GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Dynamical generation of low-energy couplings

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Functional methods in strongly correlated systems

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Motivation: QCD at low energies

Chiral symmetry

Linear realization

Linear sigma model (LSM)
Extended LSM (eLSM)
Quark-meson model

Nonlinear realization

Chiral perturbation theory (ChPT)

Integration

Tree level
and
functional renormalization group (FRG)
[PRD98, 014024; PRD99, 054023]

Low-energy limit
same as for QCD?

Effective field theories

Chiral perturbation theory (ChPT)

- Systematic analysis of **hadronic n-point functions** of QCD
  [Gasser, Leutwyler ’84,’85; Leutwyler ’94; etc.]

- Pion interactions dominate the low-energy regime $(N_f = 2)$

- **Chiral expansion:** Most general chiral-invariant Lagrangian with terms coupled by the low-energy couplings of QCD,

$$\begin{aligned}\mathcal{L}_{\mathrm{ChPT}} = \frac{1}{2}\left(\partial_\mu \vec{\pi}\right)^2 - \frac{1}{2} m_\pi^2 \vec{\pi}^{\,2} + C_{1,\mathrm{ChPT}} \left(\vec{\pi}^{\,2}\right)^2 + C_{2,\mathrm{ChPT}}\, \vec{\pi}^2 \left(\partial_\mu \vec{\pi}\right)^2 \\ + C_{3,\mathrm{ChPT}} \left(\partial_\mu \vec{\pi}\right)^2 \left(\partial_\nu \vec{\pi}\right)^2 + C_{4,\mathrm{ChPT}} \left(\partial_\mu \vec{\pi} \cdot \partial_\nu \vec{\pi}\right)^2 \\ + \mathcal{O}\left(\pi^6, \partial^6\right),\end{aligned}$$

$\vec{\pi}$: “stereographic pions”

Extended linear sigma model (eLSM)

- Linear realization of chiral symmetry ("chiral partners")
- Applications at nonzero temperature and chemical potential
- Contains all $J^P = 0^\pm, 1^\pm$ mesons up to 2 GeV in mass
- **Representations** of $U(N_f)_r \times U(N_f)_l$:

$$\Phi \sim \bar{q}_r q_l \to U_l \, \Phi \, U_r^\dagger ,$$
$$R_\mu \sim \bar{q}_r \gamma_\mu q_r \to U_r R_\mu U_r^\dagger ,$$
$$L_\mu \sim \bar{q}_l \gamma_\mu q_l \to U_l L_\mu U_l^\dagger$$

- Vector and axial-vector mesons:

$$V_\mu = \frac{1}{2} \left(L_\mu + R_\mu \right) , \qquad A_\mu = \frac{1}{2} \left(L_\mu - R_\mu \right)$$

- **Disclaimer:** No attempt to compete with ChPT

eLSM Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{eLSM}} = {} & \operatorname{tr}\left[\left(D^{\mu}\Phi\right)^{\dagger} D_{\mu}\Phi\right] - m_0^2 \operatorname{tr}\left(\Phi^{\dagger}\Phi\right) \\
& -\lambda_1 \left[\operatorname{tr}\left(\Phi^{\dagger}\Phi\right)\right]^2 - \lambda_2 \operatorname{tr}\left[\left(\Phi^{\dagger}\Phi\right)^2\right] \\
& -\frac{1}{4}\operatorname{tr}\left(L_{\mu\nu}^2 + R_{\mu\nu}^2\right) + \operatorname{tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)\left(L_{\mu}^2 + R_{\mu}^2\right)\right] \\
& + \operatorname{tr}\left[H\left(\Phi + \Phi^{\dagger}\right)\right] - c_A\left(\det\,\Phi + \det\,\Phi^{\dagger}\right) \\
& + i\frac{g_2}{2}\left(\operatorname{tr}\left\{L^{\mu\nu}\left[L_{\mu}, L_{\nu}\right]\right\} + \operatorname{tr}\left\{R^{\mu\nu}\left[R_{\mu}, R_{\nu}\right]\right\}\right) \\
& + \frac{h_1}{2}\operatorname{tr}\left(\Phi^{\dagger}\Phi\right)\operatorname{tr}\left(L_{\mu}^2 + R_{\mu}^2\right) + h_2 \operatorname{tr}\left(\left|L_{\mu}\Phi\right|^2 + \left|\Phi R_{\mu}\right|^2\right) \\
& + 2h_3 \operatorname{tr}\left(\Phi R^{\mu}\Phi^{\dagger} L_{\mu}\right) + g_3\left[\operatorname{tr}\left(L^{\mu}L^{\nu}L_{\mu}L_{\nu}\right) + \operatorname{tr}\left(R^{\mu}R^{\nu}R_{\mu}R_{\nu}\right)\right] \\
& + g_4\left[\operatorname{tr}\left(L^{\mu}L_{\mu}L^{\nu}L_{\nu}\right) + \operatorname{tr}\left(R^{\mu}R_{\mu}R^{\nu}R_{\nu}\right)\right] + g_5 \operatorname{tr}\left(L^{\mu}L_{\mu}\right)\operatorname{tr}\left(R^{\nu}R_{\nu}\right) \\
& + g_6\left[\operatorname{tr}\left(L^{\mu}L_{\mu}\right)\operatorname{tr}\left(L^{\nu}L_{\nu}\right) + \operatorname{tr}\left(R^{\mu}R_{\mu}\right)\operatorname{tr}\left(R^{\nu}R_{\nu}\right)\right],
\end{aligned}
$$

$$
D_{\mu}\Phi = \partial_{\mu}\Phi - ig_1\left(L_{\mu}\Phi - \Phi R_{\mu}\right), \qquad L_{\mu\nu} = \partial_{\mu}L_{\nu} - \partial_{\nu}L_{\mu}
$$

eLSM publications

- Introduction of the eLSM
  [Parganlija, Giacosa, Rischke ’10;
  Parganlija, Kovacs, Wolf, Giacosa, Rischke ’13]
- Chiral partner of the nucleon
  [Gallas, Giacosa, Rischke ’10; Lakaschus, Mauldin, Giacosa, Rischke ’18]
- Incorporating scalar glueball
  [Janowski, Giacosa, Rischke ’14; Giacosa, Sammet, Janowski ’17]
- Baryon multiplets
  [Olbrich, Zetenyi, Giacosa, Rischke ’16,’18]
- Nonzero-temperature study within the FRG
  [JE, Grahl, Rischke ’15]

$O(4)$ quark-meson model

- Two-flavor model, $N_f = 2$
- Specific limit of the eLSM
- **Euclidean** action:

$$
\begin{aligned}
S = \int_x \Big\{ \frac{1}{2}\left(\partial_\mu \sigma\right) \partial_\mu \sigma + \frac{1}{2}\left(\partial_\mu \vec{\pi}\right) \cdot \partial_\mu \vec{\pi} + U(\rho) - h_{\mathrm{ESB}} \sigma \\
+ \bar{\psi}\left(\gamma_\mu \partial_\mu + y\, \Phi_5\right) \psi \Big\},
\end{aligned}
$$

$$
\begin{aligned}
\Phi_5 &= \sigma\, t_0 + i\gamma_5 \vec{\pi} \cdot \vec{t}, \\
\rho &= \sigma^2 + \vec{\pi}^2
\end{aligned}
$$

- **Explicit** and **spontaneous** symmetry breaking implemented,

$$
h_{\mathrm{ESB}} \neq 0, \qquad \sigma \to \phi + \sigma
$$

Outline

- **Research objective:** Dynamical generation of (mesonic) **higher-derivative interactions** from quark-meson fluctuations

- **(Final) goals:**
  (a) Compute **low-energy couplings** of the *O(4)* quark-meson model
  (b) Determine appropriate **renormalization scales** for purely pionic models as obtained from the **FRG**

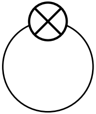
- **Exploratory works:**
  (a) Tree-level integration of resonances within the eLSM
  [Divotgey, Kovacs, Giacosa, Rischke '18]
  (b) (First) FRG study of higher-derivative pion interactions
  [JE, Divotgey, Mitter, Rischke '18]

Functional renormalization group

Functional renormalization group (FRG)

- Implementation of the **Wilsonian RG** idea
- Renormalization scale(k)-dependent **effective action** Γ_k
- FRG flow equation: [Wetterich '93]

$$\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{tr}\left[\partial_k R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1}\right] = \frac{1}{2}$$



- Regulator function R_k provides correct integration limits
- **Nonperturbative** continuum method

Flow in theory space

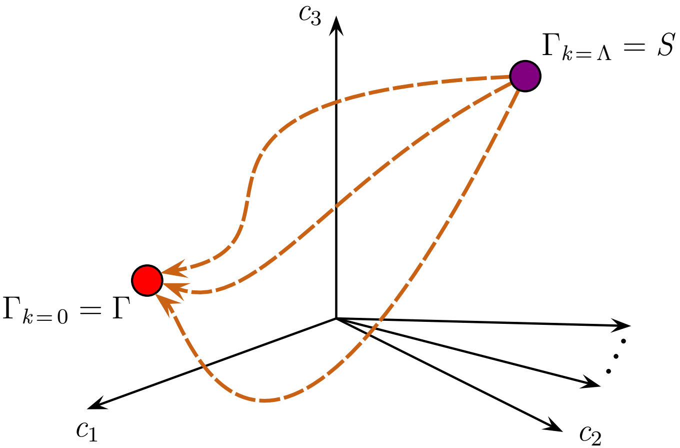


Figure 1: Theory space spanned by generic couplings.

$O(4)$ quark-meson model – truncations

- Local potential approximation (**LPA**), i.e., consider scale-dependent **effective potential**,

$$\Gamma_k = \int_x \Bigg\{ \frac{1}{2} \left(\partial_\mu \sigma\right) \partial_\mu \sigma + \frac{1}{2} \left(\partial_\mu \vec{\pi}\right) \cdot \partial_\mu \vec{\pi} + U_k(\rho) - h_{\mathrm{ESB}} \sigma \\ + \bar{\psi} \left(\gamma_\mu \partial_\mu + y_k \Phi_5\right) \psi \Bigg\}$$

- **LPA′**, i.e., include **wave-function renormalization**,

$$\Gamma_k = \int_x \Bigg\{ \frac{Z_k^\sigma}{2} \left(\partial_\mu \sigma\right) \partial_\mu \sigma + \frac{Z_k^\pi}{2} \left(\partial_\mu \vec{\pi}\right) \cdot \partial_\mu \vec{\pi} + U_k(\rho) - h_{\mathrm{ESB}} \sigma \\ + \bar{\psi} \left(Z_k^\psi \gamma_\mu \partial_\mu + y_k \Phi_5 \right) \psi \Bigg\}$$

Higher-derivative interactions

Linear effective action

- $O(4)$ field variable $\varphi = (\vec{\pi}, \sigma)$
- Introduce **higher-derivative couplings**,

$$\begin{aligned}\Gamma_k = \int_x \Big\{ & \frac{Z_k}{2} (\partial_\mu \varphi) \cdot \partial_\mu \varphi + U_k(\rho) - h_{\mathrm{ESB}} \sigma \\ & + C_{2,k} (\varphi \cdot \partial_\mu \varphi)^2 + Z_{2,k} \varphi^2 (\partial_\mu \varphi) \cdot \partial_\mu \varphi \\ & - C_{3,k} [(\partial_\mu \varphi) \cdot \partial_\mu \varphi]^2 - C_{4,k} [(\partial_\mu \varphi) \cdot \partial_\nu \varphi]^2 \\ & - C_{5,k} \varphi \cdot (\partial_\mu \partial_\mu \varphi) (\partial_\nu \varphi) \cdot \partial_\nu \varphi \\ & - C_{6,k} \varphi^2 (\partial_\mu \partial_\nu \varphi) \cdot \partial_\mu \partial_\nu \varphi \\ & - C_{7,k} (\varphi \cdot \partial_\mu \partial_\mu \varphi)^2 - C_{8,k} \varphi^2 (\partial_\mu \partial_\mu \varphi)^2 \\ & + \bar{\psi} \left(Z_k^\psi \gamma_\mu \partial_\mu + y_k \Phi_5 \right) \psi \Big\}\end{aligned}$$

- **Goal:** Compute the IR values of all scale-dependent quantities

Renormalized quantities

- Renormalized fields:

$$\tilde{\sigma} = \sqrt{Z_k^\pi}\sigma, \qquad \tilde{\vec{\pi}} = \sqrt{Z_k^\pi}\vec{\pi}, \qquad \tilde{\psi} = \sqrt{Z_k^\psi}\psi, \qquad \tilde{\bar{\psi}} = \sqrt{Z_k^\psi}\bar{\psi}$$

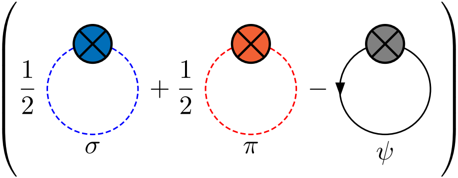
- (Squared) renormalized masses:

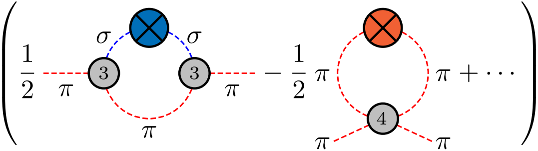
$$M_{\sigma,k}^2 = \frac{m_{\sigma,k}^2}{Z_k^\sigma}, \qquad M_{\pi,k}^2 = \frac{m_{\pi,k}^2}{Z_k^\pi}, \qquad M_{\psi,k}^2 = \frac{m_{\psi,k}^2}{\left(Z_k^\psi\right)^2}$$

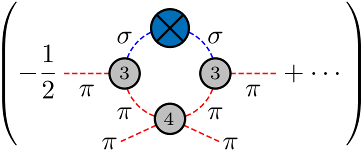
- Renormalized higher-derivative couplings:

$$\tilde{C}_{i,k} = \frac{C_{i,k}}{\left(Z_k^\pi\right)^2} \quad i = 1, \ldots, 8, \qquad \tilde{Z}_{2,k} = \frac{Z_{2,k}}{\left(Z_k^\pi\right)^2}$$

Flow equations – examples

$\partial_k U_k = \mathcal{V}^{-1}$  ,

$\partial_k Z_k^\pi = \mathcal{V}^{-1} \left. \frac{\mathrm{d}}{\mathrm{d}p^2} \right|_{p^2=0}$  ,

$\partial_k C_{2,k} = \mathcal{V}^{-1} \left. \frac{\mathrm{d}}{\mathrm{d}p^2} \right|_{p^2=0}$  ,

$\vdots$

Masses and pion decay constant

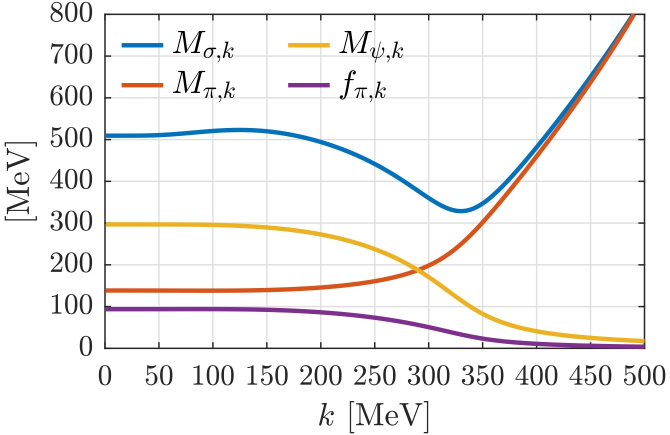


Figure 2: Scale evolution of the renormalized meson and quark masses as well as the pion decay constant; [Divotgey, JE, Mitter '19].

Higher-derivative couplings of $\mathcal{O}(\partial^2)$

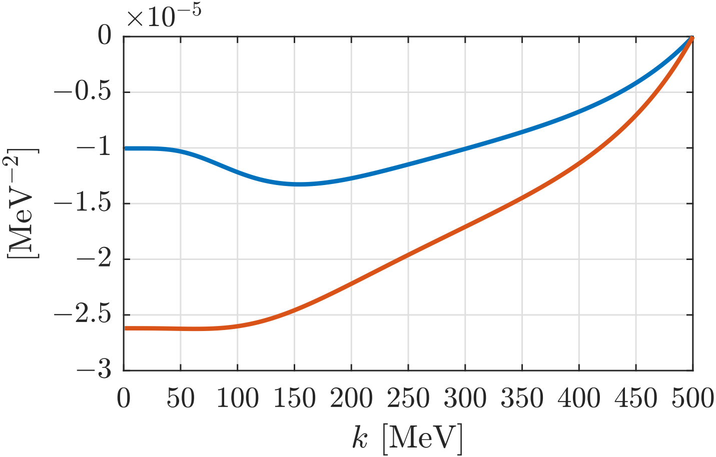


Figure 3: Scale evolution of the renormalized higher-derivative couplings of $\mathcal{O}(\partial^2)$; [Divotgey, JE, Mitter '19].

Higher-derivative couplings of $\mathcal{O}(\partial^4)$

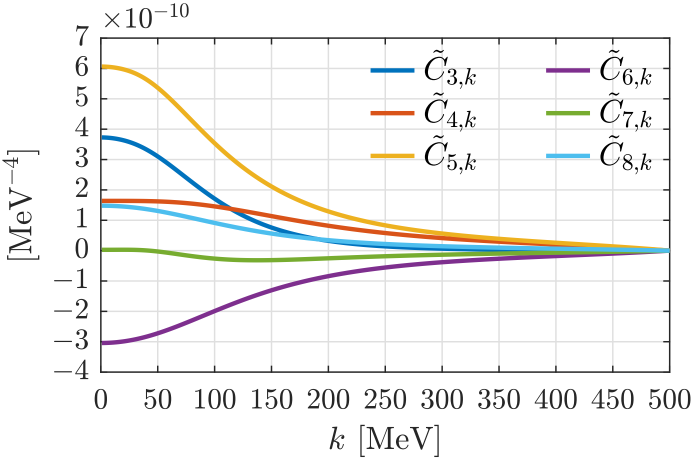


Figure 4: Scale evolution of the renormalized higher-derivative couplings of $\mathcal{O}(\partial^4)$; [Divotgey, JE, Mitter '19].

Effective pion action

Vacuum manifold

- Restrict dynamics to **vacuum manifold** $SO(4)/SO(3) \cong S^3$,

$$\tilde{\varphi} = \big(\tilde{\vec{\pi}}, \tilde{\sigma}\big) = \Sigma\big(\tilde{\zeta}\big)\tilde{\phi} \longrightarrow \Sigma\big(\tilde{\zeta}\big)\tilde{\phi}_0, \qquad \Sigma\big(\tilde{\zeta}\big) \in SO(4),$$

$$\tilde{\phi} = \big(\vec{0}, \tilde{\theta} \equiv \sqrt{\tilde{\varphi}^2}\big), \qquad \tilde{\phi}_0 = \big(\vec{0}, f_\pi\big)$$

- Choose **stereographic coordinates**,

$$\tilde{\zeta}^a = \frac{\tilde{\pi}^a}{f_\pi + \tilde{\sigma}}, \quad a = 1, 2, 3$$

Remark: Basic objects

- Construct **Maurer-Cartan form**,

$$\begin{aligned}\alpha_\mu &= \Sigma^{-1}(\tilde{\zeta})\partial_\mu \Sigma(\tilde{\zeta}) \\ &\equiv {e_\alpha}^a(\tilde{\zeta})\partial_\mu \tilde{\zeta}^\alpha x_a + {\omega_\alpha}^i(\tilde{\zeta})\partial_\mu \tilde{\zeta}^\alpha s_i, \quad \alpha = 1,2,3,\end{aligned}$$

broken generators: $x_a, \quad a = 1,2,3,$

unbroken generators: $s_i, \quad i = 1,2,3$

- **Geometry** of $SO(4)$ completely described by α_μ,

frame: ${e_\alpha}^a(\tilde{\zeta}) = \dfrac{{\delta_\alpha}^a}{1+\tilde{\zeta}^2}$, $\qquad SO(3)$-connection: ${\omega_\alpha}^i(\tilde{\zeta})$

- Define **metric** on $SO(4)/SO(3)$,

$$g_{\alpha\beta}(\tilde{\zeta}) = \delta_{ab}\, {e_\alpha}^a(\tilde{\zeta})\, {e_\beta}^b(\tilde{\zeta}) = \frac{4\delta_{\alpha\beta}}{\left(1+\tilde{\zeta}^2\right)^2}$$

Remark: Nonlinear effective action

$$
\begin{aligned}
\Gamma_k = \int_x \bigg\{ & \frac{f_\pi^2}{2}\, g_{\alpha\beta}\big(\nabla_\mu \tilde{\zeta}^\alpha\big)\nabla_\mu \tilde{\zeta}^\beta \\
& - \big(\tilde{C}_{6,k} + \tilde{C}_{8,k}\big) f_\pi^4\, g_{\alpha\beta}\big(\nabla_\mu \nabla_\mu \tilde{\zeta}^\alpha\big)\nabla_\nu \nabla_\nu \tilde{\zeta}^\beta \\
& - \big(\tilde{C}_{3,k} - \tilde{C}_{5,k} + \tilde{C}_{6,k} + \tilde{C}_{7,k} + \tilde{C}_{8,k}\big) f_\pi^4 \\
& \qquad \times g_{\alpha\beta}\, g_{\gamma\delta}\big(\nabla_\mu \tilde{\zeta}^\alpha\big)\big(\nabla_\mu \tilde{\zeta}^\beta\big)\big(\nabla_\nu \tilde{\zeta}^\gamma\big)\nabla_\nu \tilde{\zeta}^\delta \\
& - \tilde{C}_{4,k} f_\pi^4\, g_{\alpha\beta}\, g_{\gamma\delta}\big(\nabla_\mu \tilde{\zeta}^\alpha\big)\big(\nabla_\nu \tilde{\zeta}^\beta\big)\big(\nabla_\mu \tilde{\zeta}^\gamma\big)\nabla_\nu \tilde{\zeta}^\delta \\
& - \tilde{h}_{\mathrm{ESB}} f_\pi\, \frac{1 - \tilde{\zeta}^2}{1 + \tilde{\zeta}^2} \bigg\},
\end{aligned}
$$

$$
\begin{aligned}
\nabla_\mu \tilde{\zeta}^\alpha &\equiv \partial_\mu \tilde{\zeta}^\alpha, \\
\nabla_\mu \nabla_\nu \tilde{\zeta}^\alpha &\equiv \nabla_\mu \partial_\nu \tilde{\zeta}^\alpha = \partial_\mu \partial_\nu \tilde{\zeta}^\alpha + \Gamma^\alpha{}_{\beta\gamma}\big(\partial_\mu \tilde{\zeta}^\beta\big)\partial_\nu \tilde{\zeta}^\gamma
\end{aligned}
$$

Effective pion action

$$\Gamma_k = \int_x \left\{ \frac{1}{2} \left(\partial_\mu \tilde{\Pi}_a\right) \partial_\mu \tilde{\Pi}^a + \frac{1}{2} \tilde{\mathcal{M}}^2_{\Pi,k} \tilde{\Pi}_a \tilde{\Pi}^a - \tilde{\mathcal{C}}_{1,k} \left(\tilde{\Pi}_a \tilde{\Pi}^a\right)^2 \right.$$

$$+ \tilde{\mathcal{Z}}_{2,k} \tilde{\Pi}_a \tilde{\Pi}^a \left(\partial_\mu \tilde{\Pi}_b\right) \partial_\mu \tilde{\Pi}^b$$

$$- \tilde{\mathcal{C}}_{3,k} \left[\left(\partial_\mu \tilde{\Pi}_a\right) \partial_\mu \tilde{\Pi}^a\right]^2 - \tilde{\mathcal{C}}_{4,k} \left[\left(\partial_\mu \tilde{\Pi}_a\right) \partial_\nu \tilde{\Pi}^a\right]^2$$

$$- \tilde{\mathcal{C}}_{5,k} \tilde{\Pi}_a \left(\partial_\mu \partial_\mu \tilde{\Pi}^a\right) \left(\partial_\nu \tilde{\Pi}_b\right) \partial_\nu \tilde{\Pi}^b$$

$$- \tilde{\mathcal{C}}_{6,k} \tilde{\Pi}_a \tilde{\Pi}^a \left(\partial_\mu \partial_\nu \tilde{\Pi}_b\right) \partial_\mu \partial_\nu \tilde{\Pi}^b$$

$$\left. - \tilde{\mathcal{C}}_{8,k} \tilde{\Pi}_a \tilde{\Pi}^a \left(\partial_\mu \partial_\mu \tilde{\Pi}_b\right) \partial_\nu \partial_\nu \tilde{\Pi}^b \right\},$$

$$\tilde{\Pi}^a = 2 f_\pi \tilde{\zeta}^a, \quad a = 1, 2, 3$$

Low-energy couplings

- (Squared) renormalized **pion mass**:

$$\tilde{\mathcal{M}}^2_{\Pi,k} = \frac{\tilde{h}_{\mathrm{ESB}}}{f_\pi}, \qquad \tilde{h}_{\mathrm{ESB}} = \frac{h_{\mathrm{ESB}}}{\sqrt{Z^\pi_k}}$$

- Renormalized **low-energy couplings**:

$$\begin{aligned}
\tilde{\mathcal{C}}_{1,k} &= \frac{\tilde{\mathcal{M}}^2_{\Pi,k}}{8f_\pi^2}, \qquad \tilde{\mathcal{Z}}_{2,k} = -\frac{1}{4f_\pi^2}, \\
\tilde{\mathcal{C}}_{3,k} &= \tilde{C}_{3,k} - \tilde{C}_{5,k} + \tilde{C}_{7,k} + 2\left(\tilde{C}_{6,k} + \tilde{C}_{8,k}\right), \\
\tilde{\mathcal{C}}_{4,k} &= \tilde{C}_{4,k}, \qquad\qquad \tilde{\mathcal{C}}_{5,k} = 2\left(\tilde{C}_{6,k} + \tilde{C}_{8,k}\right), \\
\tilde{\mathcal{C}}_{6,k} &= -\tilde{C}_{6,k} - \tilde{C}_{8,k}, \qquad \tilde{\mathcal{C}}_{8,k} = \frac{1}{2}\left(\tilde{C}_{6,k} + \tilde{C}_{8,k}\right)
\end{aligned}$$

Mapping onto the nonlinear effective action

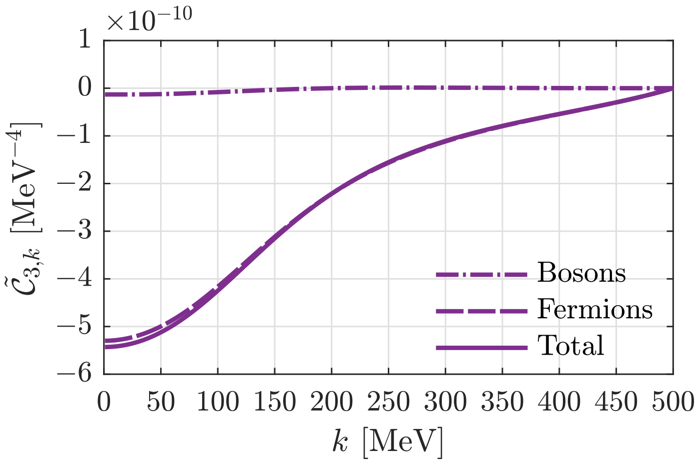


Figure 5: Scale evolution of the renormalized low-energy coupling $\tilde{\mathcal{C}}_{3,k}$; [Divotgey, JE, Mitter '19].

Summary of numerical results

- Low-energy (derivative) couplings; evaluated at $k_{\mathrm{IR}} = 1\ \mathrm{MeV}$; [Divotgey, JE, Mitter '19]

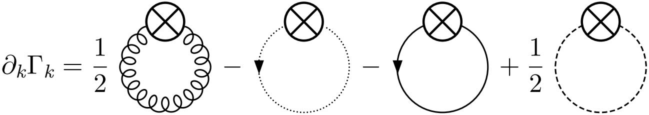
Linear model		Nonlinear model	
$\tilde{C}_2\ [1/f_\pi^2] \times 10$	-0.88	$\ldots$	
$\tilde{Z}_2\ [1/f_\pi^2] \times 10$	-2.30	$\tilde{\mathcal{Z}}_2\ [1/f_\pi^2] \times 10$	-2.50
$\tilde{C}_3\ [1/f_\pi^4] \times 10^2$	2.88	$\tilde{\mathcal{C}}_3\ [1/f_\pi^4] \times 10^2$	-4.20
$\tilde{C}_4\ [1/f_\pi^4] \times 10^2$	1.27	$\tilde{\mathcal{C}}_4\ [1/f_\pi^4] \times 10^2$	1.27
$\tilde{C}_5\ [1/f_\pi^4] \times 10^2$	4.69	$\tilde{\mathcal{C}}_5\ [1/f_\pi^4] \times 10^2$	-2.41
$\tilde{C}_6\ [1/f_\pi^4] \times 10^2$	-2.35	$\tilde{\mathcal{C}}_6\ [1/f_\pi^4] \times 10^2$	1.21
$\tilde{C}_7\ [1/f_\pi^4] \times 10^2$	0.02	$\ldots$	
$\tilde{C}_8\ [1/f_\pi^4] \times 10^2$	1.14	$\tilde{\mathcal{C}}_8\ [1/f_\pi^4] \times 10^2$	-0.60

- $\tilde{\mathcal{C}}_{1,k_{\mathrm{IR}}} = 0.27$, $\tilde{\mathcal{M}}_{\Pi,k_{\mathrm{IR}}} = 138.5\ \mathrm{MeV}$

Work in progress

Functional QCD (fQCD)

- fQCD fluctuations:



- Dynamical-hadronization technique
  [Gies, Wetterich ’02; Mitter, Pawlowski, Strodthoff ’15; Braun et al. ’16]
- **Goal:** Determine low-energy couplings from fQCD
- **First approach:** Evaluate $O(4)$ equations on fQCD solution
- In collaboration with Jan M. Pawlowski

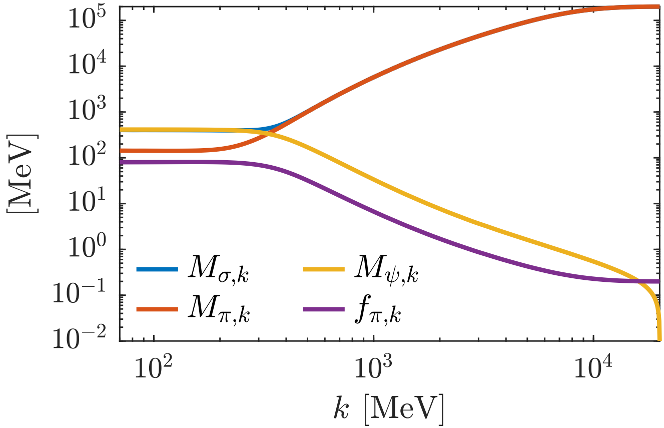


Figure 6: Renormalized meson and quark masses as well as the pion decay constant from fQCD (I).

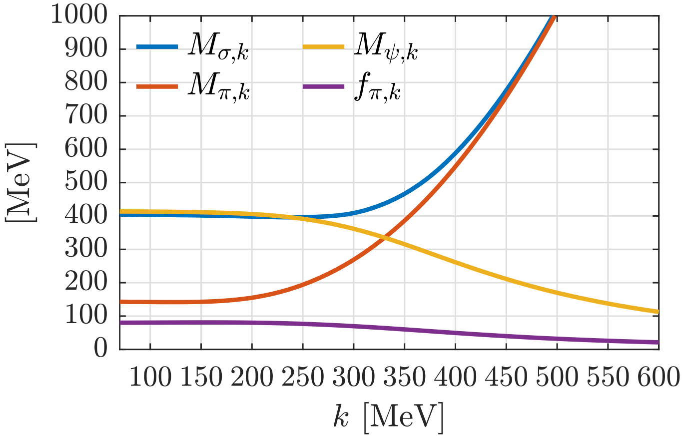


Figure 7: Renormalized meson and quark masses as well as the pion decay constant from fQCD (II).

Low-energy couplings from fQCD

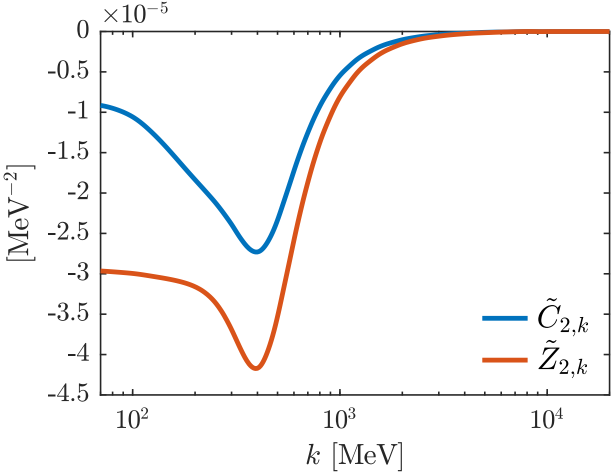


Figure 8: Low-energy couplings obtained from fQCD input (I).

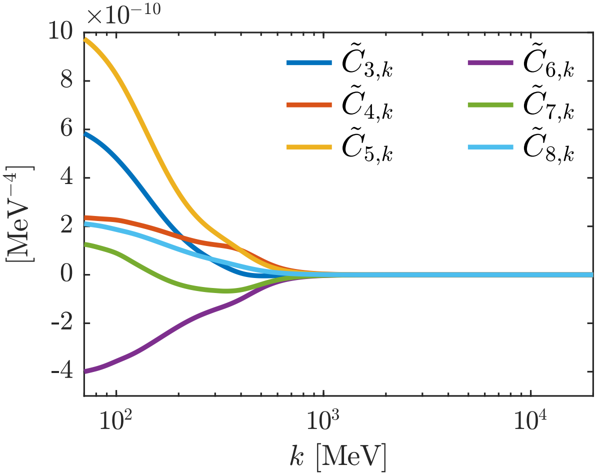


Figure 9: Low-energy couplings obtained from fQCD input (II).

Summary

Summary

Conclusions:

- Fluctuation dynamics strongly **dominated** by **fermionic** loops
- **Renormalization scales** of 50-$100\ \mathrm{MeV}$ for the purely pionic effective action as obtained from the **FRG**

Outlook:

- Compatibility analysis of **renormalization schemes**
- Compute $\boldsymbol{\pi\pi}$ **scattering** from the low-energy couplings
- Include **vector mesons**
- Determine low-energy couplings from **fQCD**
- Confront the FRG calculation with **ChPT**