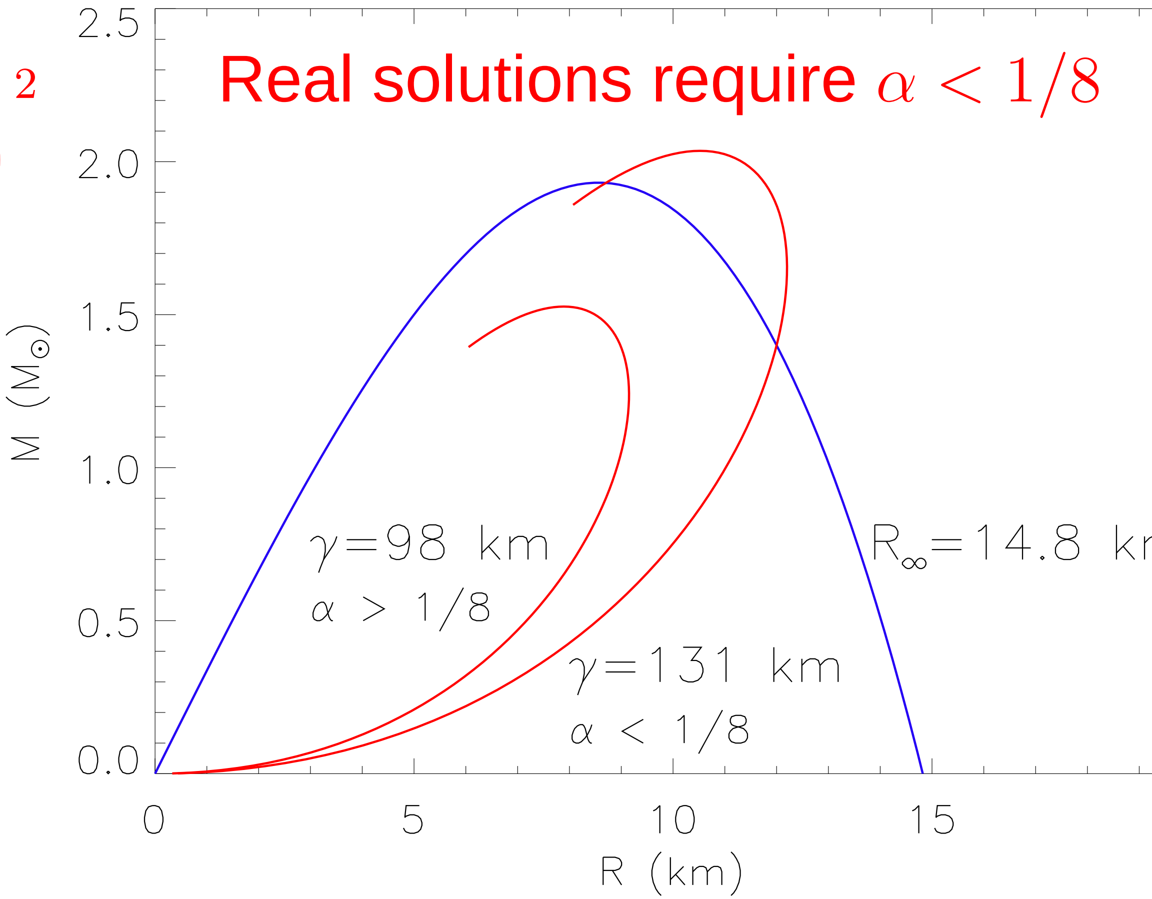
$$A = \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4}\left(\frac{R_\infty}{D}\right)^2$$

$$\alpha = \frac{F_{Edd}}{\sqrt{A}}\frac{\kappa D}{c^3 f_c^2} = \beta(1-2\beta)$$

$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd}\kappa} = \frac{R}{\beta(1-2\beta)^{3/2}}$$

$$\beta = \frac{1}{4} \pm \frac{1}{4}\sqrt{1-8\alpha}$$

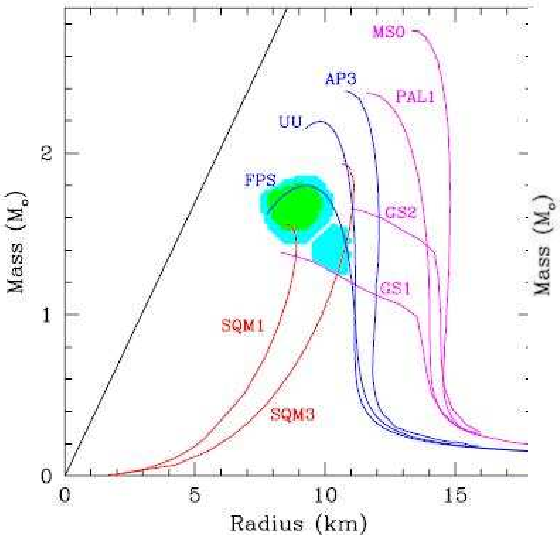
$$R_\infty = \alpha\gamma$$



$R_{ph} = R$

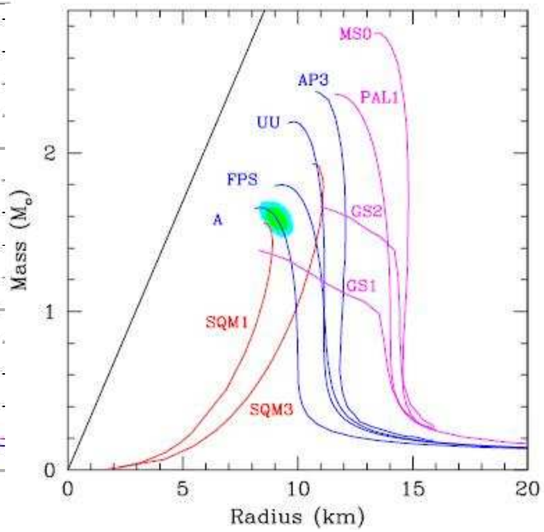
EXO 1745-248

$\alpha = 0.14 \pm 0.02$



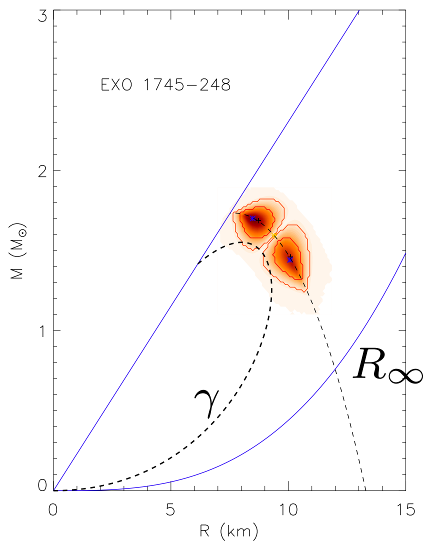
4U 1820-30

$\alpha = 0.18 \pm 0.02$



EXO 1745-248

$\alpha = 0.14 \pm 0.02$



4U 1820-30

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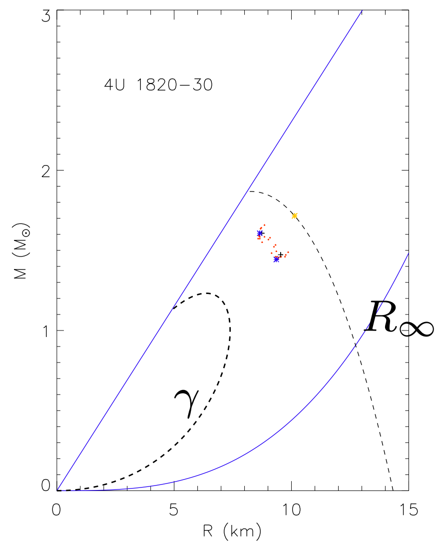
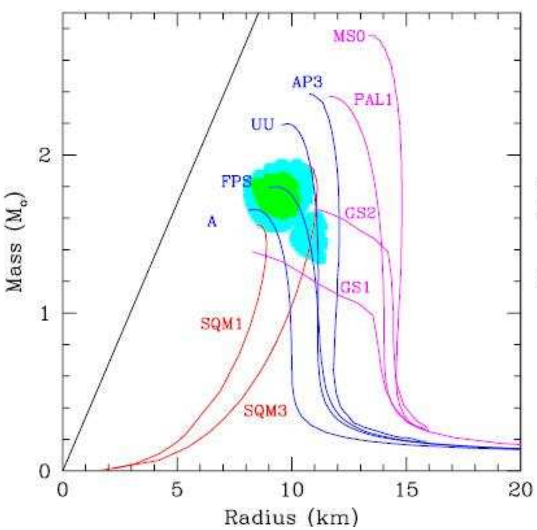
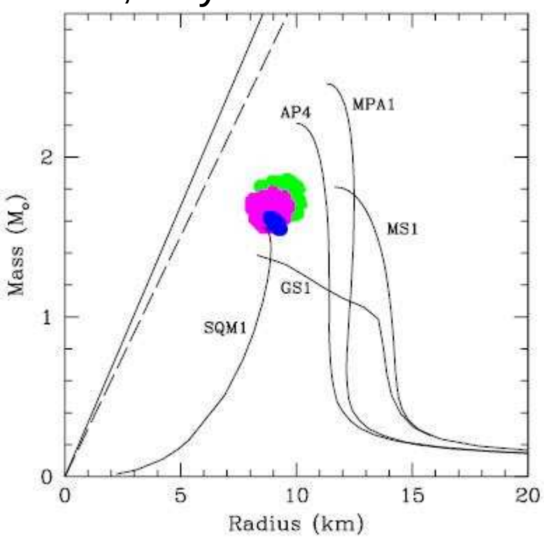


4U 1608-52

$\alpha = 0.26 \pm 0.11$



Ozel, Baym & Güver 2010



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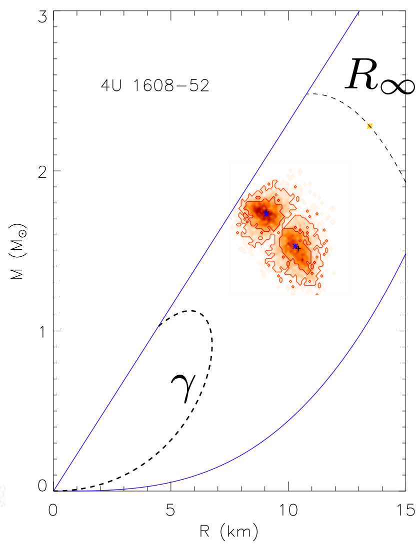
$\alpha = 0.26 \pm 0.11$



Systematics with $R_{ph} >> R$

$$F_{Edd} = \frac{GMc}{\kappa D^2}\sqrt{1 - 2\frac{GM}{R_{ph}c^2}} \simeq \frac{GMc}{\kappa D^2}$$

$$\kappa \simeq 0.2(1+X)\ \mathrm{cm^2 g^{-1}}$$

$$A = \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4}\left(\frac{R_\infty}{D}\right)^2$$

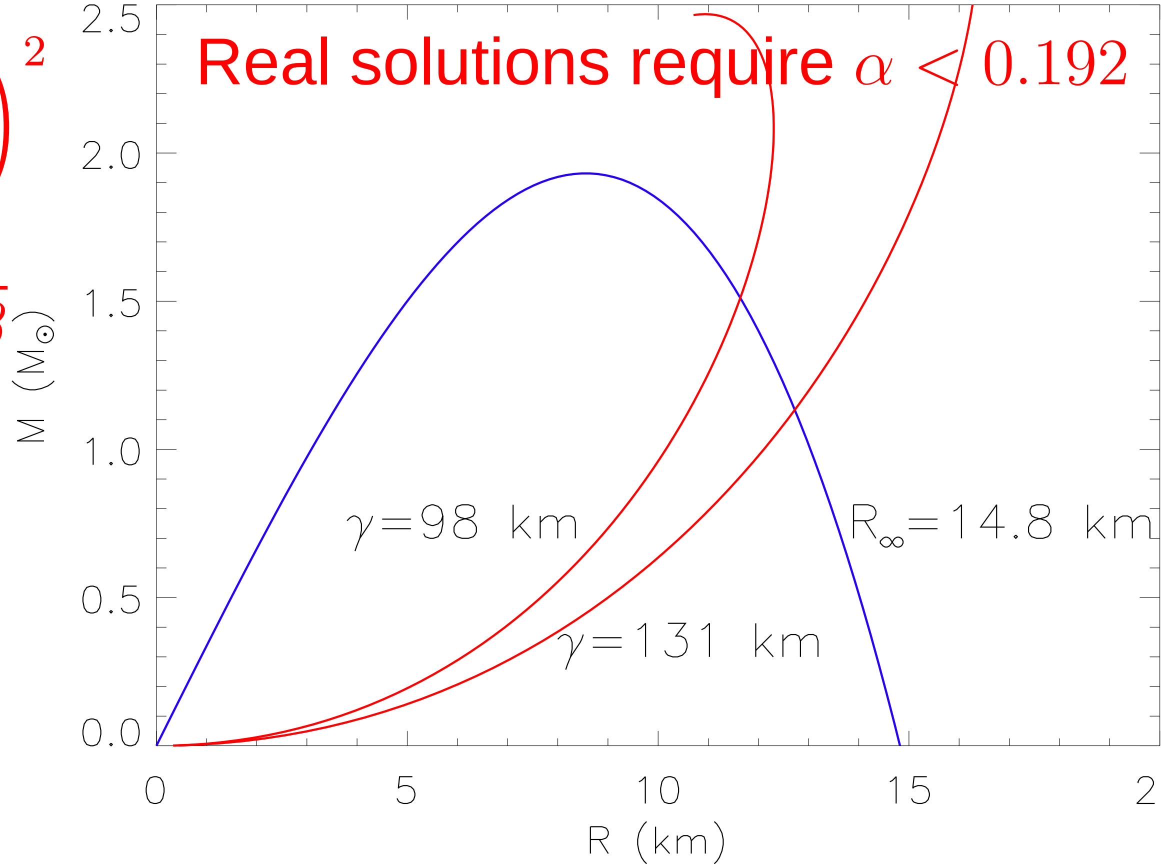
$$\alpha = \frac{F_{Edd}}{\sqrt{A}}\frac{\kappa D}{c^3 f_c^2} = \beta\sqrt{1-2\beta}$$

$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd}\kappa} = \frac{R}{\beta(1-2\beta)}$$

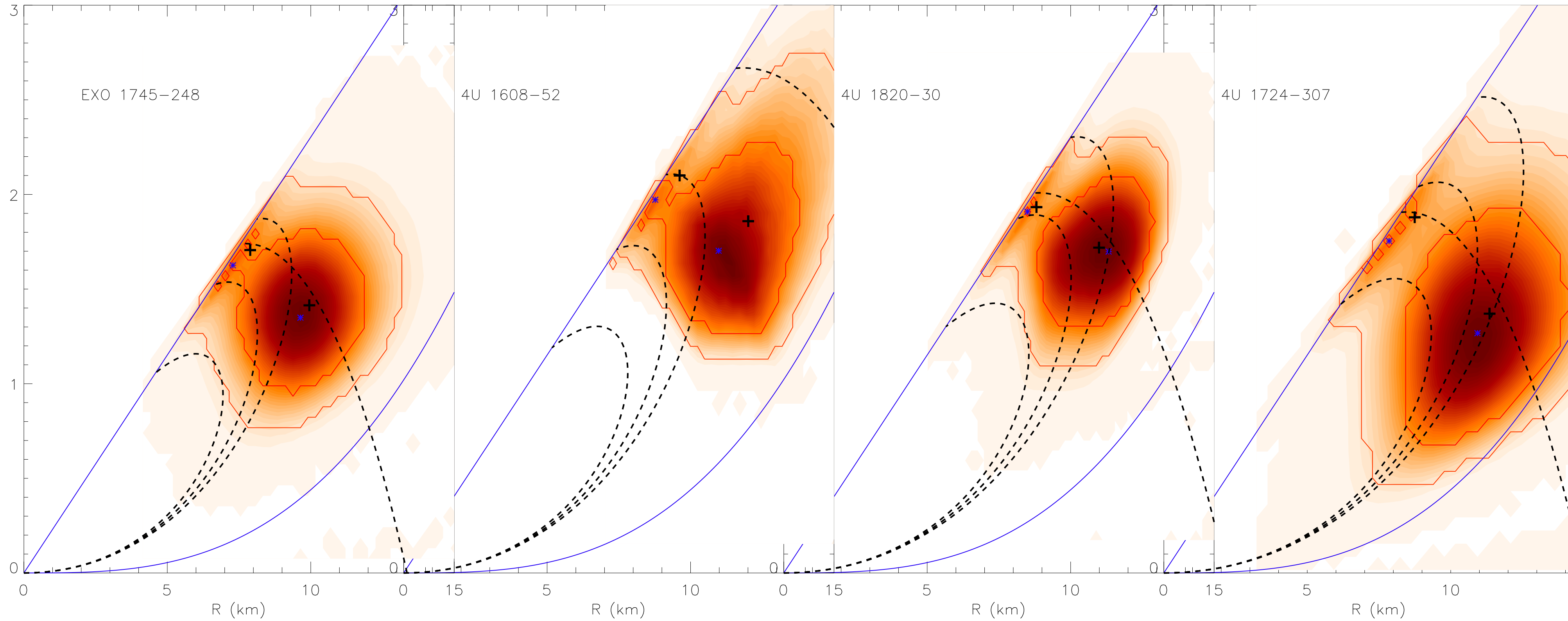
$$\beta = \frac{1}{6}\left[1 + \sqrt{3}\sin\theta - \cos\theta\right]$$

$$\theta = \frac{1}{3}\cos^{-1}(1 - 54\alpha^2)$$

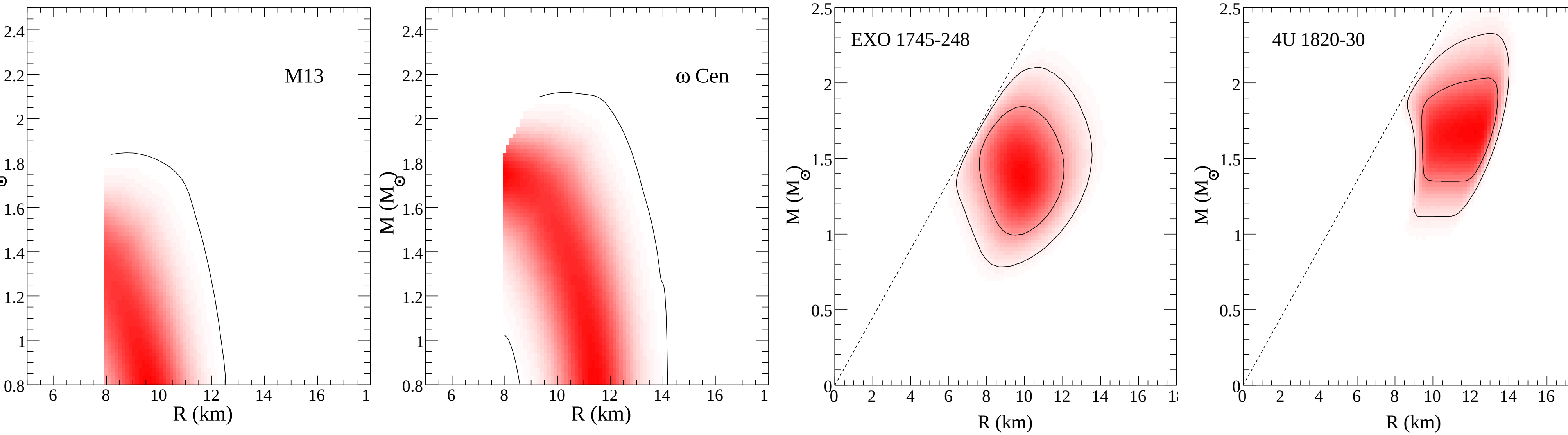
$$R_\infty = \alpha\gamma$$



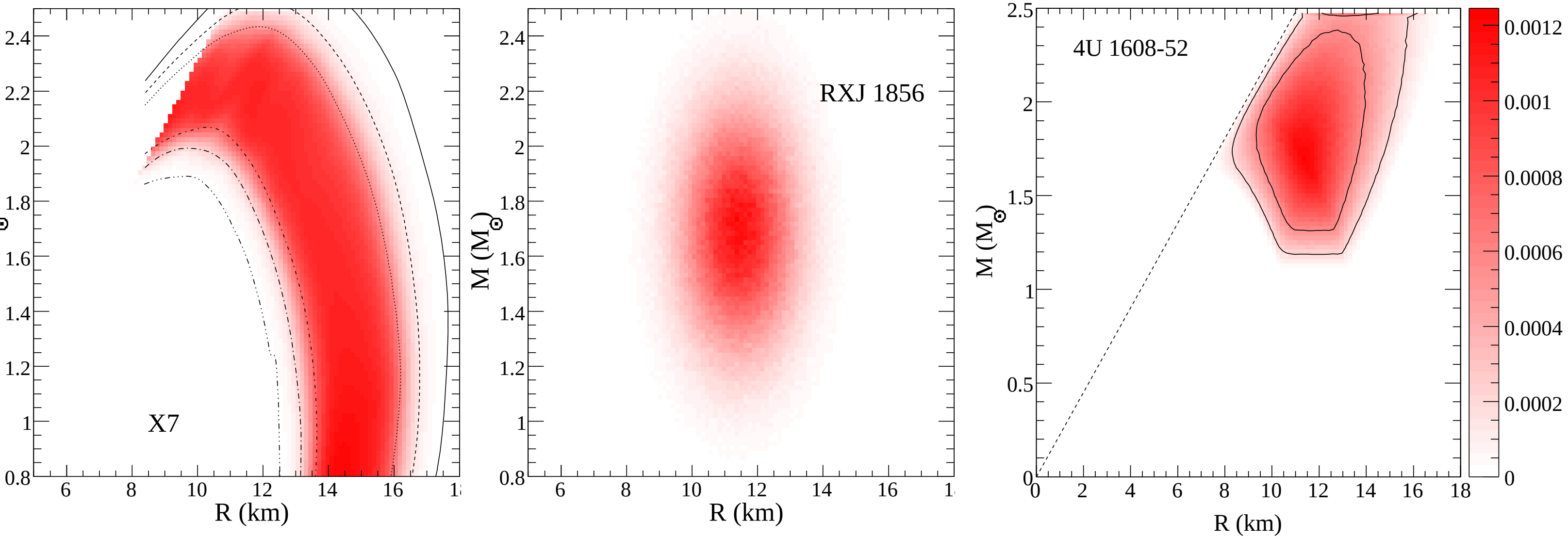
Arbitrary r_{ph}/R



M-R Probability Distributions



Steiner, Lattimer & Brown 2010



Bayesian Analysis

$P(\mathcal{M}_i|D)$: conditional probability of the model $\mathcal{M}_i$ given the data D (what we want).

$$P(\mathcal{M}_i|D) = \frac{P(D|\mathcal{M}_i)P(\mathcal{M}_i)}{\sum_k P(D|\mathcal{M}_k)P(\mathcal{M}_k)} \propto P(D|\mathcal{M}_i)\,.$$

$P(\mathcal{M}_i)$: prior probability of the model $\mathcal{M}_i$ without any information from the data D (assumed uniform).

$P(D)$: prior probability of the data D ($M-R$ probabilities).

$P(D|\mathcal{M}_i)$: conditional probability of the data D given the model $\mathcal{M}_i$. i is the EOS parameter set and the set of specific masses j.

$$P(D|\mathcal{M}_i) \propto \prod_{k,j} \mathcal{D}_{k,j}|_{M=M_j,R=R_k(M_j)}$$

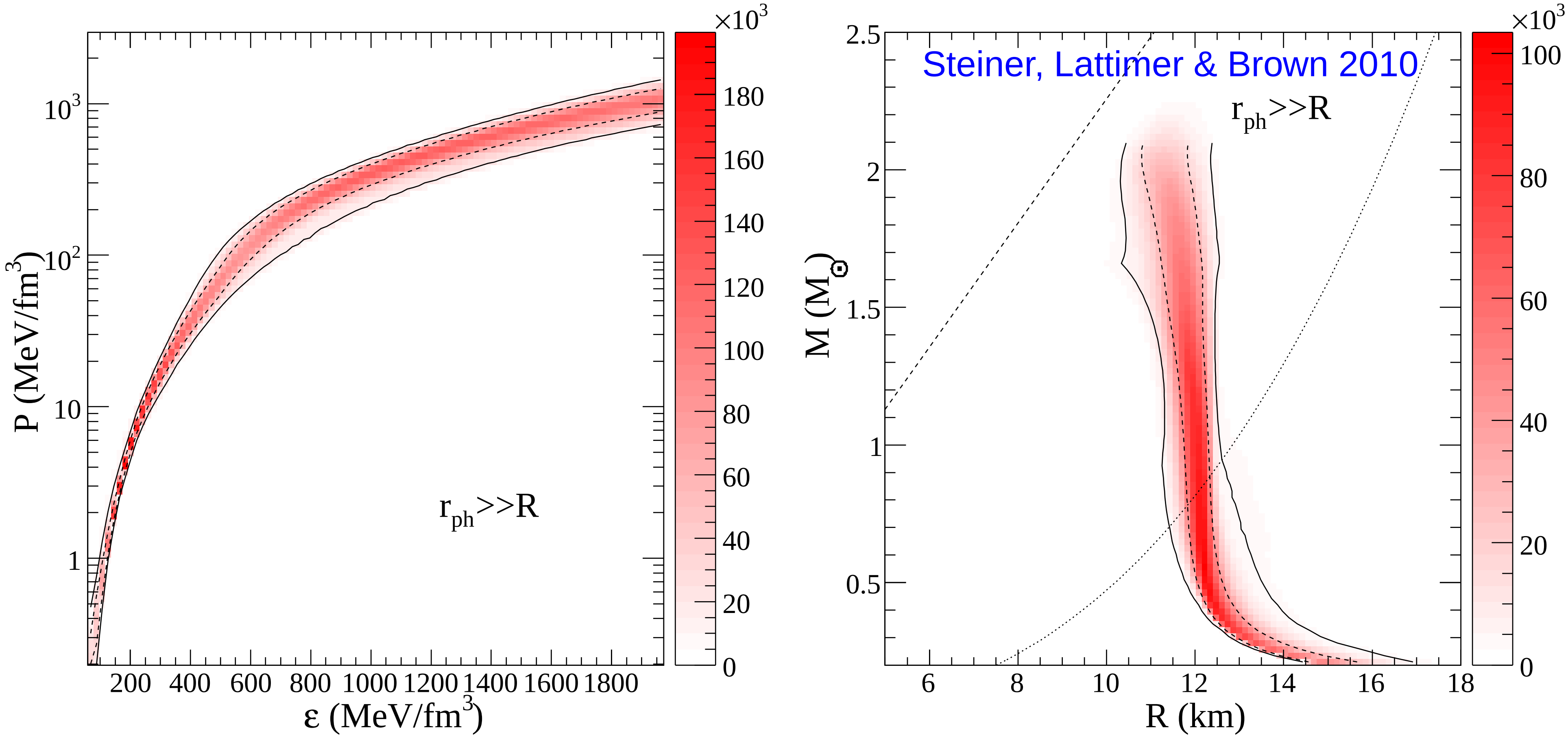
$\mathcal{D}_{k,j}$: probability of observing the radius R_j given a mass M_j in observation k.

The posterior probability for a model parameter p_i is

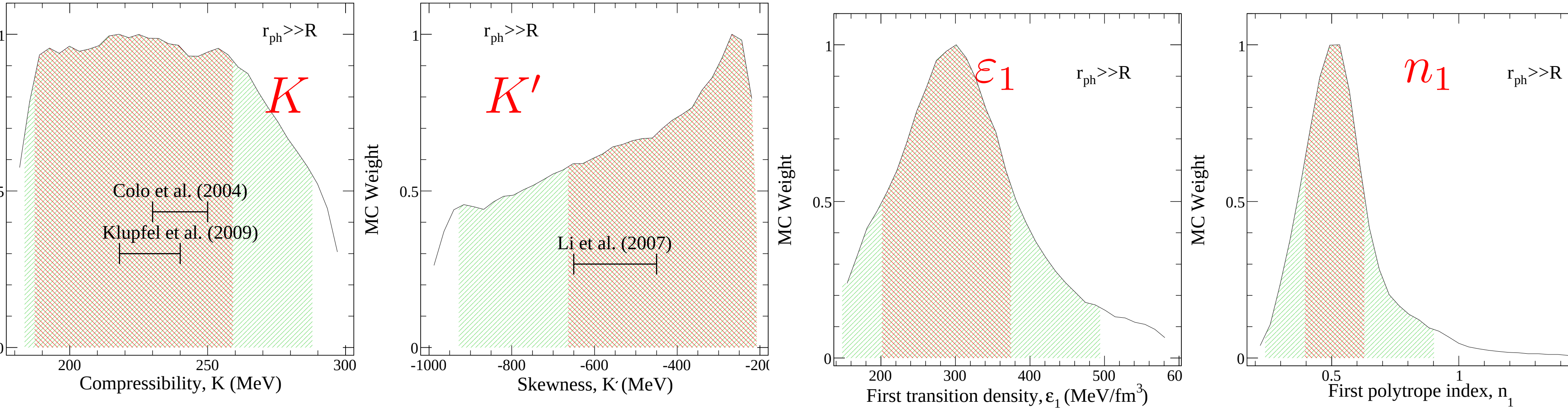
$$P(p_i|D) \propto \int P(D|\mathcal{M}_i)\, dp_1\, dp_2\, \ldots\, dp_{i-1}\, dp_{i+1}\, \ldots\, dp_{N_p}\, dM_1\, dM_2\, \ldots\, dM_{N_M}$$

Bayesian TOV Inversion

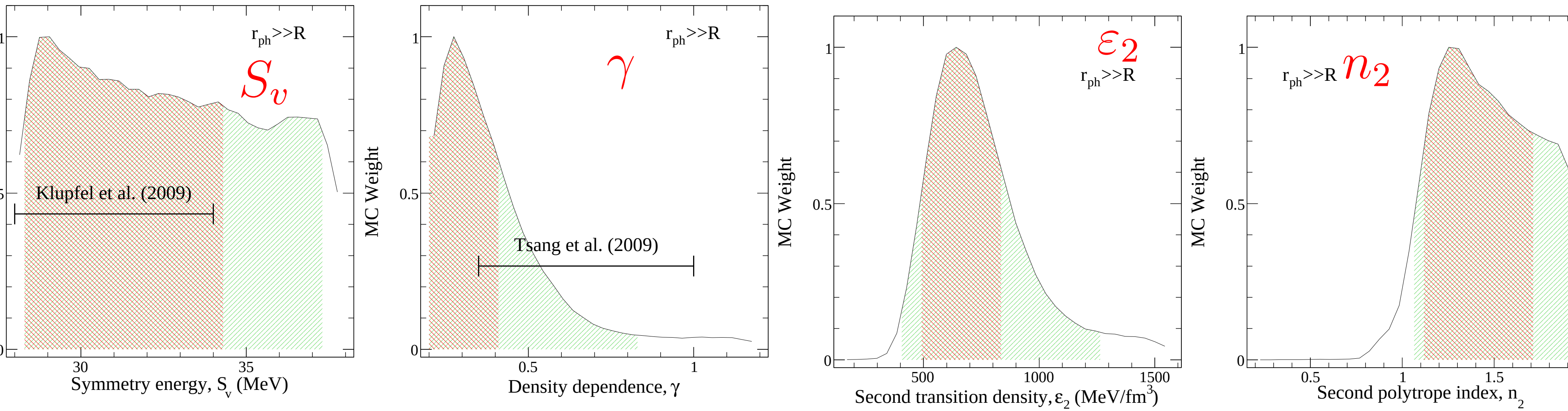
- $\varepsilon < 0.5\varepsilon_0$: EOS from BBP and NV
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- $\varepsilon_1 < \varepsilon < \varepsilon_2$: EOS is polytrope with n_1; $\varepsilon > \varepsilon_2$: EOS is polytrope with n_2
- A-priori EOS parameters ($K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$) uniformly distributed
- M and R probability distributions for 7 neutron stars treated equally ($0.8\ \mathrm{M}_\odot < M < 2.5\ \mathrm{M}_\odot$; $5\ \mathrm{km} < R < 18\ \mathrm{km}$)



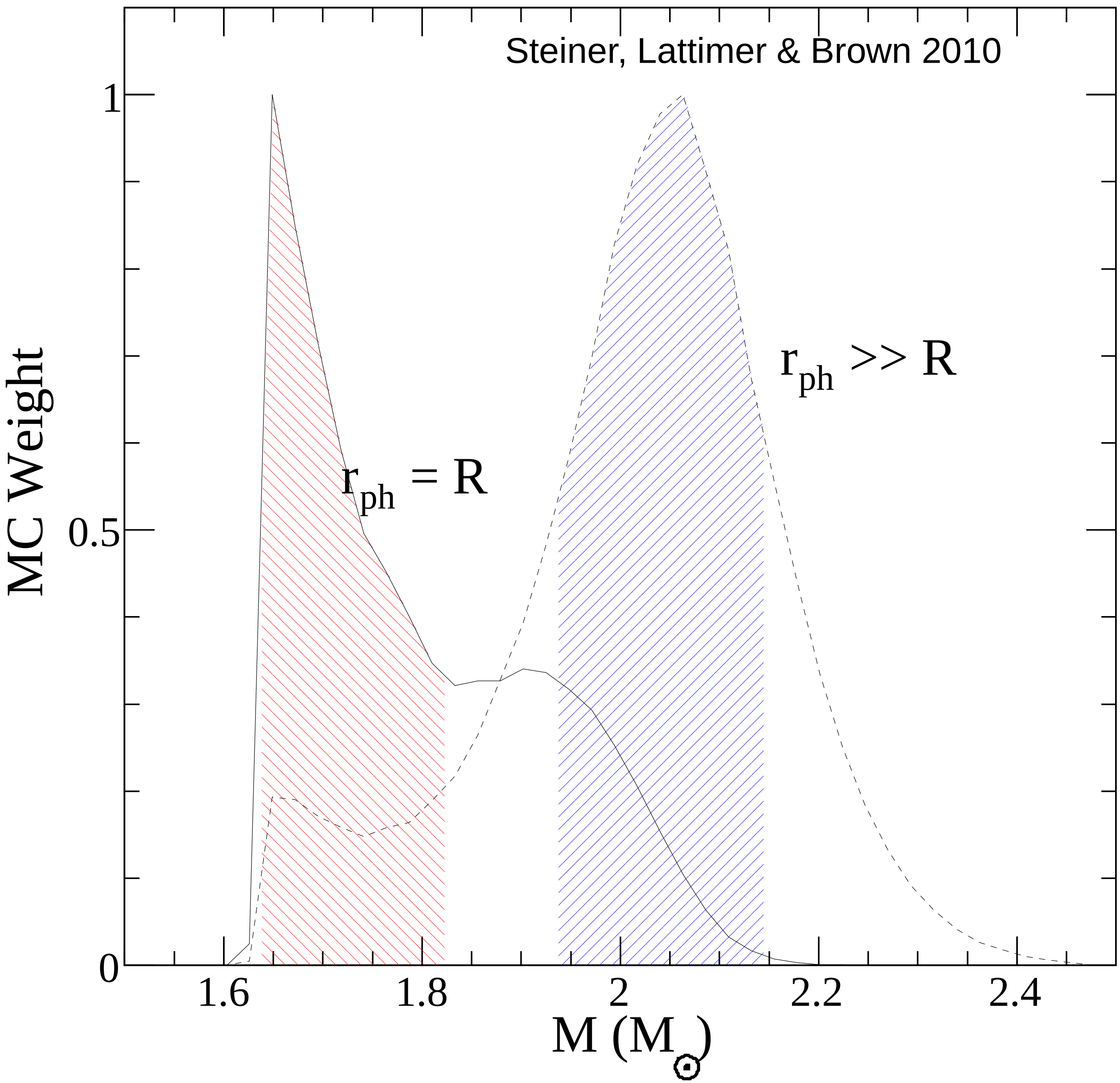
Inferred Model EOS Parameters



Steiner, Lattimer & Brown 2010



Maximum Mass Probability Distributions



Conclusions

- In spite of large estimated errors for individual stars, taken as an ensemble, they predict a remarkably tight pressure-density relation.
- Nuclear symmetry energy is predicted to be very soft, $\gamma = 0.3 \pm 0.1$
- Maximum Neutron Star Mass may be above 2 $M_\odot$
- Radii of 1.4 $M_\odot$ stars are apparently on the small side, 11.3 ± 0.3 km ($1 - \sigma$)