Shear Viscosity and the Nucleation of Antikaon Condensed Matter in Hot Neutron Star

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Plan of the Talk

- Introduction
- Shear Viscosity
- Model EoS
  Results
- Nucleation of antikaon bubbles
  Results
- Summary

Introduction

Shear viscosity (η) is the shearing force F per unit area A per unit velocity gradient in a laminar flow.

$$\frac{F}{A} = \eta \nabla_y V_x$$

Shear viscosity plays important roles in neutron star physics

- Damping gravitational wave driven instabilities (for instance r-modes).
- Essential in understanding pulsar glitches.
- Neutron viscosity was higher than the combined viscosities of e^- and μ in non-superfluid NS matter (Flowers and Itoh (ApJ230, 1979))
- Electron viscosity was larger than the neutron viscosity in a superfluid NS (Cutler and Lindblom (ApJ314, 1987) & Shternin and Yakovlev (ApJ314, 1987))
- Contribution of protons to total viscosity was neglected.
- In the presence of a negatively charged condensate, the situation changes.

Assumptions for the Calculation of Shear Viscosity

- Shear viscosity in NS cores is studied in the density range $\rho \sim (0.5 - 3)\rho_0$, where $\rho_0 = 2.8 \times 10^{14}$ g cm^{-3} .
- NS core is composed of strongly degenerate n, p, e^-, μ and $\bar{K}$.
- e^- and μ: almost ideal Fermi gases,
  n, p: a strongly interacting Fermi liquid,
  $\bar{K}$: Bose-Einstein condensates.
- p-p collisions due to both Strong and Electromagnetic(EM) interactions.
- (Anti)Kaons in the condensate do not contribute to the scattering.

R. Nandi, S. Banik and D.B., PRD80, 2009

Shear Viscosity

- The energy momentum tensor in a viscous system is given by:

$$T_{ij} = T_{ij}^{(0)} + \delta T ij$$

where the 1st term describes the perfect fluid:

$$T_{ij}^{(0)} = (P + \varepsilon)V_i V_j - P\delta_{ij}$$

and the 2nd term corresponds to a small deviation from equilibrium, which defines shear and bulk viscosities, η and ζ as

$$\begin{aligned} \delta T_{ij} &= -\eta(\nabla_i V_j + \nabla_j V_i - \frac{2}{3}\delta_{ij}\nabla \cdot V) + \zeta\delta_{ij}\nabla \cdot V \\ &= -\eta V_{ij} + \zeta\delta_{ij}\nabla \cdot V \end{aligned}$$

- The shear viscosity is calculated from a system of coupled Boltzmann kinetic equation:

$$\frac{\mathrm{d}f_c}{\mathrm{d}t} = \frac{\partial f_c}{\partial t} + \boldsymbol{v_c} \cdot \boldsymbol{\nabla f_c} + \boldsymbol{F} \cdot \boldsymbol{\nabla_p f_c} = \sum_i I_{ci} \qquad (1)$$

In absence of external force $\boldsymbol{F} = 0$ the LHS of the kinetic equation can be further simplified.

- f_c is the distribution function which slightly deviate from the equilibrium FD distribution $f_c^{(0)}$ due to the presence of a small hydrodynamic velocity field V :

$$
\begin{aligned}
f_c &= f_c^{(0)} - \Phi_c \frac{\partial f_c^{(0)}}{\partial \varepsilon_c} \\
&= f_c^{(0)} + \frac{f_{p_c}^{(0)}(1 - f_{p_c}^{(0)})}{k_B T} \Phi_c
\end{aligned}
$$

Φ measures deviation from equilibrium

- We take the simplest trial function for Φ:

$$\Phi_c = -\tau_c \left(p_{ci} p_{cj} - \frac{1}{3} p_c^2 \delta_{ij} \right) V_{ij}$$

where τ_c is taken to be independent of energy. This is known as relaxation time approximation.

- So we get the shear viscosity of carrier c as

$$\eta_c = \frac{n_c p_{Fc}^2 \tau_c}{5 m_c^*}$$

where

$$\begin{aligned} n_c &= \text{number density of the carrier} \\ p_{Fc} &= \text{Fermi momentum of the carrier} \\ m_c^* &= \text{effective mass of the carrier} \\ \tau_c &= \text{relaxation time} \end{aligned}$$

- The total shear viscosity is

$$\eta = \sum_c \eta_c$$

where $c = e, \mu, p, n$

- To know the relaxation times we have to solve the kinetic equation (1). The collision integrals on the RHS is given by:

$$I_{ci} = \frac{1}{(2\pi\hbar)^9(1+\delta_{ci})} \sum_{\sigma_1,\sigma_2,\sigma_3} \int dp_2 dp_{1'} dp_{2'} w_{ci}(12|1'2') \times [f_1' f_2'(1-f_1)(1-f_2) - f_1 f_2(1-f_1')(1-f_2')]$$

where $c, i = e, \mu, n, p$ & w_{ci} is differential transition probability.

- Linearizing the kinetic equations, multiplying them by $(p_{1i}p_{1j} - \frac{1}{3}p_1^2\delta_{ij})$, summing over σ_1 and integrating over $(2\pi\hbar)^{-3}\mathrm{d}\boldsymbol{p}_1$, we obtain a system of eqns for relaxation times,

$$1 = \sum_i \left(\nu_{ci}\tau_c + \nu'_{ci}\tau_i\right), \quad c = \mathrm{e},\ \mu,\ \mathrm{n},\ \mathrm{p} \tag{2}$$

where we introduce effective collision frequencies as:

$$\begin{aligned}
\nu_{ci} &= \frac{3\pi^2\hbar^3}{2p_{Fc}^5 k_B T m_c^*} \int \frac{\mathrm{d}\boldsymbol{p}_1\,\mathrm{d}\boldsymbol{p}_{1'}\,\mathrm{d}\boldsymbol{p}_2\,\mathrm{d}\boldsymbol{p}_{2'}}{(2\pi\hbar)^{12}} \\
&\times\ W_{ci}(12|1'2')\, f_1 f_2 (1-f_{1'})(1-f_{2'}) \\
&\times\ \left[\frac{2}{3}\, p_1^4 + \frac{1}{3}\, p_1^2 p_{1'}^2 - (\boldsymbol{p}_1\cdot\boldsymbol{p}_{1'})^2\right] \\
\nu_{ci}' &= \frac{3\pi^2\hbar^3}{2p_{Fc}^5 k_B T m_i^*} \int \frac{\mathrm{d}\boldsymbol{p}_1\,\mathrm{d}\boldsymbol{p}_{1'}\,\mathrm{d}\boldsymbol{p}_2\,\mathrm{d}\boldsymbol{p}_{2'}}{(2\pi\hbar)^{12}} \\
&\times\ W_{ci}(12|1'2')\, f_1 f_2 (1-f_{1'})(1-f_{2'}) \\
&\times\ \left[\frac{1}{3}\, p_1^2 p_{2'}^2 - \frac{1}{3}\, p_1^2 p_2^2 + (\boldsymbol{p}_1\cdot\boldsymbol{p}_2)^2 - (\boldsymbol{p}_1\cdot\boldsymbol{p}_{2'})^2\right]
\end{aligned}$$

with differential collision probability

$$\begin{aligned}
W_{ci}(12|1'2') &= 4\frac{(2\pi\hbar)^4}{\hbar^2}\,\delta(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{p}_{1'}-\boldsymbol{p}_{2'}) \\
&\times\ \delta(\varepsilon_1+\varepsilon_2-\varepsilon_{1'}-\varepsilon_{2'})\,\frac{\mathcal{Q}_{ci}}{1+\delta_{ci}}
\end{aligned}$$

The matrix equation has the following form,

$$\begin{pmatrix} \nu_e & \nu'_{e\mu} & \nu'_{ep} & 0 \\ \nu'_{\mu e} & \nu_\mu & \nu'_{\mu p} & 0 \\ \nu'_{pe} & \nu'_{p\mu} & \nu_p & \nu'_{pn} \\ 0 & 0 & \nu'_{np} & \nu_n \end{pmatrix} \begin{pmatrix} \tau_e \\ \tau_\mu \\ \tau_p \\ \tau_n \end{pmatrix} = 1,$$

where,

$$\begin{aligned} \nu_e &= \nu_{ee} + \nu'_{ee} + \nu_{e\mu} + \nu_{ep} \\ \nu_\mu &= \nu_{\mu\mu} + \nu'_{\mu\mu} + \nu_{\mu e} + \nu_{\mu p} \\ \nu_p &= \nu_{pp} + \nu'_{pp} + \nu_{pn} + \nu_{pe} + \nu_{p\mu} \\ \nu_n &= \nu_{nn} + \nu'_{nn} + \nu_{np} \ . \end{aligned}$$

Again

$$\begin{aligned} \nu_{pp} &= \nu^{s}_{pp} + \nu^{em}_{pp} \ , \\ \nu'_{pp} &= \nu'^{s}_{pp} + \nu'^{em}_{pp} \ . \end{aligned} \tag{3}$$

Model for EOS

- To calculate the collision frequencies & shear viscosity, we need to know the effective masses and Fermi momenta of particles as a function of density (EoS).
- We use the relativistic mean field model. The interaction between baryons is mediated by the exchange of scalar (σ) and vector (ω, ρ) mesons.This picture is consistently extended to include the kaons.
- The Lagrangian density for baryons is given by

$$\begin{aligned}
\mathcal{L}_B &= \sum_{B=n,p} \bar{\Psi}_B \left(i\gamma_\mu\partial^\mu - m_B^* - g_{\omega B}\gamma_\mu\omega^\mu - g_{\rho B}\gamma_\mu \mathbf{t}_B \cdot \boldsymbol{\rho}^\mu\right)\Psi_B \\
&+ \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - U(\sigma) \\
&- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu \; .
\end{aligned}$$

where $m_B^* = m_B - g_\sigma\sigma$ is the baryon effective mass.

- The Lagrangian density for (anti)kaons in the minimal coupling scheme is

$$\mathcal{L}_K = D^*_\mu \bar{K} D^\mu K - m_K^{*2} \bar{K} K \ ,$$

where the vector fields are coupled via the standard form $D_\mu = \partial_\mu + ig_{\omega K}\omega_\mu + ig_{\rho K} \mathrm{t}_K \cdot \boldsymbol{\rho}_\mu$ and the effective mass of (anti)kaons is $m_K^* = m_K - g_{\sigma K}\sigma$.

- The equation of motion for (anti)kaons is

$$(D_\mu D^\mu + m_K^*) K = 0$$

Solving the EoM within mean field approximation we get:

$$\omega_{K^-} = m_K^* - g_{\omega K}\omega_0 - \frac{1}{2} g_{\rho K}\rho_{03}. \qquad (4)$$

- Threshold condition for (anti) kaon condensation

$$\omega_{K^-} = \mu_{K^-} = \mu_e$$

(N.K. Glendenning, J. Schaffner-Bielich,Phys. Rev. **C60**, 025803 (1999) , S.B., D. Bandyopadhyay, Phys. Rev. **C64** (2001) 055805.)

Gibbs phase rules

$$P^H = P^K ,$$

$$\mu_B^H = \mu_B^K .$$

Conditions of global charge neutrality

$$(1-\chi)Q^H + \chi Q^K = 0,$$

Baryon number conservation

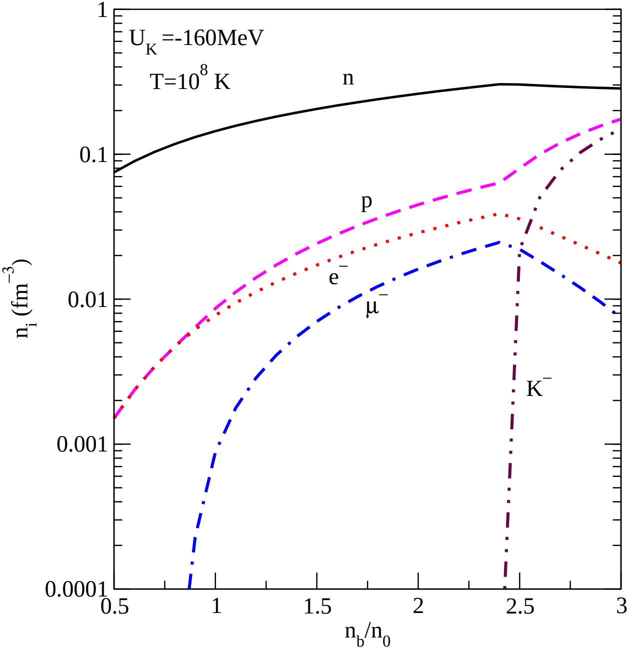
$$n_B = (1-\chi)n_B^H + \chi n_B^K$$

Total energy density

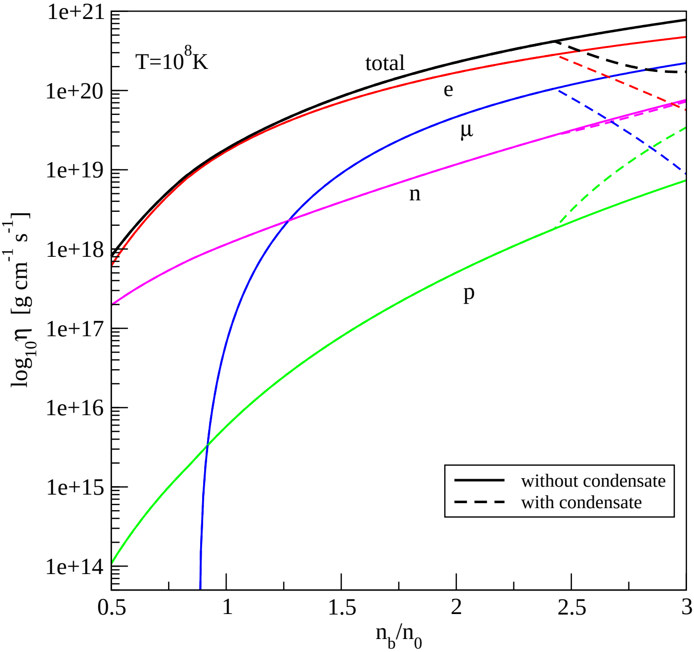
$$\epsilon = (1-\chi)\epsilon^H + \chi\epsilon^K \ .$$

Pressure

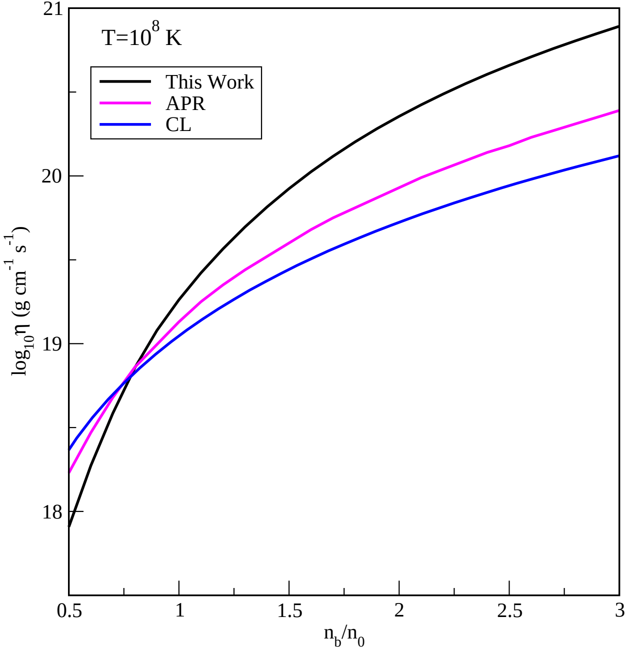
$$P = \sum_i \mu_i n_i - \varepsilon \ .$$

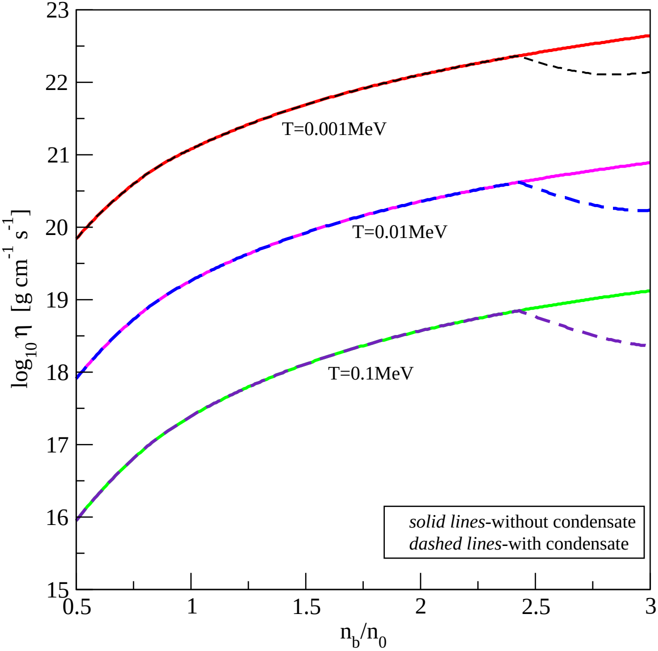


U_K =-160MeV
T=10^8 K
n
p
e⁻
μ⁻
K⁻
n_i (fm^-3)
n_b/n_0
1
0.1
0.01
0.001
0.0001
0.5
1
1.5
2
2.5
3



T=10^8K
total
e
μ
n
p
without condensate
with condensate
1e+21
1e+20
1e+19
1e+18
1e+17
1e+16
1e+15
1e+14
0.5
1
1.5
2
2.5
3
n_b/n_0
$\log_{10}\eta$ [g cm^{-1} s^{-1}]


T=10^8 K
This Work
APR
CL
21
20
19
18
0.5
1
1.5
2
2.5
3
log10η (g cm-1 s-1)
nb/n0



23
22
21
20
19
18
17
16
15
0.5
1
1.5
2
2.5
3
$\log_{10}\eta$ [g cm^{-1} s^{-1}]
n_b/n_0
T=0.001MeV
T=0.01MeV
T=0.1MeV
solid lines-without condensate
dashed lines-with condensate

The Nucleation of Antikaon Bubbles:Introduction

- A 1st order phase transition is driven by nucleation of bubbles.
- Nucleation of Quark Matter droplets in NS matter (I. Bombaci, D. Logoteta, P.K. Panda, C. Providencia and I. Vidana Phys. Lett. **B680**, 448 (2009)
  B. W. Mintz, E. S. Fraga, G. Pagliara, J. Schaffner-Bielich, Phy. Rev **D 81**, 123012)
- Nucleation in NS matter with a first order hadron to kaon condensed phase transition (T. Norsen, Phys. Rev.**C 65**, 045805)
- Shear viscosity might control the initial growth rate of a bubble.(L.P. Csernai and J.I. Kapusta, Phys. Rev. **D46**, 1379 (1992).)
- Thermal nucleation time is inversely proportional to the shear viscosity.

The Nucleation of Antikaon Bubbles

- The modern theory of homogeneous nucleation via thermal activation pioneered by Langer, yields the nucleation per unit time per unit volume

$$I = \frac{\kappa}{2\pi}\Omega_0 \exp(-\frac{\Delta F}{T}),$$

where ΔF is excess free energy of the system due to formation of the critical bubble.

- The statistical prefactor $\Omega_0 = \frac{2}{3\sqrt{3}}\left(\frac{\sigma}{T}\right)^{1.5}\left(\frac{R}{X}\right)^4$ measures the available phase space volume,
  κ is the dynamical prefactor $\kappa = \frac{2\sigma}{R_*^3(\Delta w)^2}\left[\lambda T + 2\left(\frac{4}{3}\eta + \zeta\right)\right]$
  where Δw is the difference of the enthalpy of the 2 phases,
  λ is the thermal conductivity,
  η and ζ are the shear and bulk viscosity respectively,
  σ is the surface tension for the surface separating the 2 phases.
- The dominant contribution to dynamical prefactor comes from the shear viscosity term.

- The change in free energy of the system is

$$\Delta F = \frac{4\pi}{3}(\Delta P)R^3 + 4\pi\sigma R^2$$

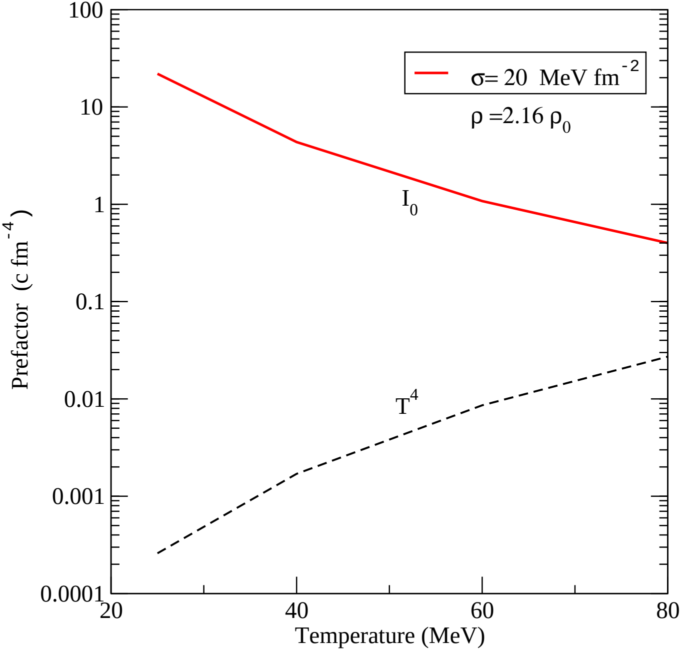
where R is the radius of the droplet.

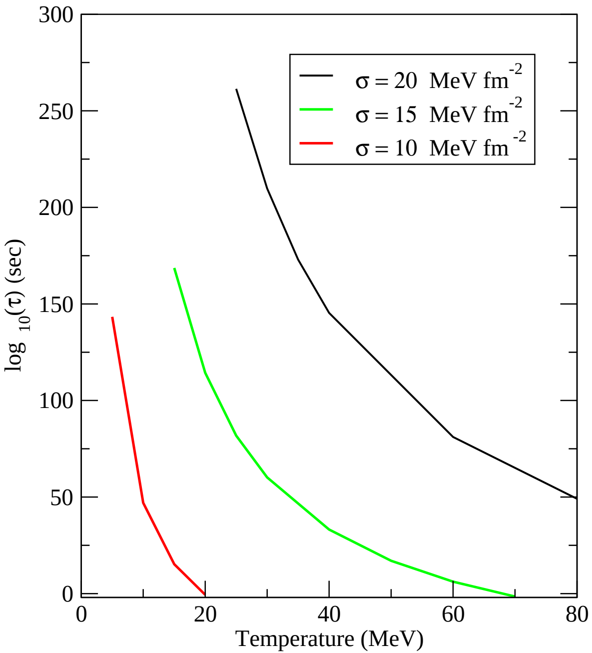
- The bubble would grow whenever $R > R_*$ and collapse otherwise.
- The free energy is maximum at this critical radius R_*.

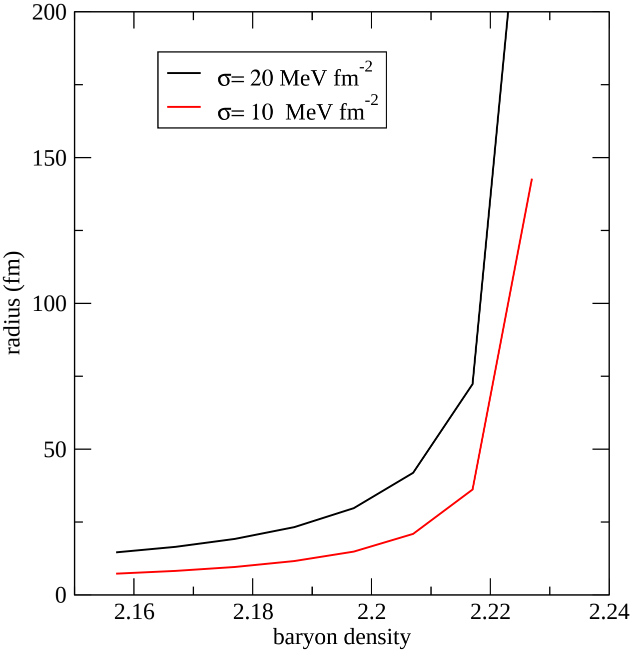
$$R_*(T) = \frac{2\sigma}{(\Delta P)}.$$

- The thermal nucleation time can be written as

$$\tau_{th} = (V_{nuc} I)^{-1}.$$


100
10
1
0.1
0.01
0.001
0.0001
20
40
60
80
Temperature (MeV)
Prefactor (c fm^{-4})
σ= 20 MeV fm^{-2}
ρ =2.16 $ρ_0$
I_0
T^4


σ = 20 MeV fm^-2
σ = 15 MeV fm^-2
σ = 10 MeV fm^-2
log 10(τ) (sec)
Temperature (MeV)
0
50
100
150
200
250
300
0
20
40
60
80



σ= 20 MeV fm-2
σ= 10 MeV fm-2
200
150
100
50
0
radius (fm)
2.16
2.18
2.2
2.22
2.24
baryon density

Summary

- Shear viscosity might affect the nucleation of antikaon bubbles in hot neutron stars
- The total shear viscosity decreases in the K^- condensed matter due to the sharp drop in the lepton shear viscosities.
- The proton shear viscosity whose contribution to the total shear viscosity was negligible compared to the leptonic one in nucleons only matter, becomes significant in the presence of theK^- condensate.
- The proton shear viscosity may even exceed the neutron as well lepton shear viscosities at higher densities.
- Shear viscosity influences the nucleation process of bubbles of K- condensed phase in NS.

Thank you......