Comparison between Different Transport Models

Pawel Danielewicz

National Superconducting Cyclotron Laboratory
Michigan State University

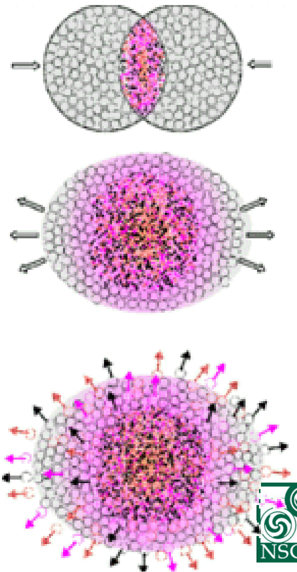
Probing Dense Baryonic Matter with Hadrons:
Status and Perspective

GSI, 11 - 13 February, 2019

Outline

1. Introduction
   - Basics
   - Types of Transport Models
2. Successes & Failures
   - E_0/A at $\rho > \rho_0$
   - $S(\rho)$ from π^-/π^+
3. Comparison Project
   - Code Comparison Effort
   - Full-Run Comparisons
   - Box Comparisons
4. Impacts: TuQMD Example
5. Conclusions

Need for Transport



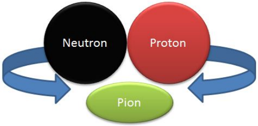
Many repeated elementary interactions outside equilibrium

- Central Nuclear Collisions
- Isotope Production
- Energetic Hadron-Nucleus Collision
- ν Detection
- Supernova Explosion
- Technological Applications
- …

Degrees of Freedom

Choice depends on energy and application

- Nucleons
- Clusters
- Pions, Baryon Resonances
- Kaons, Strange Baryons
- Photons
- …



Dominant degrees of freedom must be included; other might be treated perturbatively

Phase-space distribution (in configuration space and momentum) ⇔ Wigner function

$$f(\mathbf{p};\mathbf{R},T) = \int d\mathbf{r}\, \mathrm{e}^{-i\mathbf{p}\mathbf{r}} \langle \hat{\psi}_H^\dagger(\mathbf{R}-\mathbf{r}/2,T)\, \hat{\psi}_H(\mathbf{R}+\mathbf{r}/2,T)\rangle$$

Statistical Description

Phase-space distribution

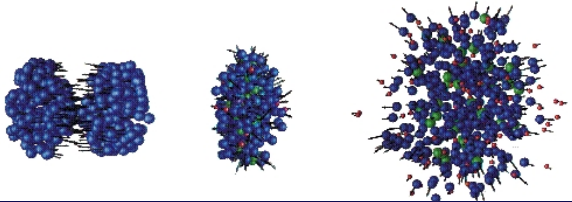
$$f(\mathbf{p};\mathbf{R},T) = \int d\mathbf{r}\,\mathrm{e}^{-i\mathbf{pr}}\,\langle\hat{\psi}_H^\dagger(\mathbf{R}-\mathbf{r}/2,T)\,\hat{\psi}_H(\mathbf{R}+\mathbf{r}/2,T)\rangle$$

Dynamics: Particles move through noisy medium: stochastic + deterministic impact of the medium on the particle - collisions + mean field

Descriptions invoke Boltzmann equation:

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{p}}\frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \epsilon}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{p}} = \mathcal{K}^{<}\,(1 \mp f) - \mathcal{K}^{>}\,f$$

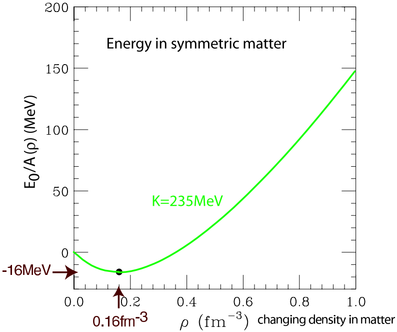
Left-hand deterministic impact Right-hand stochastic

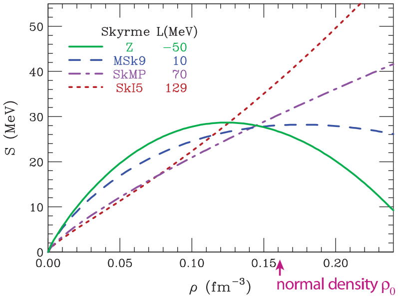


Means of Learning on EOS at $\rho > \rho_0$

$$\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + S(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)^2 + \mathcal{O}(\ldots^4)$$

symmetric matter (a)symmetry energy $\rho = \rho_n + \rho_p$





$$\frac{E_0}{A}(\rho) = -B + \frac{K}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \ldots$$

$$S(\rho) = S_0 + \frac{L}{3}\frac{\rho - \rho_0}{\rho_0} + \ldots$$

Known: $B \approx 16\,\text{MeV}$ $K \sim 235\,\text{MeV}$

Unknown: S_0 ? L ?

- Boltzmann Equation Type
  - Examples: GIBUU, IBUU, pBUU, RVUU
  - Pros: Well-defined equation, derivable from microscopic theory, solved; easy Pauli principle & mean-field
  - Cons: No fluctuations
- Molecular Dynamics
  - Examples: IQMD, CoMD, TuQMD, UrQMD
  - Pros: Good fluctuations late in reactions
  - Cons: Wrong fluctuations initially, troubles with Pauli & mean-field, too much phenomenology?
- Antisymmetrized Molecular Dynamics (AMD)
  - Pros: Excellent initial states, good mean field & Pauli
  - Cons: Troubles with final states, dose of phenomenology

EOS and Flow Anisotropies

EOS assessed through reaction plane anisotropies characterizing particle collective motion

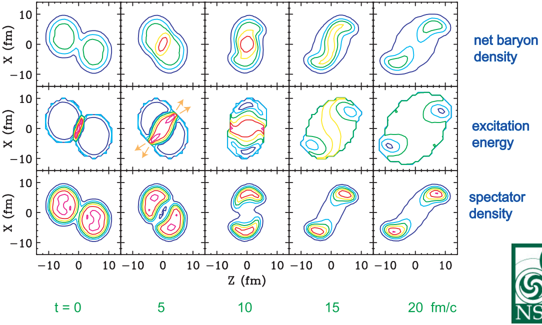
Hydro? Euler eq. in $\vec{v} = 0$ frame: $\boxed{m_N \, \rho \, \frac{\partial}{\partial t} \, \vec{v} = -\vec{\nabla} p}$

where p - pressure. From features of v, knowing Δt, we may learn about p in relation to ρ. Δt fixed by spectator motion

For high p, expansion rapid and much affected by spectators

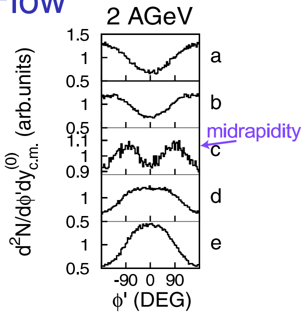
For low p, expansion sluggish and completes after spectators gone

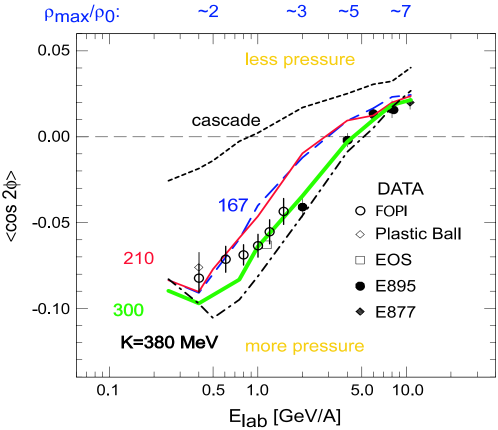
Simulation by Shi (pBUU)



2^{nd}-Order or Elliptic Flow

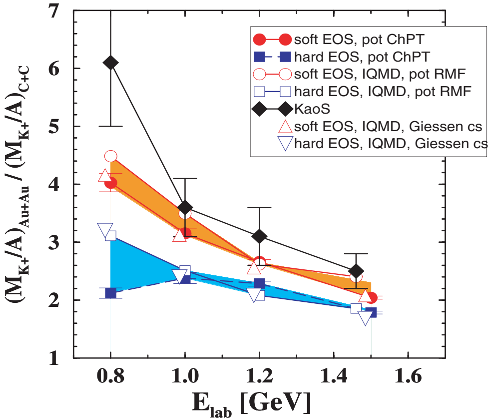
Anisotropy studied at midrapidity:
$v_2 = \langle \cos 2\phi \rangle$, where ϕ is azimuthal angle relative to reaction plane





Au+Au v_2
Excitation Function

Subthreshold Meson (K/π) Production



Ratio of kaons per participant nucleon in Au+Au collisions to kaons in C+C collisions vs beam energy

filled diamonds: KaoS data

open symbols: theory Fuchs *et al*

Kaon yield sensitive to EOS because multiple interactions needed for production, testing density

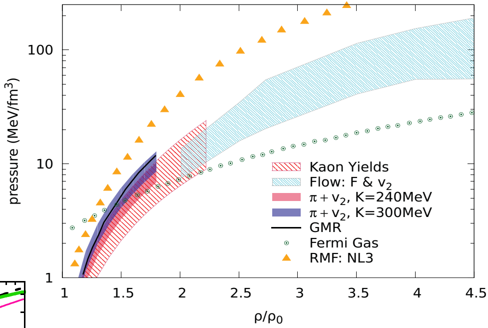
The data suggest a relatively soft EOS

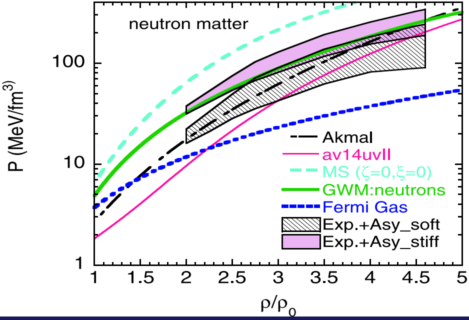
Constraints from Flow on EOS

Au+Au flow anisotropies: $\rho \simeq (2 - 4.6)\rho_0$.

No one EOS yields both flows right. Discrepancies: inaccuracy of theory

Most extreme models for EOS can be eliminated





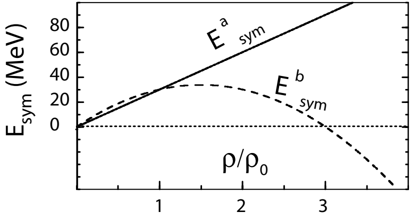
PD, Lacey & Lynch + Fuchs + Le Fevre + Hong + . . .

Neutron Matter:

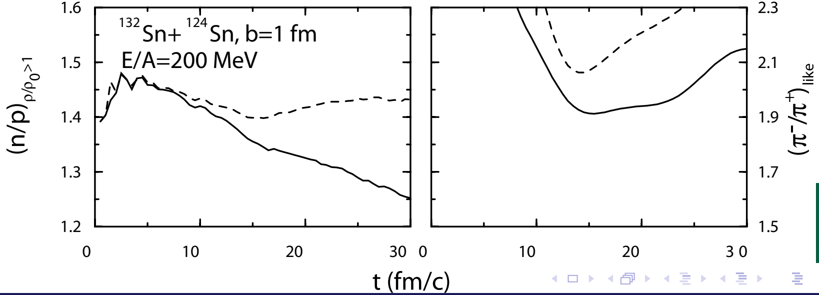
Uncertainty in symmetry energy

Charged π Probing High-ρ Symmetry Energy

B-A Li PRL88(02)192701: $S(\rho > \rho_0) \Rightarrow n/p_{\rho>\rho_0} \Rightarrow \pi^-/\pi^+$



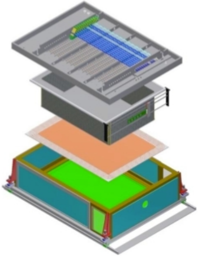
Pions originate from high ρ



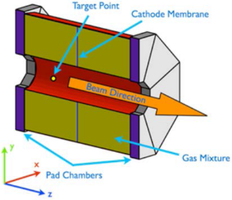
Dedicated Experimental Efforts

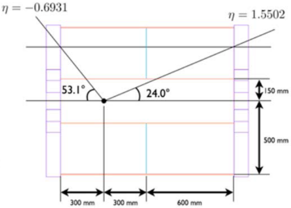
SAMURAI-TPC Collaboration (data taken; 8 countries and 43 researchers): comparisons of near-threshold π^- and π^+ and also *n*-*p* spectra and flows at RIKEN, Japan.

NSCL/MSU, Texas A&M U
Western Michigan U, U of Notre Dame
GSI, Daresbury Lab, INFN/LNS
U of Budapest, SUBATECH, GANIL
China IAE, Brazil, RIKEN, Rikkyo U
Tohoku U, Kyoto U



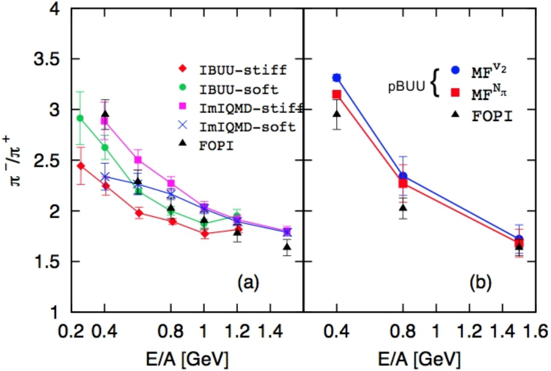
LAMPS TPC at RAON (S Korea): triple GEM, 3π sr





FOPI Au+Au π^-/π^+ Data?

Reisdorf *et al.* (FOPI) NPA781(07)459



data: black symbols

theory: colored symbols

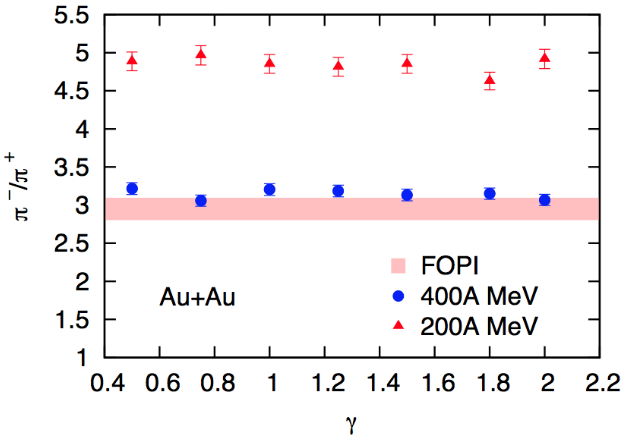
Opposing sensitivity to $S(\rho)$ claimed in transport & used to explain data!

PRL **102,** 062502 (2009) PHYSICAL REVIEW LETTERS week ending 13 FEBRUARY 2009

Circumstantial Evidence for a Soft Nuclear Symmetry Energy at Suprasaturation Densities

FOPI π^-/π^+ Reproduced by pBUU

…irrespectively of $S_{\text{int}}(\rho) = S_0\,(\rho/\rho_0)^\gamma$:



Jun Hong & PD PRC90(14)024605

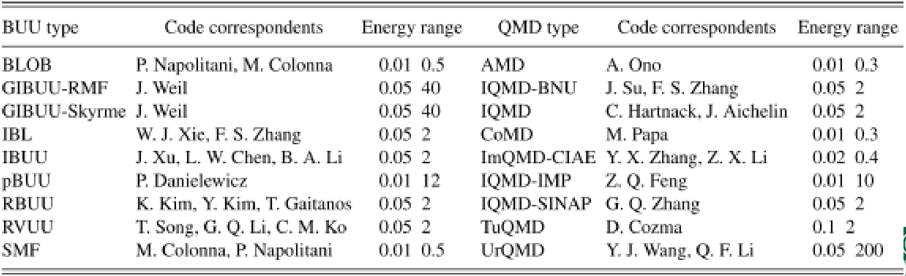
…Other probes possible, but general problem of model ambiguity remains!

Chronology

- Motivation: Discrepancies Impediment to Conclusions
- Workshops at ECT* Trento in 2004 & 2009
  - Jorg Aichelin, Christopher Hartnack, Evgeni Kolomeitsev
  - similar physics, *naive* full-run comparisons
- Second Phase $\geq$ 2014
  - Isospin physics, $\delta = (\rho_n - \rho_p)/\rho \sim 0.2$ needs more precision/consistency
  - Betty Tsang, Jun Xu, Yingxun Zhang, Akira Ono, Maria Colonna
  - similar/identical physics, *naive* restart
  - breaking problem into pieces: initial state, collisions, Pauli pcple, detailed balance, mean field. . .
- Impact on Everyday Practices

Papers & Participants

- E. E. Kolomeitsev *et al.*, J. Phys. G 31 (2005) S741
- Jun Xu *et al.* (31 authors), Phys. Rev. C 93 (2016) 044609
- Yingxun Zhang *et al.* (30 authors), Phys. Rev. C 97 (2018) 034625
- …



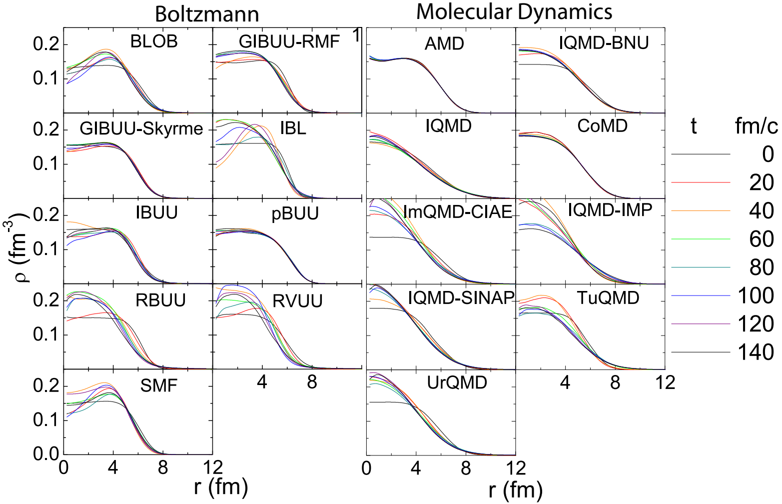
BUU type	Code correspondents	Energy range	QMD type	Code correspondents	Energy range
BLOB	P. Napolitani, M. Colonna	0.01 0.5	AMD	A. Ono	0.01 0.3
GIBUU-RMF	J. Weil	0.05 40	IQMD-BNU	J. Su, F. S. Zhang	0.05 2
GIBUU-Skyrme	J. Weil	0.05 40	IQMD	C. Hartnack, J. Aichelin	0.05 2
IBL	W. J. Xie, F. S. Zhang	0.05 2	CoMD	M. Papa	0.01 0.3
IBUU	J. Xu, L. W. Chen, B. A. Li	0.05 2	ImQMD-CIAE	Y. X. Zhang, Z. X. Li	0.02 0.4
pBUU	P. Danielewicz	0.01 12	IQMD-IMP	Z. Q. Feng	0.01 10
RBUU	K. Kim, Y. Kim, T. Gaitanos	0.05 2	IQMD-SINAP	G. Q. Zhang	0.05 2
RVUU	T. Song, G. Q. Li, C. M. Ko	0.05 2	TuQMD	D. Cozma	0.1 2
SMF	M. Colonna, P. Napolitani	0.01 0.5	UrQMD	Y. J. Wang, Q. F. Li	0.05 200

Premise

- Specify the same physics inputs for different transport codes
- Compare outputs

- Full-run comparisons
  - elastic collisions only
  - constant isotropic cross section $\sigma = 40\,\text{mb}$
  - soft EOS + momentum-independent mean-field
  - Next: π & K production
- Controlled simplified conditions
  - isolated nucleus
  - collisions in a box $\leftarrow$ approach to equilibrium
  - mean field in a box
  - Next: Δ + π production in a box. . .

Stability of Initial Density in Dynamics

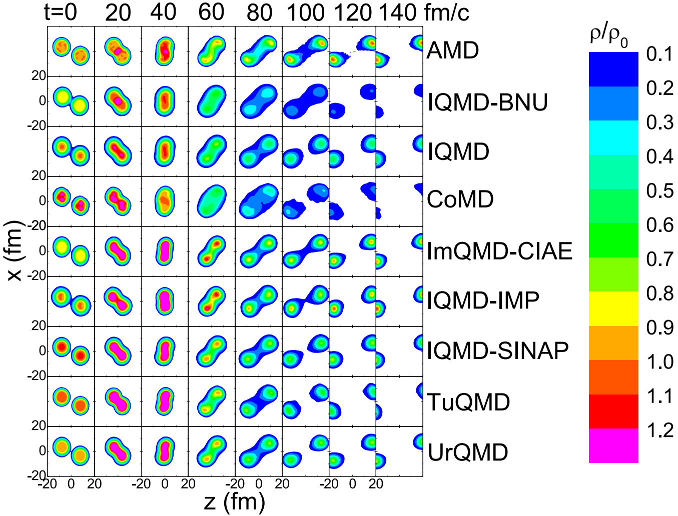


Jun Xu *et al.* PRC93(16)044609 Isolated Au nucleus

⇒ Initial state must be constructed consistently with dynamics

Density Evolutions for Molecular Dynamics

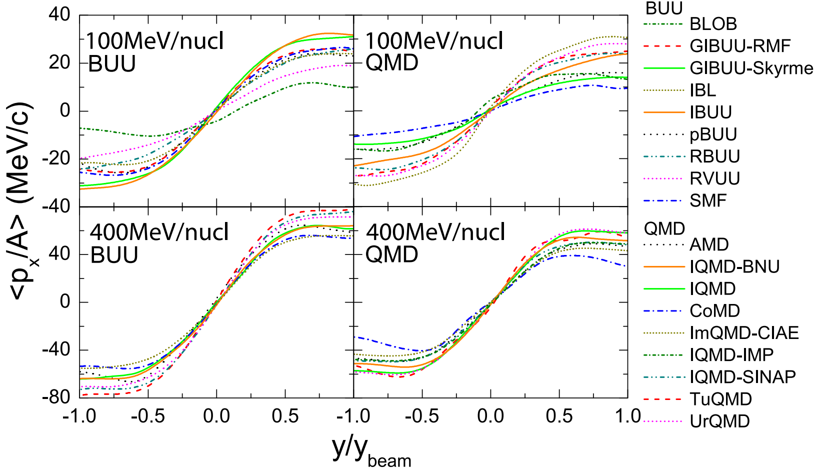
100 MeV/nucleon Au + Au at $b = 7$ fm



General characteristics the same but differences in details

From Differences in Dynamics to Observables

Au + Au at $b = 7\,\mathrm{fm}$: In-Plane Momentum vs y

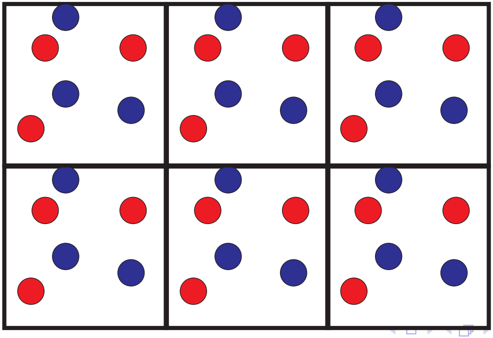


Less dispersion at high than low energy. But who is right??

Periodic Box Comparions

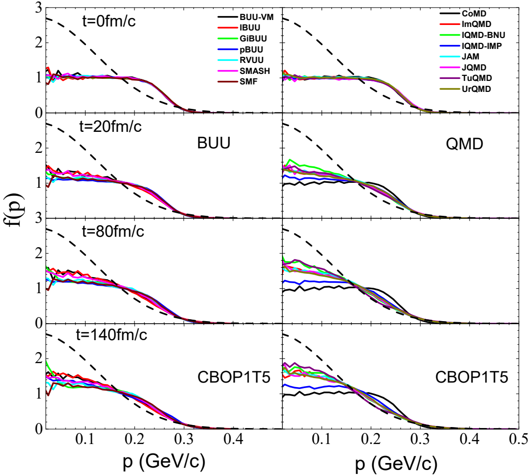
Selecting individual ingredients, testing against independently established limits, e.g.

1. Elastic Collisions Only
2. Mean-Field Only
3. Delta Production & Absorption…



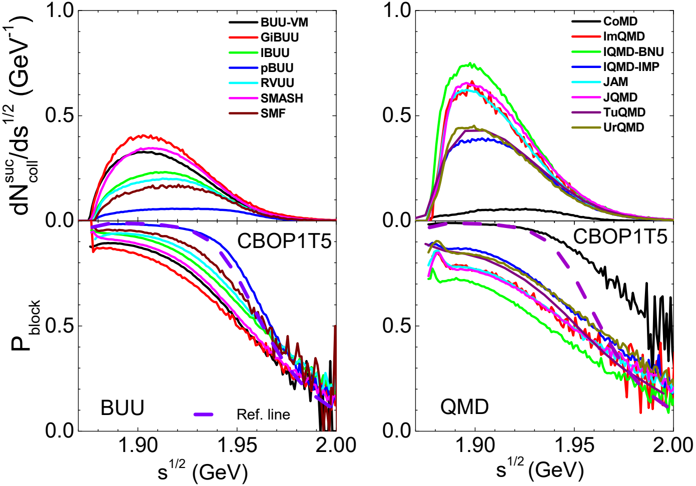
Collisions w/Pauli: Stability of Fermi-Dirac

Systems initialized with Fermi-Dirac at ρ_0 and $T = 5\,\mathrm{MeV}$



Molecular codes progress towards Boltzmann distribution (dashed line). Blocking of collisions? Zhang *et al.* PRC97(18)034625

Box: Collision Frequency



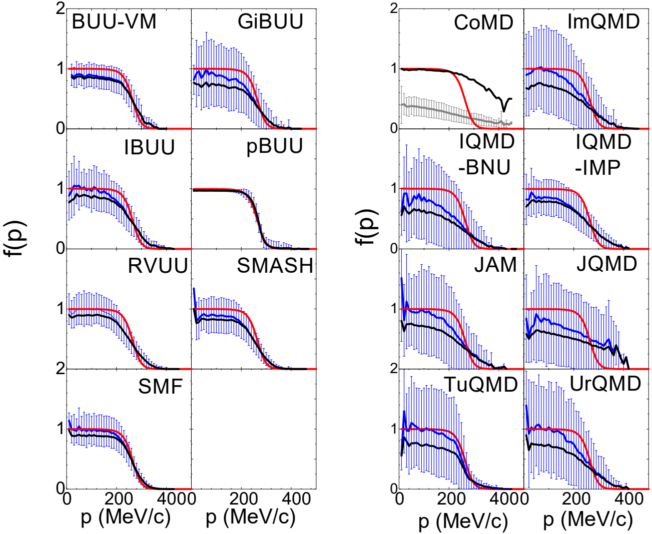
collision rate vs $\sqrt{s}$

blocking fraction

dashed line – reference

Far too many collisions allowed at low excitations ($T = 5\,$MeV)!

Box: Occupation Probabilities in Blocking Factors



red – exact

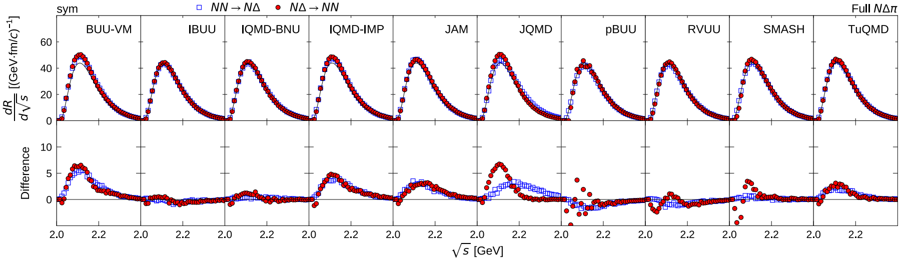
Large fluctuations in estimated probabilities!

Detailed Balance Tests

Δ & π production in a box

Rate of $N + N \to N + \Delta$ per time & energy (blue)

Rate of $N + \Delta \to N + N$ per time & energy (red)



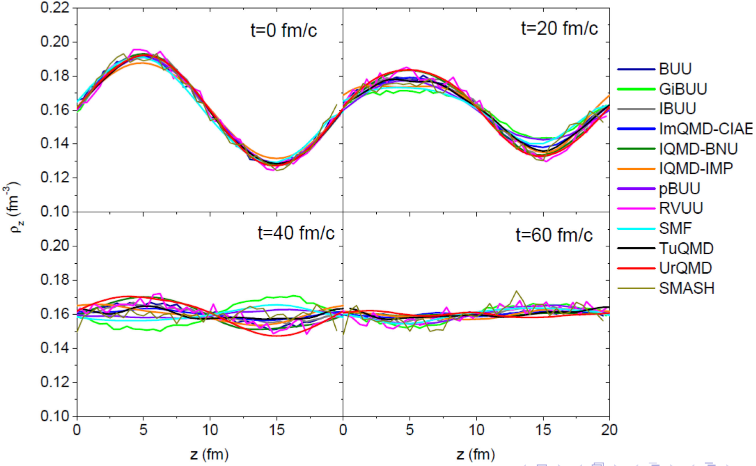
Lower panels: scaled difference

Detailed balance satisfied if rates per time & energy identical

Mean-Field Testing

Mean-field only; collisions off

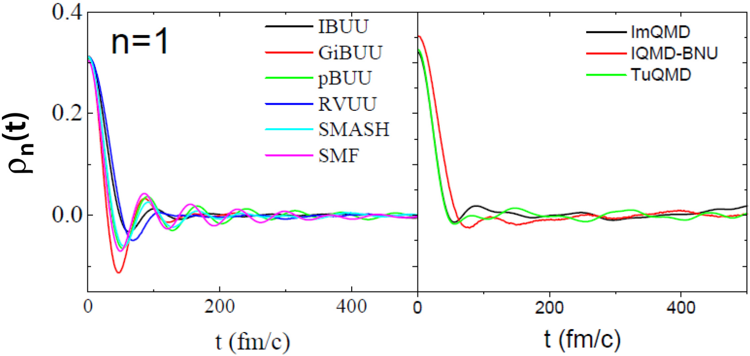
Starting density $\rho(\boldsymbol{r}, t=0) = \rho_0 + a_\rho \sin(kz)$



Mean-Field Testing - Fourier Decomposition

$\rho_n(t) = \int dx \, \sin kz \, \rho(\mathbf{r}, t) \; k = 2\pi n / L$

Starting density $\rho(\mathbf{r}, t = 0) = \rho_0 + a_\rho \sin(kz)$



Large amplitude, hence coupling between modes

Way Forward

Different codes perform differently in different tests
Some do well

After each sweep procedures are identified that lead to satisfactory performance and are recommended for all codes, e.g. initialization

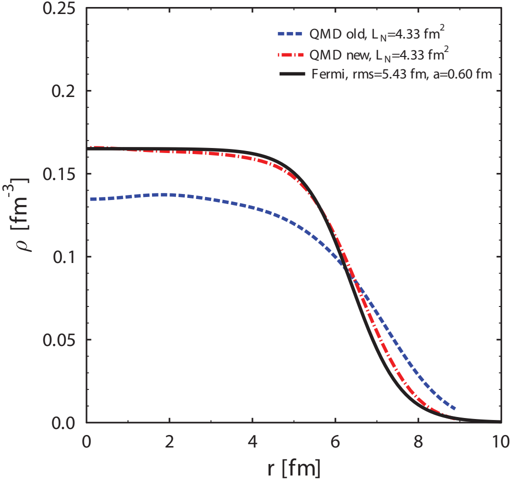
In consequence of the code comparisons, the codes are rebuilt

E.g. TuQMD

Example: Rebuilt TuQMD

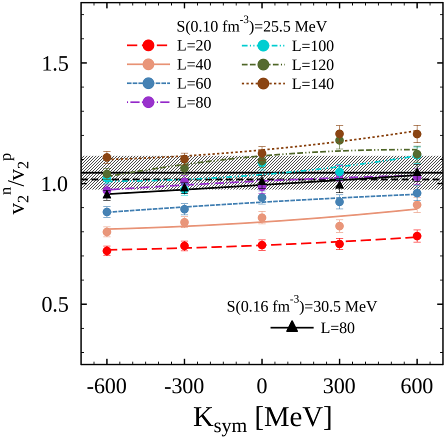
Dan Cozma EPJA54(18)23

Rebuilt density initializations and Pauli principle



FOPI-LAND & ASYEOS Elliptic-Flow Data

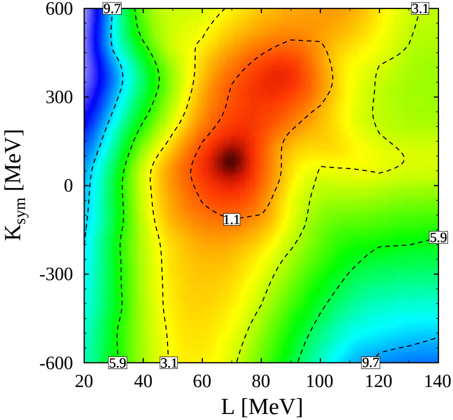
Data Cozma PRC88(13)044912



400 MeV/nucl Au + Au data above + other, particularly more differential

Constraints on Symmetry Energy Parameters

Dan Cozma EPJA54(18)23



Linear slope parameter *L* & curvature K_{sym} vs density

Conclusions

- Transport theory is indispensible in many situations
- ⇒ It is means to learn on nuclear properties at supranormal densities
- It has been used to extract constraints on nuclear pressure at supranormal densities from flow data!
- The ability to learn from finer details in data, such as on symmetry energy, calls for stringent quality control of the theory
- The community effort produces quality standards, helps to sort out the best procedures and prune out mistakes
- This helps to elevate the level of validity of conclusions reached using transport, e.g. TuQMD

Thanks to the authors participating in the code comparisons!!

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