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Desired Mechanical Accuracy of CR Plunging Electrodes

1 Description of Task

It was asked how precisely the plunging system of the CR stochastic cooling systems should work. First the signal from a pick-up and the response of kickers are discussed in a general way. Then implications of misalignment for the gain factor and the Schottky noise effect are derived.

2 Pick-ups and Kickers

2.1 Pick-up signal

The pick-up signal appears at harmonics $m\omega$ of the revolution frequency ω and at betatron sidebands $(m \pm Q_{xy})\,\omega$, where Q_{xy} is one of the Q values. Signals at higher order sidebands are usually negligible.

The voltage amplitude at the longitudinal harmonics can be written

$$V(m\omega) \propto S_p(x, y, m\omega) \tag{1}$$

where S_p is a complicated function of the particle position with respect to the electrode and of frequency. It has to be determined either from field calculations or from measurements. The sensitivity of the stripline electrodes for the CR is discussed below.

The signal at the horizontal betatron sidebands is proportional to the betatron amplitude A_x:

$$V\left([m \pm Q_x]\,\omega\right) \propto A_x \frac{\partial S_p}{\partial x} \tag{2}$$

and similarly for the vertical sidebands.

2.2 Kicker response

The change of momentum at a kicker electrode has the form

$$\delta p \propto S_k \tag{3}$$

and the change of emittance (at zero dispersion) is approximately

$$\delta A_x \propto \frac{1}{\omega} \frac{\partial S_k}{\partial x} \tag{4}$$

Here S_k is a function which is similar to the pick-up response. In fact, if pick-ups and kickers have the same geometry, the pick-up and kicker sensitivities have the same functional behaviour (up to a factor 1/2). This relationship is called reciprocity.

We now assume that at the mean position x of a particle the sensitivity can be linearized and is independent of y (this makes it easier but is otherwise unimportant)

$$S_k(x, y, \omega) = \bar{S}_k(\omega) + xS'_k(\omega) \tag{5}$$

At zero dispersion, the constant part of S does not change emittance, nor does the gradient part of S change momentum.

3 Measured sensitivities

The sensitivity of a slotline was measured by means of a field mapper. The measurements were made on paths along the beam direction above the middle of the slotline at different vertical heights (see 1). The result of integrated field measurements along the beam direction is plotted for three different frequencies in 2.

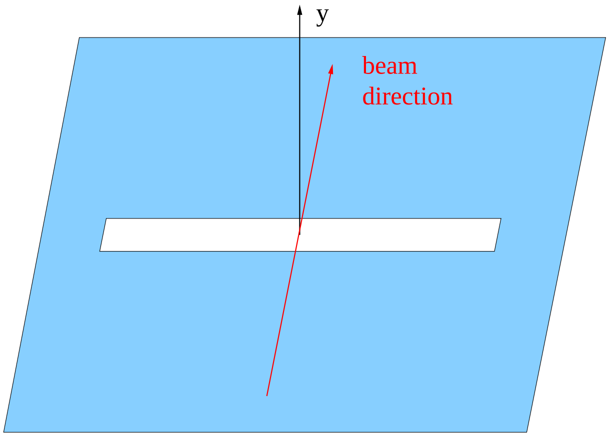

Figure 1: Sketch of path of sensitivity measurement

The measurements show that the slope of the sensitivity can be regarded as constant over a large range of betatron amplitudes. This will be the conceptual basis of the following discussion.

4 Effects of plunging and misalignment

The essential purpose of plunging is to maintain a high signal to noise ratio during the whole cooling cycle. This is important because the correction signal decreases while the momentum width and the emittance are getting smaller due

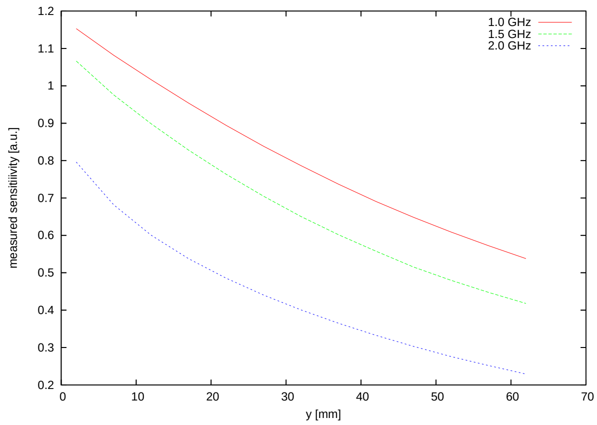

Figure 2: Measured sensitivity data

to cooling. The signal is maximized by minimizing the geometrical distance between the slotline and the beam.

In a device where both plates are operated in sum mode (longitudinal pick-up for notch filter cooling, longitudinal kicker) a misalignment of the plates has the minor effect of changing the system gain, but it does not introduce large heating.

Therefore only misalignments of transverse electrodes will change the cooling efficiency qualitatively. We assume that the zero of the difference sensitivity will be shifted by some error δ, resulting in a perturbed sensitivity

$$S(\delta) = S'\,(x - \delta) = S'x - S'\delta \tag{6}$$

i.e. the device will have an additional sum mode sensitivity $\sigma = -S'\delta$. In a pick-up this will produce additional signal at the longitudinal harmonics. This signal can lead to two effects at the transverse kicker.

1. At the beginning of the cooling cycle there can be overlap between the longitudinal and transverse bands. Only under this condition can the additional signal increase the transverse diffusion rate.

2. If there is no more any band overlap the additional signal will give rise to longitudinal heating only if there is an analog misalignment at the kicker. Hence this effect is essentially quadratic in the mean misalignment and should be negligible.

We now give the condition that the longitudinal signal from particles with momentum deviation $+\delta p/p$ begins to overlap with the transverse signal from

	η	Q_x	f_{rev}	$\delta p/p$ after debunching	$\delta p/p$ for overlap at 1 GHz	at 2 GHz
antiprotons	-0.015	4.62	1.38	$3.5 \cdot 10^{-3}$	$1.7 \cdot 10^{-2}$	$8.7 \cdot 10^{-3}$
rare isotopes	0.189	3.23	1.18	$2.0 \cdot 10^{-3}$	$7.2 \cdot 10^{-4}$	$3.6 \cdot 10^{-4}$

Table 1: Conditions for overlap between longitudinal and transverse bands

particles with momentum deviation $-\delta p/p$ (see fig. 3):

$$\frac{\delta p}{p} = \begin{cases} \dfrac{q_x}{2m\eta} & (q_x < 0.5) \\ \dfrac{1-q_x}{2m\eta} & (q_x > 0.5) \end{cases} \tag{7}$$

Here q_x is the fractional part of the Q_x value.

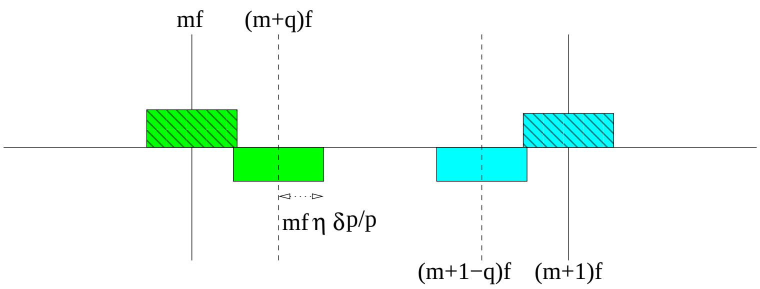

Figure 3: Band overlap condition

In the case of CR cooling, this condition is given in table 1. The overlap condition is never met for antiprotons. For the rare isotope beams, there will be substantial overlap which will finish only after a decrease of the momentum width (due to parallel momentum cooling) by a factor of 2.8 at the lower band limit and even 5.6 at the upper band limit.

In order to estimate the heating effects more quantitatively, we reexamine the power spectra involved (see [2]). These power spectra determine how large the transverse diffusion will be. The power spectra are so-called voltage power spectra, defined as the Fourier transform of the voltage autocorrelation. They are measured in units of V^2/Hz. The longitudinal power spectra are

$$C_L = \frac{(Z_l q e)^2 f_{\text{rev}}}{4|m\eta|} |S|^2 \Psi(\delta p/p) \tag{8}$$

measured at the longitudinal harmonics corresponding to a momentum deviation $\delta p/p$. Ψ is the distribution function of off-momenta $\delta p/p$, normalized to the number of particles in the beam.

The horizontal power spectra are

$$C_H = \frac{(Z_l qe)^2 f_{\mathrm{rev}} \beta_x \langle \epsilon_x(\delta p/p) \rangle}{16|m\eta|} \left| \frac{\partial S}{\partial x} \right|^2 \Psi(\delta p/p) \tag{9}$$

Here, $\langle \epsilon_x(\delta p/p) \rangle$ is the average emittance, calculated using all particles in the beam with a momentum deviation $\delta p/p$ corresponding to the betatron sideband frequency.

In order to get a reasonable estimate for the plunging error we look at the ratio C_L/C_H for transverse cooling, where the longitudinal sensitivity S is given by the plunging error δ (see eq. 6). We make the very simplifying assumption that the the momentum distributions at the momentum deviations responsible for the longitudinal and transverse harmonics are equal, as sketched in fig. 3. Then

$$\frac{C_L}{C_H} = \frac{4\delta^2}{\beta_x \langle \epsilon_x \rangle} \tag{10}$$

The most stringent condition arises when one takes the overlap condition at the upper band limit assuming that the emittance decrease is as fast as the decrease of the momentum width (transverse cooling tends to proceed more slowly). At this point, the initial emittance of $200 \cdot 10^{-6}$ m would have decreased by a factor of 5.6 to $36 \cdot 10^{-6}$ m, according to table 1. Furthermore we make the conservative assumption that the beta function at the pick-up is only 10 m (it is somewhat larger in all pick-ups). Also it is certainly sufficient to require $C_L/C_H < 1/4$. Then the additional diffusion rate due to the plunging error would be just one quarter of the rate due to transverse Schottky noise. This yields the condition

$$\frac{1}{4} > \frac{4\delta^2}{36 \times 10^{-5}\,\mathrm{m}^2} \tag{11}$$

or

$$|\delta| < 5\,\mathrm{mm} \tag{12}$$

This condition should be very easy to meet.

5 Concluding remarks

Misalignments of the plunging electrodes will give rise to substantial additional transverse heating only if they are of the order of 5 mm. Longitudinal heating effects are negligible.

The increased sensitivity due to the close distance between electrodes and beam will always be beneficial for the signal to noise ratio.

What is more important is to avoid vibrational excitations of the mechanically sensitive alumina structures which are cooled to 20 K. This requirement leads to so-called jerk-free movement profiles. These movements are presently studied experimentally in the existing test tank. Three-dimensional accelerometers are installed to measure the residual vibrations.

Apart from electrode misalignment there is another important issue which has to be unter good control during stochastic cooling: dispersion at the kickers, which introduces coupling between longitudinal and transverse cooling. If the residual dispersion at the kickers becomes too large, the equilibrium emittances and the equilibrium momentum spread will grow. Therefore a good control of this parameter is inevitable.

References

[1] FAIR technical design report

[2] F. Nolden et al., Applications of Schottky Spectroscopy at the Storage Ring ESR of GSI, Proc. Cool05, AIP Proc. 821 (2006) 211-220