Quarks & mesons at finite chemical potential

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in collaboration with Christian S. Fischer and Philipp Isserstedt

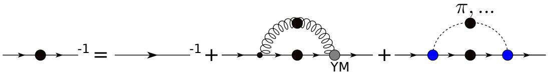
Institute for Theoretical Physics
Justus-Liebig-University Gießen

EMMI workshop on
“Functional Methods in Strongly Correlated Systems”
April 2, 2019

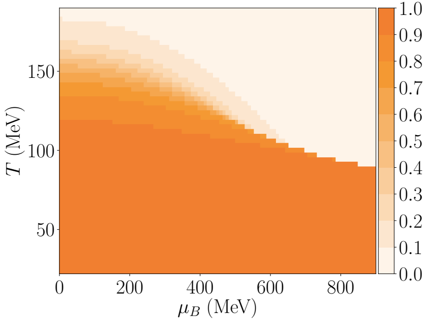
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

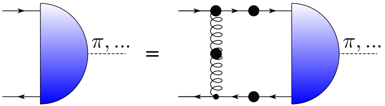
HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

Outline



1. Introduction and motivation
2. QCD phase diagram
3. Mesons at finite chem. pot.





Phase structure of QCD

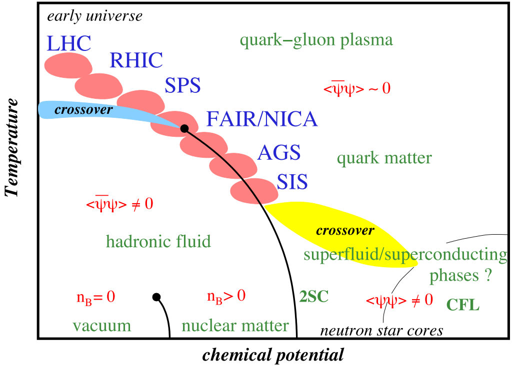


Figure: Schaefer and Wagner, Prog.Part.Nucl.Phys. 62 (2009) 381

Previous studies of our group:

QCD phase diagram with $N_f = 2+1$ and $N_f = 2+1+1$ quark flavors:

- Luecker and Fischer, Prog.Part.Nucl.Phys. 67(2), 200–205 (2012)
- Fischer, Luecker and Welzbacher, Phys.Rev. D 90(3), 34022 (2014)

Baryon effects on the location of QCD's CEP:

- Eichmann, Fischer and Welzbacher, Phys.Rev. D 93, 034013 (2016)

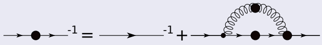
Mesonic back-coupling effects in vacuum and finite T:

- Fischer, Nickel and Wambach Phys.Rev. D 76, 094009 (2007)
- Fischer and Williams Phys.Rev. D 78, 074006 (2008)
- Fischer and Mueller Phys.Rev. D 84, 054013 (2011)

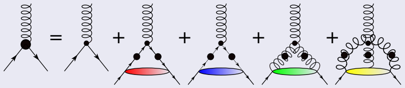
Dyson-Schwinger approach

Coupled set of Dyson-Schwinger equations (DSE):

Quark DSE:



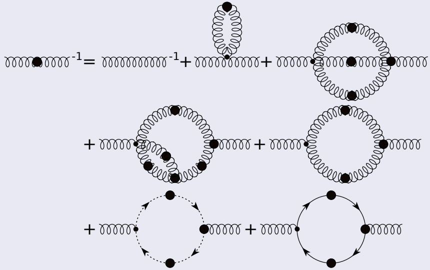
Quark-Gluon-Vertex DSE:



Further vertex and ghost DSEs:

...

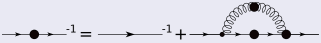
Gluon DSE:



Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step I):

Quark DSE:



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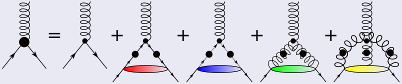
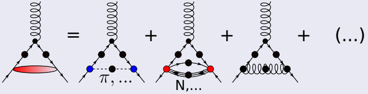
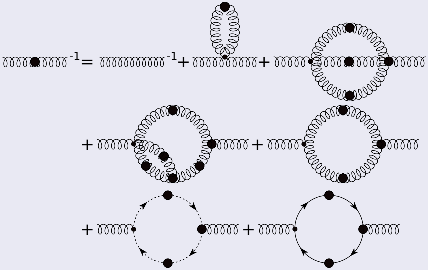


Skeleton expansion:



Gluon DSE:



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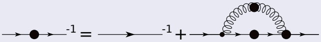
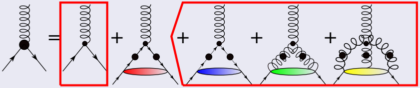


Quark-Gluon-Vertex DSE:



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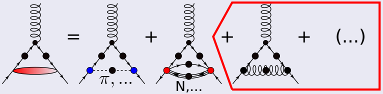
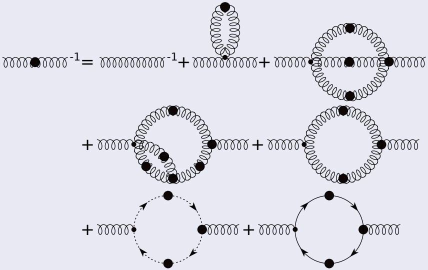


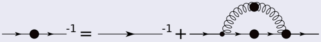
Gluon DSE:



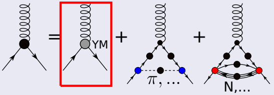
Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step II):

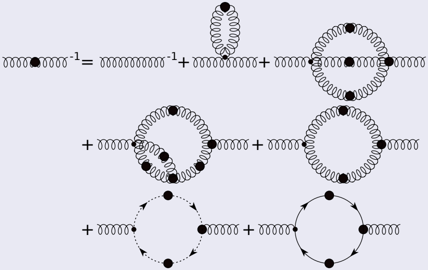
Quark DSE:



Quark-Gluon-Vertex DSE:



Gluon DSE:



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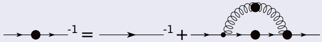
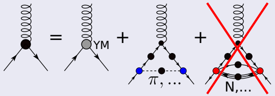


Quark-Gluon-Vertex DSE:



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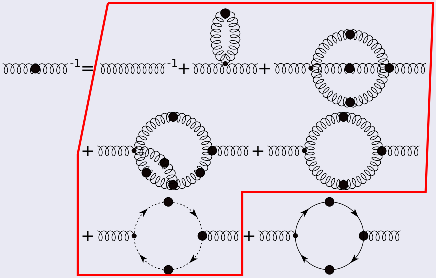
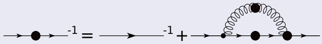


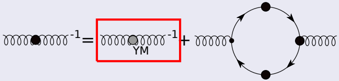
Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step III):

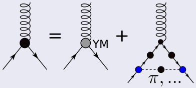
Quark DSE:



Gluon DSE:



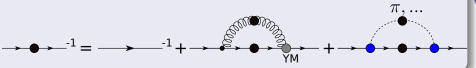
Quark-Gluon-Vertex DSE:



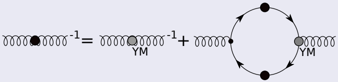
Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step IV):

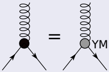
Quark DSE:



Gluon DSE:



Quark-Gluon-Vertex DSE:



Lattice data for quenched gluon:

Fischer, Maas and Müller, Eur.Phys.J. C (2010) 68: 165

Maas, Pawlowski, von Smekal, Spielmann, Phys.Rev.D 85 (2012) 034037

Truncation

Coupled set of **truncated** Dyson-Schwinger equations (DSE):

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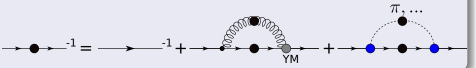

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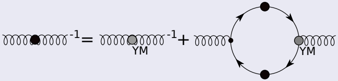

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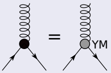
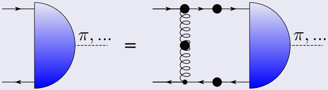


Homogeneous BSE:



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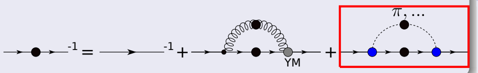

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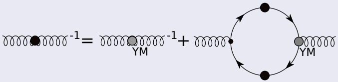
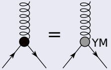


Quark-Gluon-Vertex DSE:



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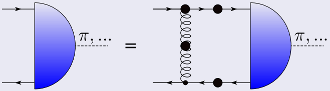

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Present QCD phase diagram

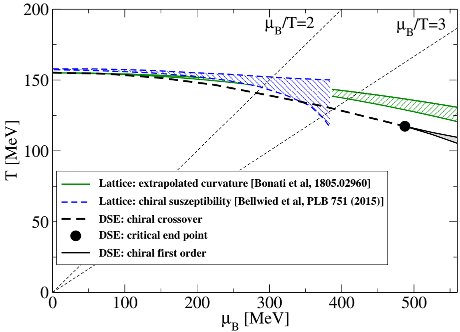
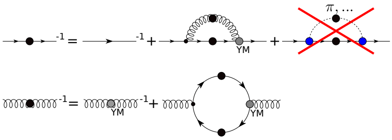
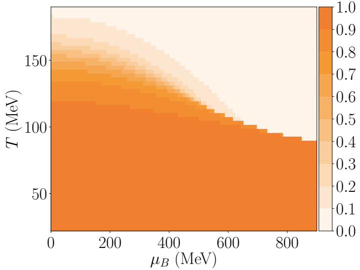


Figure: Fischer, PPNP 105 (2019) 1



→ Details: see talk by Philipp Isserstedt (Friday)

Chiral order parameter:

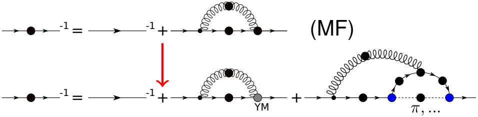


CEP at large μ_q:

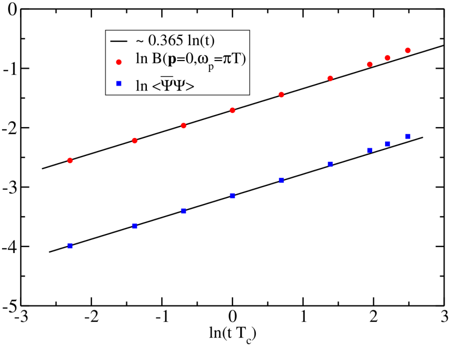
- Fischer, Luecker, PLB 718 (2013) 1036
- Fischer, Fister, Luecker, Pawlowski, PLB 732 (2014) 273
- Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022

Universality class ($N_f = 2$, chiral limit)

Importance of mesonic back-coupling effects:



$$t = \frac{T_c - T}{T_c}$$



$T = 0$: Meson corrections in order of $\sim 10 - 20\%$

$T = T_c$: Meson corrections dominant $\longrightarrow$ critical scaling ($m_q \to 0$):

$$\langle \bar{\Psi}\Psi \rangle (t) \sim t^{\nu/(2-\eta)} \sim t^{\nu/2} \sim t^{\beta}$$

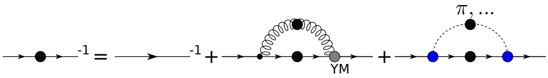
$\beta = 0.365$, $\nu = 0.73$

(Heisenberg class)

$T = 0$: Fischer, Williams, Phys.Rev. D 78, 074006 (2008)
$T = T_c$: Fischer, Mueller, Phys.Rev. D 84, 054013 (2011)

Connection to mesons at finite chemical potential

Wanted: Influence of Meson back-coupling onto QCD phase diagram and CEP



Needed: Meson propagator in medium

Meson Bethe-Salpether amplitude in medium

First step: Investigate Meson amplitudes and properties at finite chemical potential

Connection to mesons at finite chemical potential

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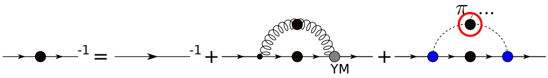

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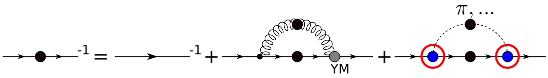

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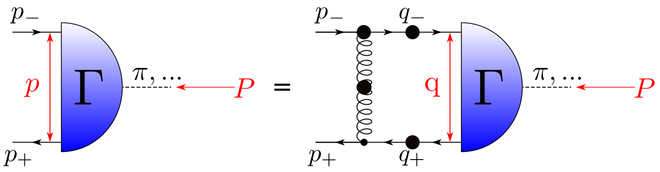
Homogeneous Bethe-Salpether equation (BSE)



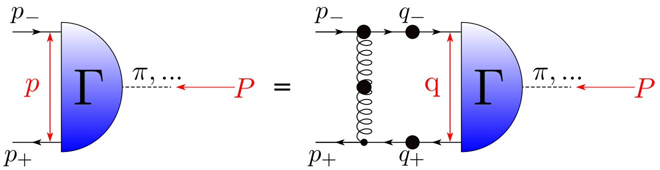
Pseudoscalar amplitude in vacuum

$$\Gamma_{\mathrm{PS}}(P,p) = \gamma_5 \left[E(P,p) - i\not{P}F(P,p) - i\not{p}(Pp)G(P,p) + \left[\not{P},\not{p}\right] H(P,p) \right]$$

Pseudoscalar amplitude in medium

$$\Gamma_{\mathrm{PS}}(P,p) = \gamma_5 \left[E(P,p) - iP_4\gamma_4 F_t(P,p) - i\vec{P}\vec{\gamma}F_s(P,p) + \ldots \right]$$

Homogeneous Bethe-Salpether equation (BSE)



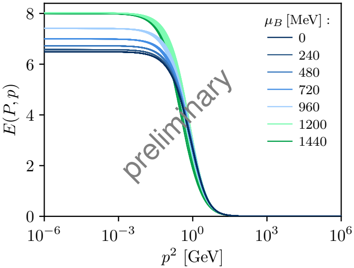
Scalar amplitude in vacuum

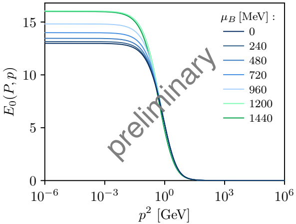
$$\Gamma_S(P,p) = \mathbb{1}\left[E(P,p) - i\not{P}(Pp)F(P,p) - i\not{p}G(P,p) + \left[\not{P},\not{p}\right] H(P,p)\right]$$

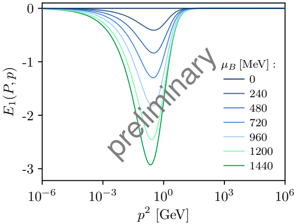
Scalar amplitude in medium

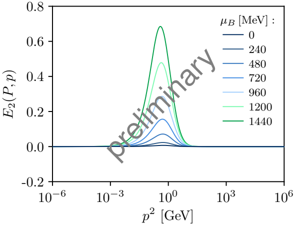
$$\Gamma_S(P,p) = \mathbb{1}\left[E(P,p) - iP_4\gamma_4(Pp)F_t(P,p) - i\vec{P}\vec{\gamma}(Pp)F_s(P,p) + \ldots\right]$$

Pion Bethe-Salpeter amplitude







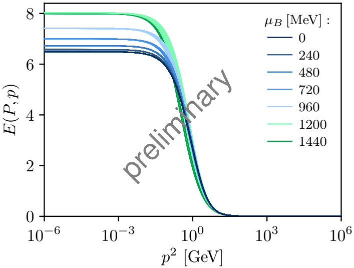


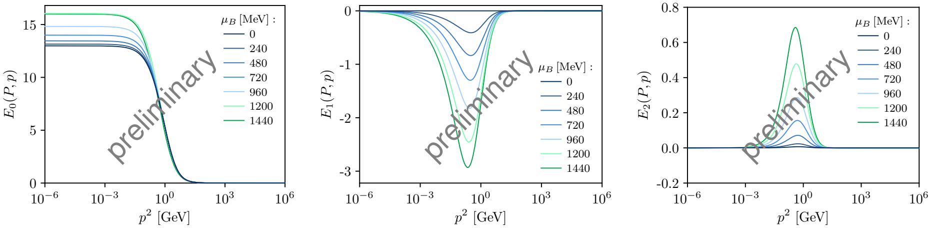
Pion Bethe-Salpeter amplitude

Chebyshev expansion:

$E(P^2, p^2, z) \approx \sum_j E_j(P^2, p^2) T_j(z)$

$z = \sphericalangle(P, p)$





Pion Bethe-Salpeter amplitude

Chebyshev expansion:

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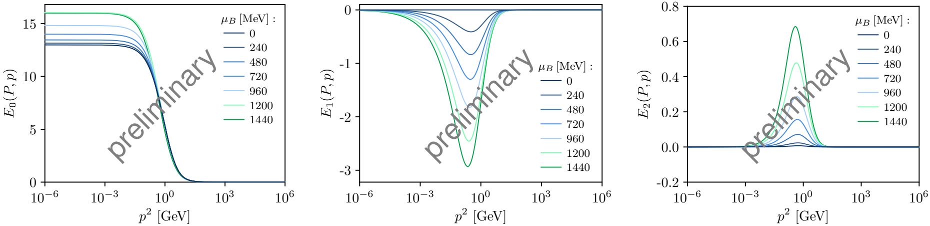
$$z = \sphericalangle(P, p)$$

Charge-conjugated pion amplitude:

$$\bar{\Gamma}_\pi(P, p) = \left[C\Gamma_\pi(P, -p)C^{-1}\right]^T$$



Pion Bethe-Salpeter amplitude

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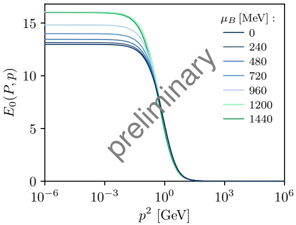
Charge-conjugated pion amplitude:

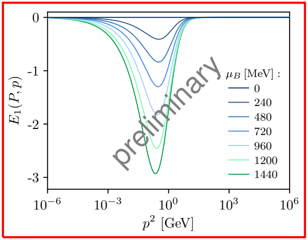
$$\bar{\Gamma}_\pi(P, p) = \left[C\Gamma_\pi(P, -p)C^{-1}\right]^T$$

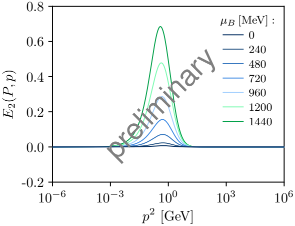
Pion: $J^{PC} = 0^{-+}$

→ odd Chebyshev coefficients vanish

→ μ_B breaks C-Parity







Meson properties

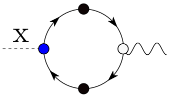
Meson mass:

$$K(P)\Gamma_x(P) = \lambda_x(P)\Gamma_x(P), \qquad \lambda_x(P_{os}) = 1 \quad \Leftrightarrow \quad P_{os}^2 = -m_x^2$$

Meson decay constants:

$x = \pi, \ldots$

$$\mathrm{f_x} P^\mu \propto$$



From vacuum to medium:

$$f_x\, P^\mu \xrightarrow{\text{medium}} \left[\, f_x^t\, \mathcal{P}^t_{\mu\nu}(v) + f_x^s\, \mathcal{P}^s_{\mu\nu}(v) \,\right] P^\nu$$

Long. $\mathcal{P}^{\mathscr{L}}_{\mu\nu}(v)$ and trans. proj. $\mathcal{P}^{\mathscr{T}}_{\mu\nu}(v)$ with assigned direction $v = (\vec{0}, 1)$

Meson propagator

Meson velocity:

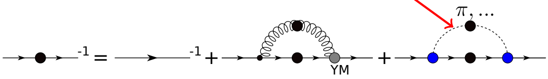
$$u_x = \frac{f_x^s}{f_x^t}$$

Meson dispersion relation:

$$\omega^2 = u^2 \left(\vec{P}^2 + m_x^2 \right)$$

Meson propagator:

$$D_x(P) = \frac{1}{P_4^2 + u_x^2 \left(\vec{P}^2 + m_x^2 \right)}$$



Son and Stephanov Phys.Rev. D, 66(7) (2002)
Fischer and Mueller Phys.Rev. D 84, 054013 (2011)

$X = \pi, \ldots$

Silver Blaze property

Definition:

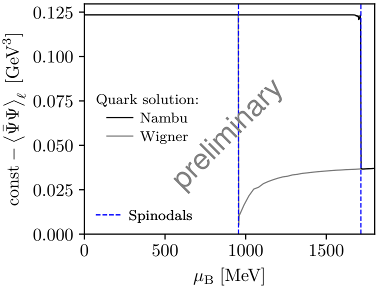
T. D. Cohen, Phys. Rev. Lett. 91 , 222001 (2003)
T. D. Cohen, arXiv:hep-ph/0405043 (2004)

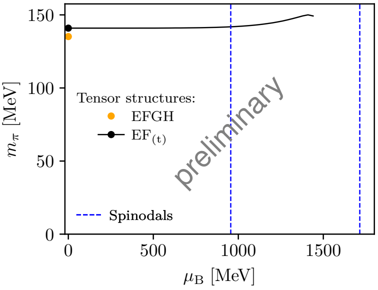
$\mu_B <$ mass gap of the system δ and $T = 0$

$\Rightarrow$ Partition function and observables independent from μ_B

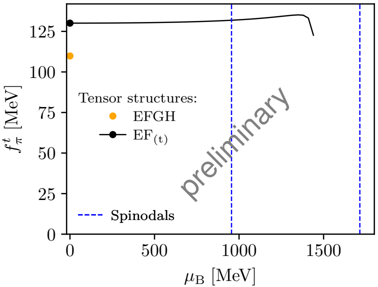
$$\delta = m_B = \text{lightest baryon mass in medium}$$

Pion properties results at finite chemical potential





- Silver Blaze property fulfilled
- No pion solution above $\mu_{B,c}^{\text{pion}}$ (eff. int)



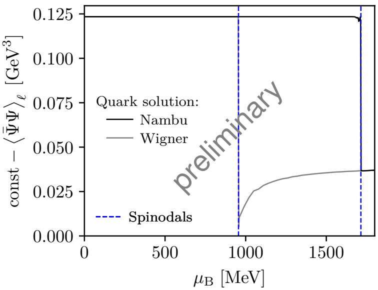
Qualitative agreement with simpler truncation schemes:

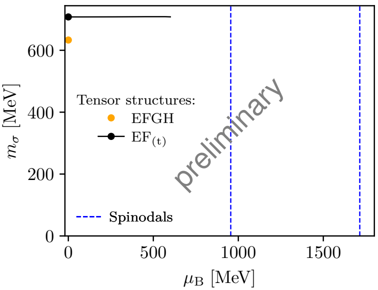
Roberts, Phys.Part.Nucl. 30:223-257 (1999)

Roberts and Schmidt, Prog.Part.Nucl.Phys. 45(1) 1-103 (2000)

Jiang, Shi, Li, Sun, and Zong, PRD 78, 116005

Sigma properties results at finite chemical potential





- Silver Blaze property fulfilled
- No pion solution above $\mu_{\mathrm{B,c}}^{\mathrm{pion}}$ (eff. int)

Qualitative agreement with simpler truncation schemes:

Roberts, Phys.Part.Nucl. 30:223-257 (1999)

Roberts and Schmidt, Prog.Part.Nucl.Phys. 45(1) 1-103 (2000)

Jiang, Shi, Li, Sun, and Zong, PRD 78, 116005

Summary

- Successfully implemented quark back coupling onto gluon

- Present QCD phase diagram for $N_f = 2 + 1$ quark flavors

  $\longrightarrow$ good curvature agreement

  $\longrightarrow$ CEP beyond lattice chiral susceptibility exclusion area

- Pion properties (m_π, f_π) at $\mu_B \neq 0$, two tensor structures

  $\longrightarrow$ $\mu_B \neq 0$ breaks C-Parity of mesons ✓

  $\longrightarrow$ $m_{\pi,...}$, $f_{\pi,...}$ fulfill Silver Blaze property ✓

  $\longrightarrow$ No pion solution above certain μ_B^c (eff. int.)

Outlook

1. Publish present results

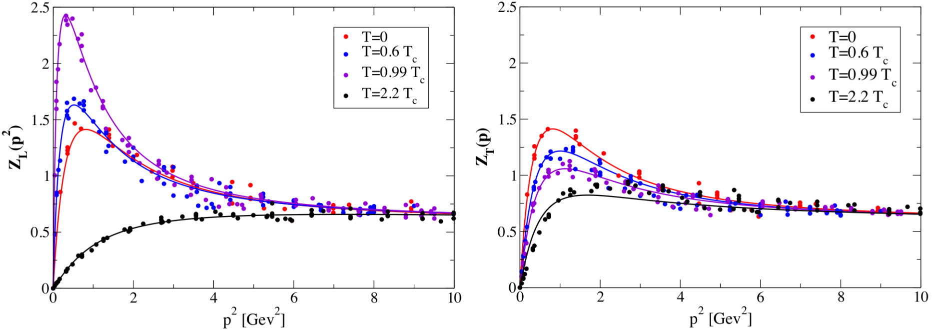
2. meson properties at finite chem. pot. & temp.

3. meson back coupling (finite chem. pot. & temp.)
   $\longrightarrow$ influence on CEP

Thank you for your attention!

Backup slides

T-dependent gluon propagator from quenched lattice simulations:



Crucial difference between transversal and longitudinal gluon
($T_c = 277$ MeV)

Cucchieri, Maas, Mendes, PRD 75 (2007)
CF, Maas, Mueller, EPJC 68 (2010)
Aouane et al., PRD 85 (2012) 034501

Cucchieri, Mendes, PoS FACESQCD 007 (2010)
Silva, Oliveira, Bicudo, Cardoso, PRD 89 (2014) 074503
FRG: Fister, Pawlowski, arXiv:1112.5440

Truncation scheme properties II

Vertex truncation: STI and perturbative behavior at large momenta constrain vertex

$$\Gamma^f_\nu(p,q,k) = \Gamma(k^2)\gamma_\nu\left(\delta_{\nu,s}\Sigma_A + \delta_{\nu,4}\Sigma_C\right)$$

$$\Gamma(k^2) = \frac{d_1}{d_2 + k^2} + \frac{k^2}{\Lambda^2 + k^2}\left(\frac{\beta_0\alpha(\mu'')\ln[k^2/\Lambda^2 + 1]}{4\pi}\right)^{2\delta}$$

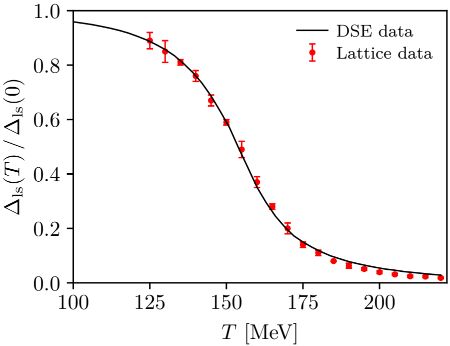
With first Ball-Chiu structure: $\Sigma_X = \frac{X(\vec{p}^2,\omega_p)+X(\vec{q}^2,\omega_q)}{2}$, $X \in \{A, C\}$

Abelian WTI: from approximated STI
Perturbation theory
Infrared ansatz: d_2 fixed to match gluon input, d_1 fixed via quark condensate

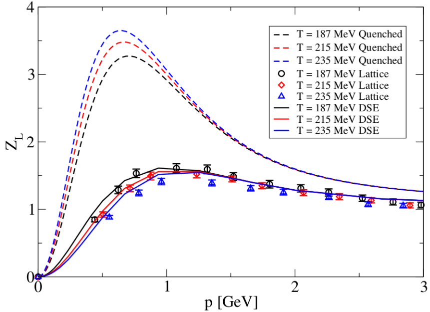
Fischer and Mueller, Phys.Rev. D 80, 074029 (2009)
Fischer, Phys.Rev.Lett. 103, 052003 (2009)

Truncation scheme properties III

Determination of d_1 and prediction for unquenched gluon:



Lattice: Borsanyi et al. [Wuppertal-Budapest], JHEP 1009(2010) 073

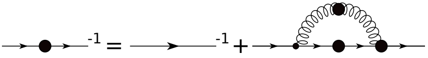


Lattice: Aouane, Burger, Ilgenfritz, Müller-Preussker, Sternbeck, PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]

Quantitative agreement: DSE results verified by lattice

Order parameter

Chirality:



Quark condensate

$$\left\langle \bar{\Psi}\Psi \right\rangle^{f} = -Z_m Z_2 \int_q \mathrm{tr}_{DC}\left[S^f(p)\right]$$

Regularized quark condensate

$$\Delta_{f'f} = \left\langle \bar{\Psi}\Psi \right\rangle^{f'} - \frac{m_B^{f'}}{m_B^{f}} \left\langle \bar{\Psi}\Psi \right\rangle^{f}$$

Deconfinement: $\langle L[A] \rangle \propto e^{-\frac{F_q}{T}}$, static quark free energy F_q

Dressed Polyakov loop[1]

$$\Sigma = -\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} \left\langle \bar{\Psi}\Psi \right\rangle_{\varphi}$$

Polyakov loop potential[2]

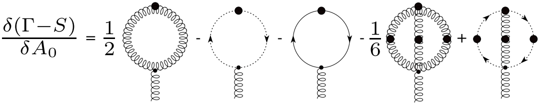
$$L[A] := \frac{1}{N_c} \mathrm{tr}_C \left(\mathcal{P} e^{i\int d\tau A_0(\vec{x},\tau)} \right)$$

1:

Synatschke, Wipf, Wozar, PRD 75, 114003 (2007)

Bilgici, Bruckmann, Gattringer, Hagen, PRD 77 094007 (2008)

Fischer, PRL 103 052003 (2009)



2:

Braun, Gies, Pawlowski, PLB 684, 262 (2010)

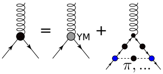
Braun, Haas, Marhauser, Pawlowski, PRL 106 (2011)

Fister, Pawlowski, PRD 88 045010 (2013)

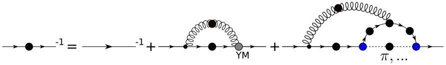
Fischer, Fister, Luecker, Pawlowski, PLB 732

Skeleton expansion

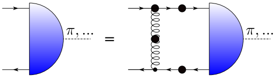
Only mesonic contributions:



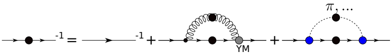
Inserting vertex into quark:

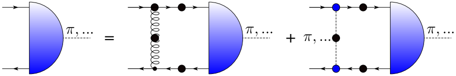


BSE:



Assumption: Only Yang-Mills part present in BSE ⇒ rewrite Quark DSE by inserting DSE into second diagram





Approximation justified if BSE vertex function with and without pion interaction term do not differ strongly

Fischer, Nickel and Wambach, Phys.Rev. D 76(9) (2007)

Curvature

Definition:

first coefficient (κ) in taylor series expansion of the transition line in terms of $\frac{\mu_q}{T_c(0)}$

$$\frac{T_c(\mu_q)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_q}{T_c(0)} \right)^2 + O\left[\left(\frac{\mu_q}{T_c(0)} \right)^4 \right]$$

Remark:

Curvature depends on choice of pesudo-critical temperature definition in crossover region

Silver Blaze property example

Definition:

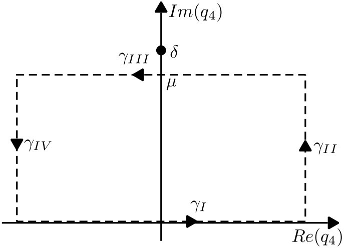
T. D. Cohen, Phys. Rev. Lett. 91 , 222001 (2003)
T. D. Cohen, arXiv:hep-ph/0405043 (2004)

$\mu_B <$ mass gap of the system δ and $T = 0$

$\Rightarrow$ Partition function and observables independent from μ_B

$$\delta = m_B = \text{lightest baryon}$$

Example:



Substitution:

$$\langle \bar{\Psi}\Psi \rangle \sim \int_q S(\vec{q}^2, q_4 + i\mu_q)$$

$$\overset{q_4 \to q_4 + i\mu_q}{=} \int_q S(\vec{q}^2, q_4) \sim \langle \bar{\Psi}\Psi \rangle_{vac}$$

Condition: No singularity in γ