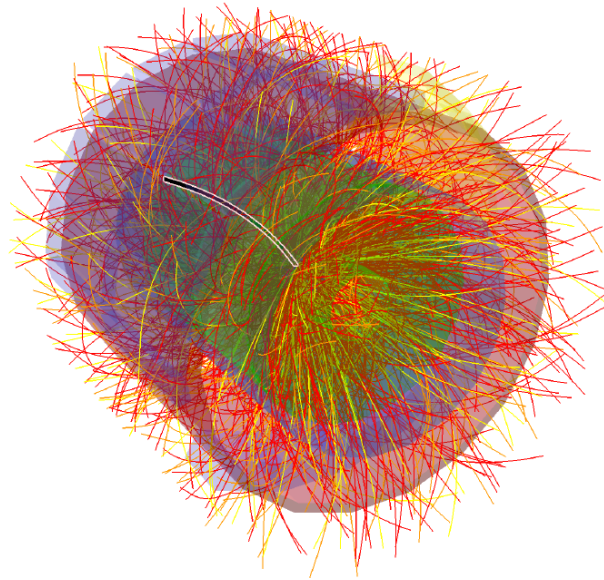


# Statistical hadron production



11.02.2019

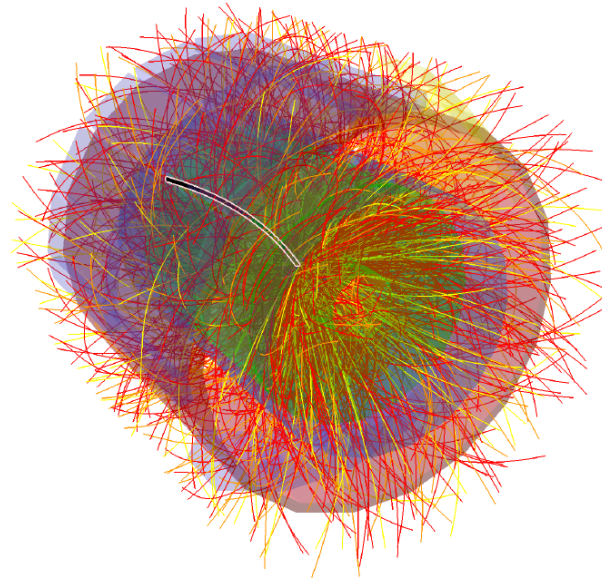
EMMI Workshop

Probing dense baryonic matter with hadrons: Status and Perspective  
GSI

**Benjamin Dönigus**

Institut für Kernphysik  
Goethe Universität Frankfurt

# Statistical hadron production



Thanks to  
A. Andronic  
P. Braun-Munzinger  
M. Lorenz  
V. Vovchenko

11.02.2019

EMMI Workshop

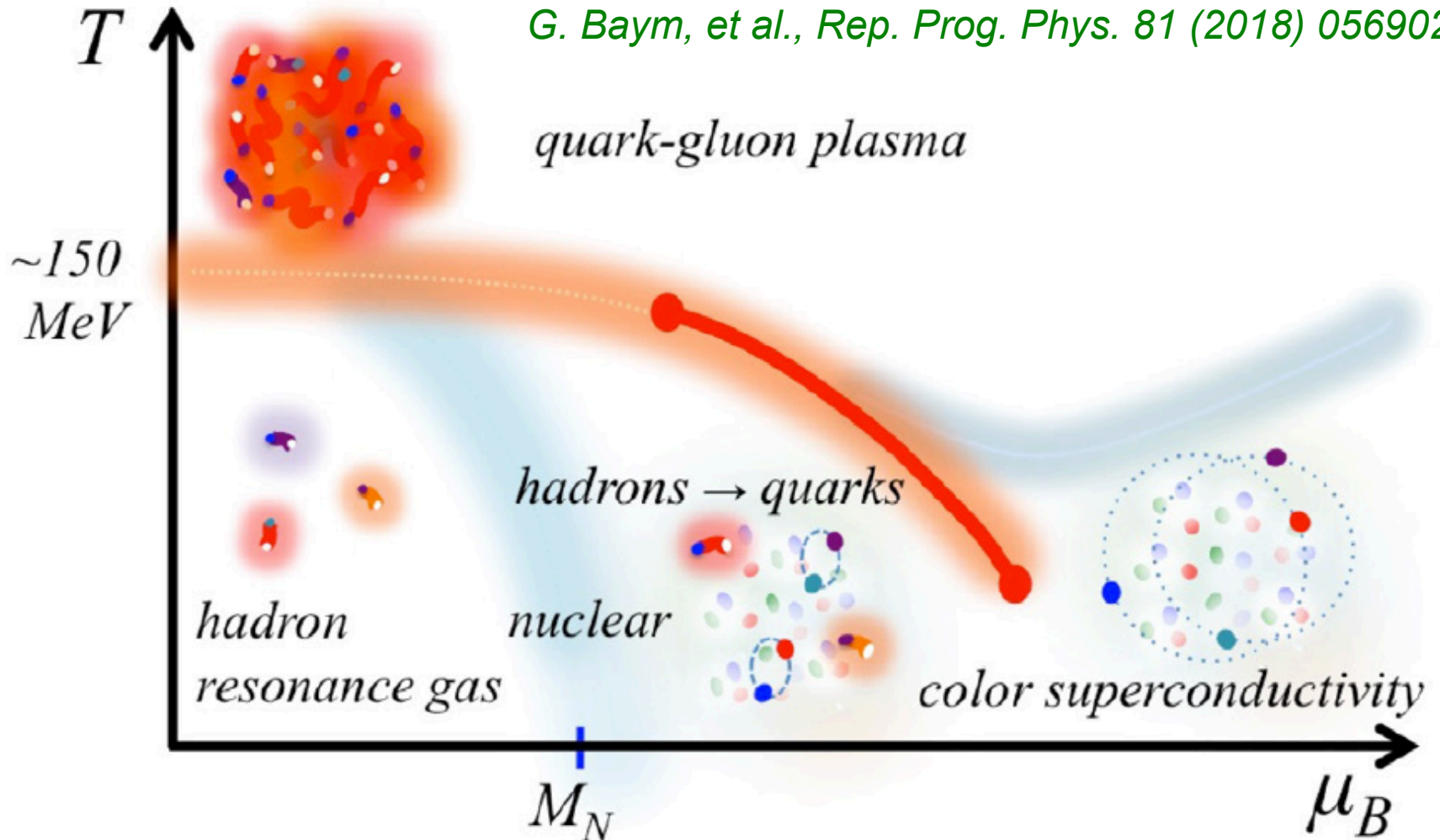
Probing dense baryonic matter with hadrons: Status and Perspective  
GSI

# Outline

- Introduction
- From LHC to HADES
- Systematics
- Light nuclei and exotica at FAIR
- Summary

# Phase diagram

*G. Baym, et al., Rep. Prog. Phys. 81 (2018) 056902*

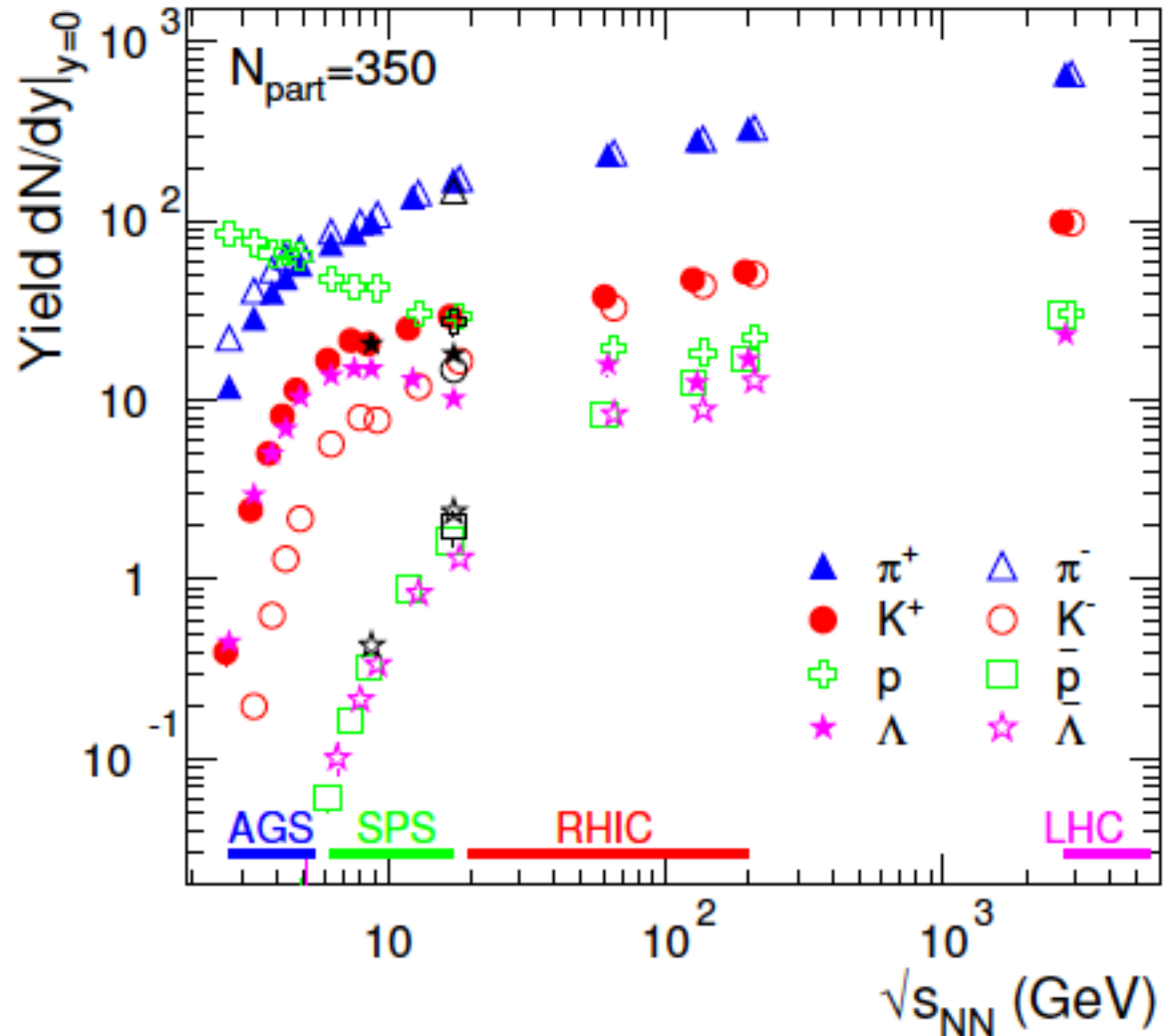


Phase diagram accessible through measurement of hadron production yields

# Hadron production yields

A. Andronic

- Large amount of particles measured, many of those newly produced ( $E = mc^2$ )
- Large variety of hadrons
  - $\pi^\pm$  ( $u\bar{d}$ ,  $d\bar{u}$ ),  $m=140$  MeV
  - $K^\pm$  ( $u\bar{s}$ ,  $\bar{u}s$ ),  $m=494$  MeV
  - $p$  ( $uud$ ),  $m=938$  MeV
  - $\Lambda$  ( $uds$ ),  $m=1116$  MeV
  - also:  $\Xi(dss)$ ,  $\Omega(sss)$ ...
- Three decades of energy corresponding to three decades of experimental investigation



# Thermal model

Simplest approach using a grand-canonical ensemble and its corresponding partition function for specie  $i$  (pions, kaons, protons, etc.):

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$g_i = (2J_i + 1)$  spin degeneracy factor;  $T$  temperature;

$E_i = \sqrt{p^2 + m_i^2}$  total energy; + for fermions – for bosons

$\mu_i = \mu_B B_i + \mu_{I_3} I_{3i} + \mu_S S_i + \mu_C C_i$  chemical potentials

$\mu$  ensure conservation (on average) of quantum numbers, fixed by “initial conditions”

i) isospin:  $V_{cons} \sum_i n_i I_{3i} = I_3^{tot}$ , with  $V_{cons} = N_B^{tot} / \sum_i n_i B_i$   
 $I_3^{tot}$ ,  $N_B^{tot}$  isospin and baryon number of the system ( $\simeq 0$  at high energies)

ii) strangeness:  $\sum_i n_i S_i = 0$

iii) charm:  $\sum_i n_i C_i = 0$ .

# Thermal model: yields

Particle densities of each species can be extracted from the partition function as

$$n_i = N_i/V = -\frac{T}{V} \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}$$

In practise the thermal model codes usually use a particle listing (PDG based) as input and compare the yields with the „best“ values of  $T, \mu_B$  and  $V$ , by minimising

$$\chi^2 = \sum_i \frac{(N_i^{exp} - N_i^{therm})^2}{\sigma_i^2}$$

$N_i$ : hadron yield,  $\sigma_i$ : experimental uncertainty (statistical and systematical)

→  $(T, \mu_B, V)$  tests chemical freeze-out (chemical equilibrium)

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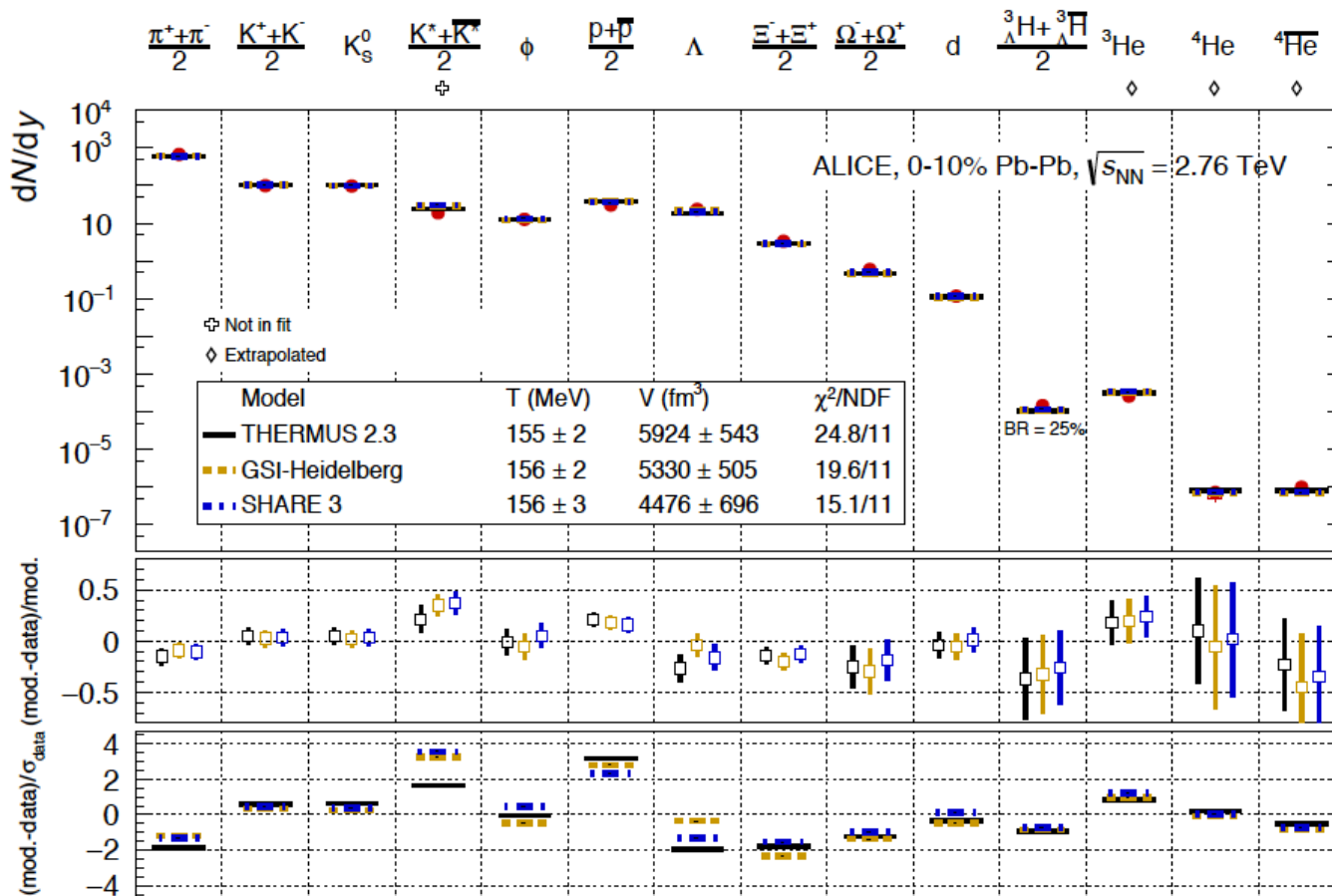
→  $(T, \mu_B, V)$  tests chemical freeze-out (chemical equilibrium)

Particle listing used in the model is one possible systematic uncertainty of the codes → other important ones in the following



# Thermal model fits

THERMUS: S. Wheaton, et al., CPC 180, 84 (2009)  
 GSI-Heidelberg: A. Andronic, et al., PLB 697, 203 (2011); PLB 673, 142 (2009) 142  
 SHARE3: G. Torrieri, et al., CPC 167, 229 (2005); CPC 175, 635 (2006); CPC 185, 2056 (2014)

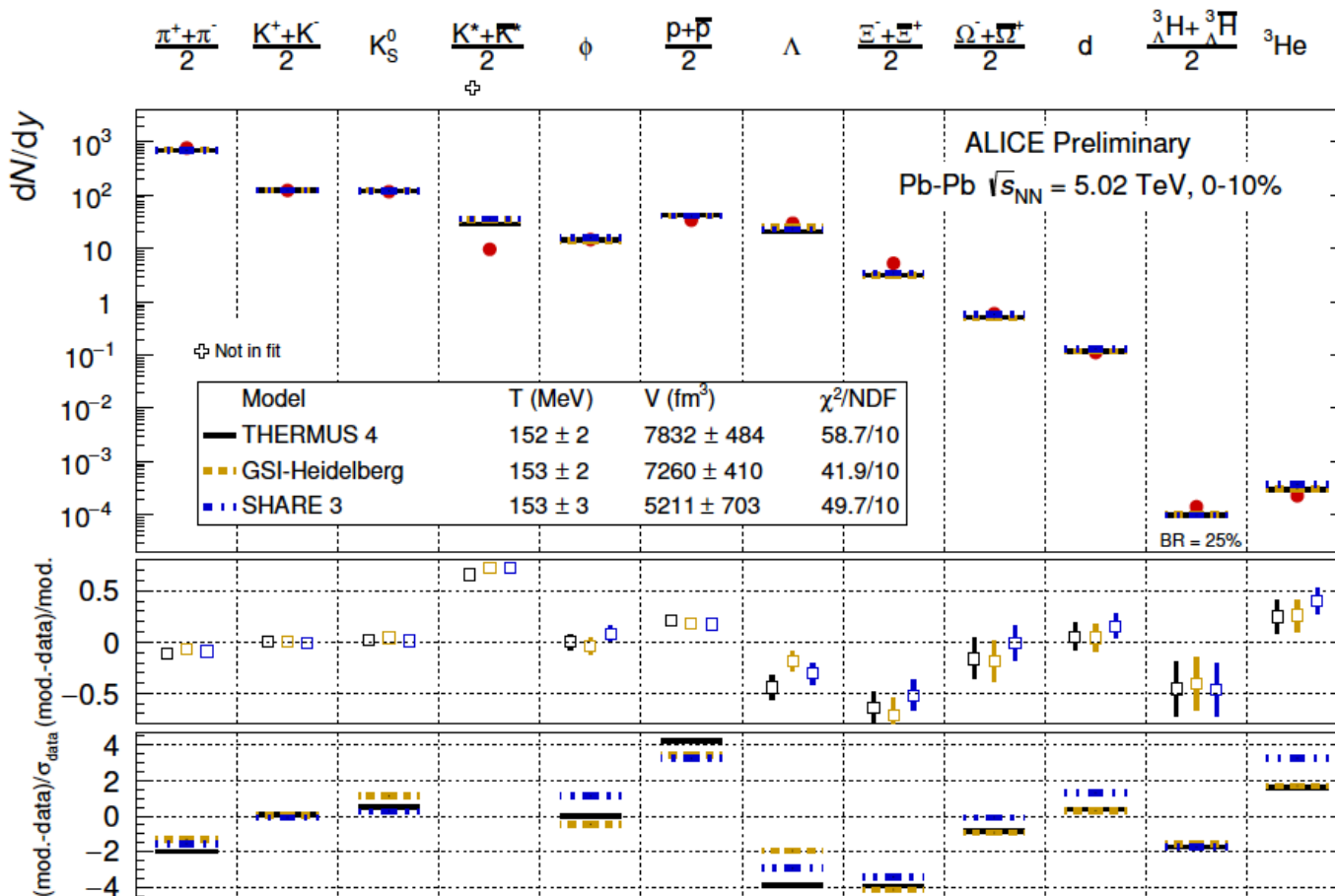


- Different models describe particle yields including light (hyper-)nuclei well with  $T_{ch}$  of about 156 MeV
- Including nuclei in the fit causes no significant change in  $T_{ch}$

ALICE Collaboration, arXiv:1710.07531,  
 NPA 971, 1 (2018)

# Thermal model fits

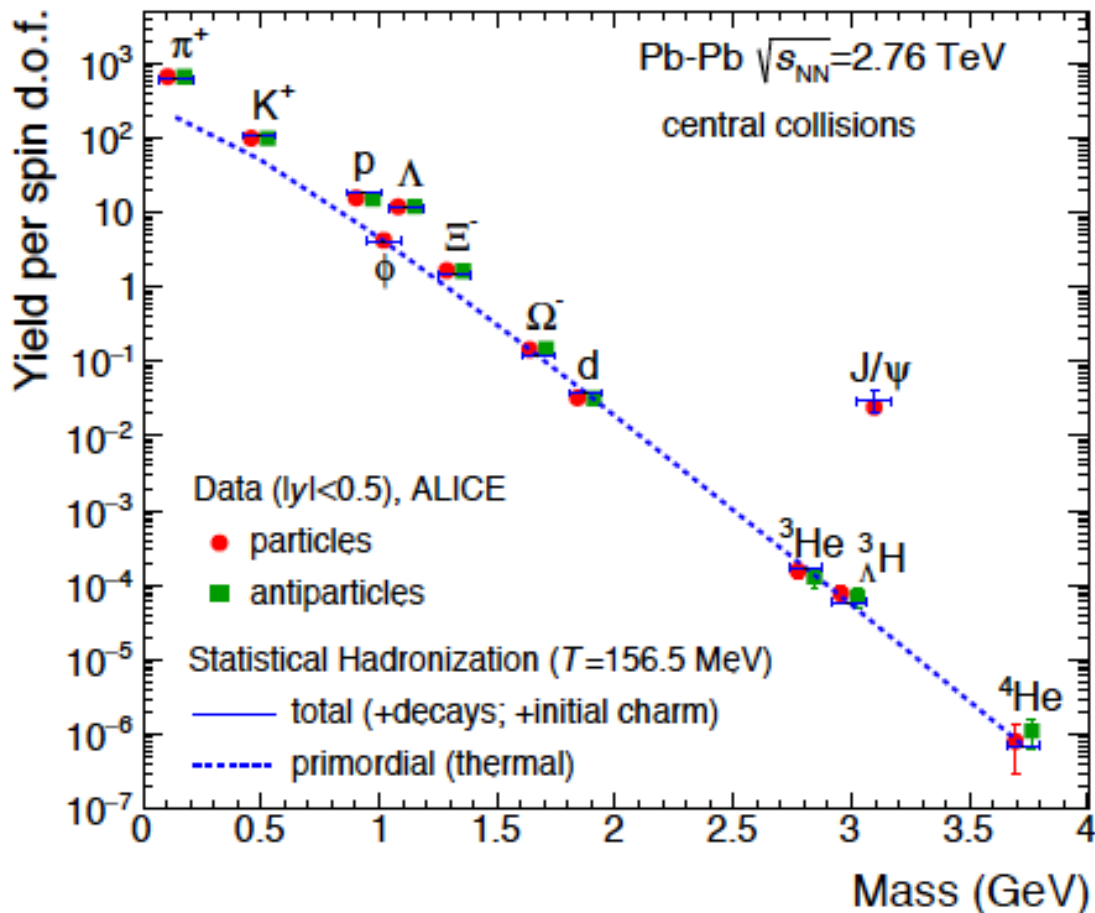
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 SHARE3: G. Torrieri, et al., CPC 167, 229 (2005); CPC 175, 635 (2006); CPC 185, 2056 (2014)



- Different models describe particle yields including light (hyper-)nuclei slightly worse at higher collision energy with a  $T_{ch}$  of about 153 MeV
- Including nuclei in the fit causes no significant change in  $T_{ch}$

# Fits: different view

## A. Andronic



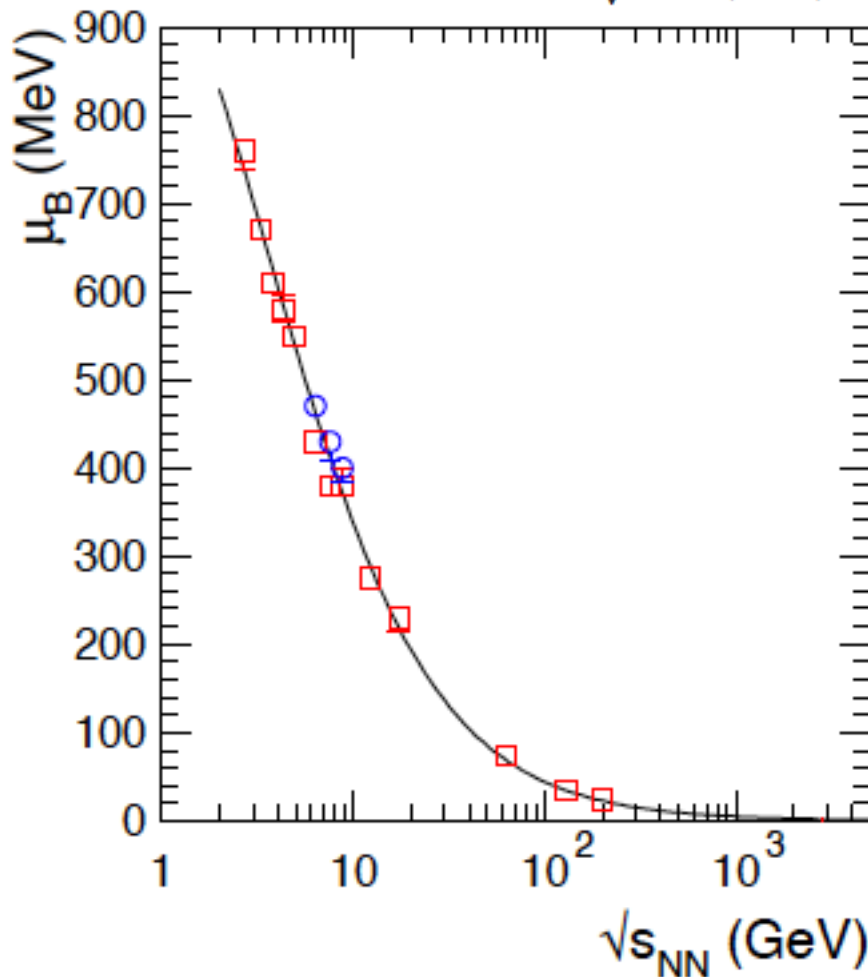
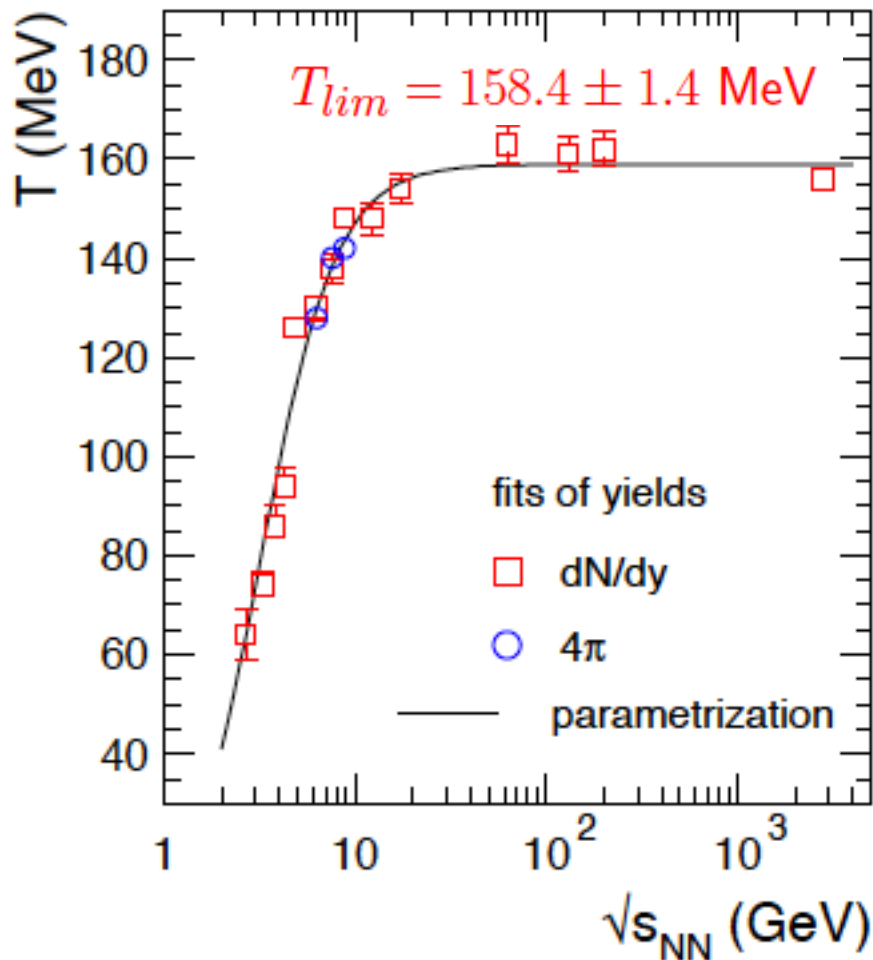
- Excellent agreement over 9 orders of magnitude
- Contribution from resonances significant and depending on the particle type
- Fit of  $\phi$ ,  $\Omega$ ,  $d$ ,  ${}^3\text{He}$ ,  ${}^3\Lambda\text{H}$ ,  ${}^4\text{He}$ :  
 $T_{ch} = 156.0 \pm 2.5$  MeV  
( $\chi^2/\text{ndf} = 7.4/8$ )
- Fit of nuclei ( $d$ ,  ${}^3\text{He}$ ,  ${}^4\text{He}$ ):  
 $T_{ch} = 159 \pm 5$  MeV
- 3-4 MeV upper bound of syst. uncertainty due to hadron spectrum

charm quarks, out of chemical equilibrium, undergo statistical hadronization

# Parameterization

$$T = T_{lim} \frac{1}{1 + \exp(2.60 - \ln(\sqrt{s_{NN}}(\text{GeV}))/0.45)}$$

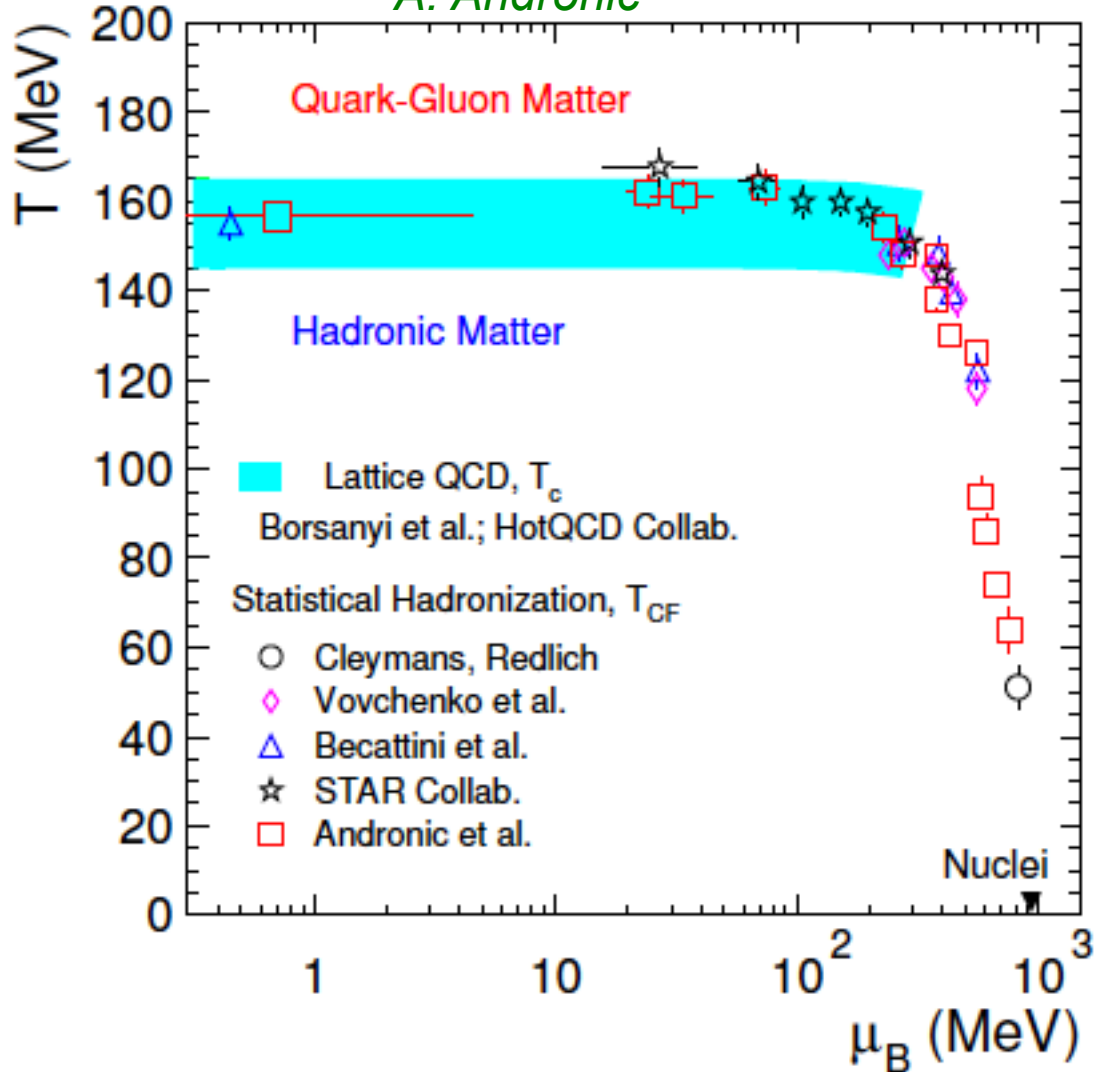
$$\mu_B[\text{MeV}] = \frac{1307.5}{1 + 0.288 \sqrt{s_{NN}}(\text{GeV})}$$



A. Andronic, et al., Phys. Lett. B 673 (2009) 142, with updates

# Fits $\rightarrow$ phase diagram

A. Andronic



- At low  $\mu_B$  the extracted freeze-out temperatures, and thus  $T_{lim}$ , coincide with the pseudo-critical temperature extracted through lattice QCD

Lattice QCD:

*Borsanyi et al., JHEP 1009 (2010) 073, JHEP 1208 (2012) 053*  
*HotQCD, PRD 90 (2014) 094503, PRD 83 (2011) 014504*

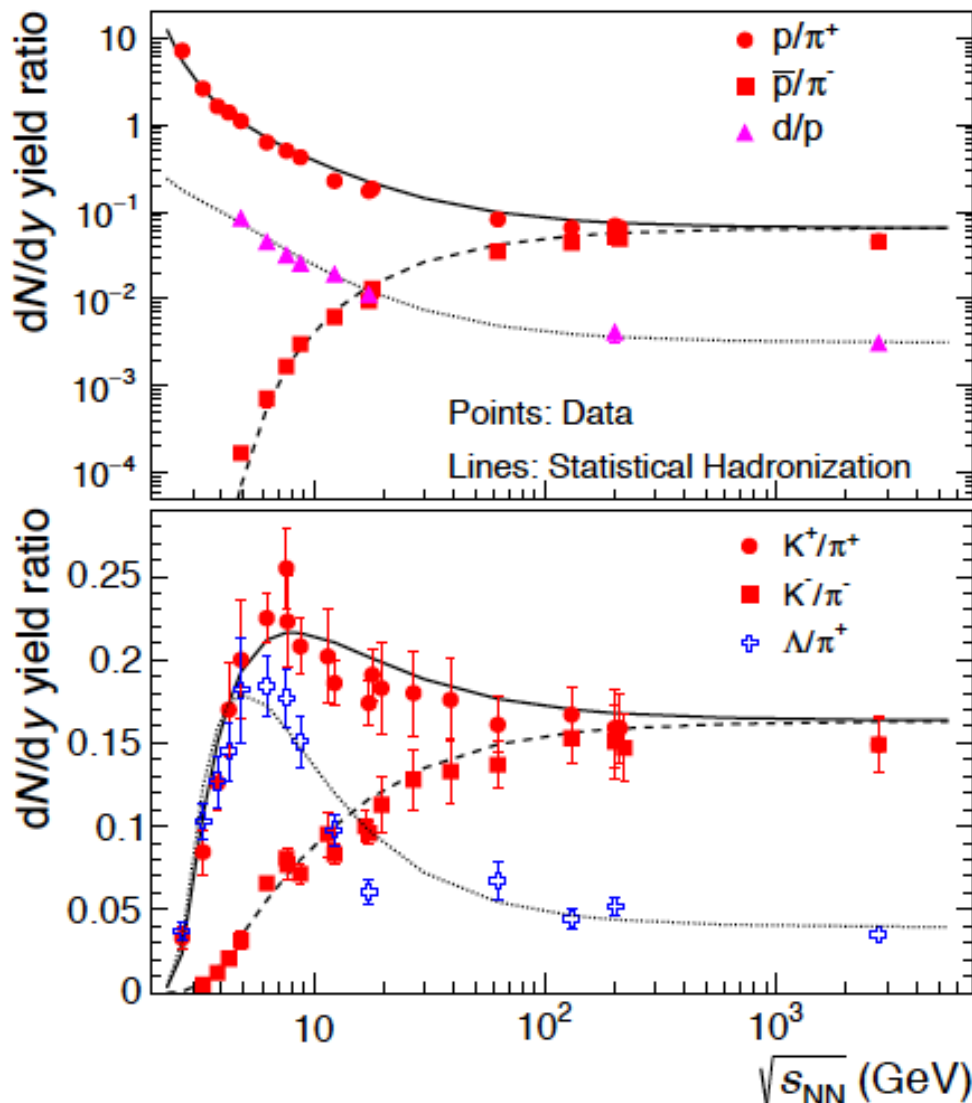
Statistical model:

*Cleymans, Redlich, PRC 59 (1999) 1663*  
*Vovchenko et al., PRC 93 (2016) 064906*  
*Becattini et al., PLB 764 (2017) 241*  
*STAR, PRC 96 (2017) 044904*

- In the CBM regime (high- $\mu_B$ ) one faces a much stronger energy dependence of  $T$

# Ratios

## A. Andronic



Data:

AGS: E895, E864, E866, E917, E877

SPS: NA49, NA44

RHIC: STAR, BRAHMS

LHC: ALICE

NB: no contribution from weak decays

„structures" described by SHM  
determined by  $T$ ,  $\mu_B$  and  
strangeness conservation

„features" are not necessarily (1st  
order) phase transition signatures  
(or chiral symmetry restoration)

# Proton anomaly

Possible explanations:

- incomplete hadron spectrum
- chemical non-equilibrium at freeze-out
- modification of hadron abundancies
- separate freeze-out temperatures for strange and non-strange hadrons
- excluded volume interactions
- energy dependent Breit-Wigner  $T = 155 \pm 1.7$  MeV
- replace Breit-Wigner by phase shift analysis  
 $T = 155.0$  MeV

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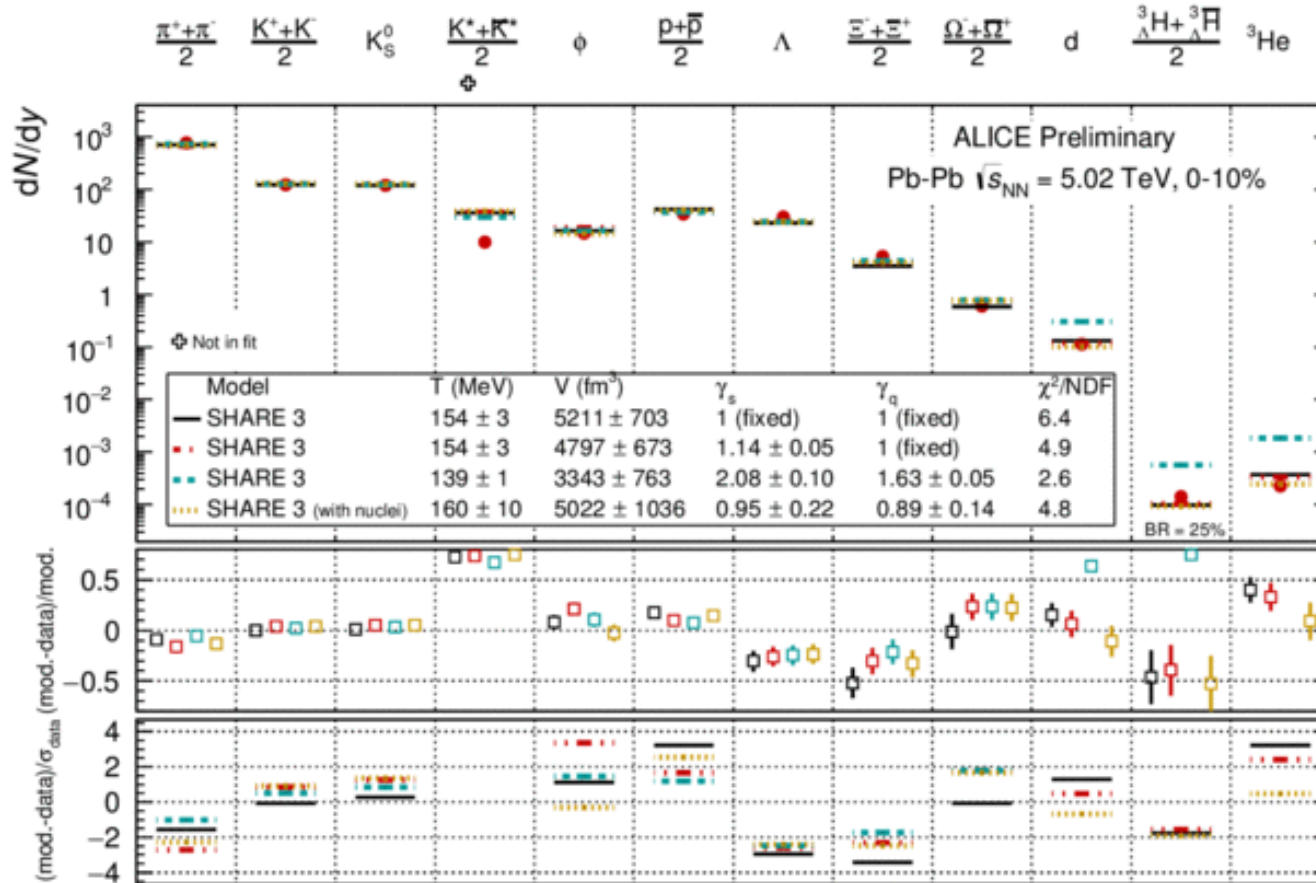
$T = 155.0$  MeV

→ All connected to systematic uncertainties of thermal model approaches



# Thermal model: SHARE

SHARE3: G. Torrieri, et al., CPC 167, 229 (2005); CPC 175, 635 (2006); CPC 185, 2056 (2014)

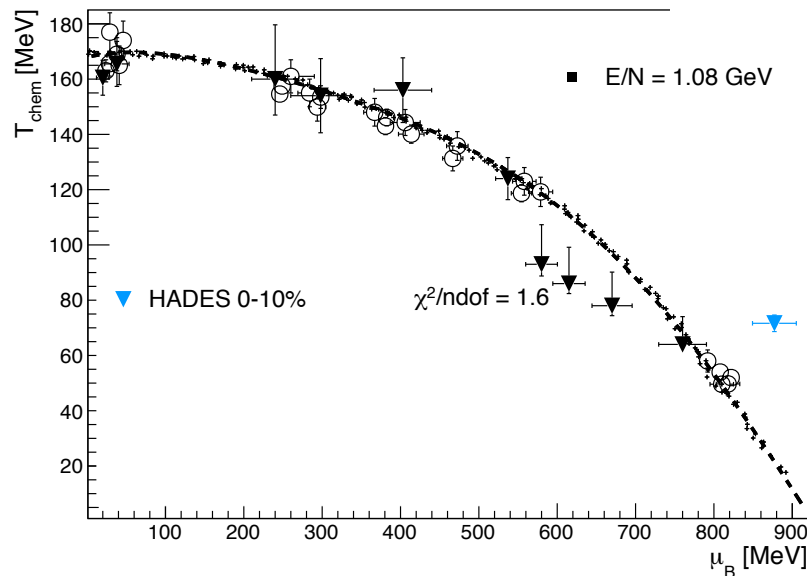
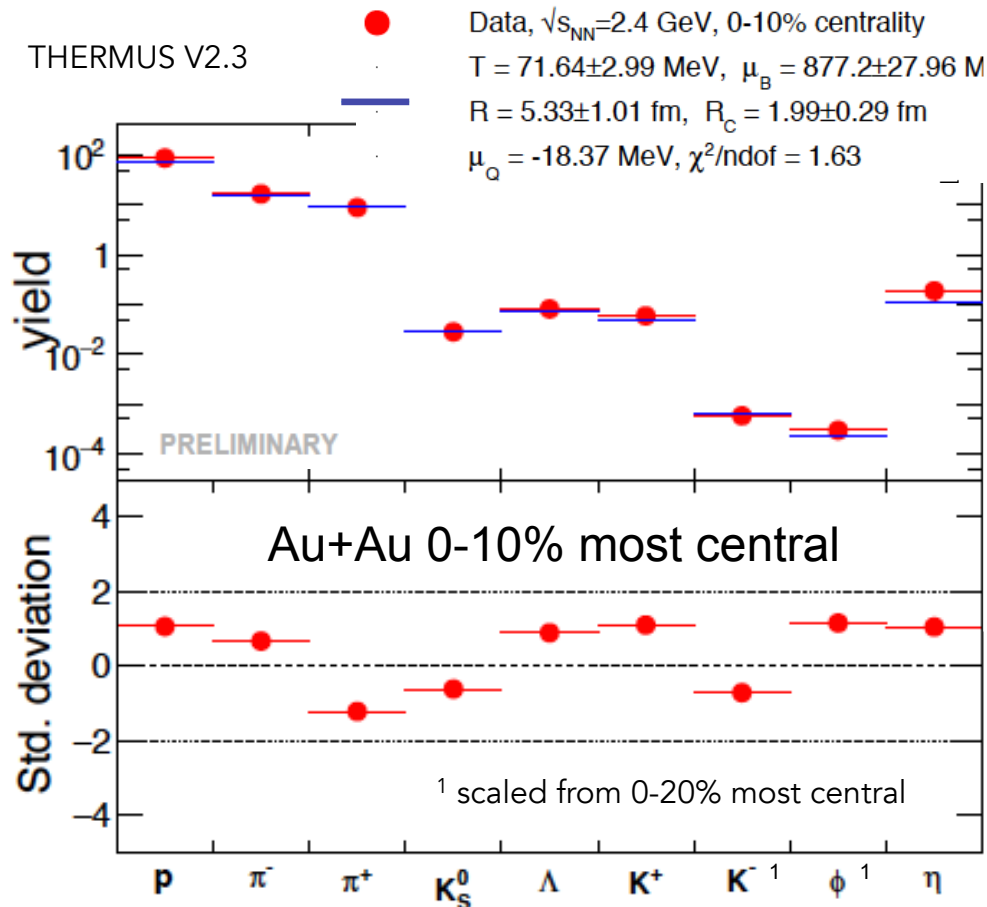


- Non-equilibrium thermal approach leads to better fit
- Observations similar to QM2014 results
- Including nuclei drives a non-equilibrium fit towards the equilibrium values

# HADES Au-Au

M. Lorenz

PRELIMINARY

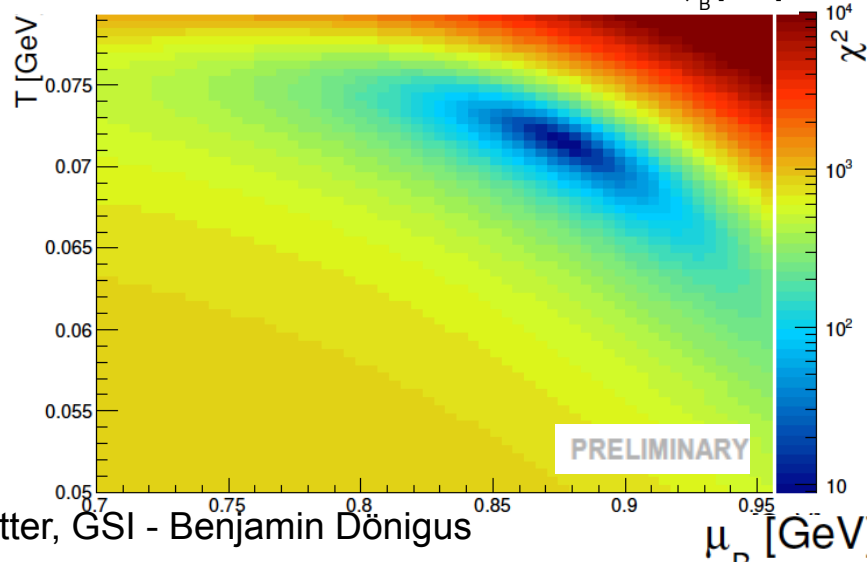
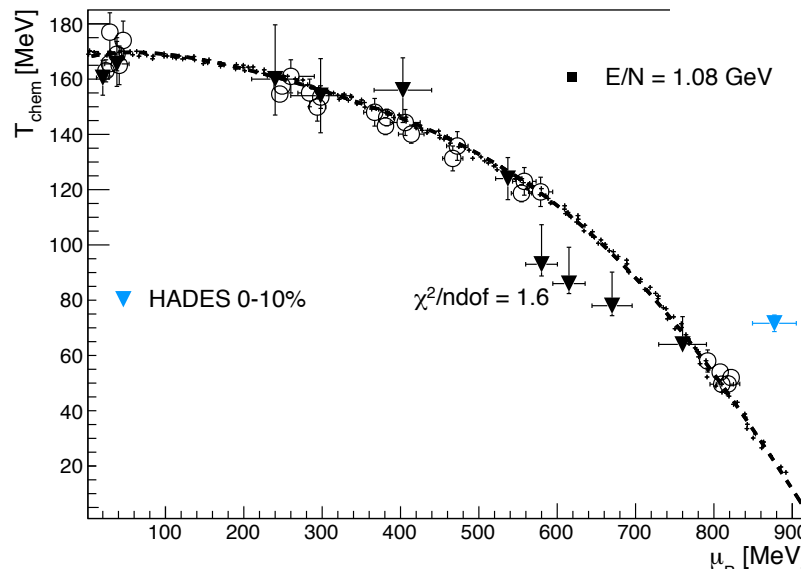
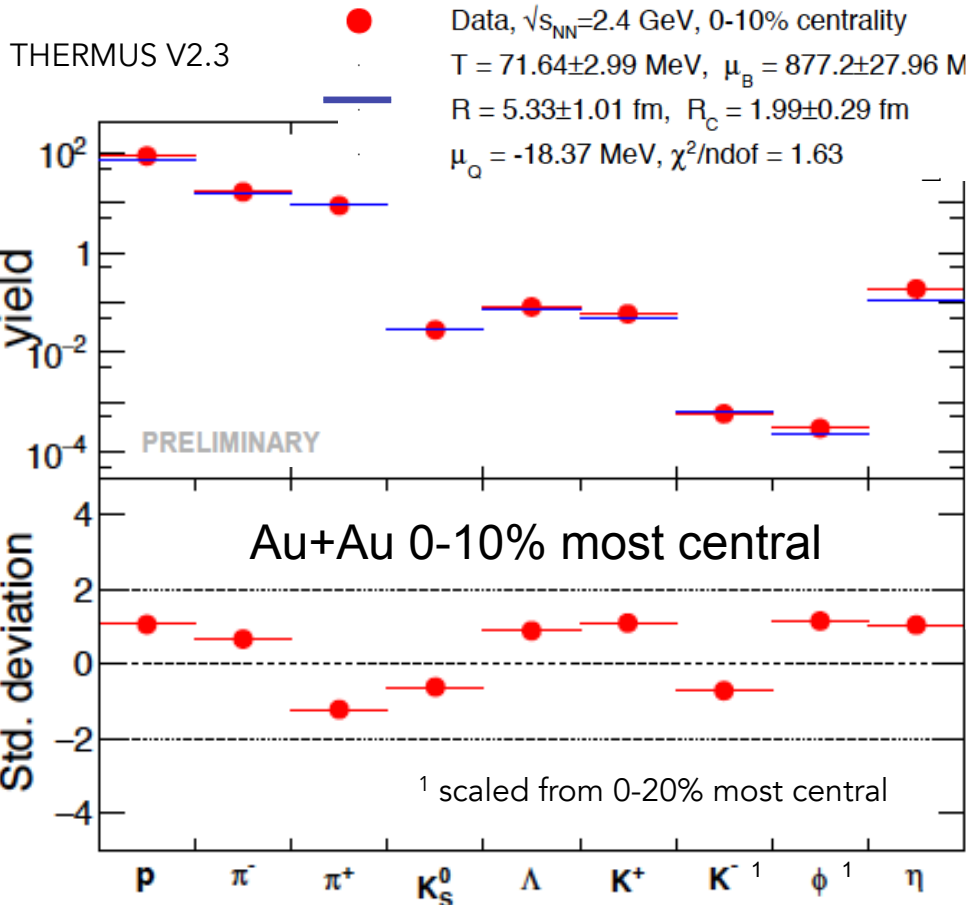


- Freeze-out point stays at higher  $T$  and  $\mu_B$  also for 0-10% most central events

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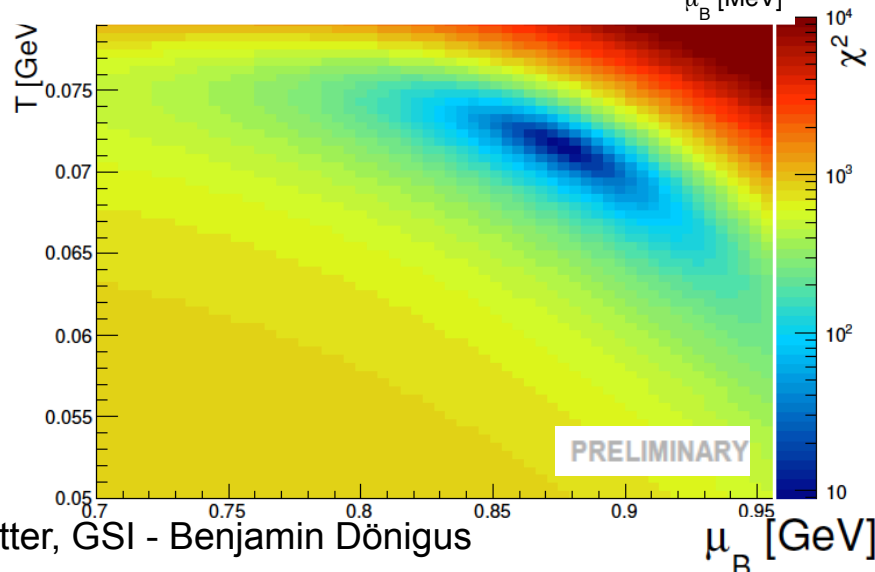
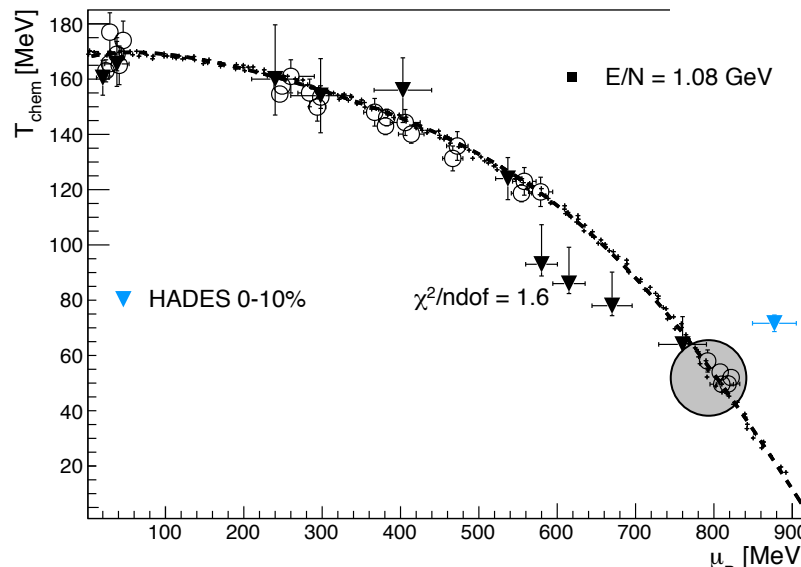
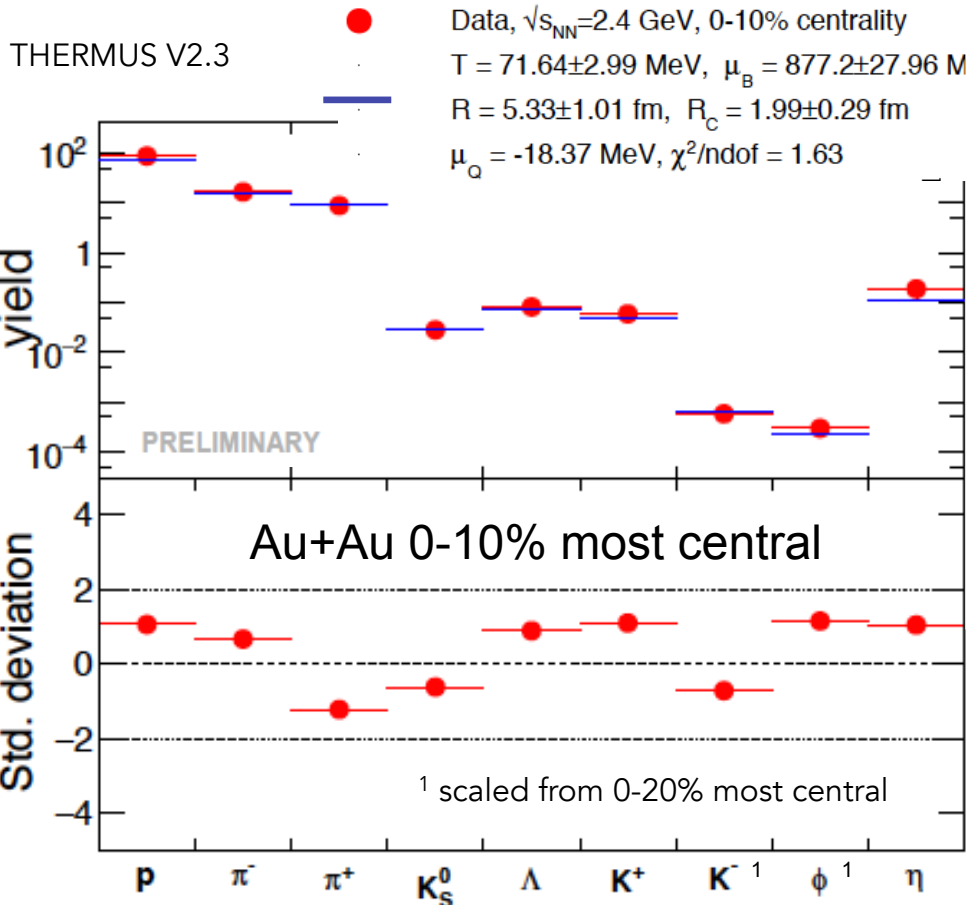


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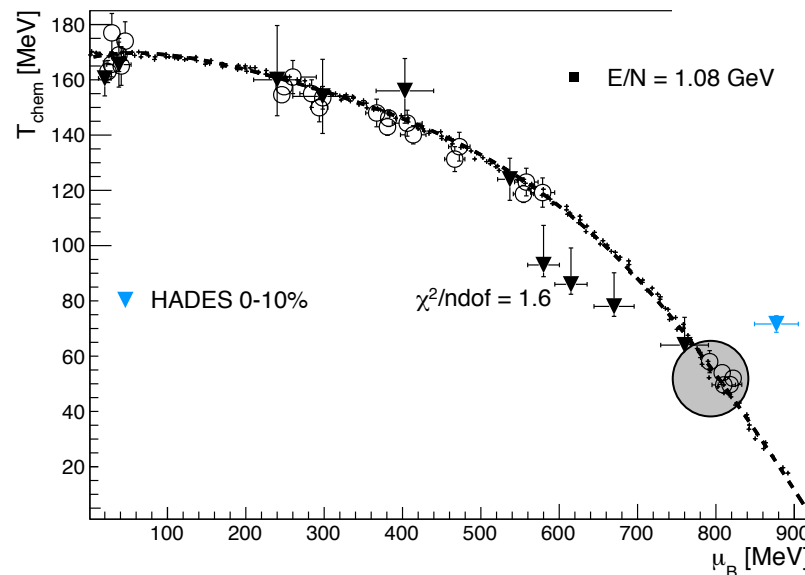
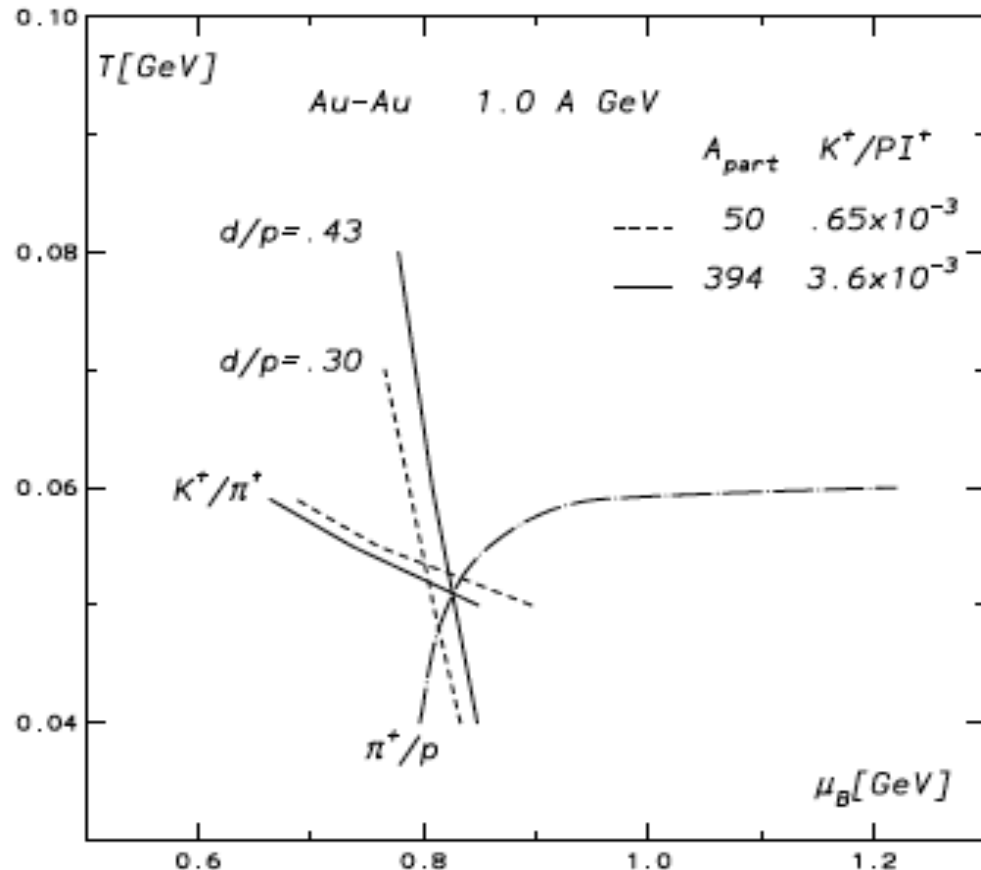


- Freeze-out point stays at higher  $T$  and  $\mu_B$  also for 0-10% most central events
- $\chi^2$  scan: no minimum near expected freeze-out
- Which hadron yields drive the fit away?

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M. Lorenz

PRELIMINARY



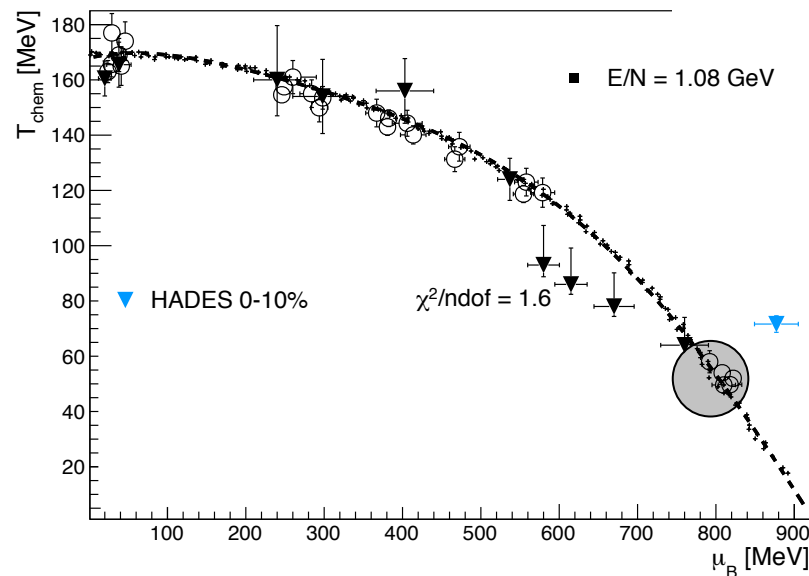
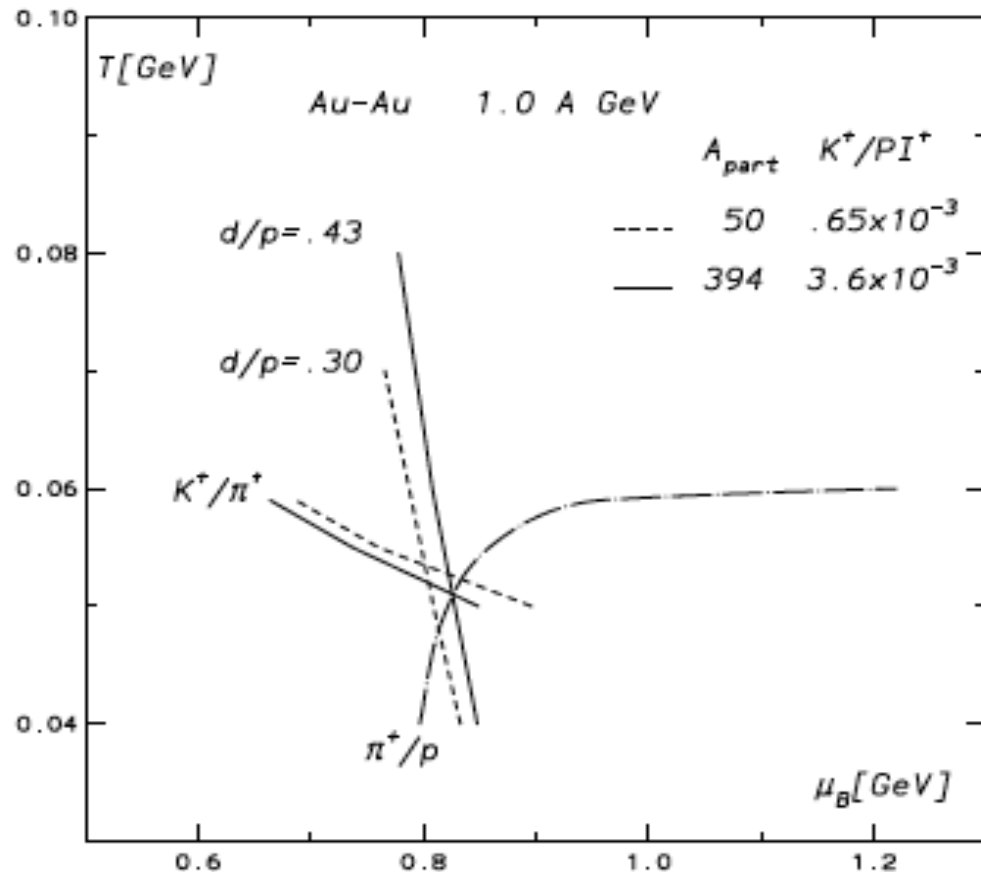
J. Cleymans, H. Oeschler, K. Redlich, *Phys.Rev. C59* (1999)

- Freeze-out points previously estimated based on ratios of  $p$ ,  $d$ ,  $K^+$ ,  $\pi^+$

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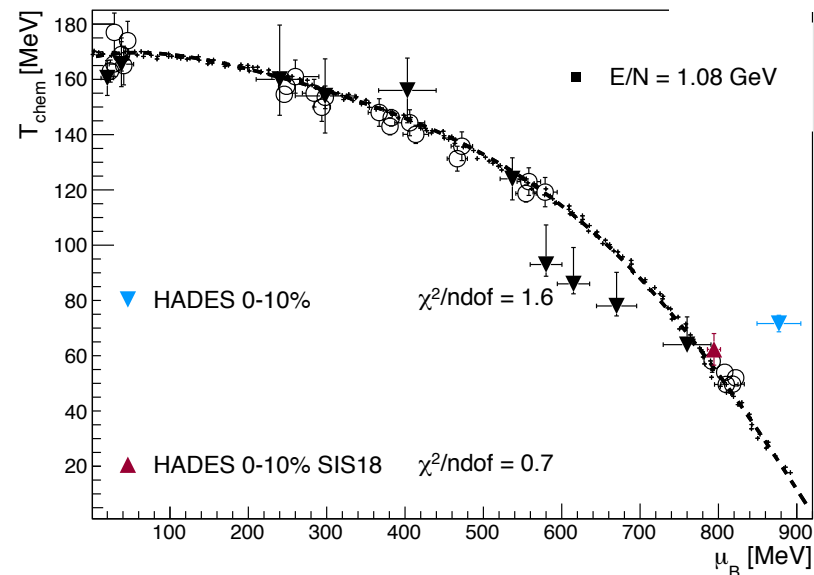
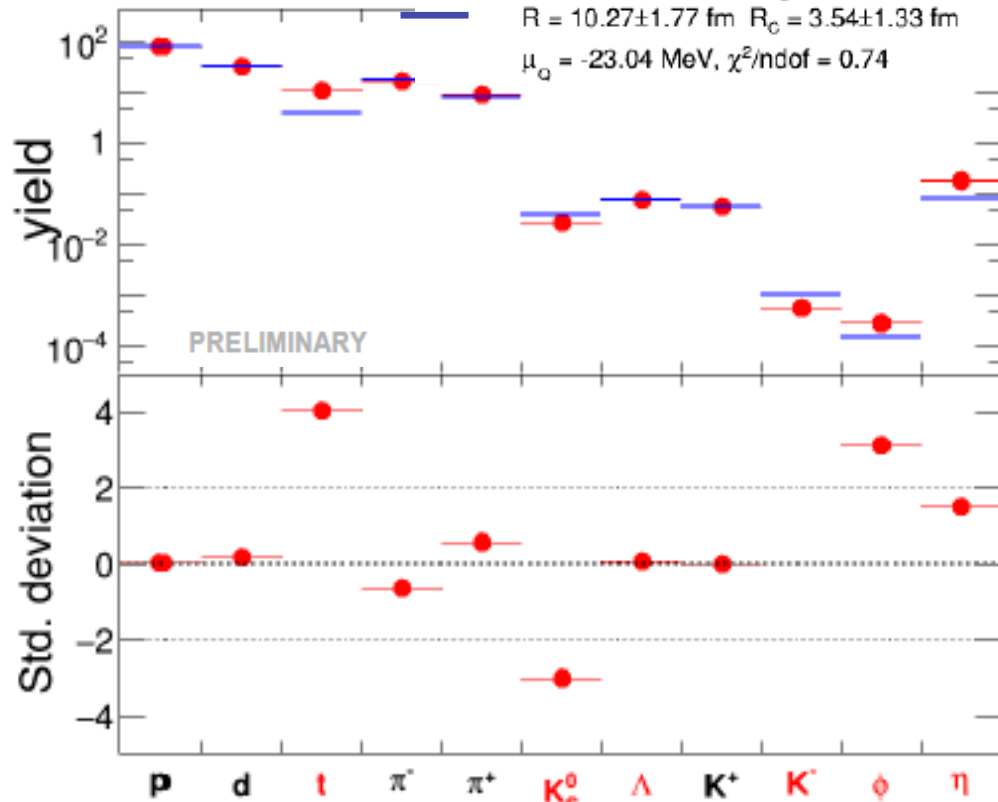
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M. Lorenz

PRELIMINARY

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 $R = 10.27 \pm 1.77$  fm  $R_C = 3.54 \pm 1.33$  fm  
 $\mu_Q = -23.04$  MeV,  $\chi^2/\text{ndof} = 0.74$



- Fit to HADES data consistent with previous works when same hadron yields are used

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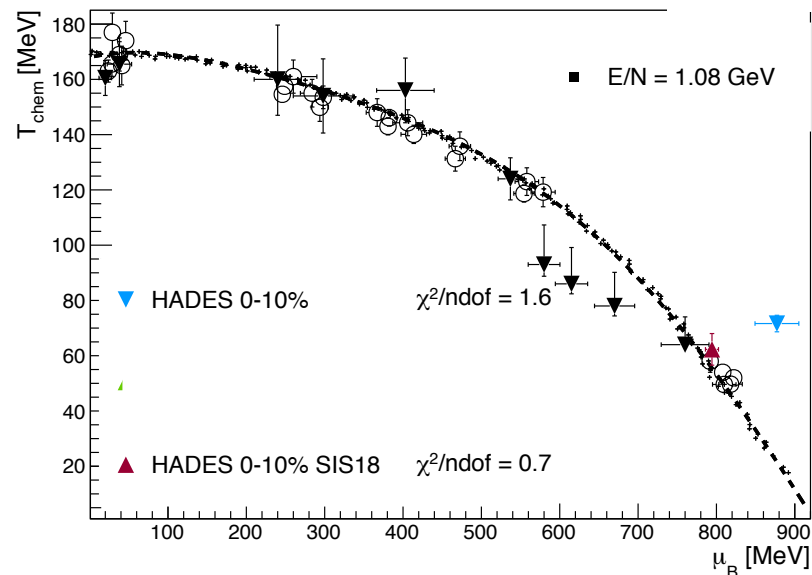
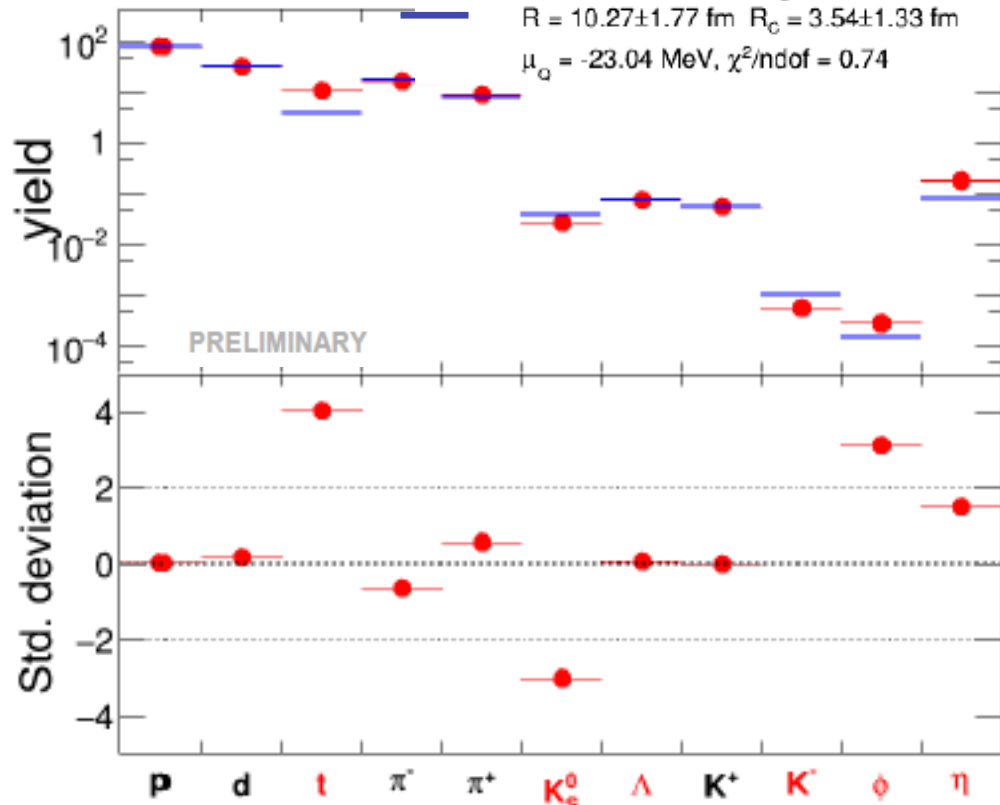
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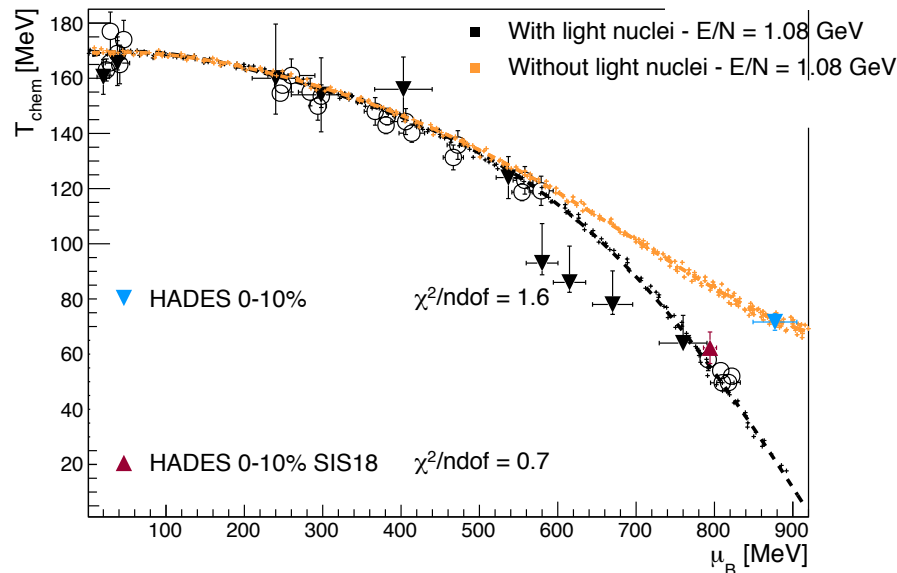
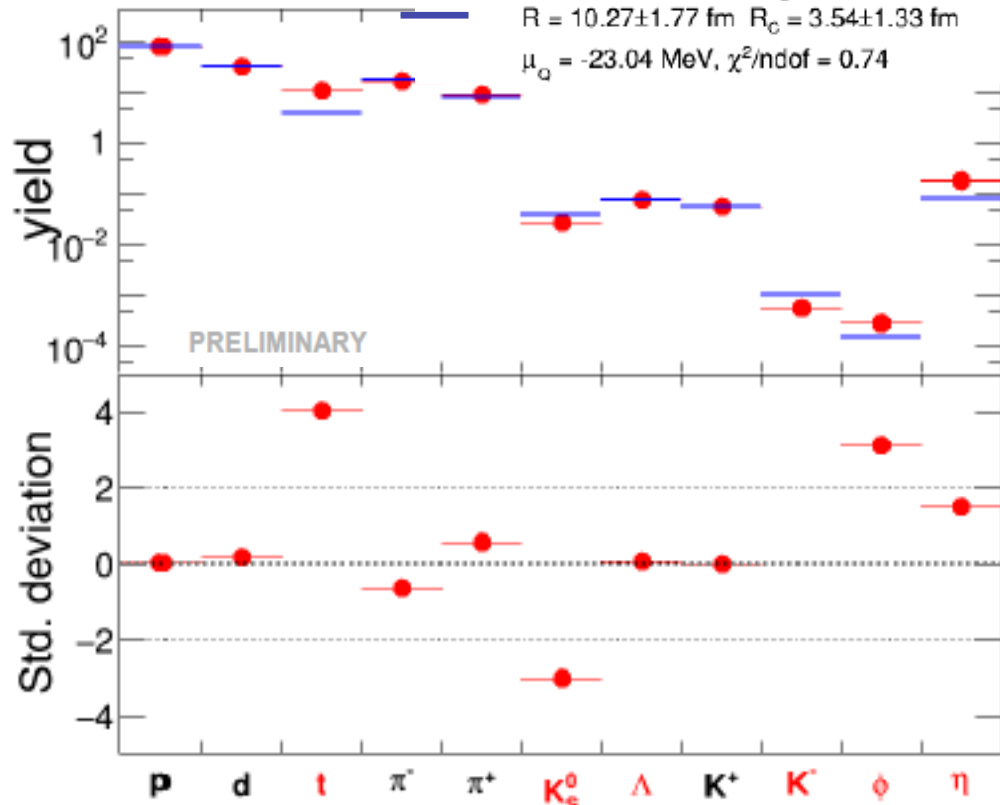
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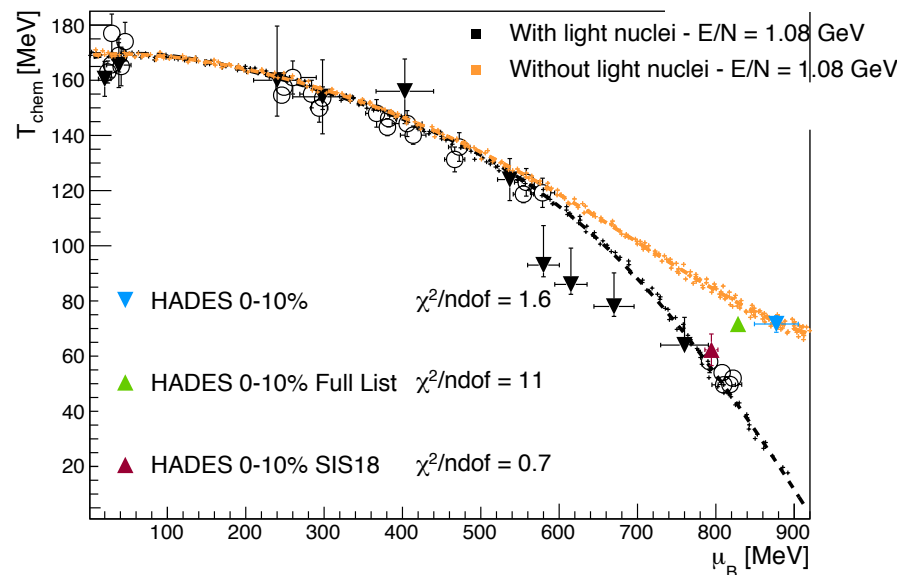
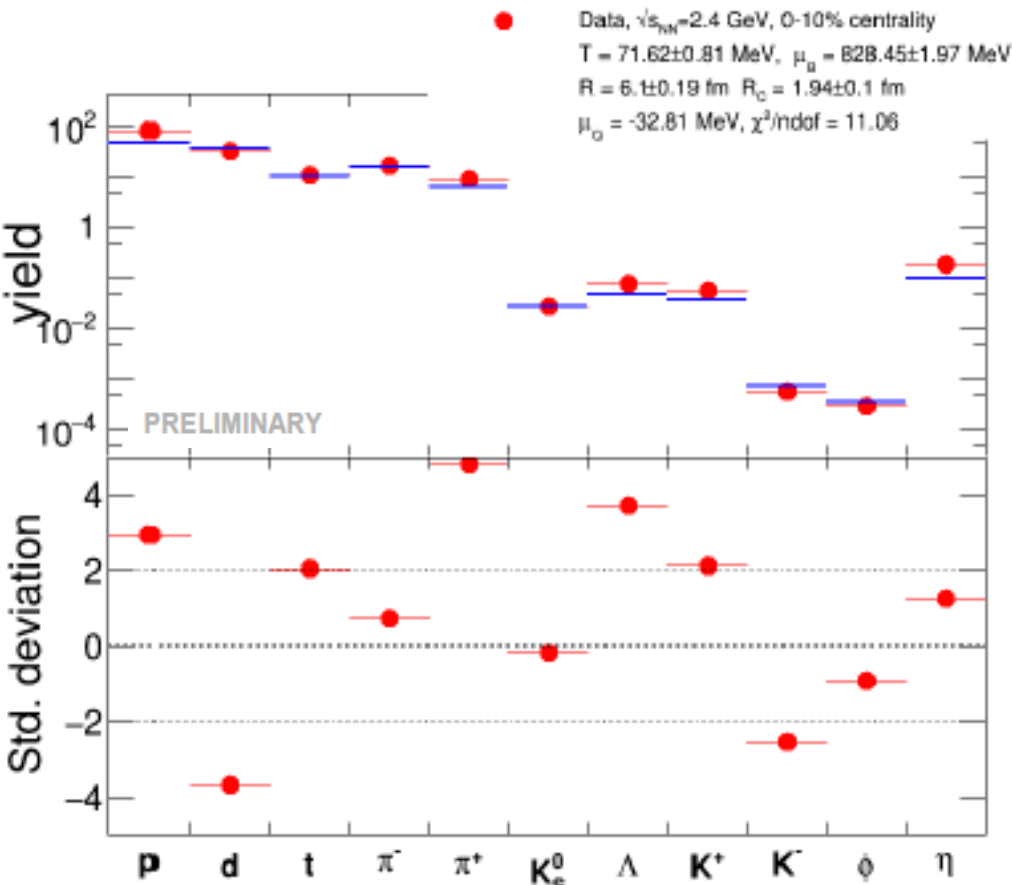
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M. Lorenz

PRELIMINARY



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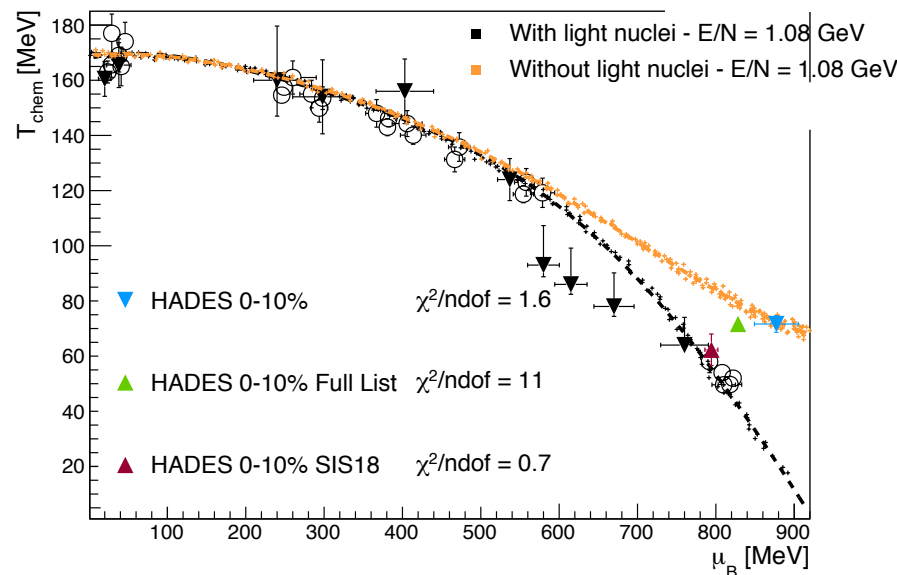
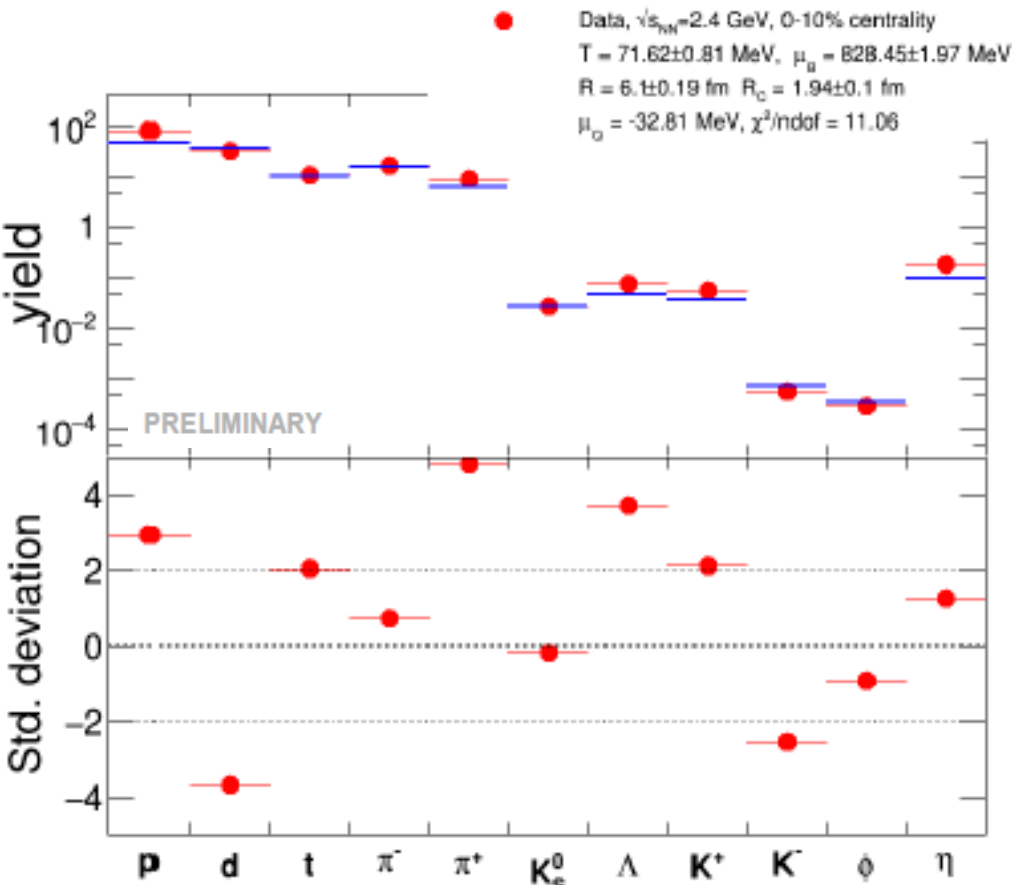
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M. Lorenz

PRELIMINARY



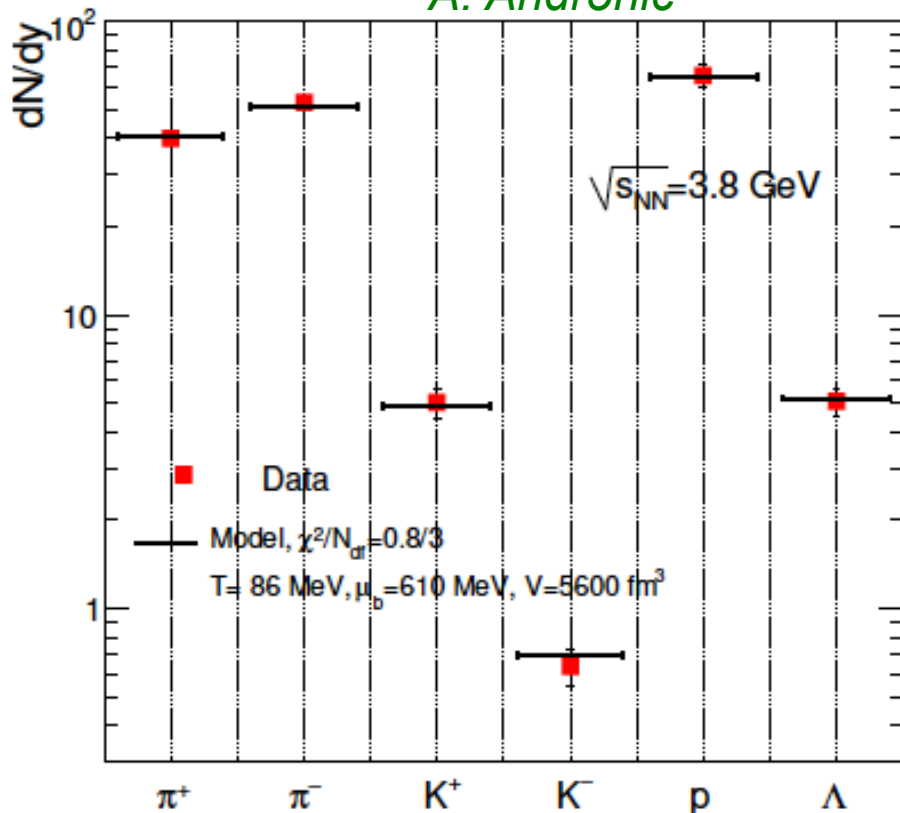
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- Fit to complete hadron set gives bad  $\chi^2$  (very preliminary triton yield)
- Inclusion of repulsive interactions<sup>1</sup> needed?

<sup>1</sup>V. Vovchenko, H. Stöcker  
 J.Phys. G44 (2017) no.5, 055103  
 A. Andronic et al. arXiv:1808.03102

- J. Cleymans, H. Oeschler, K. Redlich, *Phys.Rev. C59* (1999))
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# SIS100 energy regime

A. Andronic

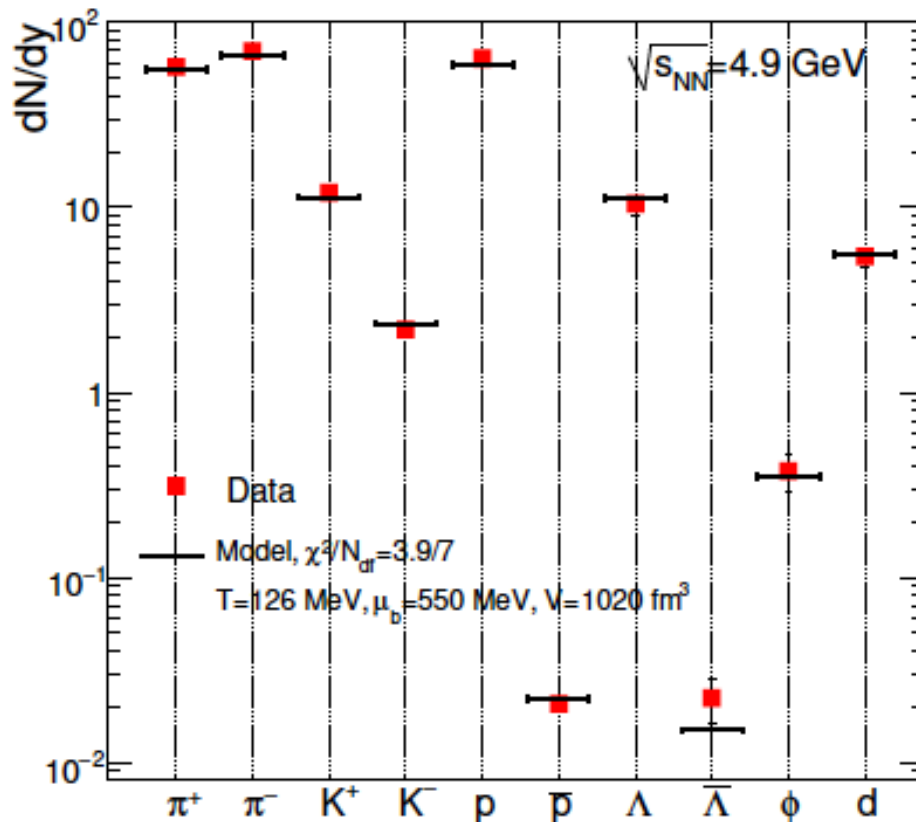
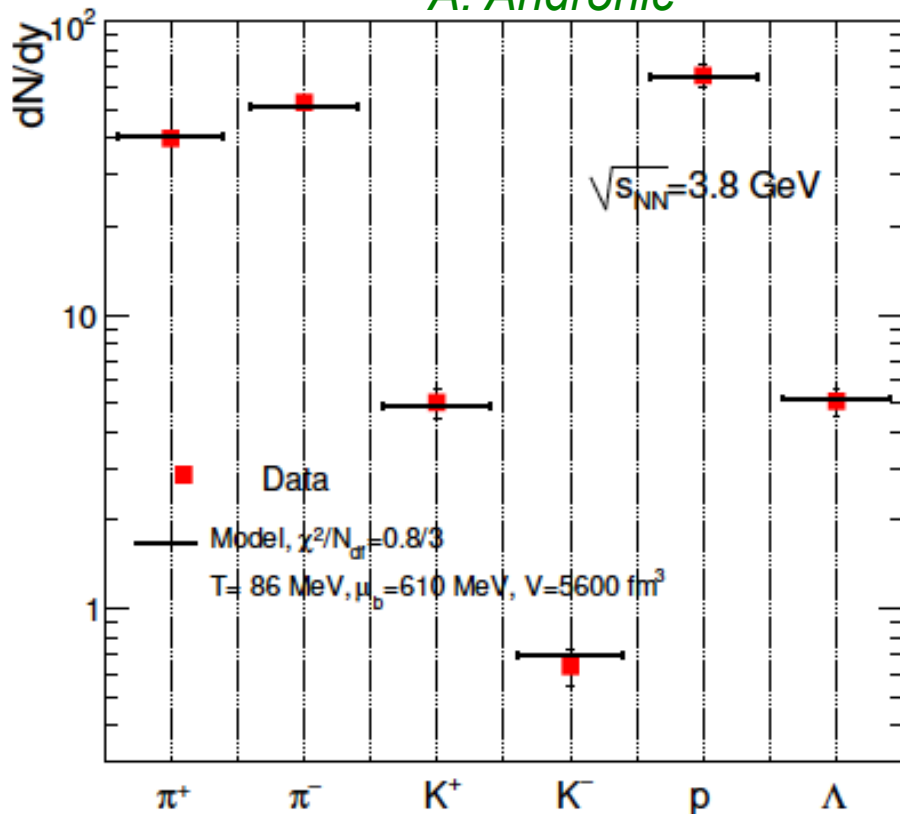


Data situation in the SIS100 energy regime rather scarce  
Nevertheless, fits are of good quality

→ precision and number of hadrons should be increased

# SIS100 energy regime

A. Andronic

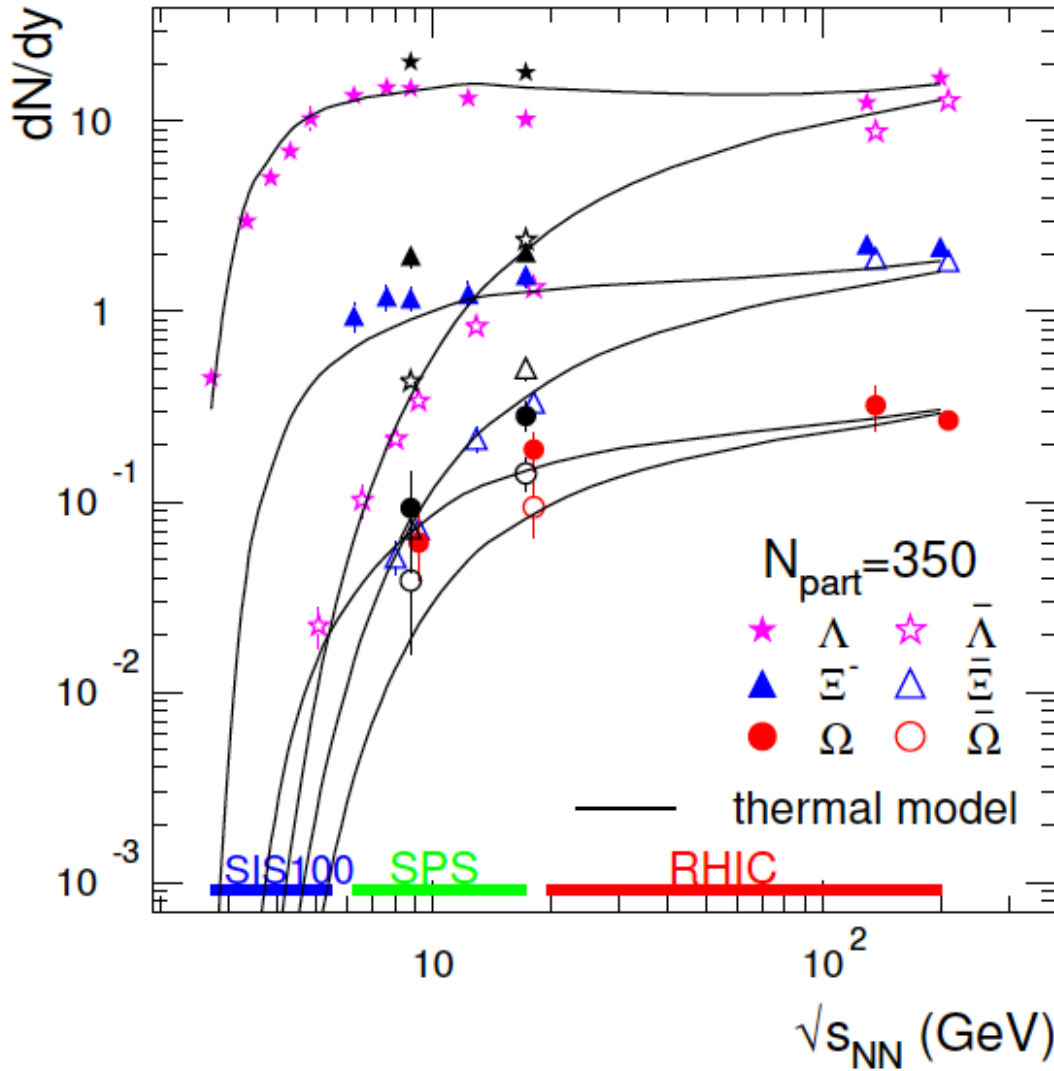


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# Predictions

A. Andronic



Example:

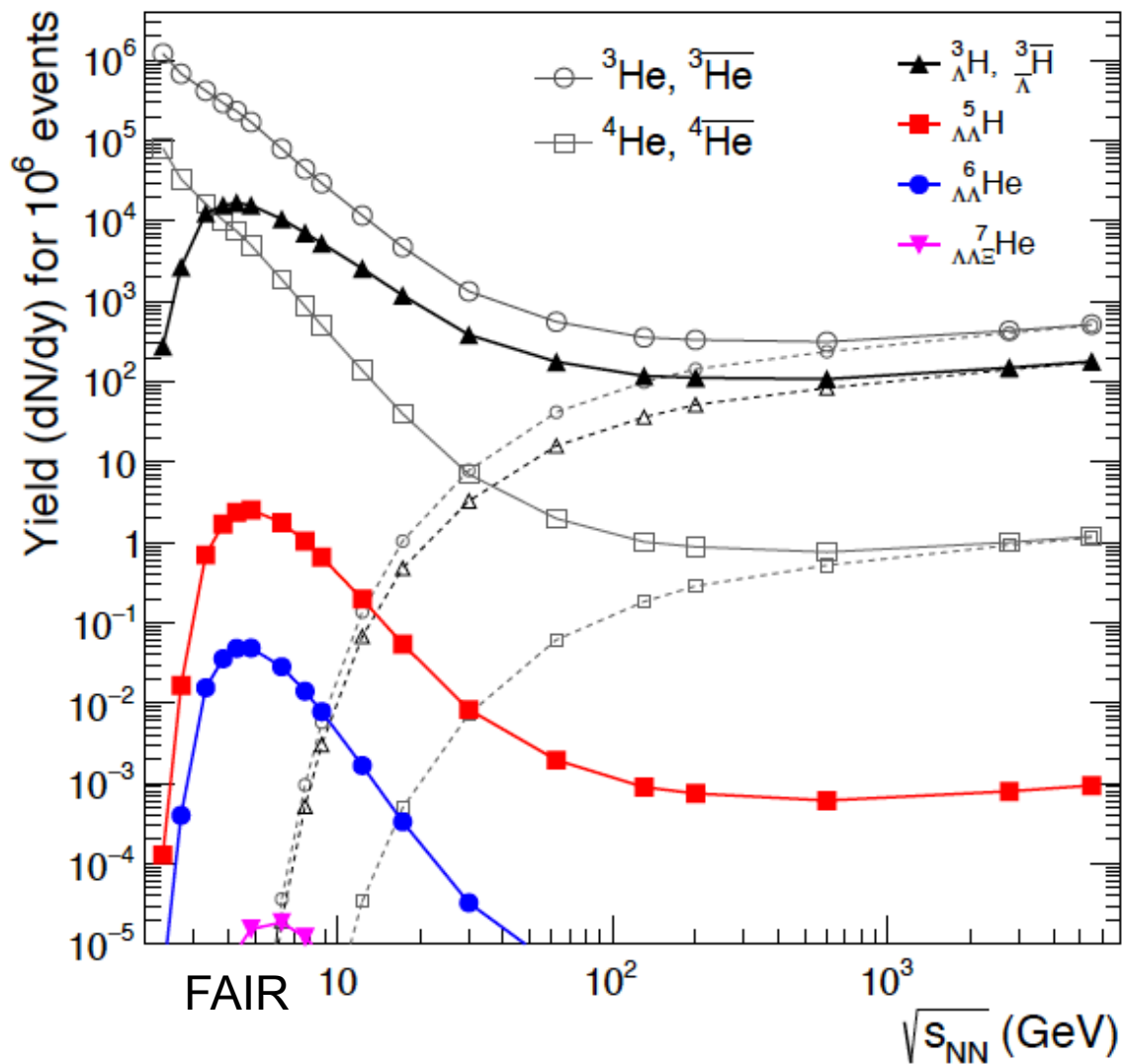
$dN/dy|_{y=0}$  yields at 8 AGeV:

$\Lambda$ : 9.0	$\bar{\Lambda}$ : $1.4 \cdot 10^{-3}$
$\Xi^-$ : 0.27	$\bar{\Xi}^-$ : $3.2 \cdot 10^{-4}$
$\Omega^-$ : $5.4 \cdot 10^{-3}$	$\bar{\Omega}^-$ : $5.4 \cdot 10^{-5}$



# Hypernuclei

A. Andronic

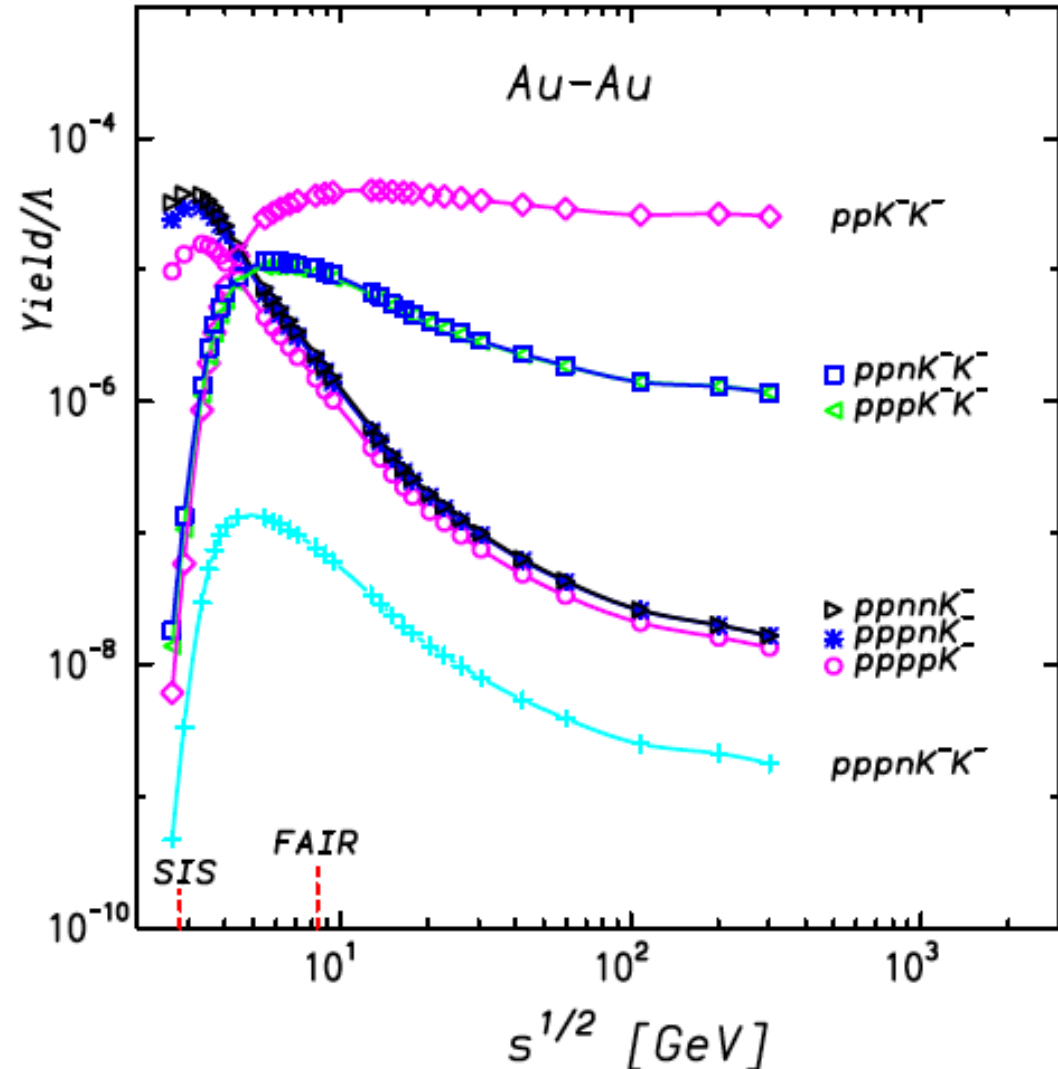


- Hypernuclei production maximum around FAIR energies
  - Roughly two orders of magnitude higher  ${}^3_{\Lambda}\text{H}$  production compared to LHC
  - Even hypernuclei with higher strangeness content will be in reach
- FAIR will be a hypernuclei factory

# Exotica

*A. Andronic, et al., NPA 765 (2006) 211*

- Exotica searches feasible at FAIR energies
- Kaonic bound states have highest production probability in the SIS100 region
- Strange dibaryons of all kinds are also good candidates for searches



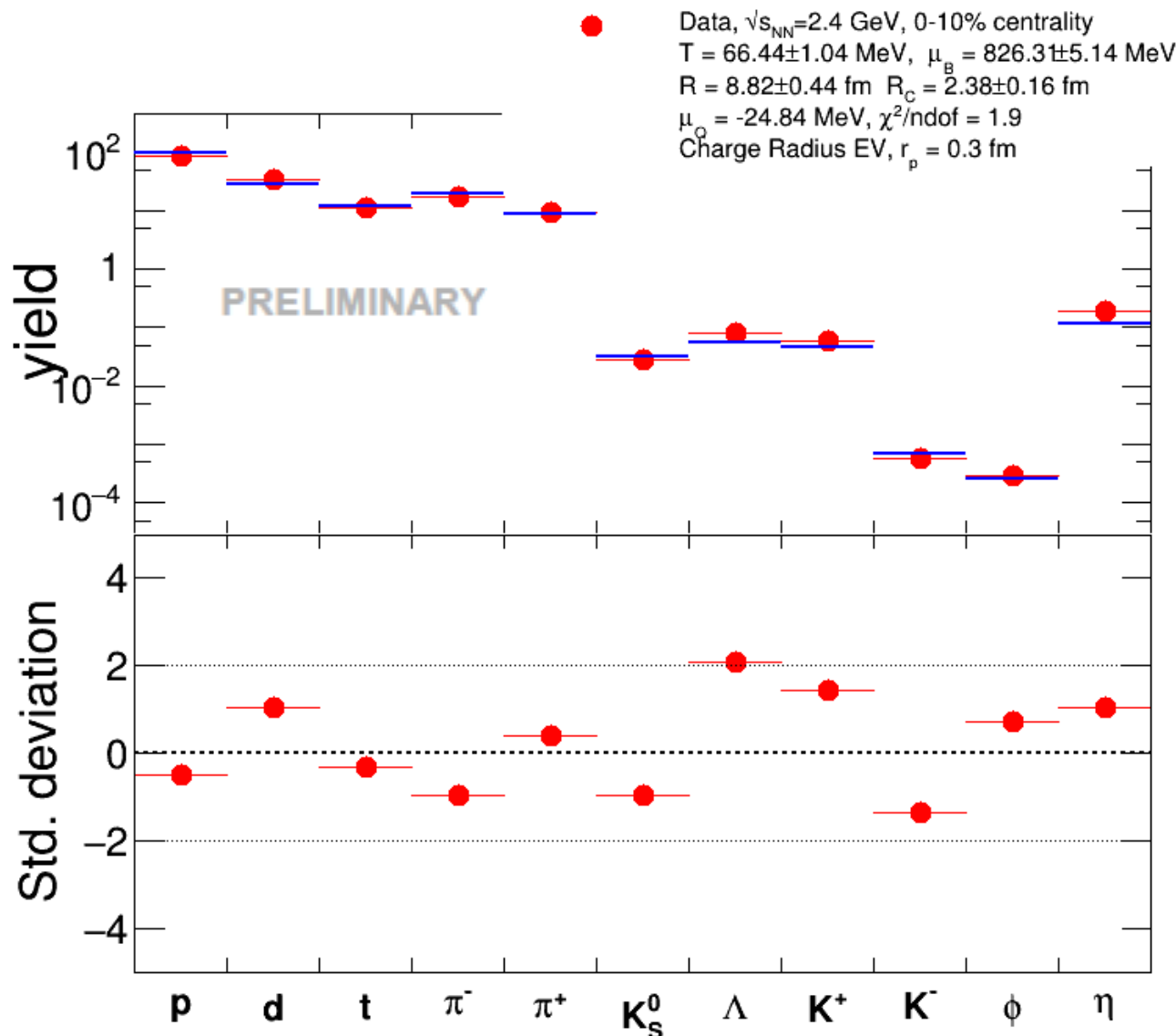


# Summary

- The thermal model provide a clear way to obtain „experimental points“ in the QCD phase diagram (via fits of  $u, d, s$  hadron yields)
- Thermal fits work well (AGS – LHC) with 3 parameters ( $T, \mu_B, V$ ) and can describe the current data with a precision of about 10%
- Canonical models work reasonably well in small systems and below threshold (HADES)
- The improved precision of the new data sets are testing the thermal model much stronger than before
- Systematics of the thermal model can lead to better or worse description of the high precision data
- Improved models using eigenvolume corrections, phase shifts or energy dependent Breit-Wigner lead to better fits of the current data (better  $\chi^2/\text{ndf}$ )
- Abundant production of (hyper)nuclei and exotica can be expected from predictions from the thermal model

# HADES Au-Au

*M. Lorenz*



- SHM fit including EV-corrections based on the charge radius of the nuclei describes the same data set rather well

Master thesis J. Stumm

# Canonical approach

In particular for small systems as  $e^+e^-$  and  $pp$  one often uses the canonical ensemble instead of the grand canonical ensemble, i.e. one or more quantum number is no longer conserved on average but needs to be conserved exactly

Often only strangeness-canonical treatment is used which can be modelled by a conservation or correlation Radius  $R_C$  in which the quantum number (strangeness) is exactly conserved