QCD physics beyond the Faddeev-Popov model

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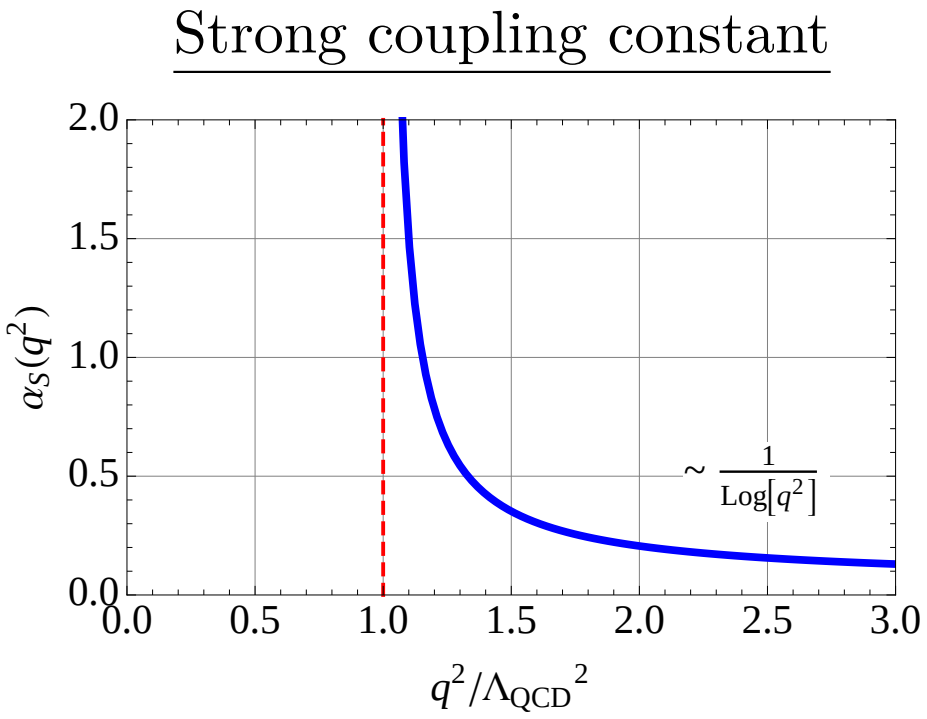
Disclaimer

This talk is an invitation to look at the low energy properties of QCD from a different perspective.

Main message to be conveyed: unlike what is usually accepted to be the standard lore, the pure gauge sector of the theory could be accessible through perturbative methods and some QCD properties from a small parameter(s) expansion.

The aim is not to compete with functional methods but to bring a complementary view on certain aspects of the problem.

Standard lore



Perturbation theory is well justified at high energies but looses its meaning at low energies.

Based on this observation, two families of nonperturbative approaches have been deployed to tackle QCD:

- (discrete) lattice simultations;
- (continuum) functional methods.

In fact ...

Perturbation theory requires choosing a gauge.

In practice: one does not work with the original QCD action but with a gauge-fixed version of it, obtained via the Faddeev-Popov procedure

$$S_{QCD} \to S_{FP} = S_{QCD} + \delta S_{GF}$$

This procedure is not free of ambiguities. Therefore, the failure of the standard perturbative expansion at low energies may be due either to the perturbative expansion itself or to the Faddeev-Popov procedure.

Outline

1. Beyond Faddeev-Popov: why and how?
2. The Curci-Ferrari model.
3. Yang-Mills phase structure.
4. QCD phase structure in the heavy-quark regime.
5. QCD phase structure with light quarks.

1. Beyond Faddeev-Popov: why and how?

Faddeev-Popov gauge fixing

Consider (Euclidean) Yang-Mills theory:

$$S_{YM}[A] = \frac{1}{4g^2} \int_x F^a_{\mu\nu} F^a_{\mu\nu} \,, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu$$

It is invariant under gauge transformations:

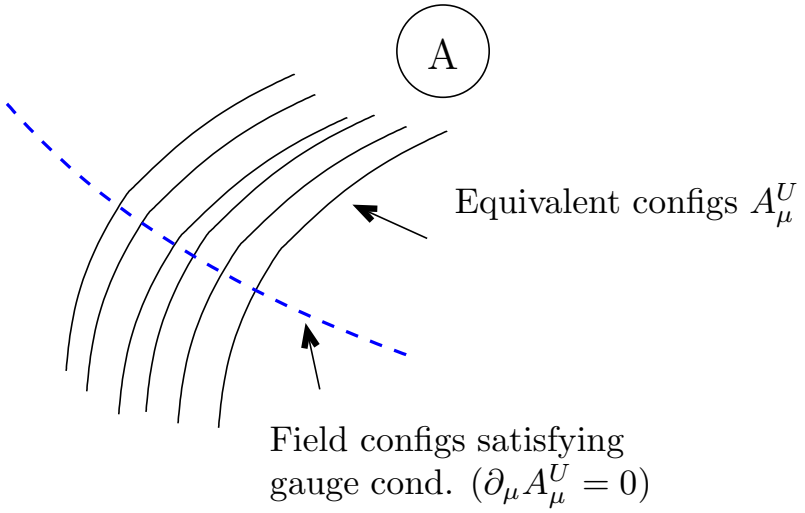
$$S_{YM}[A^U] = S_{YM}[A] \quad \text{with} \quad (A^a_\mu t^a)^U = U A^a_\mu t^a U^\dagger + i U \partial_\mu U^\dagger$$

Faddeev-Popov gauge fixing

Observables are expectation values of gauge-invariant functionals:

$$\langle \mathcal{O}[A] \rangle = \frac{\int \mathcal{D}A\, \mathcal{O}[A]\, \exp\{-S_{YM}[A]\}}{\int \mathcal{D}A\, \exp\{-S_{YM}[A]\}} \quad \text{with} \quad \mathcal{O}[A^U] = \mathcal{O}[A]$$

Redundant integration over the orbits of the gauge group:



Faddeev-Popov gauge fixing

In the Landau gauge ($\partial_\mu A^a_\mu = 0$) considered in this talk, one obtains

$$\begin{aligned}\mathcal{D}A &\to \mathcal{D}A\,\delta(\partial_\mu A_\mu)\det(-\partial_\mu D_\mu)\\ &= \mathcal{D}A\int \mathcal{D}[c,\bar{c},h]\,\exp\left\{-\int_x ih^a\partial_\mu A^a_\mu + \bar{c}^a\partial_\mu D_\mu c^a\right\}\end{aligned}$$

The original action S_{YM} is replaced by the gauge-fixed action

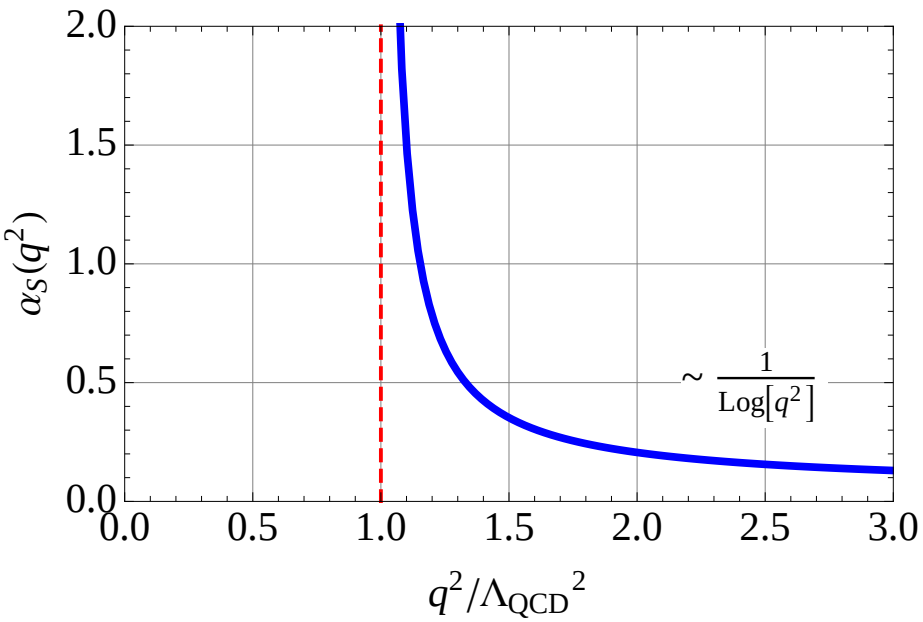
$$S_{FP} = S_{YM} + \int_x \left\{ih^a\partial_\mu A^a_\mu + \bar{c}^a\partial_\mu(D_\mu c)^a\right\}$$

Faddeev-Popov gauge fixing

Gauge symmetry is explicitly broken but is replaced by another powerful symmetry:

$$sA^a_\mu = D_\mu c^a,\ sc^a = \frac{1}{2} f^{abc} c^b c^c,\ s\bar{c}^a = ih^a,\ sih^a = 0 \quad \text{[BRST]}$$

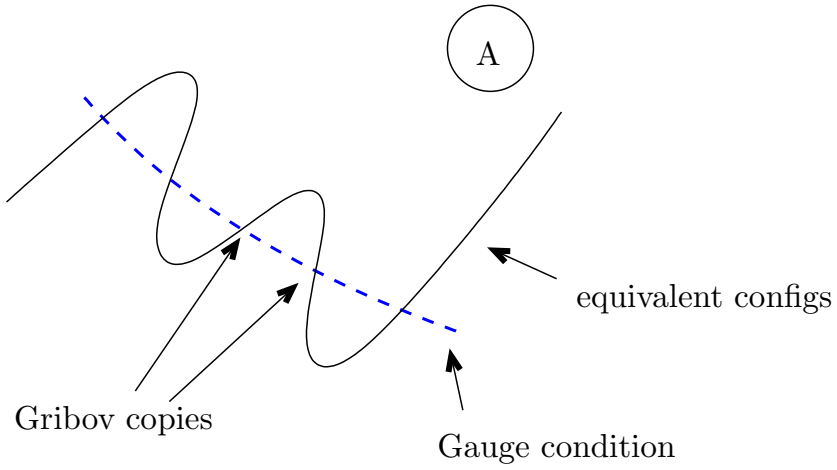
ensuring the renormalizability of the Faddeev-Popov action and leading to the perturbative expansion as we know it:



However ...

The FP procedure is believed to be invalid/incomplete:

1) It ignores the presence of Gribov copies (multiple solutions to $\partial_\mu A^a_\mu = 0$ along a given orbit). Fine at high energies but could become delicate at low energies.

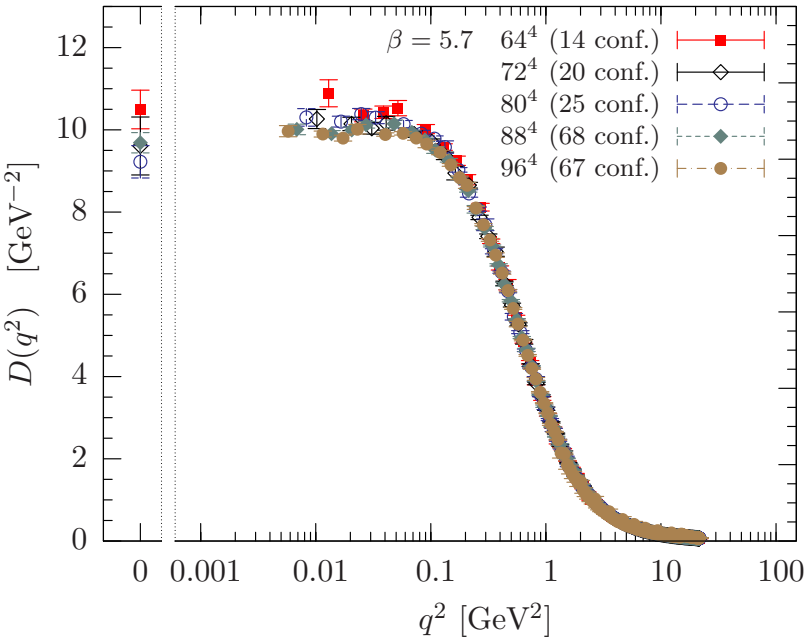


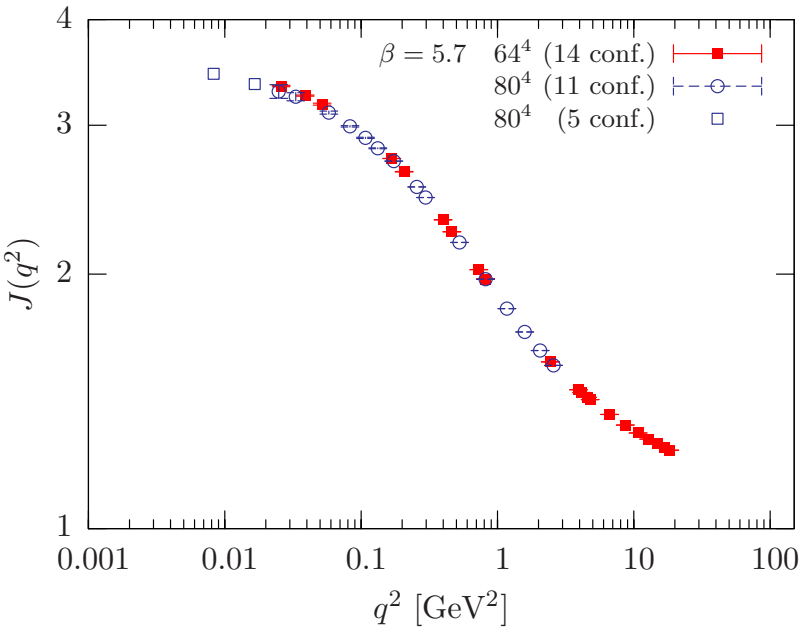
2) Similar procedure in Statistical Physics (Parisi-Sourlas) is known to be invalid for $d < d_c$, due to a breaking of the corresponding BRST symmetry. [Tarjus and Tissier, Phys. Rev. Lett. 107, 041601 (2011)]

However ...

The FP procedure is believed to be invalid/incomplete:

3) It predicts (through BRST) a scaling behavior of the correlation functions, at odds with the decoupling behavior seen on the lattice:





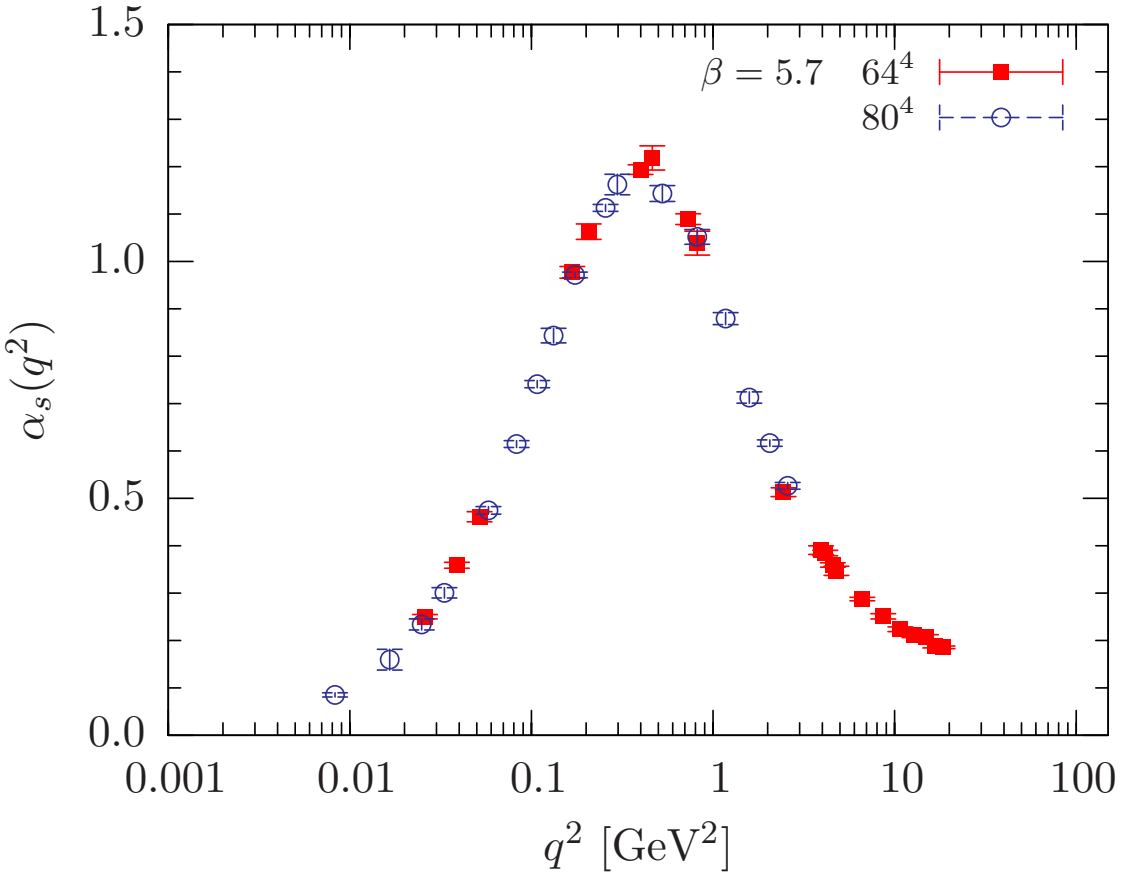
[I. L. Bogolubsky, E. M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, PLB 676, 69 (2009)]

$0 < D(p \to 0), J(p \to 0) < \infty$ instead of $D(p) \sim p^{\alpha}$ and $J(p) \sim p^{\beta}$

However ...

The FP procedure is believed to be invalid/incomplete:

4) Finally, lattice simulations show no evidence for a Landau pole:



[I. L. Bogolubsky, E. M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, Phys. Lett. B676, 69 (2009).]

Beyond the Faddeev-Popov approach

From the above considerations, it seems that

- the FP action needs to be extended;
- the extension could possibly break BRST;
- the extension could lead to a new perturbative regime!

How to find the appropriate extension of the FP action?

- semi-constructive approaches: Gribov-Zwanziger, ...;
- phenomenological approaches: include new operators to the Faddeev Popov model and try to constraint their couplings, or even discard them, using experiments/lattice simulations.

2. The Curci-Ferrari Model

The Curci-Ferrari (CF) model

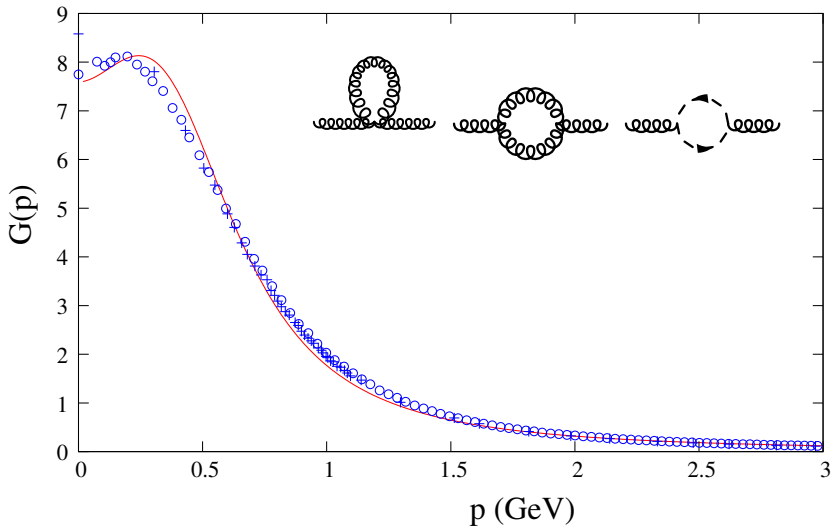
The Curci-Ferrari model is a an example of such a phenomenological approach. It is the simplest extension of the FP action that breaks BRST symmetry while maintaining renormalizability.

$$S_{FP} \to S_{CF} = S_{FP} + \int_x \frac{m_0^2}{2} A_\mu^a A_\mu^a$$

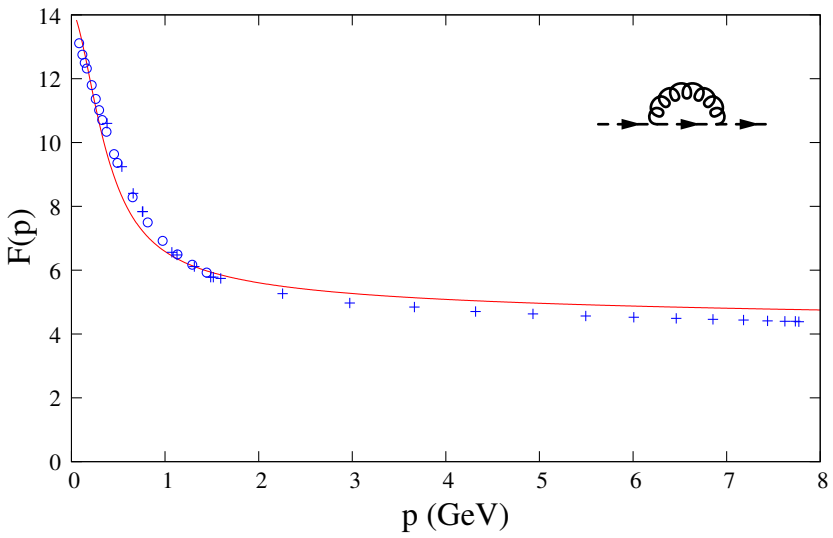
One (and only one) additional parameter m_0 to be dealt with. Fixed by fitting lattice data in the vacuum.

One-loop correlation functions

$G(p) \equiv D(p)$



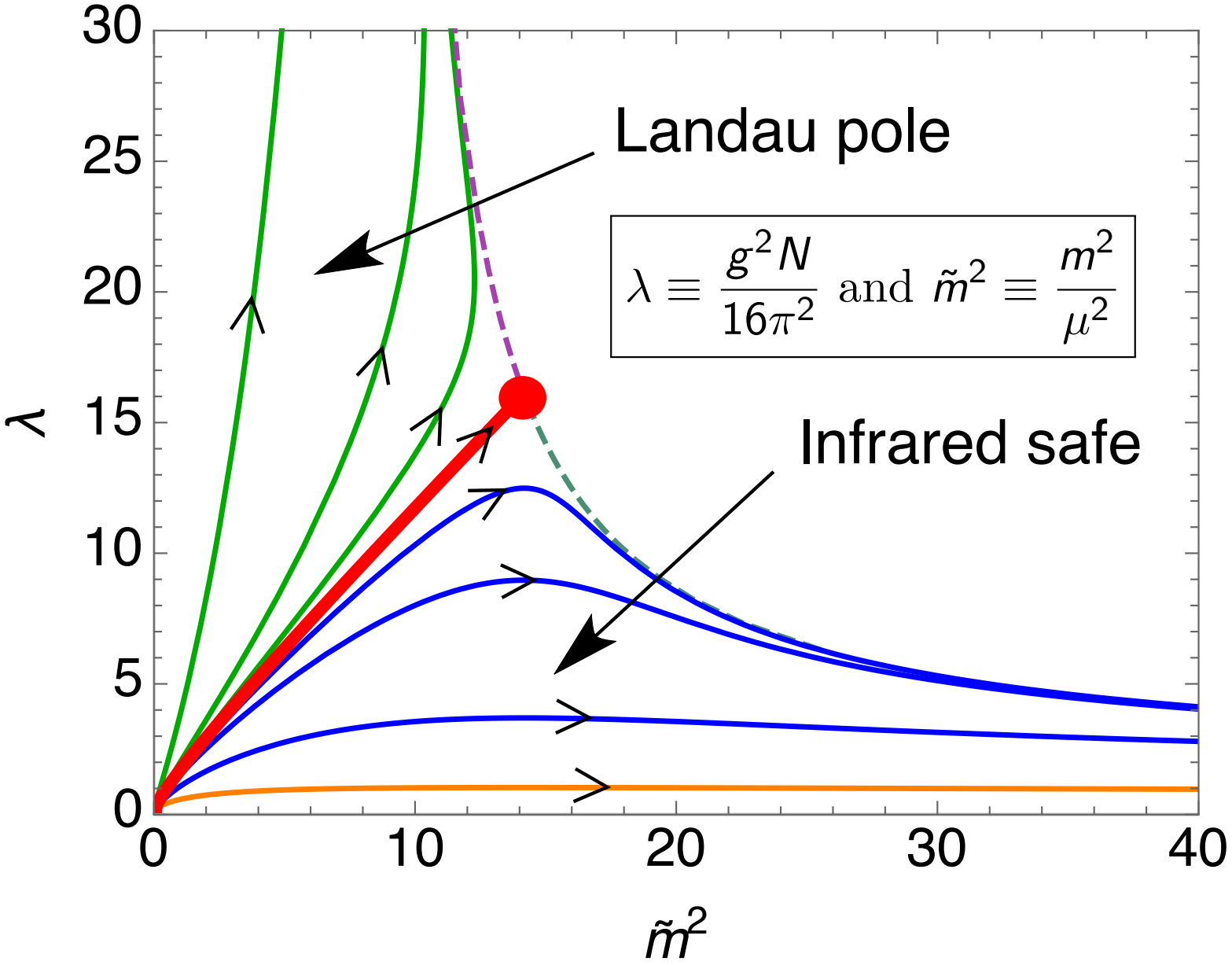
$F(p) \equiv J(p)$



Tissier and Wschebor, Phys. Rev. D84 (2011);
[For 3-point functions, see also: Peláez, Tissier, Wschebor, Phys. Rev. D88 (2013)]
[With quarks, see also: Peláez, Tissier, Wschebor, Phys. Rev. D90 (2014) & Phys. Rev. D92 (2015)]

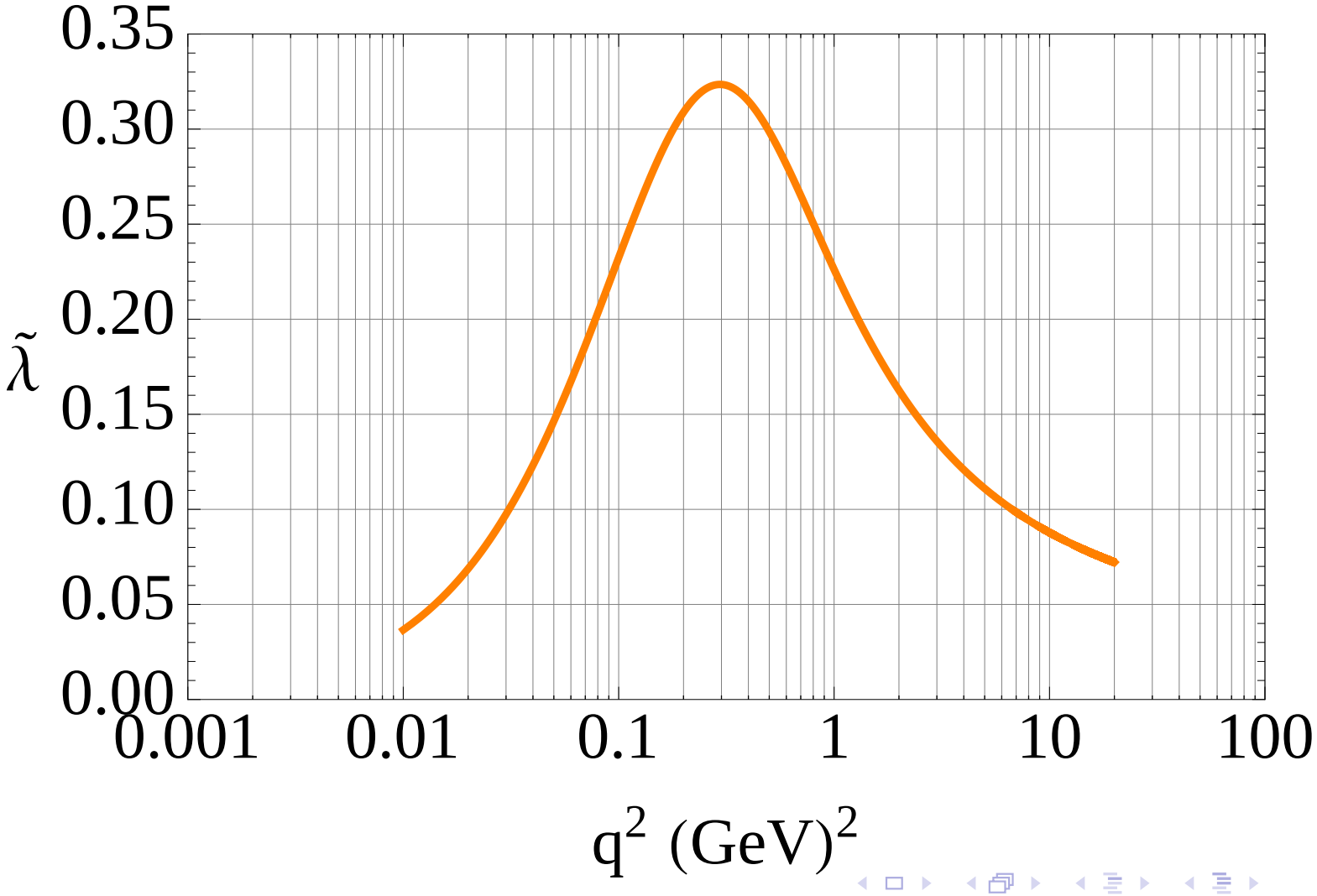
$$m_0 \simeq 500\,\mathrm{MeV}$$

Flow diagram of the Curci-Ferrari model

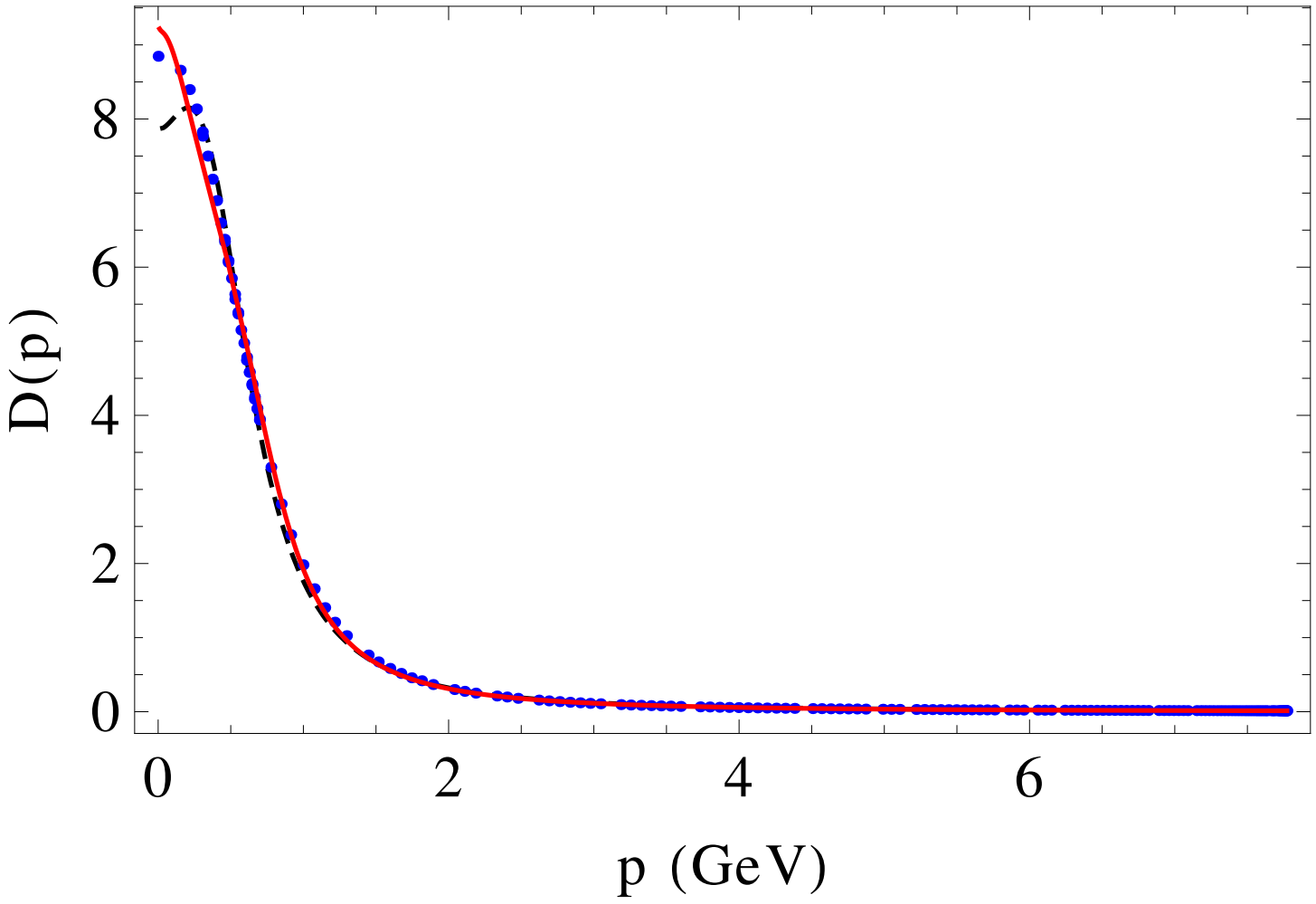


Best fitting trajectory

relevant expansion parameter: $\tilde{\lambda}(q^2) = \frac{g^2(q^2)N}{16\pi^2} \frac{1}{1 + m^2/q^2}$

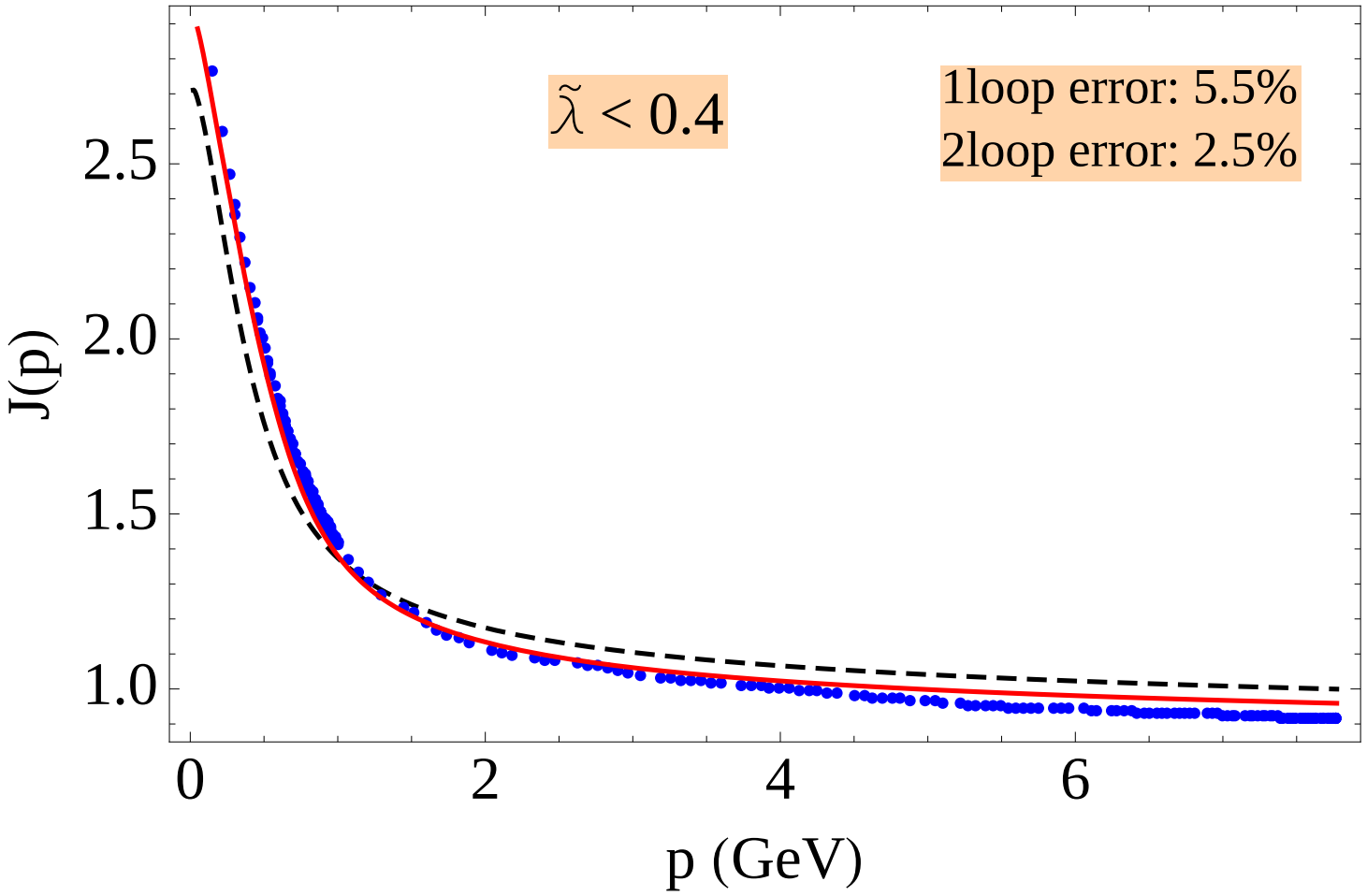


Two-loop corrections



[J.A. Gracey, M. Peláez, U. Reinosa, M. Tissier, in preparation]

Two-loop corrections



[J.A. Gracey, M. Peláez, U. Reinosa, M. Tissier, in preparation]

Link to functional approaches

Functional implementations of the (Landau gauge) FP action necessarily break the BRST symmetry of the action.

This is not a problem per se but requires one to work in a larger theory space, including in particular an operator $m_\Lambda^2 A_\mu^a A_\mu^a$.

The question is how to fix the extra parameters $m_\Lambda, \ldots$ in terms of g_Λ? This depends on the fate of BRST symmetry in the continuum.

Link to functional approaches

For all practical purposes (so far), functional approaches to YM can be seen as nonperturbative implementations of the CF model.

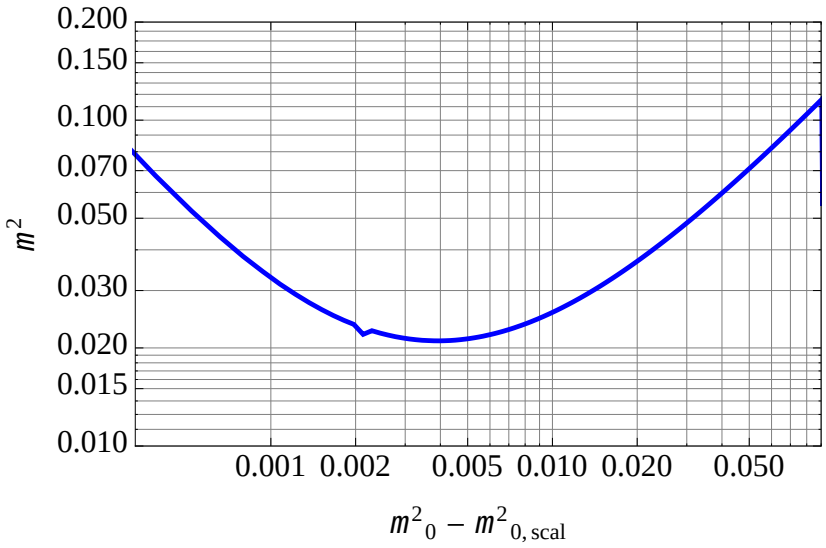
The pCF (perturbative Curci-Ferrari) can allow disentangling what is genuinely non-perturbative from what could become perturbative once the Gribov ambiguity is lifted.

Link to functional approaches

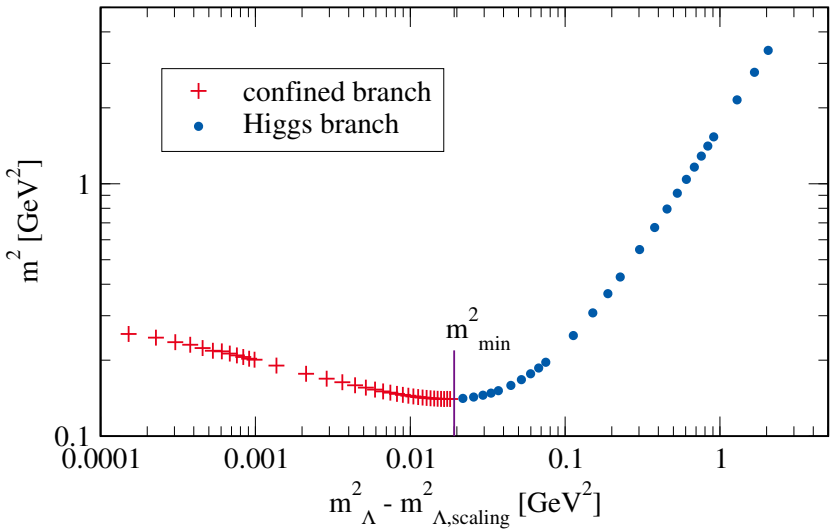
Our fixed point corresponds to one of the two scaling solutions found in functional approaches [Located at large coupling but exponents fixed by symmetries]

We miss the second scaling solution [Probably also a large coupling solution]

For decoupling solutions, we find qualitative agreement with functional approaches. In particular the m_0-dependence of the gluon screening mass $m^2 \equiv D^{-1}(0)$ mimics the m_Λ-dependence obtained within the fRG:



[pCF collaboration, Phys. Rev. D96, 014005 (2017)]

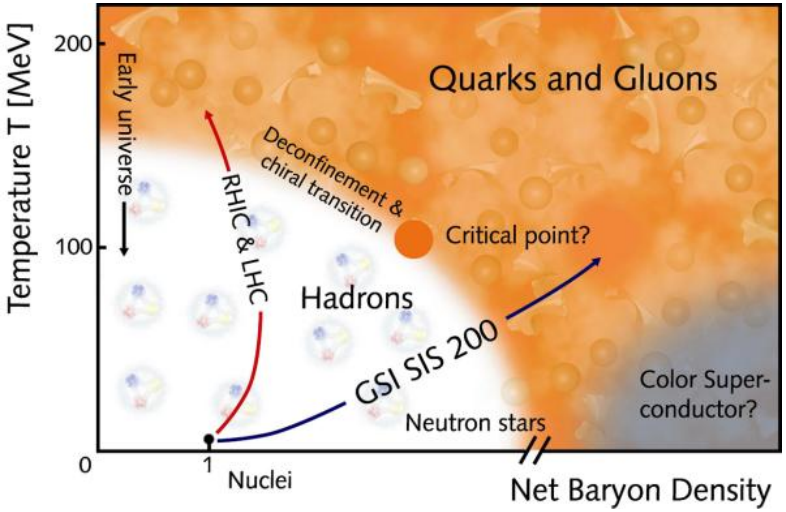


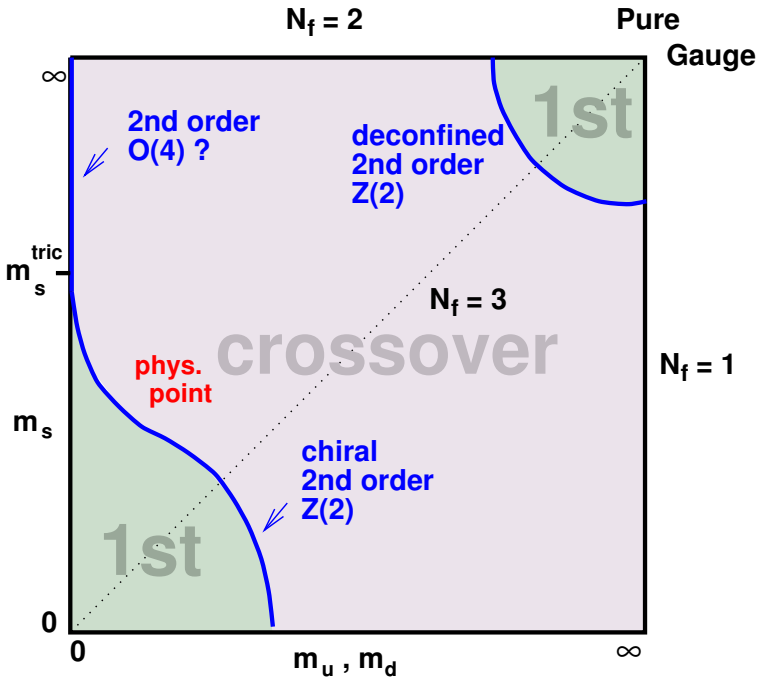
[Cyrol et al, Phys. Rev. D94, 054005 (2016)]

3. Yang-Mills phase structure

QCD phase transitions

What are the predictions of the CF model regarding the confinement/deconfinement transition and chiral symmetry breaking?





Deconfinement and Center symmetry breaking

In the pure Yang-Mills theory, the order parameter for deconfinement is the Polyakov loop:

$$\ell_q \equiv \frac{1}{N_c} \left\langle \mathrm{tr}\, P\, e^{ig \int_0^\beta d\tau\, A_0^a(\tau,\vec{x}) t^a} \right\rangle \sim e^{-\beta \Delta F_q}$$

- If $\ell_q = 0$ then $\Delta F_q = \infty$ (confined phase);
- If $\ell_q \neq 0$, $\Delta F_q < \infty$ (deconfined phase).

How could ℓ_q vanish?

Deconfinement and Center symmetry breaking

Because of center symmetry. The YM action is invariant under

$$(A^U)^a_\mu(\tau,\vec{x})t^a \equiv U(\tau,\vec{x})A^a_\mu(\tau,\vec{x})t^a U^\dagger(\tau,\vec{x}) + \frac{i}{g}U(\tau,\vec{x})\partial_\mu U^\dagger(\tau,\vec{x})$$

At finite $T \equiv 1/\beta$, the periodicity of the gauge field implies

$$U(\tau+\beta,\vec{x}) = Z\,U(\tau,\vec{x}) \quad \text{with} \quad Z \in \{\mathbb{1}, e^{i2\pi/3}\mathbb{1}, e^{i4\pi/3}\mathbb{1}\} \simeq \mathbb{Z}_3$$

The Polyakov loop transforms as $\ell_q \to Z\ell_q$. If center symmetry is realized, then $\ell_q = Z\ell_q$ and therefore $\ell_q = 0$.

Center symmetry and Gauge-fixing

Can one study center symmetry breaking within a continuum setting?

Problem: Continuum approaches require gauge-fixing.
In such a context, center symmetry is usually not explicit.

Solution: Generalize the gauge-fixing by including a background.
[J. Braun, H. Gies, J.M. Pawlowski, Phys.Lett. B684 (2010) 262-267]

The Landau-DeWitt (LDW) gauge

Trick: replace the Landau gauge-fixing condition $0 = \partial_\mu A^a_\mu$ by a covariant version of it

$$0 = \bar{D}^{ab}_\mu (A^b_\mu - \bar{A}^b_\mu) \equiv (\partial_\mu \delta^{ab} + g f^{acb} \bar{A}^c_\mu)(A^b_\mu - \bar{A}^b_\mu)$$

with $\bar{A}^a_\mu$ some given field configuration (background).

The gauge-fixed action reads

$$S_{\bar{A}}[A] = \int d^4x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{D}_\mu \bar{c}^a (D_\mu c)^a + i h^a \bar{D}_\mu (A - \bar{A})^a_\mu \right\}$$

Center symmetry is manifest in the sense that: $S_{\bar{A}^U}[A^U] = S_{\bar{A}}[A]$.

This property is inherited by the quantum action: $\Gamma_{\bar{A}^U}[A^U] = \Gamma_{\bar{A}}[A]$.

Background extended CF model

We extend the Faddeev-Popov action into a background extended Curci-Ferrari model:

$$\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{D}_\mu \bar{c}^a (D_\mu c)^a + i h^a \bar{D}_\mu (A - \bar{A})^a_\mu + \frac{1}{2} m^2 (A - \bar{A})^a_\mu (A - \bar{A})^a_\mu$$

The mass term is such that $S_{\bar{A}^U}[A^U] = S_{\bar{A}}[A]$ and $\Gamma_{\bar{A}^U}[A^U] = \Gamma_{\bar{A}}[A]$.

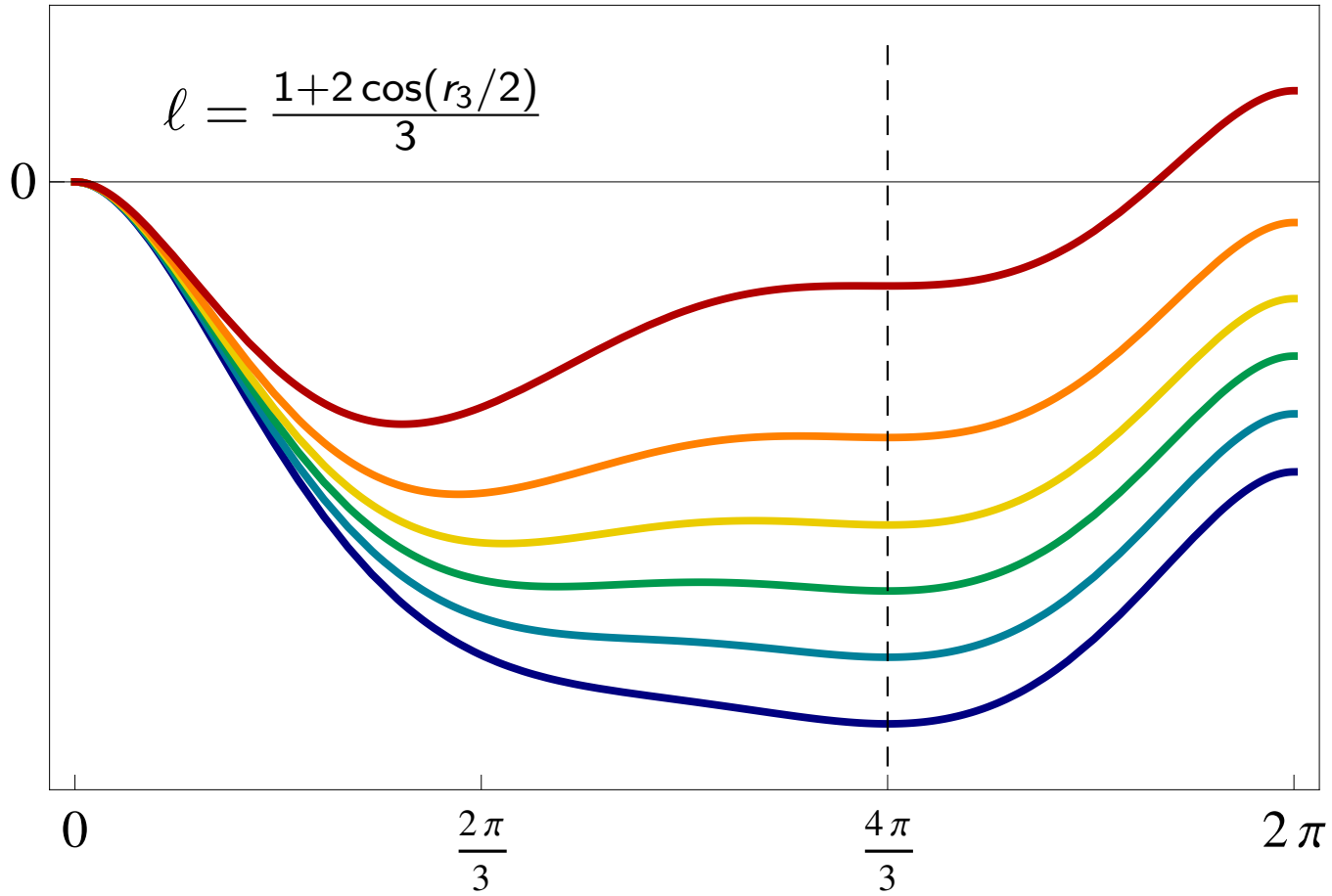
Equilibrium: we restrict to homogeneous and isotropic backgrounds, in the diagonal part of the algebra:

$$g \bar{A}_\mu = T \left(r_3 \frac{\lambda_3}{2} + r_8 \frac{\lambda_8}{2} \right) \delta_{\mu 0}$$

The effective action boils down to a potential $V(r_3, r_8)$.

Pure YM: background CF model at one-loop

Manifest charge conjugation invariance $\Rightarrow r_8 = 0$:



[UR, J. Serreau, M. Tissier and N. Wschebor, Phys.Lett. B742 (2015) 61-68]

Pure YM: background CF model at one-loop

Why is this working?

$$V(r) = \frac{3}{2}\sum_{\kappa} T \sum_{n\in\mathbb{N}} \int \frac{d^3q}{(2\pi)^3} \ln\left[(\omega_n + T\kappa\cdot r)^2 + q^2 + m^2\right] \leftarrow \text{gluons}$$

$$- \frac{1}{2}\sum_{\kappa} T \sum_{n\in\mathbb{N}} \int \frac{d^3q}{(2\pi)^3} \ln\left[(\omega_n + T\kappa\cdot r)^2 + q^2\right] \leftarrow \text{ghosts}$$

$T \gg m$: Weiss potential, the confining point is the absolute maximum

$$\left(\frac{3}{2} - \frac{1}{2}\right)\sum_{\kappa} T \sum_{n\in\mathbb{N}} \int \frac{d^3q}{(2\pi)^3} \ln\left[(\omega_n + T\kappa\cdot r)^2 + q^2\right]$$

$T \ll m$: Inverted Weiss potential, the confining point becomes the minimum

$$-\frac{1}{2}\sum_{\kappa} T \sum_{n\in\mathbb{N}} \int \frac{d^3q}{(2\pi)^3} \ln\left[(\omega_n + T\kappa\cdot r)^2 + q^2\right]$$

In line with the confinement scenario of [J. Braun, H. Gies, J.M. Pawlowski]

Pure YM: Summary of one-loop results

order	lattice	fRG	variational	CF at 1-loop
SU(2)	2nd	2nd	2nd	2nd
SU(3)	1st	1st	1st	1st
SU(4)	1st	1st	??	1st
Sp(2)	1st	1st	??	1st

T_c (MeV)	lattice	fRG(*)	variational(**)	CF at 1-loop(***)
SU(2)	295	230	239	238
SU(3)	270	275	245	185

(*) L. Fister and J. M. Pawlowski, Phys.Rev. D88 (2013) 045010.

(**) M. Quandt and H. Reinhardt, Phys.Rev. D94 (2016) no.6, 065015.

(***) UR, J. Serreau, M. Tissier and N. Wschebor, Phys.Lett. B742 (2015) 61-68.

Pure YM: Summary of two-loop results

order	lattice	fRG	variational	CF at 1-loop	CF at 2-loop
SU(2)	2nd	2nd	2nd	2nd	2nd
SU(3)	1st	1st	1st	1st	1st
SU(4)	1st	1st	??	1st	1st
Sp(2)	1st	1st	??	1st	1st

T_c (MeV)	lattice	fRG$^{(*)}$	variational$^{(**)}$	CF at 1-loop	CF at 2-loop$^{(***)}$
SU(2)	295	230	239	238	284
SU(3)	270	275	245	185	254

$^{(*)}$ L. Fister and J. M. Pawlowski, Phys.Rev. D88 (2013) 045010.

$^{(**)}$ M. Quandt and H. Reinhardt, Phys.Rev. D94 (2016) no.6, 065015.

$^{(***)}$ UR, J. Serreau, M. Tissier and N. Wschebor, Phys.Rev. D93 (2016) 105002.

4. QCD in the heavy-quark regime

Adding quarks: Symmetries

$$S = S_{\mathrm{bgCF}} + \sum_{f=1}^{N_f} \int_x \left\{ \bar{\psi}_f (\not{\partial} - ig\not{A}^a t^a + M_f - \mu\gamma_0)\psi_f \right\}$$

Quarks transform in the fundamental representation:

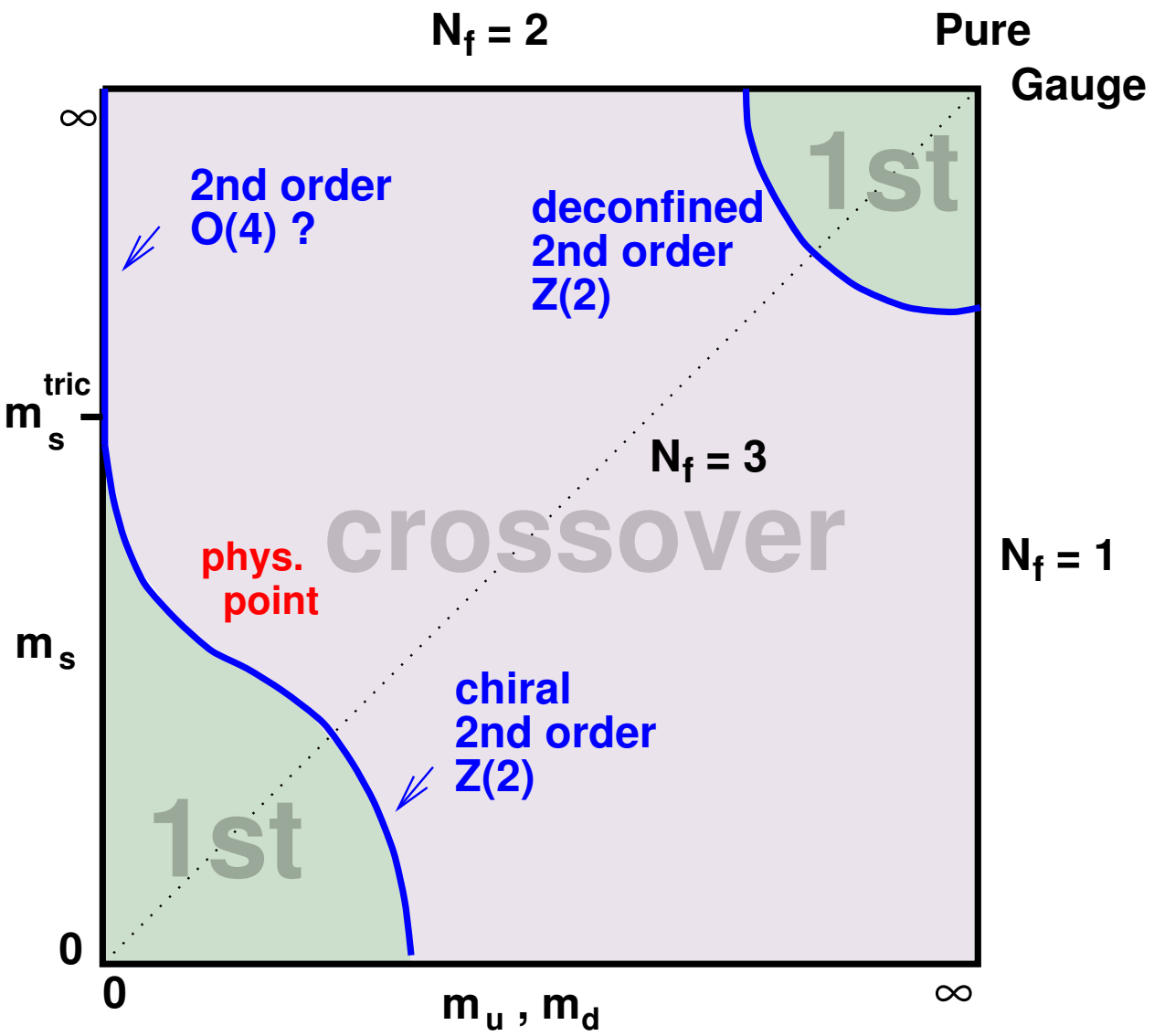
$$\psi^U(\tau, \vec{x}) = U(\tau, \vec{x})\psi(\tau, \vec{x})$$

Center symmetry is broken due to the boundary conditions:

$$U(\beta, \vec{x}) = e^{i2\pi/3} U(0, \vec{x}) \Rightarrow \psi^U(\beta, \vec{x}) = -e^{i2\pi/3}\psi^U(0, \vec{x})$$

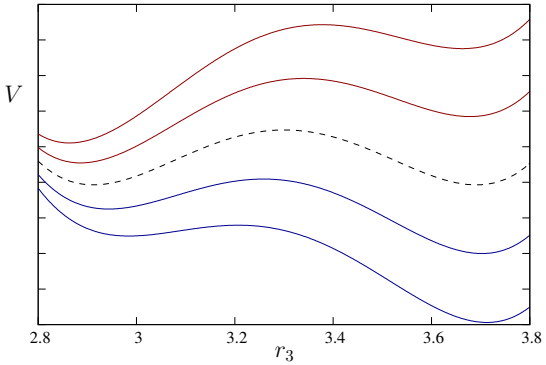
Charge conjugation is also broken as soon as $\mu \neq 0$. Then $r_8 \neq 0$.

Adding quarks: Columbia Plot at $\mu = 0$



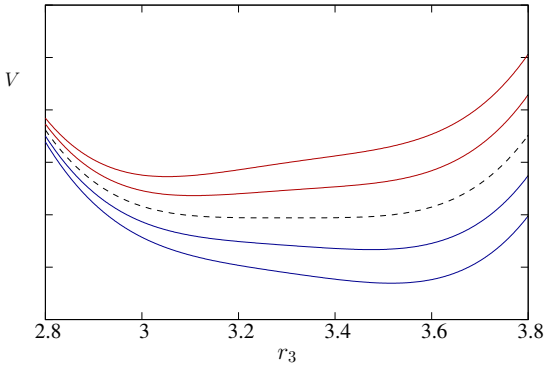
$\mu = 0$: background CF model at one-loop

$\underline{M > M_c}$



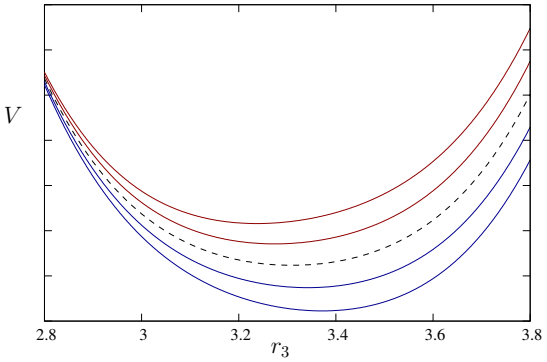
first order

$\underline{M = M_c}$



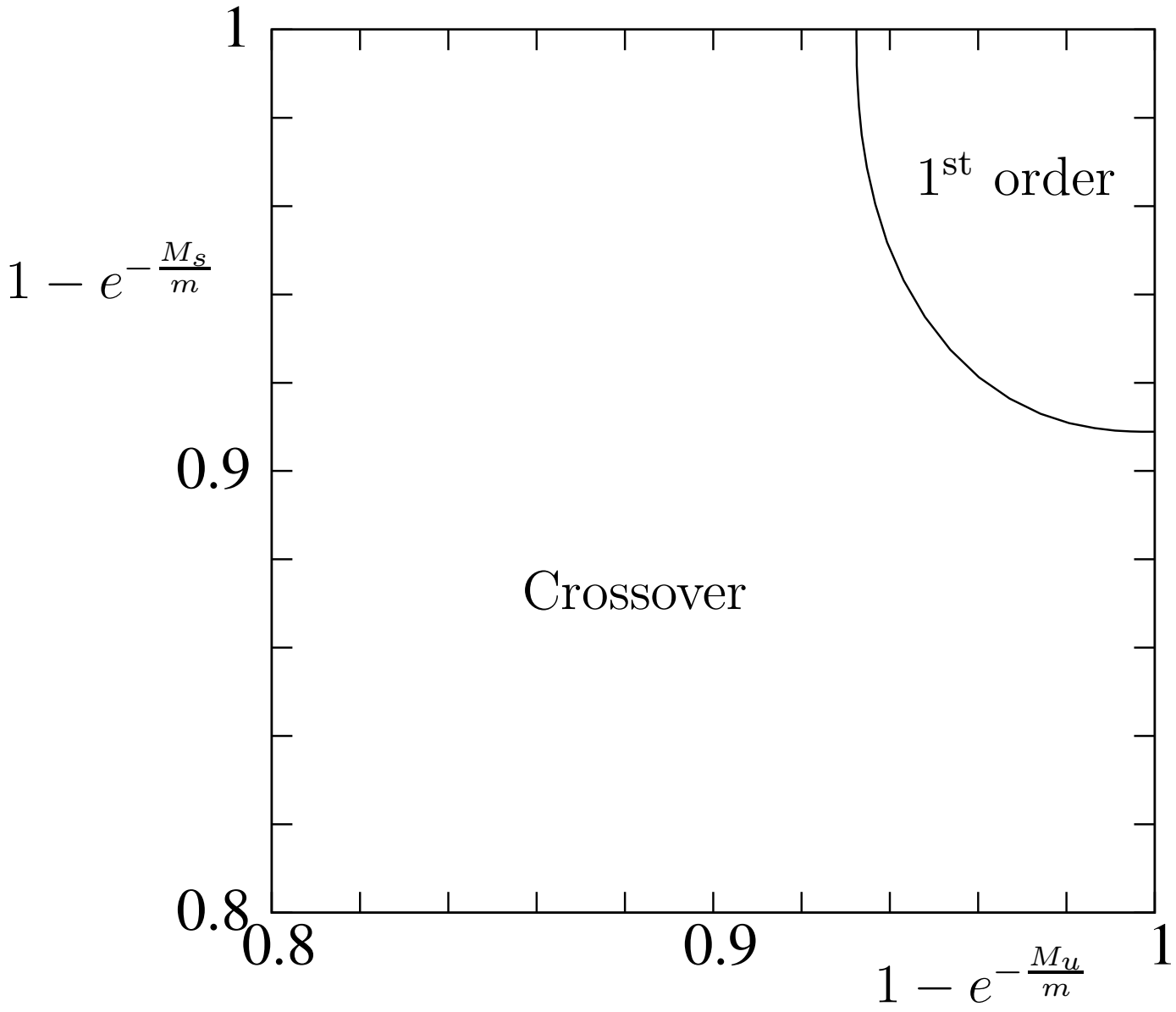
second order

$\underline{M < M_c}$



crossover

$\mu = 0$: background CF model at one-loop



$\mu = 0$: background CF model at one-loop

$R_{N_f} \equiv M_c/T_c$	$N_f = 1$	$N_f = 2$	$N_f = 3$
Lattice[(i)]	7.23	7.92	8.33
CF[(ii)]	6.74	7.59	8.07
GZ[(iii)]	7.09	7.92	8.40
Matrix[(iv)]	8.04	8.85	9.33
DSE[(v)]	1.42	1.83	2.04

(i) M. Fromm, J. Langelage, S. Lottini and O. Philipsen, JHEP 1201 (2012) 042.
(ii) UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015).
(iii) J. Maelger, UR, J. Serreau, Phys.Rev. D98 (2018) 094020.
(iv) K. Kashiwa, R. D. Pisarski and V. V. Skokov, Phys.Rev. D85 (2012) 114029.
(v) C. S. Fischer, J. Luecker and J. M. Pawlowski, Phys.Rev. D91 (2015) 1, 014024.

$\mu = 0$: background CF model at two-loop

At two-loop order, the comparison to lattice data is tricky since the quark mass M is scheme dependent. To reduce scheme dependences we can compare ratios of $R_{N_f} = M_c(N_f)/T_c(N_f)$ at various values of N_f.

$R_{N_f} \equiv M_c/T_c$	$N_f = 1$	$N_f = 2$	$N_f = 3$	R_2/R_1	R_3/R_1
Lattice[(i)]	7.23	7.92	8.33	1.10	1.15
CF 1-loop[(ii)]	6.74	7.59	8.07	1.12	1.20
CF 2-loop[(iib)]	7.53	8.40	8.90	1.11	1.18
GZ[(iii)]	7.09	7.92	8.40	1.12	1.19
Matrix[(iv)]	8.04	8.85	9.33	1.10	1.16
DS[(v)]	1.42	1.83	2.04	1.29	1.43

(i) M. Fromm, J. Langelage, S. Lottini and O. Philipsen, JHEP 1201 (2012) 042.

(ii) UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015).

(iib) J. Maelger, UR and J. Serreau, Phys.Rev. D97 (2018) 074027.

(iii) J. Maelger, UR, J. Serreau, Phys.Rev. D98 (2018) 094020.

(iv) K. Kashiwa, R. D. Pisarski and V. V. Skokov, Phys.Rev. D85 (2012) 114029.

(v) C. S. Fischer, J. Luecker and J. M. Pawlowski, Phys.Rev. D91 (2015) 1, 014024.

Imaginary chemical potential

$$S = S_{\mathrm{bgCF}} + \sum_{f=1}^{N_f} \int_x \left\{ \bar{\psi}_f (\not{\partial} - ig\not{A}^a t^a + M_f - (\quad \mu \qquad\qquad)\gamma_0)\psi_f \right\}$$

Center symmetry is explicitely broken: $\psi^U(\beta, \vec{x}) = -e^{i2\pi/3}\psi^U(0, \vec{x})$.

However this can be compensated by an abelian transformation

$$(\psi^U)'(\tau, \vec{x}) = e^{-i(2\pi/3)(\tau/\beta)}\psi^U(\tau, \vec{x})$$

The generated abelian gauge field corresponds to a shift of μ.

Combining this with a C transformation that changes μ to $-\mu$, one finds a symmetry $\mu \to i2\pi/3 - \mu$ with a fixed-point at $\mu = i\pi/3$.
$\Rightarrow$ Roberge Weiss symmetry.

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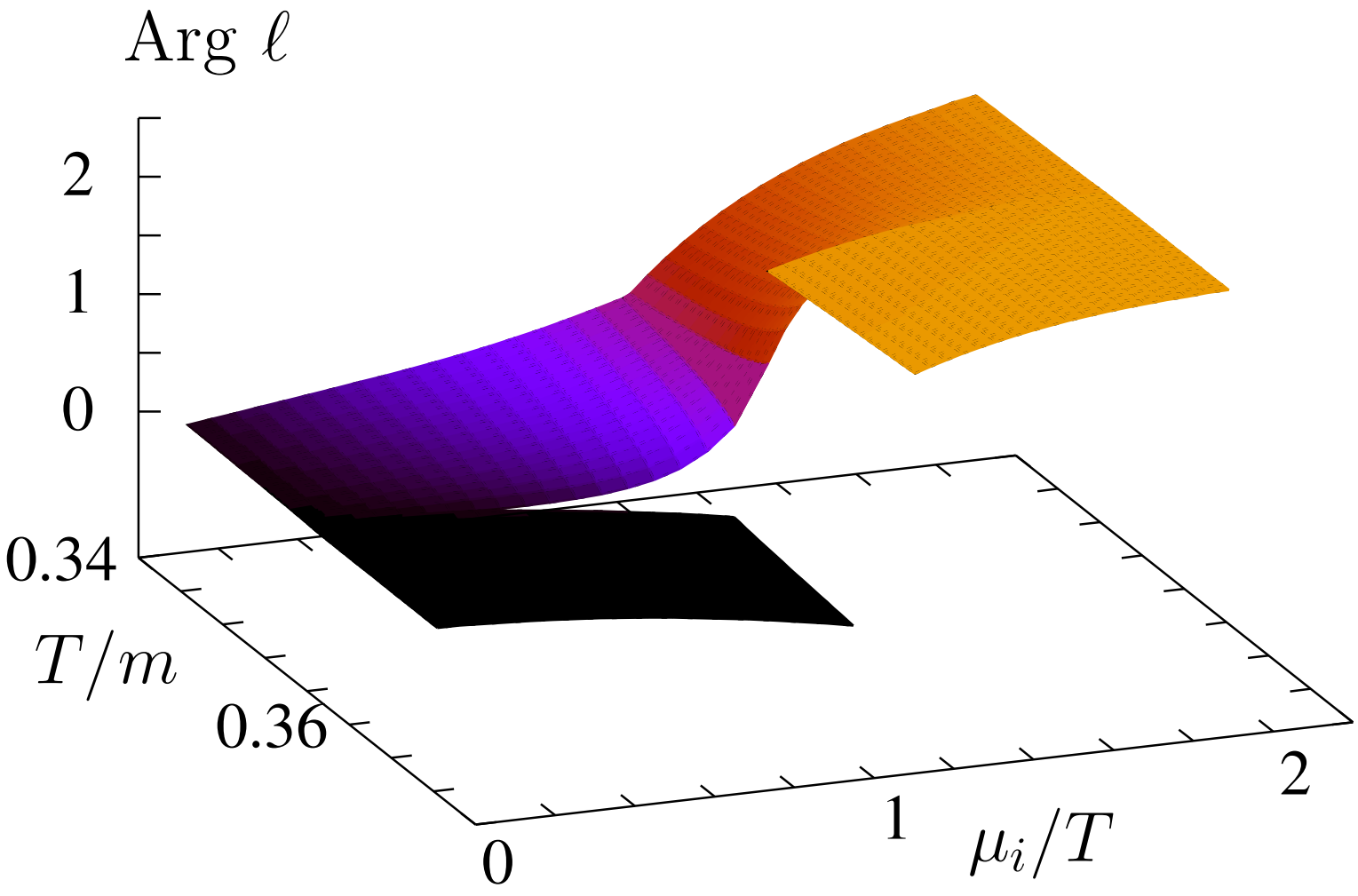
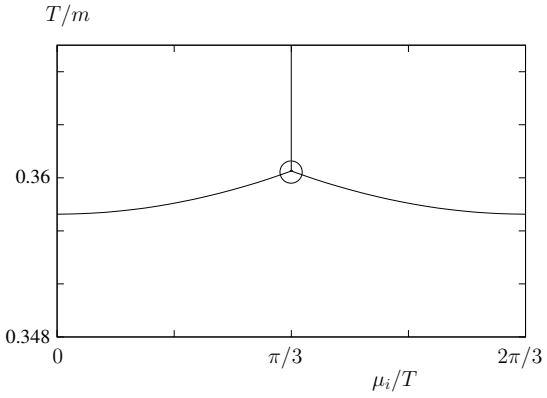
Imaginary μ: background CF model at one-loop

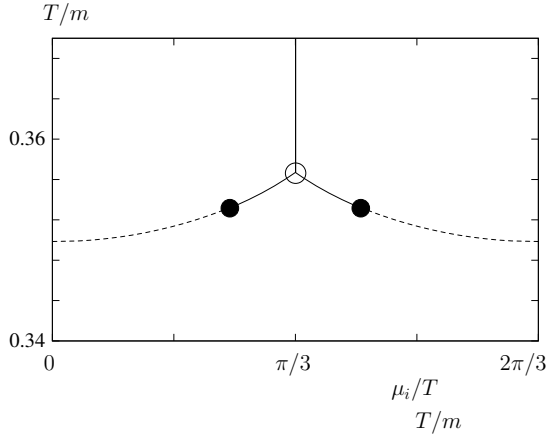


Imaginary μ: background CF model at one-loop

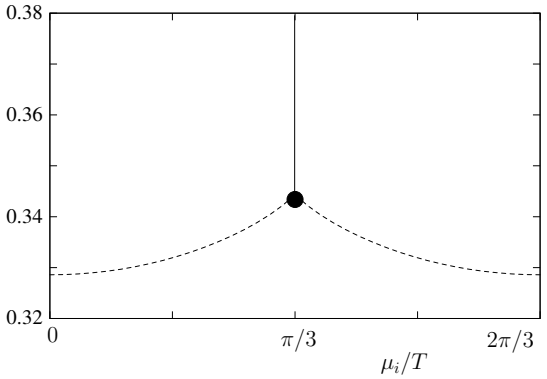
$M > M_c(0)$:



$M \in [M_c(i\pi/3), M_c(0)]$:



$M = M_c(i\pi/3)$:



Agrees with lattice [P. de Forcrand, O. Philipsen, Phys.Rev.Lett. 105 (2010)]

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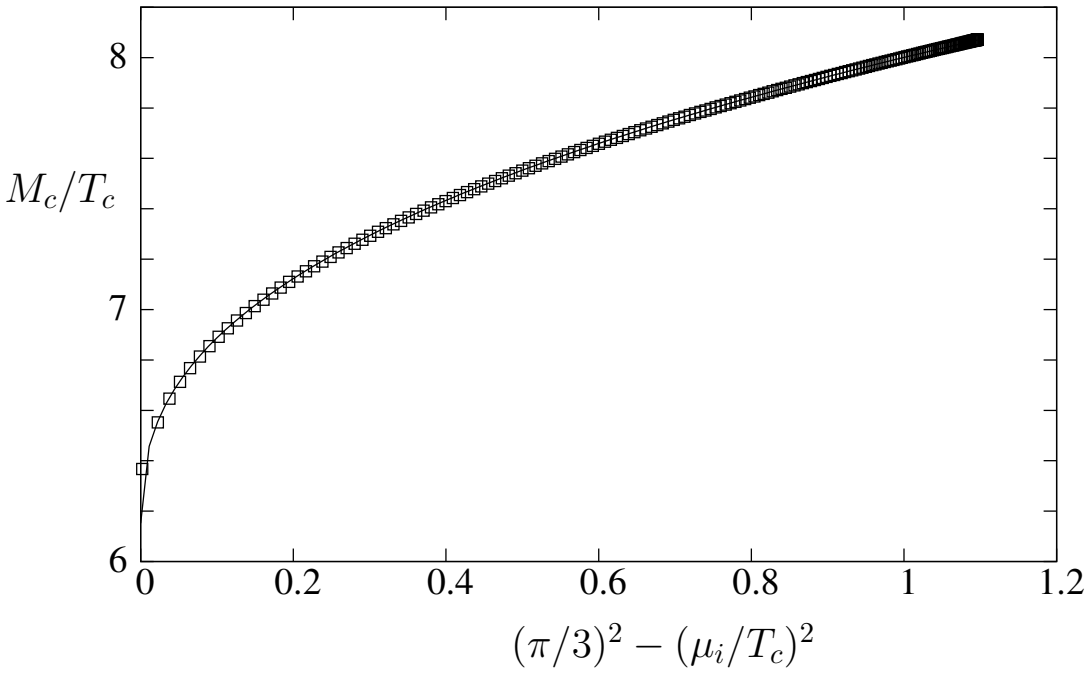


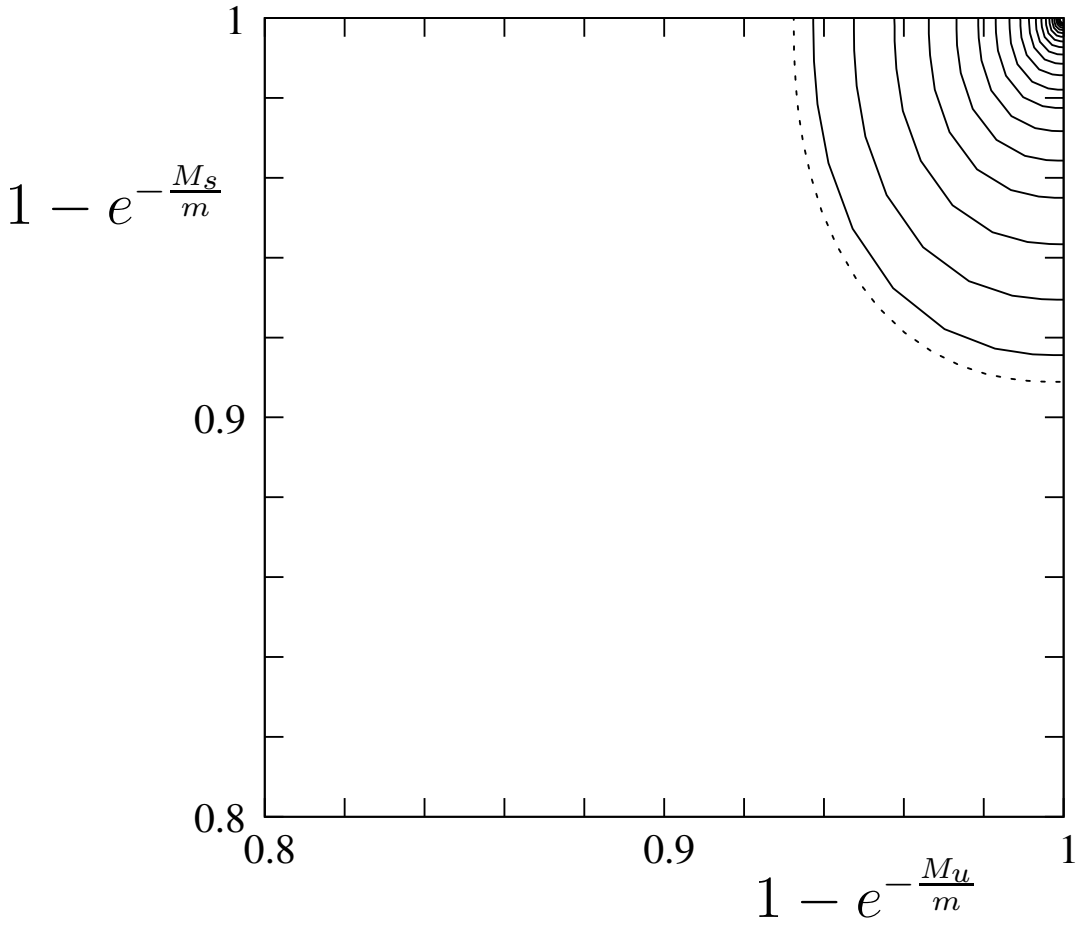
$$\frac{M_c}{T_c} = \frac{M_{\text{tric.}}}{T_{\text{tric.}}} + K\left[\left(\frac{\pi}{3}\right)^2 + \left(\frac{\mu}{T}\right)^2\right]^{2/5}$$

	our model(*)	lattice(**)	SD(***)
K	1.85	1.55	0.98
$\frac{M_{\text{tric.}}}{T_{\text{tric.}}}$	6.15	6.66	0.41

(*) UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015).
(**) Fromm et.al., JHEP 1201 (2012) 042.
(***) Fischer et.al., Phys.Rev. D91 (2015) 1, 014024.

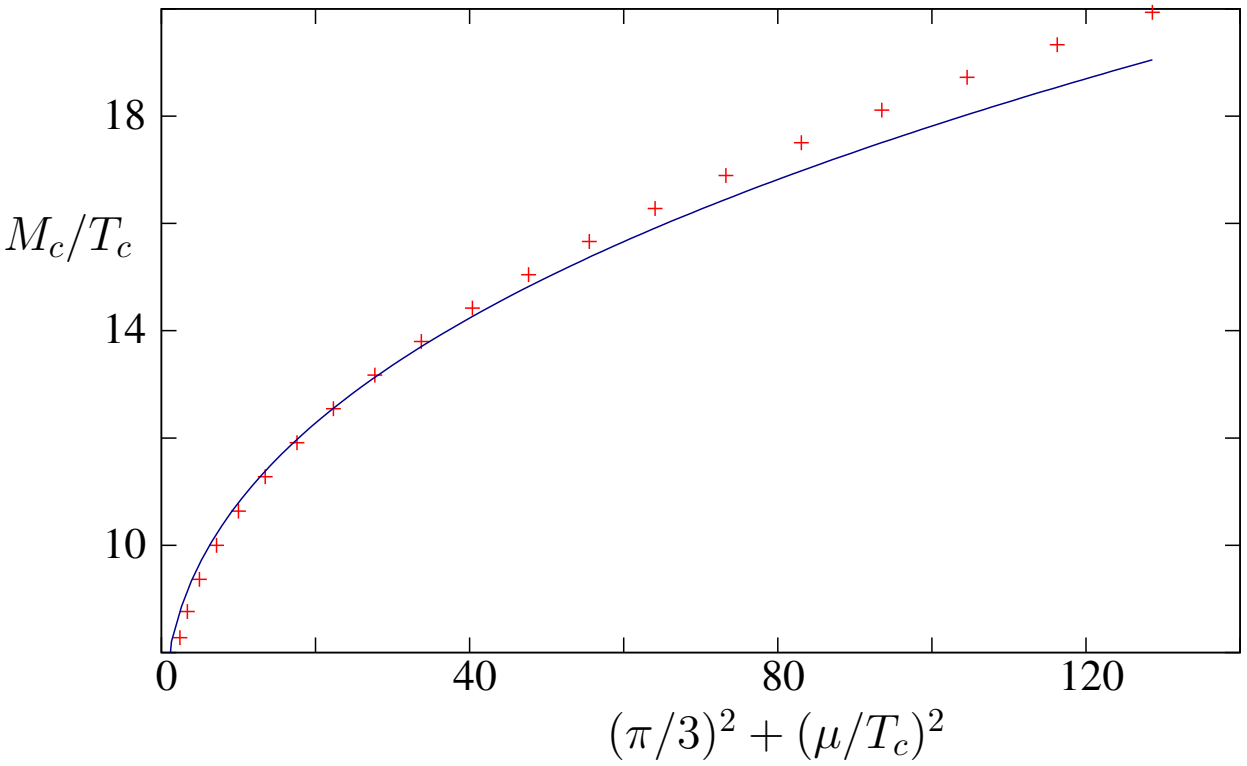
Real μ: background CF model at one-loop

The boundary line in the Columbia plot moves towards larger quark masses:
[UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015)]

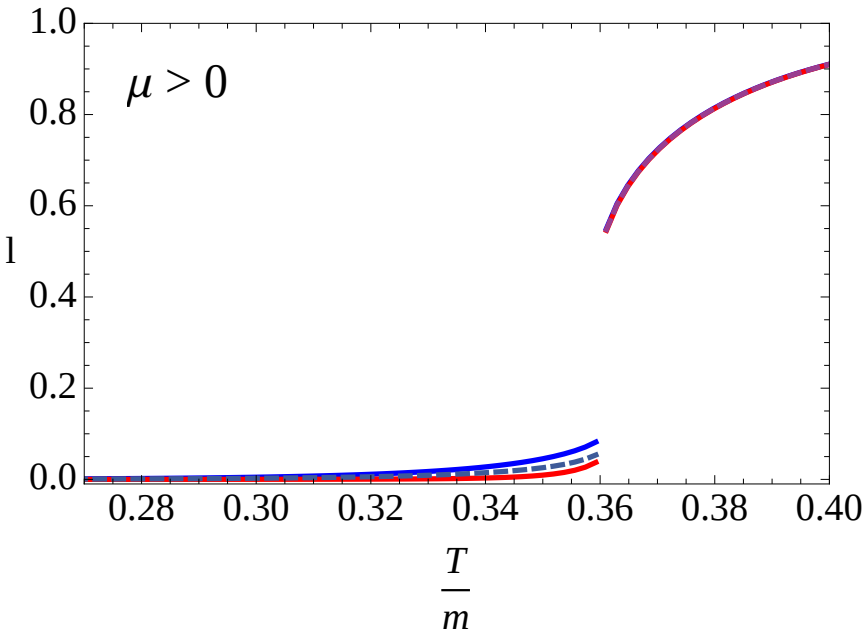


Real μ: background CF model at one-loop

The tricritical scaling survives deep in the $\mu^2 > 0$ region:
[UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015)]



Real μ: background CF model at one-loop



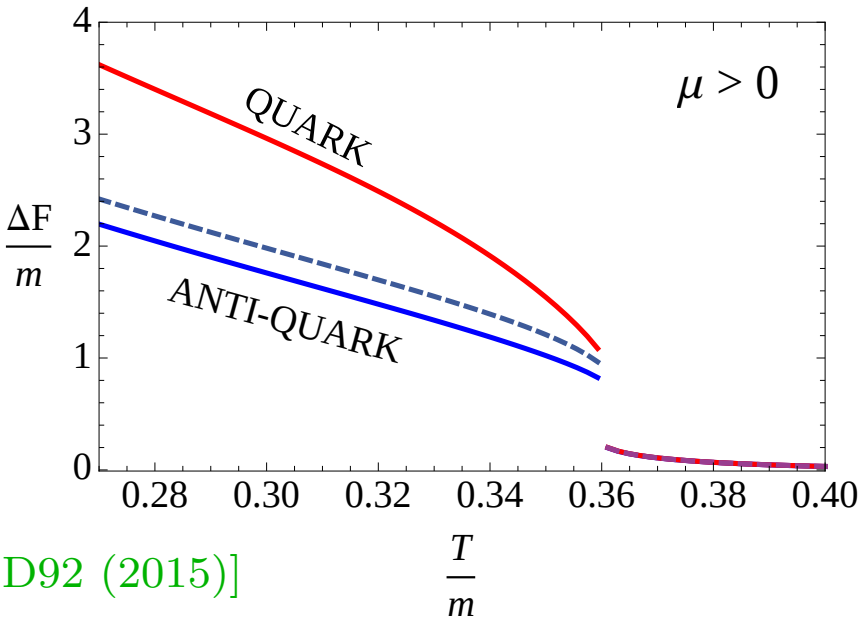
$\mu \in \mathbb{R}^* \Rightarrow$ broken $C \Rightarrow r_8 \neq 0$

$r_8 \in \mathbb{R} \Rightarrow V, \ell, \bar{\ell} \in \mathbb{C}$

$r_8 = 0 \Rightarrow V \in \mathbb{R}$ but $\ell = \bar{\ell}$

$r_8 \in i\mathbb{R} \Rightarrow V \in \mathbb{R}$ and $\ell, \bar{\ell} > 0$

$$\Delta F_{q,\bar{q}} = -T \ln \ell_{q,\bar{q}}$$



[UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015)]

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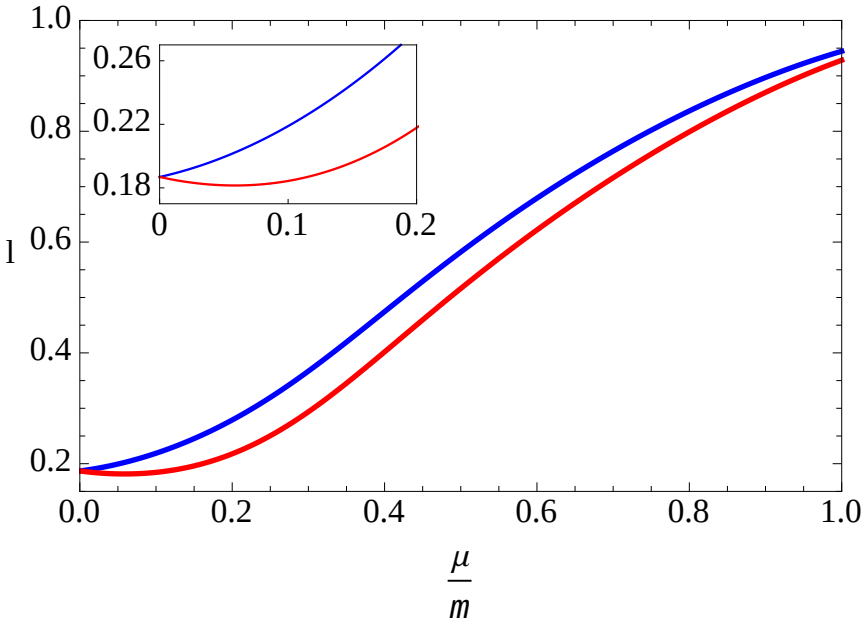
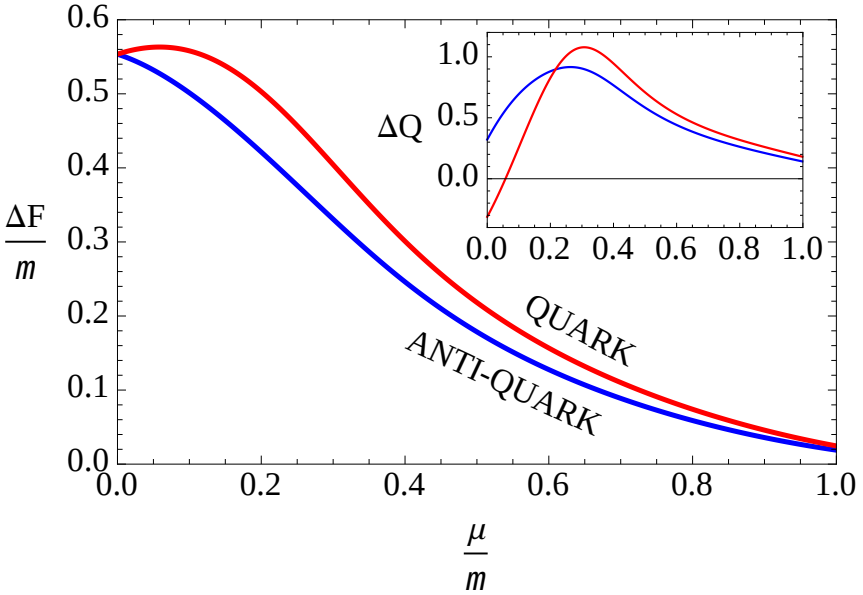


$$\left.\frac{\partial \Delta F}{\partial \mu}\right|_{\mu=0} = -\langle Q \rangle \begin{cases} > 0 \text{ (quark)} \\ < 0 \text{ (anti-quark)} \end{cases}$$

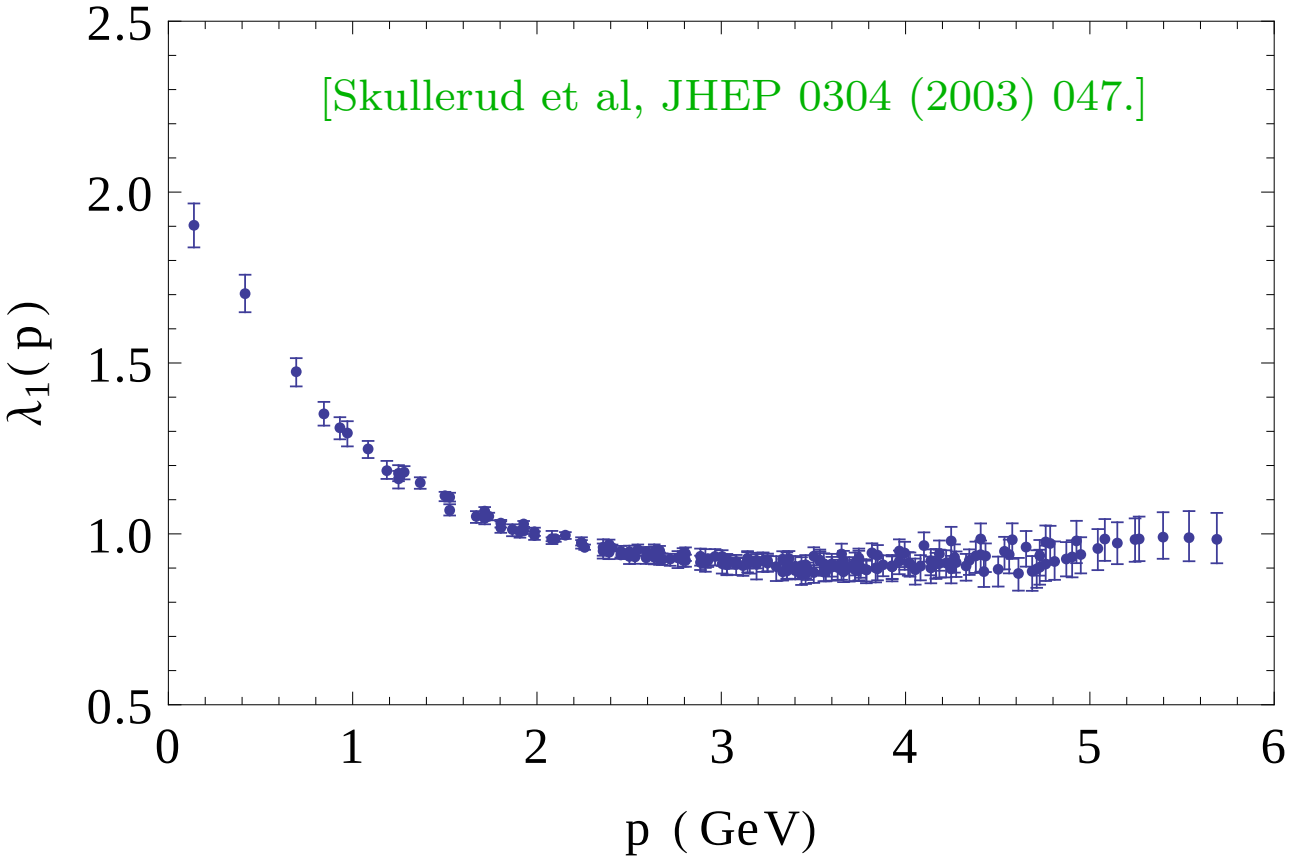


[J. Maelger, UR and J. Serreau, Phys.Rev. D97 (2018) 074027]

5. QCD phase structure with light quarks

Infrared couplings

For light quark masses, the quark-gluon vertex coupling $g_q(p)$ is up to 2× larger in the infrared than the pure YM coupling $g_g(p)$:

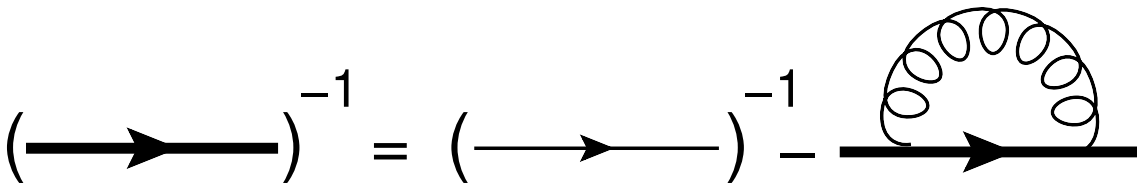


Perturbation theory is not justified in this case.

Small parameter expansion

We can however use the knowledge that the pure gauge sector of the CF model is perturbative, to construct a systematic double parameter expansion in powers of g_g and $1/N_c$.

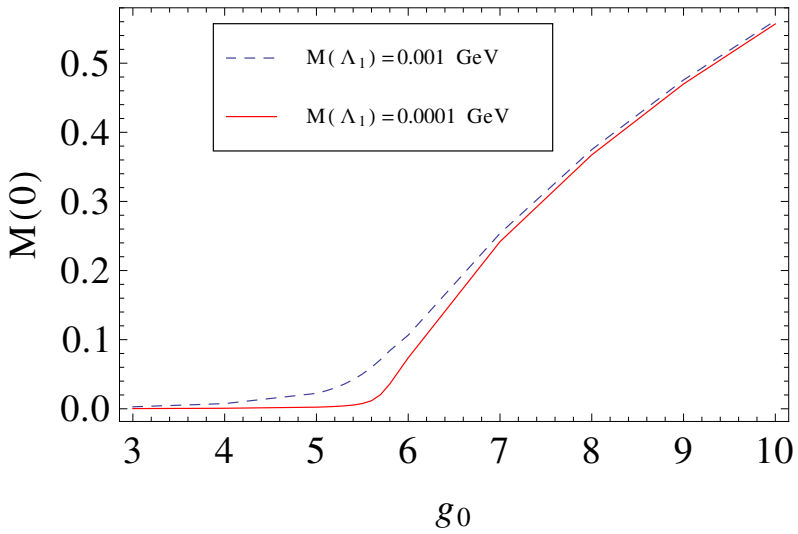
At leading order, it reproduces the well known "rainbow equation" for the quark propagator, with however a CF gluon propagator:

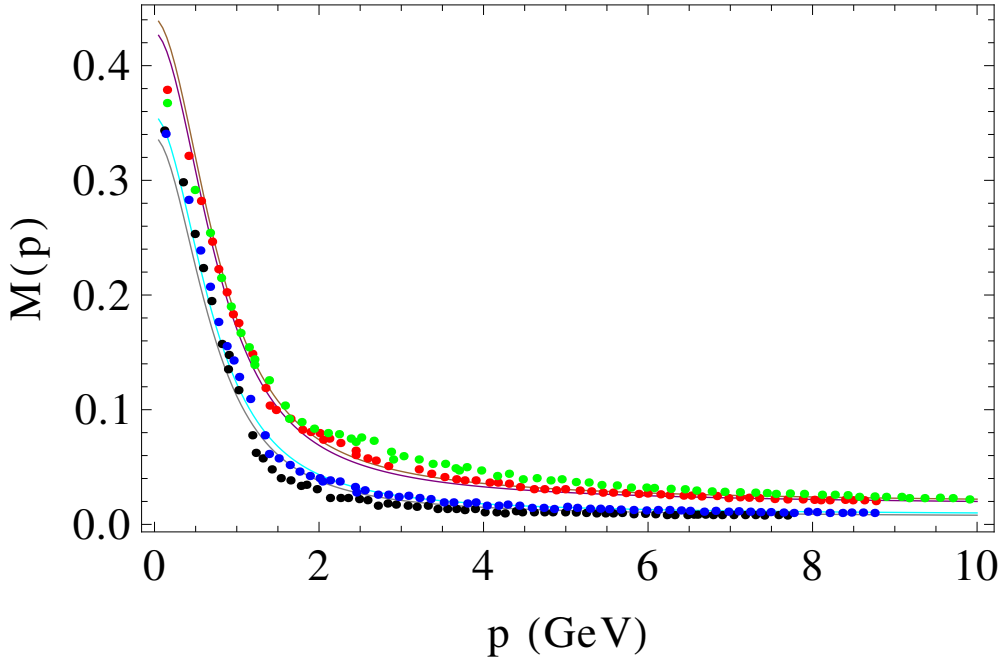


Our double expansion allows for the systematic inclusion of corrections, including RG-improvements.

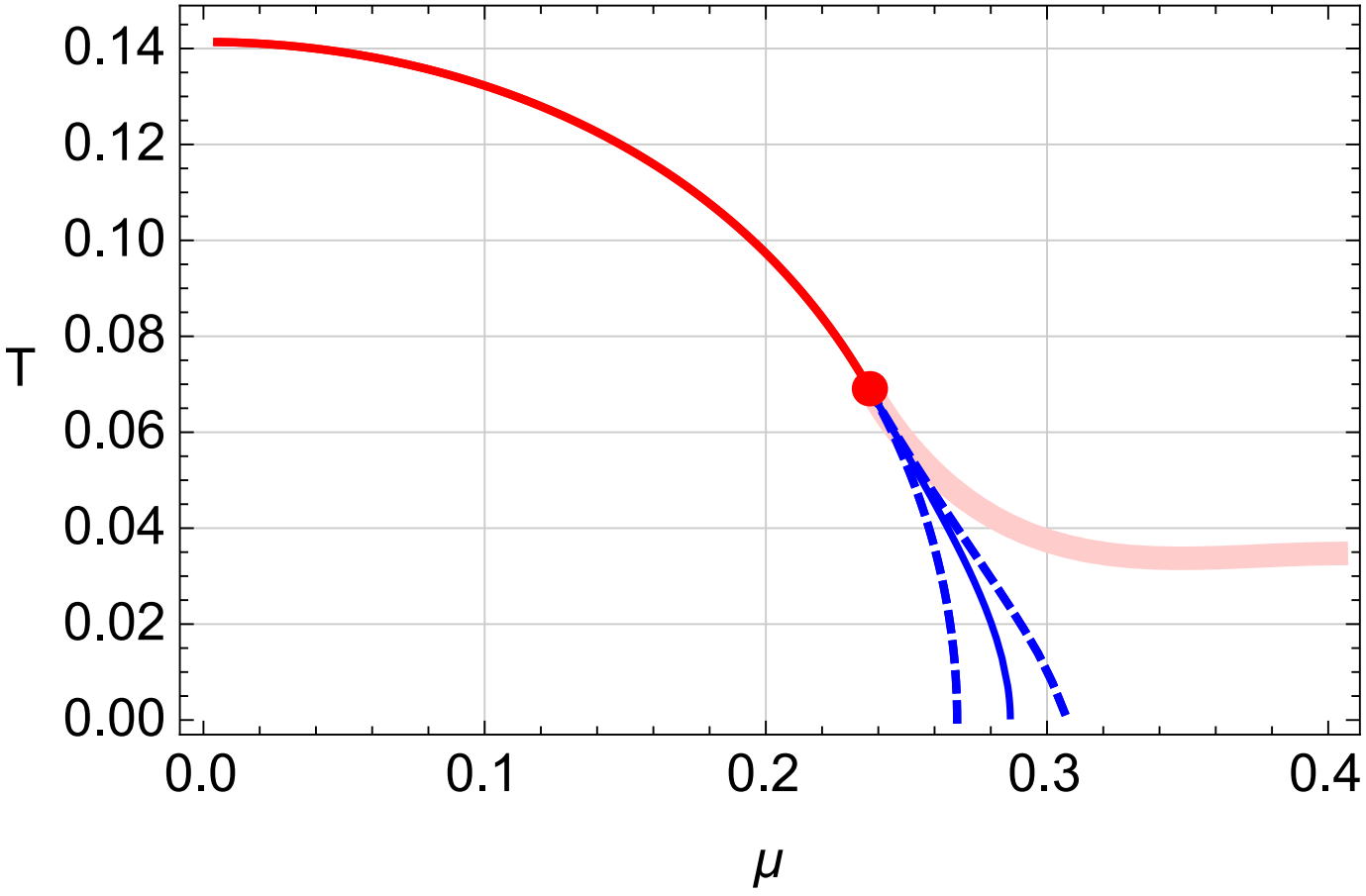
Good grasp on SχSB in the vacuum

[Peláez, Reinosa, Serreau, Tissier, Wschebor, Phys.Rev. D96 (2017) no.11, 114011.]





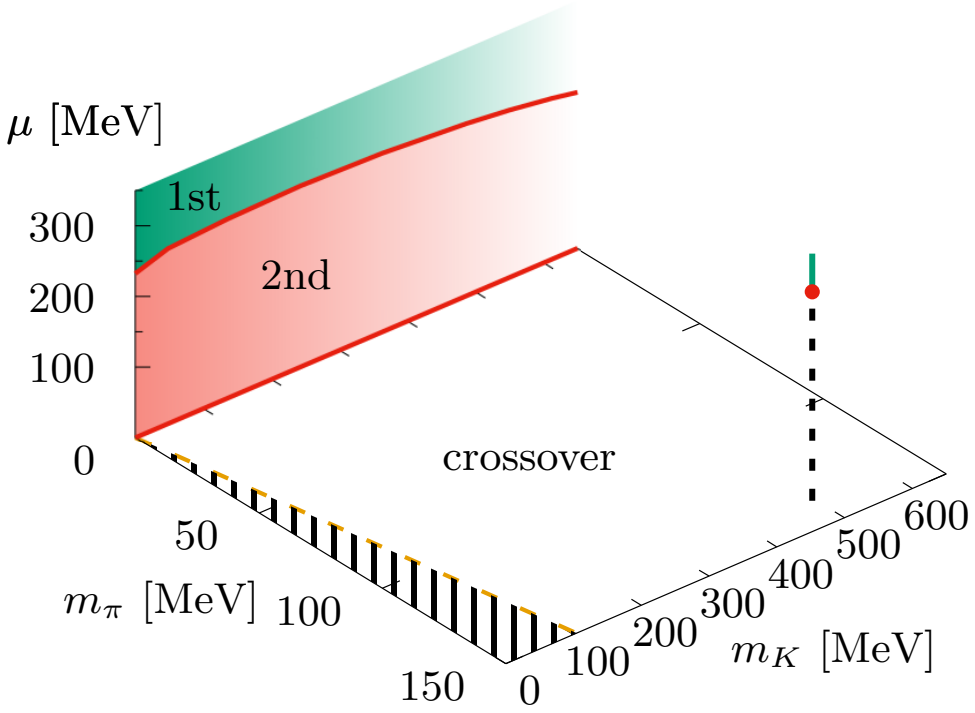
Phase diagram



[J. Maelger, UR, J. Serreau, arXiv:1903.04184.]

Phase diagram

Similar result as with quark-meson effective models at mean-field (extended) level without anomaly, as expected in a leading order $1/N_c$ calculation.



[S. Resch, F. Rennecke, B.-J. Schaefer, arXiv:1712.07961.]

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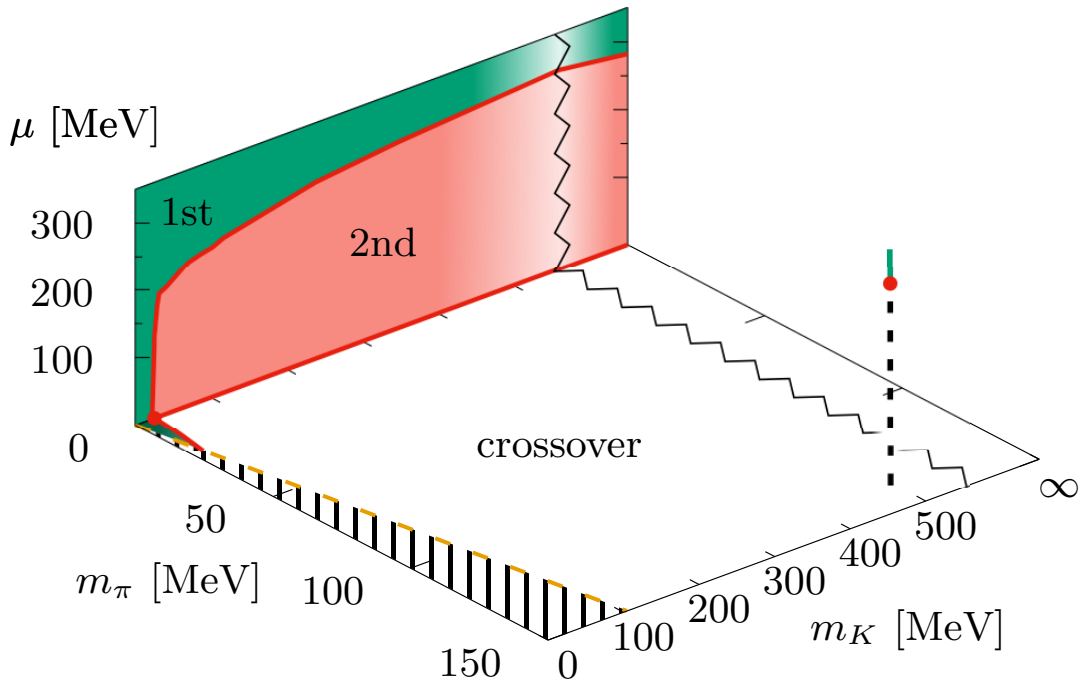

[S. Resch, F. Rennecke, B.-J. Schaefer, arXiv:1712.07961.]

CONCLUSIONS

- A simple model beyond the Faddeev-Popov recipe seems to render perturbation theory viable at all scales in the pure YM theory.
  - Lattice correlators are reproduced quite satisfactorily with a simple one-loop calculation. Two-loop corrections are tiny and improve the fits.
  - The phase structure of pure Yang-Mills is reproduced.
  - This applies also to QCD in the heavy quark-limit.
  - Certain aspects require the inclusion of two-loop corrections but the results seem to converge.
- Perturbation theory is not applicable in the presence of light quarks but a double parameter expansion in powers of g_g and $1/N_c$ gives a good account of chiral symmetry breaking already at leading order.
- TODO: NLO in the double parameter expansion to gain better grasp on the QCD phase diagram.