Mass hierarchies near quantum phase transitions of Dirac fermions

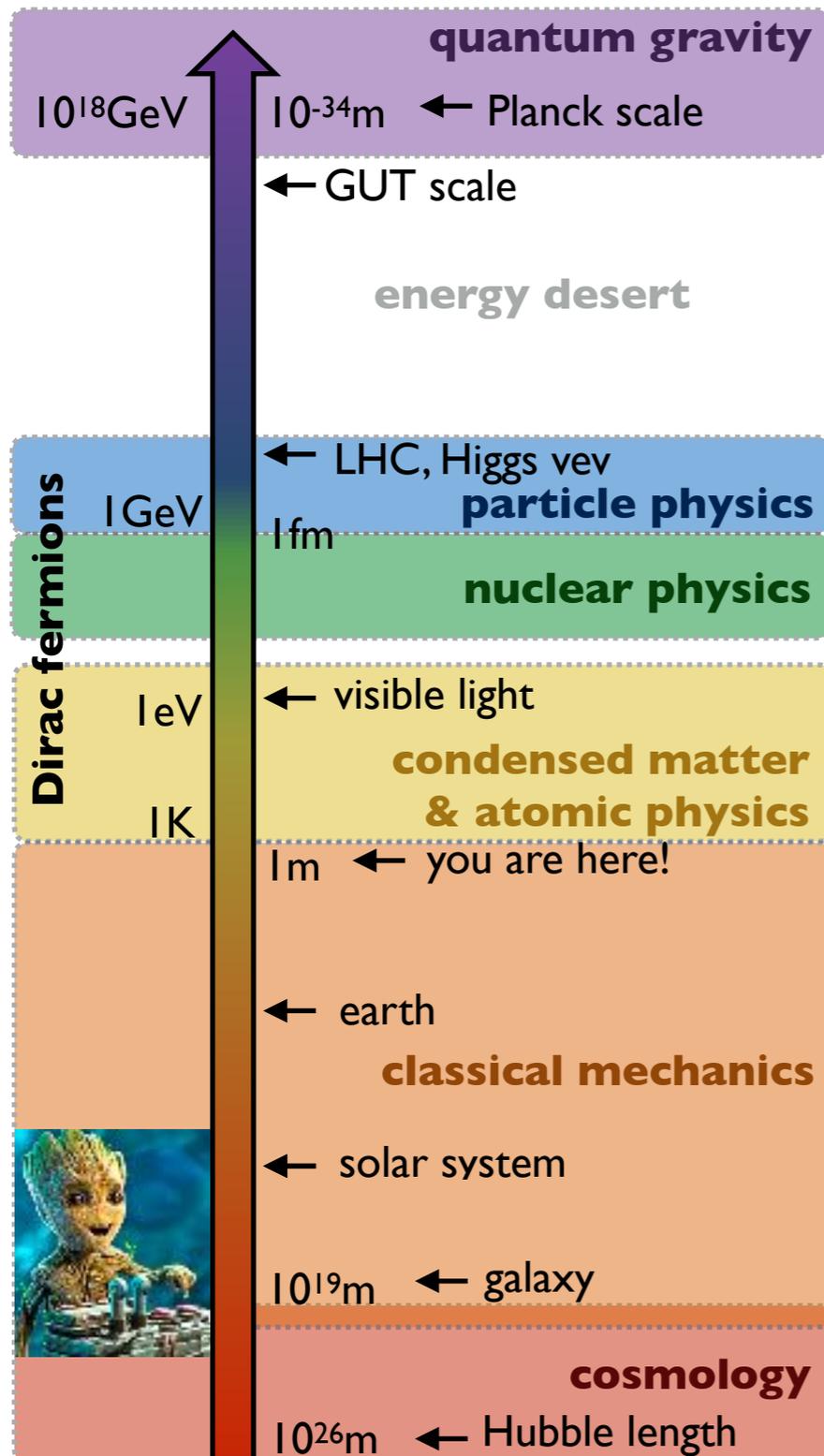**Michael M. Scherer**

Institute for Theoretical Physics, Cologne University

April 5, 2019 @ Hirschegg

Relativistic fermions in condensed matter

- **Dirac, Weyl & Majorana** quasiparticles emerge at low energies in materials

relativistic-like dispersion in non-relativistic crystals

- ▸ **3d** and **2d** realizations

- ▸ **universal properties:**

  specific heat, transport, …

- ▸ **Dirac fermions** appear in

  - *d-wave superconductors*

  - *graphene*

  - *topological insulators*

  - …

Labels in the left diagram:

- quantum gravity
- 10^{18}GeV
- 10^{-34}m ← Planck scale
- ← GUT scale
- energy desert
- ← LHC, Higgs vev
- 1GeV
- 1fm
- **particle physics**
- **nuclear physics**
- ← visible light
- 1eV
- **condensed matter & atomic physics**
- 1K
- 1m ← you are here!
- ← earth
- **classical mechanics**
- ← solar system
- 10^{19}m ← galaxy
- **cosmology**
- 10^{26}m ← Hubble length
- **Dirac fermions**

Outline

- **Introduction & motivation**

  ‣ One origin of Dirac fermions in condensed matter physics — graphene

  ‣ Quantum phase transitions of Dirac fermions — ordering patterns/scenarios

  ‣ Universal critical behavior: (F)RG, QMC, conformal bootstrap — number crunching

- **The Kekulé transition and fluctuation-induced criticality**

  ‣ First-order vs. second-order transition

  ‣ FRG fixed-points and scaling

  ‣ Symmetry-broken phase and emergence of two mass scales

- **Mechanism of mass hierarchies near quantum phase transitions**

  ‣ Application to multicomponent superconductors

  ‣ Some bold speculations about particle physics
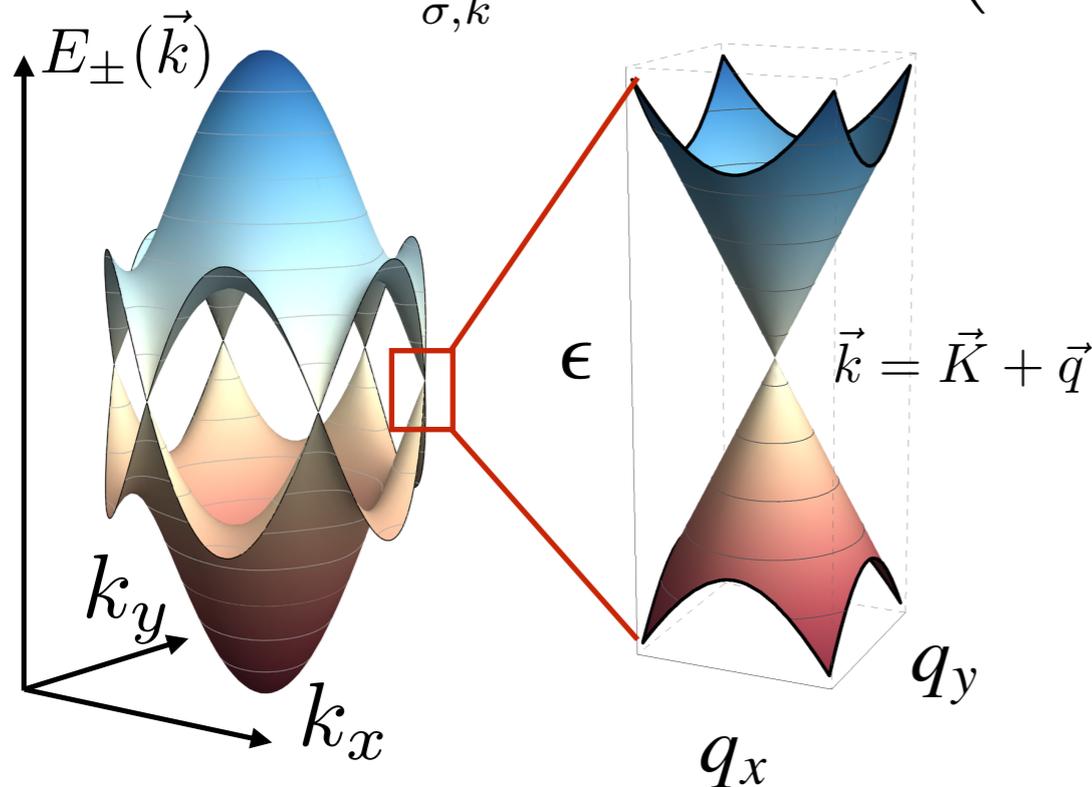
Electrons on the honeycomb lattice

- **2D Dirac materials** (graphene)

sublattice A

sublattice B
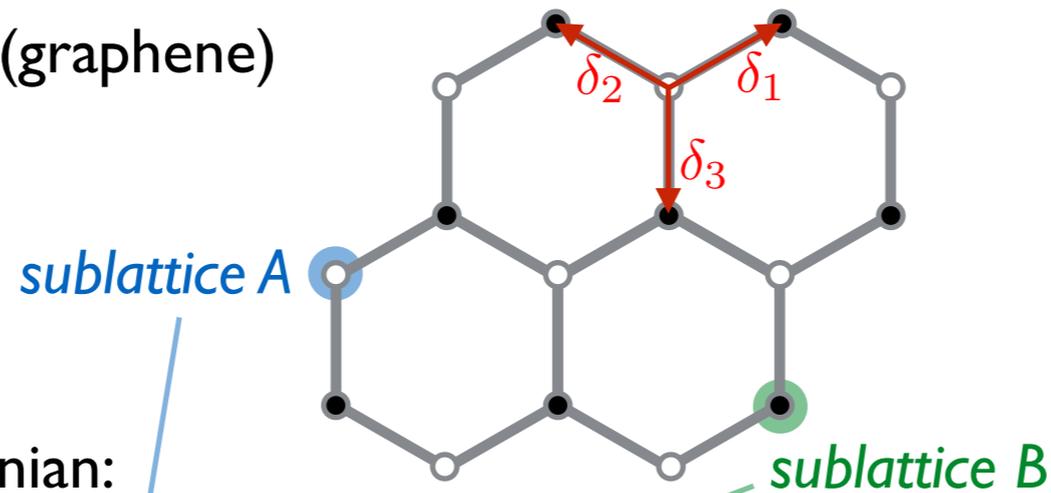
▸ tight-binding Hamiltonian:

$$H_0 = -t \sum_{\sigma, \vec{R}, \vec{\delta}_i} \left[u_\sigma^\dagger(\vec{R}) v_\sigma(\vec{R} + \vec{\delta}_i) + \text{h.c.} \right]$$

$$= \sum_{\sigma, \vec{k}} \left(u_\sigma^\dagger(\vec{k}), v_\sigma^\dagger(\vec{k}) \right) \begin{pmatrix} 0 & -t\, d(\vec{k}) \\ -t\, d^*(\vec{k}) & 0 \end{pmatrix} \begin{pmatrix} u_\sigma(\vec{k}) \\ v_\sigma(\vec{k}) \end{pmatrix} \quad \text{where} \quad d(\vec{k}) = \sum_{\vec{\delta}_i} e^{i \vec{k} \cdot \vec{\delta}_i}$$

➡ energy dispersion: $\quad E_\pm(\vec{k}) = \pm t\, |d(\vec{k})|$

▸ no gap & vanishing density of states
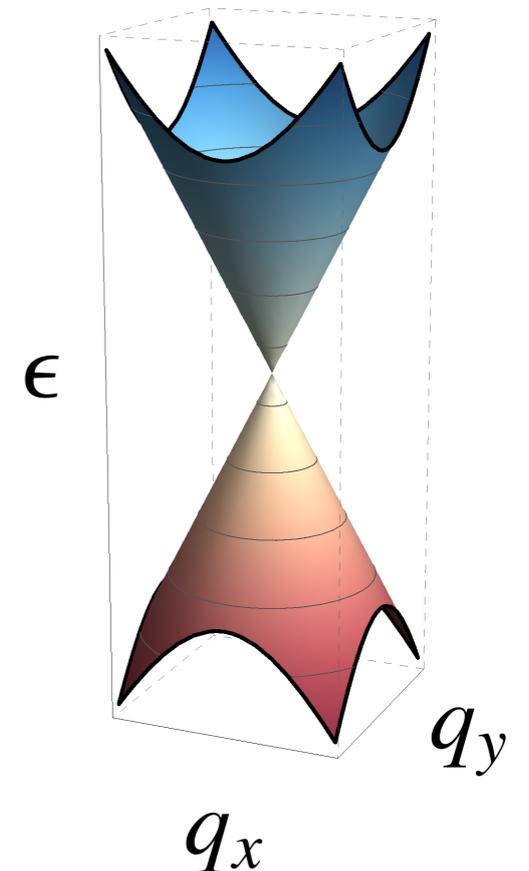
▸ **semi-metallic** behavior

Effective theory for fermions on the honeycomb lattice

$$H_0 = -t \sum_{\vec{R},i} \left[u^\dagger(\vec{R}) v(\vec{R} + \vec{\delta}_i) + \text{h.c.} \right]$$

▸ energy: linear & isotropic near **K, K'**

▸ retain only Fourier components around **K, K'**

▸ *low-energy effective* Hamiltonian:

Dirac matrices *α*

$$H_{\text{eff}} = -iv_F \sum_{\sigma=\pm} \int dxdy \left[\psi_\sigma^\dagger \vec{\alpha} \cdot \vec{\nabla} \psi_\sigma \right]$$

▸ 8-component spinor

$$\psi_\sigma^\dagger(\vec{x}, \tau) = T \sum_{\omega_n} \int^\Lambda \frac{d\vec{q}}{(2\pi a)^2} e^{i\omega_n + i\vec{q}\cdot\vec{x}} \left[u^\dagger(\vec{K}+\vec{q}, \omega_n), v^\dagger(\vec{K}+\vec{q}, \omega_n), u^\dagger(-\vec{K}+\vec{q}, \omega_n), v^\dagger(-\vec{K}+\vec{q}, \omega_n) \right]$$

▸ generalization: *N*-component spinor

ϵ

q_y

q_x

# Dirac **fermions** and **critical phenomena**

- strong interactions may induce **phase transitions** of **gapless Dirac fermions**

  - *Dirac electrons in singlelayer graphene*: AFM/Kekulé/staggered order

  - *Dirac cones in twisted bilayer graphene*: correlated insulator transition

Dirac SM   *ordered/gapped phase*

r_c *tuning parameter **r***
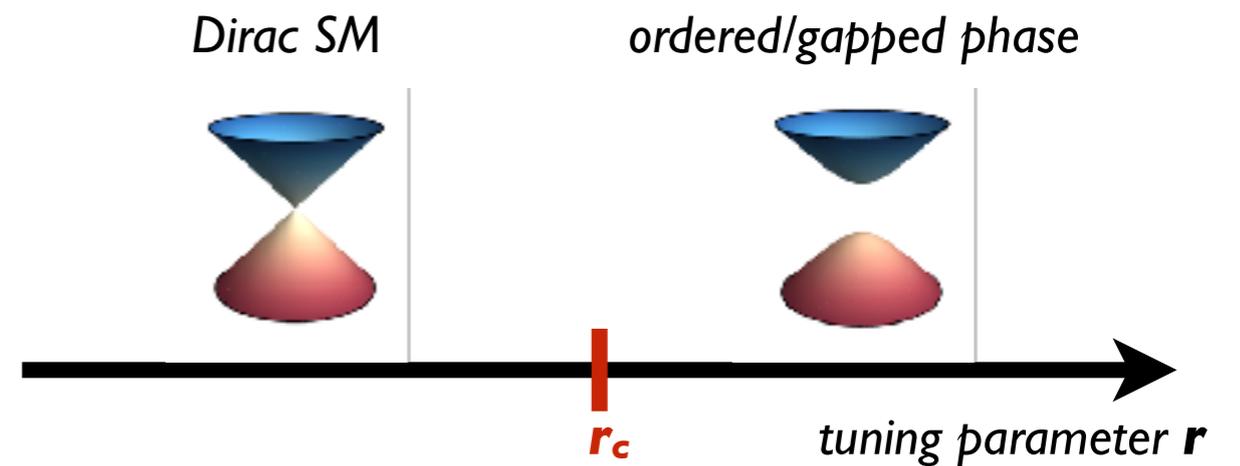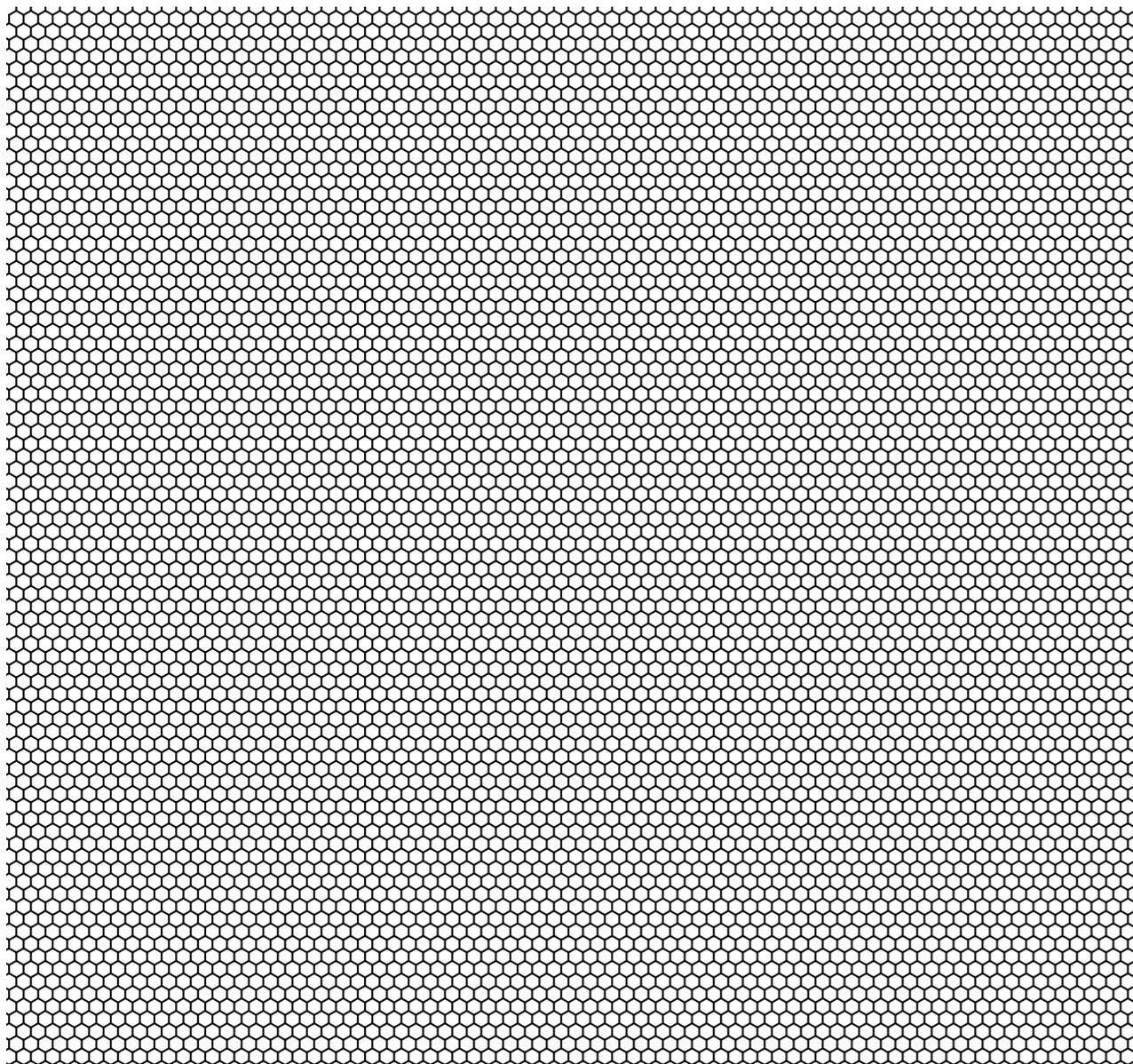
▸ **quantum critical point**

▸ (2+1)D **fermionic universality classes!**

Recap: **Phase transitions** and **critical phenomena**

- near critical point of continuous phase transition: **universality**

- **correlation length** diverges for T → T$_c$: $\xi(t) \propto |t|^{-\nu}$

- order parameter **correlation function** at T$_c$: $G(\vec{r}) \propto \dfrac{1}{|\vec{r}|^{D-2+\eta}}$

order parameter
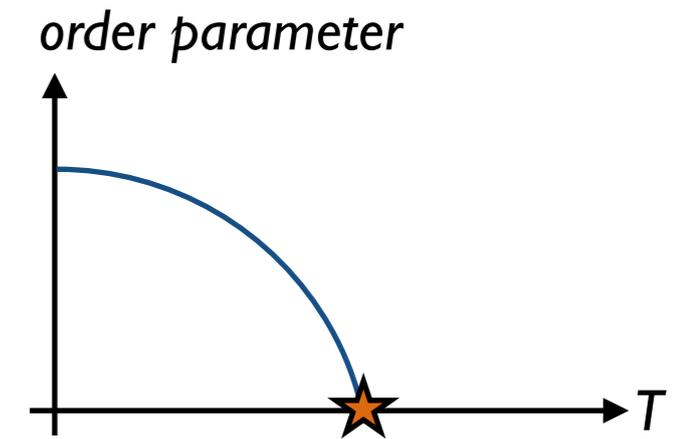
- **3D Ising universality class:**

Method	ν	η	
conformal bootstrap	0.629971(4)	0.036298(2)	Kos *et al.* (2016)
Monte Carlo	0.63002(10)	0.03627(10)	Hasenbusch (2010)
pRG, 4-ϵ, 6th order	0.6292(5)	0.0362(2)	Panzer & Kompaniets (2017)
functional RGs, DE	0.630(5)	0.034(5)	Litim & Zappala (2010)
experiment (fluid mixture)	0.629(3)	0.032(13)	Sengers & Shanks (2009)

✦ **gapless Dirac fermions *not* in Ising/O(N) universality classes!**

Effective theories for phase transitions in **Dirac** systems

- Critical point described by simple *continuum* field theory in $D = 2+1$ dimensions Herbut (2006)

  ▸ *Gross-Neveu model:* $\mathcal{L}_{\mathrm{GN}} = \bar{\psi}_i \gamma_\mu \partial_\mu \psi_i + g(\bar{\psi}_i \psi_i)^2$

  - simplest *fermionic theory* with *critical point* (relativistic, no Fermi surface,…)

  - example: **charge density wave** transition of Dirac electrons in graphene

  - *bosonized version* of model…

  ▸ *Gross-Neveu-Yukawa model:* $\mathcal{L}_{\mathrm{GNY}} = \bar{\psi}_i (\gamma_\mu \partial_\mu + \sqrt{y}\phi)\psi + \frac{1}{2}\phi(m^2 - \partial_\mu^2)\phi + \lambda\phi^4$

- both models have **critical point** in **2 < D < 4** and lie in **same universality class**

  ➡ **Gross-Neveu/chiral/fermionic universality class**

- *precision determination* of universality classes of Dirac fermions *now within reach…*
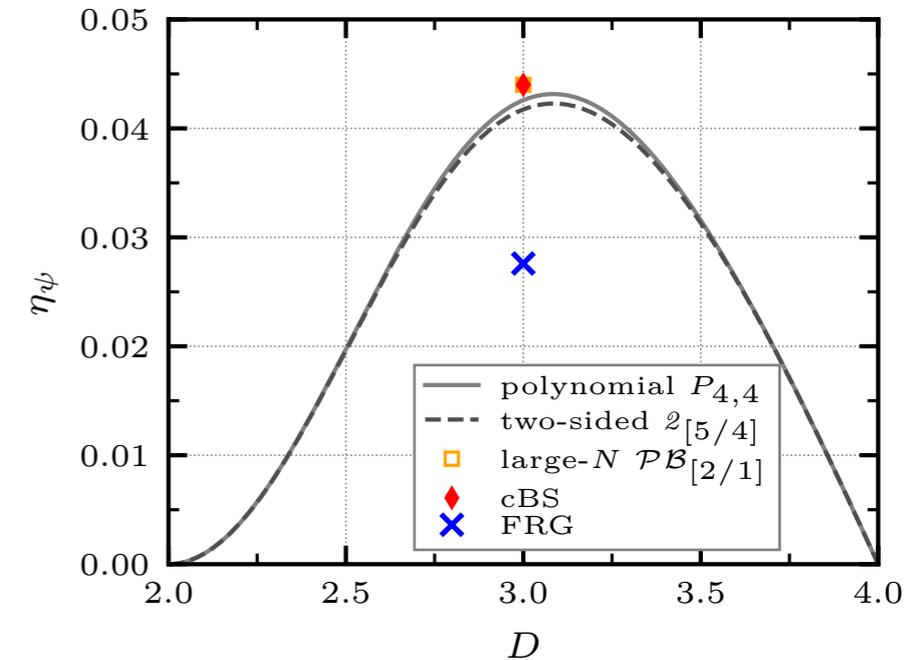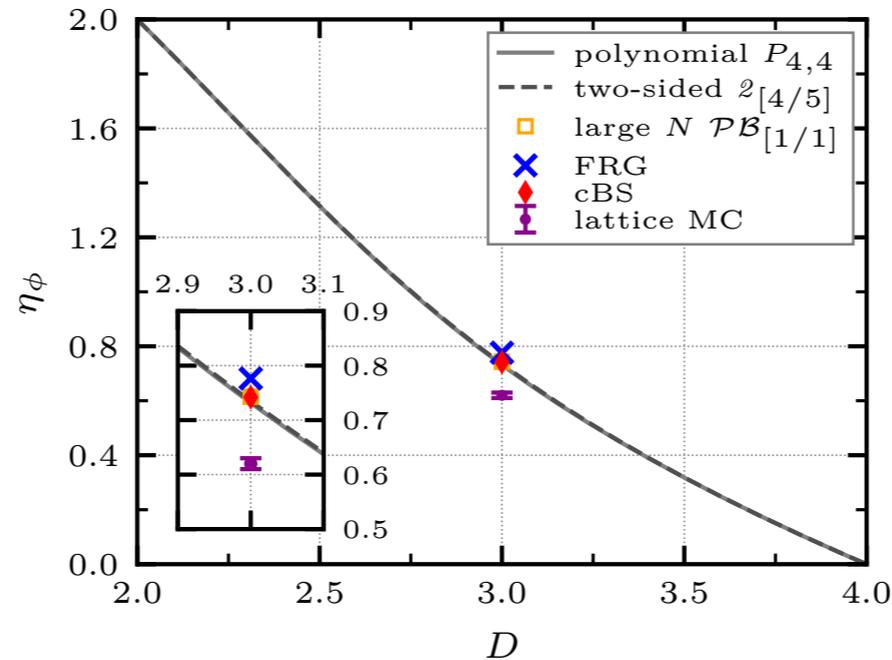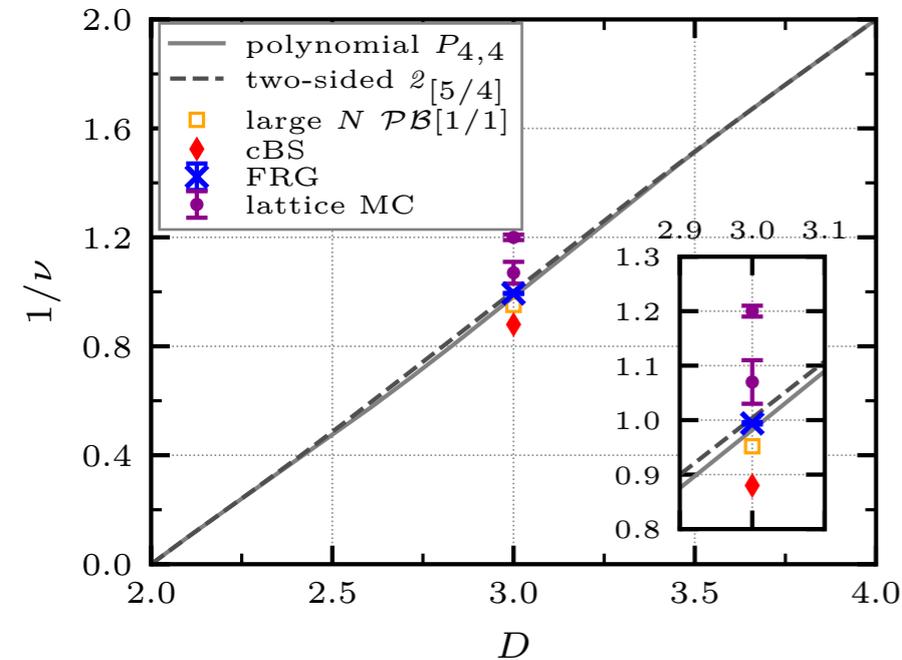
Critical exponents of Dirac systems

from perturbative renormalization

with Bernhard Ihrig and Luminita Mihaila

Quantum critical behavior of Gross-Neveu model (*N=8*)

- Critical exponents for graphene case from 4-loop RG in (2+ε)D and (4-ε)D:

📑 Gracey, Luthe & Schroeder (2016)
📑 Zerf, Mihaila, Marquard, Herbut, MMS (2017)

Method	$1/\nu$	η_B	η_F	
4-loop pert. RG, ϵ^4	0.99	0.731	0.043	📑 Ihrig, Mihaila, MMS (2018)
conformal bootstrap	0.88	0.742	0.044	📑 Iliesiu et al. (2017)
functional RG	0.994	0.7765	0.0274	📑 Knorr (2016)
Monte Carlo	1.20(1)	0.62(1)	0.38(1)	📑 Chandrasekharan & Li (2013)
Monte Carlo	1.07(4)	-	-	📑 Schmidt (2017)
Monte Carlo	1.00(4)	0.754(8)	-	📑 Karkkainen et al. (1994)

Fluctuation-induced quantum criticality
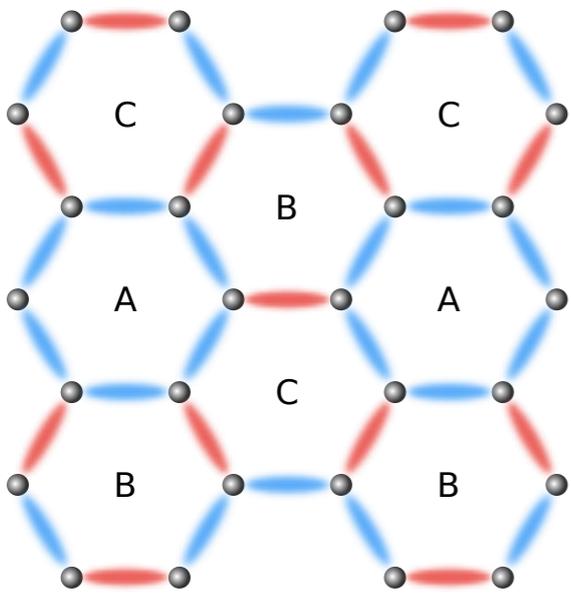
emergent U(1) symmetry and two mass scales

with Emilio Torres, Laura Classen and Igor Herbut

Laura Classen, Igor Herbut, MMS, Phys. Rev. B 96, 115132 (2017) Editors' Suggestion

Emilio Torres, Laura Classen, Igor Herbut, MMS, Phys. Rev. B 97, 125137 (2018) Editors' Suggestion

Symmetry breaking and **mass gaps** from bond **dimerization**

- single-particle Hamiltonian:
$$H_0 = -t \sum_{\vec{R},i} \left[u^\dagger(\vec{R}) v(\vec{R} + \vec{\delta}_i) + \text{h.c.} \right]$$

▸ introduce **dimerization pattern** (C$_6$ → C$_3$):

$$\Delta H_0 = - \sum_{\vec{R},i} \left[\Delta t_{\vec{R},i} u^\dagger(\vec{R}) v(\vec{R} + \vec{\delta}_i) + \text{h.c.} \right]$$

▸ with $\Delta t_{\vec{R},i} = \phi(\vec{R}) e^{i\vec{K}\cdot\vec{\delta}_i} e^{i(\vec{K}-\vec{K}')\cdot\vec{R}} / 3 + \text{c.c.}$

▸ $\phi = \phi_1 + i\,\phi_2$ is complex-valued OP with Z$_3$ symmetry

- **Kekulé valence bond solid**

- mass gap: $E_\pm \propto \pm\sqrt{|\vec{q}|^2 + |\phi_0|^2}$

- mechanisms to induce **Kekulé** order: phonons, substrate, electron-electron interactions

Effective theory for **Kekulé** transition

- **coupling** between **fermions** and **OP** field: $\mathcal{L}_y = y \left(\phi_1 \bar{\psi} \, i\gamma_3 \psi + \phi_2 \bar{\psi} \, i\gamma_5 \psi \right)$
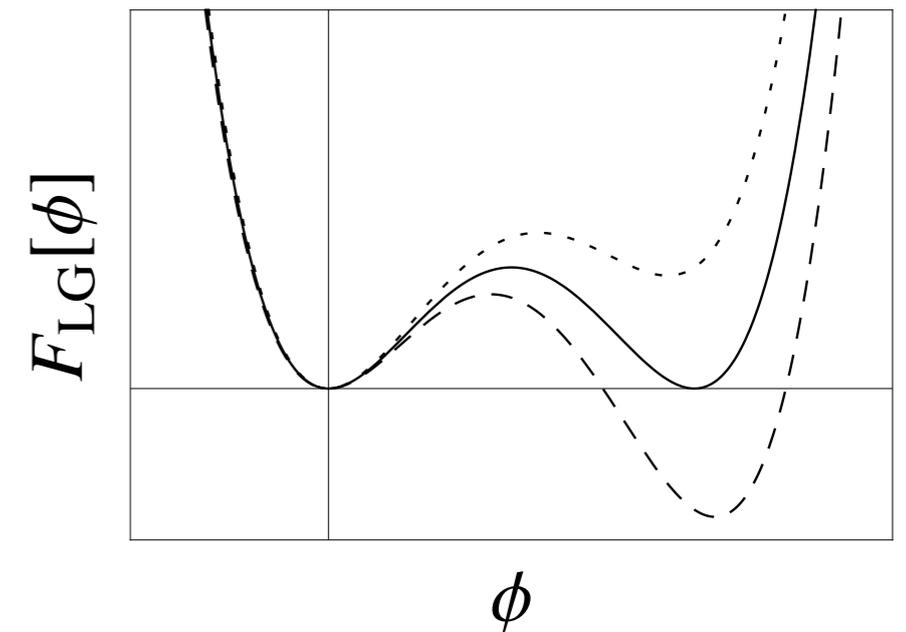
  where $\gamma_3 = \sigma_x \otimes \sigma_y$ and $\gamma_5 = \sigma_y \otimes \sigma_y$

- **order parameter**: $\mathcal{L}_\phi = \phi^* \left(\partial_\tau^2 - \nabla^2 + m^2 \right) \phi + g \left(\phi^3 + \phi^{*3} \right) + \lambda |\phi|^4$

  relevant   marginal

  ▸ Z_3 symmetry allows presence of **cubic OP terms**

- **scaling near critical dimension** $D_c = 4$:

  ▸ marginal couplings → irrelevant at NGFP below D_c

  ▸ cubic self-coupling of bosonic OP field ($D = 4 - \epsilon$)

$$[g] = 1 + \frac{\epsilon}{2}$$

- canonically relevant → **2 tuning parameters!**

- tuning of one parameter not sufficient for critical behavior

- expect **1st order transition**

RG approach: ε expansion & β functions

- tree level: ψ ——→ ϕ ············· g λ y

▸ **RG flow of cubic coupling:** $\beta_g = \dfrac{dg}{d\ln b} = \left(1 + \dfrac{\epsilon}{2} - \dfrac{3\eta_\phi}{2}\right) g - 3g\lambda$

$$= (1 + \epsilon/2 - \bigcirc\!\!\bigcirc - \triangleleft\!\!\triangleright -)g - \triangleleft\!\!\blacksquare$$

- **$g_* = 0$** even when other couplings are finite at FP: **$\mathbf{Z_3 \rightarrow U(1)}$**

▸ **RG scaling** of **cubic coupling:** $\theta = \dfrac{\partial \beta_g}{\partial g}\bigg|_{\mathrm{FP}} = 1 + \dfrac{\epsilon}{2} - \dfrac{3N}{8}y_*^2 - 3\lambda_*$
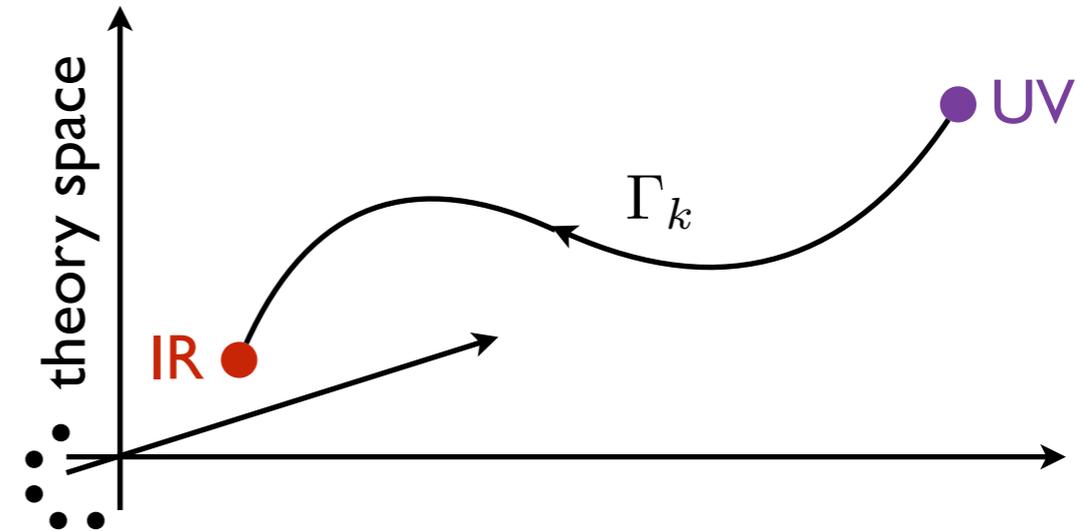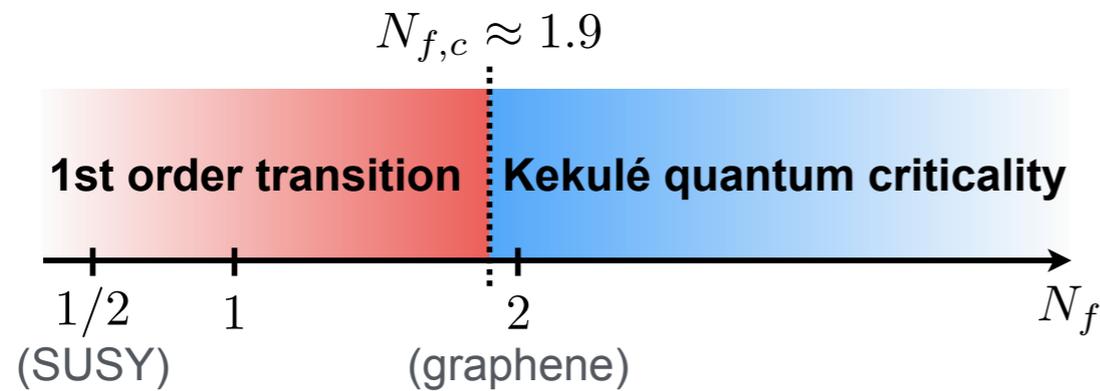
- **cubic** coupling may become **irrelevant** induced by fermion fluctuations!

- **fluctuation-induced QCP:** putative 1st order transition is turned 2nd order

Non-perturbative RG approach: **FRG**

$F_{\mathrm{LG}}[\phi]$ / ϕ

▸ Employ functional RG approach: *non-perturbative scaling, SSB regime*

▸ LPA' results for critical behavior:

$N_{f,c} \approx 1.9$

| **1st order transition** | Kekulé quantum criticality |

1/2   1   2 N_f
(SUSY)   (graphene)

theory space

● UV

Γ_k

IR ●

▸ subleading exponent close to 0:

| N_f	θ_1	θ_2	θ_3
1	0.8365	**+0.1667**	-0.8245
2	0.8639	**-0.00311**	-0.8789
3	0.9020	**-0.02350**	-0.9205
4	0.9239	**-0.02629**	-0.9414
5	0.9377	**-0.02552**	-0.9537
6	0.9473	**-0.02394**	-0.9617
7	0.9543	**-0.02224**	-0.9674
8	0.9596	**-0.02065**	-0.9716
9	0.9638	**-0.01921**	-0.9749
10	0.9672	**-0.01792**	-0.9775
20	0.9831	**-0.01054**	-0.9889
50	0.9931	**-0.00465**	-0.9956
∞	1	**0**	-1

▸ cubic scaling is generally small in D=2+1

▸ expect large **corrections to scaling**:

$$\xi \sim A|\Delta|^{-\nu}\left(1 + B|\Delta|^{-\theta_2} + \dots\right)$$

– challenge for QMC simulations!

Classen, Herbut, MMS (2017)

# **Spontaneously** symmetry-broken **regime**

- U(1) vs Z_3 symmetric model
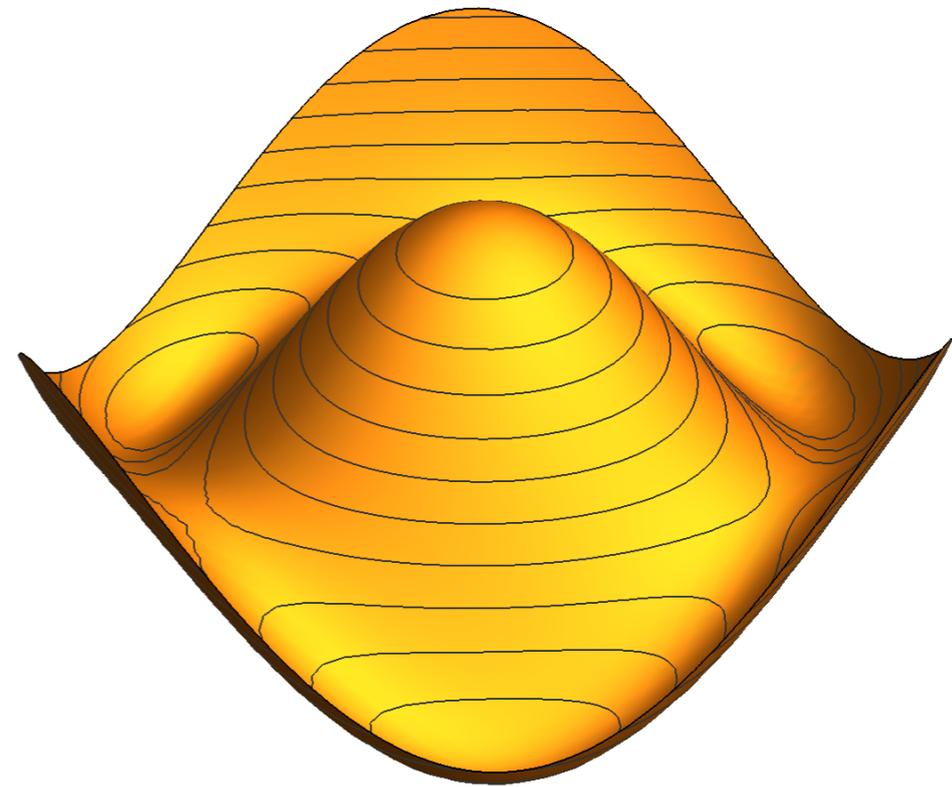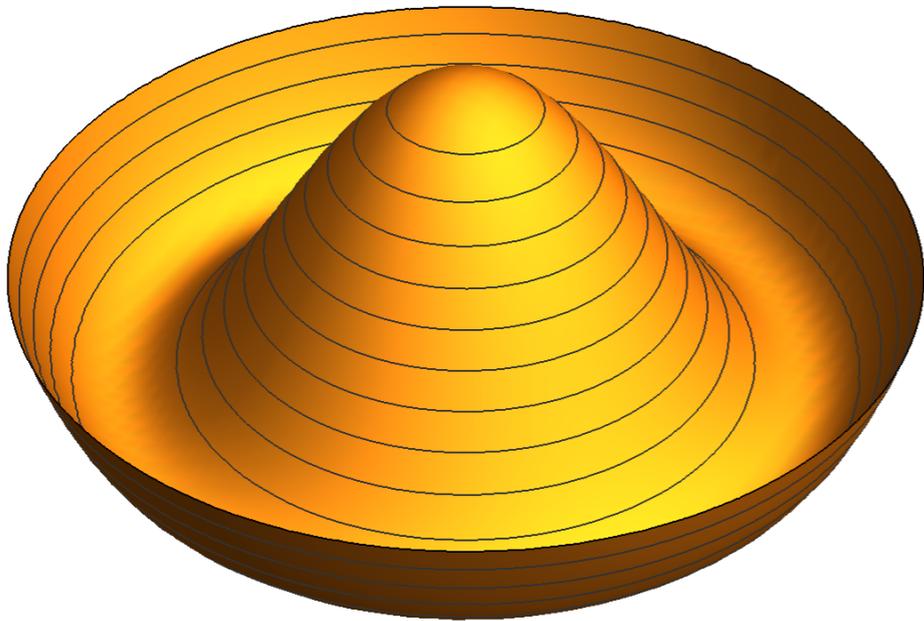
▸ *discrete symmetry breaking* - **no Goldstone modes**

▸ system has **one** transition with **longitudinal** mass and **transversal** mass

- **two** length/mass scales below T_c

Quantum criticality with two mass scales

- ▶ flow starts in SYM regime - *fine-tuned*

- ▶ approaches QCP - *scaling regime*

- ▶ at QCP g is irrelevant & becomes small: $g \to g_* = 0$

- ▶ flow departs from QCP - in SSB: Nambu-Goldstone FP

- ▶ at Nambu-Goldstone FP: g is relevant and increases again
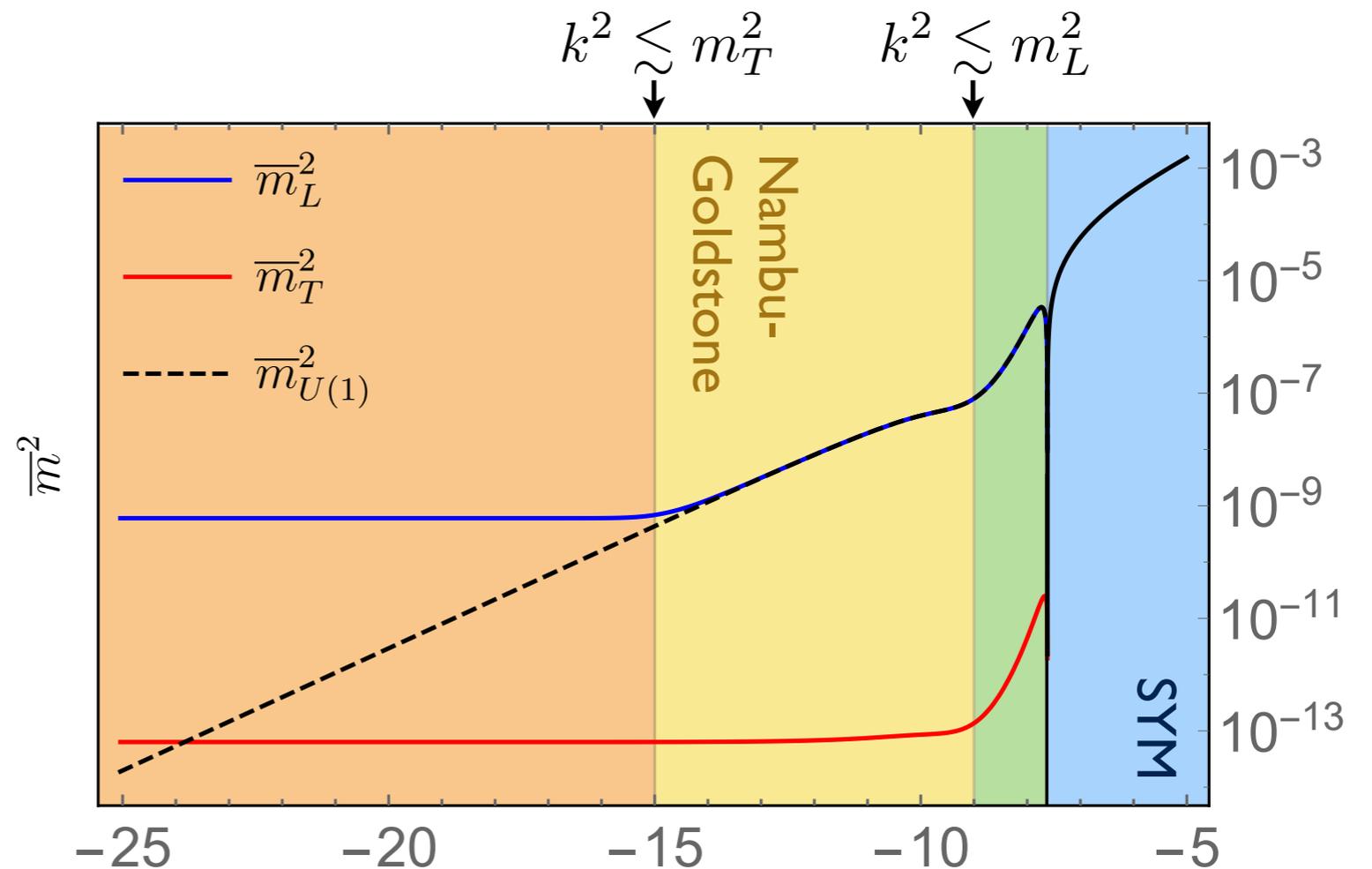
Torres, Classen, Herbut, MMS (2018)
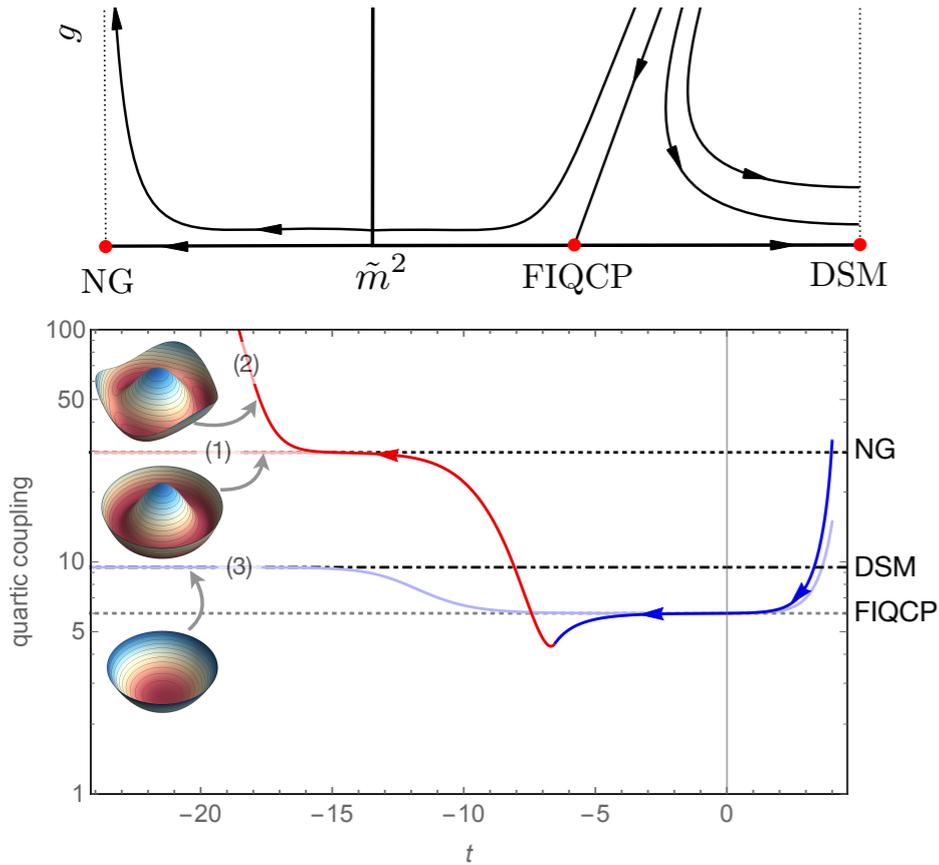
- **FRG flow of boson masses:**

$$m_L^2 \propto \lambda \cdot vev^2$$

$$m_T^2 \propto g \cdot vev$$

- ▶ transversal mass small as

$g \to g_* = 0$ near transition!

Mechanism behind **mass hierarchy**

Leonard, Delamotte, Wschebor (2018)

Jian, MMS, Yao (2018)

- **Ingredients:**

  ‣ action with two scalar fields $\phi = (\phi_1, \phi_2)$ and O(2) symmetry

  $$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{8}(\phi^2)^2$$

  ‣ **break O(2) symmetry** down to $\mathbb{Z}_n$ by including corresponding terms in action
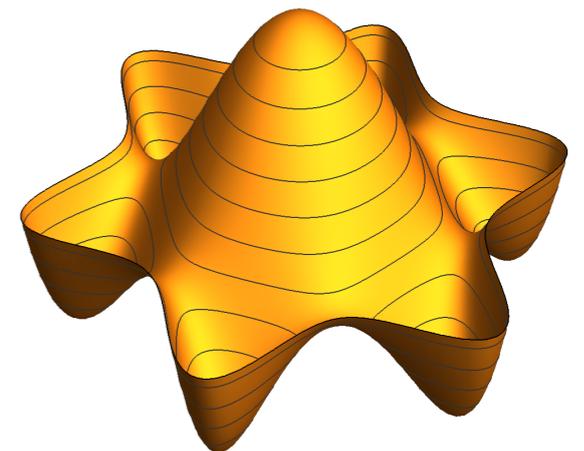
  - Kekulé transition, n=3: $\Delta\mathcal{L} \propto \phi^3$

  - hexagonal anisotropy, n=6: $\Delta\mathcal{L} = \frac{\lambda_6}{8}(\phi_1 - \phi_2)^2(\phi_1 + 4\phi_1\phi_2 + \phi_2)^2$

  ‣ make sure that additional terms are **irrelevant at CP**

  ✓ Kekulé & Dirac fermions, D=2+1: $\theta_g|_{QCP} < 0$

  ✓ hexagonal anisotropy: canonical scaling $[\lambda_6] = 6 - 2D$

Mechanism behind **mass hierarchy**

- **Symmetry-broken regime** ($\mathbb{Z}_6$) with vev $\sqrt{2\kappa}$

  ▸ potential $U = \dfrac{\lambda}{2}(\rho - \kappa)^2 + \lambda_6 \tau$ with $\rho = \dfrac{\phi^2}{2}$ and $\tau = (\phi_1 - \phi_2)^2(\phi_1 + 4\phi_1\phi_2 + \phi_2)^2/8$

  ▸ masses: $m_L^2 = 2\lambda\kappa$ and $m_T^2 = 18\lambda_6\kappa^2$

  ▸ **mass ratio** $\boxed{\dfrac{m_T^2}{m_L^2} \propto \dfrac{\lambda_6\kappa}{\lambda}}$
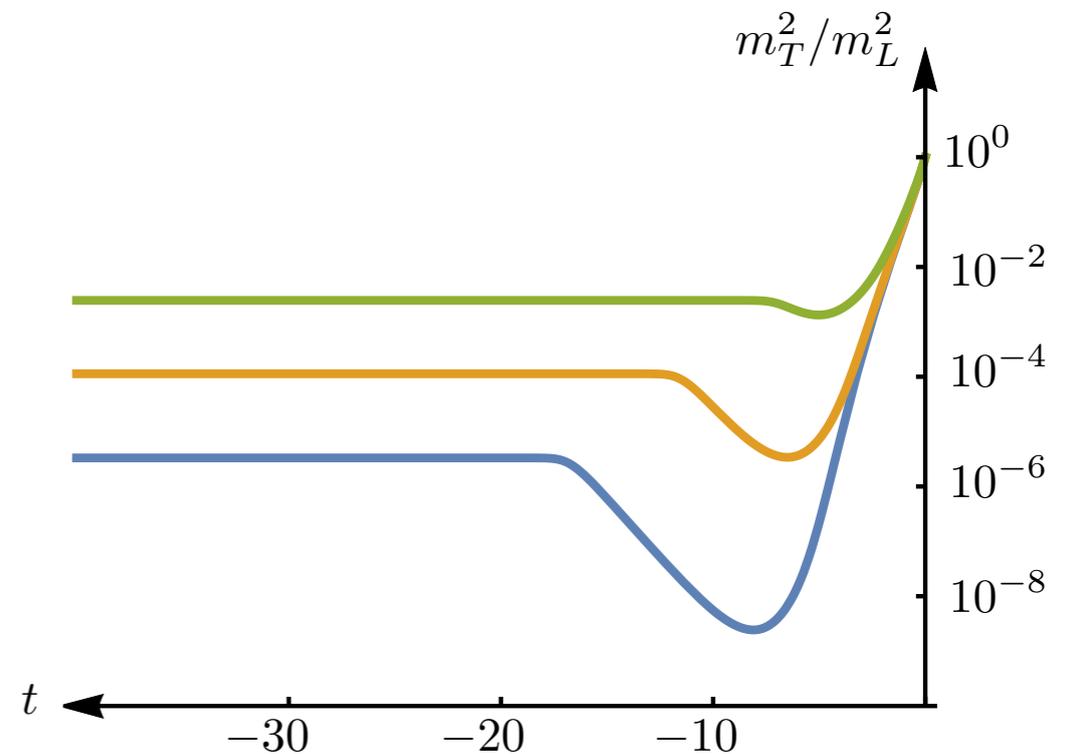
- **RG near fixed point**

  ▸ **CP scaling regime:**

    - dimensionless κ and λ stay nearly constant

    - λ_6 flows rapidly to small values

  ▸ **infrared regime:**

    - both masses freeze out and give small constant mass ratio

   - closer to criticality mass hierarchy is larger

Mass hierarchies in multi-component superconductors

with Shaokai Jian and Hong Yao

# **Multicomponent** superconductors and the **Leggett mode**

- Pair-density wave (PDW) superconducting OP: $\quad \Delta(\vec{r}) = \Delta_+(\vec{r})e^{i\vec{Q}\cdot\vec{r}} + \Delta_-(\vec{r})e^{-i\vec{Q}\cdot\vec{r}}$

  ‣ two SC condensates

  ‣ U(1) trafo from charge conservation shifts all components

  ‣ **Leggett mode** corresponds to relative phase fluctuations between condensates

  ‣ **Josephson coupling** between two condensates → discrete $\mathbb{Z}_n$ with term $\propto (\Delta_+\Delta_-)^3$

  ‣ **Dirac electrons** on the honeycomb lattice

mass

hierarchy

$$m_\psi^2 = 2h^2\kappa$$

$$m_+^2 = 2\lambda\kappa$$
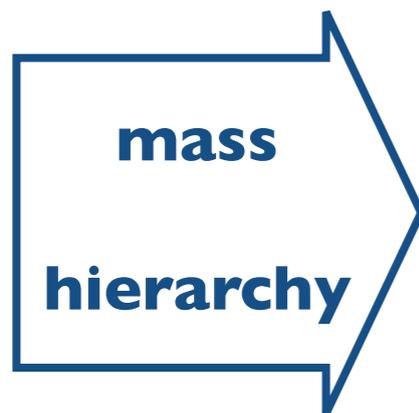
$$m_{\mathrm{Legg}} = 18\lambda_6\kappa^2$$
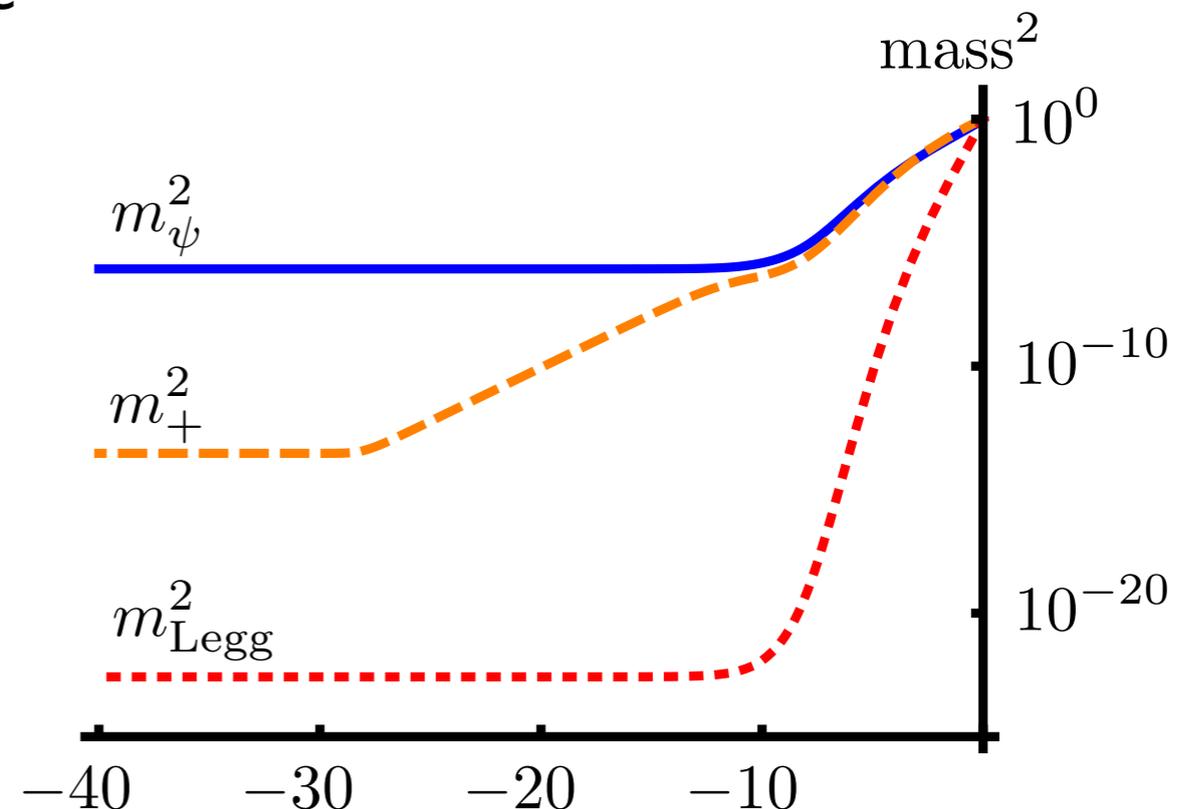
Summary & **conclusions**

◉ **Fermion-induced quantum critical points**

  ‣ *continuous transition* where discontinuous one is expected

  ‣ *emergent U(1)* symmetry at quantum critical point

  ‣ *two divergent length/mass scales due to discrete SB*

  ‣ *natural mechanism to generate small masses!*