Fractionalization in correlated spin systems

An fRG perspective

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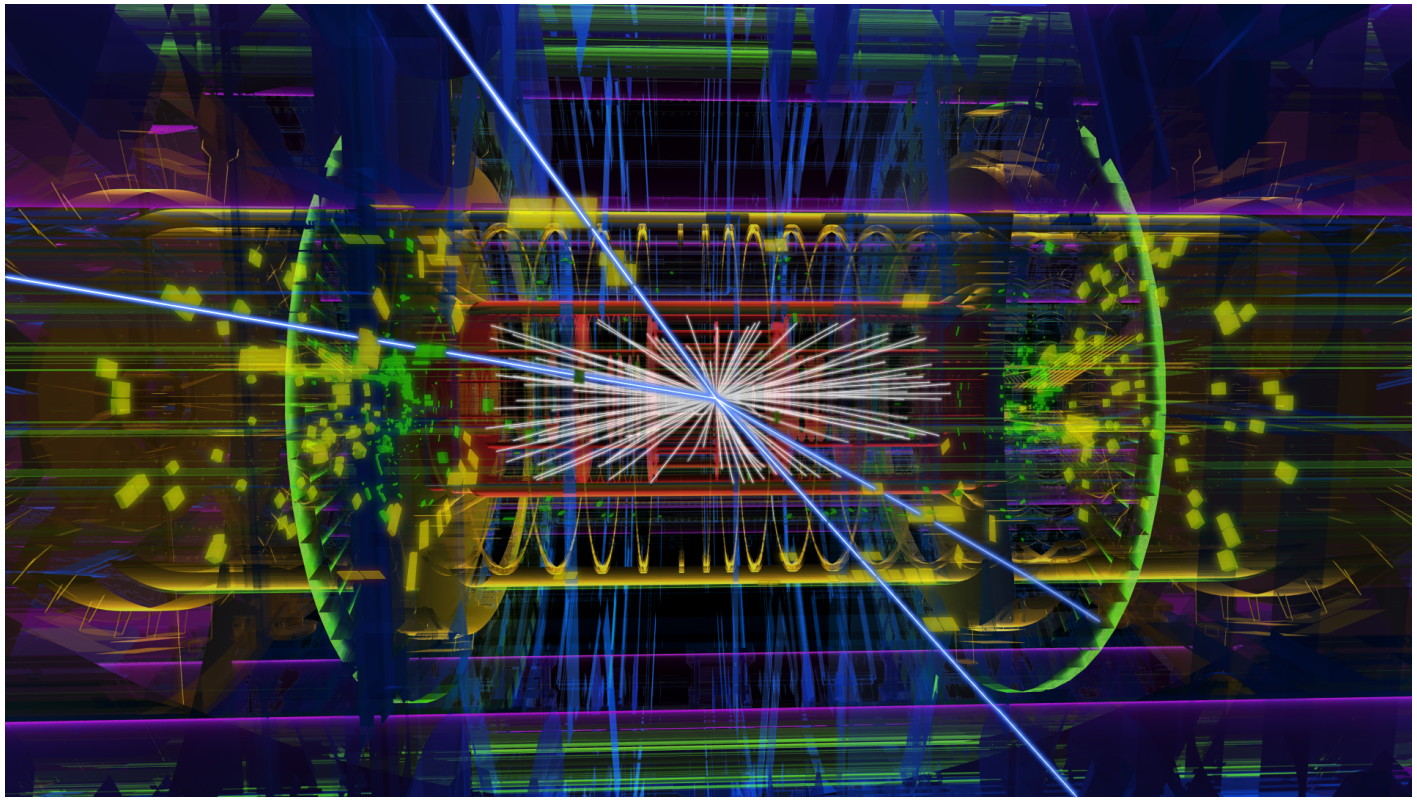
Institute for Theoretical Physics / Universität zu Köln

Hirschegg Workshop 2019
April 1st, 2019

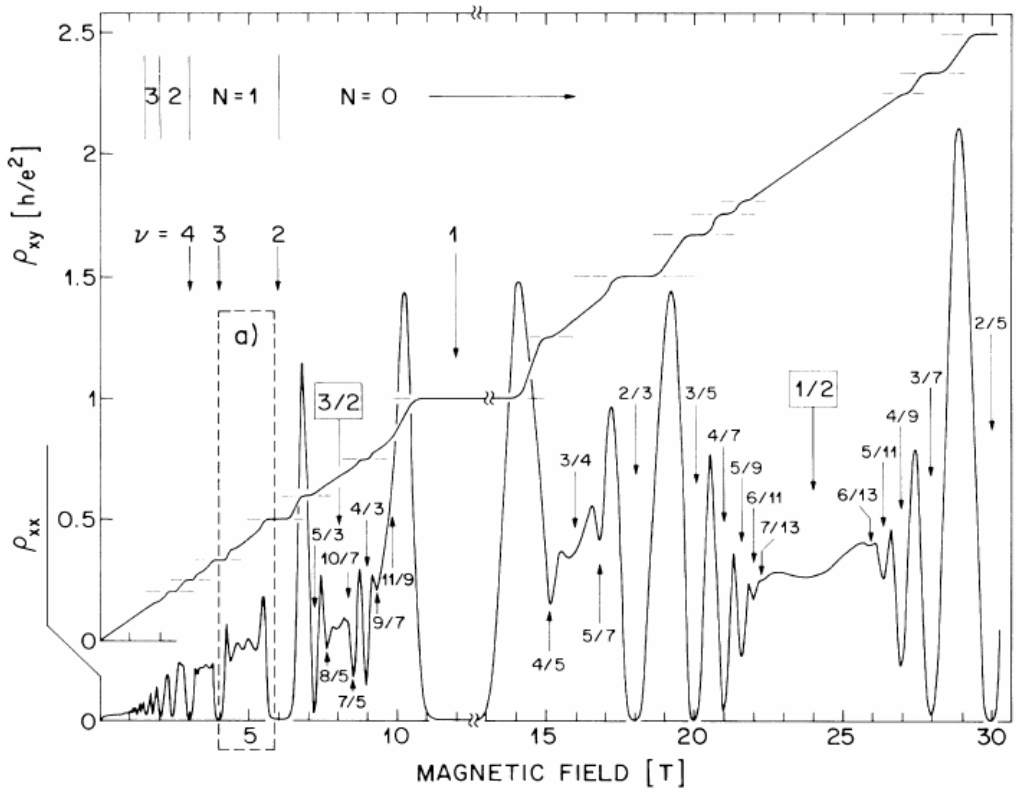
High energy... “fractionalization”



ATLAS Experiment © 2016 CERN

Shattering bound states by brute force

Fractionalization in solids



[R. Willet, J.P. Eisenstein, H.L. Störmer, D.C. Tsui, A.C. Gossard, J.H. English, '87]

Low-energy Collective Effect

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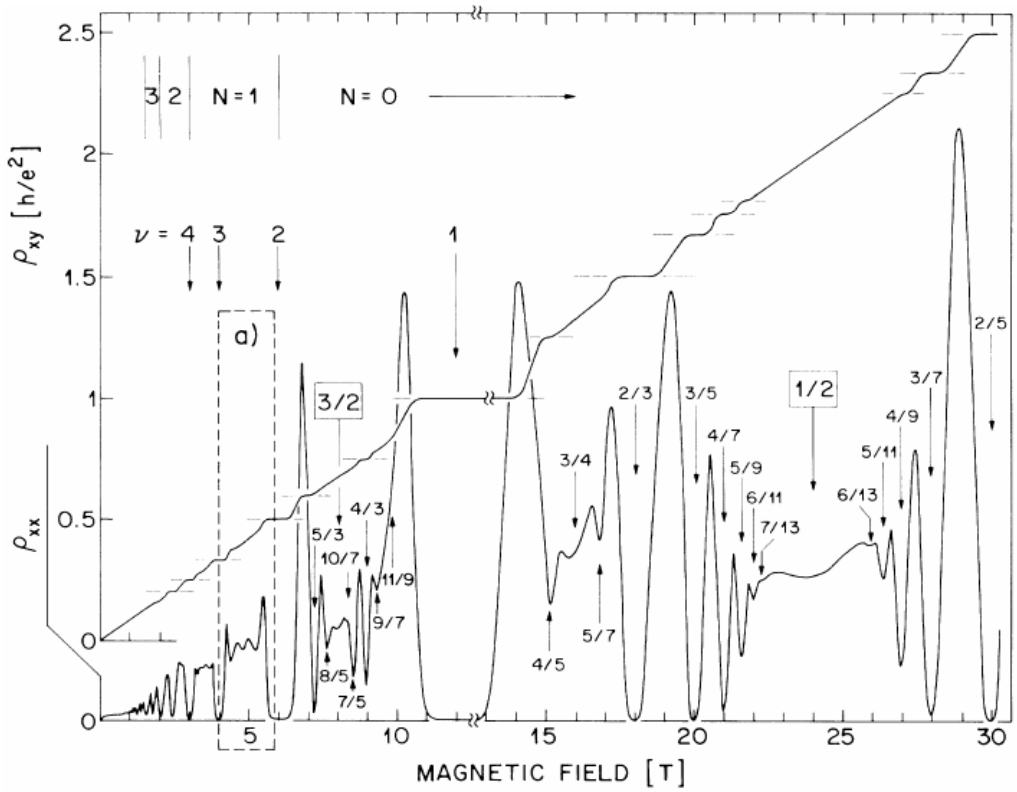

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Low-energy Collective Effect

- Seems perfectly suited for field theory & (f)RG
- Extremely rich/confusing field: anyons, majoranas, gauge fields...
- Actual physical observables/interpretation?

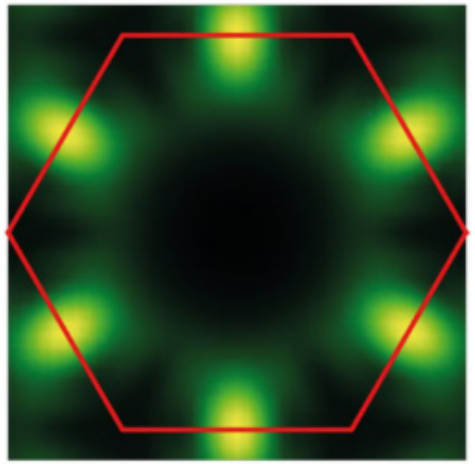
Spin systems & Spin liquids

Heisenberg model:

$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} S_i^\mu \cdot S_j^\mu$$

Magnetic phases (SU(2) symmetry breaking):

$$\sum_i \langle S_i^\mu \rangle \neq 0$$



[F.L. Buessen, S. Trebst, '16]

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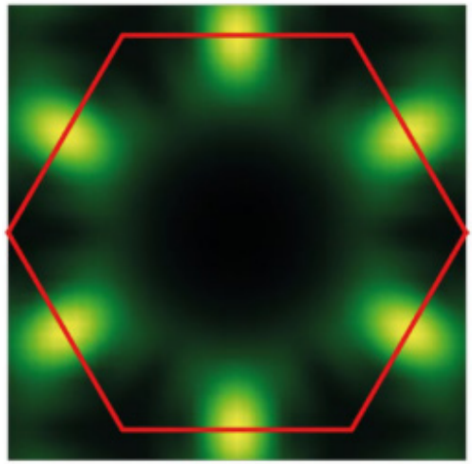

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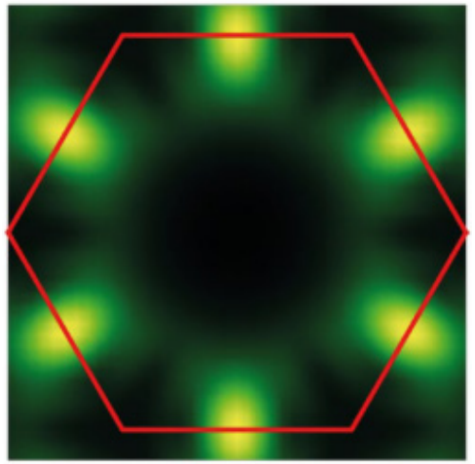

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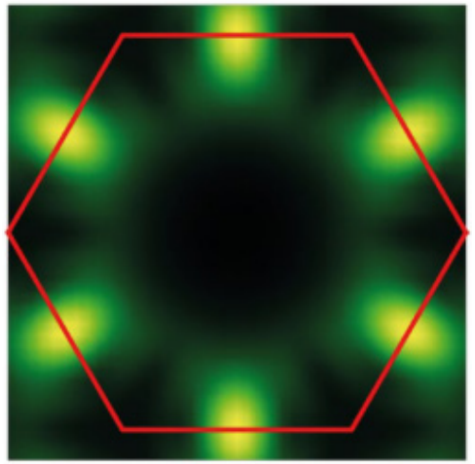

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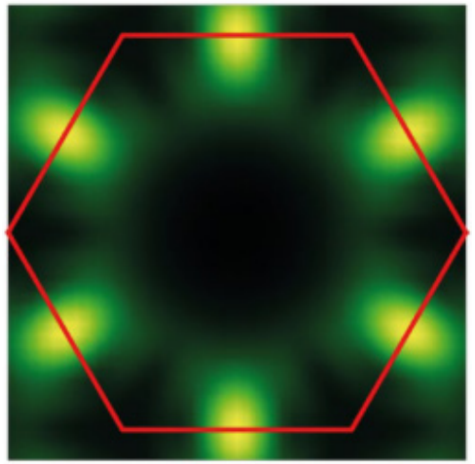

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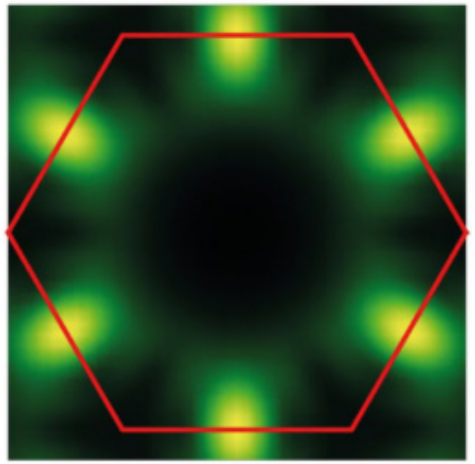

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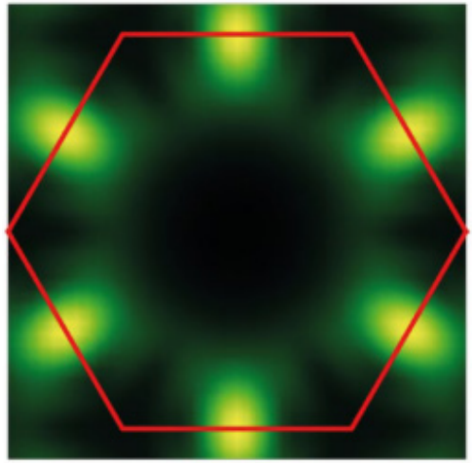

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Spin systems with non-magnetic but non-trivial ground states

- Long-range entanglement
- Topological order
- Fractionalization

Pseudofermion representation

Spin decomposition [A.A. Abrikosov, '65]:

$$S_i^\mu \equiv \frac{1}{2} f_{i\alpha}^\dagger \sigma_{\alpha\beta}^\mu f_{j\beta}$$

"Microscopic" Spin model:

$$\mathcal{H}^{\mathrm{UV}} = J \sum_{\langle i,j \rangle} S_i^\mu \cdot S_j^\mu \simeq -\frac{J}{2} \sum_{\langle i,j \rangle} f_{i\alpha}^\dagger f_{j\alpha} f_{j\beta}^\dagger f_{i\beta}$$

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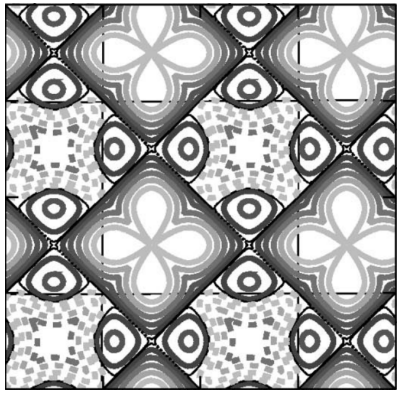
Low-energy Spin liquid model [X.-G. Wen, '02]:

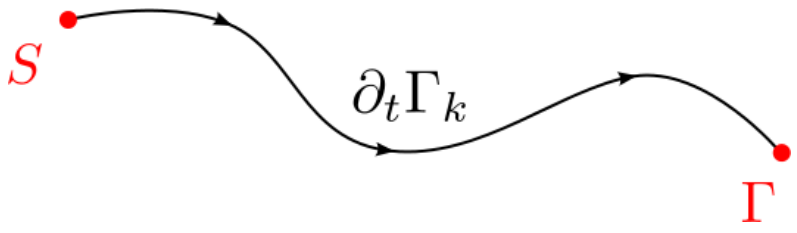
$$\mathcal{H}^{\mathrm{IR}} \sim \sum_{\langle i,j \rangle} \left[Q_{ij} f_{i\alpha}^\dagger f_{j\alpha} + \Delta_{ij} \epsilon_{\alpha\beta} f_{i\alpha}^\dagger f_{j\beta}^\dagger + h.c. + \ldots \right]$$

- 246 different classes (symmetries of Q, Δ)
- Fractionalization, "Topological order"
- Alternatively: SSB of (local) U(1)



Pseudofermion functional RG

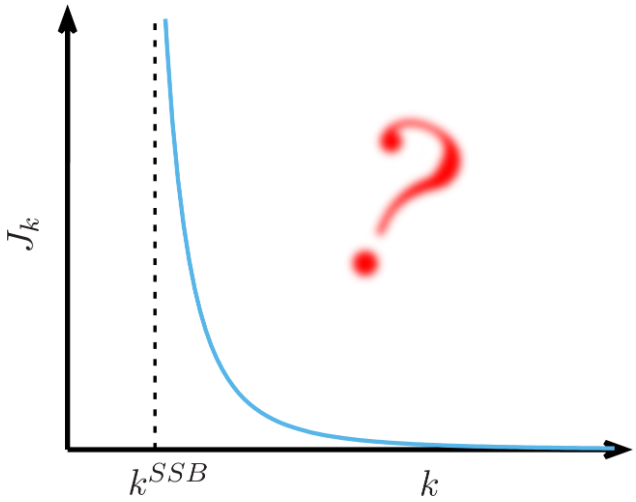
$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\left[\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}\right]$$

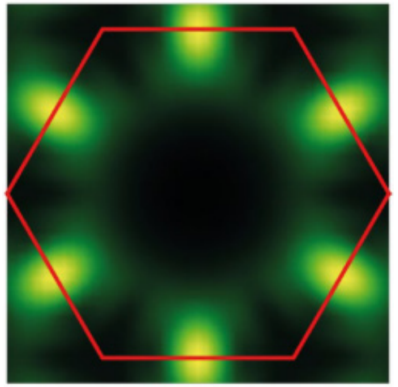


[C. Wetterich, '93]

$$\Gamma_k = \int_\tau \left[\sum_i f_{i\alpha}^\dagger (i\partial_\tau) f_{i\alpha} - \frac{J_k}{2}\sum_{\langle i,j\rangle} f_{i\alpha}^\dagger f_{j\alpha} f_{j\beta}^\dagger f_{i\beta}\right]$$

Signature of order in fermionic RG flow: $J_{k_{\mathrm{SB}}} \to \infty$





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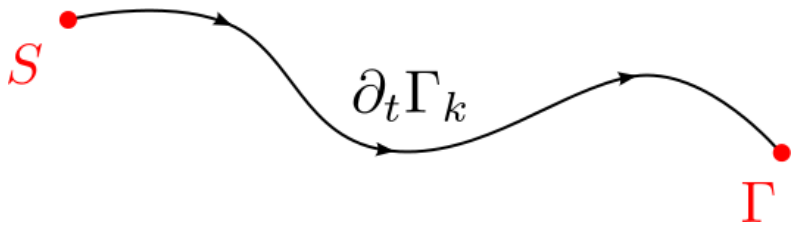

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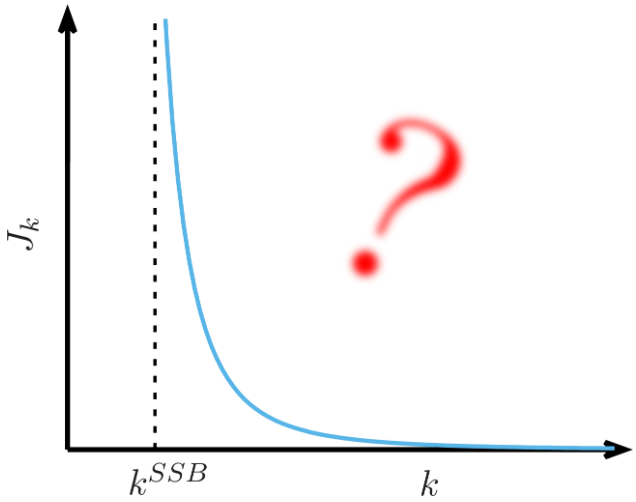
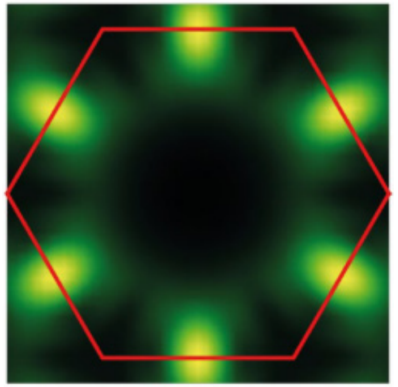
Bosonization?





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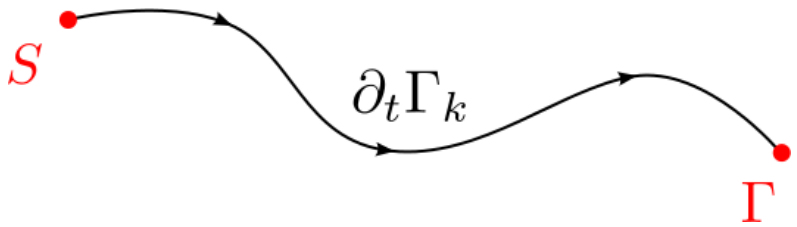

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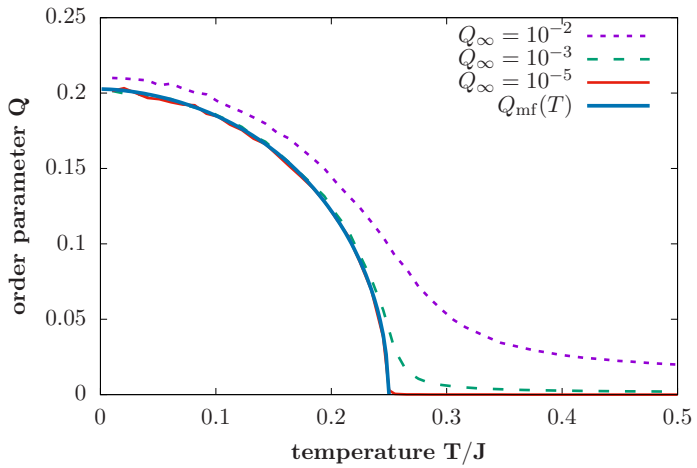
Infinitesimal explicit symmetry breaking:

[M. Salmhofer, C. Honerkamp, W. Metzner, O. Lauscher, '04]

- Minimal bias
- New vertices (Fierz-completeness!)
- Spatial symmetry breaking by compactification
- Exact limiting case SU(N)

[DR, F.L. Buessen, M.M. Scherer, S. Trebst, S. Diehl, '18]

[→ Talk by Nico Gneist]



Subtlety: Fermion number constraint

Hilbert spaces

Spin operator: $\{|\uparrow\rangle, |\downarrow\rangle\}$ $\overset{?}{\longleftrightarrow}$ *Pseudo*fermions: $\{|0,0\rangle, |0,1\rangle, |1,0\rangle, |1,1\rangle\}$

Need to enforce $f^\dagger_{i\alpha} f_{i\alpha} = 1$

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[I. Affleck, Z. Zou, T. Hsu, P.W. Anderson '88]

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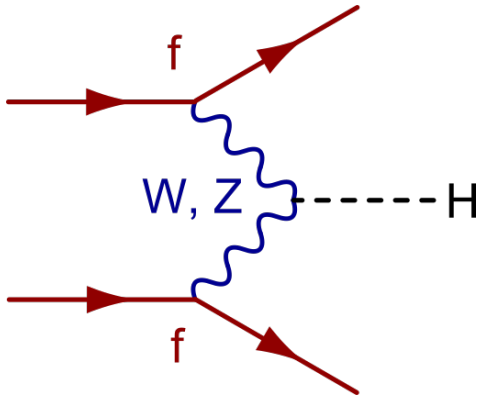

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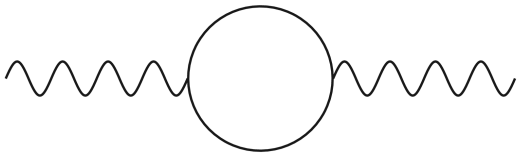

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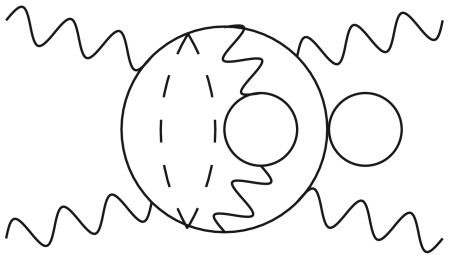

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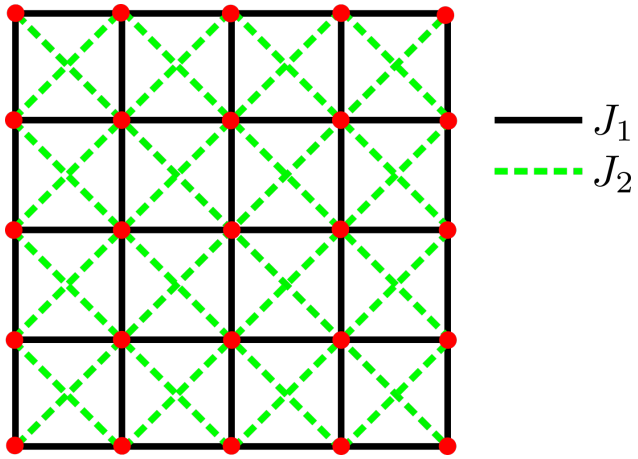
Option 2: add $\mu_{\mathrm{PF}} = \frac{i}{2}\pi T$

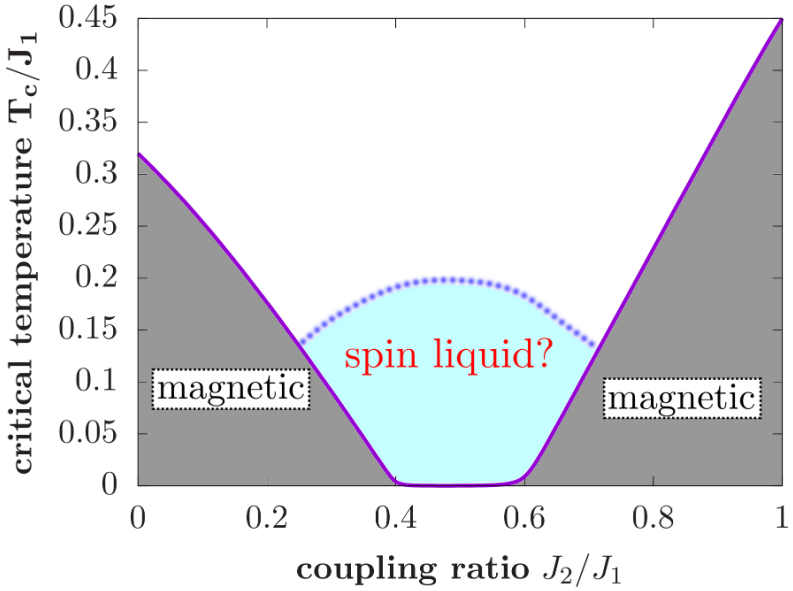
[V.N. Popov, S.A. Fedotov '88]

- Simple implementation
- Straightforward truncation
- Physical interpretation!?

Results: $J_1 - J_2$ Heisenberg model

arXiv:1904.xxxxx





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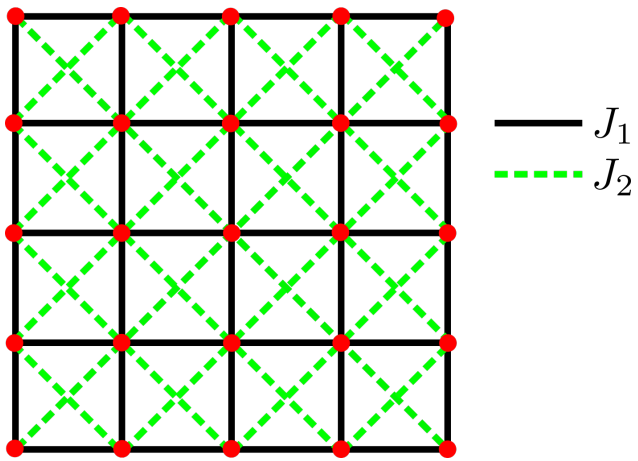

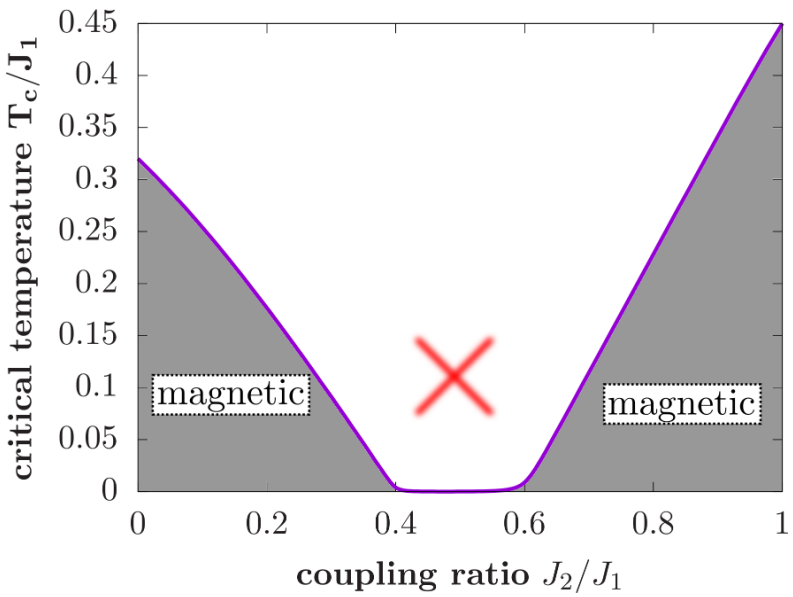

- Localization of magnetic phases consistent with literature
- Non-magnetic regime is *not* a spin liquid of the type presented here
- fRG currently not sensitive to 4/6/8... fermion order parameters

Conclusions & Outlook

Achievements of spin liquid fRG:

- Systematic emergence of spin liquids from microscopic spin models
- Clear picture of underlying ordering mechanisms
- Systematic inclusion of the fermion number constraint
- Successful proof of principle [Talk by Nico Gneist]
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