Clusters in Nuclear Matter

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Maria Voskresenskaya (GSI Darmstadt)

EMMI workshop: Neutron Matter in Astrophysics
From Neutron Stars to the r-Process

Outline

- **Motivation**
- **Thermodynamical Conditions**
- **Dense Matter and Nuclear Matter**
- **Theoretical Approaches**
- **Equation of State (EoS) with Light Clusters**
- **Liquid-Gas Phase Transition**
- **Heavy Clusters**
- **Symmetry Energy**
- **Summary and Outlook**

for details see Phys. Rev. C **81**, 015803 (2010) and Phys. Rev. Lett. **104**, 202501 (2010)

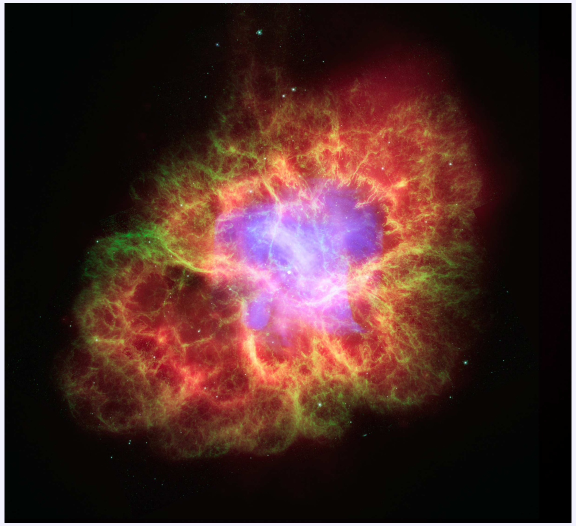
Motivation: The Life and Death of Stars

when the fuel for nuclear fusion reactions is consumed:

- last phases in the life of a massive star ($8 M_{\rm sun} \lesssim M_{\rm star} \lesssim 30 M_{\rm sun}$)

  $\Rightarrow$ core-collapse supernova

  $\Rightarrow$ neutron star or black hole



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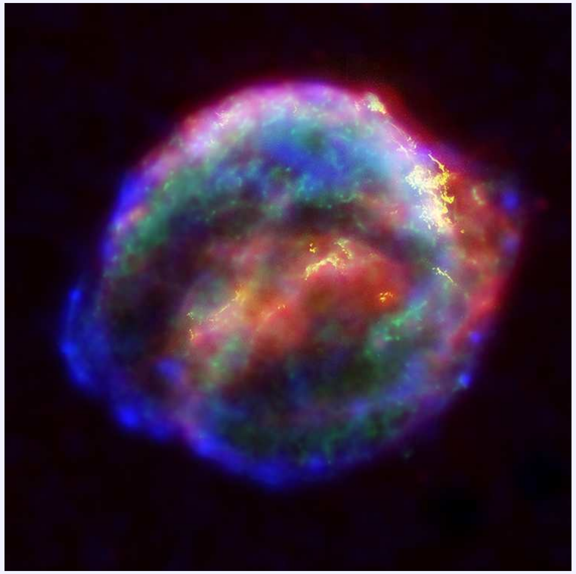

NASA/ESA/R.Sankrit & W.Blair (Johns Hopkins Univ.)

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- essential ingredient in astrophysical
  model calculations:

  equation of state (EoS) of dense matter

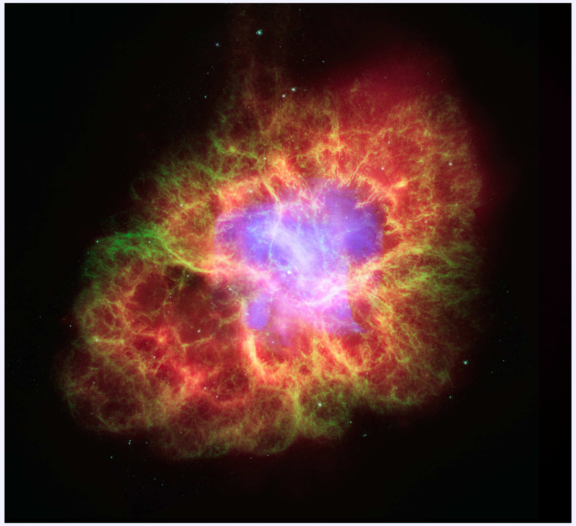
  $\Rightarrow$ dynamical evolution of supernova

  $\Rightarrow$ static properties of neutron star

  $\Rightarrow$ conditions for nucleosynthesis

  $\Rightarrow$ energetics, chemical composition,

  transport properties, . . .



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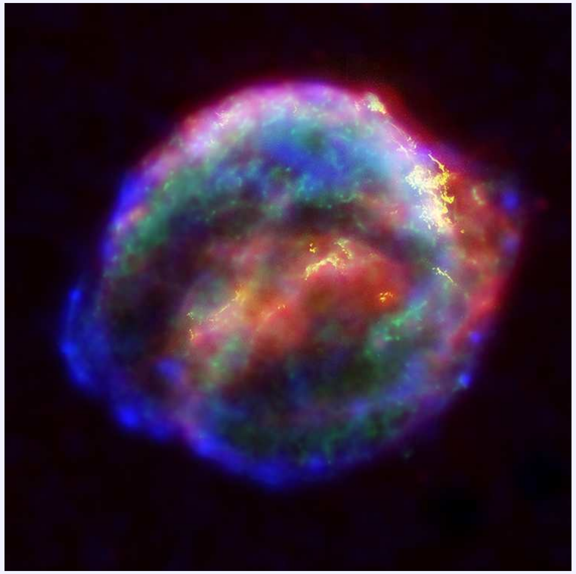

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  – nucleons, α particle and representative heavy nucleus
  – suppression of α at high densities: excluded volume mechanism
  – simple parametrization of density distribution of heavy nucleus

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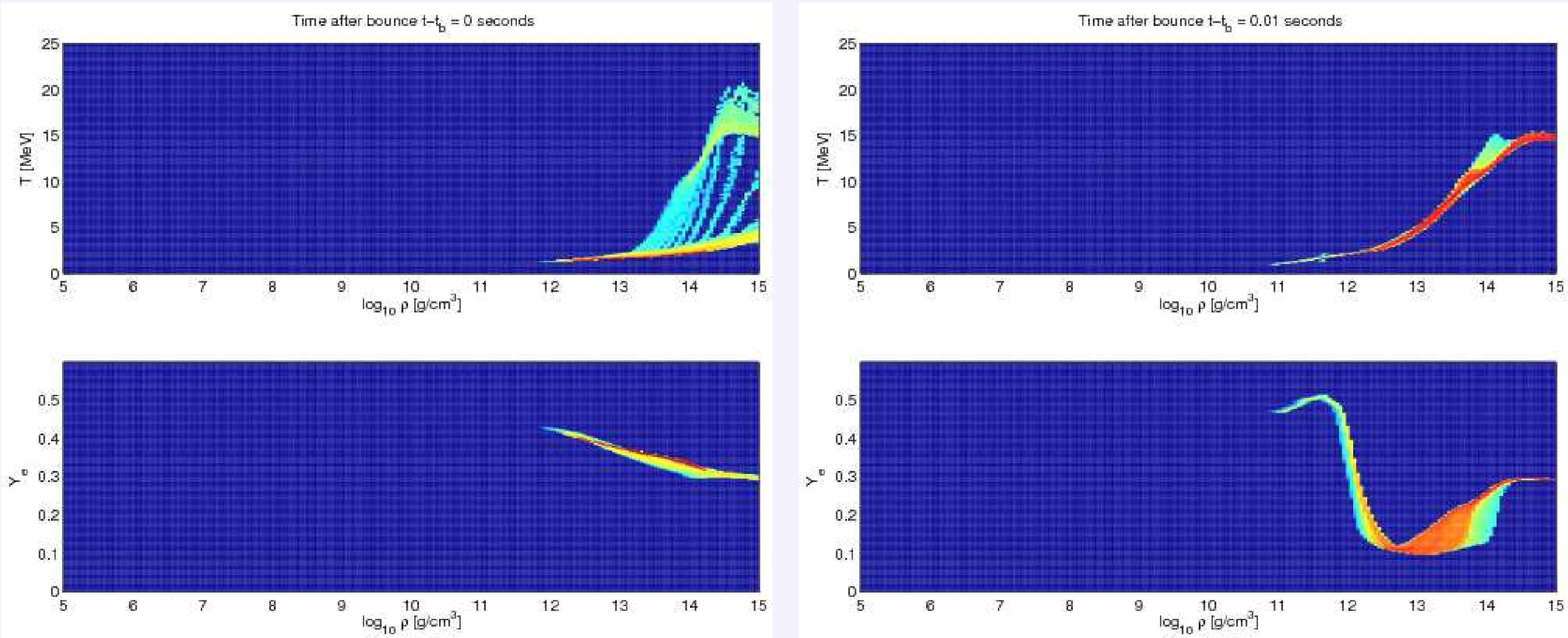
  more recent EoS for astrophysical applications:
  ∘ M. Hempel, J. Schaffner-Bielich (Nucl. Phys. A 837 (2010) 210)
  ∘ A.S. Botvina, I.N. Mishustin (Nucl. Phys. A 843 (2010) 98)
  – mixture of nucleons and all nuclei
  – statistical equilibrium, excluded volume mechanism

Thermodynamical Conditions

- **densities**: $10^{-9} \lesssim \varrho/\varrho_{\rm sat} \lesssim 10$ ($\varrho_{\rm sat} \approx 2.5 \cdot 10^{14}$ g/cm^3)
- **temperatures**: 0 MeV $\leq k_B T \lesssim 25$ MeV ($\hat{=}\ 2.9 \cdot 10^{11}$ K)
- **electron fraction**: $0 \leq Y_e \lesssim 0.6$



M. Liebendörfer, R. Käppeli, S. Scheidegger, Universität Basel

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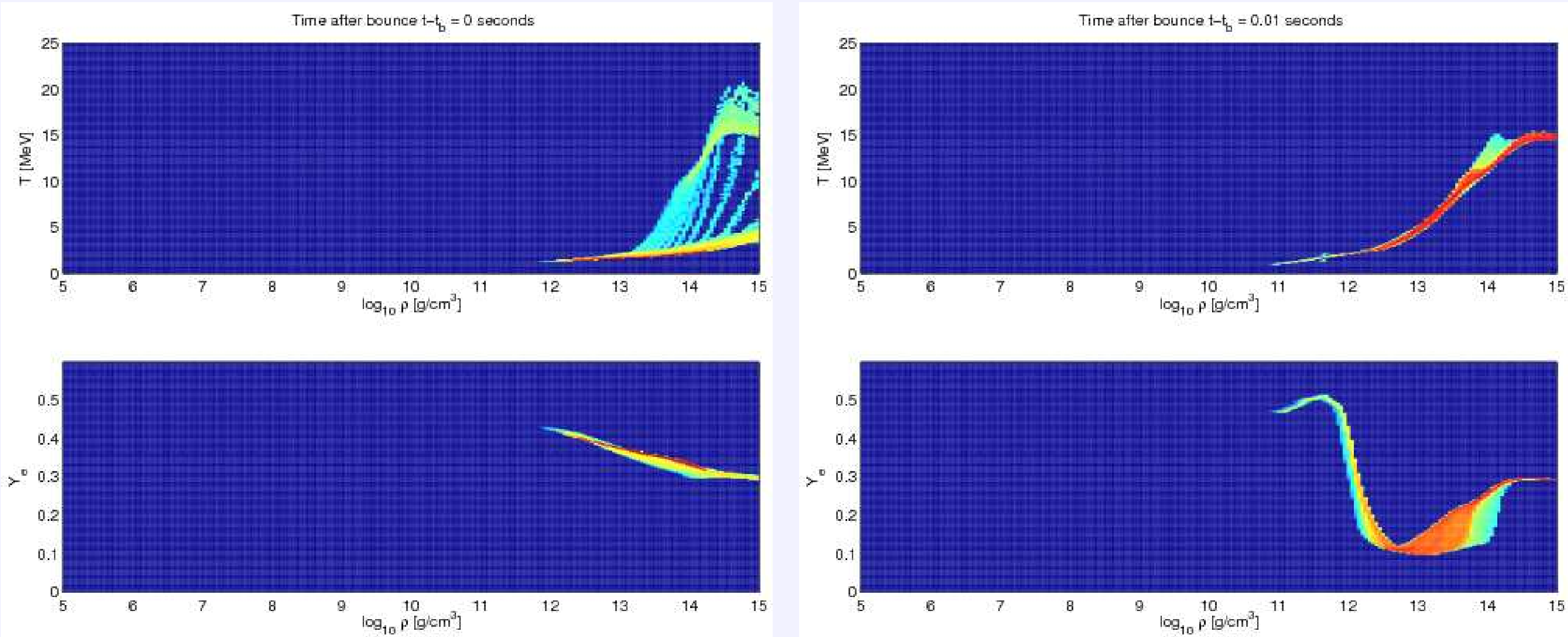
⇒ global theoretical description of matter properties is required



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Constituents of Dense Matter

large variety of possible
particle species, e.g.

- nuclei
- hadrons
  - baryons
    - nucleons
    - hyperons
  - mesons
- quarks
- leptons
  - electrons, muons
  - neutrinos
- photons
- . . .

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- in general:
  consider all particles that cannot decay
  (depends on conditions)
  and all interactions between them

- most difficult problem:
  description of strongly interacting
  subsystem (hadronic or quark matter)

- in this talk:
  - nuclear matter at densities
    around and below nuclear saturation
  - formation of "clusters"

Composition of Nuclear Matter I

- depends strongly on density, temperature and neutron-proton asymmetry
- affects thermodynamical properties

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- theoretical models: different points of view
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    problems:
    - properties of constituents independent of medium
    - interaction between particles
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  - **physical picture:**
    correlations of nucleons ⇒ formation of bound states
    problems:
    - treatment of three-, four-, . . . many-body correlations difficult
    - choice of interaction

⇒ combination of approaches?

Composition of Nuclear Matter II

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  - models with nuclei in statistical equilibrium

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  - surface effects and long-range Coulomb interaction
  - inhomogeneous matter
  - formation of "pasta" phases/lattice structures

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interpolation between low-density and high-density limit needed

Theoretical Approaches - Low Densities I

- nuclear statistical equilibrium (NSE)
  - ideal mixture of nucleons (p, n) and nuclei (X) in chemical equilibrium

$$Zp + Nn \Leftrightarrow {}^{A}_{Z}X_{N} \qquad Z\mu_p + N\mu_n = \mu_X$$

    with relativistic chemical potentials μ_i
  - interaction between particles not considered, no medium effects

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- virial equation of state in classical description (with potentials)
  - expansion of grand-canonical partition function $\mathcal{Z}(V, T, \tilde{\mu}_i)$ in powers of fugacities $z_i = \exp(\tilde{\mu}_i/T) \ll 1$ with chemical potentials $\tilde{\mu}_i$ (non-relativistic)
  - Maxwell-Boltzmann statistics
  - two-body correlations encoded in second virial coefficient b_{ij}
  - limitation $n_i \lambda_i^3 \ll 1$ with density n_i and thermal wavelength $\lambda_i = \sqrt{2\pi/(m_i T)}$

Theoretical Approaches - Low Densities II

- virial equation of state in quantum mechanical generalization
  (G. E. Beth and E. Uhlenbeck Physica 3 (1936) 729, Physica 4 (1937) 915)
  - second virial coefficient

$$b_{ij} = \frac{1+\delta_{ij}}{2} \left(\frac{m_i + m_j}{\sqrt{m_i m_j}} \right)^3 \int dE \, D_{ij}(E) \exp\left(-\frac{E}{T}\right)$$

with two-body density of states

$$D_{ij}(E) = \sum_k g_k^{(ij)} \delta(E - E_k^{(ij)}) + \sum_l \frac{g_l^{(ij)}}{\pi} \frac{d\delta_l^{(ij)}}{dE}$$

with contributions of bound states at energies $E_k^{(ij)} < 0$
and scattering states with phase shifts $\delta_l^{(ij)}$

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    with contributions of bound states at energies $E_k^{(ij)} < 0$
    and scattering states with phase shifts $\delta_l^{(ij)}$
  - corrections for Bose/Fermi statistics possible
  - with experimental bound state energies/phase shifts
    $\Rightarrow$ low-density behaviour of EoS established model-independently
    (e.g. C. J. Horowitz, A. Schwenk, Nucl. Phys. A 776 (2006) 55)

Theoretical Approaches - High Densities

phenomenological approaches

- nonrelativistic Hartree-Fock calculations
  with, e.g., Skyrme/Gogny interaction
- relativistic mean-field models
  with nonlinear meson self-interactions
  or density dependent meson-nucleon couplings

parameters fitted to properties of finite nuclei

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"ab-initio" approaches

with given microscopic nucleon-nucleon interaction, e.g.

- (non)relativistic Brueckner-Hartree-Fock calculations

two-body correlations in continuum included, but not bound states,

only homogeneous, isotropic matter

$\Rightarrow$ nucleon self-energies

Quantum Statistical Approach I

- nonrelativistic finite-temperature Green's function formalism
- starting point: nucleon number densities ($\tau = p, n$)

$$n_\tau(T, \tilde{\mu}_p, \tilde{\mu}_n) = 2 \int \frac{d^3k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \, f_\tau(\omega) S_\tau(\omega)$$ with Fermi distribution $f_\tau(\omega)$

  and spectral function $S_\tau(\omega)$ depending on self-energy Σ_τ
- expansion of spectral function beyond quasiparticle approximation

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$\Rightarrow$ **generalized Beth-Uhlenbeck descripton** with

- ◦ medium dependent self-energy shifts/binding energies
- ◦ generalized scattering phase shifts from in-medium T-matrix

- T, n_p, $n_n \Rightarrow \tilde{\mu}_p$, $\tilde{\mu}_n \Rightarrow$ free energy $F(T, n_p, n_n)$ by integration $\left(\left.\frac{\partial(F/V)}{\partial n_\tau}\right|_{T,n_{\tau'}} = \tilde{\mu}_\tau\right)$

  $\Rightarrow$ thermodynamically consistent derivation of EoS

Quantum Statistical Approach II

medium modifications

- single nucleon properties
  - self-energy shift of quasiparticle energy
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  (calculation with effective
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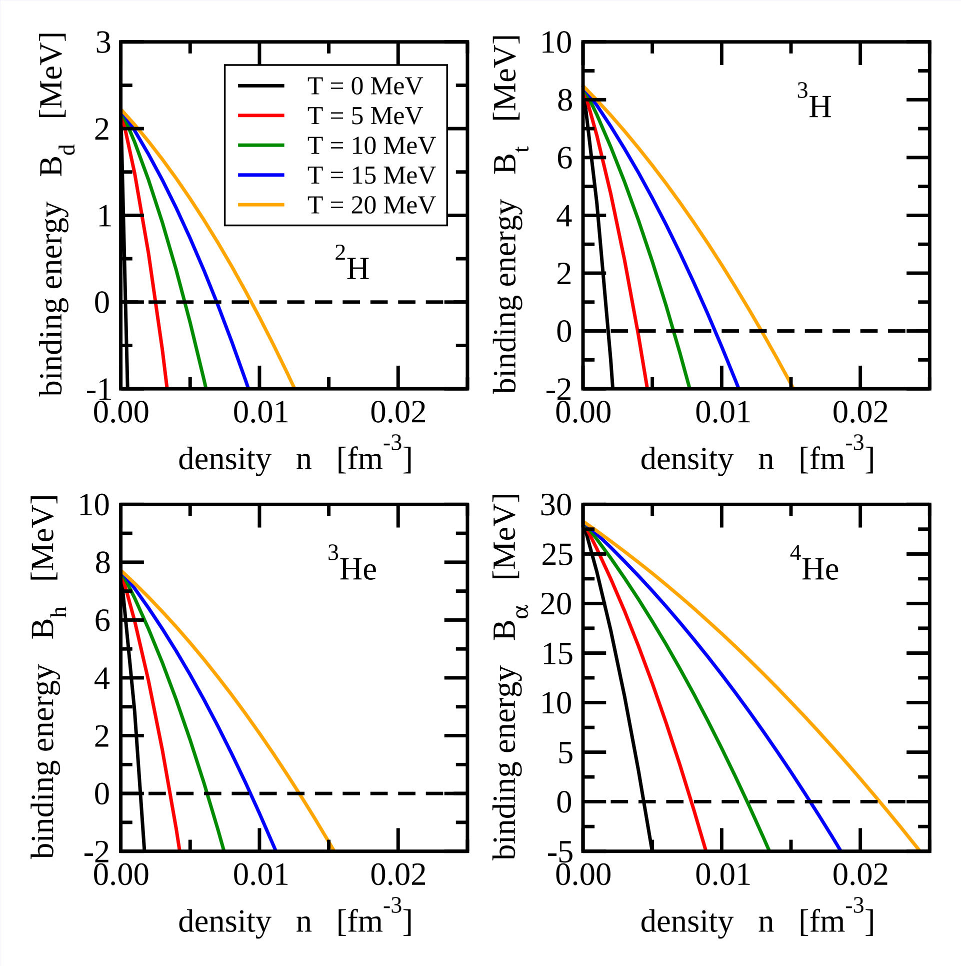
symmetric nuclear matter



parametrization used in generalized RMF model

Generalized Relativistic Mean-Field (RMF) Model

- extended **relativistic Lagrangian density** of Walecka type
  with nucleons (ψ_p, ψ_n), deuterons (φ_d^μ), tritons (ψ_t), helions (ψ_h), α-particles (φ_α), mesons ($\sigma, \omega_\mu, \vec{\rho}_\mu$), electrons ($\psi_e$) and photons ($A_\mu$) as degrees of freedom

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  - only minimal (linear) meson-nucleon couplings
  - density-dependent meson-nucleon couplings Γ_i
    - functional form as suggested by Dirac-Brueckner calculations of nuclear matter
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    - in total **10 free parameters** (highly correlated)
  - cluster binding energies
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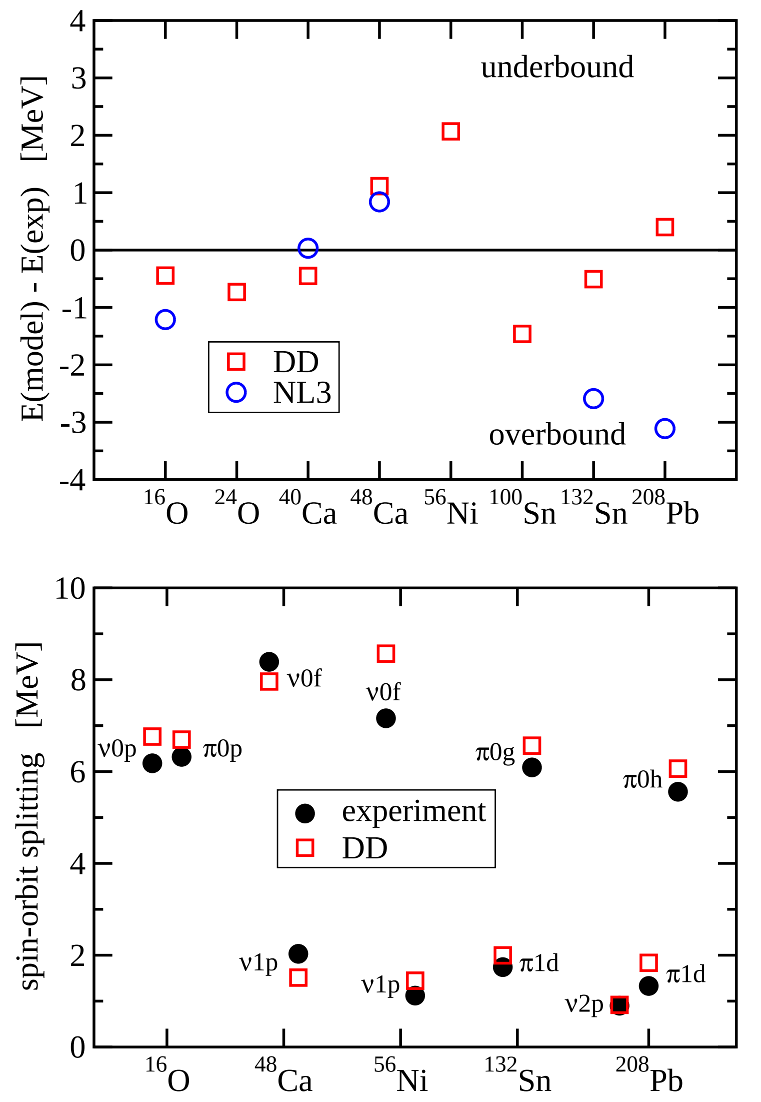
$\Rightarrow$ nucleon/cluster/meson/photon field equations,
solved selfconsistently in mean-field approximation
(Hartree approximation, no-sea approximation, classical meson/photon fields)

Generalized RMF Model - Constraints

- from **finite nuclei**
  - binding energies, spin-orbit splittings
  - charge/diffraction radii
  - surface/neutron skin thickness

  ⇒ couplings near nuclear saturation density

  (used in fit of present parametrization)



NL3: G.A. Lalazissis, J. König, P. Ring, Phys. Rev. C 55 (1997) 540

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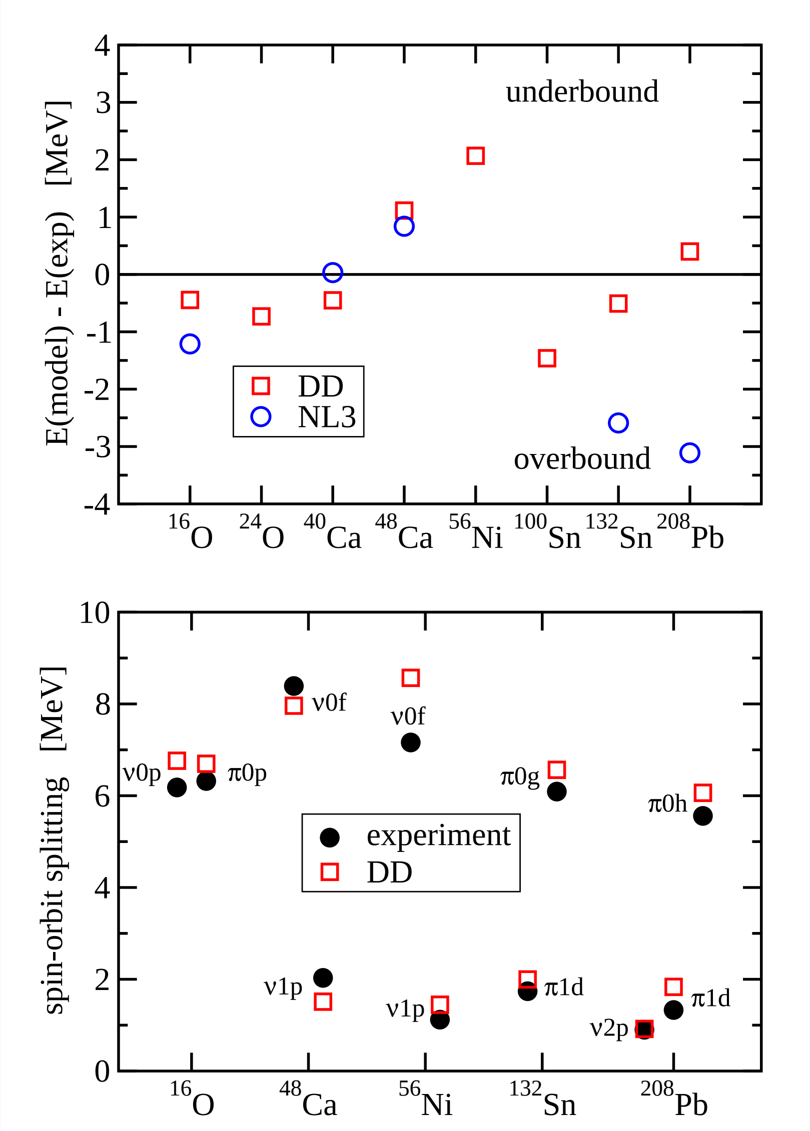
- from **nucleon-nucleon scattering**
  - low-energy s-wave phase shifts
  - scattering lengths
  - virial coefficients

  $\Rightarrow$ couplings at zero density

  (currently explored)



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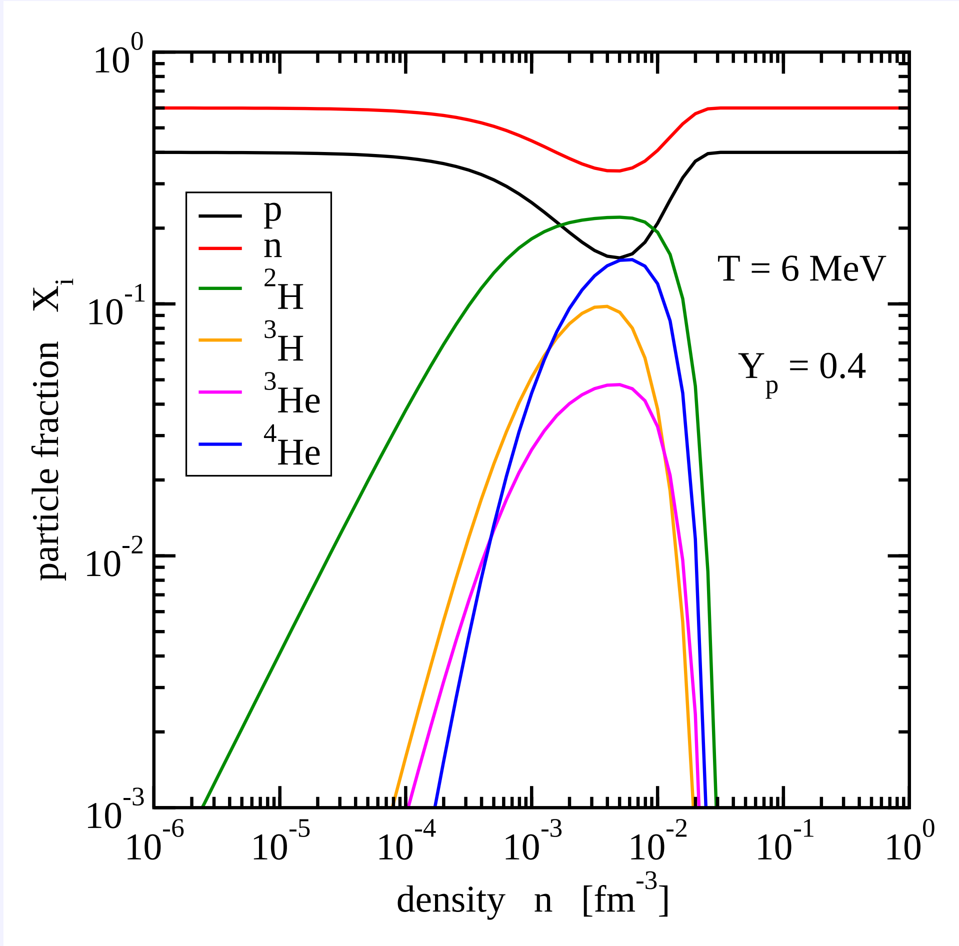
QS Approach vs. Generalized RMF Model

Relation and Differences of Models

Quantum Statistical Approach		Generalized RMF Model
empirical nucleon-nucleon potential $\Downarrow$ medium dependence of cluster binding energies	$\Rightarrow$	parametrization of binding energy shifts
parametrization of nucleon self-energy and effective mass in nonrelativistic approximation	$\Leftarrow$	phenomenological meson-nucleon interaction $\Downarrow$ scalar/vector nucleon self-energies
no effect of cluster formation on nucleon mean fields		medium-dependent change of cluster properties induces change of mean fields
correction for contribution of scattering states in deuteron channel		only cluster bound states

EoS with Light Clusters - Generalized RMF Model

- consider 2-, 3-, and 4-body correlations in the medium
  - presently only bound states (deuterons, tritons, helions, and alphas)
  - scattering contributions neglected so far
- Mott effect: clusters dissolve at high densities
- correct limits at low and high densities



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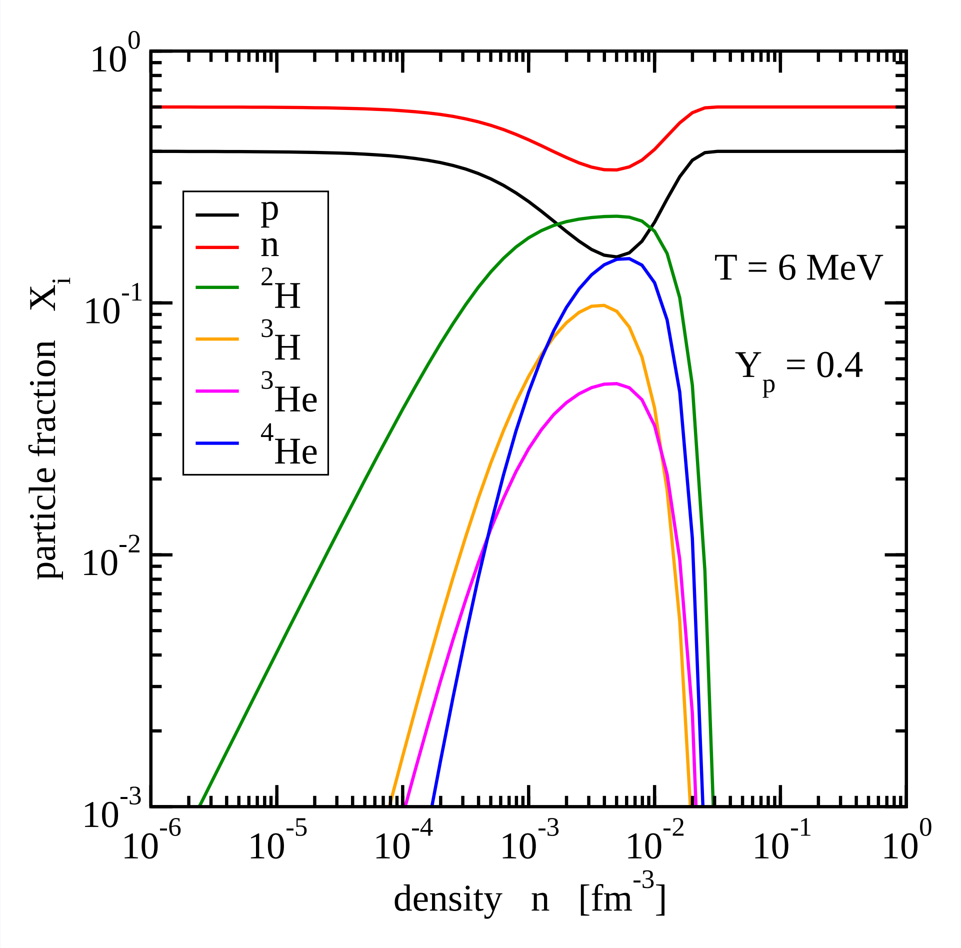
- no heavy clusters/phase transition included here



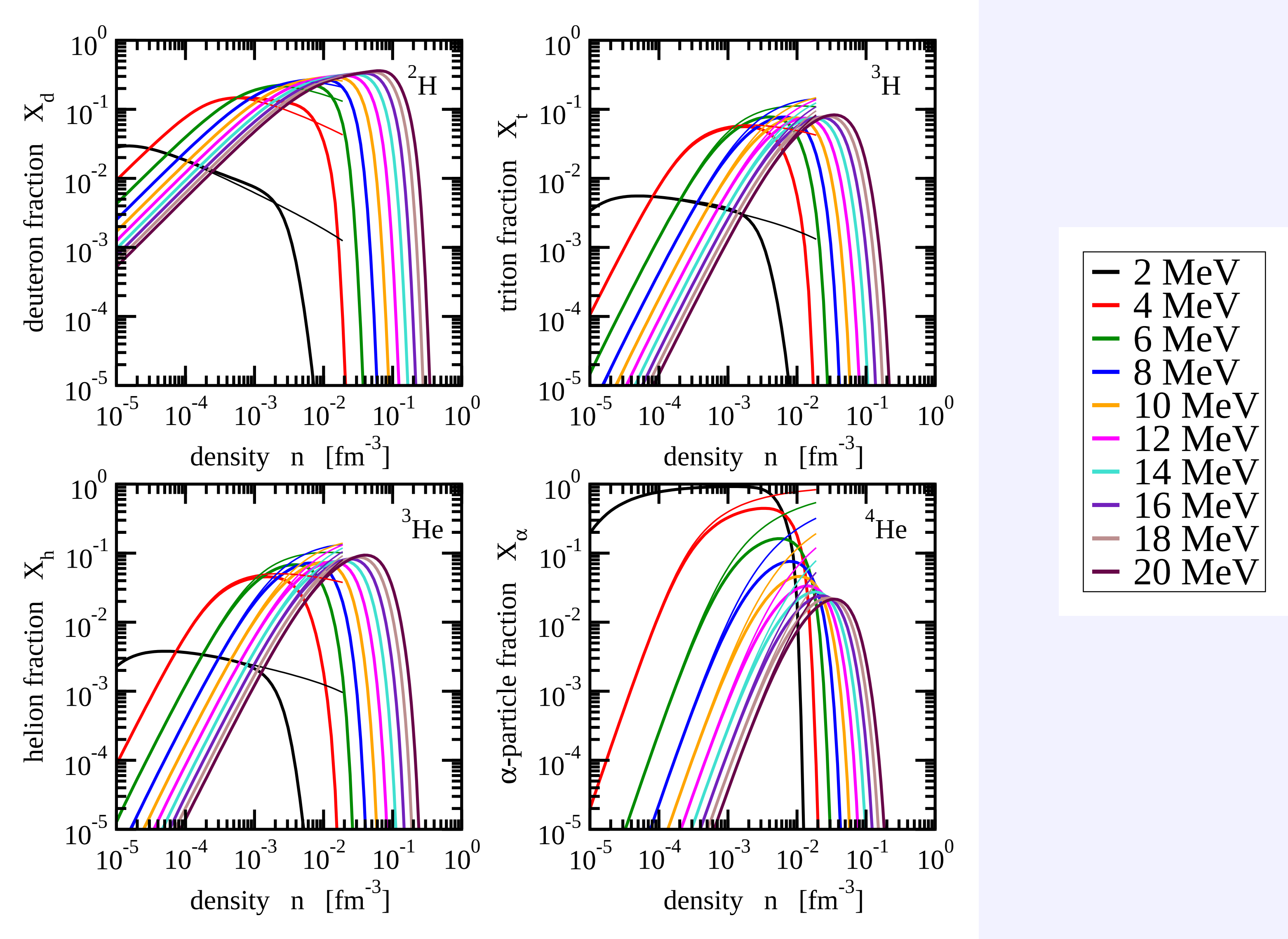
- medium dependence of couplings and binding energies

  ⇒ "rearrangement" contributions in self-energies and source densities essential for thermodynamical consistency

EoS with Light Clusters - Cluster Fractions

symmetric nuclear matter

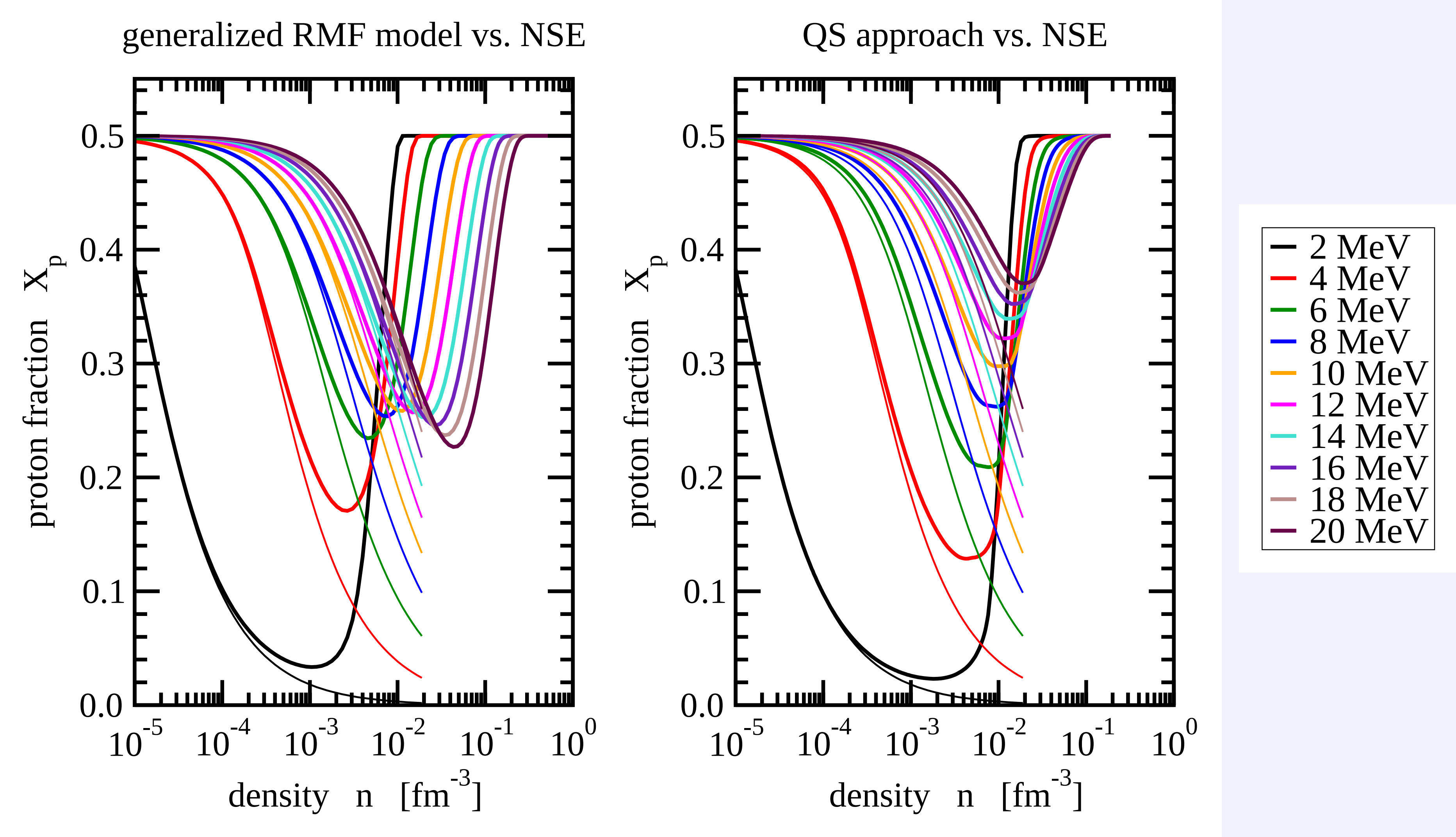
generalized RMF model vs. NSE (thin lines)



EoS with Light Clusters - Fraction of Free Protons

symmetric nuclear matter

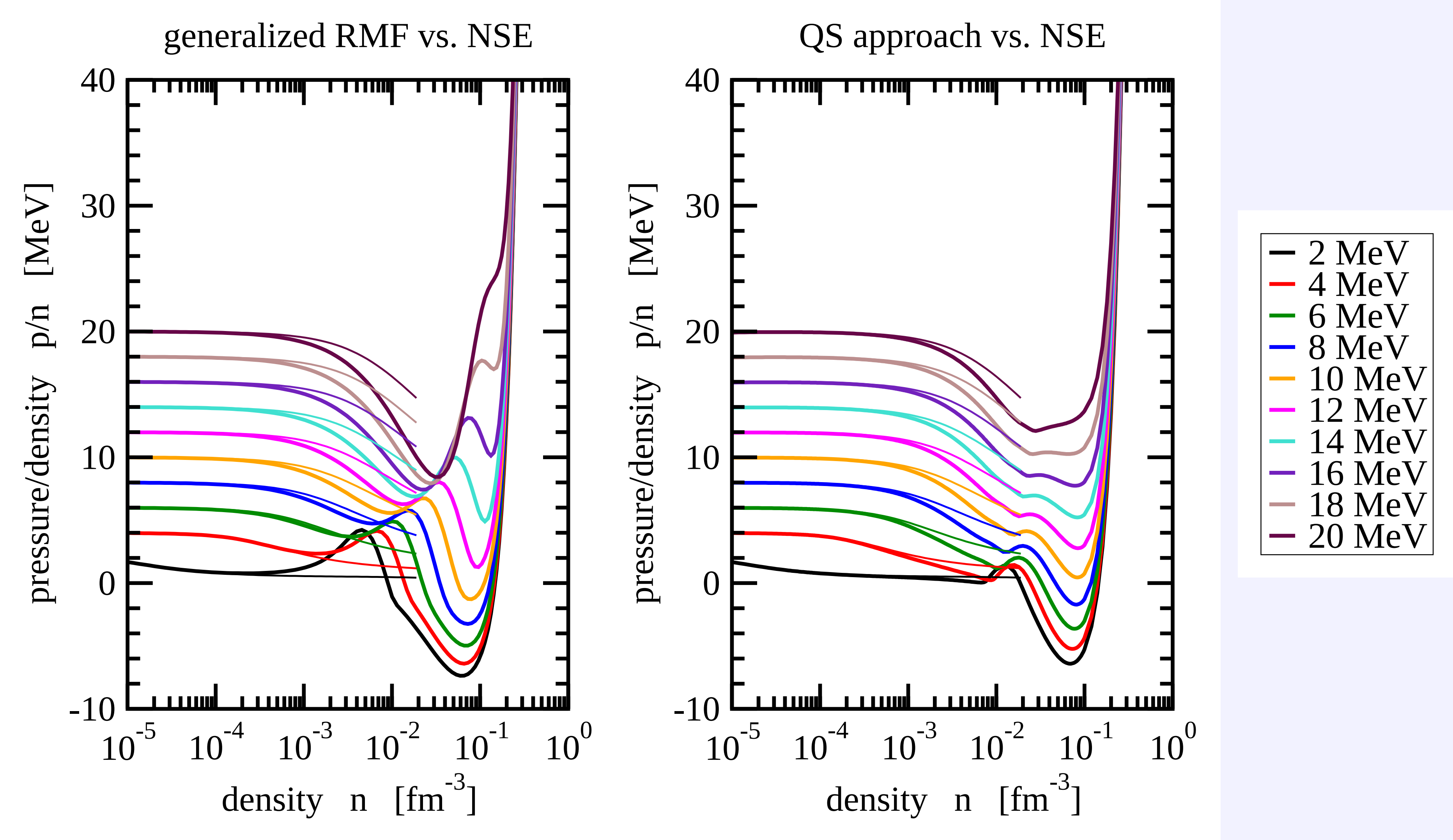
thin lines: NSE



EoS with Light Clusters - Pressure/Density

symmetric nuclear matter

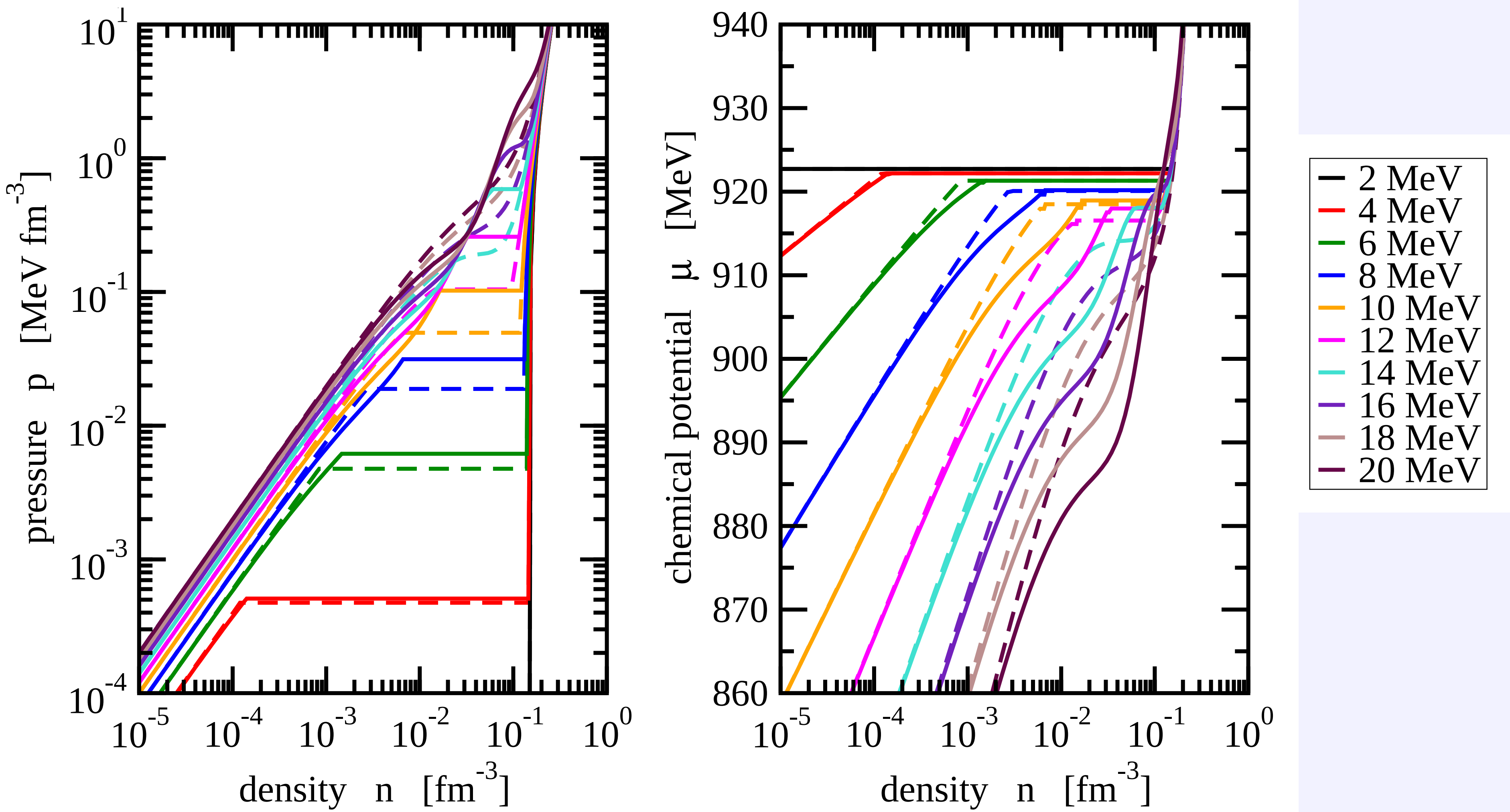
$\lim_{n\to 0}(p/n) = T$ (ideal gas)



Phase Transition - Pressure and Chemical Potential

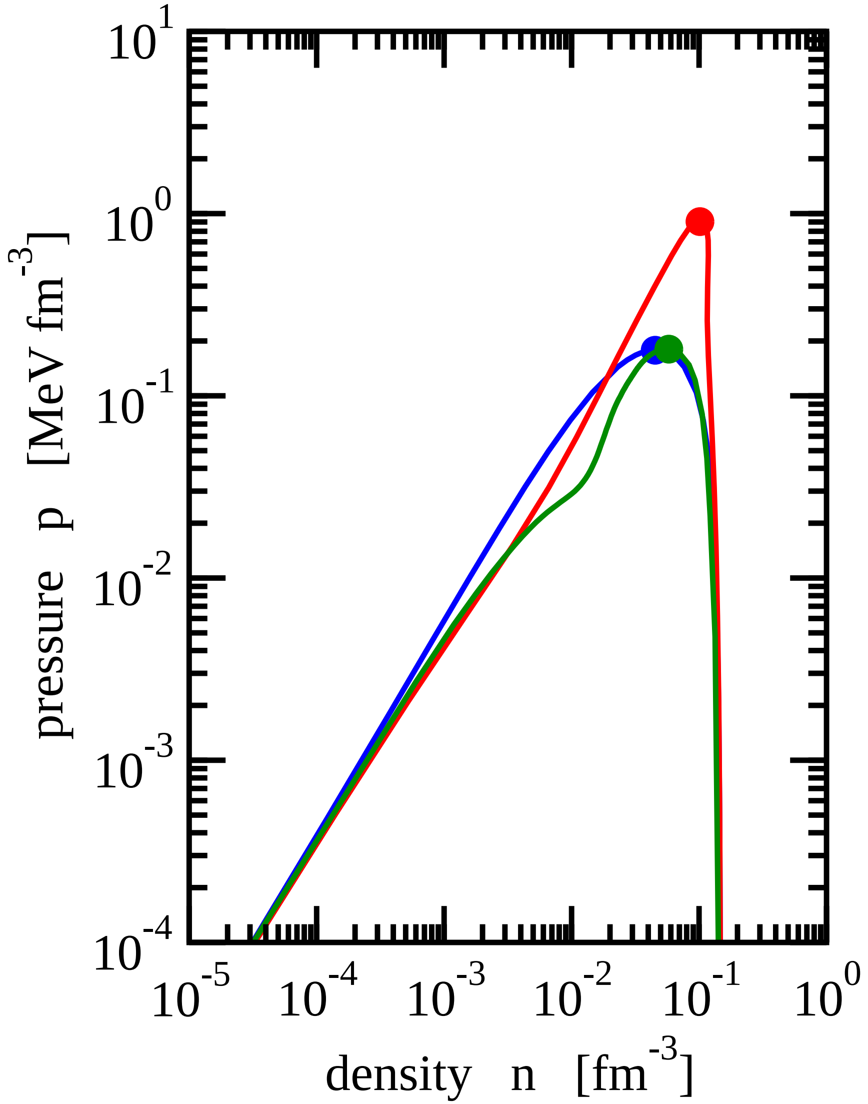
symmetric nuclear matter (Maxwell construction sufficient)

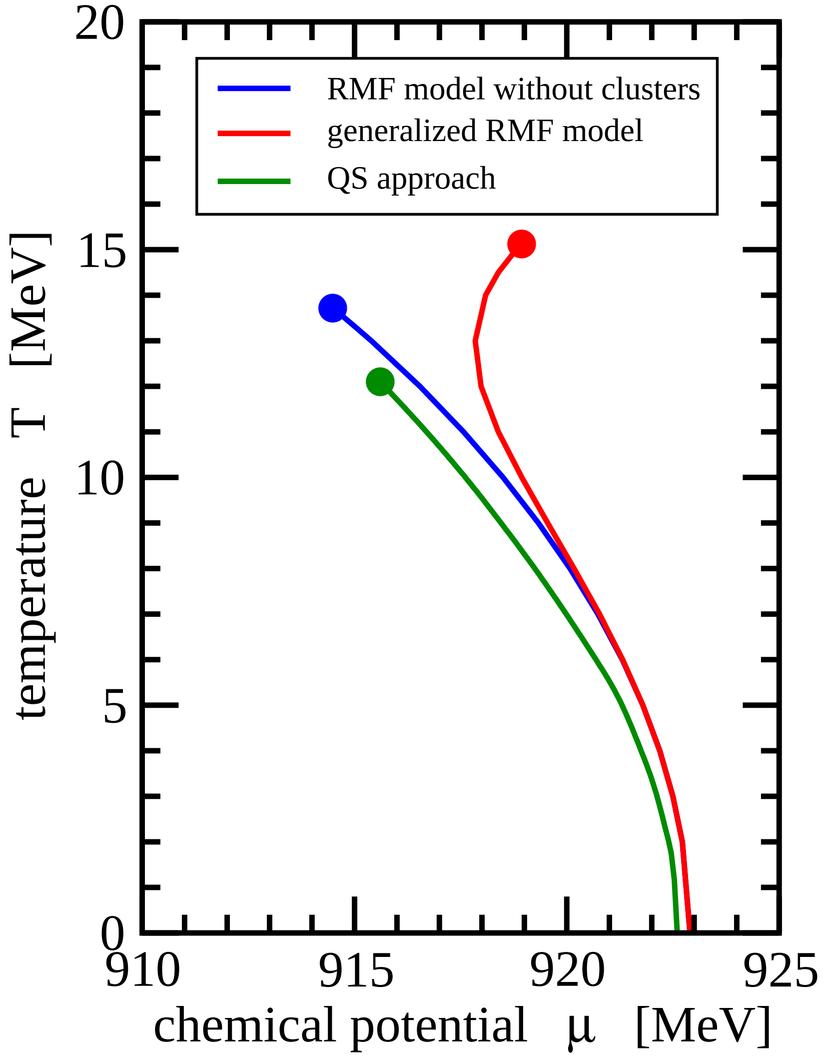
RMF model without (dashed lines) and with (solid lines) clusters



Phase Transition - Binodals and Phase Diagram

symmetric nuclear matter





Heavy Clusters

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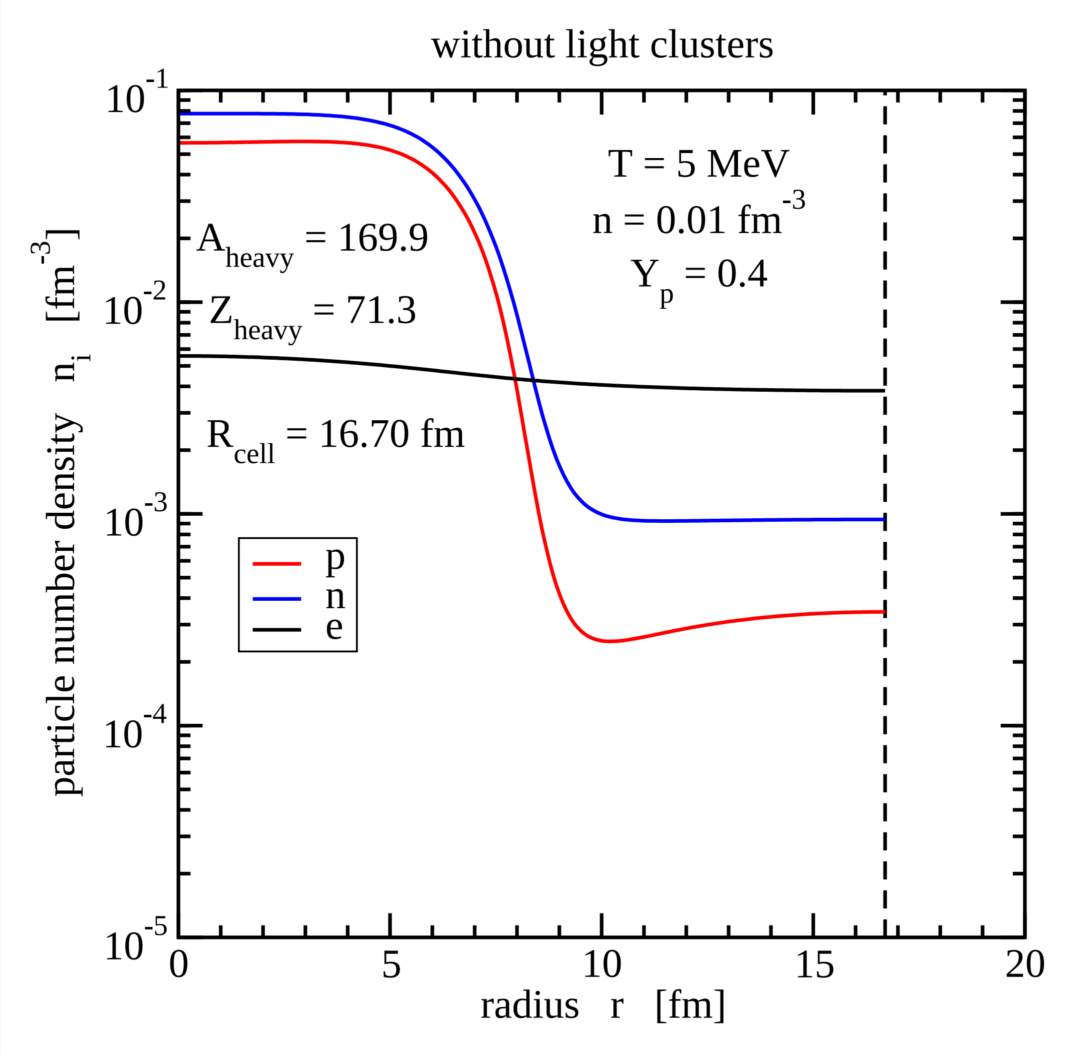
- first step in improvement:
  spherical Wigner-Seitz cell calculation
  - generalized RMF model
  - Thomas-Fermi approximation
  - electrons for charge compensation
  - heavy nucleus surrounded by
    gas of nucleons and light clusters



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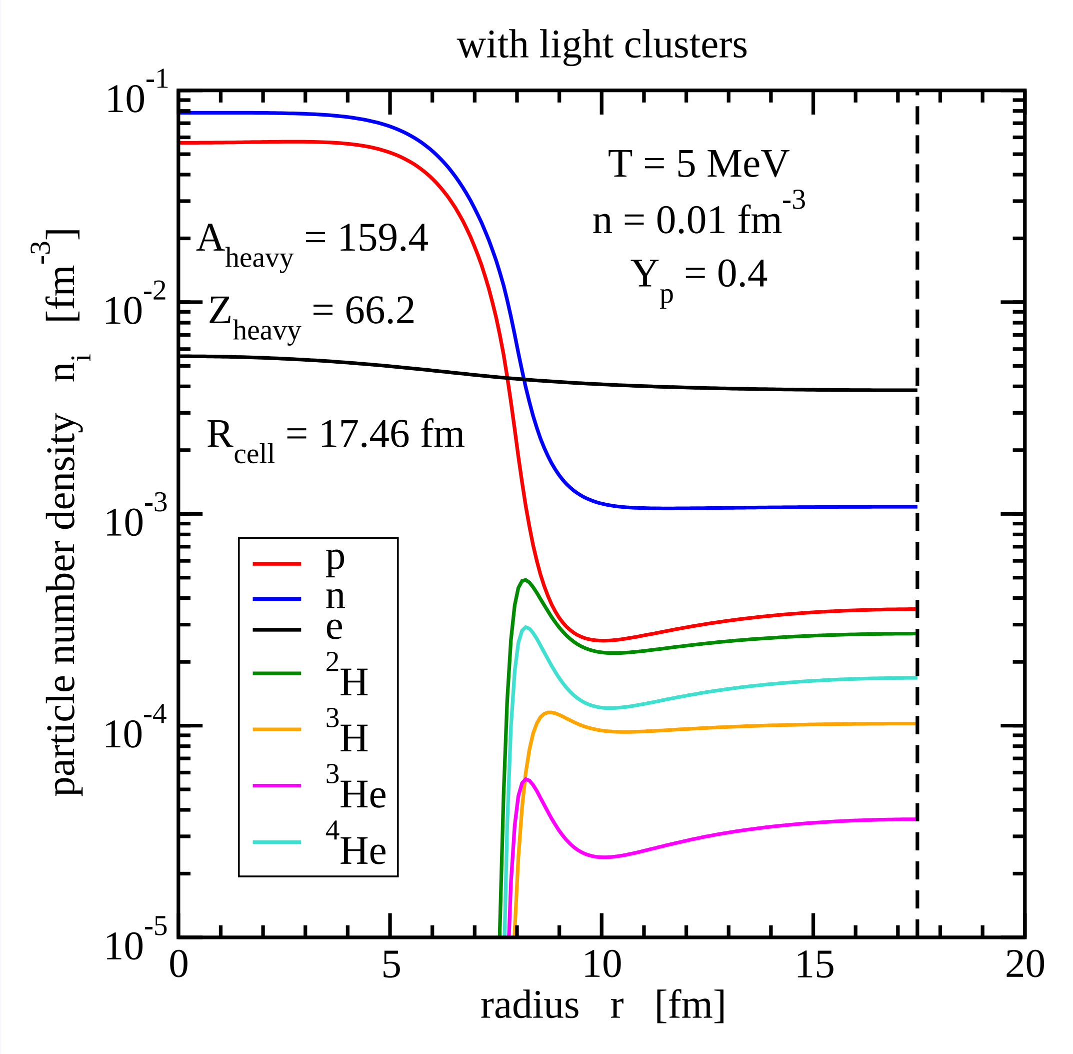
- first self-consistent calculation with
  interacting nucleons, light clusters
  and electrons



Symmetry Energy I

- general definition for zero temperature:

$$E_s(n) = \frac{1}{2}\frac{\partial^2}{\partial\beta^2}\frac{E}{A}(n,\beta)\Big|_{\beta=0} \qquad \beta = \frac{n_n-n_p}{n_n+n_p}$$

$\Rightarrow$ nuclear matter parameters

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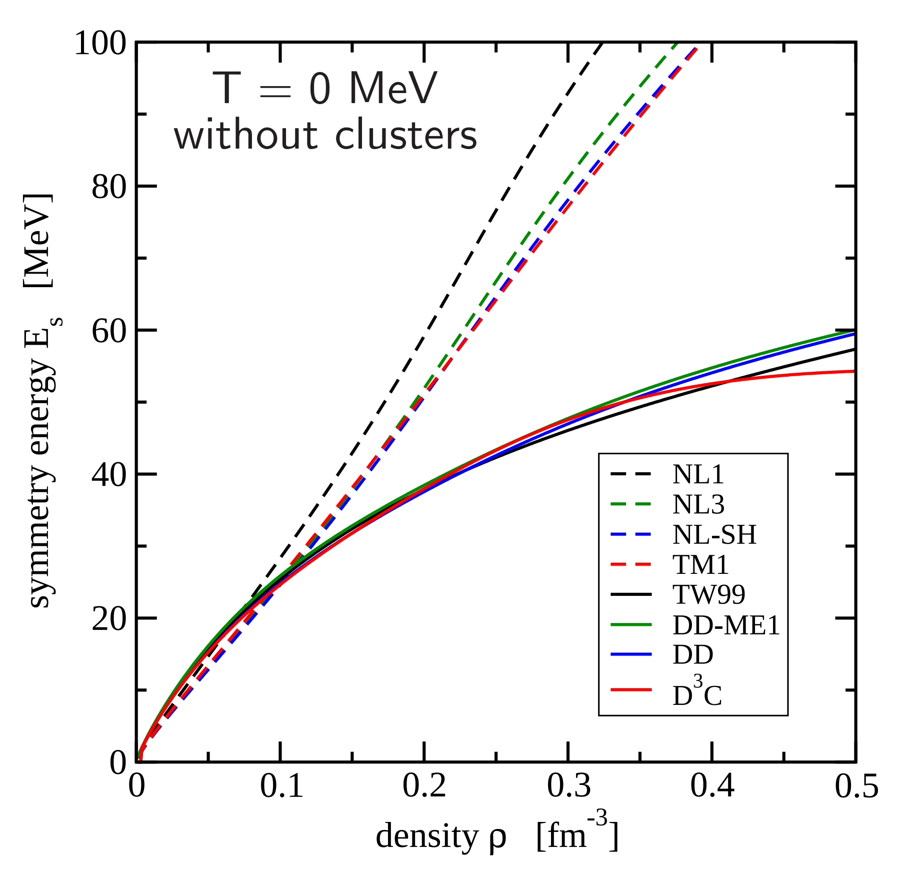
- correlation: neutron skin thickness

$\Leftrightarrow$ slope of neutron matter EoS ($\Leftrightarrow L$)

B. A. Brown, Phys. Rev. Lett. 85 (2000) 5296,

S. Typel, B. A. Brown, Phys. Rev. C 64 (2001) 027302



Symmetry Energy I

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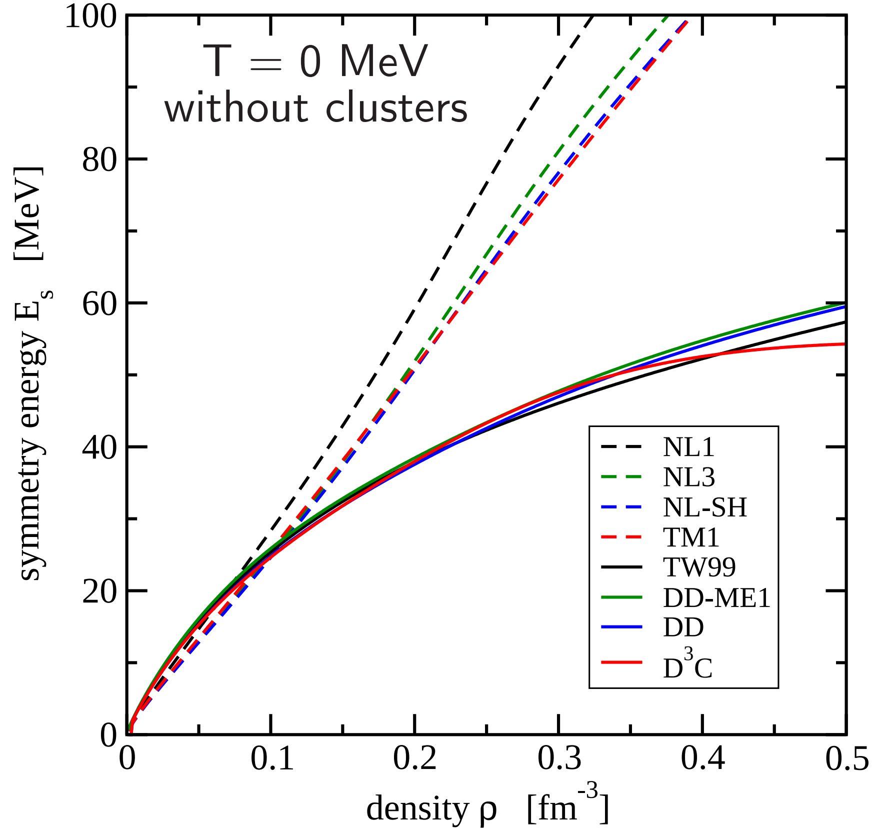


- with clusters and at finite temperatures:
  - ○ use finite differences

$$E_{\text{sym}}(n) = \frac{1}{2}\left[\frac{E}{A}(n,1) - 2\frac{E}{A}(n,0) + \frac{E}{A}(n,-1)\right]$$

  - ○ distinguish free symmetry energy F_{sym} and internal symmetry energy E_{sym}

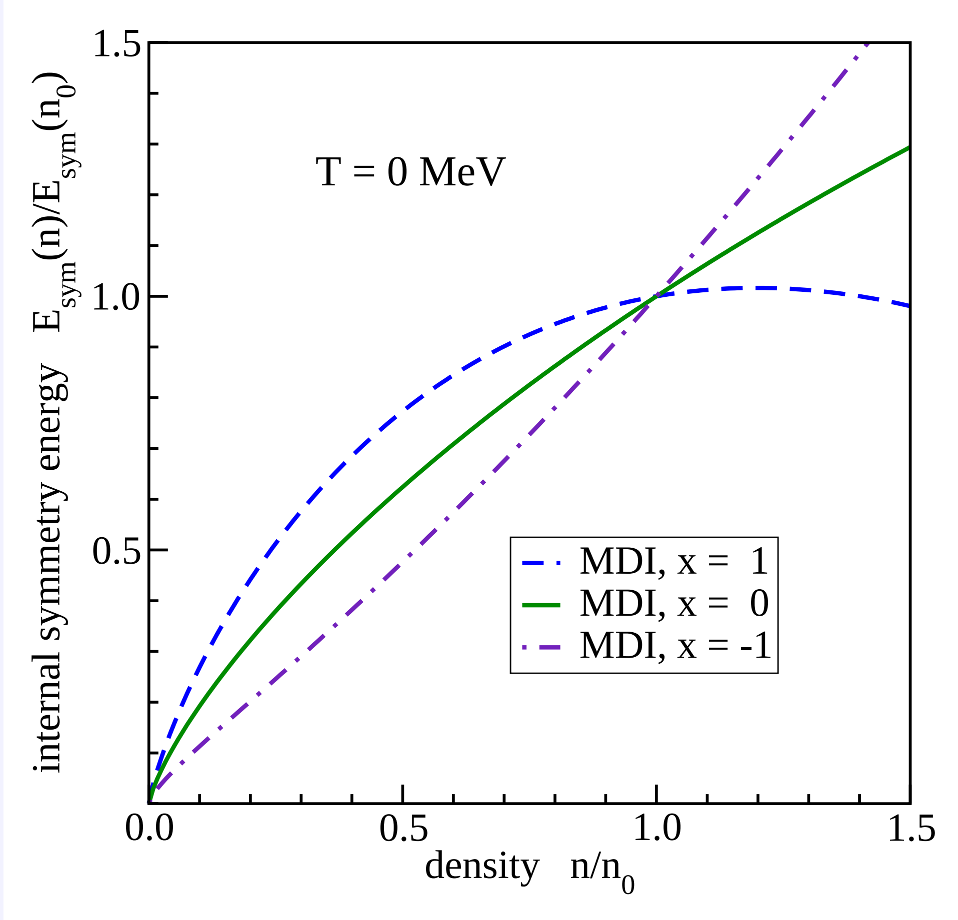
Symmetry Energy II

temperature $T = 0$ MeV

- mean-field models without clusters

  e.g. model with momentum-dependent interaction (MDI), parameter x controls density dependence of E_{sym}
  (B. A. Li et al., Phys. Rep. 464 (2008) 113)

  $\Rightarrow$ low-density behaviour not correct



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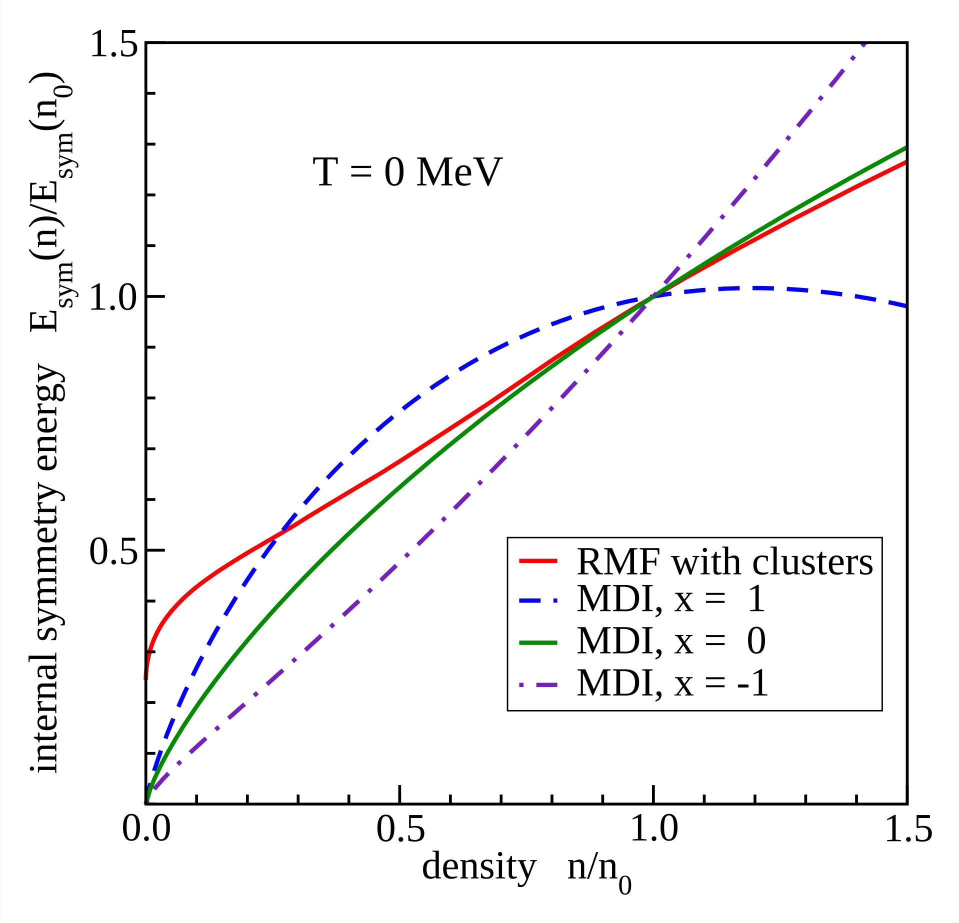
- RMF model with (heavy) clusters

  $\Rightarrow$ increase of E_{sym} at low densities due to formation of clusters

  $\Rightarrow$ finite symmetry energy in the limit $n \to 0$



Symmetry Energy III

finite temperature

- experimental determination of symmetry energy
  - heavy-ion collisions of ^{64}Zn on ^{92}Mo and ^{197}Au at $35\ A$ MeV
    temperature, density, free symmetry energy derived as functions of parameter $v_{\rm surf}$ (measures time when particles leave the source)
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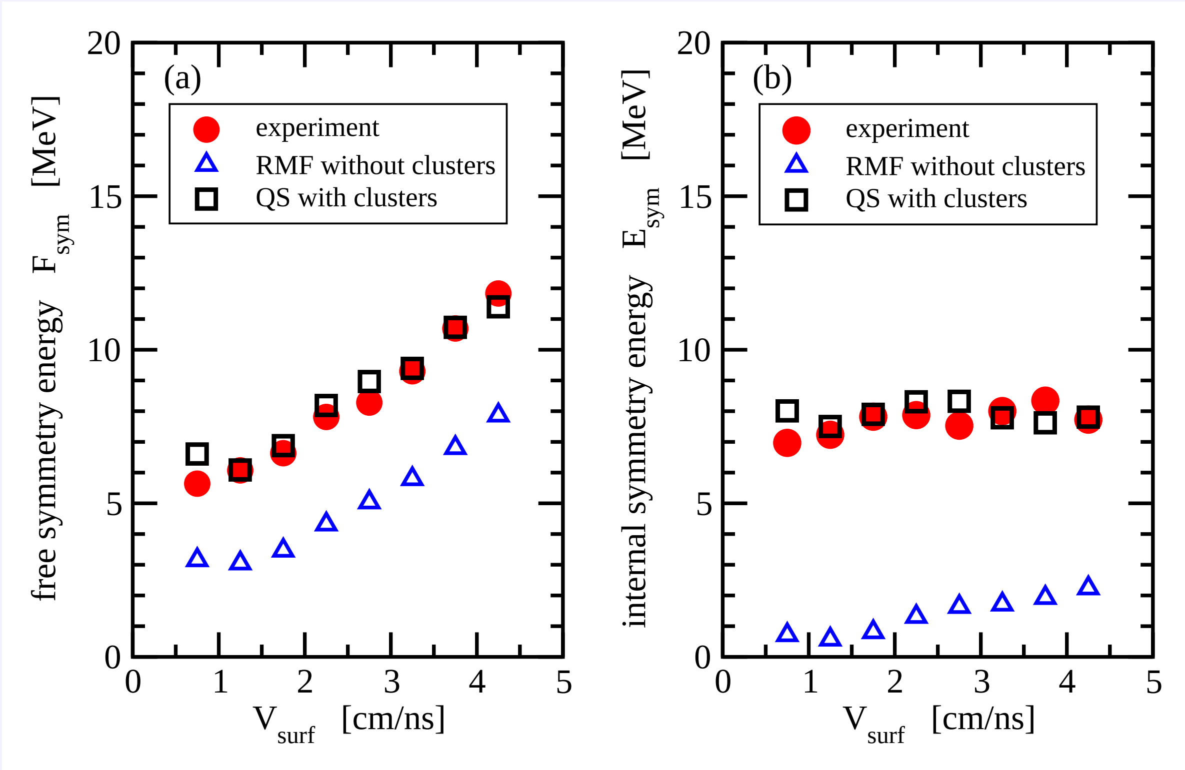
- symmetry energies in RMF calculation without clusters are too small

- very good agreement with QS calculation with light clusters



Summary and Outlook

- **theoretical models of EoS with clusters**
  - quantum statistical approach (QS)
  - generalized relativistic mean-field model (gRMF)
  - both thermodynamically consistent
  - correct limits at low and high densities
  - difference in details

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  - formation of clusters with medium dependent properties
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- **future**
  - improvement of RMF parametrization (low-density limit)
  - include formation of heavy clusters fully
  - application to astrophysical models