Dimensional crossover in ultracold Fermi gases

from Functional Renormalisation

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Hirschegg, Austria

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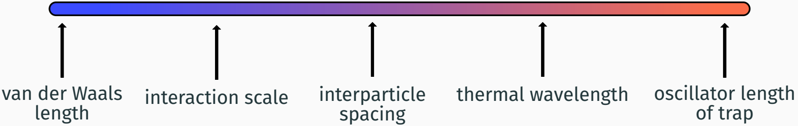
Physics of ultracold atoms

Scale hierarchy & Hamiltonian

adapted from Boettcher et al. Nuclear Physics B (2012)



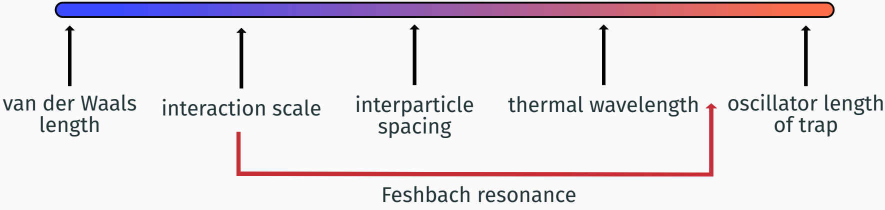
Effective Hamiltonian valid on scales $\gg \ell_{\text{vdW}}$:

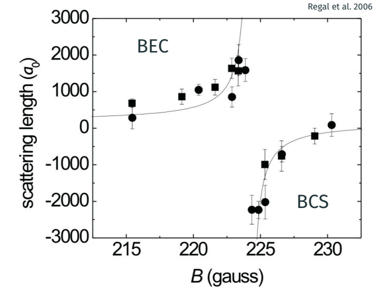
$$\hat{H} = \int_{\vec{x}} \left[\hat{a}^\dagger(\vec{x}) \left(-\frac{\hbar\,\Delta}{2\,M} + V_{\text{ext}}(\vec{x}) \right) \hat{a}(\vec{x}) + g_\Lambda\, \hat{n}(\vec{x})^2 \right]$$

with $\hat{n} = \hat{a}^\dagger\, \hat{a}$ and $g_\Lambda = \frac{4\,\pi\,\hbar^2}{M}\, a$

Feshbach resonances

adapted from Boettcher et al. Nuclear Physics B (2012)



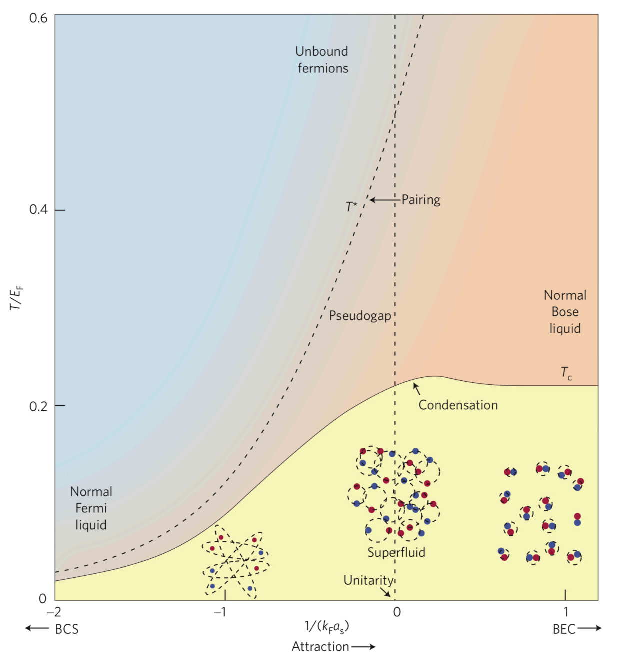


$$\nu(B) = \Delta\mu\,(B - B_0)$$

$$a = a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

with $\nu \to 0$ @ resonance

The BCS-BEC crossover in 3D



$k_F = (3\,\pi^2\,n)^{1/3}$

Randeria Nature (2010)

BCS-BEC physics from Functional Renormalisation

Action and truncation

[I. Boettcher, S. Diehl, F. Floerchinger, H. Gies, M. Scherer, J. M. Pawlowski, C. Wetterich, ...]

Microscopic action

$$S = \int_X \left\{ \psi^* \left(\partial_\tau - \Delta - \mu\right) \psi + \phi^* \left(\partial_\tau - \frac{\Delta}{2} + \nu - 2\mu\right) \phi - h \, \left(\phi^* \, \psi_1 \, \psi_2 - \phi \, \psi_1^* \, \psi_2^*\right) \right\}$$

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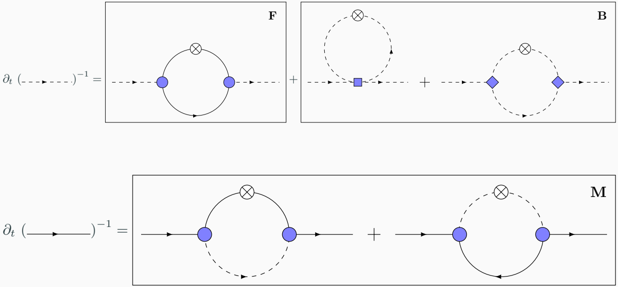

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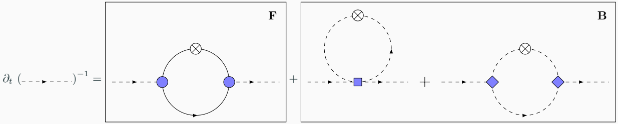
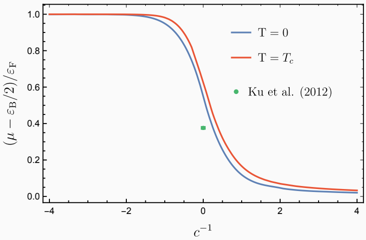


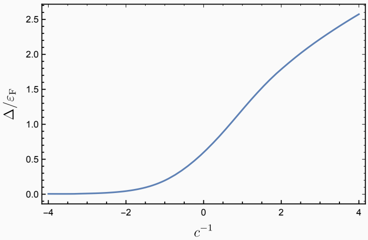
- Truncation: derivative expansion
- Litim-type regulator (fermions: regularise around Fermi surface)

$$U(\rho) = \sum_{n=1}^{2} \frac{u_n}{n!} \left(\rho - \rho_0\right)^n - n_k \left(\mu - \mu_0\right) + \alpha \left(\mu - \mu_0\right) \left(\rho - \rho_0\right)$$

3D BCS-BEC crossover

$T = 0$: (with $c = k_F a$)





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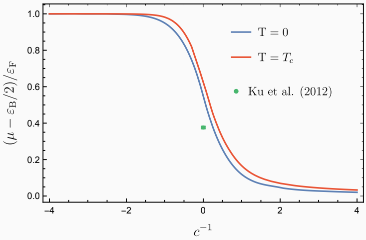
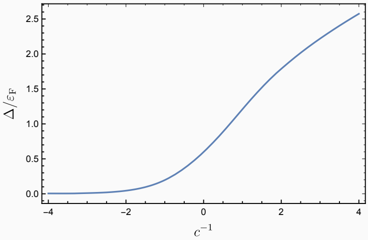
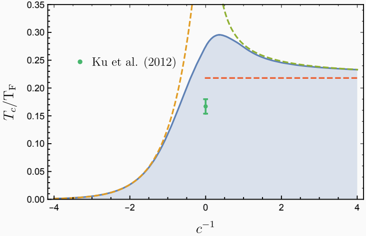




$T > 0$:



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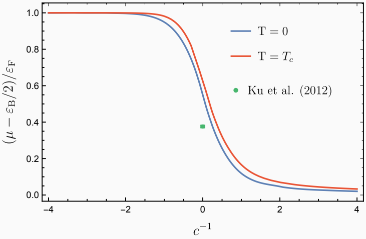

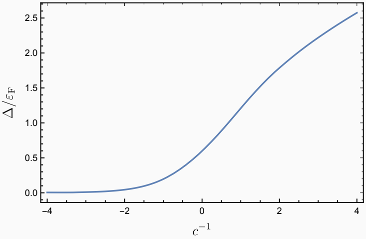

$\xi_{\text{exp}} = 0.376(5), \quad \xi_{\text{calc}} \simeq 0.55$

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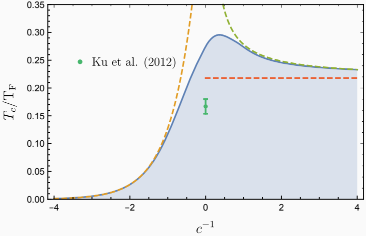
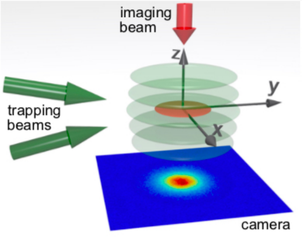


Dimensional crossover

Why quasi-2D systems?

- promising materials: graphene, high T_c-superconductors, layered semiconductors
- pronounced influence of quantum fluctuations
- experimental accessibility via highly anisotropic trapping potentials
- for insufficient anisotropy $\longrightarrow$ dimensional crossover

$\longrightarrow$ disentangle dimensionality from many-body physics



[Bosons: S. Lammers, I. Boettcher, C. Wetterich (2016)]

Zürn et al. PRL (2015)

Boundary conditions

- delimit z-direction by potential well of length L

$$V_{\text{box}}(z) = \begin{cases} 0 & 0 \le z \le L \\ \infty & \text{else} \end{cases}$$

- impose **periodic** boundary conditions: $\Psi = \{\psi, \phi\}$

$$\Psi(\tau, x, y, z = 0) = \Psi(\tau, x, y, z = L)$$

$\longrightarrow$ quantisation of momentum in z-direction:

$$q_z \to k_n = \frac{2\,\pi\,n}{L}, \qquad n \in \mathbb{N}$$

- spatial Matsubara sum

$$\int \frac{d^d q}{(2\,\pi)^d} = \frac{1}{L} \sum_{k_n} \int \frac{d^{d-1} q}{(2\,\pi)^{d-1}}$$

Regulator and method

Litim-type regulator (as before) with $\vec{q} = \hat{\vec{q}} + q_z \to \hat{\vec{q}} + k_n$

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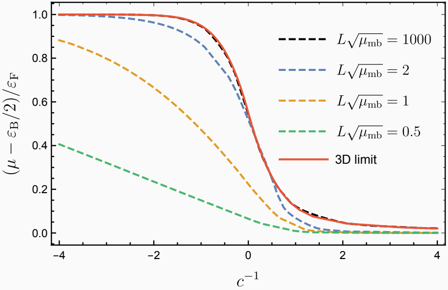
Idea:

- initialise RG flow at UV scale $k = \Lambda$ where $\Gamma_\Lambda = S_{3D}$
- L introduces new length scale to 3D system
- following RG flow successively integrates out 3rd dimension

Zero temperature [BFC, J. M. Pawlowski, C. Wetterich in preparation]

- correct 3D-limit



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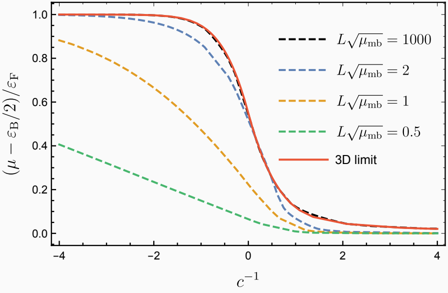
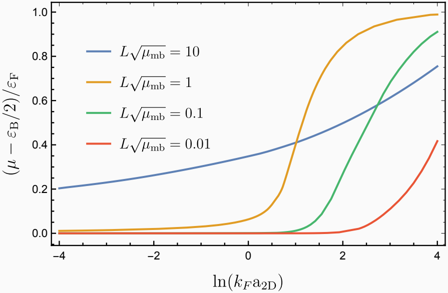


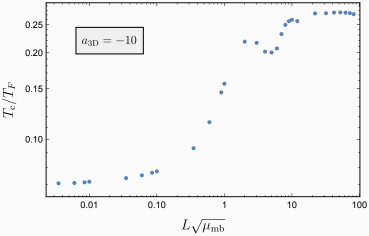
- quasi-2d scattering length:

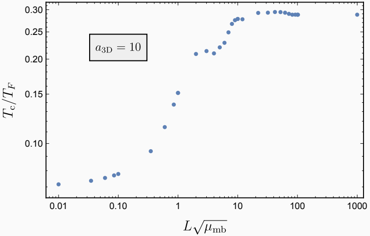
$$a_{2D}^{(\text{pbc})} = L \exp\left(-\frac{1}{2}\,\frac{L}{a_{3D}}\right)$$

- qualitatively correct behaviour of equation of state (except very small confinements)



Finite temperature & phase diagram





[cf. A. Fischer and M. Parish (2014) for mean-field study (BCS-limit)]

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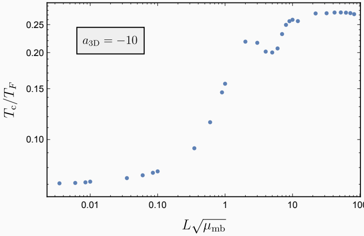

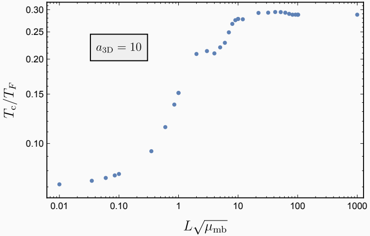

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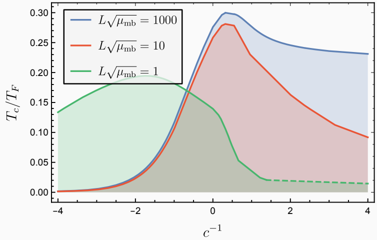

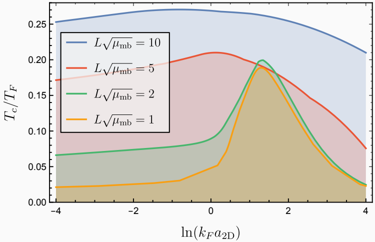

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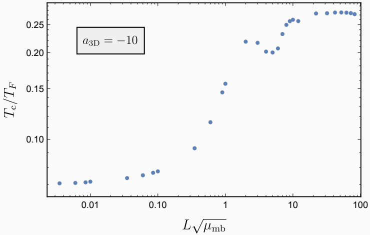

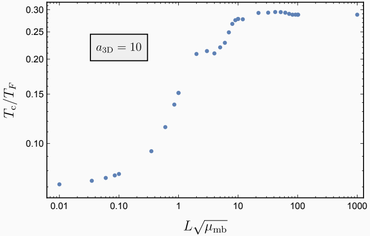

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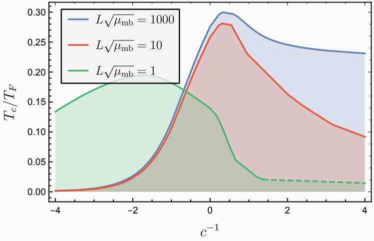
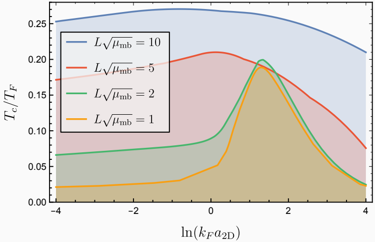
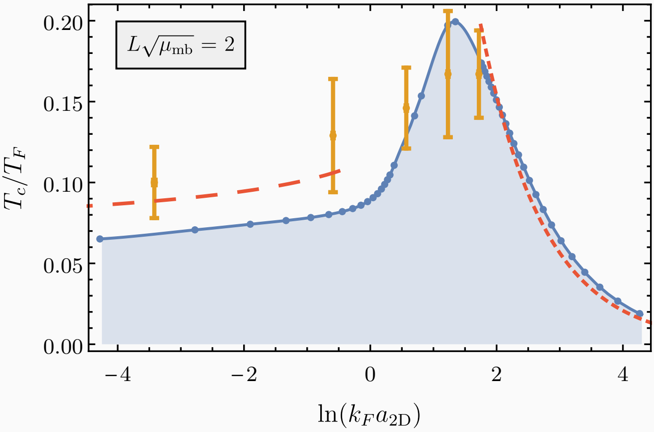




- 3D phase diagram reproduced for large L
- increased T_c/T_F around $\ln(k_F a_{2D}) \sim 1$

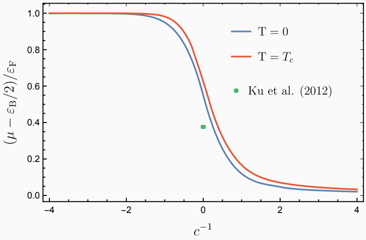
Comparison to experiment

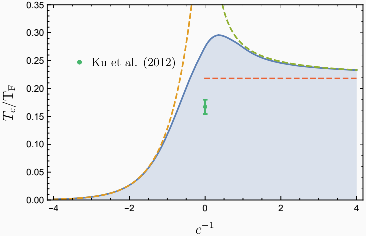


experimental data from Ries et al. PRL (2015)

Determination of the density

μ-dependence of initial conditions





μ-dependence of initial conditions

Free energy density, f, normalised in the vacuum,

$$\partial_t f(T,\mu) := \left(\frac{\partial_t \Gamma_k[\phi_{\mathrm{EoS},k}; T, \mu]}{\mathcal{V}_T} - \frac{\partial_t \Gamma_k[\phi_{\mathrm{EoS},k}; 0, 0]}{\mathcal{V}_0} \right) ,$$

has polynomial growth with k,

$$\partial_t f(T,\mu) \to c_{d_f-2} k^{d_f} \hat{k}^{-2} + c_{d_f-4} k^{d_f} \hat{k}^{-4} + k^{d_f} O(\hat{k}^{-6}) \,.$$

with $\hat{k} = k/\sqrt{\mu}$ (non-rel.), $\hat{k} = k/\mu$ (rel).

$\longrightarrow$ initial conditions for couplings g_i with scaling dimension $d_{g_i} \geq 2$ μ-dependent,

i.e. fermionic mass parameter **(including the density itself)**

Density [W. Fu, J. M. Pawlowski (2015)]

- For non-relativistic case, equation of density (from its flow)

$$\partial_t n = \frac{1}{\mathrm{Vol}} \frac{d\,\partial_t \Gamma_k}{d\mu} \to c_{n,3} k^3 + c_{n,1} \mu\, k + O(\hat{k}^{-1})\,,$$

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- while flow of second μ-derivative tends to zero

$$\partial_t \partial_\mu^2 n = \frac{\partial_\mu^3 \partial_t \Gamma_k}{\text{Vol}} \to O(\hat{k}^{-1})\,.$$

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- Write flow of density as

$$\partial_t n_k = \frac{d\partial_t \Gamma_k}{d\mu} = \partial_\mu\Big|_{\vec{g}} \partial_t \Gamma_k + \frac{dg_i}{d\mu} \partial_{g_i} \partial_t \Gamma_k\,.$$

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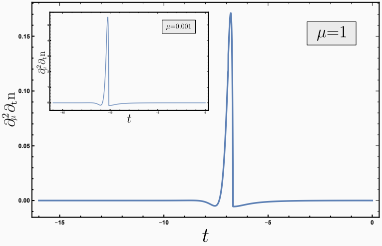
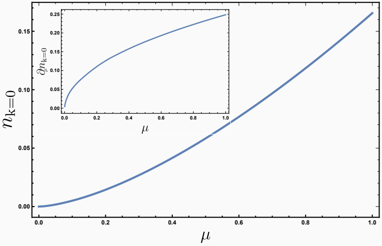
- Each partial μ- and $dg_i/d\mu\,\partial_{g_i}$-derivative lowers effective k-dimension by two.
- The coefficients $g_i^{(1)} = dg_i/d\mu$ follow from their flow
- Can be iteratively extended to higher derivatives

Preliminary results [BFC, J. M. Pawlowski, C. Wetterich in preparation]

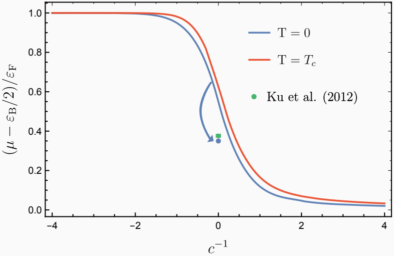
3d BCS-BEC crossover ($T = 0$):





Bertsch parameter: $\xi = \frac{\mu}{\varepsilon_F}|_{a\to\infty}$

- $\xi_{\text{exp}} = 0.376(5)$ [Zwierlein et al. (2012)]
- Preliminary: $\xi_{\text{calc}} \simeq 0.35$



Summary and Outlook

Conclusion

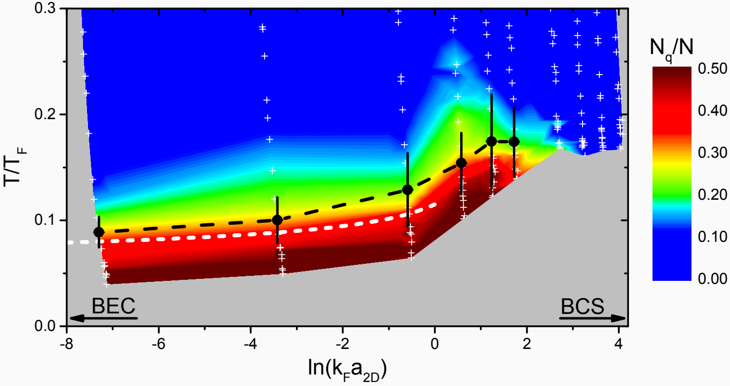
- dimensional crossover in Fermi gas with FRG
- qualitatively comparable to experiments
- preliminary results for density

Outlook

- iterative calculation of density for whole crossover (3d, 2d-3d)
- employ harmonic trapping potential
- explore scale anomaly in dimensional crossover

Thank you for your attention!

Phase diagram from experiment



Ries et al.
PRL (2015)