Quark and baryon number fluctuations from Dyson-Schwinger equations

Philipp Isserstedt [a]

In collaboration with:
Michael Buballa [b], Christian S. Fischer [a], Pascal Gunkel [a]

[a] Institut für Theoretische Physik, Justus-Liebig-Universität Gießen, Germany
[b] Theoriezentrum, Institut für Kernphysik, Technische Universität Darmstadt, Germany

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

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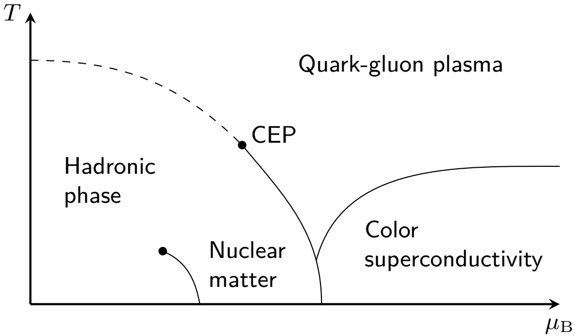
Phase transitions in QCD

Theoretical approaches

- Lattice QCD ... limited to $\mu_B/T \lesssim 3$ due to sign problem
- Effective models ... generalizable?
- Functional methods ... all QCD degrees of freedom & no sign problem (but truncations are necessary)



Dyson-Schwinger equations

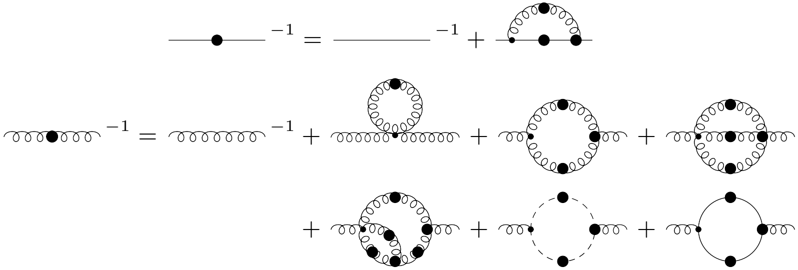
Master equation

$$0 = \int \mathcal{D}\vec{\varphi}\, \frac{\delta}{\delta\varphi_k} \exp\left(-S_{\mathrm{E}}[\vec{\varphi}\,] + \int \mathrm{d}^4x\, \vec{J}\cdot\vec{\varphi} \right)$$

$$\Downarrow$$

$$\frac{\delta\Gamma_{\mathrm{1PI}}}{\delta\widetilde{\varphi}_k} = \frac{\delta S_{\mathrm{E}}}{\delta\varphi_k}\left[\varphi_\ell \to \pm\left(\frac{\delta}{\delta J_\ell} + \widetilde{\varphi}_\ell \right) \right]$$

DSEs for quark and gluon propagators:



How to truncate?

Quark DSE

Main focus on quark propagator:

- Source for order parameters (chiral symmetry, confinement)
- Starting point for fluctuations

Dressed quark-gluon vertex:

- Studied in vacuum

  Fischer, Williams; PRL 103, 122001 (2009)
  Mitter, Pawlowski, Strodthoff; PRD 91, 054035 (2015)
  Williams; EPJA 51, 53 (2015)
  Williams, Fischer, Heupel; PRD 93, 034026 (2016)
  Sternbeck *et al.*; PoS (LATTICE2016) 349

- $T \neq 0$: Ansatz based on STI and known perturbative behavior

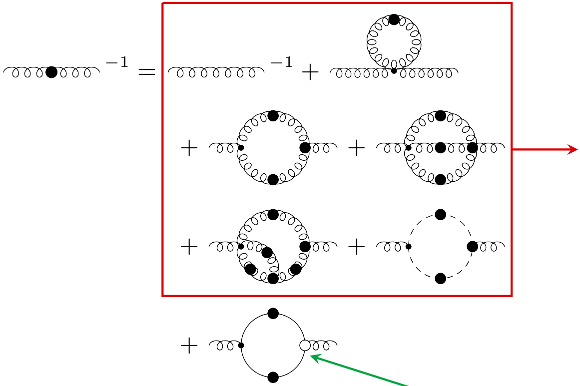
Dressed gluon propagator:

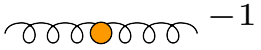
- Two strategies:
  - Model gluon propagator

    Qin, Chang, Chen, Liu, Roberts; PRL 106, 172301 (2011)
    Gao, Liu; PRD 94, 076009 (2016)

  - Explicit treatment of gluonic sector
- Here: Use the latter; consistent mass and flavor dependencies

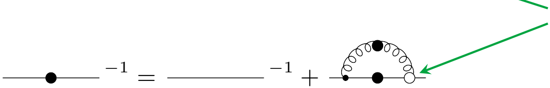
How to truncate?





quenched, T-dependent lattice gluon propagator

Fischer, Maas, Müller; EPJC 68, 165 (2010)
Maas, Pawlowski, von Smekal, Spielmann; PRD 85, 034037 (2012)



(T, μ)-dependent ansatz for quark-gluon vertex

Fischer, Luecker, Welzbacher; PRD 90, 034022 (2014) (and references therein)

More details: Talk by Pascal Gunkel

Ansatz for quark-gluon vertex

$$S_f^{-1}(p) = \mathrm{i}\tilde{p}_4\gamma_4 C_f(p) + \mathrm{i}\vec{\not p} A_f(p) + B_f(p)$$

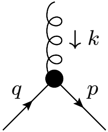
Vertex ansatz:

$$\Gamma_\nu^f(q,p,k) = \widetilde{Z}_3\, \Gamma(k^2)\, \gamma_\nu \left(\delta_{4\nu} \frac{C_f(q) + C_f(p)}{2} + (1-\delta_{4\nu}) \frac{A_f(q) + A_f(p)}{2} \right)$$

Phenomenological dressing function:

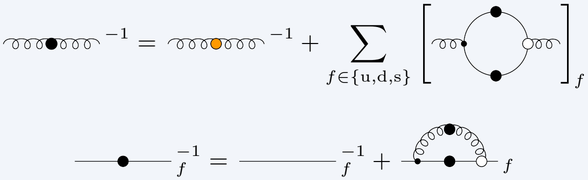
$$\Gamma(k^2) = \frac{d_1}{d_2 + k^2} + \frac{1}{1 + k^2/\Lambda^2} \left(\frac{\alpha_s \beta_0}{4\pi} \log\left(1 + k^2/\Lambda^2\right) \right)^{2\delta}$$

- Abelian STI (leading term of Ball-Chiu vertex)
  Ball, Chiu; PRD 22, 2542 (1980)
- Perturbative running in the ultraviolet
- Ansatz for IR
  - d_1 fixed via T_c
  - d_2 fixed to match scale of quenched lattice gluon data



Three-flavor QCD with Dyson-Schwinger equations

Final set of truncated DSEs



- Quenched lattice gluon propagator as input & unquenching via quark loops
- Non-trivial coupling between different quark flavors
- Vertex ansatz built along STI and perturbation theory

Phase structure: $N_{\mathrm{f}} = 2+1$ at physical point

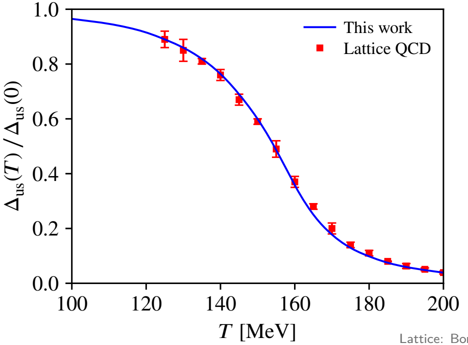
Chiral order parameter:

Quark condensate

$$\langle \bar{\psi}\psi \rangle_f = -Z_2^f Z_m^f \, \mathrm{Tr}[S_f]$$

Subtracted condensate

$$\Delta_{ff'} = \langle \bar{\psi}\psi \rangle_f - \frac{m_f}{m_{f'}} \langle \bar{\psi}\psi \rangle_{f'}$$



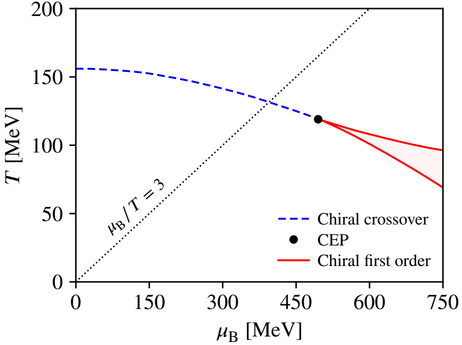
Lattice: Borsányi *et al.*; JHEP 2010, 09:073

Phase structure: $N_{\mathrm{f}} = 2 + 1$ at physical point

Main result

Second order CEP at large chemical potential:

$$\mu_{\mathrm{B}}^{\mathrm{CEP}} = 495\,\mathrm{MeV}\,, \quad T^{\mathrm{CEP}} = 119\,\mathrm{MeV}$$



- Ratio: $\mu_{\mathrm{B}}^{\mathrm{CEP}} / T^{\mathrm{CEP}} \approx 4.16$
- Crossover temperature: $T_{\mathrm{c}}^{(\mu=0)} = 156\,\mathrm{MeV}$

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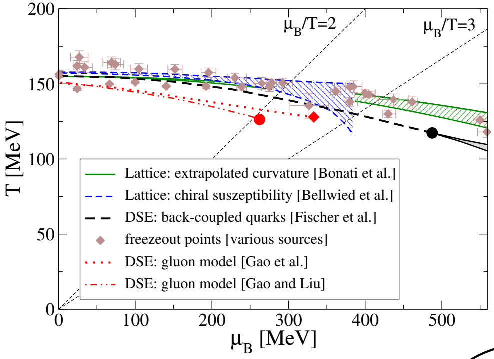

Figure taken from:
Fischer; PPNP 105, 1 (2019)

- Ratio: $\mu_\mathrm{B}^\mathrm{CEP} / T^\mathrm{CEP} \approx 4.16$
- Crossover temperature: $T_\mathrm{c}^{(\mu=0)} = 156\,\mathrm{MeV}$

backcoupling important!

Contact to experiment: Fluctuations

Fluctuations from QCD's grand-canonical potential

$$\chi_{ijk}^{\mathrm{uds}} = -\frac{1}{T^{4-(i+j+k)}} \frac{\partial^{i+j+k}\,\Omega}{\partial\mu_{\mathrm{u}}^{i}\,\partial\mu_{\mathrm{d}}^{j}\,\partial\mu_{\mathrm{s}}^{k}}$$

Relation to conserved charges:
("quark basis ↔ phenomenological basis")

$$\mu_{\mathrm{u}} = \mu_{\mathrm{B}}/3 + 2\mu_{\mathrm{Q}}/3$$
$$\mu_{\mathrm{d}} = \mu_{\mathrm{B}}/3 - \mu_{\mathrm{Q}}/3$$
$$\mu_{\mathrm{s}} = \mu_{\mathrm{B}}/3 - \mu_{\mathrm{Q}}/3 - \mu_{\mathrm{S}}$$

Ratios related to experimental quantities, e.g.:

$$\frac{\chi_2^{\mathrm{B}}}{\chi_1^{\mathrm{B}}} = \frac{\sigma_{\mathrm{B}}^2}{M_{\mathrm{B}}}\,, \qquad \frac{\chi_4^{\mathrm{B}}}{\chi_2^{\mathrm{B}}} = K_{\mathrm{B}}\,\sigma_{\mathrm{B}}^2$$

Sensitive to phase structure: $\chi_2^{\mathrm{B}} \sim \xi^c$ (with $c > 0$) and $\xi \to \infty$ at CEP

Review: Luo, Xu; Nucl. Sci. Tech. 28, 112 (2017)

Fluctuations from DSEs: Formalism

Grand-canonical potential from 2PI formalism:

Cornwall, Jackiw, Tomboulis; PRD 10, 2428 (1979)

$$\Omega = -\frac{T}{V}\left(\mathrm{Tr}\log\frac{S^{-1}}{T} - \mathrm{Tr}\left[\mathbb{1} - S_0^{-1}S\right] + \Phi_{\mathrm{int}}[S]\right) + \Omega_{\mathrm{YM}}$$

First order fluctuation (density):

$$\rho_f = T^3\chi_1^f = -\partial\Omega/\partial\mu_f = -Z_2^f\,\mathrm{Tr}[\gamma_4 S_f]$$

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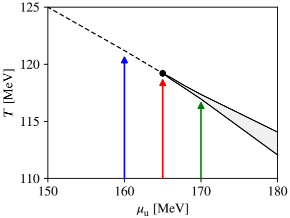
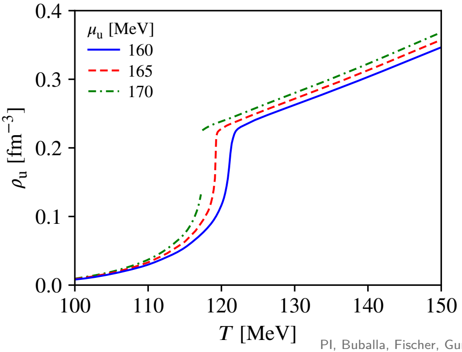
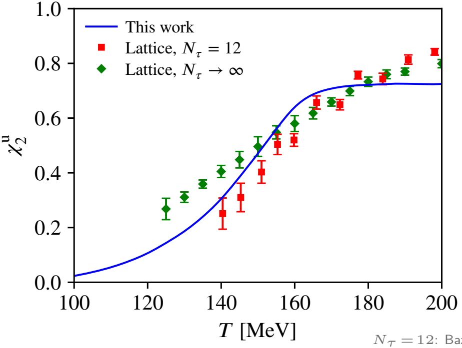
Behavior around CEP:
Consistent with effective models

e.g. Schaefer, Wambach; PRD 75, 085015 (2007)
Buballa; Phys. Rep. 407, 205 (2005)





PI, Buballa, Fischer, Gunkel; in prep.

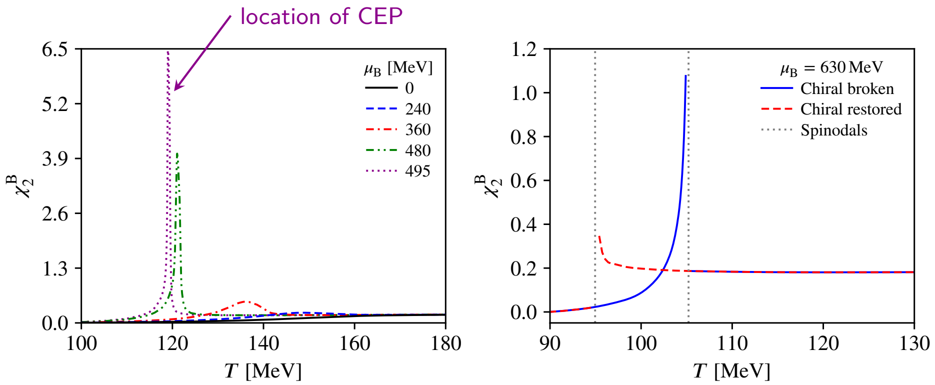


$N_\tau = 12$: Bazavov *et al.*; PRD 85, 054503 (2012)
$N_\tau \to \infty$: Borsányi *et al.*; JHEP 2012, 01:138

- Same ballpark as lattice results
- Saturation below Stefan-Boltzmann limit: Truncation artifact

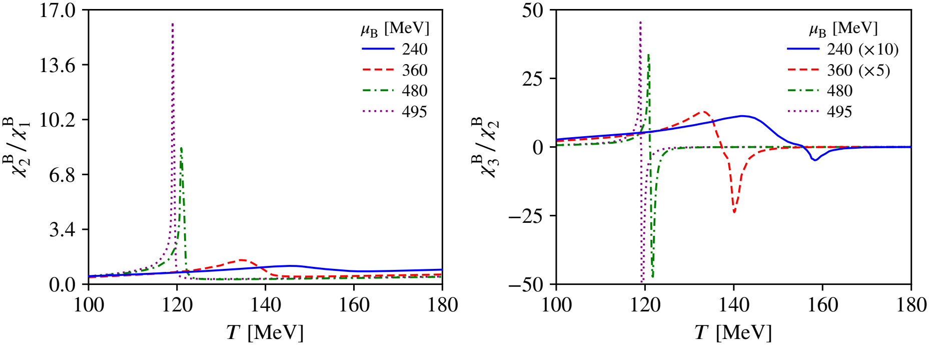
Fluctuations from DSEs: χ_2^{B} at $\mu_{\mathrm{B}} \neq 0$

PI, Buballa, Fischer, Gunkel; in preparation



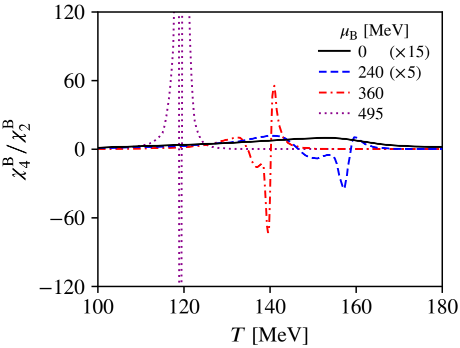
- Divergence of fluctuation indicates CEP
- Jump at phase boundary in first order region

Fluctuations from DSEs: Ratios at $\mu_B \neq 0$



- Approach to CEP is clearly visible in ratios
- Higher order fluctuations more sensitive to phase structure

Fluctuations from DSEs: Kurtosis at $\mu_{\mathrm{B}} \neq 0$



- Qualitative agreement with effective models
  e.g. Fu, Wu; PRD 82, 074013 (2010)
- Very sensitive to CEP

Summary and outlook

QCD phase diagram with DSEs ($2+1$ flavors at physical point):

- Backcoupling of quarks onto gluons important
- CEP at large chemical potential: $\mu_{\mathrm{B}}^{\mathrm{CEP}}/T^{\mathrm{CEP}} \approx 4.16$
- Combined DSE/FRG result: No CEP at $\mu_{\mathrm{B}}/T < 3$

Fluctuations with DSEs:

- First calculation in a truncation with reasonable CEP location
- $\mu = 0$: Same ballpark as lattice results
- $\mu \neq 0$: Qualitative agreement with effective models
- Critical behavior clearly seen
- Track singularity behavior $\Leftrightarrow$ Map phase diagram & locate CEP

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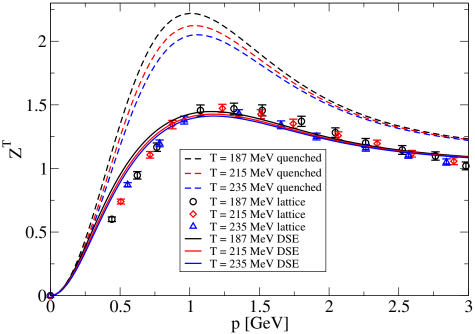
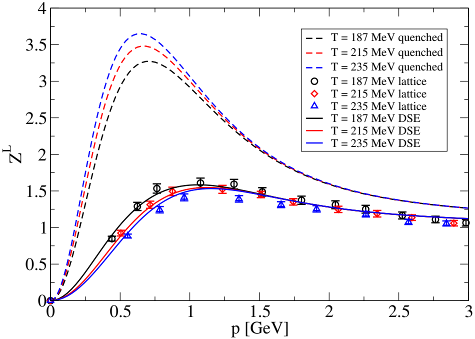
Work in progress:

- Critical exponents
- Plot vs. $\sqrt{s}$ and compare with experiment

Backup slides





- Very good agreement of DSE and lattice results
- DSE prediction verified by lattice

Figures taken from: Fischer; PPNP 105, 1 (2019)
DSE results: Fischer, Luecker; PLB 718, 1036 (2013)
Lattice results: Aouane, Burger, Ilgenfritz, Müller-Preussker, Sternbeck; PRD 87, 114502 (2013)

Ball-Chiu vertex in vacuum

$$\Gamma_\nu^{f,\mathrm{BC}}(p,q) = \frac{A_f(p^2)+A_f(q^2)}{2}\,\gamma_\nu + \mathrm{i}\,\frac{B_f(p^2)-B_f(q^2)}{p^2-q^2}\,(p+q)_\nu + \frac{1}{2}\,\frac{A_f(p^2)-A_f(q^2)}{p^2-q^2}\,(p+q)_\nu\,(\not{p}+\not{q})$$

Ball, Chiu; PRD 22, 2542 (1980)