FRG analysis of Quantum Spin Liquids in Heisenberg models

Nico Gneist

3.4.2019

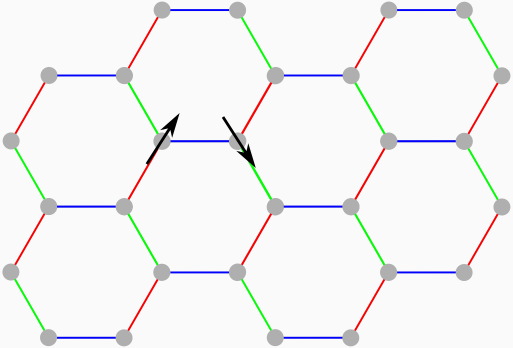
Institute for Theoretical Physics Cologne

Quantum Spin Liquids

Quantum Spin Liquid features

- Macroscopical entanglement
- Topological order
- Strong short-range correlations
- Fractionalization
- ...

Working definition: spin systems with non-trivial ground states which are not magnetic

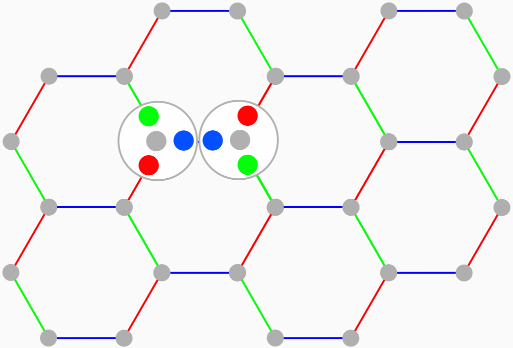


Quantum Spin Liquids

Quantum Spin Liquid features

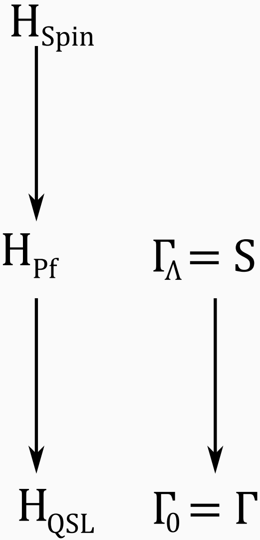
- Macroscopical entanglement
- Topological order
- Strong short-range correlations
- Fractionalization
- ...

Working definition: spin systems with non-trivial ground states which are not magnetic



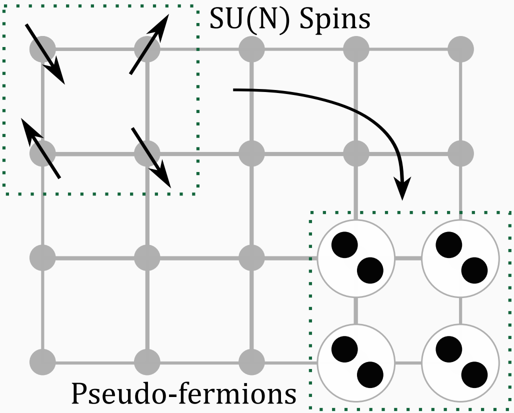
Spin Systems, pseudo-fermions and parton construction

- $\mathcal{H} = -J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j$
- Pseudo-fermions: $S_i^\alpha = \frac{1}{2} f_{i\mu}^\dagger T_{\mu\nu}^\alpha f_{i\nu}$
- Constraint for enlarging Hilbert space:
  $\{|\uparrow\rangle, |\downarrow\rangle\} \rightarrow \{|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle\}$
- $\mathcal{H} = \frac{-J}{2} \sum_{<ij>} f_{i\alpha}^\dagger f_{j\beta}^\dagger f_{j\alpha} f_{i\beta}$
- Parton construction [X.-G. Wen 2002]:
  $\mathcal{H}_{\text{eff}} =$
  $\sum_{<ij>} \left[t_{ij}^{\alpha\beta} f_{i\alpha}^\dagger f_{j\beta} + \Delta_{ij}^{\alpha\beta} f_{i\alpha}^\dagger f_{j\beta}^\dagger + h.c. \right]$



SU(N) Heisenberg Model [Arovas,Auerbach, 1988]

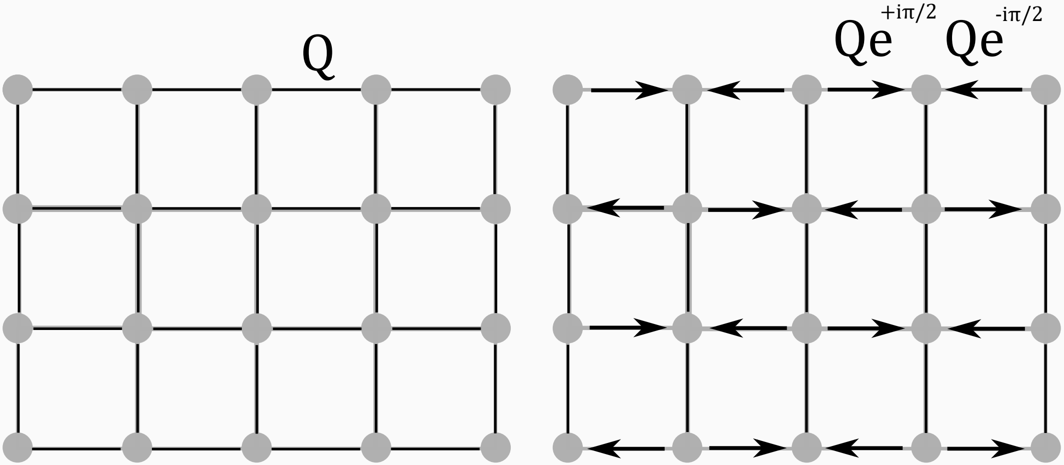
- $\mathcal{H} = \frac{J}{N} \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j$
- Pseudo-fermions: $S_i^\alpha = \frac{1}{2} f_{i\mu}^\dagger T_{\mu\nu}^\alpha f_{i\nu}$
- $S[f] = \int_\tau \sum_i f_{i\alpha}^\dagger \dot{f}_{i\alpha} - \frac{J}{N} \sum_{<i,j>} f_{i\alpha}^\dagger f_{j\alpha} f_{j\beta}^\dagger f_{i\beta}$
- Introduce $Q_{ij} \propto f_{j\alpha}^\dagger f_{i\alpha}$ via HS: $S_{HS}[f,Q] = \int_\tau \sum_i f_{i\alpha}^\dagger \dot{f}_{i\alpha} + \sum_{<i,j>} \left(Q_{ij}^\dagger Q_{ij} + \left(Q_{ij}^\dagger f_{j\alpha}^\dagger f_{i\alpha} + h.c.\right)\right)$



Large N Mean Field results [Arovas,Auerbach, 1988]

Mean-field ansatz for Q_{ij}: $Q_{ij} = Q\exp(i\theta)$, with $\theta = 0$ on y-bonds

Two phases: BZA (all $\theta = 0$) and π-flux ($\theta = \pm\pi/2$)



Ansatz and Projection

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\frac{\partial_k R_k}{\Gamma_k^{(2)}+R_k} \qquad \Gamma_k = \int_\tau \sum_i f_{i\alpha}^\dagger \dot{f}_{i\alpha} - \frac{J_k}{N}\sum_{<ij>} f_{i\alpha}^\dagger f_{j\alpha} f_{j\beta}^\dagger f_{i\beta}$$

Momentum space:

$$\Gamma_k = \int_1 f_{1\alpha}^\dagger f_{1\alpha} - \frac{J_k}{N}\int_{1,2,3,4} f_{1\alpha}^\dagger f_{2\alpha} f_{3_\beta}^\dagger f_{4_\beta}\left[\cos(k_{2,x}-k_{3,x})+\cos(k_{2,y}-k_{3,y})\right]\delta_{1234}$$

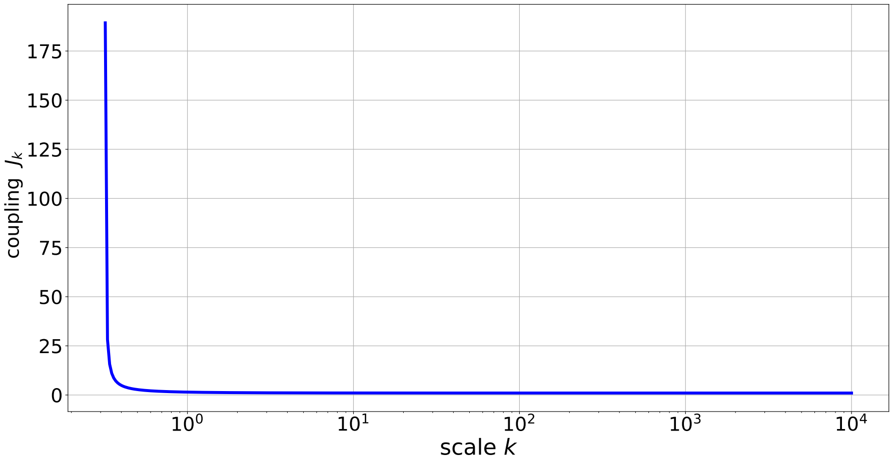
Point-like projection s.t. vertices are only spin-dependent:

$$f_{a\alpha}^\dagger = f_\alpha^\dagger \delta(\omega_a)\delta(k_a)$$

Derive flow:

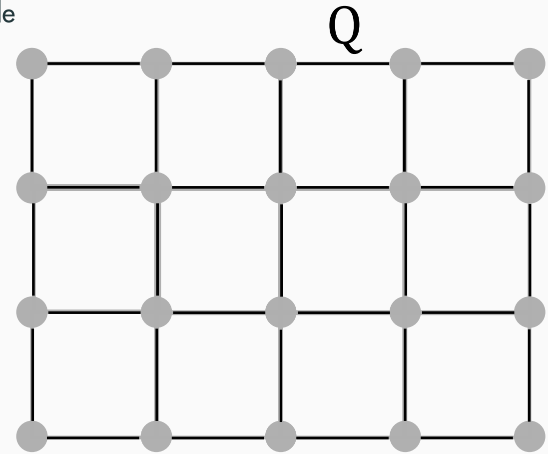
$$\partial_k J_k = \beta_{J_k}(J_k)$$

Result for J_k



Explicit symmetry breaking

- Add order parameter explicitely $\rightarrow$ enable flow into phase [Salmhofer et al., 2004]
- Suspect from mean-field: $Q_{ij} \propto f^{\dagger}_{j\alpha} f_{i\alpha}$
  $\rightarrow \int_{\tau} \sum_{<i,j>} Q_k \left[f^{\dagger}_{i\alpha} f_{j\alpha} + f^{\dagger}_{j\alpha} f_{i\alpha} \right]$

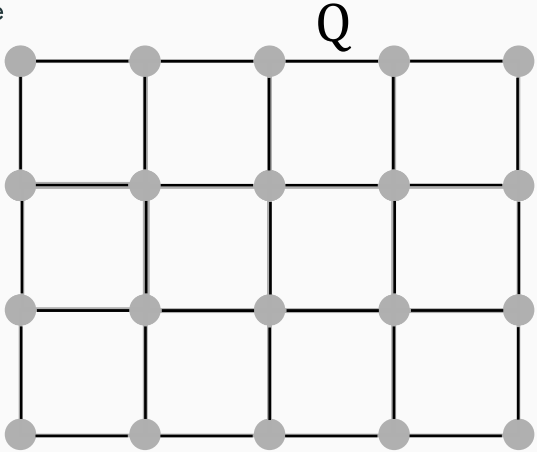


Explicit symmetry breaking

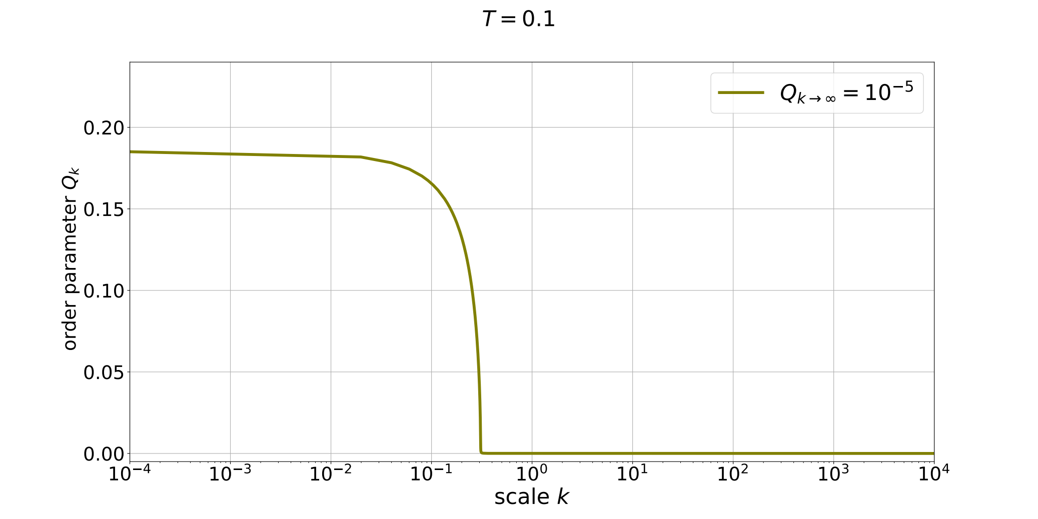
- Add order parameter explicitely $\rightarrow$ enable flow into phase [Salmhofer et al., 2004]

- Suspect from mean-field: $Q_{ij} \propto f^\dagger_{j\alpha} f_{i\alpha}$
$$\rightarrow \int_\tau \sum_{<i,j>} Q_k \left[f^\dagger_{i\alpha} f_{j\alpha} + f^\dagger_{j\alpha} f_{i\alpha} \right]$$

- New ansatz:
$$\int_\tau \Big\{ \sum_i f^\dagger_{i\alpha} \dot{f}_{i\alpha} - \frac{J_k}{N} \sum_{<ij>} f^\dagger_{i\alpha} f_{j\alpha} f^\dagger_{j\beta} f_{i\beta} + Q_k \sum_{<i,j>} \left[f^\dagger_{i\alpha} f_{j\alpha} + f^\dagger_{j\alpha} f_{i\alpha} \right] - \frac{I_k}{N} \sum_{<ij>} f^\dagger_{i\alpha} f_{j\alpha} f^\dagger_{i\beta} f_{j\beta} \Big\}$$

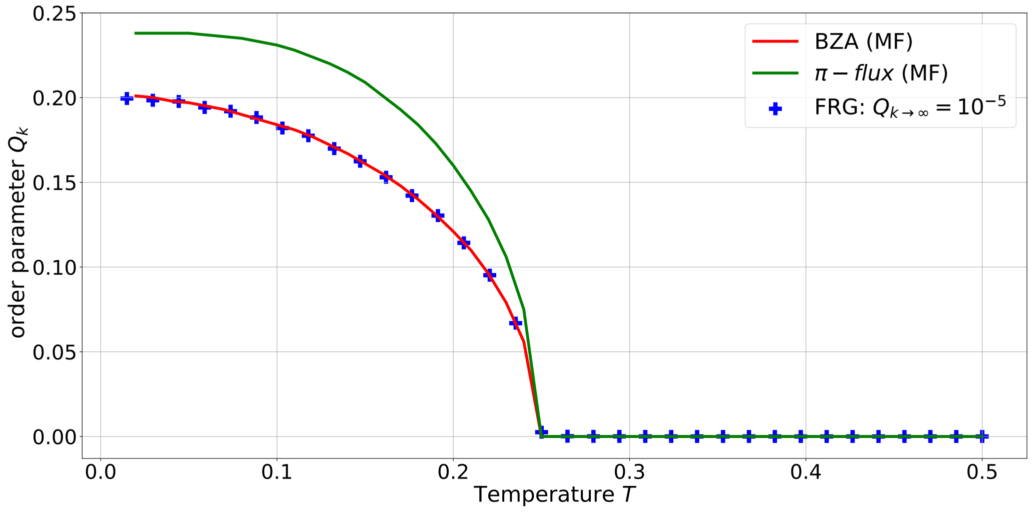


BZA Result



$T = 0.1$
$Q_{k\to\infty} = 10^{-5}$
order parameter Q_k
0.00
0.05
0.10
0.15
0.20
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
scale k

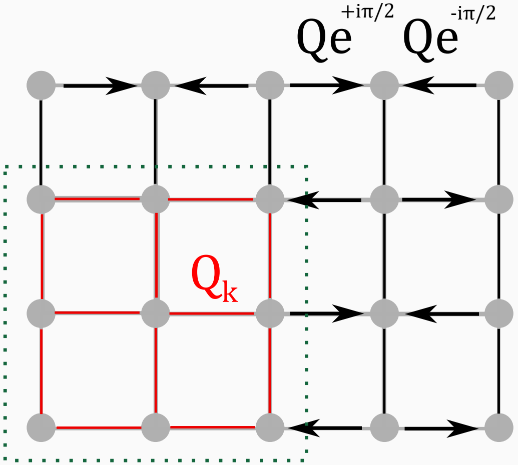
BZA Result



BZA (MF)
$\pi - flux$ (MF)
FRG: $Q_{k\to\infty} = 10^{-5}$
order parameter Q_k
Temperature T
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.1
0.2
0.3
0.4
0.5

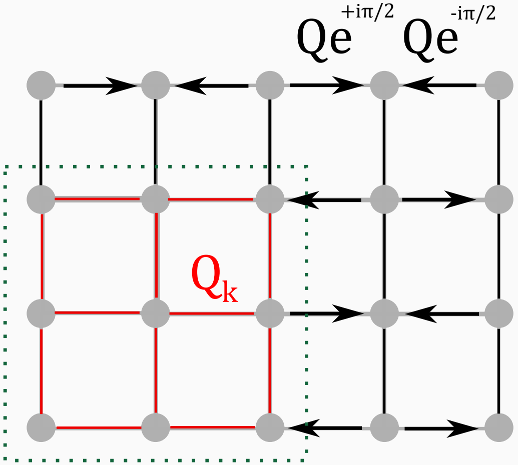
Accessing the π-Flux phase - Problem

- Flux phase requieres non-uniform bonds
- $\int_\tau \sum_{<i,j>} Q_k \left[f^\dagger_{i\alpha} f_{j\alpha} + f^\dagger_{j\alpha} f_{i\alpha} \right]$ fixes same value on each bond



Accessing the π-Flux phase - Problem

- Flux phase requieres non-uniform bonds
- $\int_\tau \sum_{<i,j>} Q_k \left[f^\dagger_{i\alpha} f_{j\alpha} + f^\dagger_{j\alpha} f_{i\alpha} \right]$ fixes same value on each bond
- introduce anisotropic couplings $Q_k^{(*),1,2,3,4}$
- need sublattice structure!



Accessing the π-Flux phase - Solution

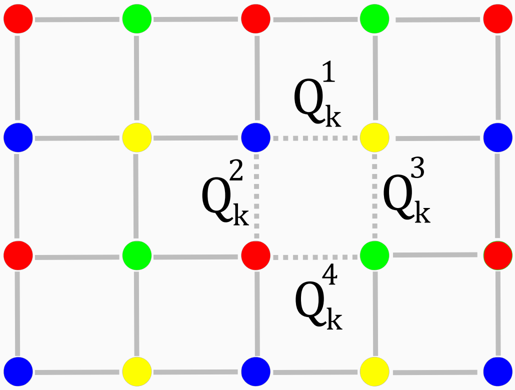
- 4 Sublattices: ABCD , e.g. $f_{i\alpha} \to f_{i\alpha}^{A}$

- Couplings example:

$$\int_\tau \sum_{<i,j>} Q_k^{(2)} f_{i\alpha}^{\dagger(A)} f_{j\alpha}^{(D)}$$

$$\int_\tau \sum_{<i,j>} Q_k^{*(2)} f_{j\alpha}^{\dagger(D)} f_{i\alpha}^{(A)}$$

- same for $Q_k^{(*)(2,3,4)}$ and $J_k^{(1,2,3,4)}$



Complete ansatz for π-Flux phase

- $$\Gamma_k = \int_\tau \Big\{ \sum_i f^\dagger_{i\alpha} \dot{f}_{i\alpha}$$
$$- \frac{J_k^{(1)}}{N} \sum_{<ij>} f^{\dagger(A)}_{i\alpha} f^{(C)}_{j\alpha} f^{\dagger(C)}_{j\beta} f^{(A)}_{i\beta} - \frac{J_k^{(2)}}{N} \sum_{<ij>} f^{\dagger(A)}_{i\alpha} f^{(D)}_{j\alpha} f^{\dagger(D)}_{j\beta} f^{(A)}_{i\beta}$$
$$- \frac{J_k^{(3)}}{N} \sum_{<ij>} f^{\dagger(B)}_{i\alpha} f^{(C)}_{j\alpha} f^{\dagger(C)}_{j\beta} f^{(B)}_{i\beta} - \frac{J_k^{(4)}}{N} \sum_{<ij>} f^{\dagger(B)}_{i\alpha} f^{(D)}_{j\alpha} f^{\dagger(D)}_{j\beta} f^{(B)}_{i\beta}$$

INITIAL COUPLING

Complete ansatz for π-Flux phase

- $$\Gamma_k = \int_\tau \Big\{ \sum_i f^\dagger_{i\alpha} \dot{f}_{i\alpha}$$
$$- \frac{J_k^{(1)}}{N} \sum_{<ij>} f^{\dagger(A)}_{i\alpha} f^{(C)}_{j\alpha} f^{\dagger(C)}_{j\beta} f^{(A)}_{i\beta} - \frac{J_k^{(2)}}{N} \sum_{<ij>} f^{\dagger(A)}_{i\alpha} f^{(D)}_{j\alpha} f^{\dagger(D)}_{j\beta} f^{(A)}_{i\beta}$$
$$- \frac{J_k^{(3)}}{N} \sum_{<ij>} f^{\dagger(B)}_{i\alpha} f^{(C)}_{j\alpha} f^{\dagger(C)}_{j\beta} f^{(B)}_{i\beta} - \frac{J_k^{(4)}}{N} \sum_{<ij>} f^{\dagger(B)}_{i\alpha} f^{(D)}_{j\alpha} f^{\dagger(D)}_{j\beta} f^{(B)}_{i\beta}$$

INITIAL COUPLING

$$+ \sum_{<i,j>} Q_k^{(1)} f^{\dagger(A)}_{i\alpha} f^{(C)}_{j\alpha} + \sum_{<i,j>} Q_k^{(2)} f^{\dagger(A)}_{i\alpha} f^{(B)}_{j\alpha}$$
$$+ \sum_{<i,j>} Q_k^{(3)} f^{\dagger(B)}_{i\alpha} f^{(C)}_{j\alpha} + \sum_{<i,j>} Q_k^{(4)} f^{\dagger(B)}_{i\alpha} f^{(C)}_{j\alpha}$$

SYMMETRY BREAKING TERM

+ h.c. order parameters

Complete ansatz for π-Flux phase

- $\Gamma_k = \int_\tau \Big\{ \sum_i f^\dagger_{i\alpha} \dot{f}_{i\alpha}$

$$- \frac{J_k^{(1)}}{N} \sum_{<ij>} f^{\dagger(A)}_{i\alpha} f^{(C)}_{j\alpha} f^{\dagger(C)}_{j\beta} f^{(A)}_{i\beta} - \frac{J_k^{(2)}}{N} \sum_{<ij>} f^{\dagger(A)}_{i\alpha} f^{(D)}_{j\alpha} f^{\dagger(D)}_{j\beta} f^{(A)}_{i\beta}$$

INITIAL COUPLING

$$- \frac{J_k^{(3)}}{N} \sum_{<ij>} f^{\dagger(B)}_{i\alpha} f^{(C)}_{j\alpha} f^{\dagger(C)}_{j\beta} f^{(B)}_{i\beta} - \frac{J_k^{(4)}}{N} \sum_{<ij>} f^{\dagger(B)}_{i\alpha} f^{(D)}_{j\alpha} f^{\dagger(D)}_{j\beta} f^{(B)}_{i\beta}$$

$$+ \sum_{<i,j>} Q_k^{(1)} f^{\dagger(A)}_{i\alpha} f^{(C)}_{j\alpha} + \sum_{<i,j>} Q_k^{(2)} f^{\dagger(A)}_{i\alpha} f^{(B)}_{j\alpha}$$

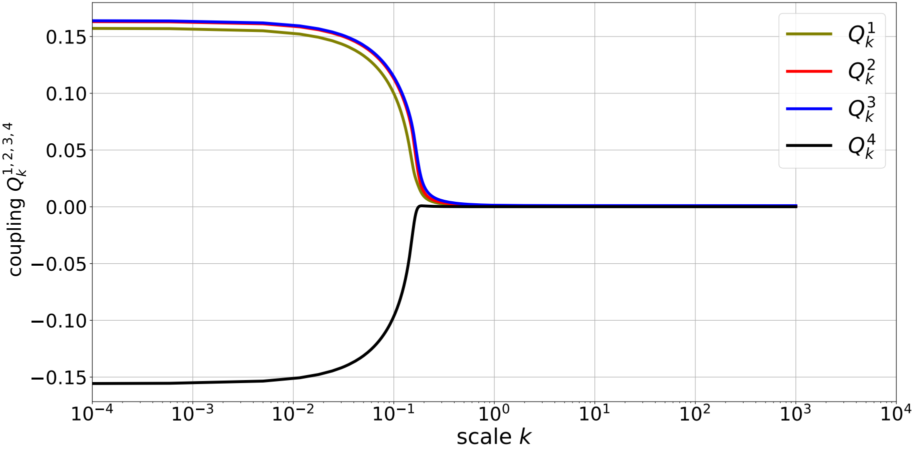
SYMMETRY BREAKING TERM

$$+ \sum_{<i,j>} Q_k^{(3)} f^{\dagger(B)}_{i\alpha} f^{(C)}_{j\alpha} + \sum_{<i,j>} Q_k^{(4)} f^{\dagger(B)}_{i\alpha} f^{(C)}_{j\alpha}$$

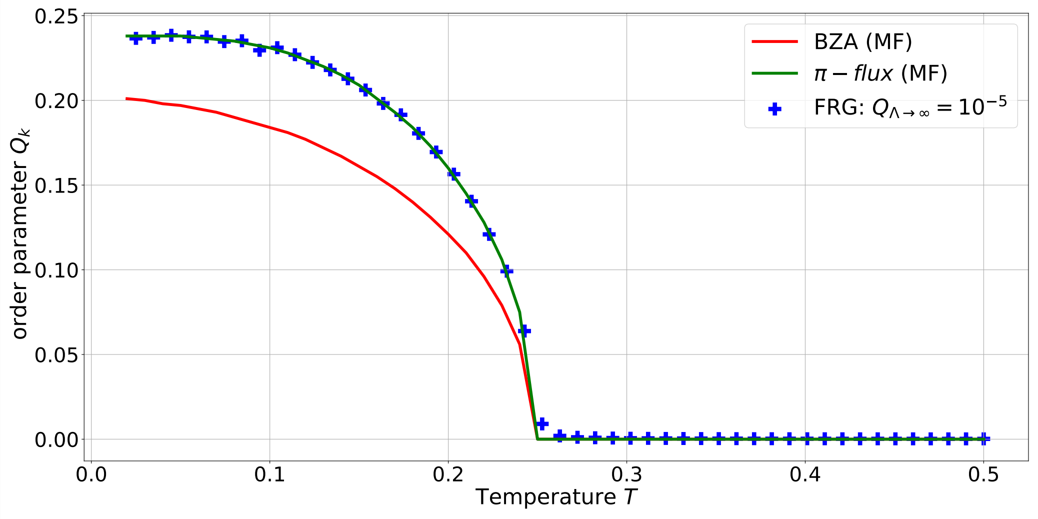
+ h.c. order parameters

+ 32 new couplings GENERATED COUPLINGS

Results for π-flux ansatz



Results for π-flux ansatz



Summary and Outlook

- Introducing FRG approach for Spin Liquid phases
- Reproduces both known phases of Large N SU(N)-model on square lattice
- Sublattice approach generalizable for other geometries (Kagome, Honeycomb,...)
- Constraint can be implemented by Popov-Fedotov
- Next stop: Kitaev model