Studienstiftung des deutschen Volkes

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

Role of Fluctuations in Compact Objects

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IN COLLABORATION WITH

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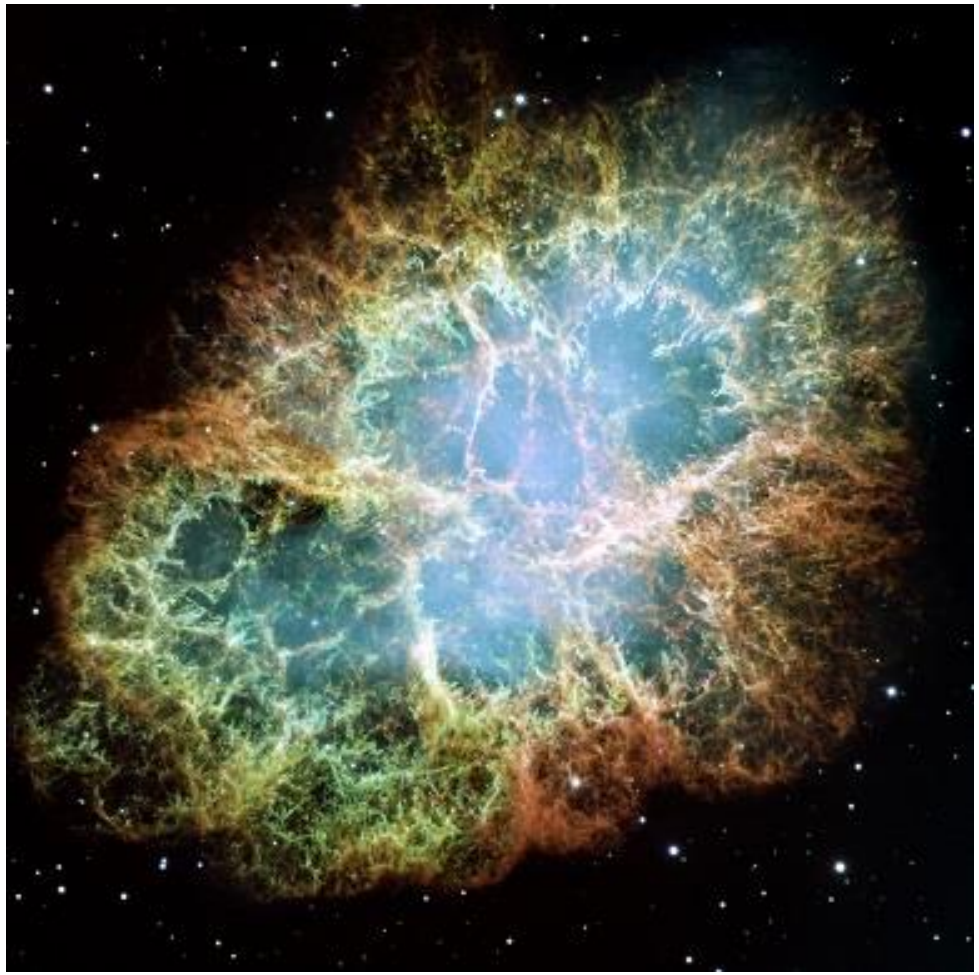


[NASA]

Motivation

- Phase diagram of Quantum Chromodynamics (QCD)

  → access only with non-perturbative methods

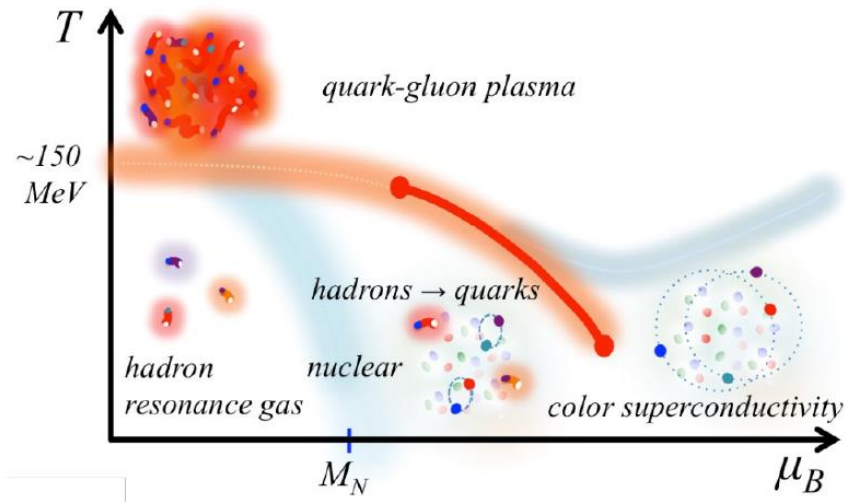
- Complex phase structure in cold and dense region

  → first-order chiral phase transition?

  → color superconducting phases (CFL, 2SC, …)

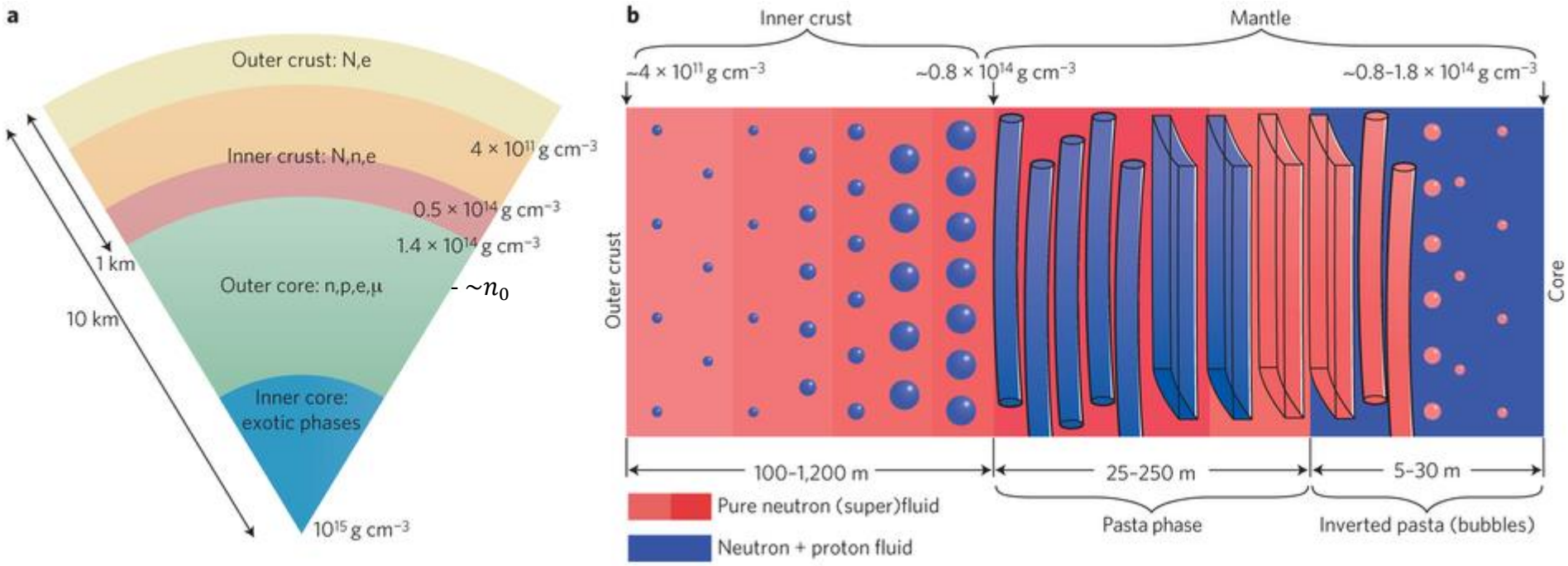
  → inhomogeneities? (see Martin Steil, Adrian Königstein)

- Composition of neutron stars (quark matter?, role of strangeness?)



[Baym et al., Rept. Prog. Phys. 81 '18]

Neutron Stars



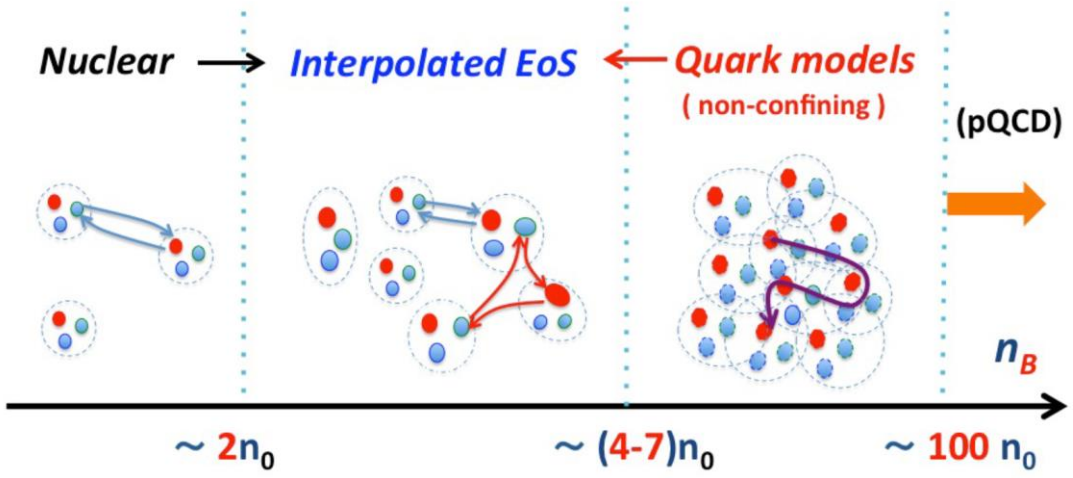
compstar.uni-frankfurt.de/wp-content/uploads/2015/12/pastaNewton.jpg]

→ **Interested in inner core! (pure quark matter star at first…)**

Solve Tolman-Oppenheimer-Volkoff equations:

$$\frac{dp(r)}{dr} = -G\,\frac{\big(\varepsilon(r)+p(r)\big)\big[m(r)+4\pi r^3 p(r)\big]}{r[r-2Gm(r)]}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r)$$



[Baym et al., Rept. Prog. Phys. 81 '18]

→ Obtain mass-radius relationship

→ Single-parameter curve: central pressure $p_0 = p(0)$ as free parameter

→ But: need equation of state $p(\varepsilon)$

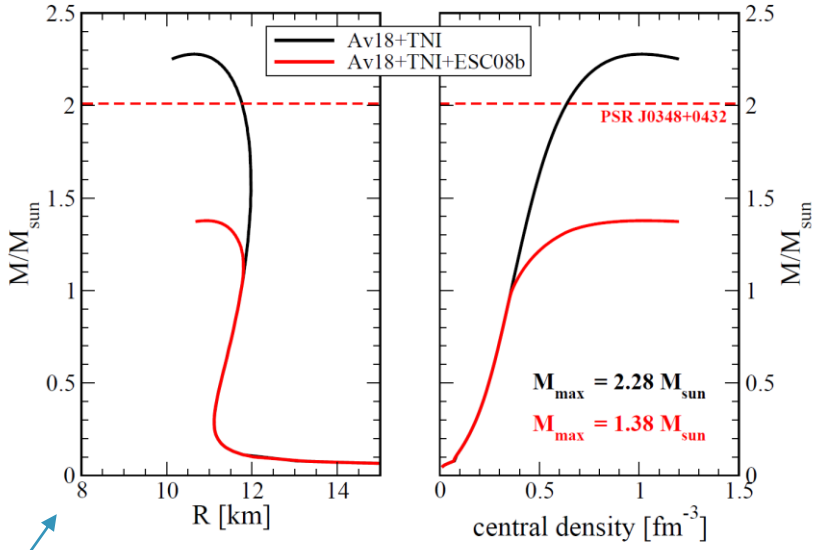
General problems:

- Many EoS look similar or produce similar mass-radius curves

  → "masquerade problem" [discussion: Alvarez-Castillo, Blaschke '14]

- Onset of strangeness in hadronic phase, quark phase, or not at all?

  → "hyperon puzzle" [hyperon model calculations: e.g. Djapo, Schaefer, Wambach '10; Bombaci '16]



Macroscopic constraints: $R \sim$ 10 - 14 km $M > 2M_{\odot}$ constraints on tidal deformability [Most et al. '18]

Nuclear phase: restrictions on EoS at low densities from nuclear physics [e.g. Hempel, Schaffner-Bielich '10]

Quark phase: mostly mean-field calculations in NJL-type or phenomenological models, known/suspected relevant channels (diquark) [e.g. Christian et al. '18; Pagliara, Schaffner-Bielich '08]

FRG Approach

Using quark-meson model in LPA:

$$\Gamma_k = \int_x \left[\begin{array}{c} \bar{q}\left(\not{\partial} + gT_a(\sigma_a + i\gamma_5\pi_a)\right)q + \mathrm{Tr}\left(\partial_\mu \Phi^\dagger \partial_\mu \Phi\right) + U_k\left(\rho_1, \dots, \rho_{N_f}\right) \\ -c\left(\det \Phi^\dagger + \det \Phi\right) - \mathrm{Tr}\left(H\left(\Phi + \Phi^\dagger\right)\right) \end{array} \right]$$

with

$$\rho_n = \mathrm{Tr}\left[\left(\Phi^\dagger \Phi\right)^n\right]$$

→ UV cutoff $\Lambda = 1\ \mathrm{GeV}$

→ $\boldsymbol{N_f = 2}$ **and** $\boldsymbol{N_f = 2 + 1}$

$$E_\sigma^2 = k^2 + 2U_k' + 4\sigma^2 U_k''$$
$$E_\pi^2 = k^2 + 2U_k'$$
$$E_q^2 = k^2 + g^2\sigma^2$$

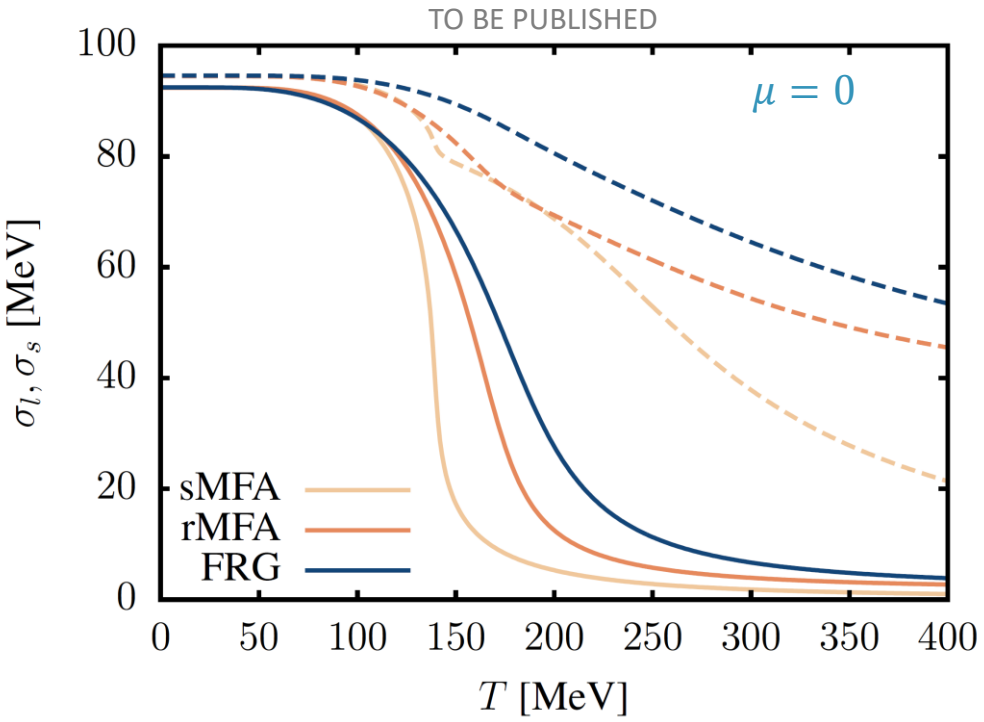
Flow equation for U_k with 3-d Litim regulators:

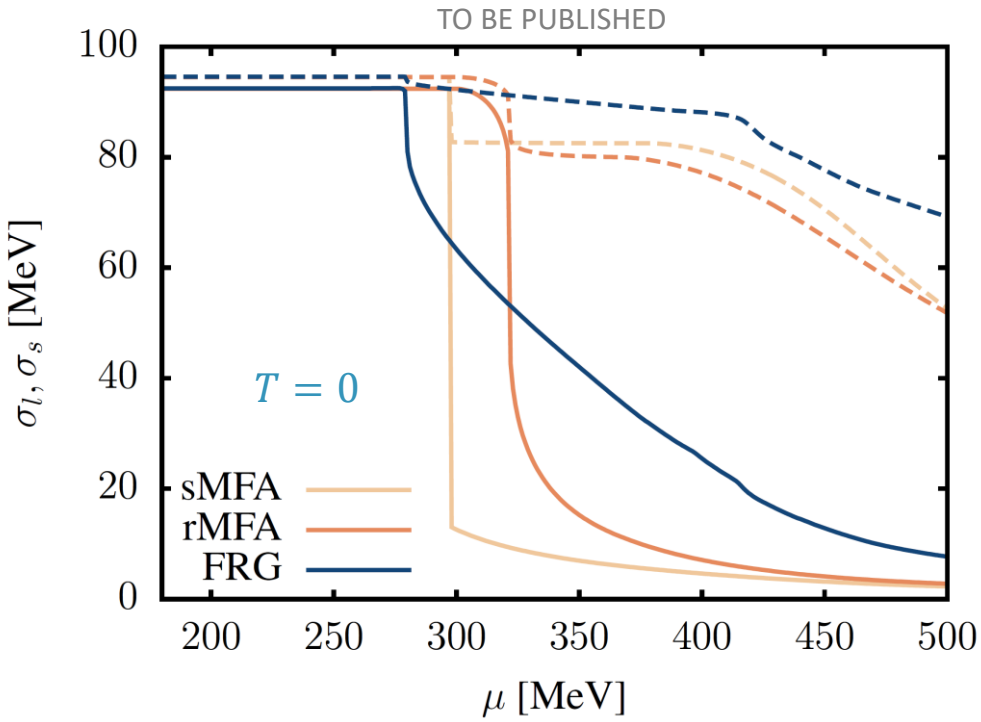
$$\partial_t U_k = \frac{k^5}{12\pi^2}\left[\sum_b \frac{1}{E_b}\coth\left(\frac{E_b}{2T}\right) - 2N_c\sum_f \frac{1}{E_f}\left(\tanh\left(\frac{E_f-\mu}{2T}\right) + \tanh\left(\frac{E_f+\mu}{2T}\right)\right)\right]$$

Comparison to mean-field results:

→ Standard mean-field approximation (sMFA) without vacuum term

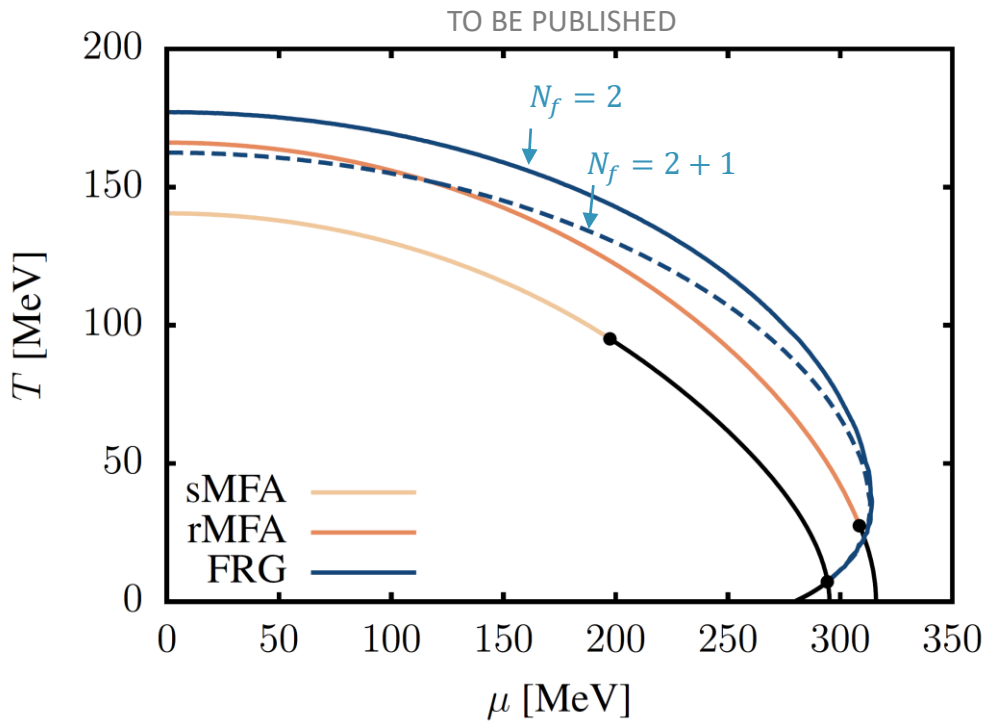
→ Renormalized mean-field approximation (rMFA): solve only fermionic flow

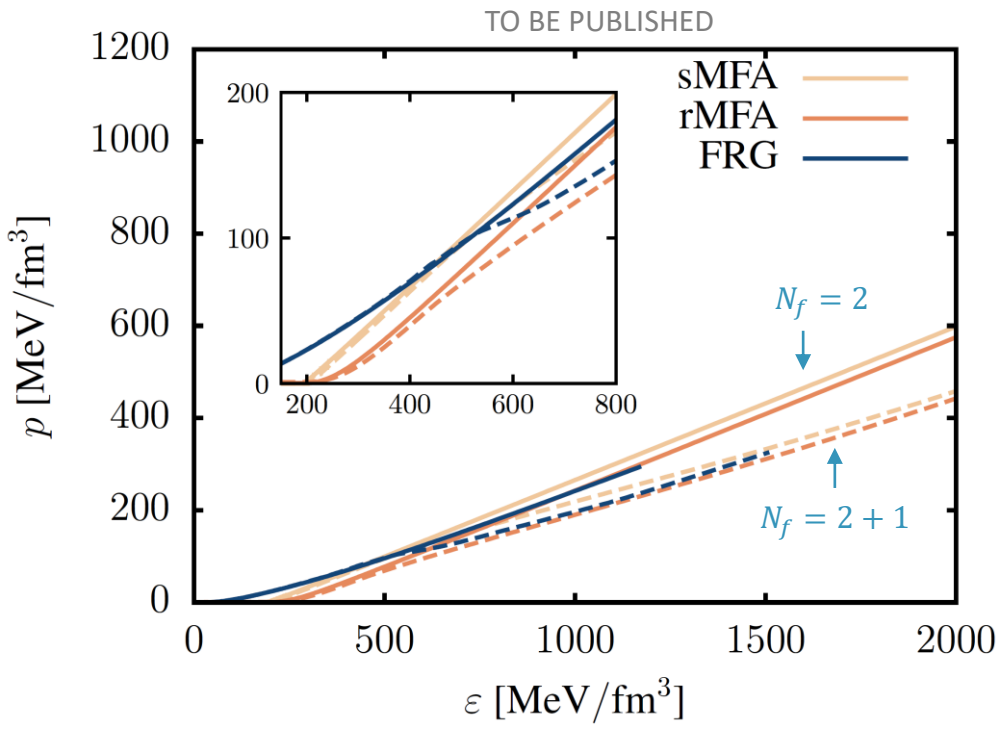




→ Crossover transition along temperature axis

→ First-order transition along chem. potential axis





→ Critical endpoint (CEP) at highest T_c in sMFA, lowest T_c in FRG

→ Inclusion of strange matter reduces T_c in crossover region

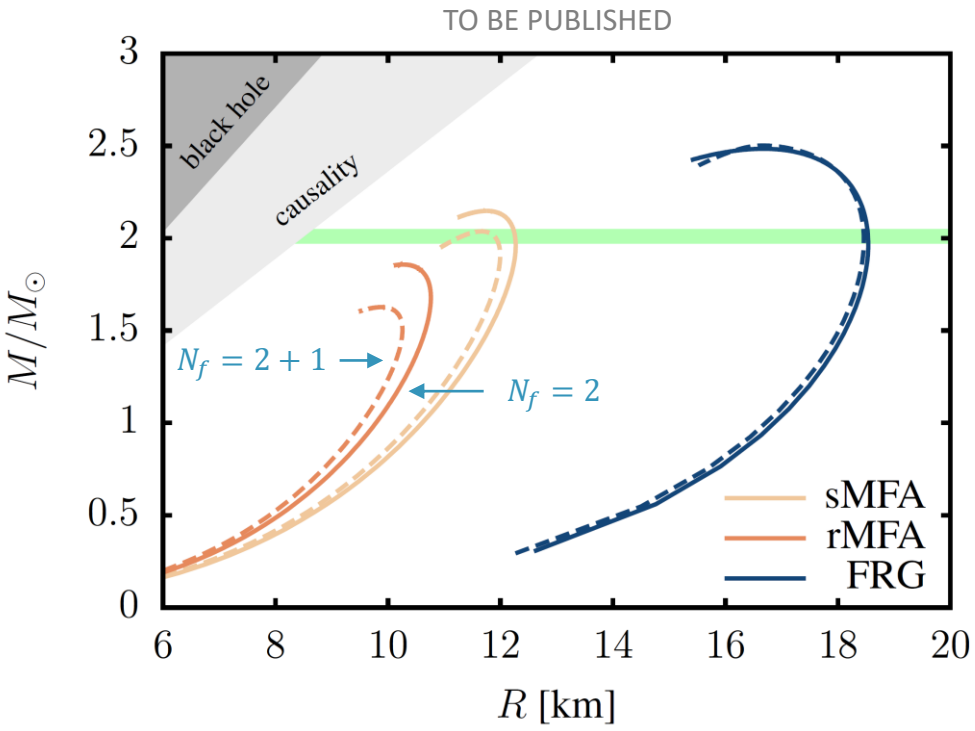
→ Back-bending of FRG line at low T

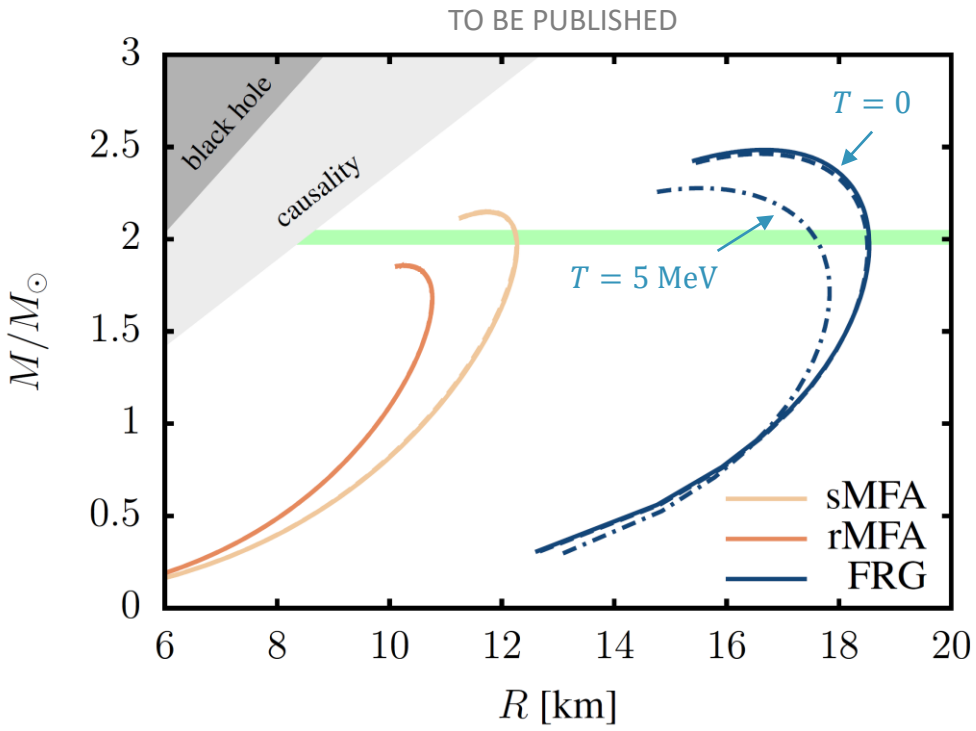
[Tripolt, Schaefer, von Smekal, Wambach '18]

→ As expected: strangeness reduces stiffness of EoS

→ Influence on neutron stars?

Depends on maximum central pressure in stable stars!

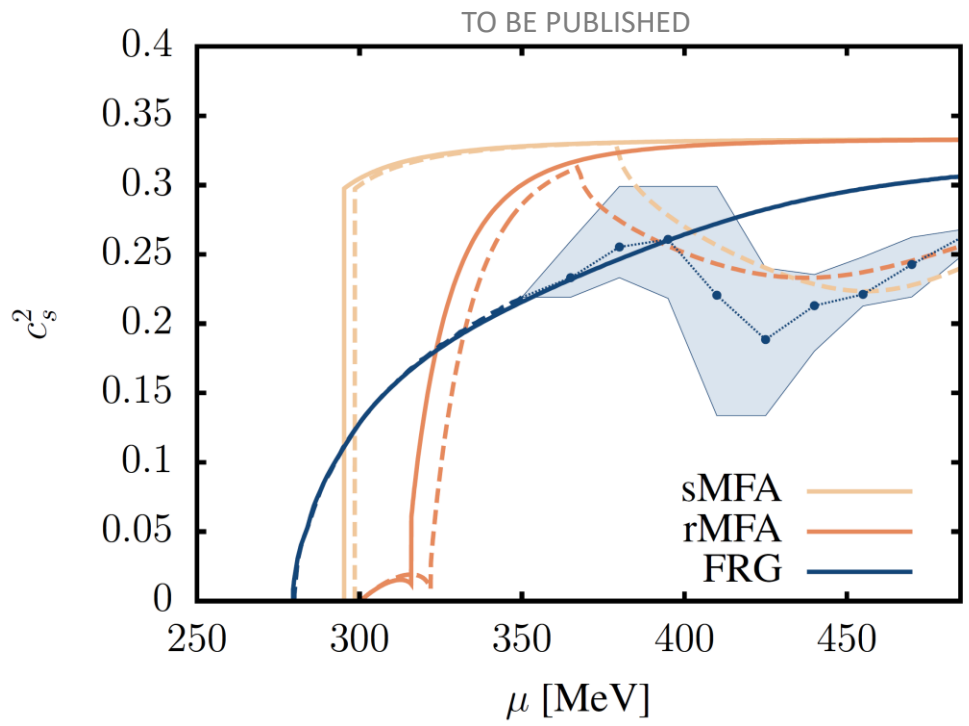


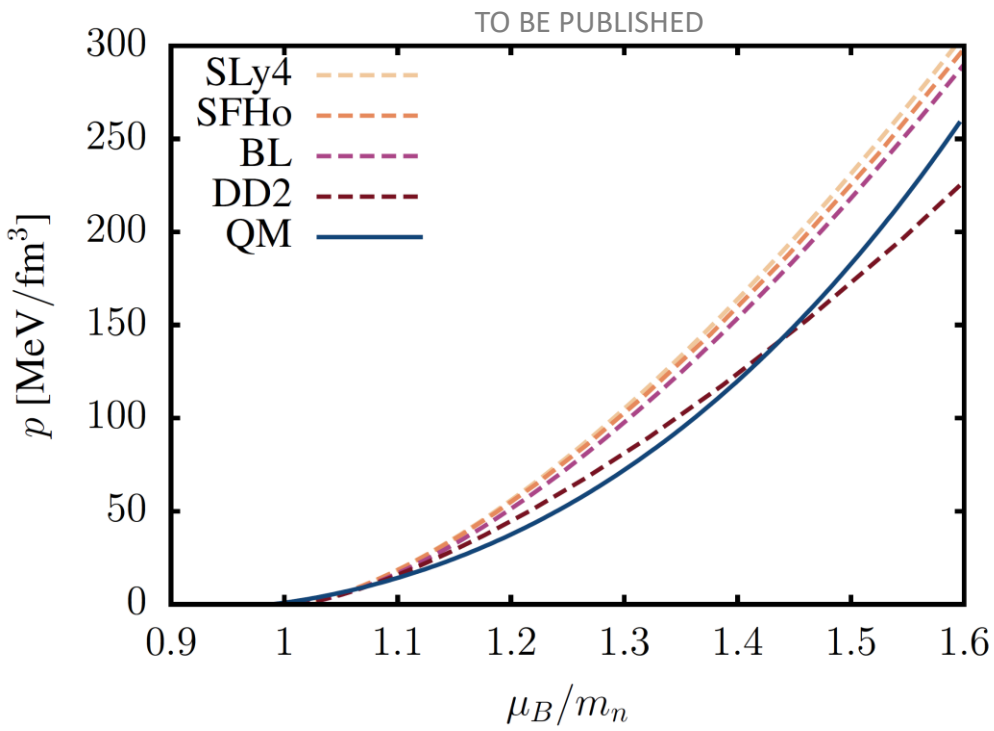


→ No influence of strangeness visible in FRG, but in sMFA and rMFA

→ Strong dependence on approximation scheme!

→ Influence of temperature only visible at $T \gtrsim 5$ MeV and only in FRG





→ Mean-field quickly converges to expected result $c_s^2 = 1/3$ at $N_f = 2$ in comparison to FRG

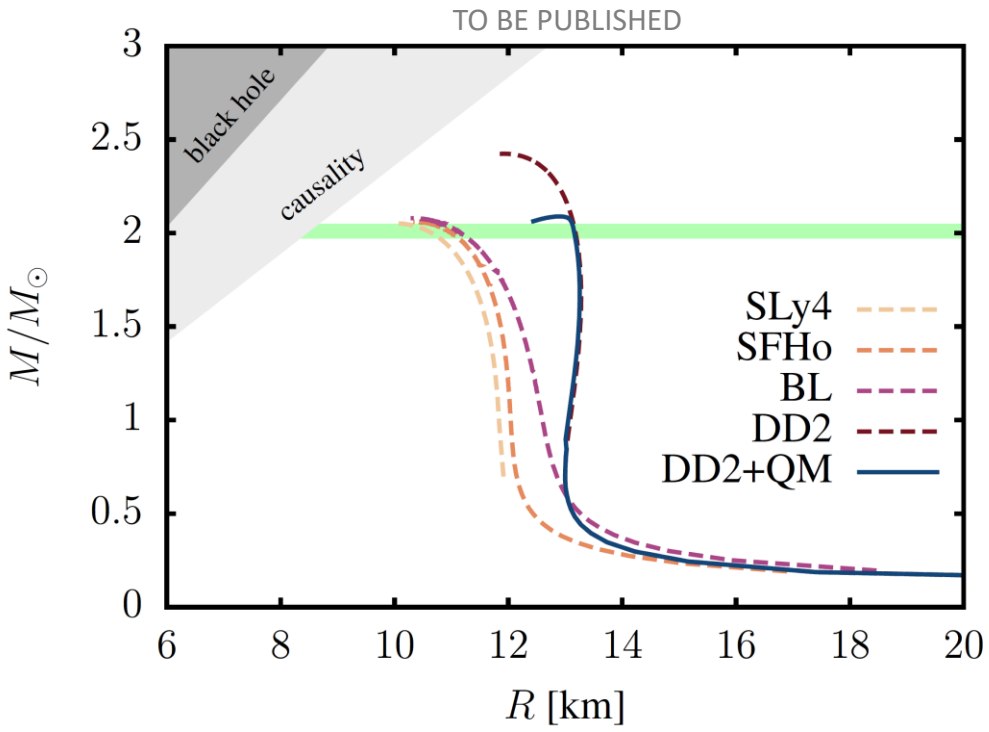
→ Dip at onset of strangeness

→ Similar behavior in FRG, with generally lower speed of sound

→ Matching with nuclear EoS (DD2) possible (Maxwell construction)

→ But: had to increase vacuum quark mass to ≈ 370 MeV

→ Have to ignore intersection at low p



→ Fulfills solar mass constraints…

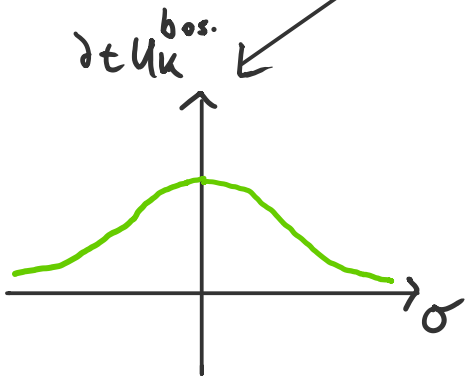
… but not very realistic description of cold and dense (quark) matter!

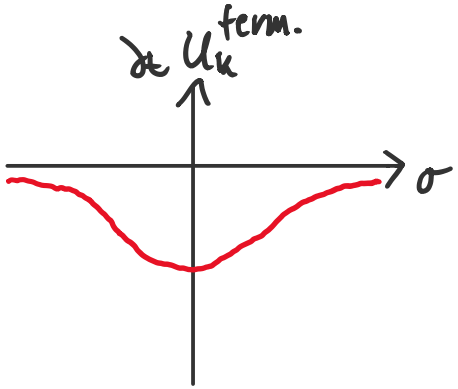
→ Need to improve quark matter sector! Let‘s start with the truncation…

→ Therefore, first take a step back

Consider $N_f = 2$ and vacuum flow:

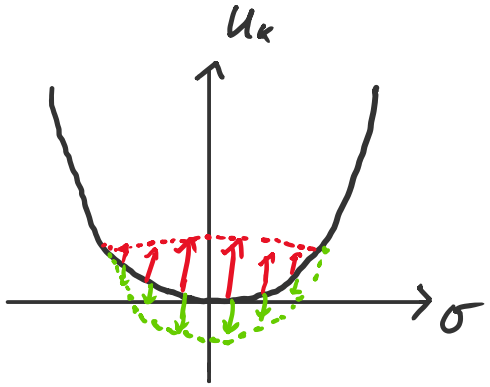
$$\partial_t U_k \sim k^5 \left(\underbrace{\frac{1}{E_\sigma} + \frac{3}{E_\pi}} - 24 \frac{1}{E_q} \right)$$



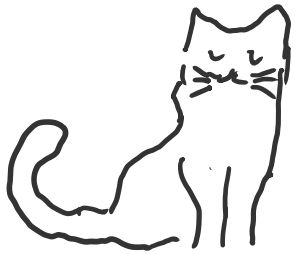


step $-\delta t$

Fermions drive the flow towards χ_{SB}…



… while bosons try to restore chiral symmetry!



What happens at finite temperature?

→ At high T (and fixed energy):

$$\coth\left(\frac{E_b}{2T}\right) \sim \frac{T}{E_b}$$

$$\tanh\left(\frac{E_f}{2T}\right) \sim \frac{E_f}{T}$$

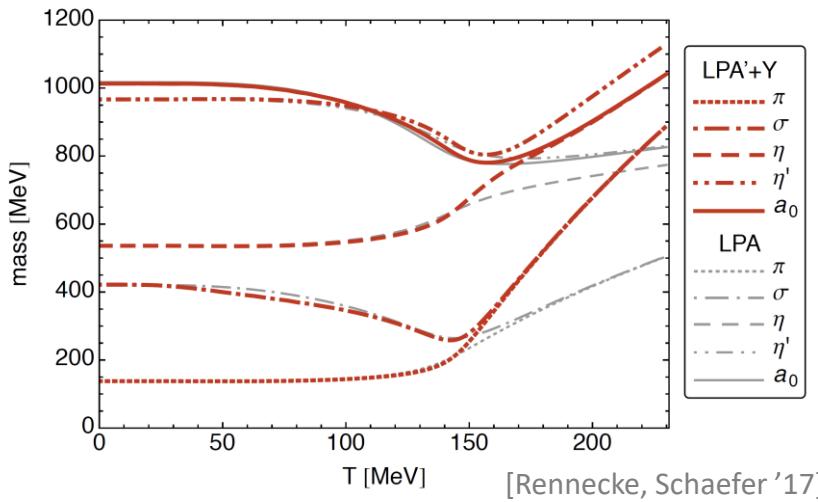
→ Mesonic fluctuations dominate, chiral symmetry is restored

(high μ: fermionic flow stops running for low values of σ, mesons dominate and push the potential down → first-order transition)

But: mesons should decouple from the flow at high energies!

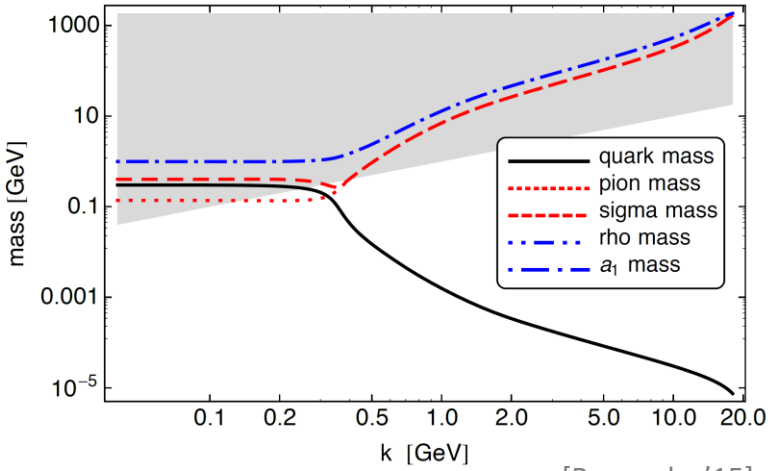
→ Seen in studies including running wave function renormalization

[e.g. Pawlowki, Rennecke ’14; Rennecke ’15; Braun et al. ’16; Rennecke, Schaefer ‘17]



[Rennecke, Schaefer ’17]

→ Running Z_k (and g_k) is necessary criterion to capture this and incorporate d.o.f. dynamically (dynamical hadronization)



[Rennecke ’15]

Outlook
Not Yet Understood (by me…)

- Field dependence of Z_ks:
  - $Z_k(\rho) \approx Z_k(\rho_{0,k})$ [e.g. Tetradis, Wetterich '94]
  - Specifically: dependence on expansion point, …
  - Influence of splitting: include $\frac{1}{4} Y_k(\rho)\left(\partial_\mu \rho\right)^2$ (see Alexander Stegemann)
- Influence of higher orders in derivative expansion (complicated) [Divotgey, Eser, Mitter '19]