



# Light nuclei formation in FRIGA



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- ▶ A clusterisation approach...
- ▶ An application: the hypernucleus production.
- ▶ How are influenced the (hyper-)isotope yields and phase space distributions by:
  - ▶ the clusterisation time,
  - ▶ the cluster binding energy,
  - ▶ the ingredients (EOS, in-medium properties) of the transport model.
- ▶ Spectator versus fireball cluster formation: cold-static versus hot-sequential clustering



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## Fragment Recognition In General Applications



Frigg / Friga, spinning the clouds

Friga (Frigg), goddess of harmonious weddings and alliances, setting order in the chaos, in the old Germanic mythology.



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- \* Prediction of (light and heavy) (hyper)isotope yields and full phase space distribution.



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- ➔ Because, apart from emitted elementary particles, **they carry the only information** that the experimental instruments can measure.
- ❖ Making clusters is **not an easy task**, because it involves, in a complex environment:
  - ▶ the fundamental nuclear properties,
  - ▶ quantum effects,
  - ▶ and variable timescales.



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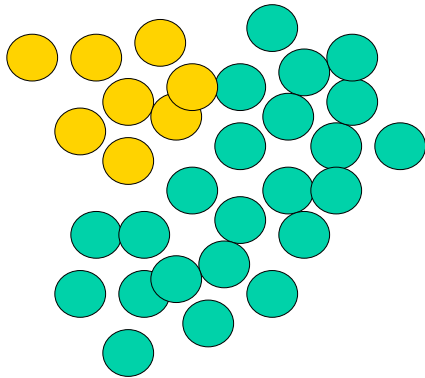
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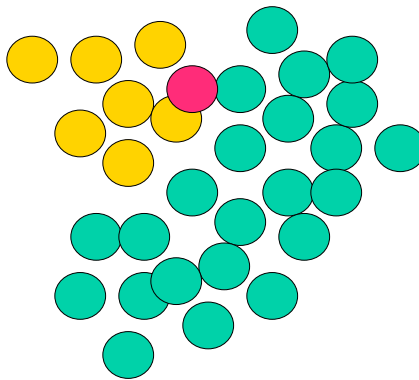
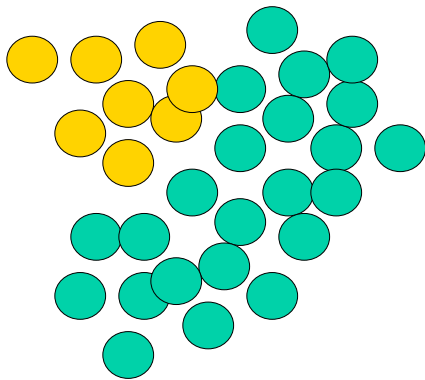
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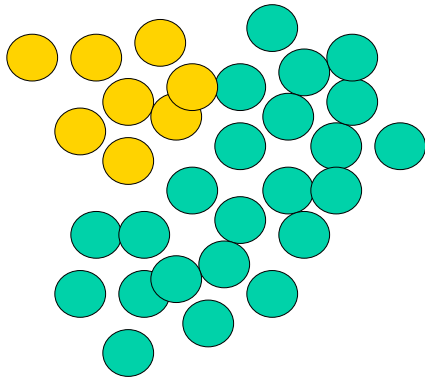
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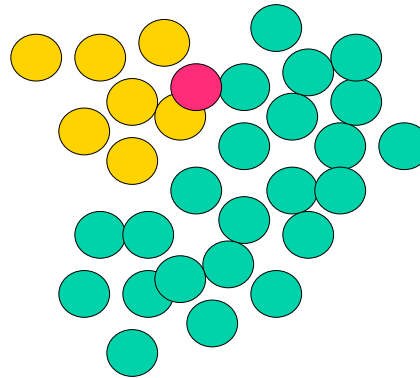
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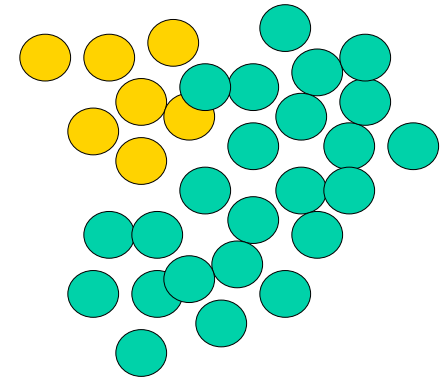
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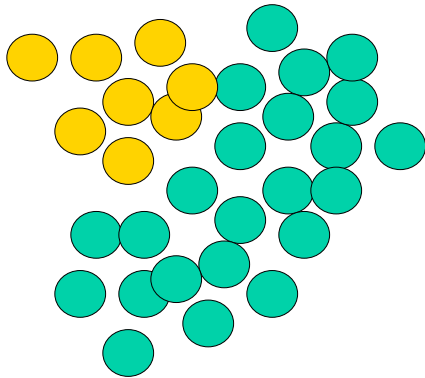




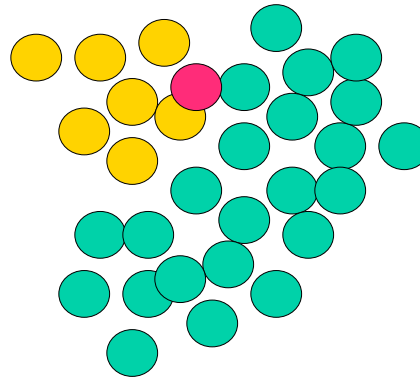
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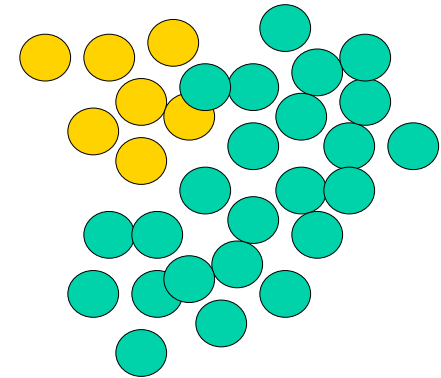


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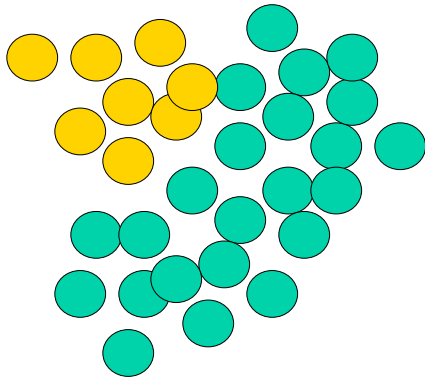




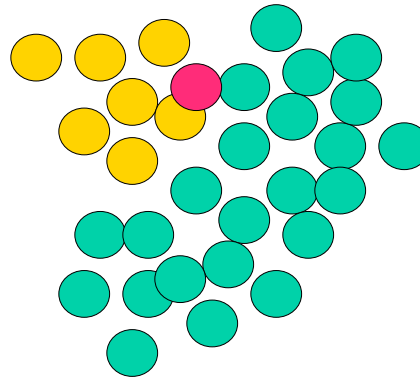
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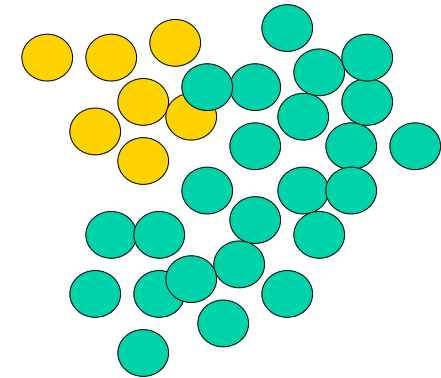
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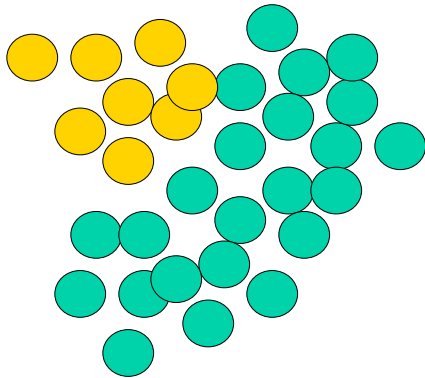




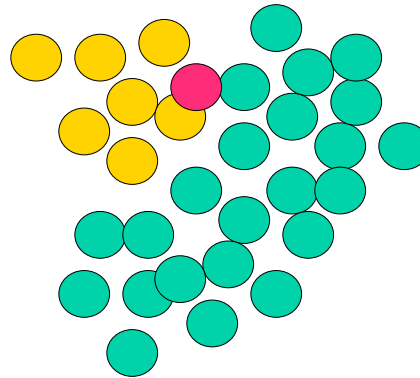
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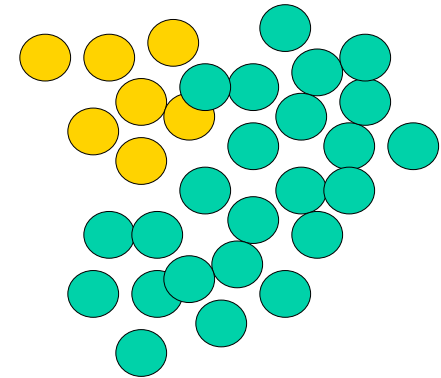


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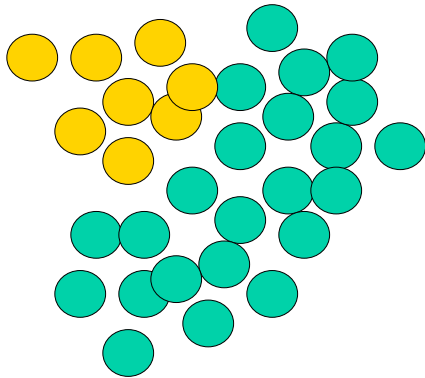




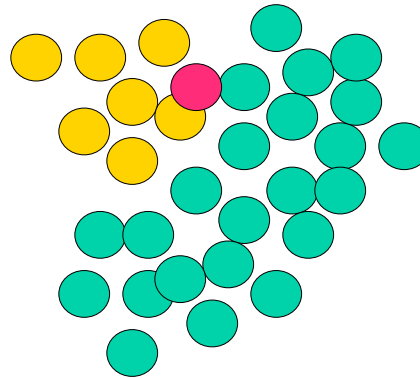
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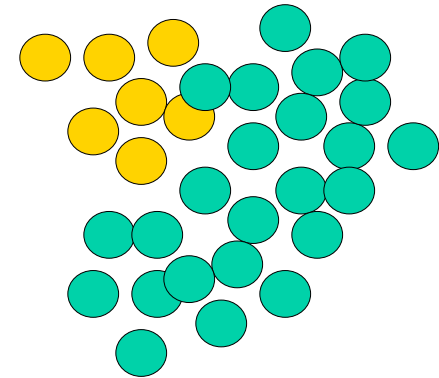
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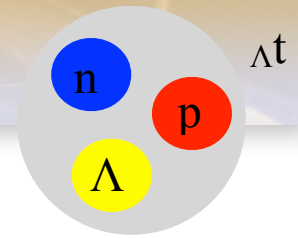
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It leads automatically to **the most bound configuration**.



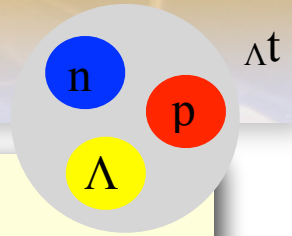


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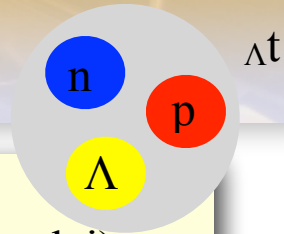
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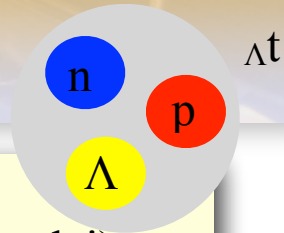


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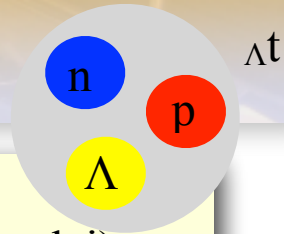


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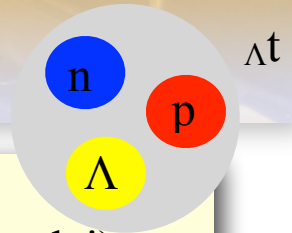


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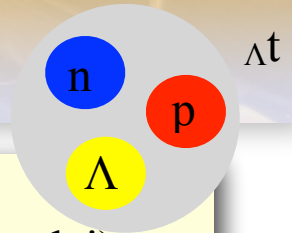


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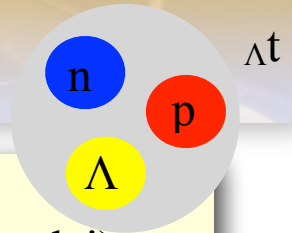


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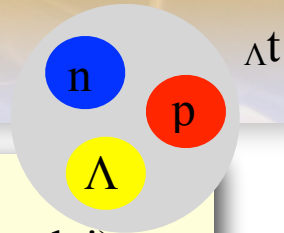
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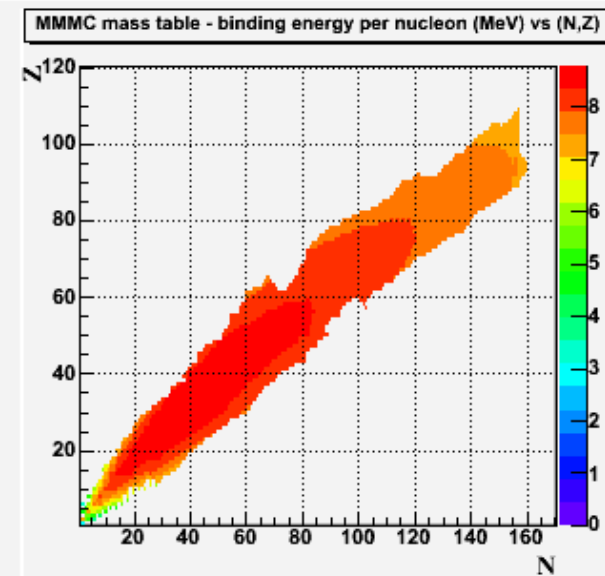
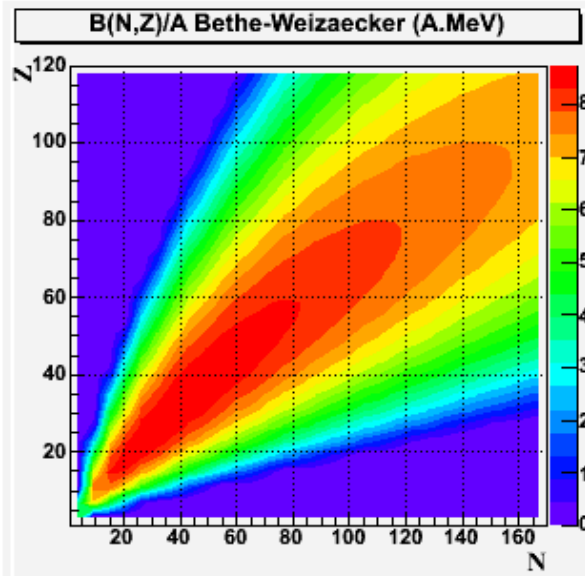
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## Remarks:

- The clusterisation has to happen **quite early** (passing time) such as to produce **hypernuclei**.
- **Λ yields and phase space distribution** as regard to the hadronic matter has to be realistic  $\Rightarrow$  influence of the EOS, in medium-properties, etc. of the transport model.

# More detailed structure corrections to apply

☞ In order to account for all major structure effects which make the binding energy deviate from the liquid drop model, for each nucleus  $(N,Z)$ , what we call «shell» binding energy will be the difference in binding energy between experimental measurements (hypernuclei included) and the Bethe-Weizsäcker formula (without pairing).

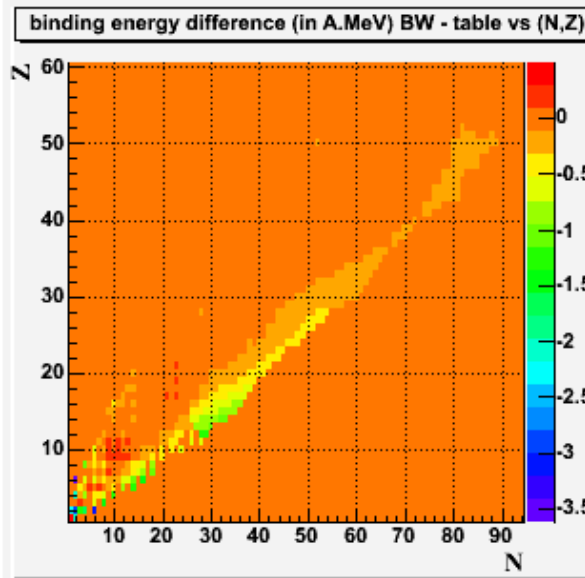
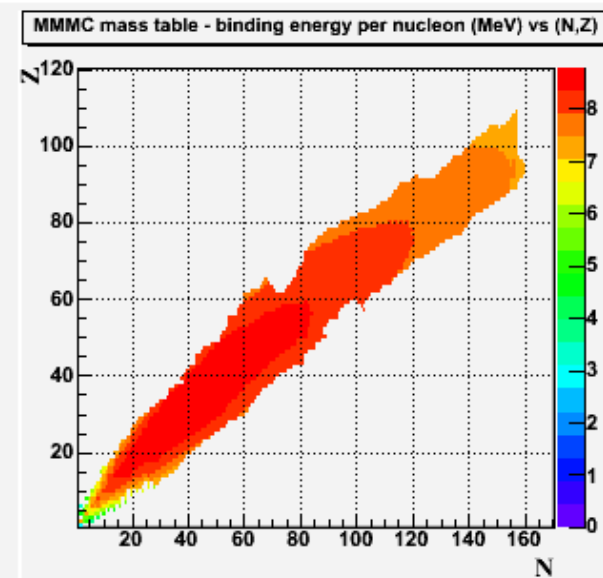
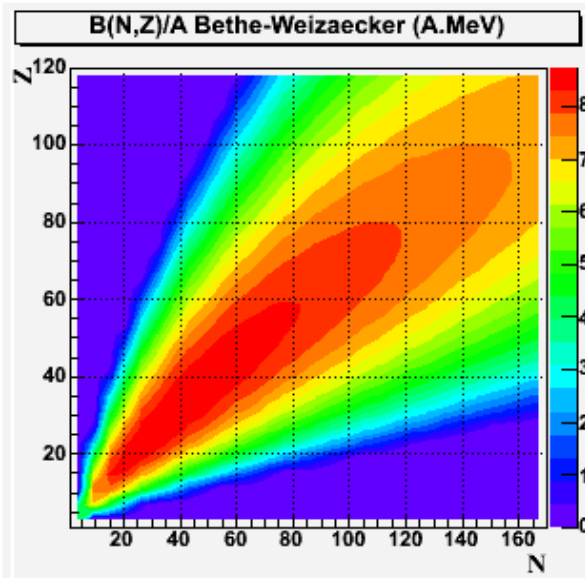


# More detailed structure corrections to apply

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→  $\Delta B_{\text{shell}}(N,Z,\rho_0)$ .

Strategy adopted in FRIGA: whatever the cluster density  $\rho$ ,  $\Delta B_{\text{pairing}}(N,Z,\rho)$  is determined from the assumption of a fixed proportion  $\Delta B_{\text{shell}}/B_{\text{surf.}+\text{vol.}}$ .

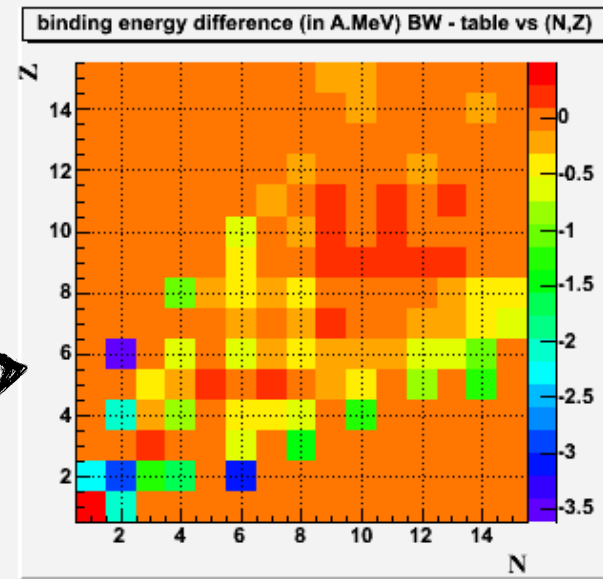
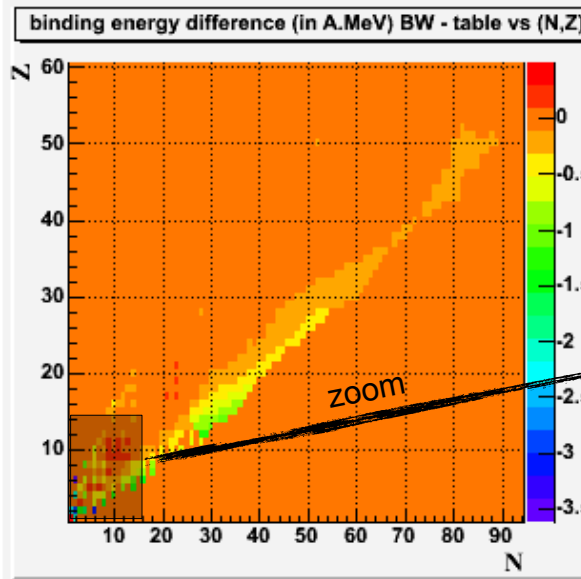
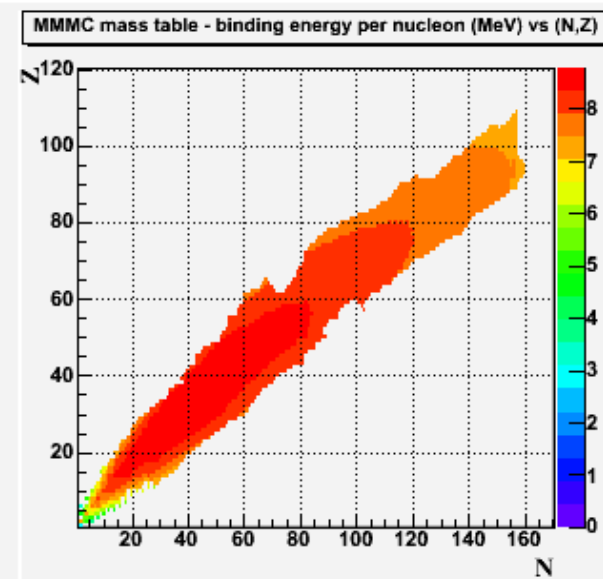
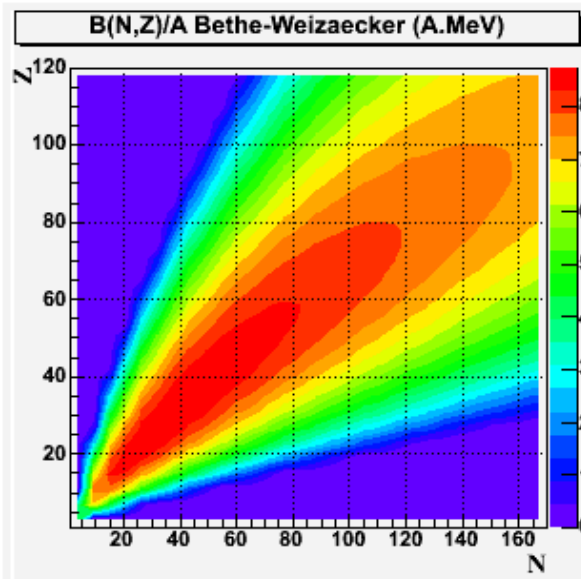


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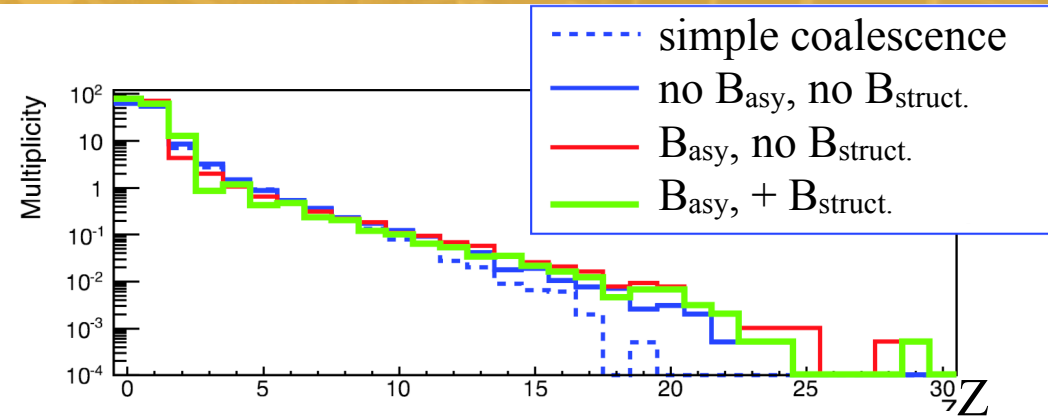
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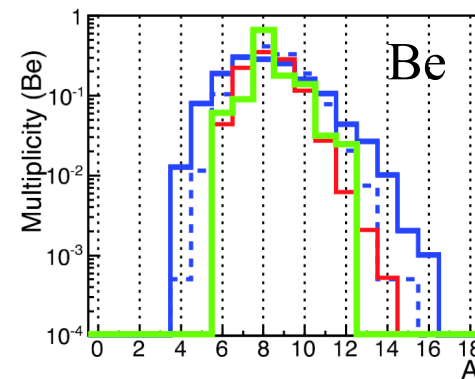
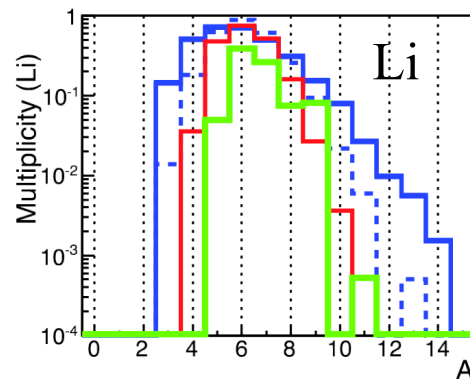
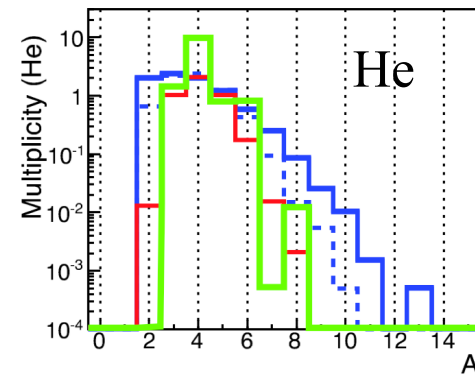
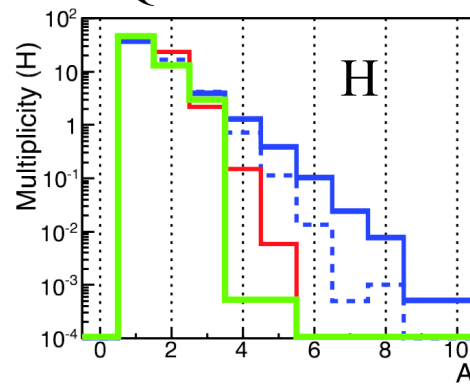




# FRIGA: a cluster



IQMD\*-FRIGA central Xe+Sn @ 100 A.MeV



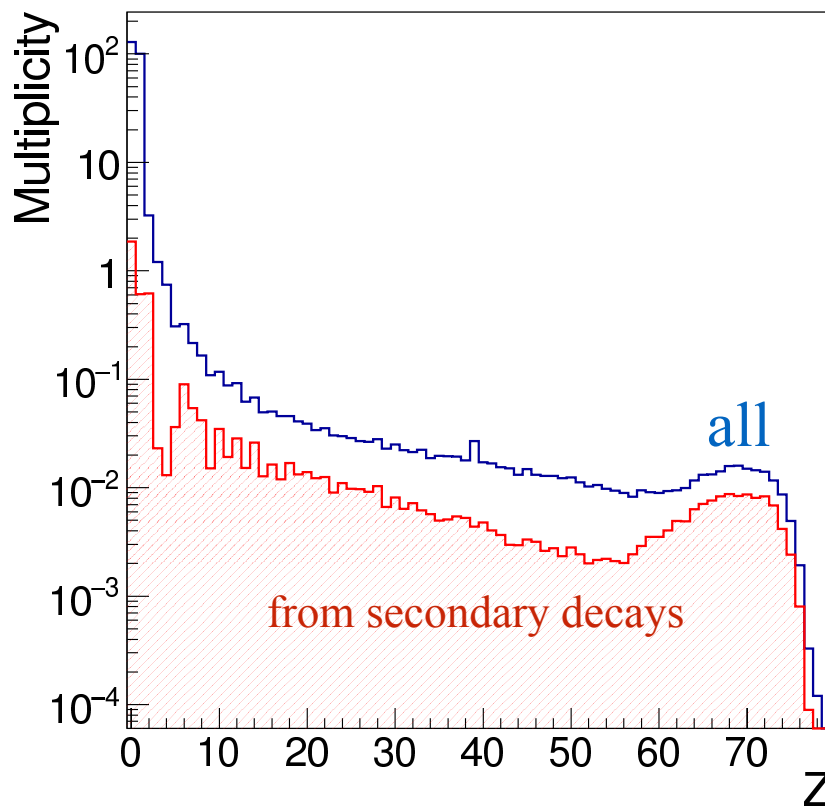
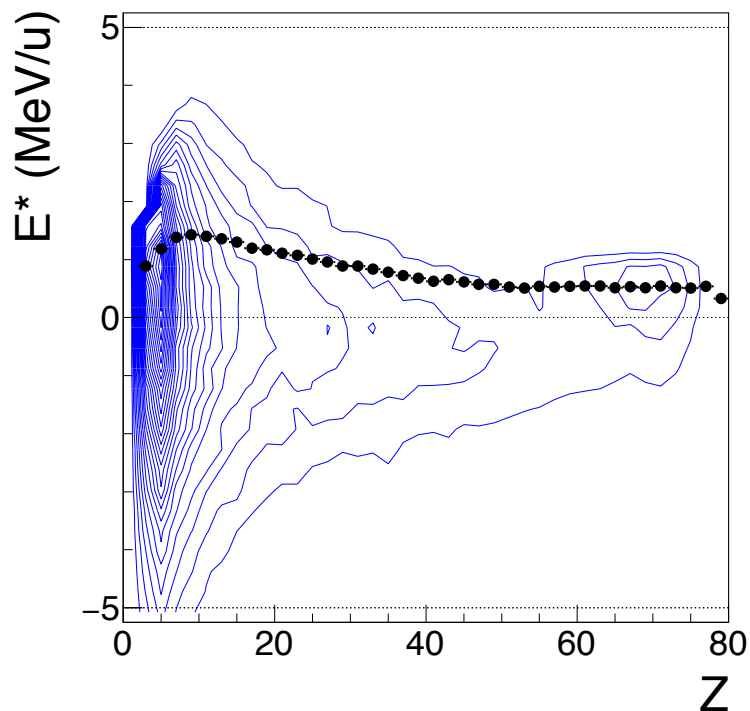
\*: Ch.Hartnack et al., Eur. Phys. J. A 1(1998) 151.



# Influence of the secondary decays on light isotope yields

An example: Au+Au @ 600 A.MeV (min. bias),  $b < 6$  fm (passing time =  $2 t_{\text{pass}}$ )  
from BQMD\*+FRIGA

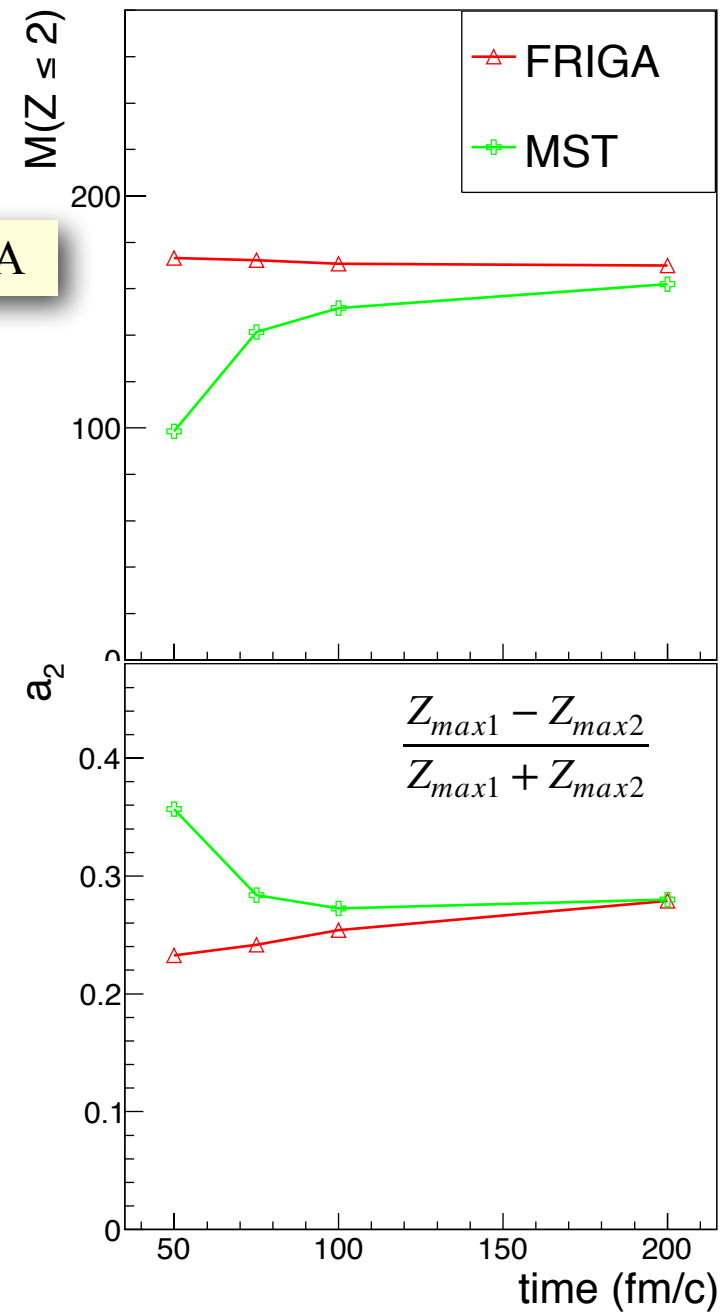
Primary cluster excitation energy



\*: J. Aichelin. Phys. Reports 202, 233 (1991).

# FRIGA versus coalescence (Minimum Spanning Tree)

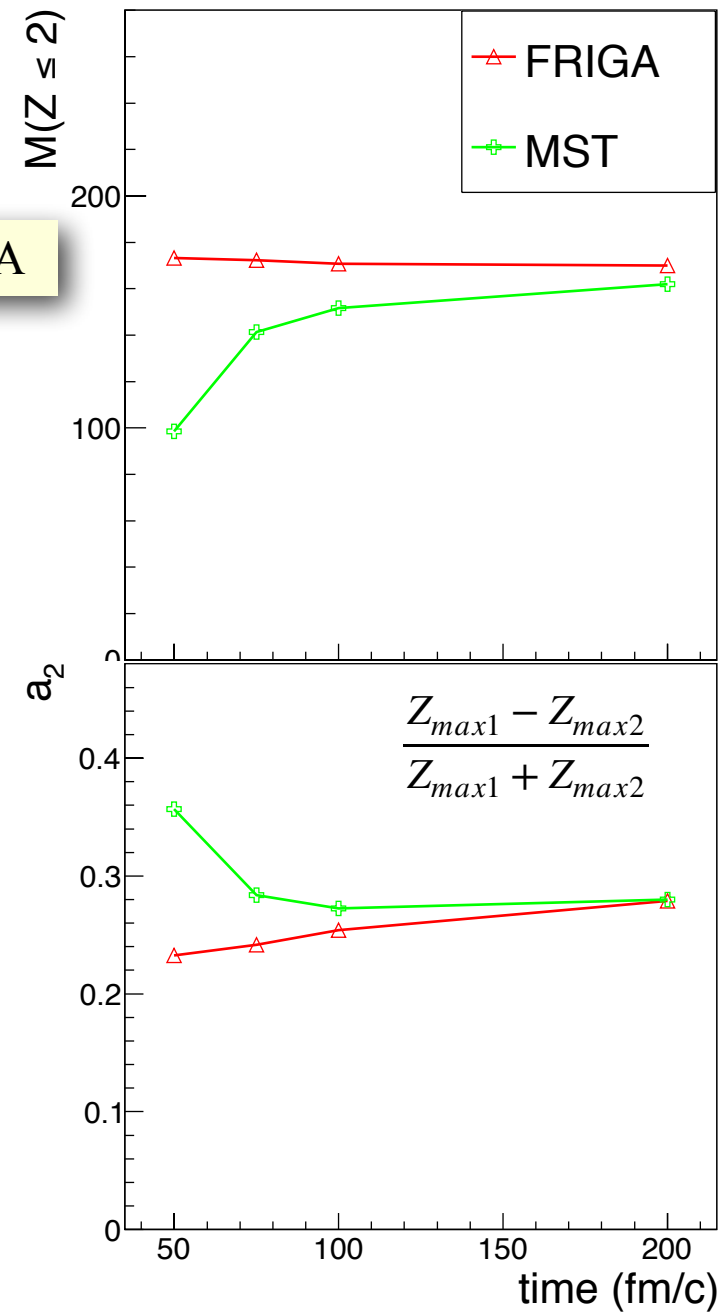
Au+Au @ 600 A.MeV,  $b < 6$  fm from BQMD\*+FRIGA



# FRIGA versus coalescence (Minimum Spanning Tree)

Au+Au @ 600 A.MeV,  $b < 6$  fm from BQMD\*+FRIGA

Unlike **FRIGA**, **MST** is not able to describe the early formation of fragments.



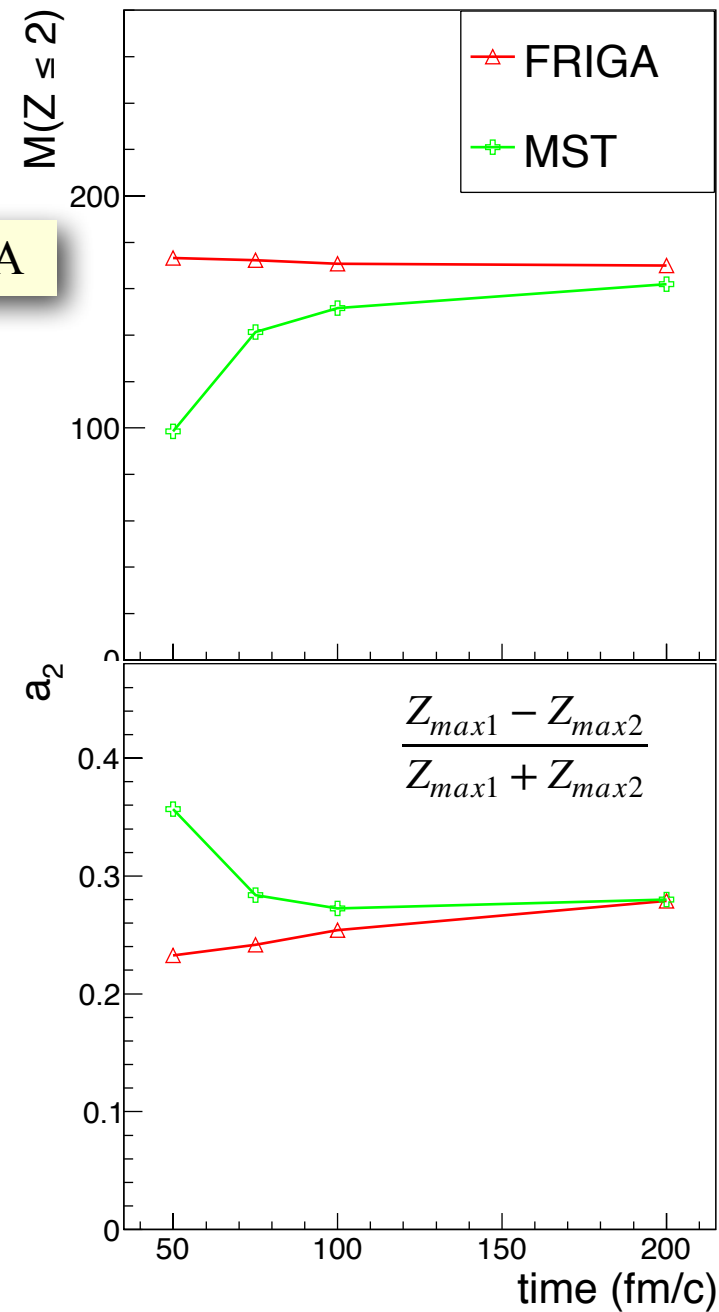


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➔ With **MST**, one has to consider necessarily later times (typically 200-400 fm/c), where the dynamical conditions are no longer the same.



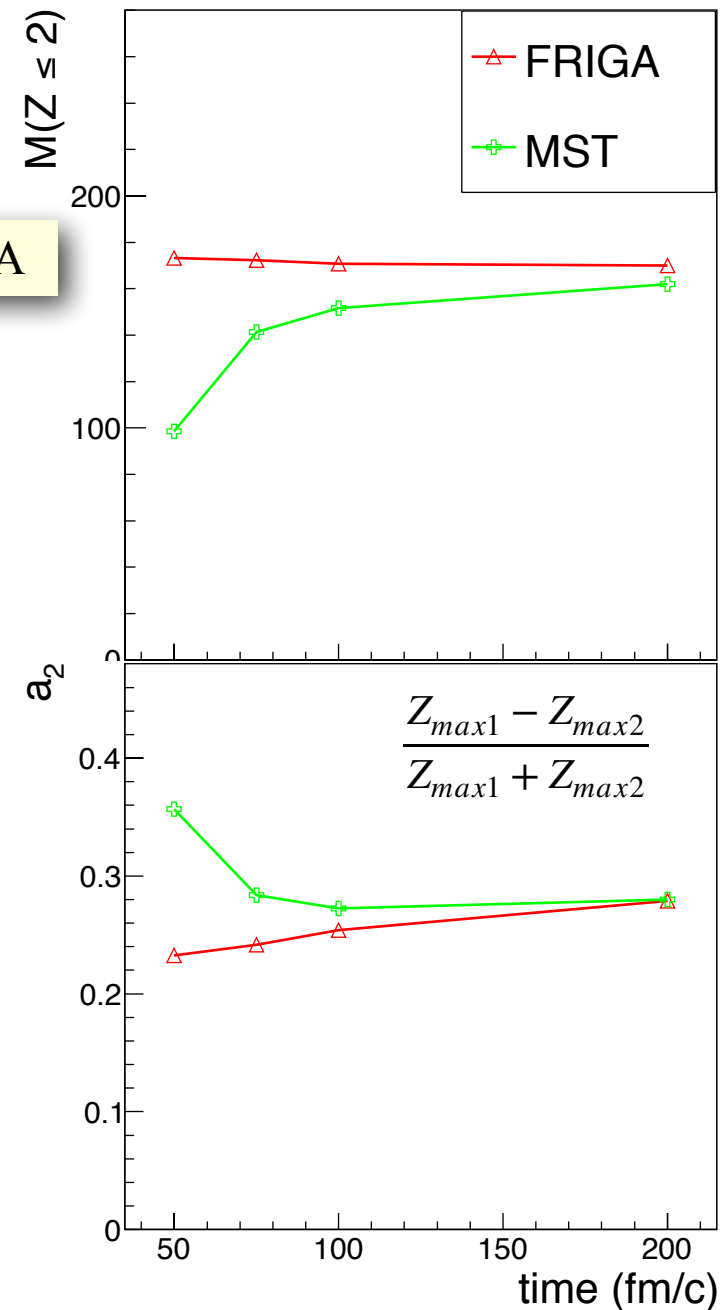
# FRIGA versus coalescence (Minimum Spanning Tree)

Au+Au @ 600 A.MeV,  $b < 6$  fm from BQMD\*+FRIGA

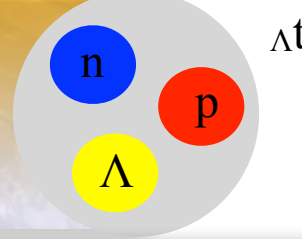
Unlike **FRIGA**, **MST** is not able to describe the early formation of fragments.

➔ With **MST**, one has to consider necessarily later times (typically 200-400 fm/c), where the dynamical conditions are no longer the same.

▶ Advantage of **FRIGA**: the fragment partitions can reflect the **early dynamical conditions** (Coulomb, density, flow details, strangeness...), which is particularly **important for the hypernucleus formation**.



# Clusterisation time influence on hypernuclei (phase space and yields)



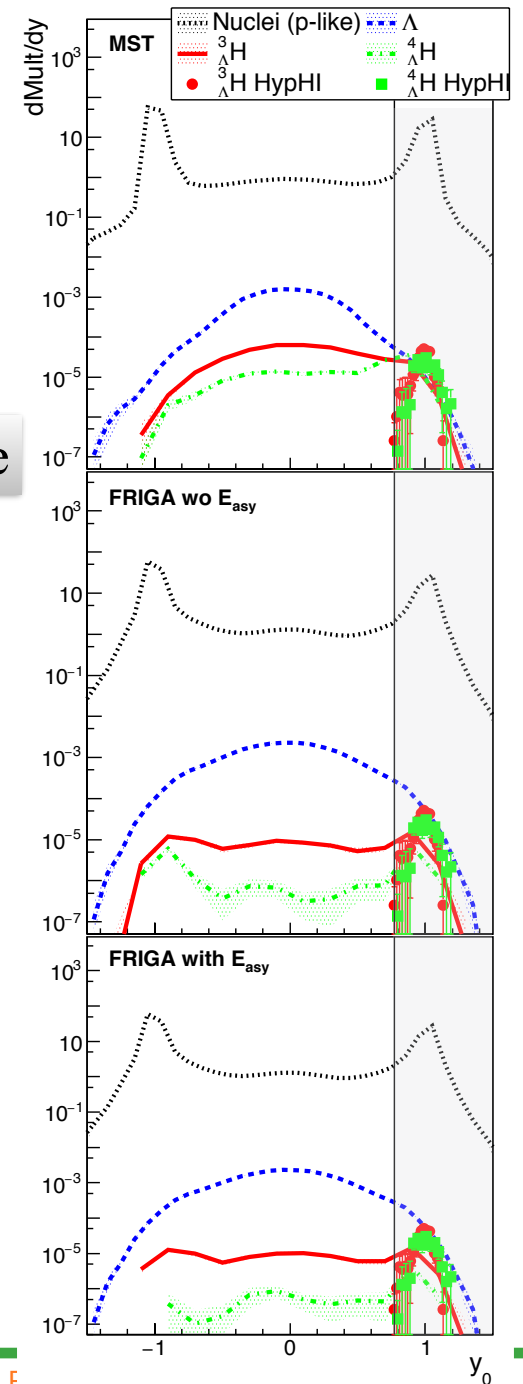
An example: HypHI experiment,  ${}^6\text{Li}+{}^{12}\text{C}$  @ 2A GeV (SIS),  $b=6$  fm (passing time = 4.4 fm/c) from IQMD+FRIGA

2 x passing time

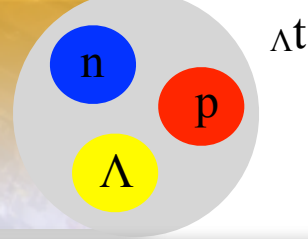
HyPHI experiment @ GSI  
Ch. Rappold et al.,  
PLB 747 (2015) 129-13:

$$R = Y(\Lambda^3\text{H}) / Y(\Lambda^4\text{H}) = 1.4 \pm 0.8$$

IQMD-FRIGA :  
without Easy:  $R = 2.7 \pm 0.5$   
with Easy:  $R = 4.6 \pm 1.1$

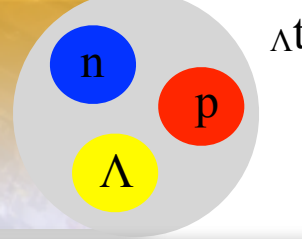


# Clusterisation time influence on hypernuclei (phase space and yields)



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# Clusterisation time influence on hypernuclei (phase space and yields)



An example: HypHI experiment,  ${}^6\text{Li}+{}^{12}\text{C}$  @ 2A GeV (SIS),  $b=6$  fm (passing time = 4.4 fm/c) from IQMD+FRIGA

## Remarks:

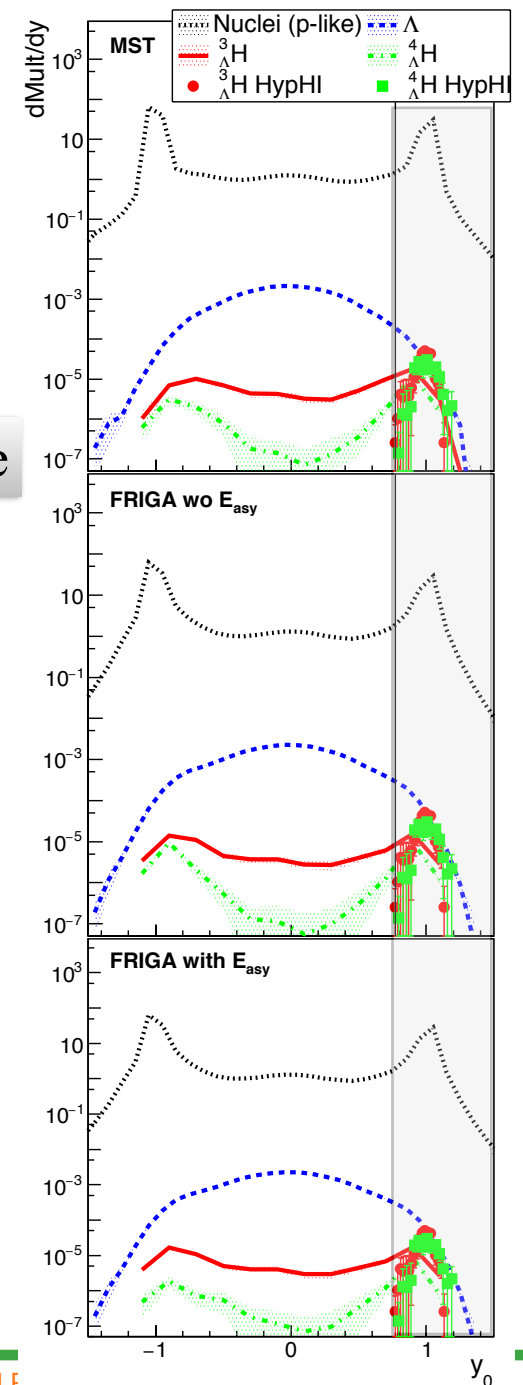
- At mid-rapidity (fireball), the instantaneous hyper-light cluster yields are strongly changing up to at least 4 times the passings
- In the spectator region, they are stabilising faster

4 x passing time

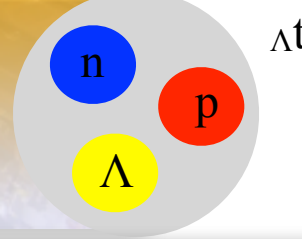
HyPHI experiment @ GSI  
Ch. Rappold et al.,  
PLB 747 (2015) 129-13:

$$R = Y(\Lambda^3\text{H}) / Y(\Lambda^4\text{H}) = 1.4 \pm 0.8$$

**IQMD-FRIGA :**  
without Easy:  $R = 2.0 \pm 0.4$   
with Easy:  $R = 2.5 \pm 0.5$



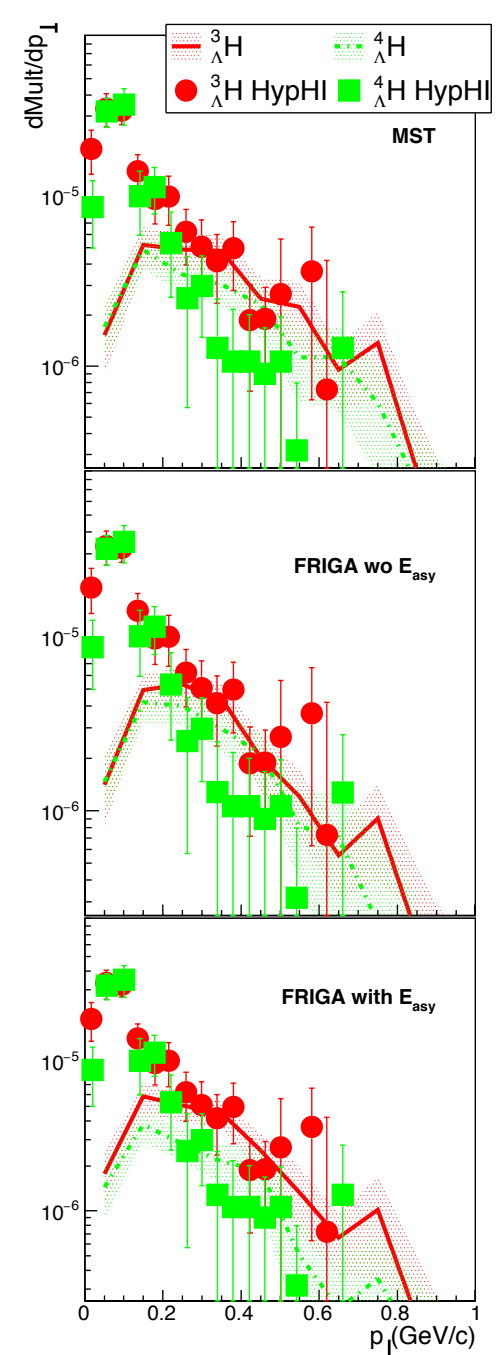
# Clusterisation time influence on hypernuclei (phase space and yields)



An example: HypHI experiment,  ${}^6\text{Li}+{}^{12}\text{C}$  @ 2A GeV (SIS),  $b=6$  fm (passing time = 4.4 fm/c) from IQMD+FRIGA

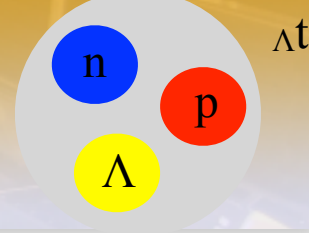
## Remarks:

- Yields and transverse momentum distributions are well reproduced
- Except at very low  $p_t$  where predicted Lambdas are not numerous enough to induce large hyper-cluster yields.





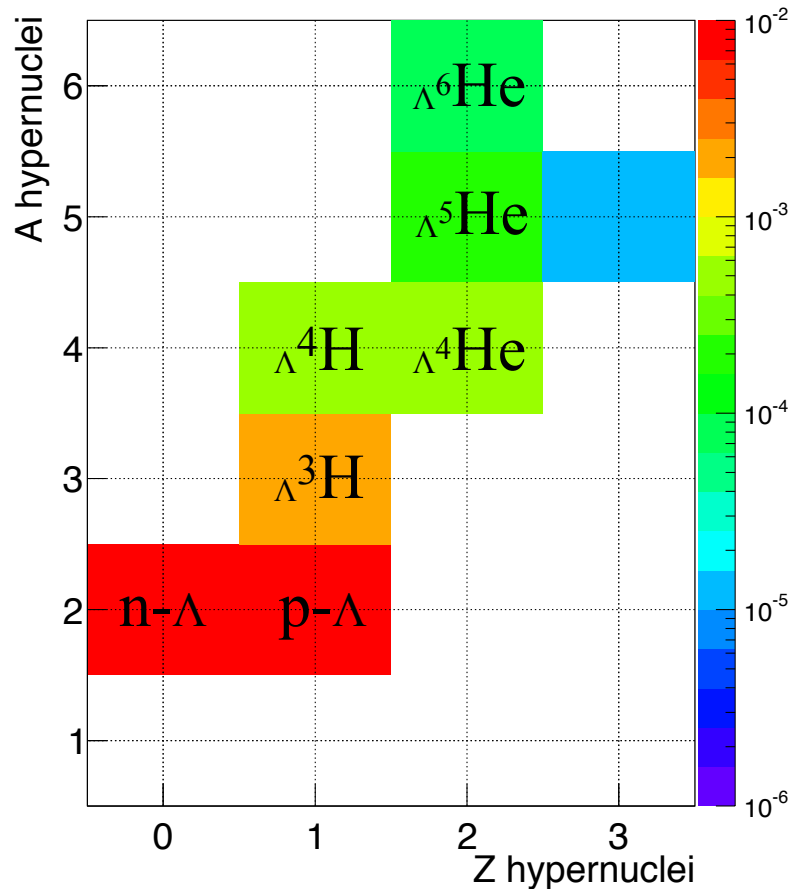
# EOS, in medium-properties and hypernuclei yields



## FOPI system

IQMD\*+FRIGA  
 $^{58}\text{Ni}+^{58}\text{Ni}$  @1.91A.GeV  
 $b < 6$  fm  
( $t_{\text{passing}}=8.7$  fm/c)  
 $t_{\text{cluster.}}=20$  fm/c

IQMD+SACA  $^{58}\text{Ni}+^{58}\text{Ni}$  at 1.93 A.GeV ( $b < 6$  fm,  $t_{\text{cluster.}} = 20$ fm/c) - soft no mdi, kaon pot.

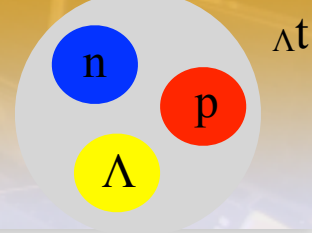


Soft EOS  
no m.d.i.  
with Kaon pot.

\*: Ch.Hartnack et al.,Eur. Phys. J. A 1(1998) 151.



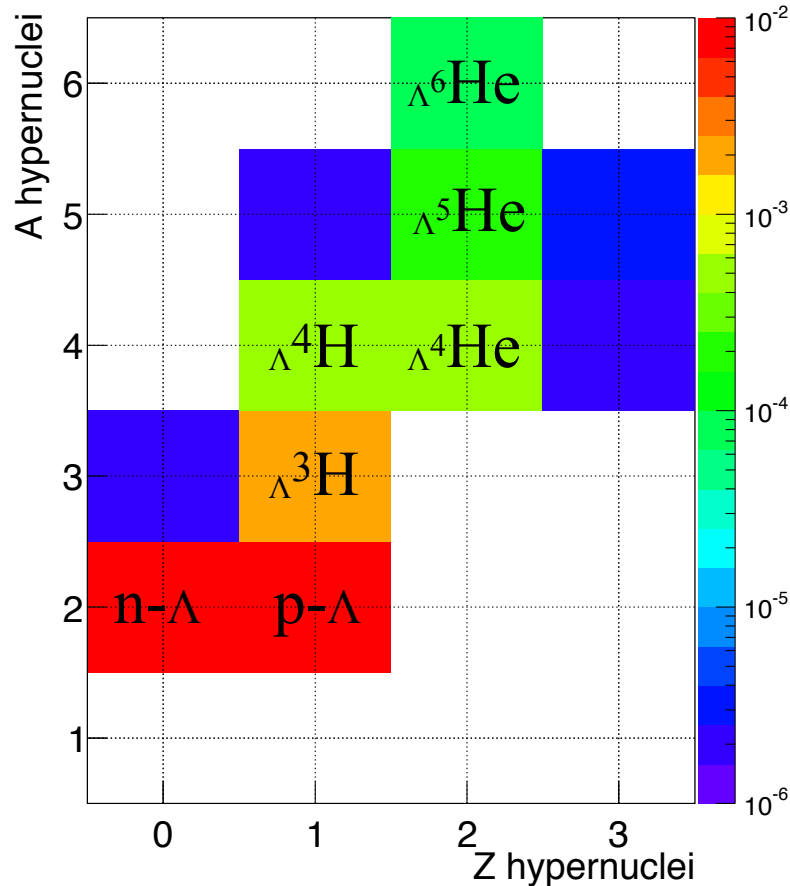
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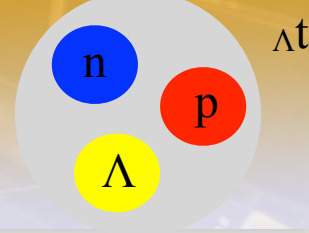
Soft EOS  
 no m.d.i.  
 no Kaon pot.

\*: Ch.Hartnack et al.,Eur. Phys. J. A 1(1998) 151.



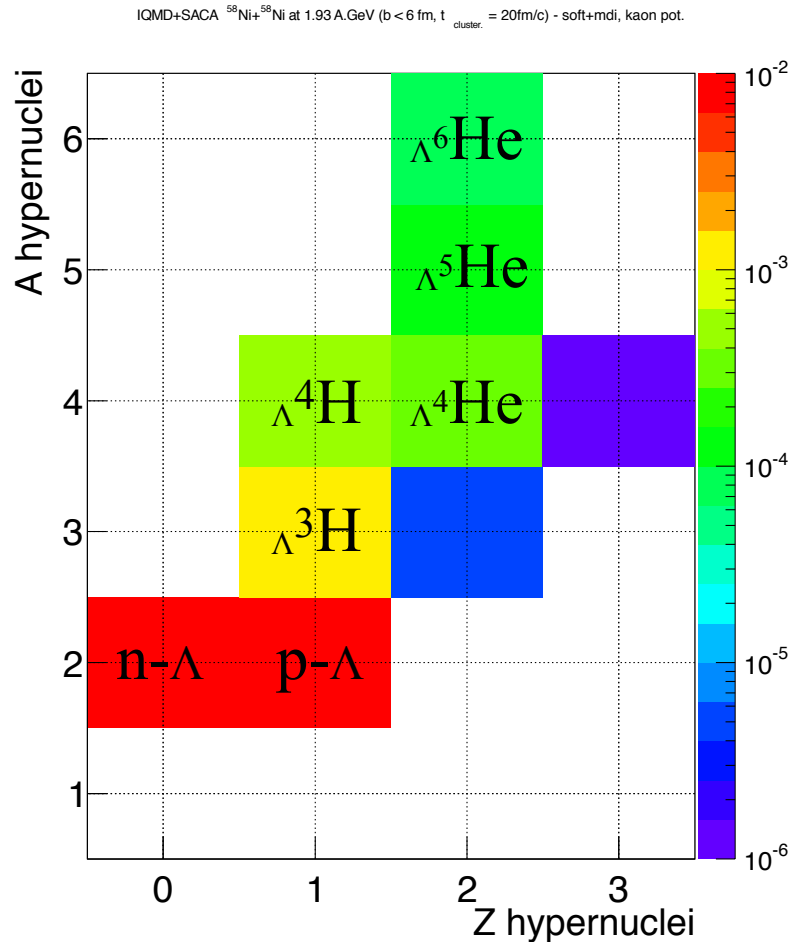


# EOS, in medium-properties and hypernuclei yields



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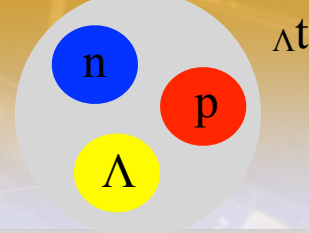


Soft EOS  
with m.d.i.  
with Kaon pot.

\*: Ch.Hartnack et al.,Eur. Phys. J. A 1(1998) 151.



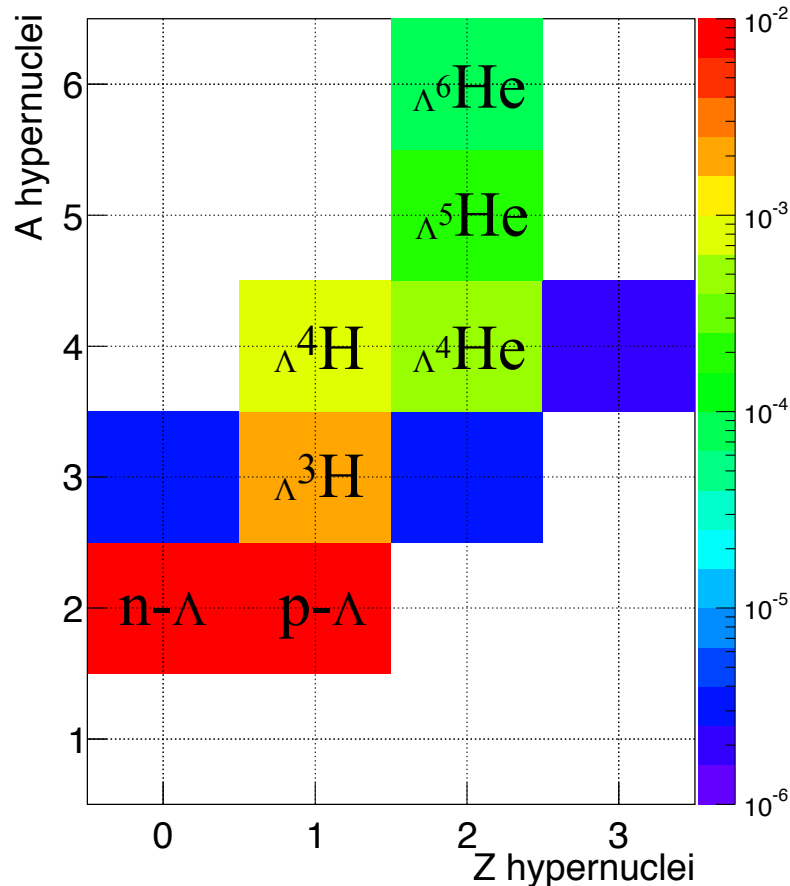
# EOS, in medium-properties and hypernuclei yields



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 $^{58}\text{Ni}+^{58}\text{Ni}$  @1.91A.GeV  
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IQMD+SACA  $^{58}\text{Ni}+^{58}\text{Ni}$  at 1.93 A.GeV ( $b < 6$  fm,  $t_{\text{cluster.}} = 20$ fm/c) - soft+mdi, no kaon pot.

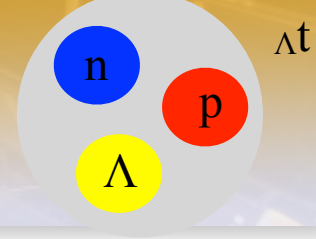


Soft EOS  
 with m.d.i.  
 no Kaon pot. .

\*: Ch.Hartnack et al.,Eur. Phys. J. A 1(1998) 151.



# EOS, in medium-properties and hypernuclei yields



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IQMD\*+FRIGA

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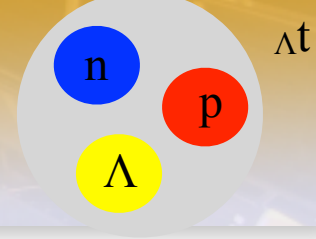
$b < 6$  fm

( $t = 2.3 t_{\text{pass}}$ )

\*: Ch.Hartnack et al.,Eur. Phys. J. A 1(1998) 151.



# EOS, in medium-properties and hypernuclei yields



FOPI system

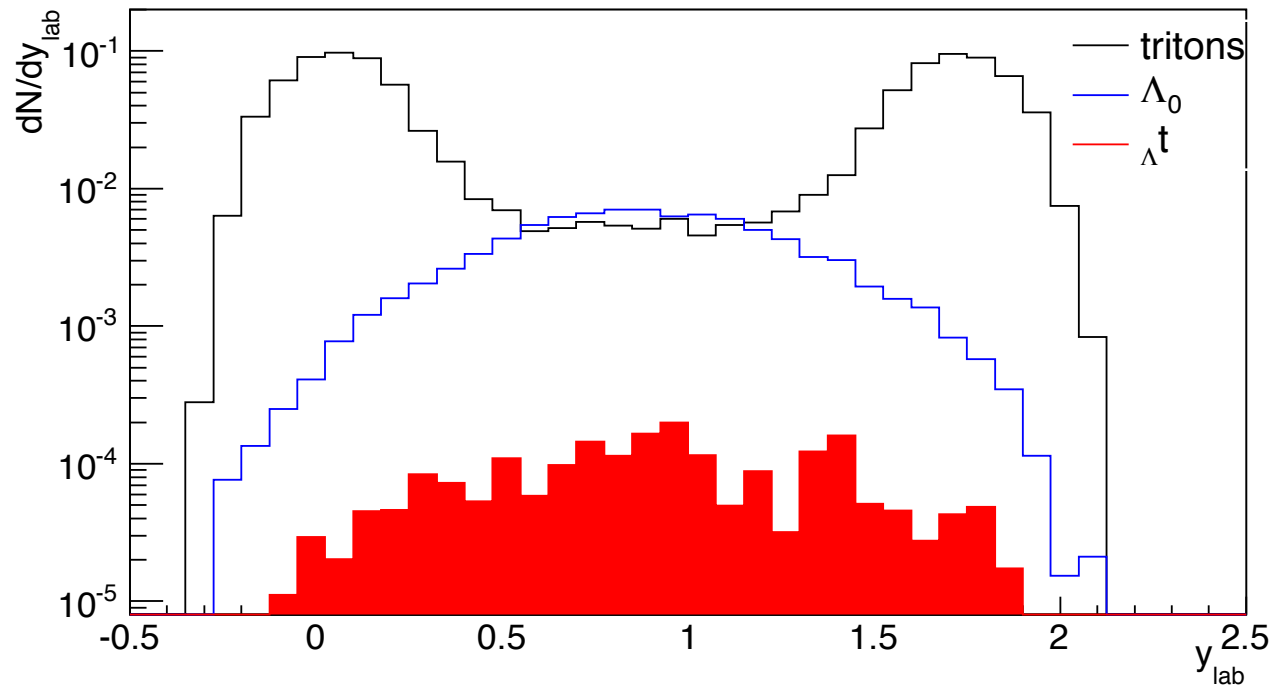
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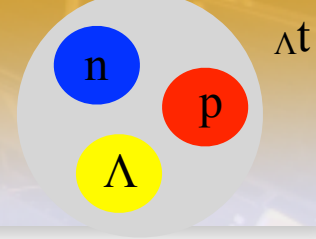
Soft EOS, no m.d.i., with Kaon pot.



\*: Ch.Hartnack et al.,Eur. Phys. J. A 1(1998) 151.



# EOS, in medium-properties and hypernuclei yields



FOPI system

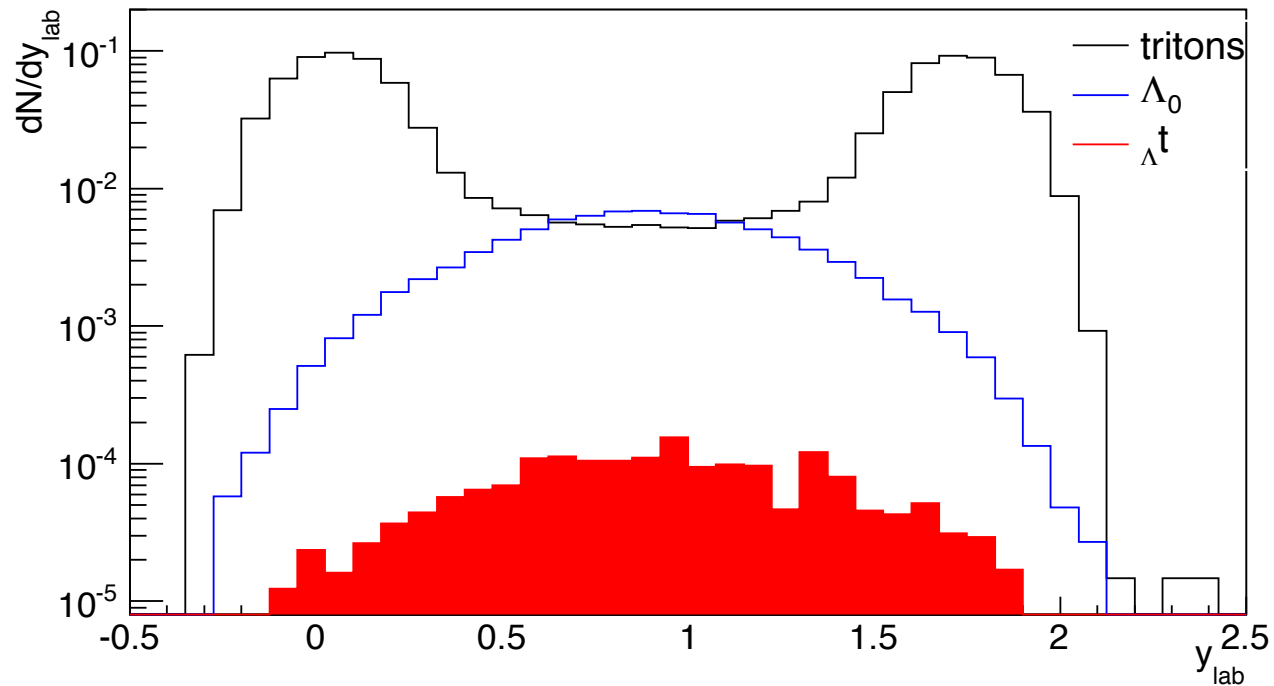
IQMD\*+FRIGA

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( $t = 2.3 t_{\text{pass}}$ )

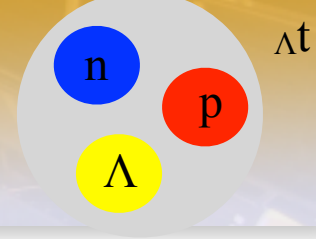
Soft EOS, no m.d.i., no Kaon pot.



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FOPI system

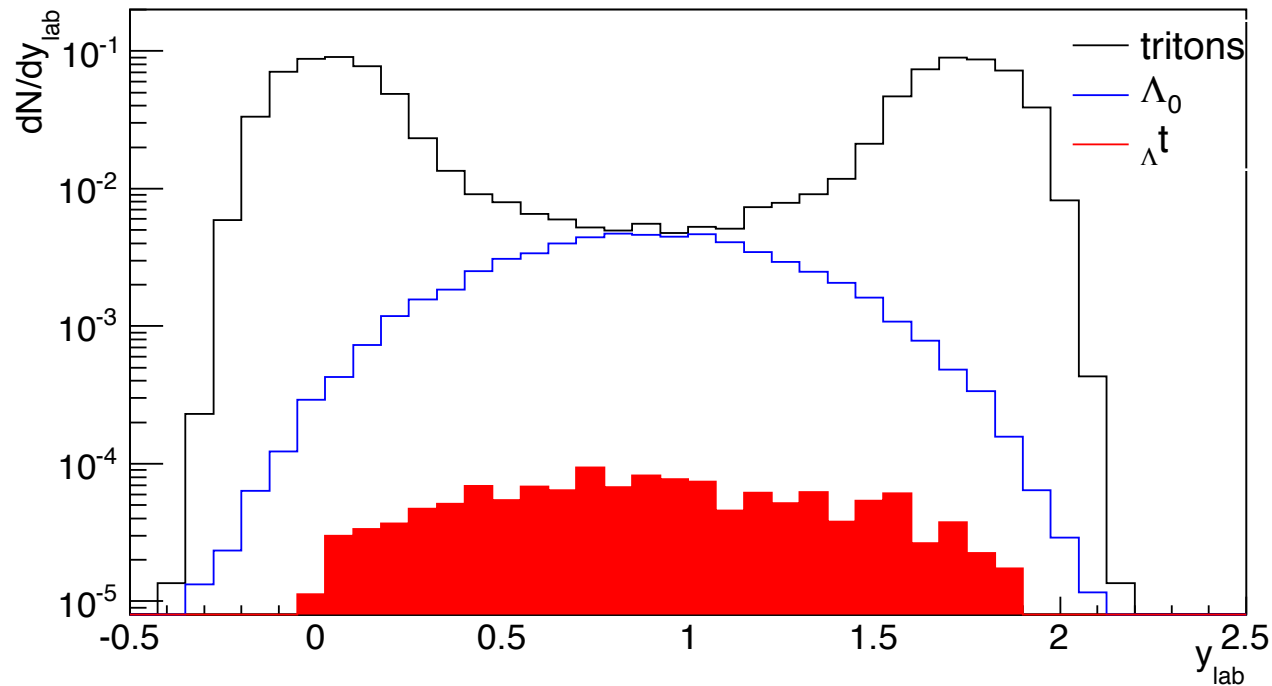
IQMD\*+FRIGA

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$b < 6$  fm

( $t = 2.3 t_{\text{pass}}$ )

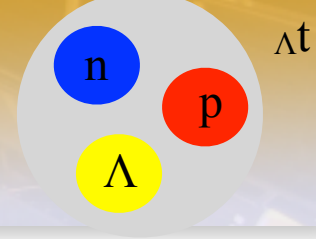
Soft EOS, no m.d.i., with Kaon pot.



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# EOS, in medium-properties and hypernuclei yields



FOPI system

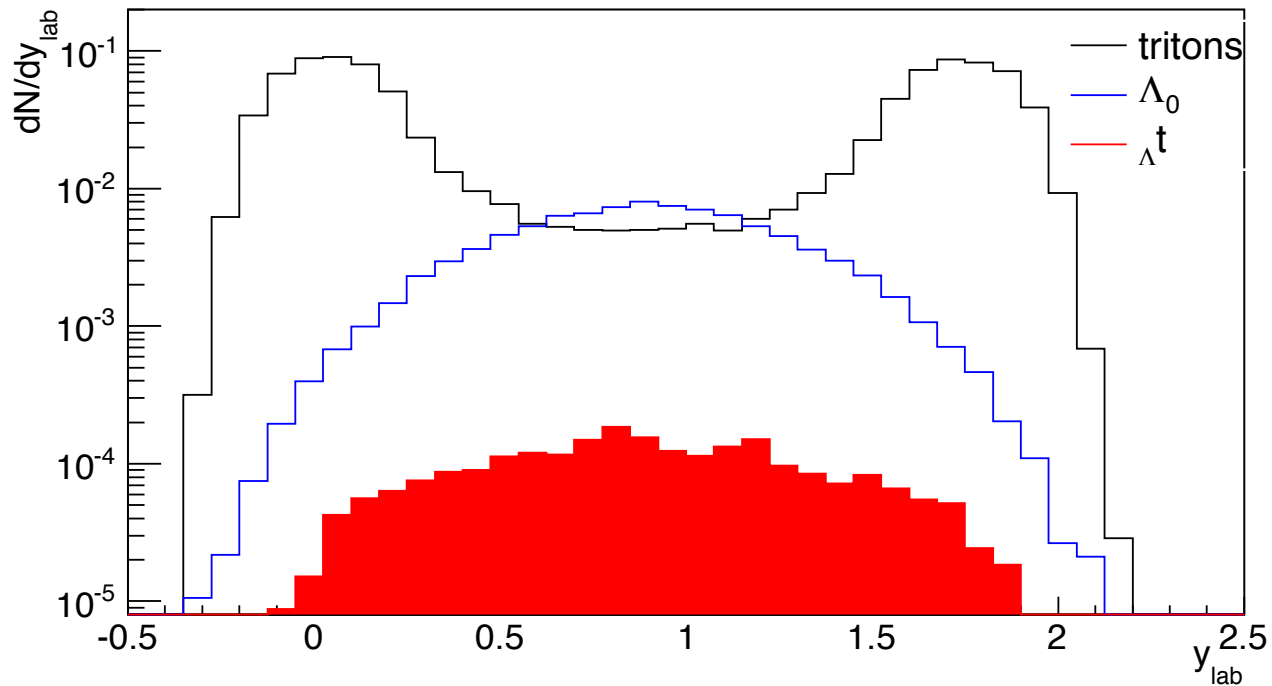
IQMD\*+FRIGA

$^{58}\text{Ni}+^{58}\text{Ni}$  @1.91A.GeV

$b < 6$  fm

( $t = 2.3 t_{\text{pass}}$ )

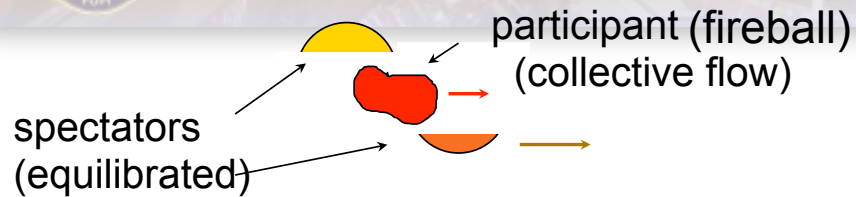
Soft EOS, no m.d.i., no Kaon pot.



\*: Ch.Hartnack et al.,Eur. Phys. J. A 1(1998) 151.



# Some successful applications in the spectator regime

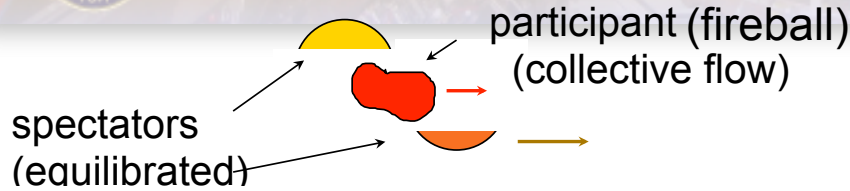


← centrality

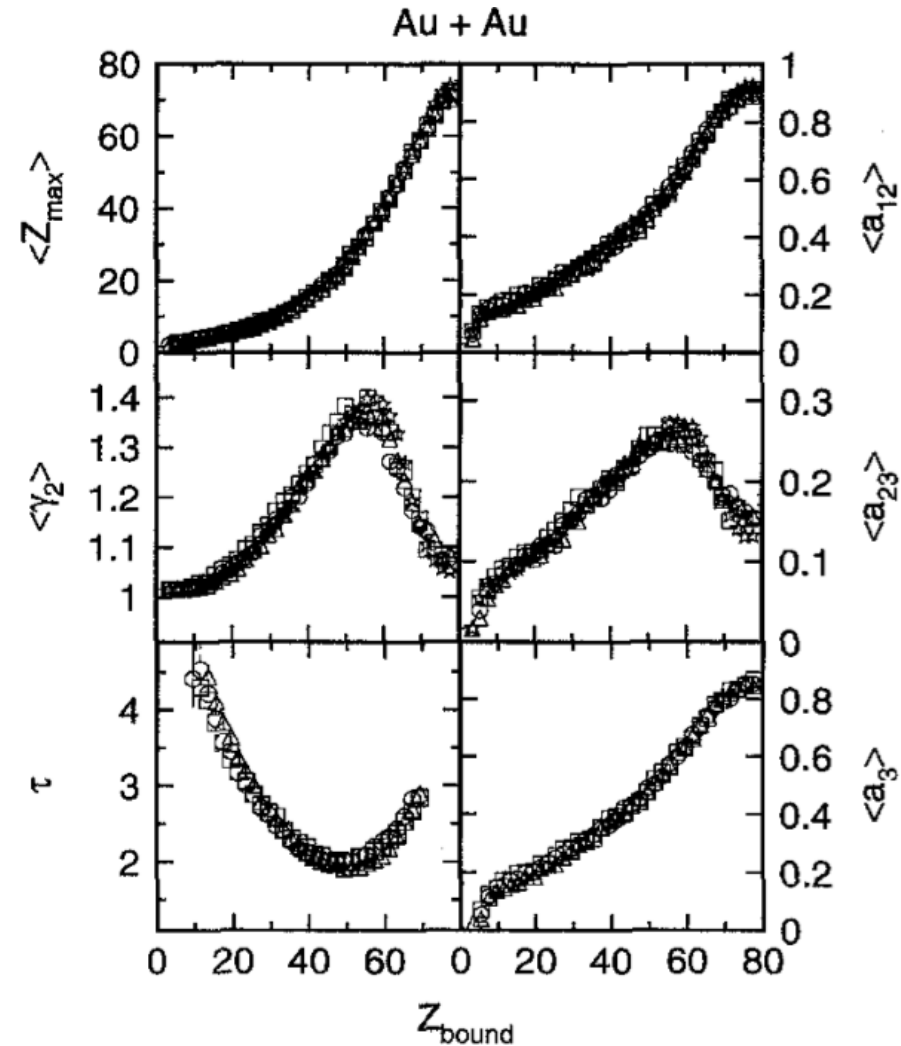
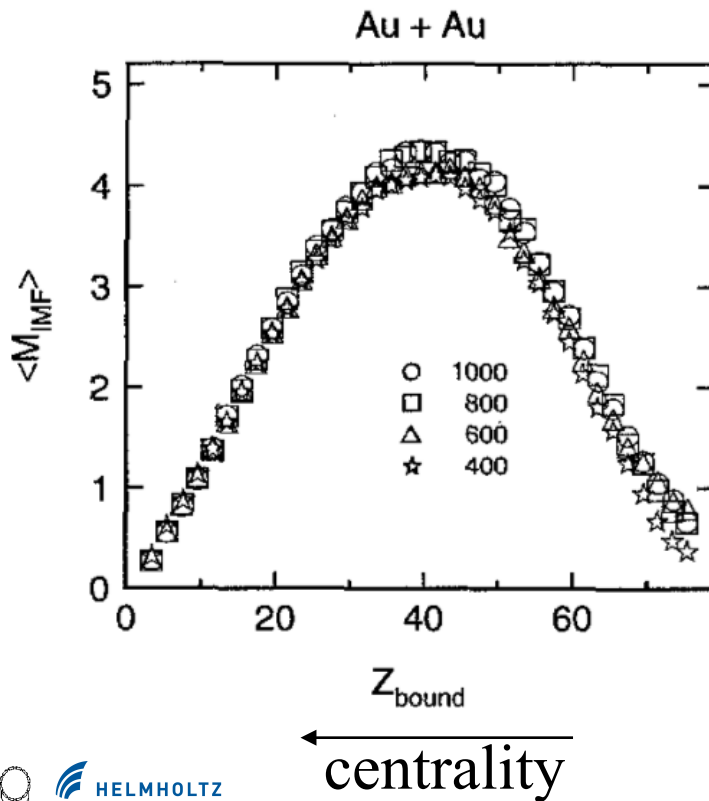




# Some successful applications in the spectator regime

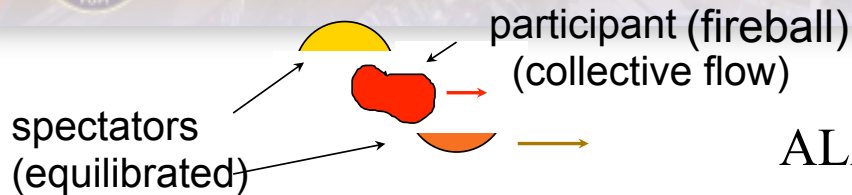


A. Schüttauf et al. / Nuclear Physics A 607 (1996) 457-486

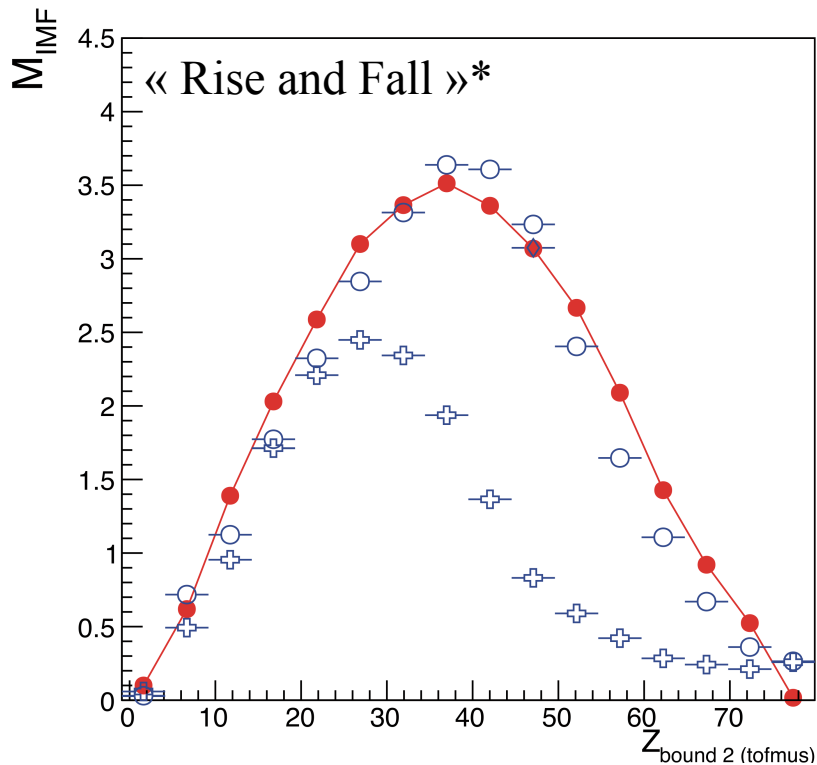




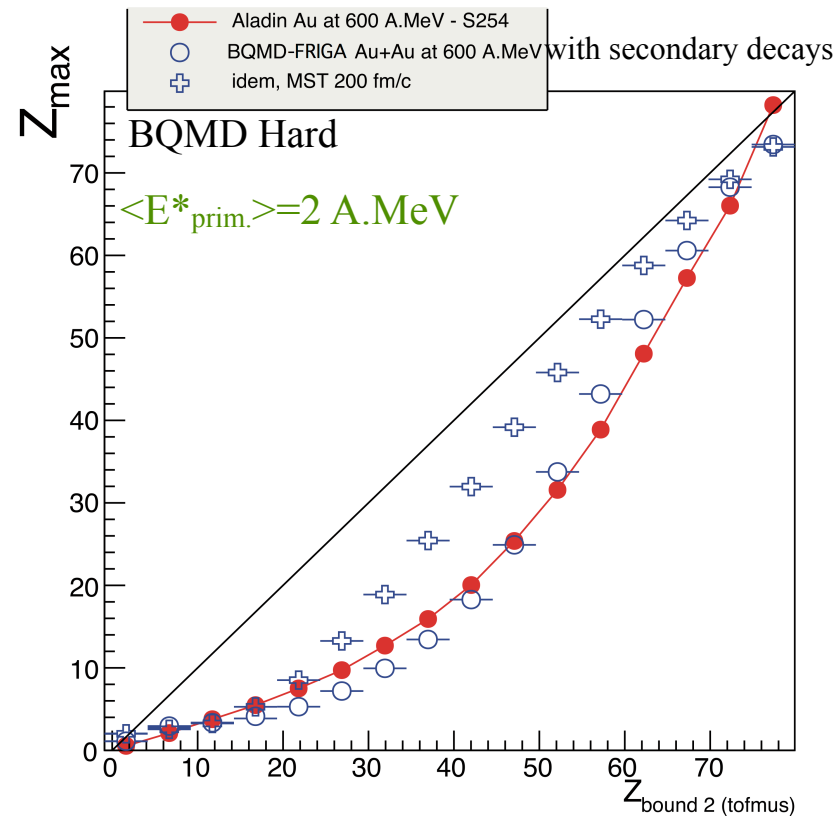
# Some successful applications in the spectator regime



ALADiN Au+Au @ 600 A.MeV (S254 exp., 2003)

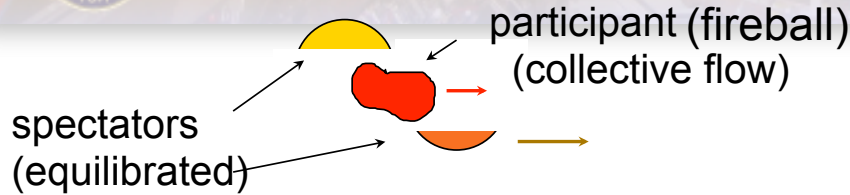


← centrality





# Another world: the fireball regime



Central

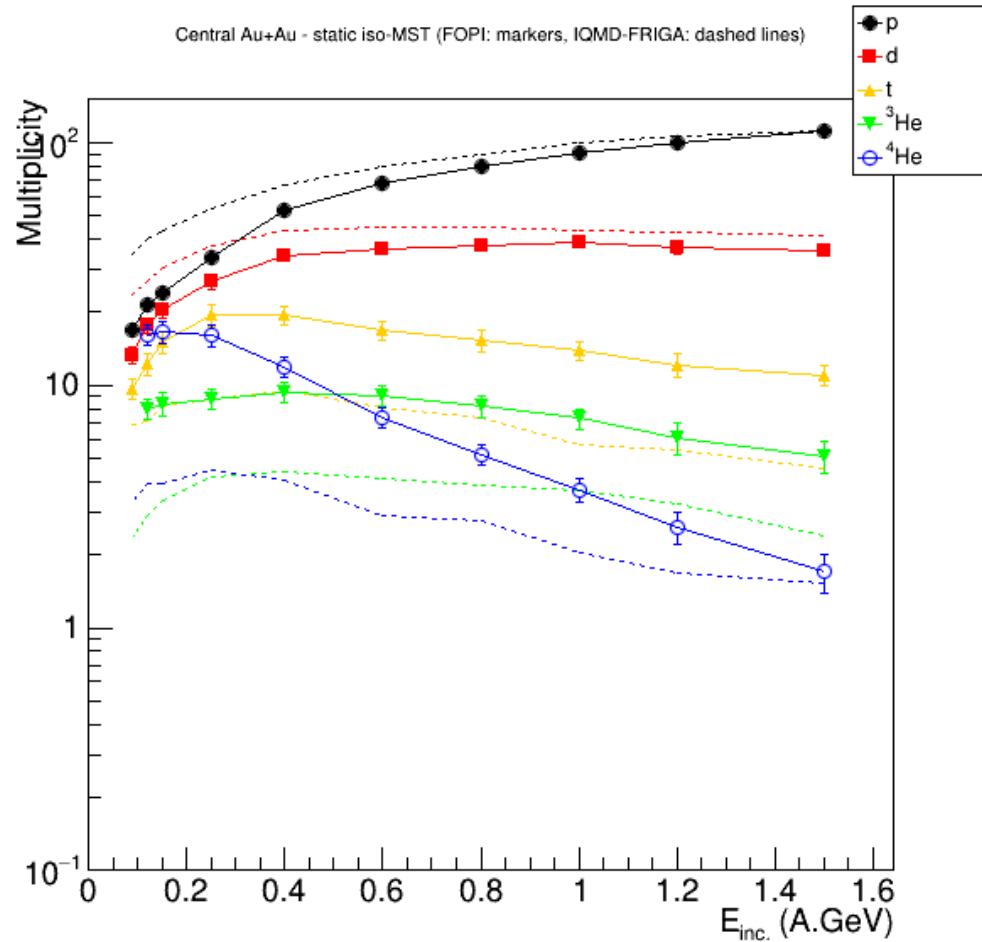
Au+Au:

IQMD-FRIGA (dashed lines)

vs

FOPI data (markers)\*

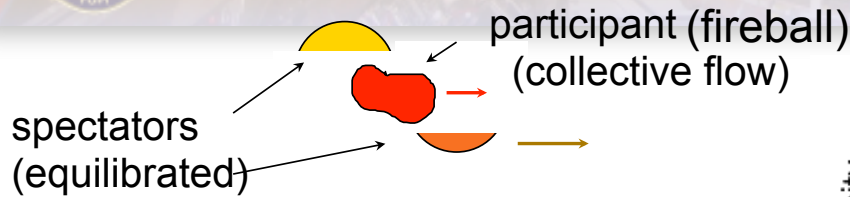
MST  
(200 fm/c)



\*: W. Reisdorf *et al.*, FOPI Collaboration / Nuclear Physics A 848 (2010) 366–427



# Another world: the fireball regime



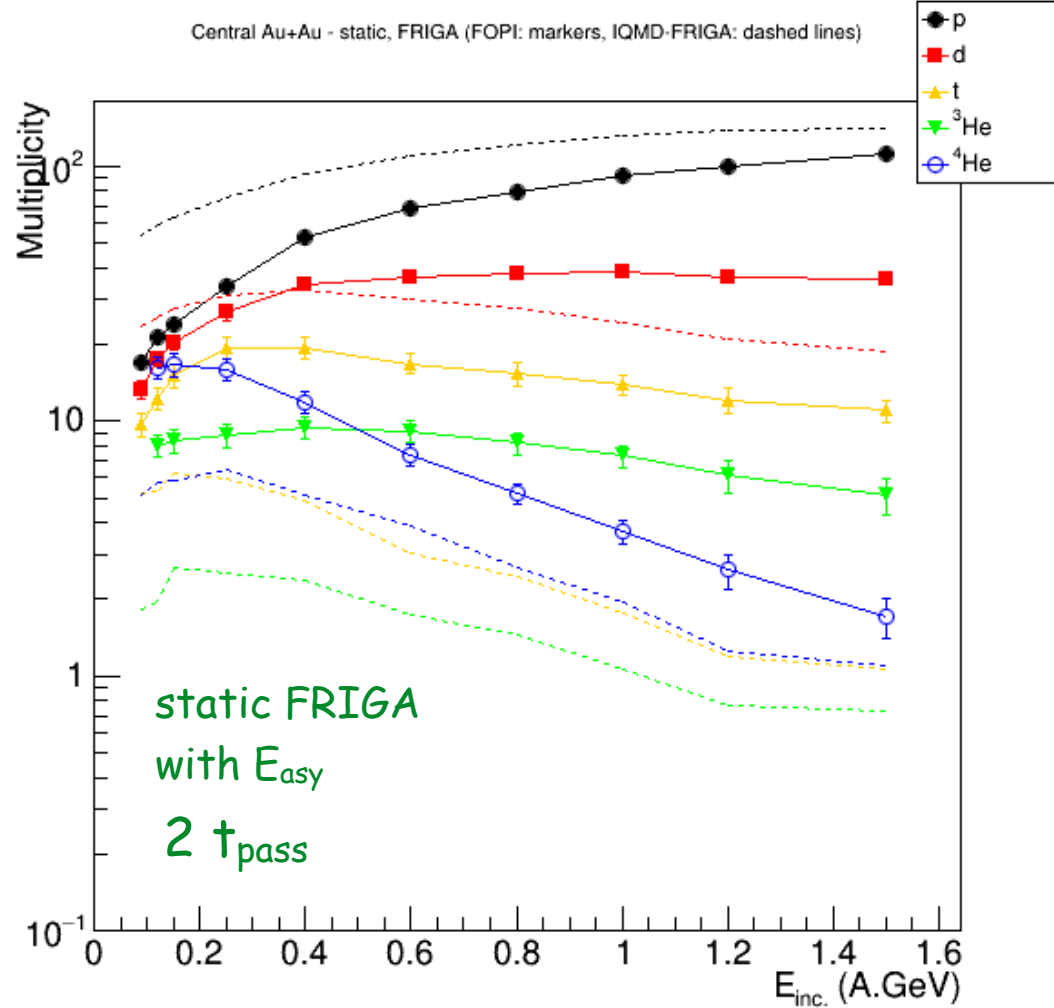
Central

Au+Au:

IQMD-FRIGA (dashed lines)

vs

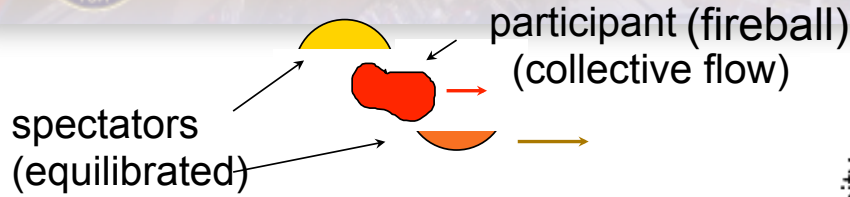
FOPI data (markers)\*



\*: W. Reisdorf et al., FOPI Collaboration / Nuclear Physics A 848 (2010) 366–427



# Another world: the fireball regime



Central

Au+Au:

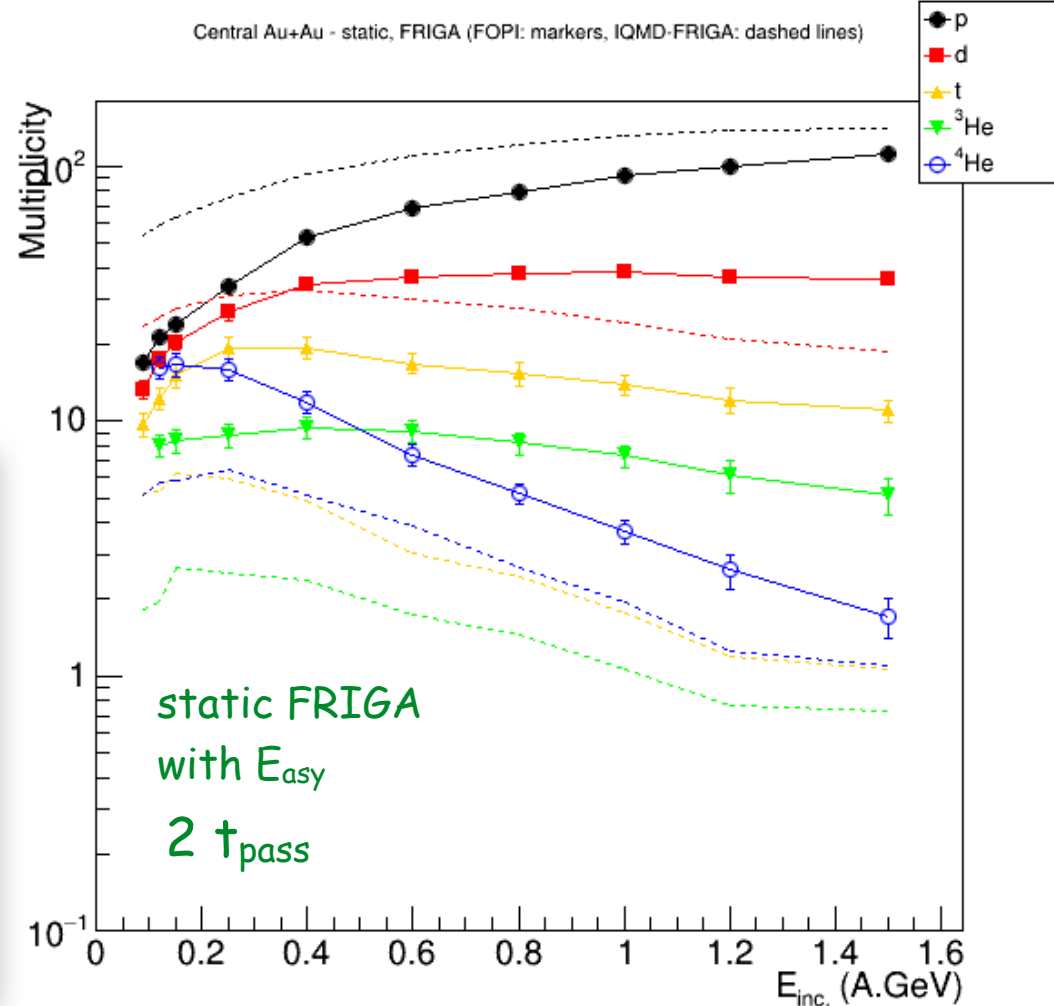
IQMD-FRIGA (dashed lines)

vs

FOPI data (markers)\*

⇒ In central collisions, the static/instantaneous FRIGA strategy (including asymmetry and structure binding energies) does not provide accurate light isotope yields at large incident energies.

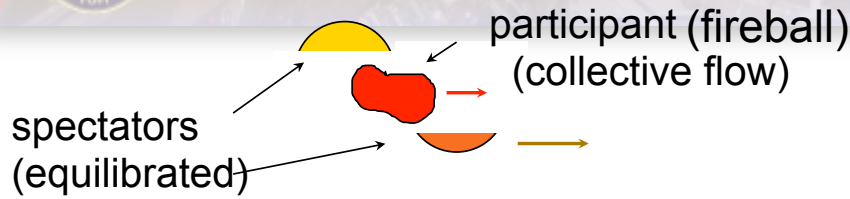
⇒ Conversely, the static MST coalescence approach fails at these lowest energies, and is more reliable so far (but not the final word) at the highest energies.



\*: W. Reisdorf et al., FOPI Collaboration / Nuclear Physics A 848 (2010) 366–427



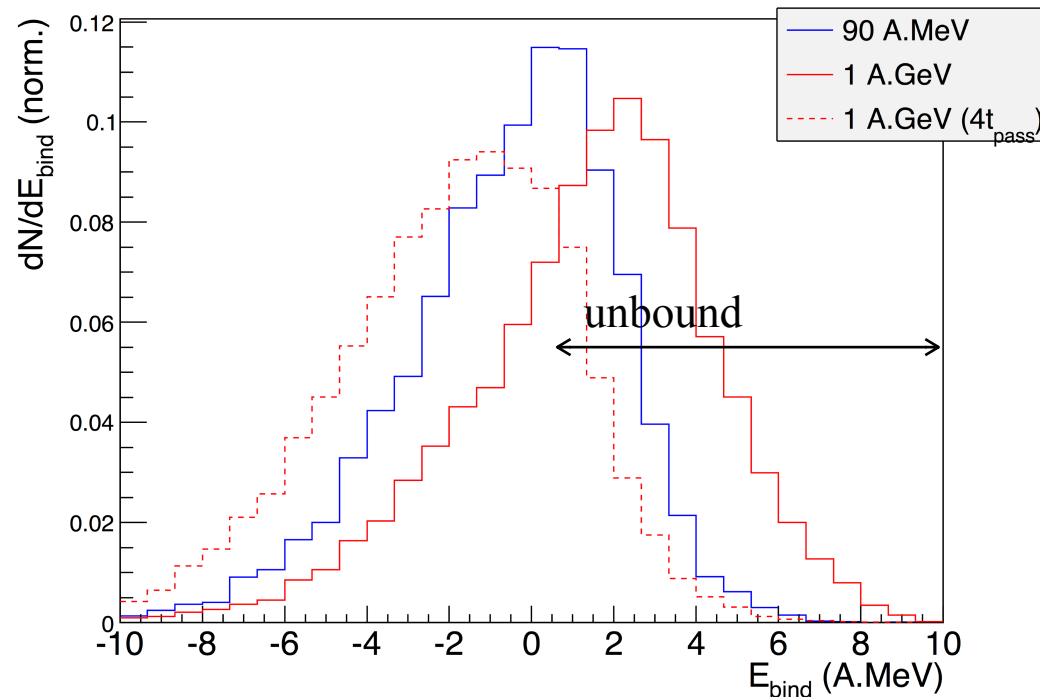
# Still to be worked on: the fireball regime



Central  
Au+Au:  
IQMD-FRIGA

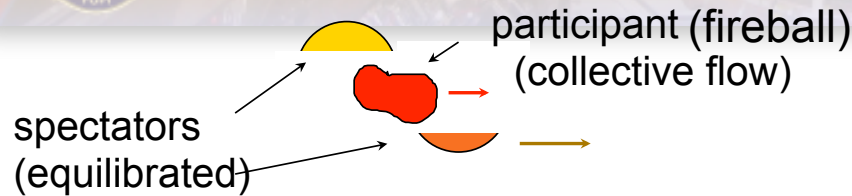
$B_{asy}$  on  
 $B_{struct}$  on

Binding energy of early ( $2 t_{pass}$ ) tritons  
from coalescence (MST)





# Still to be worked on: the fireball regime

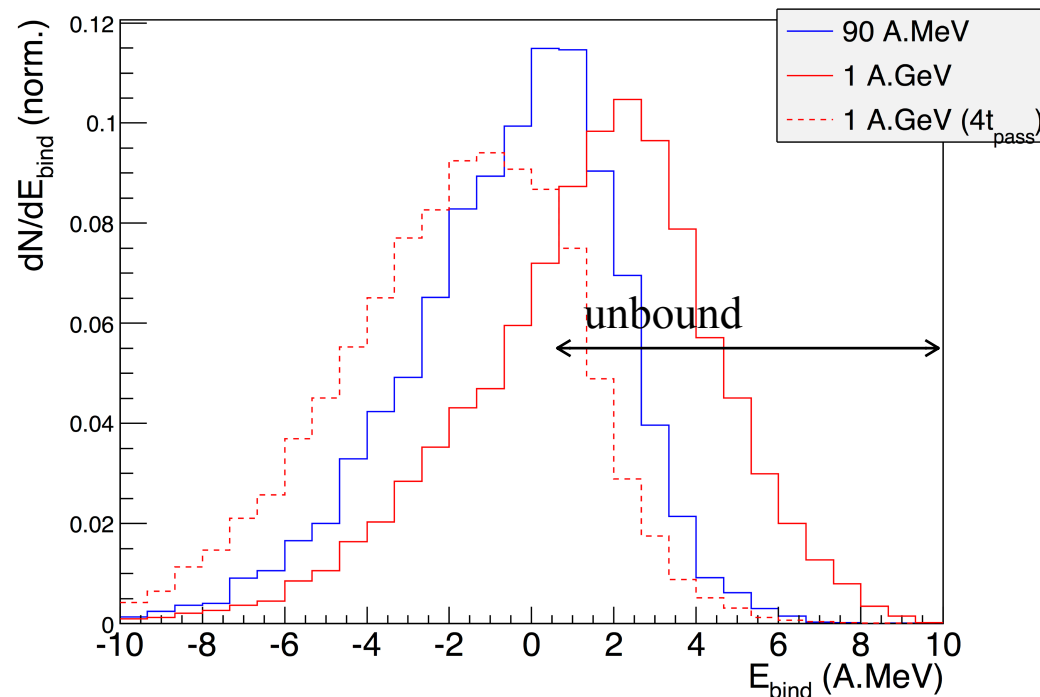


Central  
Au+Au:  
IQMD-FRIGA

In the contrary of the rather cool central source of intermediate energies (and spectator), in the hot fireball, early pre-fragments are mostly hot and unbound.

⇒ Invalidity of FRIGA as an early « afterburner » in the fireball regime. Better alternative: follow the process of cluster formation up to a relatively longer time.

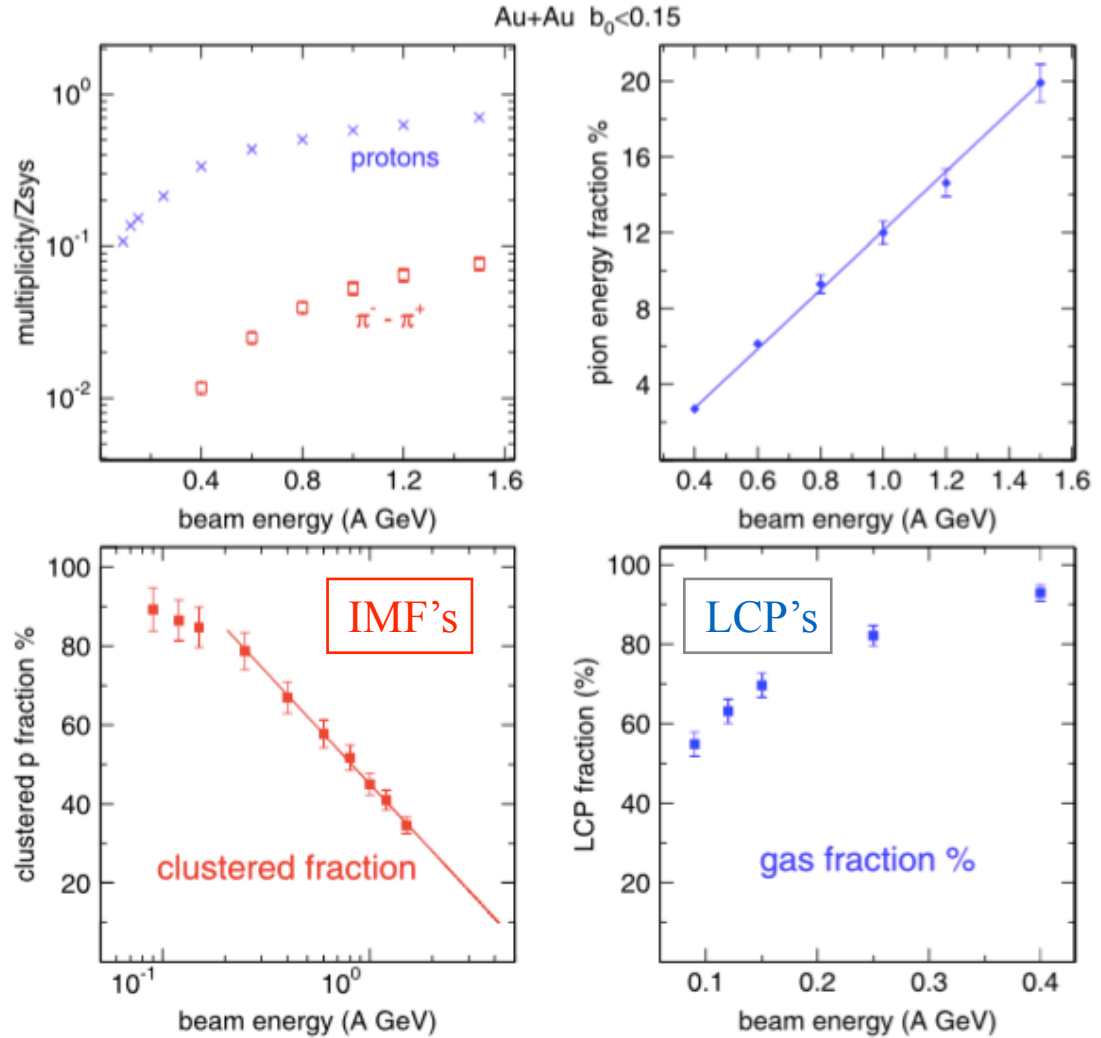
Binding energy of early ( $2 t_{\text{pass}}$ ) tritons from coalescence (MST)





# The fireball regime: a high degree of clusterisation

FOPI Collaboration / Nuclear Physics A 848 (2010) 366–427







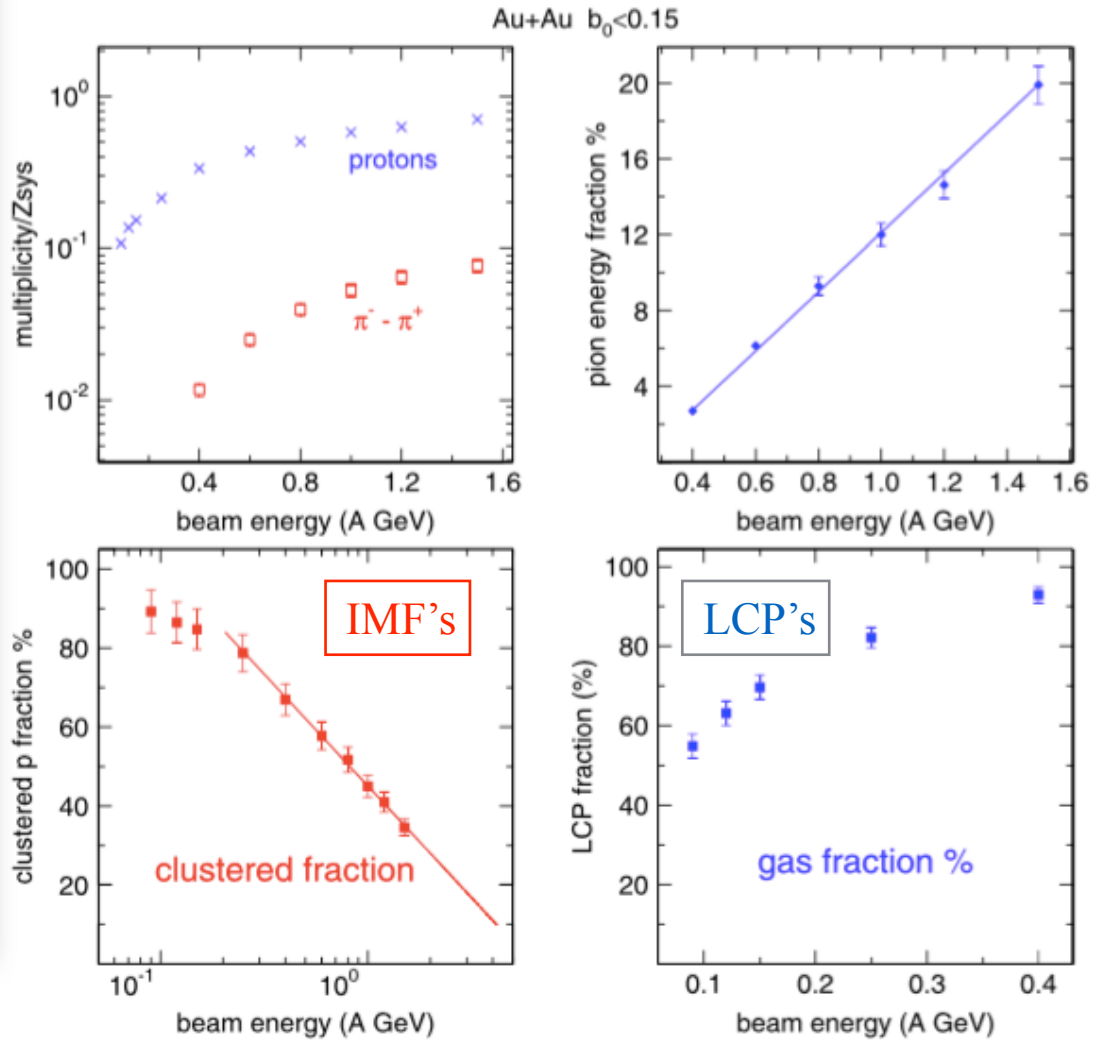
# The fireball regime: a high degree of clusterisation

FOPI Collaboration / Nuclear Physics A 848 (2010) 366–427

Extrapolation -> clustered fraction >10% up to 4A GeV. Persistence of a significant probability to clusterize at freeze-out up to an available energy per nucleon more than two orders of magnitude higher than typical nucleonic binding energies

⇒ Signal of local cooling accompanying the fireball expansion

⇒ Strong constraint on the associated entropy.





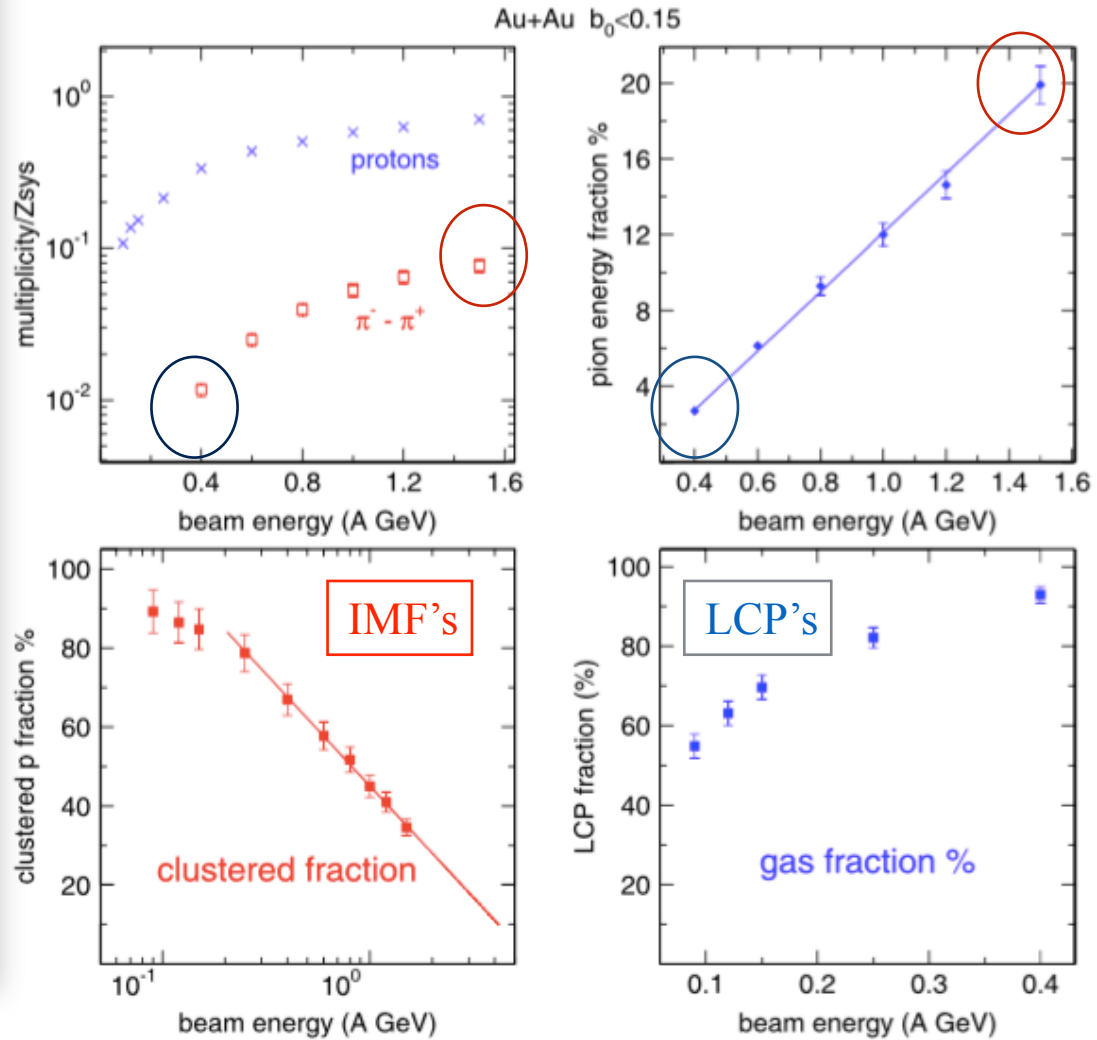
# The fireball regime: a high degree of clusterisation

FOPI Collaboration / Nuclear Physics A 848 (2010) 366–427

Pions in a large system:

At low energy: perturbative, low incidence on the dynamics and available N/Z.

At high energy: non-perturbative, modify the available N/Z (1.49  $\rightarrow$  1.26 : 8 protons converted into neutrons), and modify the dynamics (carry 20% of the total energy)





# The fireball regime: a high degree of clusterisation

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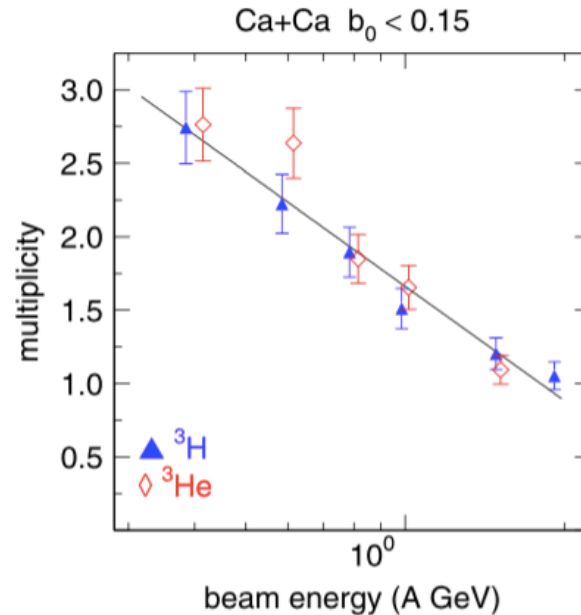


Fig. 42.  $^3\text{He}$  and  $^3\text{H}$  multiplicities in central  $^{40}\text{Ca} + ^{40}\text{Ca}$  collisions as function of the incident beam energy. The straight line is a common fit linearly decreasing with the logarithm of the energy.

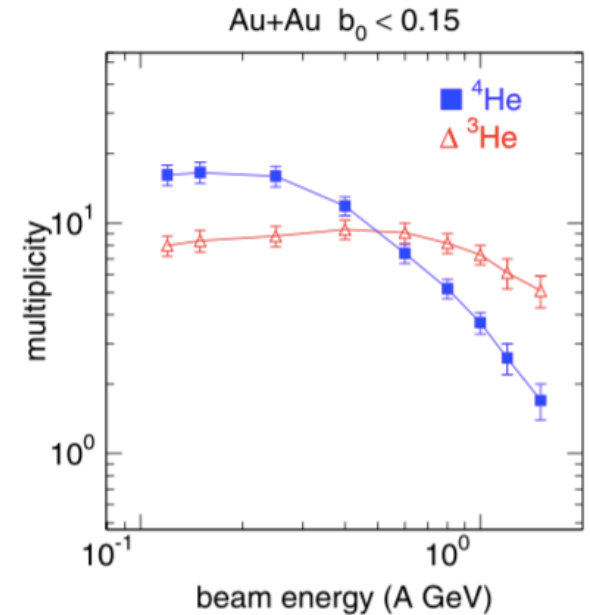


Fig. 43.  $^3\text{He}$  and  $^4\text{He}$  multiplicities in central Au + Au collisions as function of the incident beam energy. The 'crossing' energy is 0.5A GeV.



# The fireball regime: a high degree of clusterisation

Small symmetric system:

⇒ The available  $N/Z$  for cluster stay unchanged up to high energies.

412

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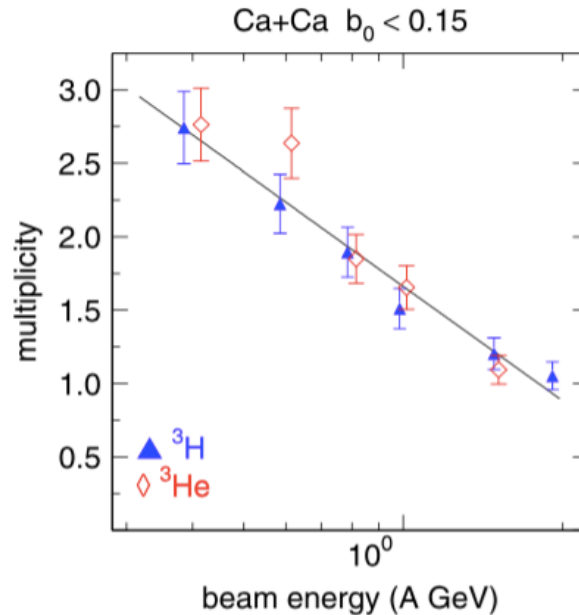


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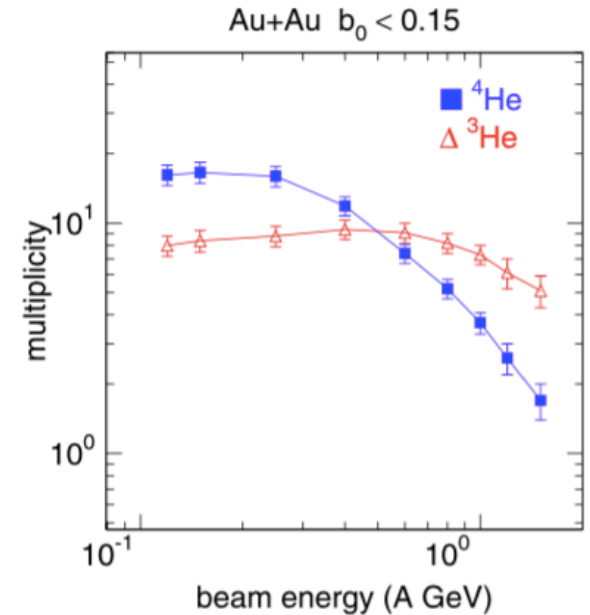


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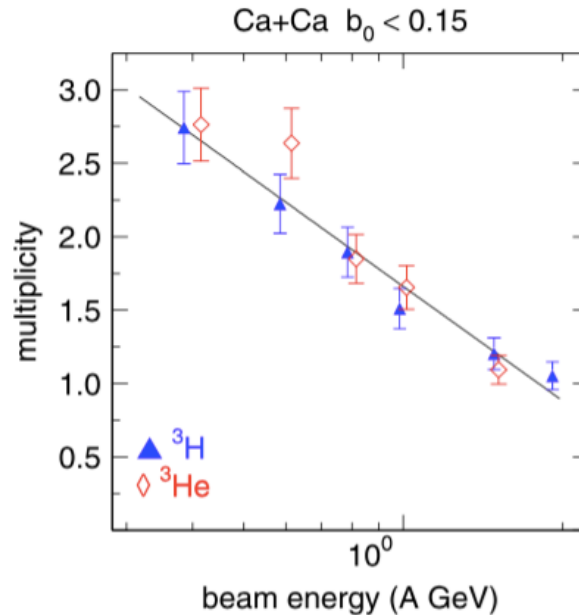


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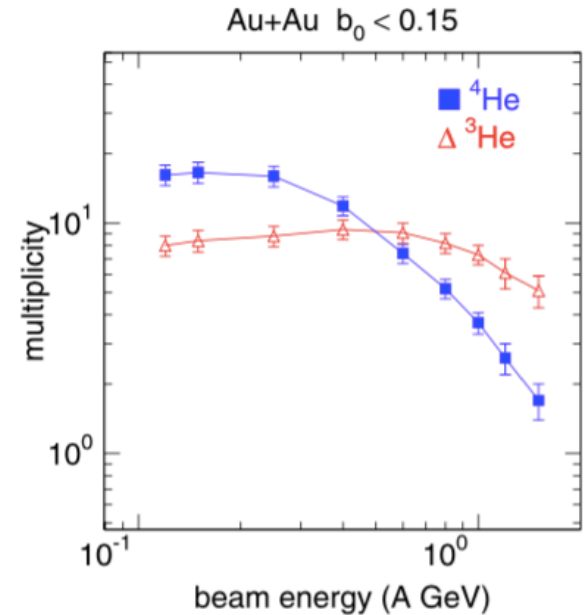


Fig. 43.  ${}^3\text{He}$  and  ${}^4\text{He}$  multiplicities in central Au + Au collisions as function of the incident beam energy. The ‘crossing’ energy is  $0.5A$  GeV.

A simple perturbative coalescence model cannot explain this behavior.



# The fireball regime: droplet formation?

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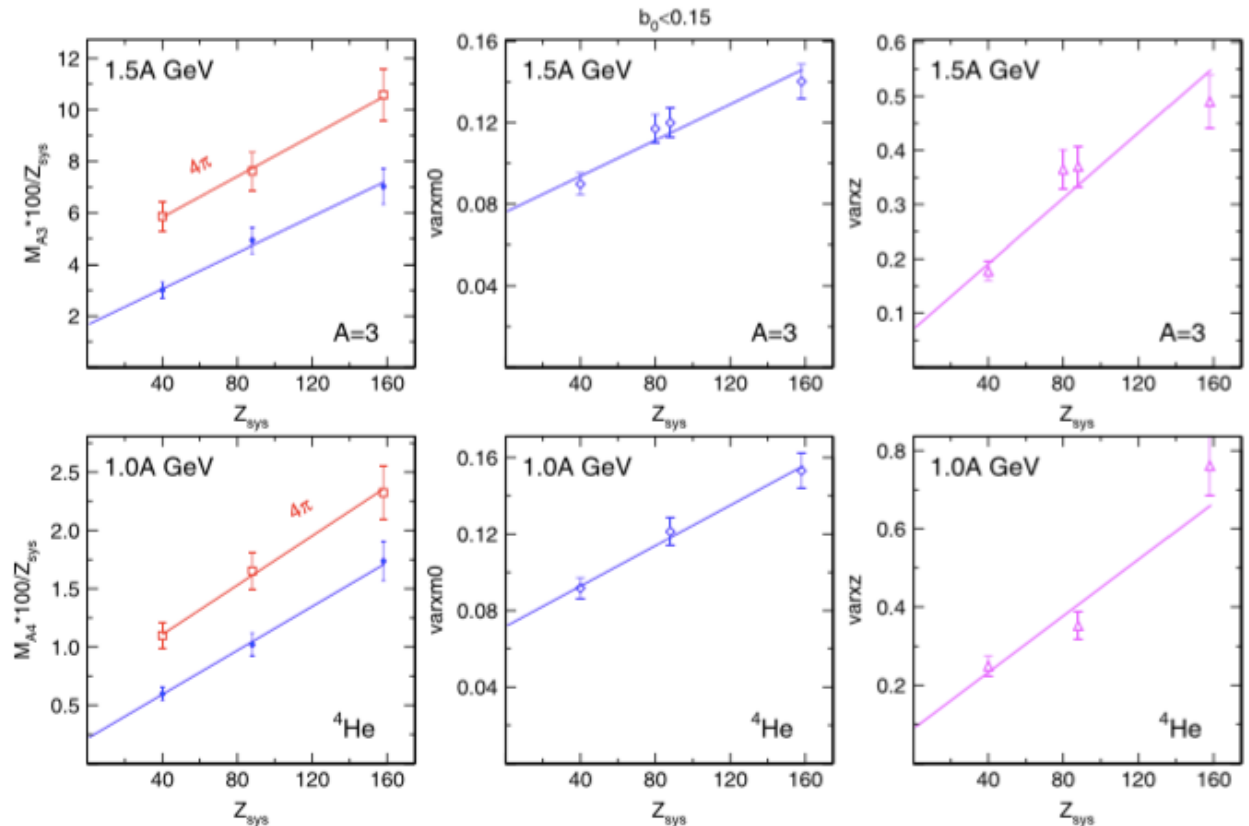


Fig. 48. System size dependences of symmetric central collisions at incident beam energies of 1.0A GeV (lower panels,  ${}^4\text{He}$  fragments) and 1.5A GeV (upper panels, average of  ${}^3\text{He}$  and  ${}^3\text{H}$  fragments). All straight lines are linear fits. Left panels: multiplicities (reduced to 100 protons). The data marked  $4\pi$  with open square symbols are  $4\pi$  integrated, while the closed symbols represent midrapidity yields (constrained to  $|y_{z0}| < 0.5$ ). Middle panels: variance  $\text{varxm0}$  of the scaled constrained transverse rapidity distributions ( $dn/dy_{xm0}$ ). Right panels: stopping  $\text{varxz}$ .



# The fireball regime: droplet formation?

An interpretation:

Increased stopping (right panels)  $\leftrightarrow$  increased compression.

Increasing constrained transverse rapidity variances  $varxm0$  (middle panels)  $\leftrightarrow$  increasing radial flow developed thereafter in the expansion phase coupled to increased cooling

('droplet formation')

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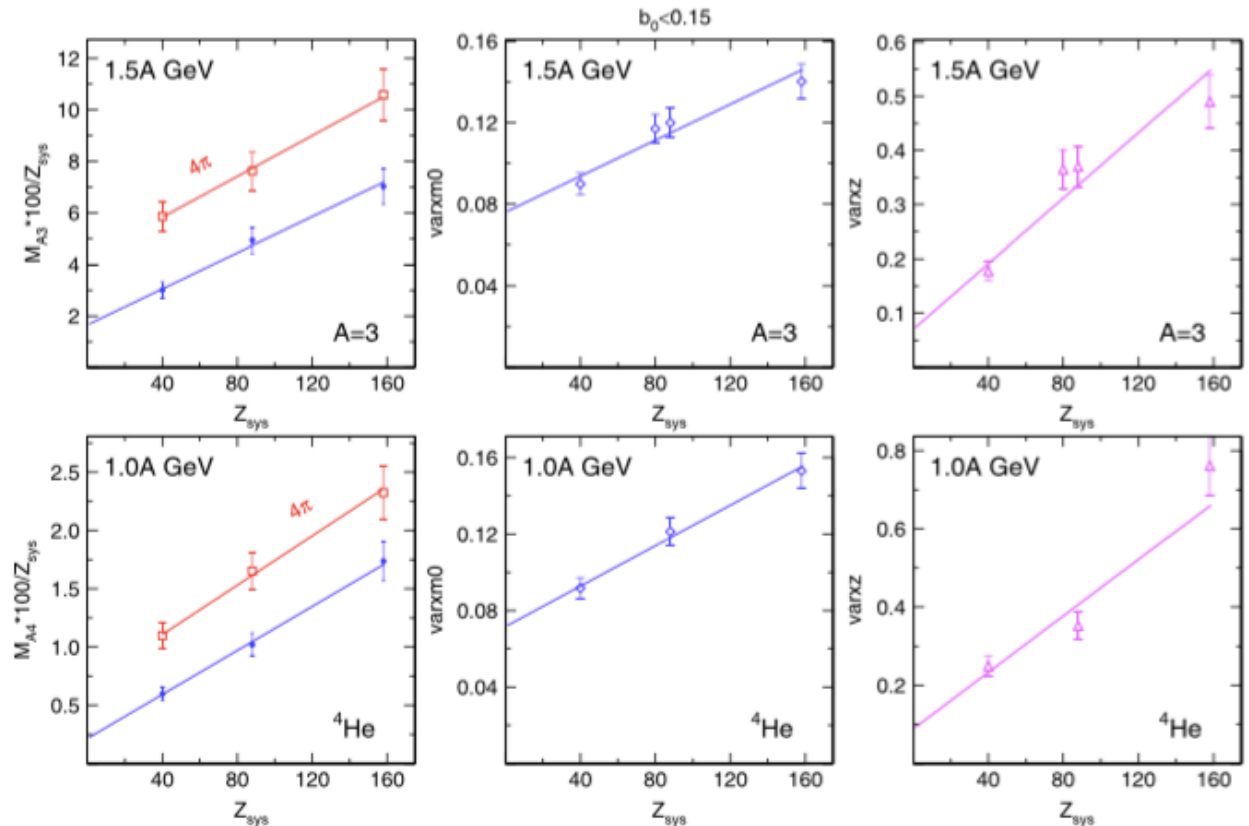


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# A new non-static approach: « sequential » FRIGA

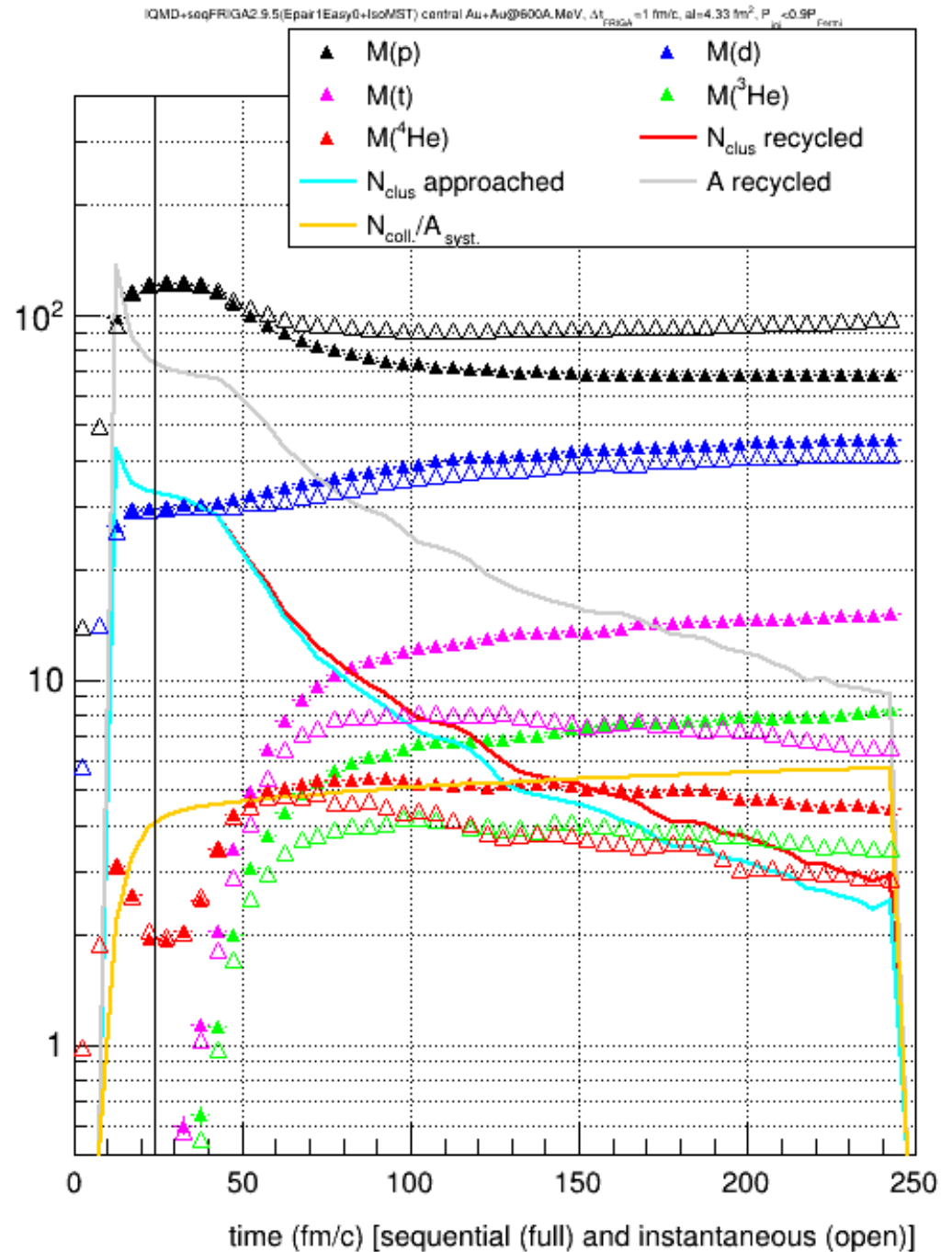
- **Start from a first time step of the collision (typically when the 2 nuclei start to collide):**  
Pre-detect a partition of clusters with FRIGA/MST
- **At each subsequent time step of the collision (typically every fm/c):**
  - 1) Let survive clusters of the previous time step that have not suffered from any **collision** and that have not been **approached** (at coalescence proximity) by an external hadron in the meanwhile.
  - 2) Otherwise set all its constituents as free
  - 3) Process with MST/FRIGA free hadrons only.
- **Follow the process until the cluster partition has stabilised**



# A new non-static approach: « sequential » FRIGA

Central

IQMD Au+Au 600A MeV:  
sequential FRIGA  
versus  
static FRIGA



\*: W. Reisdorf *et al.*, FOPI

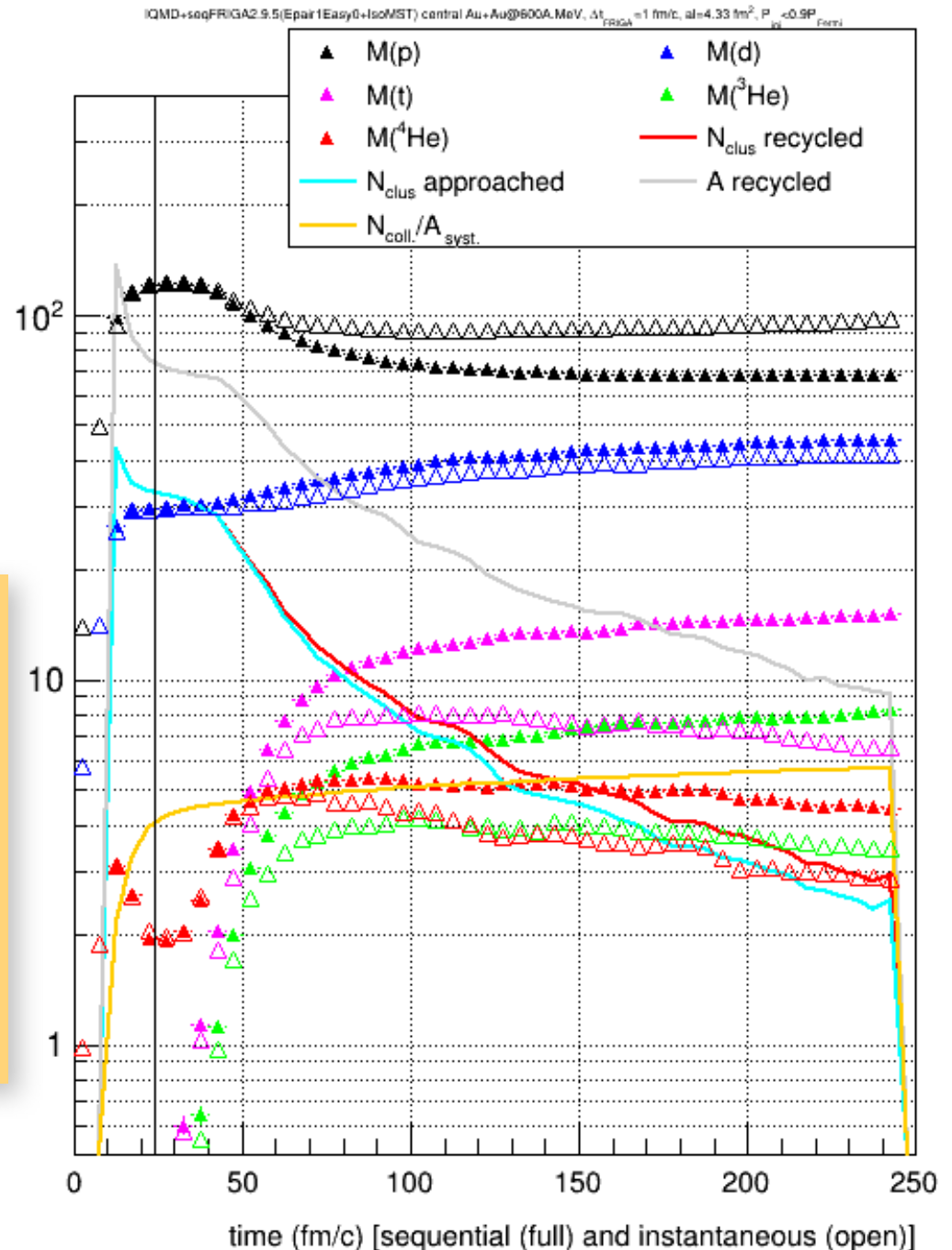
# A new non-static approach: « sequential » FRIGA

## Central

IQMD Au+Au 600A MeV:  
sequential FRIGA  
versus  
static FRIGA

⇒ Sequential strategy: light cluster yields grow over a long period of time, during the expansion of the fireball

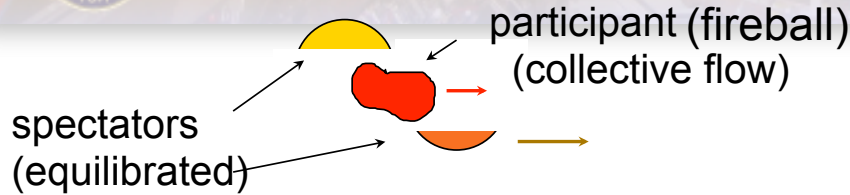
⇒ Reversely, with the static approach, the yields saturate earlier, at lower values.



\*: W. Reisdorf at al., FOPI



# A new non-static approach: « sequential » FRIGA



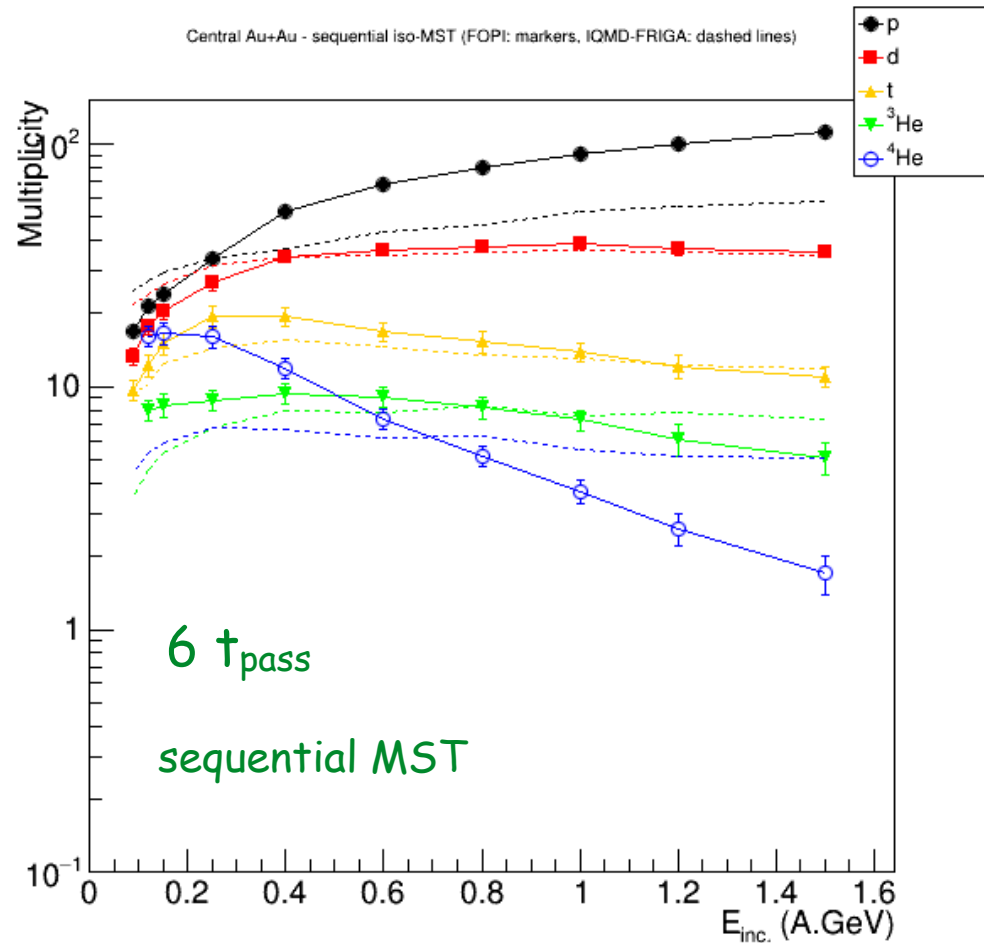
Central

Au+Au:

IQMD-FRIGA (dashed lines)

vs

FOPI data (markers)\*

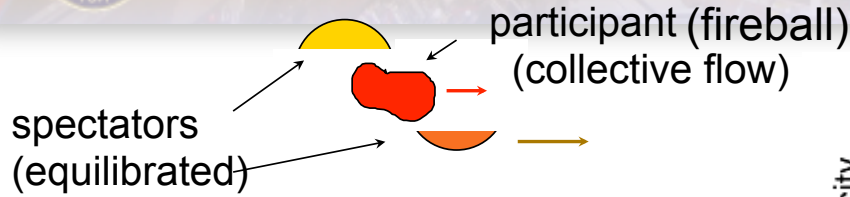


6  $t_{pass}$   
sequential MST

\*: W. Reisdorf et al., FOPI Collaboration / Nuclear Physics A 848 (2010) 366–427



# A new non-static approach: « sequential » FRIGA



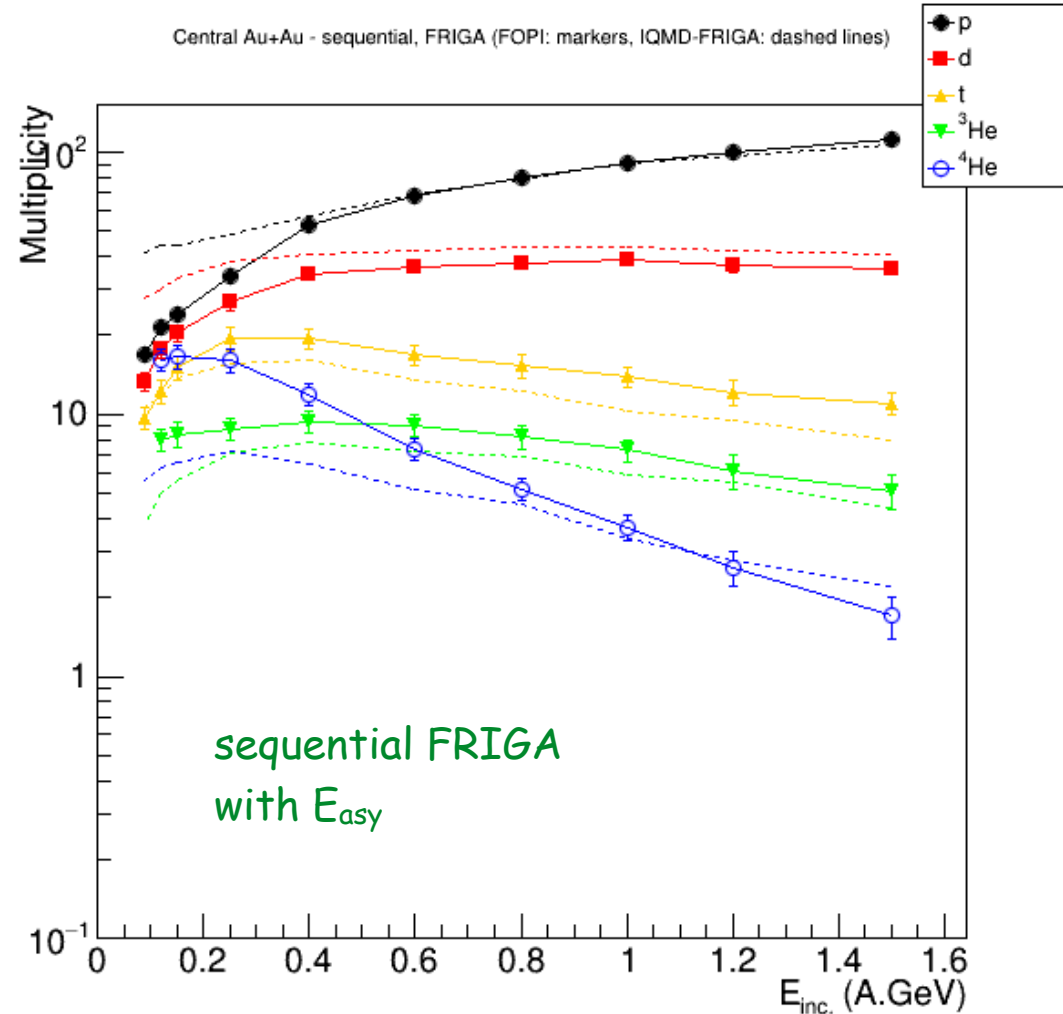
Central

Au+Au:

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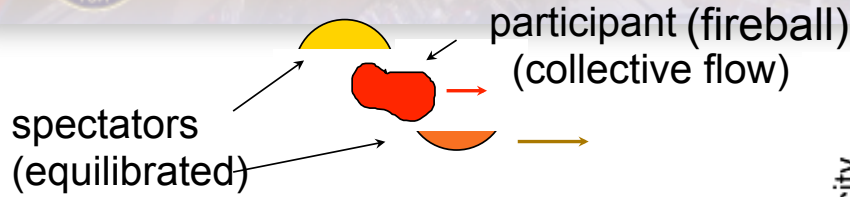
FOPI data (markers)\*



\*: W. Reisdorf et al., FOPI Collaboration / Nuclear Physics A 848 (2010) 366–427



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Au+Au:

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vs

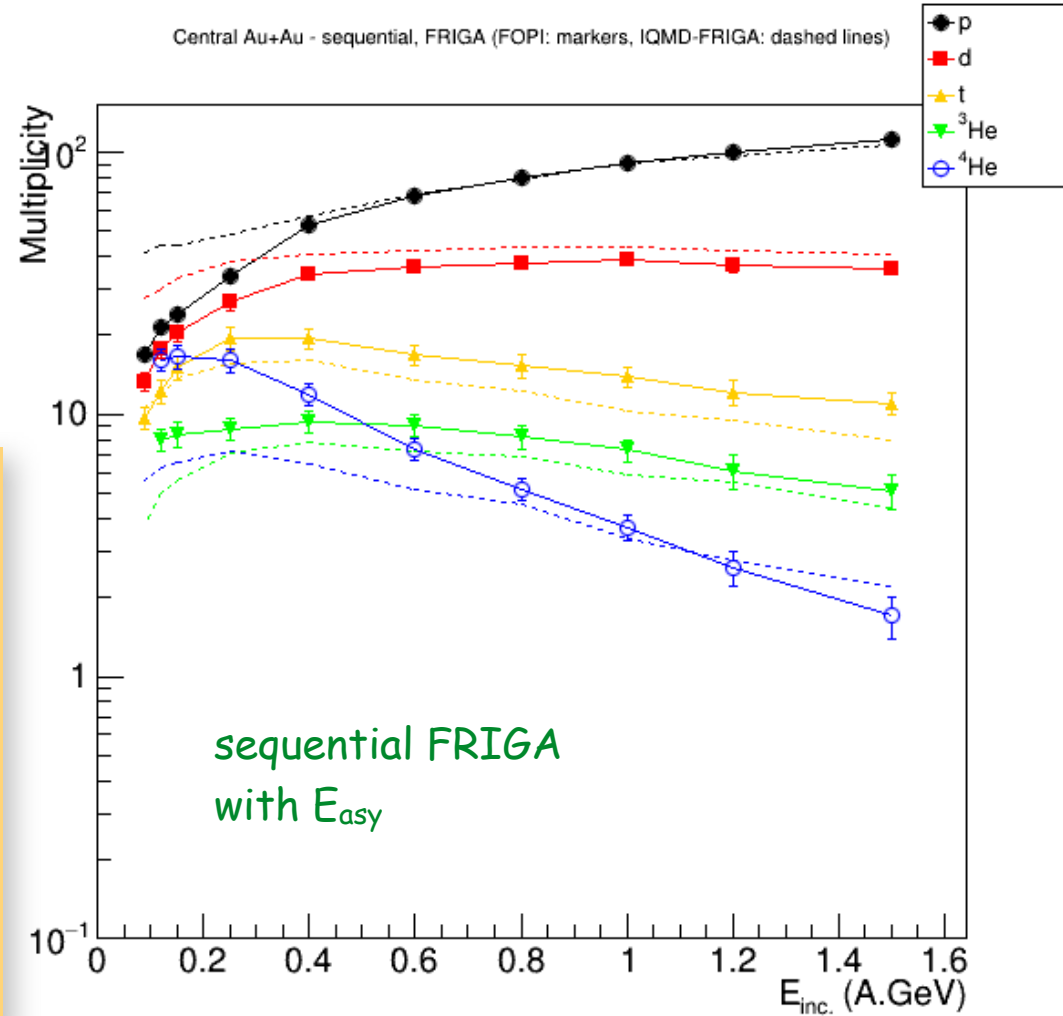
FOPI data (markers)\*

⇒ Sequential strategy: exhibit light cluster yields in much better agreement with experimental data

⇒ The FRIGA approach remains better than the simple coalescence method.

⇒ Still discrepancies at the lowest incident energies: d yields too large, 4He yields too small.

⇒ Extra coalescence channels with unusually large cross-sections to implement:  $d+d \rightarrow 4\text{He}$ ,  $n+3\text{He} \rightarrow 4\text{He}$



\*: W. Reisdorf et al., FOPI Collaboration / Nuclear Physics A 848 (2010) 366–427



# Summary and perspectives



# Summary and perspectives

## Summary:

- ❖ Supplying FRIGA with a more precise description of nuclei binding energy at abnormal density allows promising, realistic predictions of absolute isotope yields, and hypernuclei.
- ❖ The **clusterisation time** has a strong influence on the heavy hypernucleus yields and momentum distributions.
- ❖ The **secondary decays** and **structure effects** should not be neglected.
- ❖ Whereas the early clustering with FRIGA gives good results in the spectator / intermediate energy regime, the **fireball regime needs a relatively longer time to pre-form clusters (droplets?)**.



# Summary and perspectives

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- ❖ Whereas the early clustering with FRIGA gives good results in the spectator / intermediate energy regime, the **fireball regime needs a relatively longer time to pre-form clusters (droplets?)**.

## New developments:

- ❖ **Sequential/Hot clustering**: allow clustering to be done all along the expansion phase coupled to increased cooling.
- ❖ Promising description of light cluster yields in the fireball regime at SIS energies.
- ❖ Still some refinements to implement
  - ❖ regarding alpha particles
  - ❖ so far, or clusters do not interact as state of their own with the rest of the system during the dynamical development; necessary?
- ❖ For doing this: compulsory in transport models: **a stable nuclear matter + enough fluctuations**.
- ❖ The fragment formation in the fireball at SIS energies (overclustering) = opening a field of understanding of clustering in ultra-relativistic energies.





# Some successful applications at intermediate energies

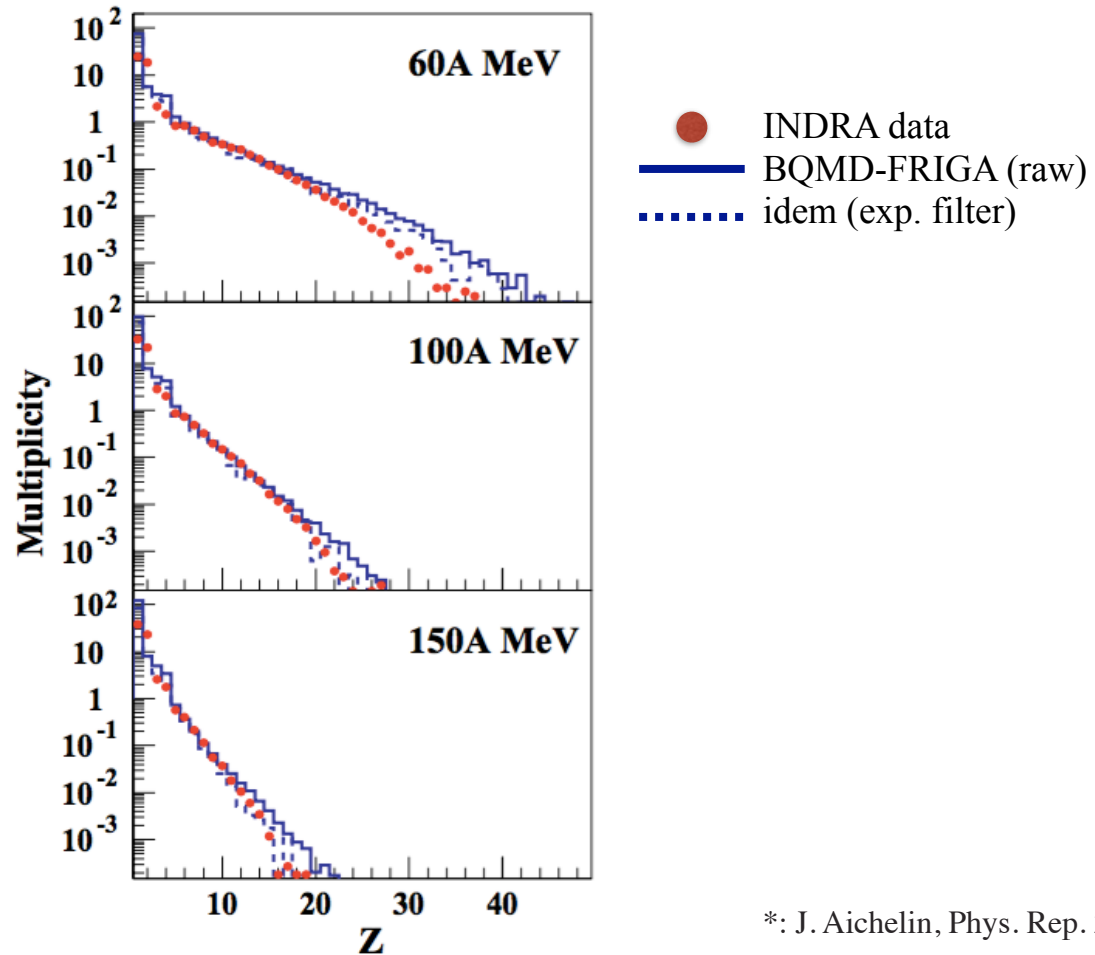
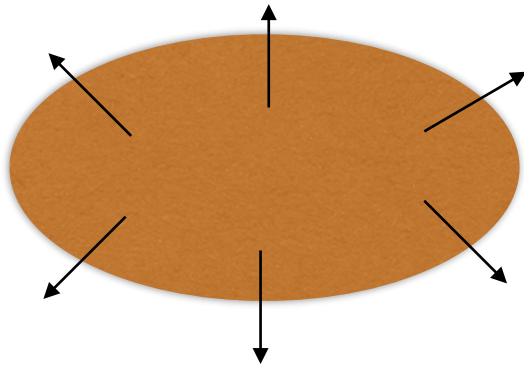
INDRA systems

K. Zbiri et al., PHYSICAL REVIEW C **75**, 034612 (2007)

BQMD\*+FRIGA

**central Au+Au**

( $t = 2 t_{\text{pass}}$ )



\*: J. Aichelin, Phys. Rep. **202**, 233 (1991).



# Some successful applications at intermediate energies

INDRA systems

A. Le Fèvre and J. Aichelin - PRL 100, 042701 (2008)

BQMD\*+FRIGA

**Au+Au**

( $t = 2 t_{\text{pass}}$ )

=> **Bimodality\*\* =**  
**a mechanical instability**  
**(critical phenomenon)**

fragment size asymmetry

$$a_2 = (Z_1 - Z_2) / (Z_1 + Z_2)$$

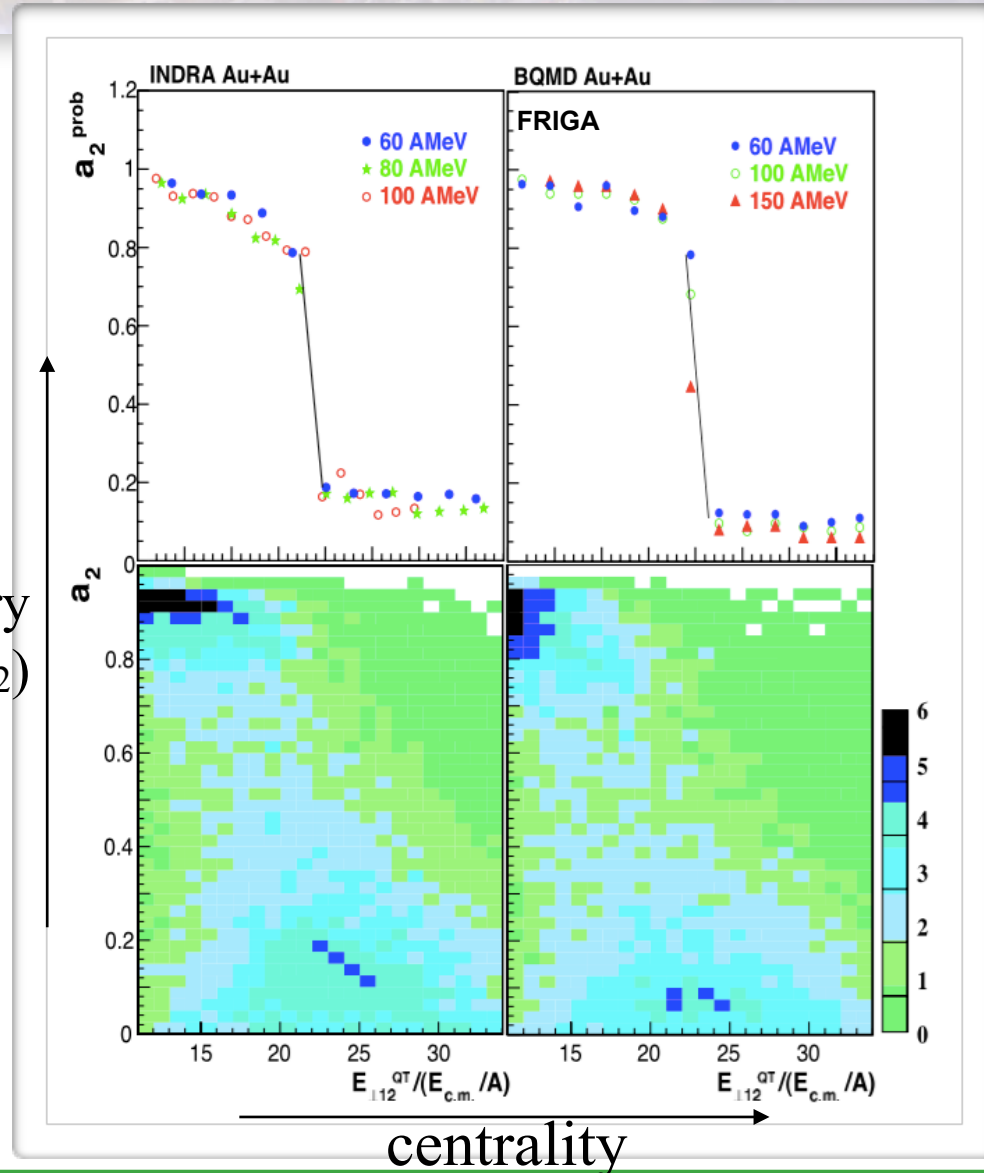
=> **sufficient fluctuations**  
**necessary in transport models**

\*: J. Aichelin, Phys. Rep. **202**, 233 (1991).

\*\* : also found in Xe+Sn system in

*M. Pichon et al,*

*INDRA, ALADIN Coll. / Nuclear Physics A 779 (2006) 267–296*



centrality