Thermal splitting of wave-function renormalisations within the FRG

Progress report

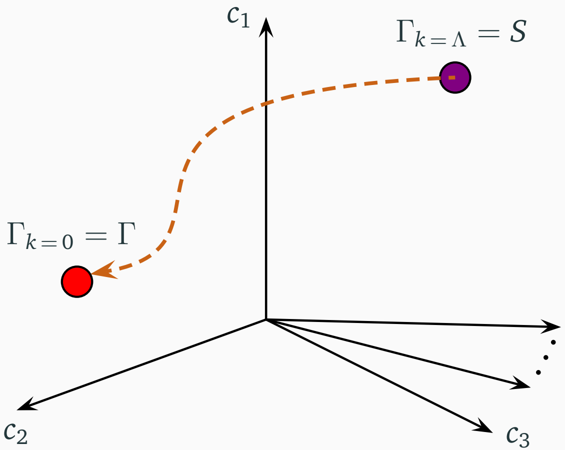
Alexander Stegemann

with Dirk Rischke and Lorenz von Smekal

Functional Methods in Strongly Correlated Systems — Hirschegg — 04 April 2019

Functional renormalisation group

FRG (1)



FRG (2)

- (Exact) Wetterich equation

$$\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_k R_k\right]$$

- Regulator R_k ensures correct integration limits

$$\Gamma_k \xrightarrow{k\to\Lambda} S \qquad\qquad \Gamma_k \xrightarrow{k\to 0} \Gamma$$

- In practice, truncations are needed

$\rightarrow$ Derivative expansion for a scalar field

$$\Gamma_k = \int \mathrm{d}^4x \left[U_k(\phi^2) + \frac{1}{2}Z_k(\phi^2)\big(\partial_\mu\phi\big)^2 + \frac{1}{2}Y_k(\phi^2)\big(\phi\partial_\mu\phi\big)^2 + \dots\right]$$

Quark-meson model in LPA

Quark-meson model in LPA

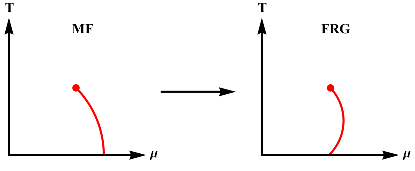
- The quark-meson model is a low-energy effective model for two-flavour QCD
- Ansatz for the effective average action

$$\Gamma_k = \int \mathrm{d}^4 x \left(\bar{\psi}(\partial\!\!\!/ - \mu\gamma_0 + h\Sigma_5)\psi + \frac{1}{2}(\partial_\mu \phi)^2 + U_k(\phi^2) - c\sigma \right)$$

$$\Sigma_5 \; = \; (\sigma + \mathrm{i}\gamma_5 \vec{\pi}\vec{\tau}), \qquad \phi \; = \; (\sigma, \vec{\pi})^{\mathrm{T}}$$

Why go beyond LPA?

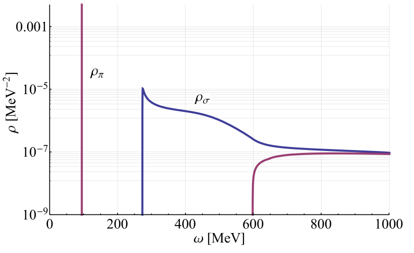
- “Wrong” shape of the first-order transition line



[R.-A. Tripolt *et al.*, Phys. Rev. D 97, 034022 (2018)]

- Pion curvature and pole masses show a large discrepancy in the vacuum

$$m_{\pi,\mathrm{curv}} \approx 138\ \mathrm{MeV}$$
$$m_{\pi,\mathrm{pole}} \approx 100\ \mathrm{MeV}$$



[J. Wambach *et al.*, PoS CPOD2017, 077 (2018)]

Quark-meson model in LPA$'$

Quark-Meson Model in LPA′

- Ansatz for the effective average action

$$\Gamma_k = \int \mathrm{d}^4x \bigg(\bar{\psi}(\partial\!\!\!/ - \mu\gamma_0 + h\Sigma_5)\psi + \frac{1}{2} Z_k \big(\partial_\mu \phi\big)^2 + \frac{1}{2} Y_k \big(\phi \partial_\mu \phi\big)^2 + U_k(\phi^2) - c\sigma \bigg)$$

- Effective wave-function renormalisations

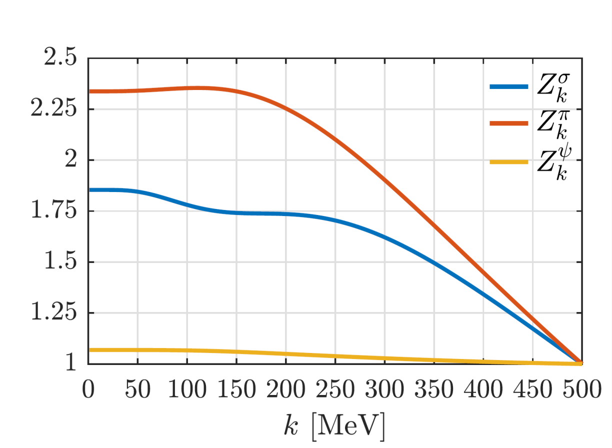
$$Z_\sigma = Z_k + \sigma^2 Y_k$$

$$Z_\pi = Z_k$$

- Z_ψ is neglected (usually found to be close to 1)

Z_ψ in comparison to Z_σ and Z_π

- Vacuum study of the quark-meson model



[F. Divotgey *et al.*, Phys.Rev. D99 no.5, 054023 (2019)]

$\rightarrow Z_\psi$ stays small in comparison to Z_σ and Z_π

Including a finite temperature

- Compactification of the time direction, Matsubara frequencies

$$\int \mathrm{d}^4x \longrightarrow \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^3x \qquad\qquad \int \frac{\mathrm{d}^4p}{(2\pi)^4} \longrightarrow T\sum_{n\in\mathbb{Z}} \int \frac{\mathrm{d}^3p}{(2\pi)^3}$$

$$\omega_n = 2\pi T \cdot n \qquad \text{(bosons)}$$

$$\nu_n = 2\pi T \cdot \left(n + \frac{1}{2}\right) \qquad \text{(fermions)}$$

- Ansatz for the effective average action

$$\Gamma_k = \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^3x \Big(\bar{\psi}(\partial\!\!\!/ - \mu\gamma_0 + h\Sigma_5)\psi + \frac{1}{2} Z_{k,\parallel} (\partial_0\phi)^2 + \frac{1}{2} Z_{k,\perp} (\partial_i\phi)^2$$
$$+ \frac{1}{2} Y_{k,\parallel} (\phi\partial_0\phi)^2 + \frac{1}{2} Y_{k,\perp} (\phi\partial_i\phi)^2 + U_k(\phi^2) - c\sigma \Big)$$

Expectation ($Y_k = 0$)

- Vacuum

$$Z_\phi \equiv Z_k$$

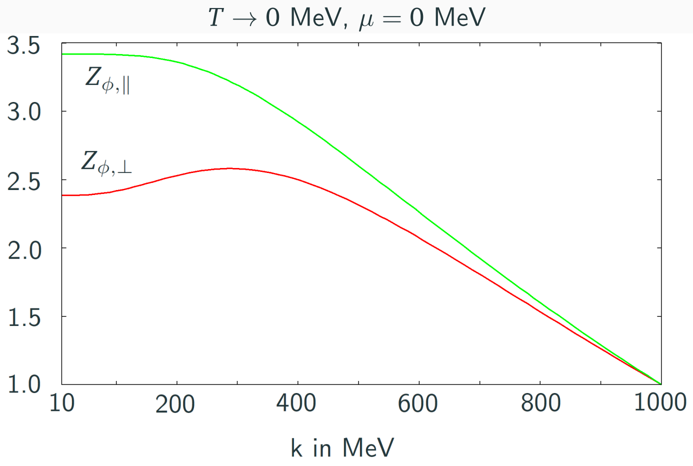
- Finite temperature

$$Z_{\phi,\parallel} \overset{T>0}{\neq} Z_{\phi,\perp} \qquad\qquad Z_{\phi,\parallel} \overset{T\to 0}{=} Z_{\phi,\perp}$$

- A 3-dimensional regulator induces an artificial splitting

  $\rightarrow$ Is a 3-dimensional regulator suited?

Results ($Y_k = 0$, $3d$ Litim regulator)



Specifying the problem

- Possible reasons for the large splitting in the vacuum
  - 3-dimensional regulator
  - Error(s) in this calculation
  - ...?

- Contributions to the flow equations in the limit $T \to 0$

$$\frac{(\partial_k Z_{\phi,\perp})_{\mathrm{B}}}{(\partial_k Z_{\phi,\parallel})_{\mathrm{B}}} \overset{T\to 0}{\approx} 1 \qquad\qquad \frac{(\partial_k Z_{\phi,\perp})_{\mathrm{F}}}{(\partial_k Z_{\phi,\parallel})_{\mathrm{F}}} \xrightarrow{T\to 0} 1 - \frac{M_\psi^2}{k^2}$$

$\rightarrow$ Is there an error in the derivation?

Possible solution

- Including $Z_{\psi,\parallel}$ and $Z_{\psi,\perp}$

  $\rightarrow \left(\partial_k Z_{\phi,\parallel}\right)_{\mathrm{F}}$ and $(\partial_k Z_{\phi,\perp})_{\mathrm{F}}$ get modified by

$$Z_{\psi,\parallel}, \qquad Z_{\psi,\perp}, \qquad \eta_\psi \equiv -k\partial_k \ln Z_{\psi,\perp}$$

  $\rightarrow \partial_k Z_{\phi,\perp}$ and $\partial_k Z_{\psi,\perp}$ are mutually coupled

  $\rightarrow \frac{\partial_k Z_{\phi,\perp}}{\partial_k Z_{\phi,\parallel}} \overset{T\rightarrow 0}{\approx} 1$ becomes possible

Outlook

Outlook

$$\Gamma_k = \int_0^{1/T} \mathrm{d}\tau \int_x \Big[\bar{\psi}\big(Z_{\psi,\parallel}\gamma_0(\partial_0+\mu) + Z_{\psi,\perp}\gamma_i\partial_i + h_k\Sigma_5\big)\psi$$

$$+ \frac{1}{2}Z_{k,\parallel}\big(\partial_0\phi\big)^2 + \frac{1}{2}Z_{k,\perp}\big(\partial_i\phi\big)^2 + \frac{1}{2}Y_{k,\parallel}\big(\phi\partial_0\phi\big)^2 + \frac{1}{2}Y_{k,\perp}\big(\phi\partial_i\phi\big)^2$$

$$+ \frac{1}{2}X_{k,\parallel}\phi^2\big(\partial_0\phi\big)^2 + \frac{1}{2}X_{k,\perp}\phi^2\big(\partial_i\phi\big)^2 + U_k(\phi^2) - c\sigma\Big]$$

- Effective wave-function renormalisations

$$Z_{\sigma,\parallel} = Z_{k,\parallel} + \sigma^2\big(X_{k,\parallel} + Y_{k,\parallel}\big) \qquad Z_{\sigma,\perp} = Z_{k,\perp} + \sigma^2\big(X_{k,\perp} + Y_{k,\perp}\big)$$

$$Z_{\pi,\parallel} = Z_{k,\parallel} + \sigma^2 X_{k,\parallel} \qquad Z_{\pi,\perp} = Z_{k,\perp} + \sigma^2 X_{k,\perp}$$