Hadron production in JAM: Effects of mean-field

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- Introduction

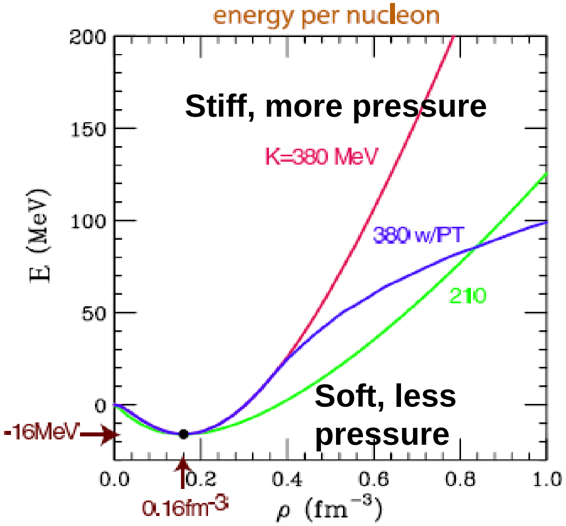
- QMD with scalar potential (RQMDs)
- QMD with vector potential (RQMDv)

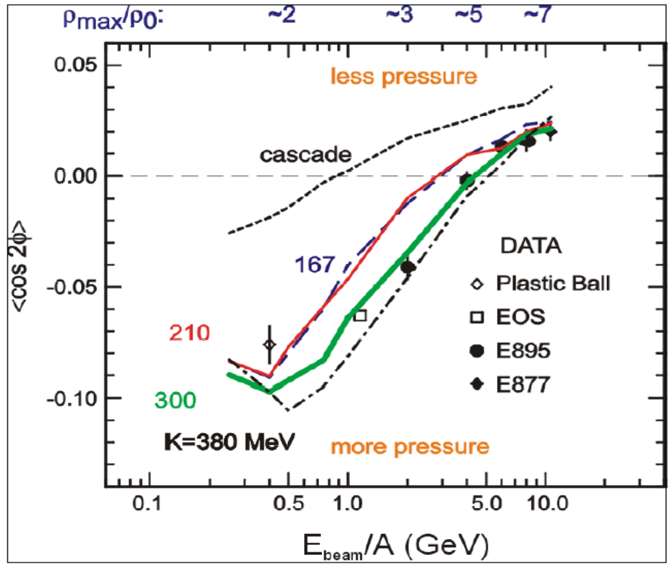
- Results : dNdy, pt, collective flows

EMMI Workshop “Probing dense baryonic matter with hadrons:
Status and Perspective Feb. 11-13, 2019 GSI

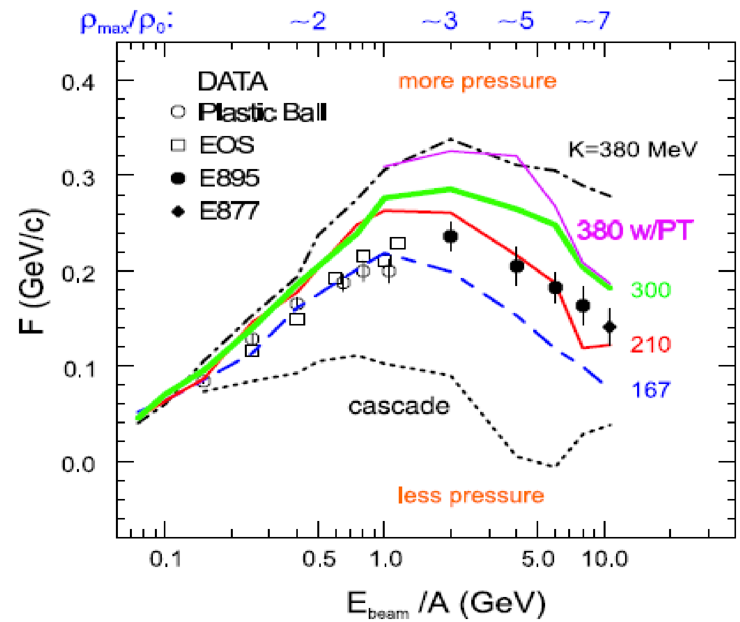
Determination of EOS at high density from an anisotropic flow in heavy ion collisions

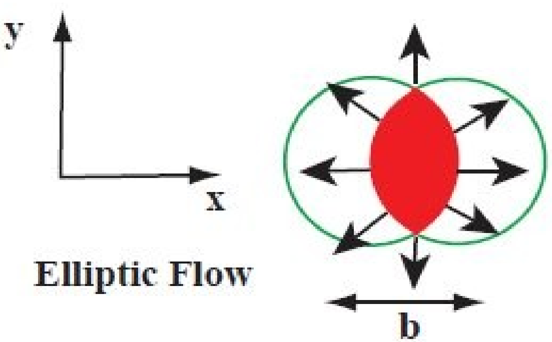
P. Danielewicz, R. Lacey, W.G. Lynch, Science 298 (2002) 1592



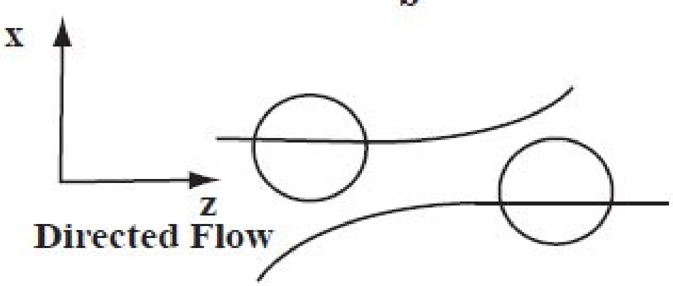


In-plane flow, v1





$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle \qquad F = \left. \frac{d\langle p_x/A \rangle}{d(y/y_{cm})} \right|_{y/y_{cm}=1}$$



BUU Transport model predicts strong sensitivities of EOS on the directed and elliptic flows.

See recent work on v3:P.Hillmann, J.Phys.G45(2018)085101

JAM microscopic transport model

- space-time propagation of particles based on cascade method
- Resonance (up to 2GeV) and string excitation and decays
- Re-scattering among all hadrons
- DPM type string excitation law as in HIJING.
- Use Pythia6 for string fragmentation
- Nuclear cluster formation and its statistical decay

- Propagation by the hadronic mean-fields within RQMD/S formulation
- EoS controlled collision term (2017)
- Dynamical coupling of Fluid dynamics through source terms (2018) (Hydro + hadronic cascade)
- Hydrodynamic Quantum Molecular Dynamics (HQMD) approach (2019)

Potentials

We use the Skyrme density dependent
and Lorentzian momentum dependent potential:

$$V_i = \frac{\alpha}{2\rho_0}\rho_i + \frac{\beta}{(1+\gamma)\rho_0^\gamma}\rho_i^\gamma + \sum_{k=1,2}\frac{C_{ex}^{(k)}}{2\rho_0}\sum_{j\neq i}\frac{1}{1+[p_{ij}/\mu_k]^2}\rho_{ij}$$

In the non-relativistic QMD approach, density is computed by

$$\rho_i = \sum_{j\neq i}\rho_{ij}, \qquad \rho_{ij} = \frac{1}{(4\pi L)^{3/2}}\exp\left(\frac{-(\boldsymbol{r}_i-\boldsymbol{r}_j)^2}{4L}\right),$$

We would like to develop a relativistic version of QMD approach in a effective way.

Arguments of potential $\boldsymbol{r}_i - \boldsymbol{r}_j$ and $\boldsymbol{p}_i - \boldsymbol{p}_j$ are replaced by
the distances in the two-body c.m.

The RQMD model (1989)

Relativistic extension of QMD (RQMD) was developed by H. Sorge based on the constrained Hamiltonian dynamics:
H. Sorge, H. Stoecker, W. Greiner, Ann. Phys. 192, 266 (1989).

Manifestly covariant way: four-vectors $q_i^\mu, \quad p_i^\mu \ (i = 1, N)$

For the description of N-particle system, we have 8N dimension.
In order to reduced the dimension from 8N to 6N, we need 2N constraints.

On-mass shell condition:

$$H_i = p_i^2 - m_i^2 - 2m_i V_i = 0, \quad (i = 1, N)$$

Time fixation:

$$\chi_i = \sum_{j \neq i} \frac{\exp(q_{ij}^2 / L_c)}{q_{ij}^2 / L_c} q_{ij} p_{ij} = 0, \quad (i = 1, \cdots, N)$$

We need to evaluate the inversion of the (N x N) matrix at every time-step to solve Equation of motion.

Simplifed RQMD approach

Simplified version of RQMD was proposed by T. Maruyama (1996)
T. Maruyama, et. al. Prog. Theor. Phys. 96, 263 (1996).

On-mass shell condition:

$$H_i = p_i^2 - m_i^2 - 2m_i V_i = 0, \quad (i = 1, N)$$

Time fixation to equate the all time coordinate of the particles:

$$\chi_i = \hat{a} \cdot (q_i - q_N) = 0 \quad (i = 1, \cdots, N-1)$$

$$\chi_N = \hat{a} \cdot q_N - \tau = 0$$

$$\hat{a} = (1, 0, 0, 0) \text{ in a reference frame}$$

We also assume that time-component of the momentum coordinate is replaced by the kinetic energy in the argument of the potential.

These approximations yield that it is equivalent to solve the following Hamiltonian system:

$$H = \sum_i^N \sqrt{\boldsymbol{p}_i^2 + m_i^2 + 2m_i V_i}$$

$$\dot{\boldsymbol{r}}_i = \frac{\boldsymbol{p}_i}{p_i^0} + \sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \boldsymbol{p}_i}, \quad \dot{\boldsymbol{p}}_i = -\sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \boldsymbol{r}_i} \qquad p_i^0 = \sqrt{\boldsymbol{p}_i^2 + m_i^2 + 2m_i V_i}$$

The RQMDs model

$$H = \sum_{i}^{N} \sqrt{\boldsymbol{p}_i^2 + m_i^2 + 2m_i V_i}$$

In the original RQMD by Sorge and Maruyama model (RQMDs),
The argument of the potential is replaced
by the distance in the two-body c.m. frame:

$$q_{Tij} = q_{ij} - (q_{ij} \cdot P_{ij}) P_{ij} / P_{ij}^2, \quad P_{ij} = p_i + p_j$$

Marty and Aichelin model
(R. Marty and J. Aichelin PRC87 (2013) 034912)
assumes that distance in the N-body c.m. frame is used:

$$q_{Tij} = q_{ij} - (q_{ij} \cdot P_{ij}) P_{ij} / P_{ij}^2, \quad P_{ij} = p_i + p_j + \cdots + p_N$$

Vector potential implementation

Let us consider only the time-component of the vector potential:

$$H = \sum_{i}^{N} \sqrt{\boldsymbol{p}_i^2 + m_i^2} + V_i$$

Niita model in JQMD (PRC52 (1995) 2620) assumes that

$$\rho_i = \sum_{j \neq i} \rho_{ij}, \qquad \rho_{ij} = \frac{1}{(4\pi L)^{3/2}} \exp\left(\frac{q_{Tij}^2}{4L}\right),$$

So in this model, potential is Lorentz scalar.
The difference between RQMDs and RQMDv in the EoM is the factor 1/gamma_j .

$$\dot{\boldsymbol{r}}_i = \frac{\boldsymbol{p}_i}{p_i^0} + \sum_j \underline{\frac{m_j}{p_j^0}} \frac{\partial V_j}{\partial \boldsymbol{p}_i}, \quad \dot{\boldsymbol{p}}_i = -\sum_j \underline{\frac{m_j}{p_j^0}} \frac{\partial V_j}{\partial \boldsymbol{r}_i}$$

However, this model underestimates the interaction density, since density is close to the scalar density instead of a baryon density. i.e. normalization of Gaussian is not correct.

Vector potential (RQMDv)

For the correct normalization, one needs to include gamma factor:

$$\rho_i = \sum_{j \neq i} B_j \rho_{ij}, \qquad \rho_{ij} = \frac{\gamma_{ij}}{(4\pi L)^{3/2}} \exp\left(\frac{q_{Tij}^2}{4L}\right), \quad \gamma_{ij} = \frac{P_{ij}^0}{\sqrt{P_{ij}^2}}$$

B_j is the baryon number of a jth particle.

In RQMDv, the invariant baryon density is used in the density of the Skyrme potential as implemented in pBUU, GiBUU, and SMASH:

$$J_i^\mu = \sum_j B_j \frac{p_j^\mu}{p_j^0} \rho_{ij}, \quad \rho_{Bi} = \sqrt{J_i^\mu J_{i\mu}}$$

But what is the consistent modification in the momentum dependent potential?

JAM Mean-field mode summary

EoS is included by mean-field within the quantum molecular dynamics framework.

1. RQMDs mode: potential is included as a scalar:

Hamiltonian of the system is given by

$$H = \sum_{i}^{N} \sqrt{\boldsymbol{p}_i^2 + m_i^2 + 2m_i V_i(q_{Tij})}$$

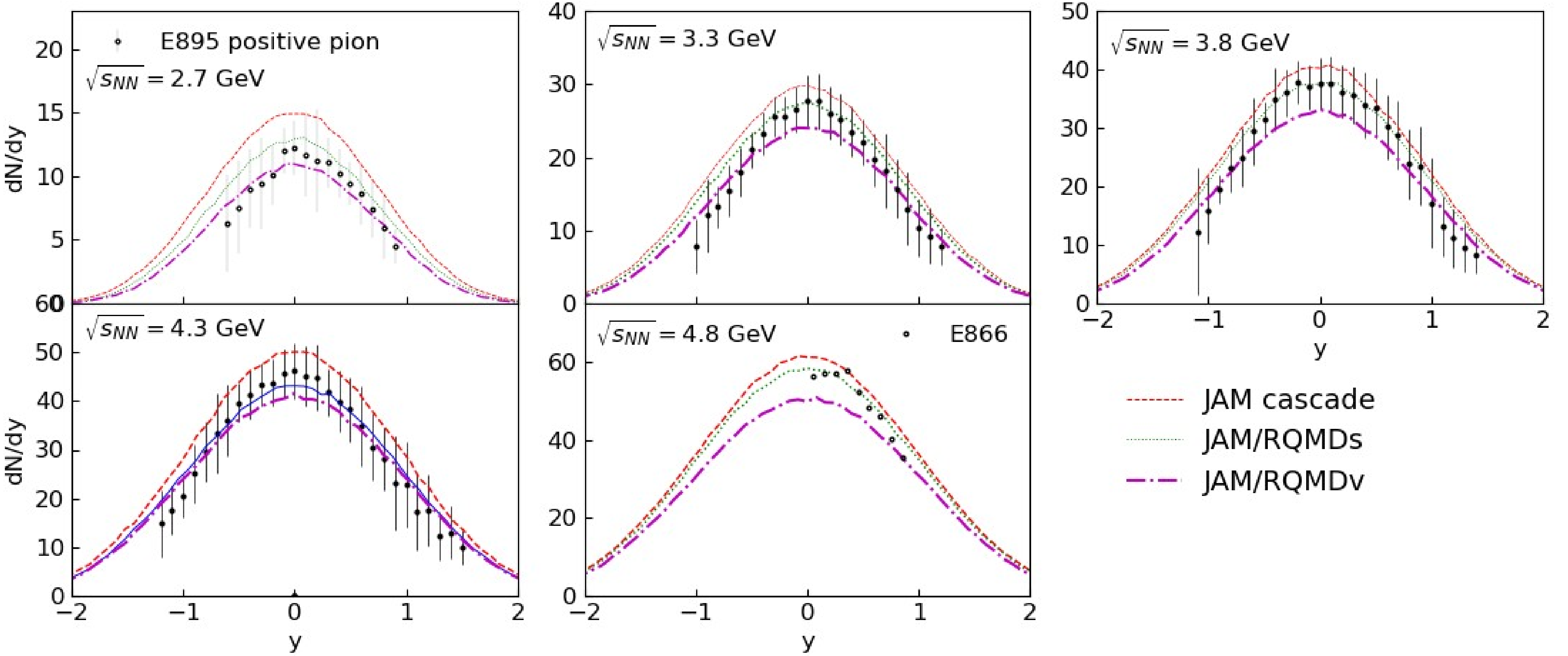
2. RQMDv mode: : potential is included as a vector (only time-component)

$$H = \sum_{i}^{N} \left(\sqrt{\boldsymbol{p}_i^2 + m_i^2} + V_i \right)$$

$$\rho_{Bi} = \sqrt{J_i^{\mu} J_{i\mu}}, \quad J_i^{\mu} = \sum_{j} B_j \frac{p_j^{\mu}}{p_j^0} \rho_{ij}$$

dN/dy for pion at AGS energies

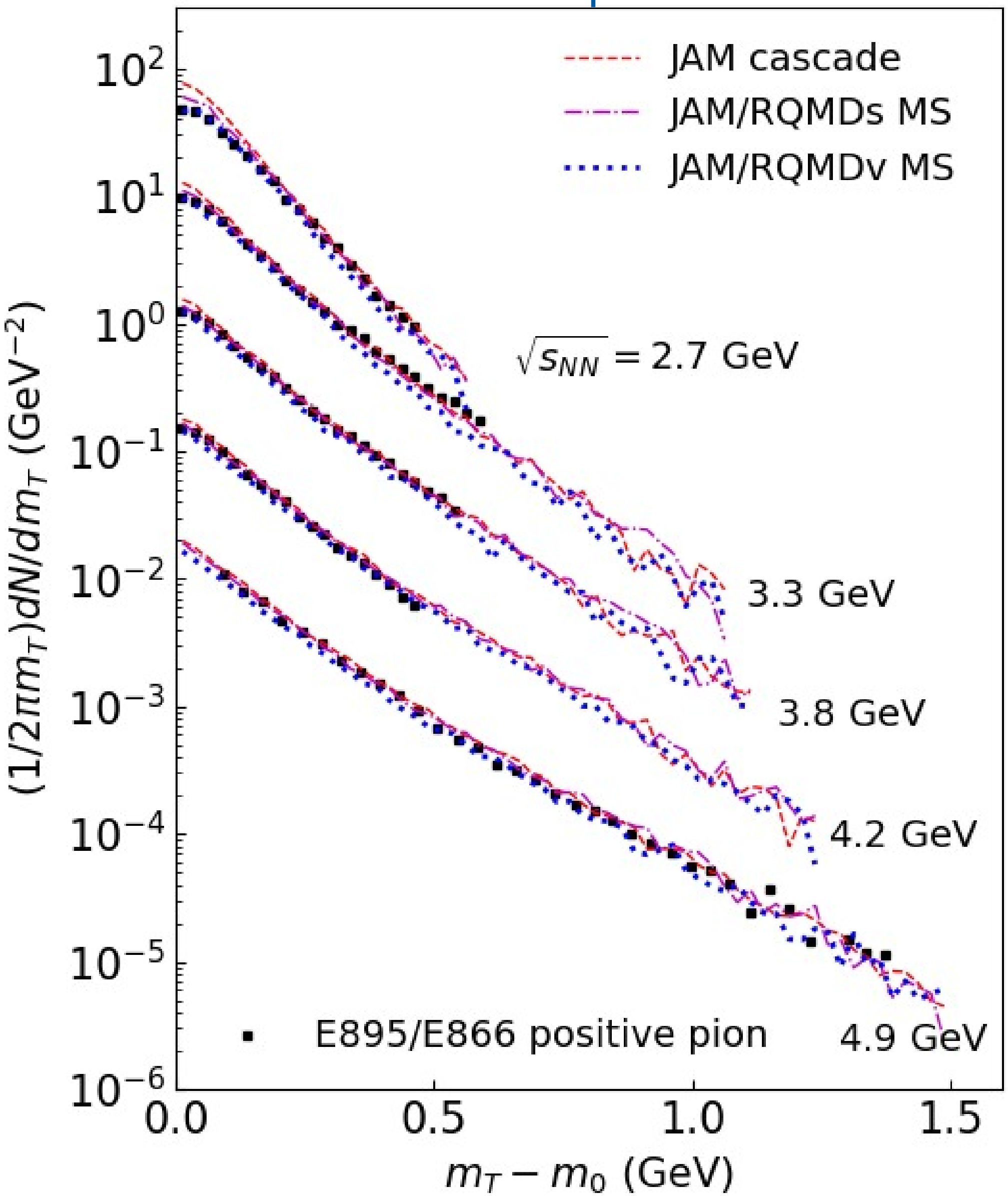
Mom. Dep. Soft (K=270MeV)



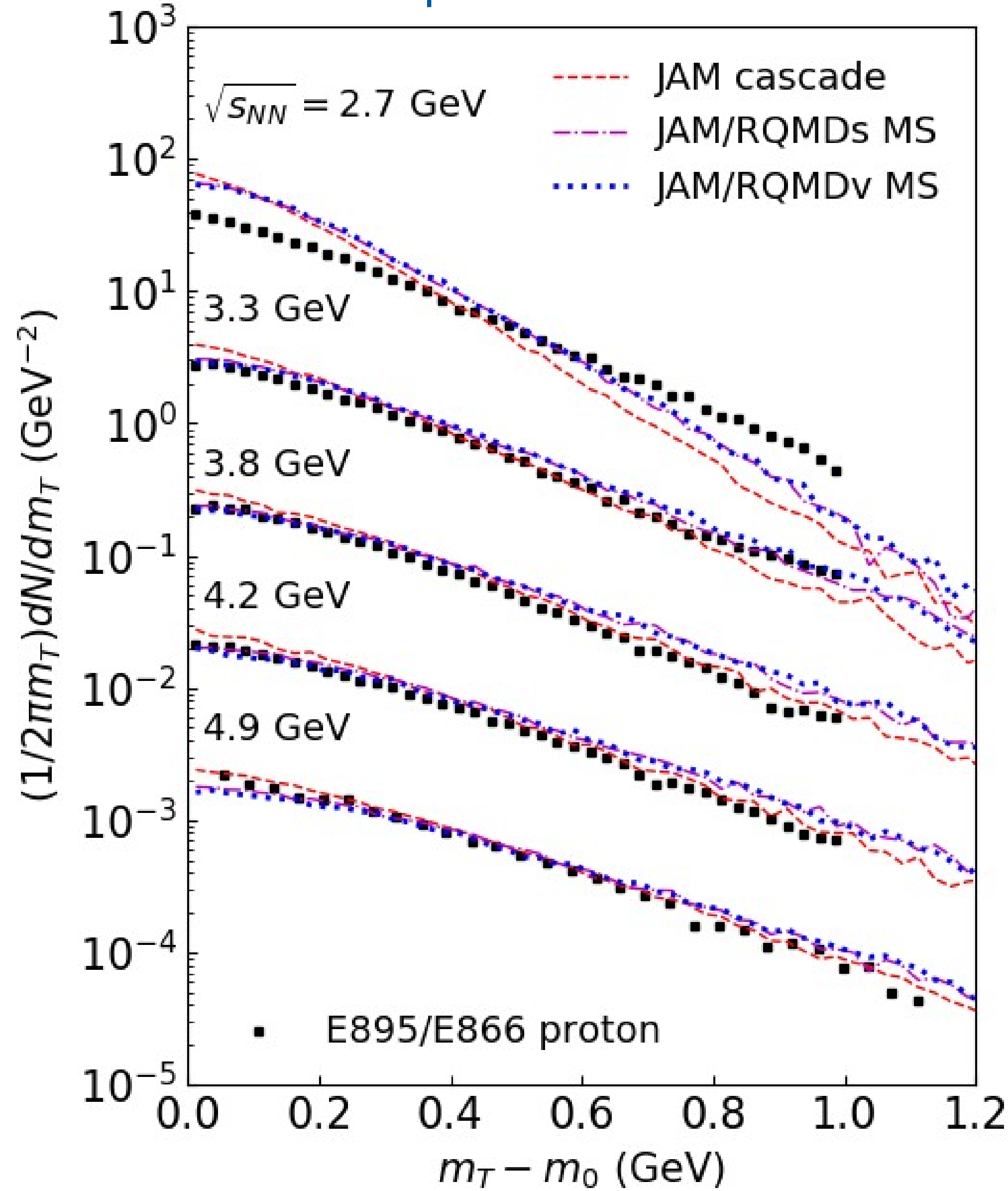
Mom. Dep. Soft (K=270MeV)

dN/dmt at AGS energies

Positive pions

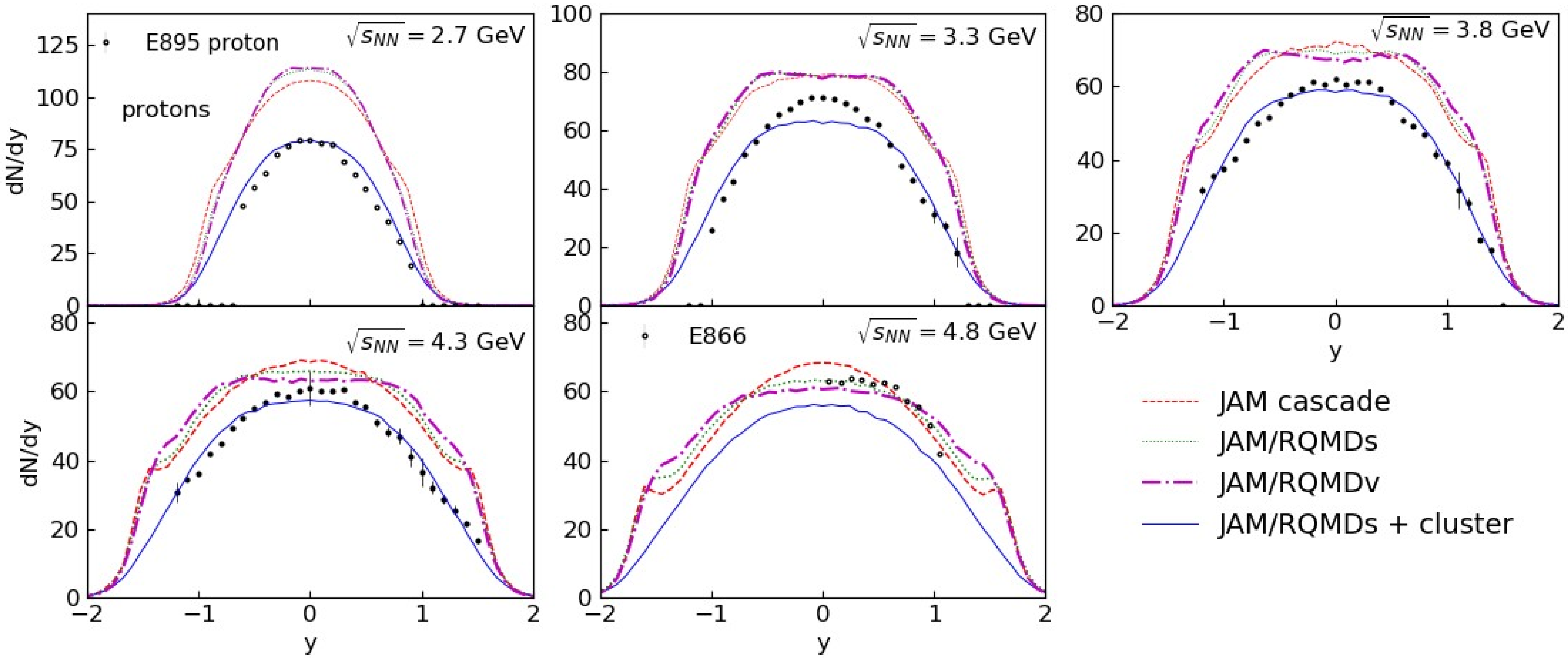


protons



dN/dy for proton at AGS energies

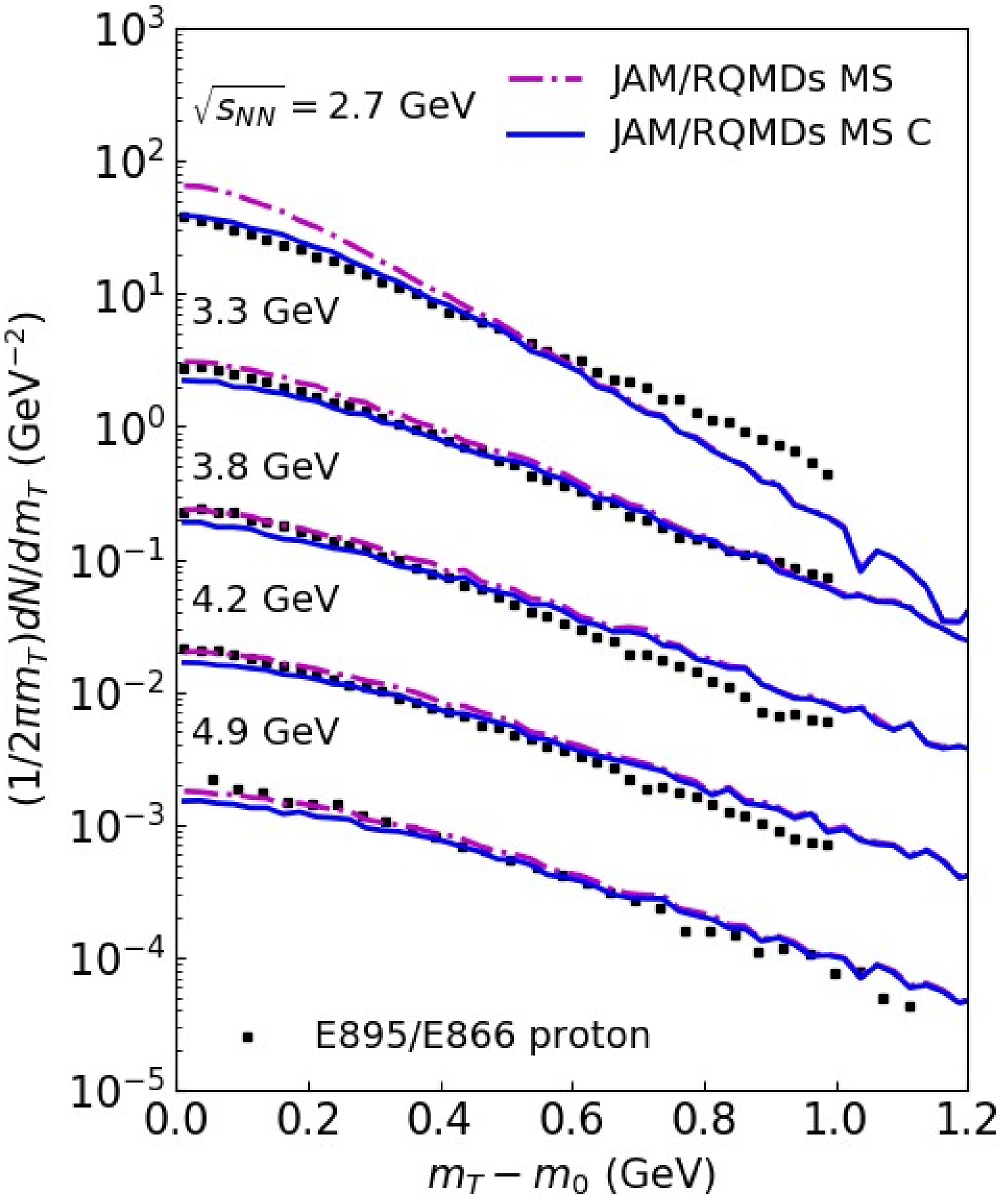
Mom. Dep. Soft (K=270MeV)



Effects of nuclear cluster formation is very large.

Mom. Dep. Soft (K=270MeV)

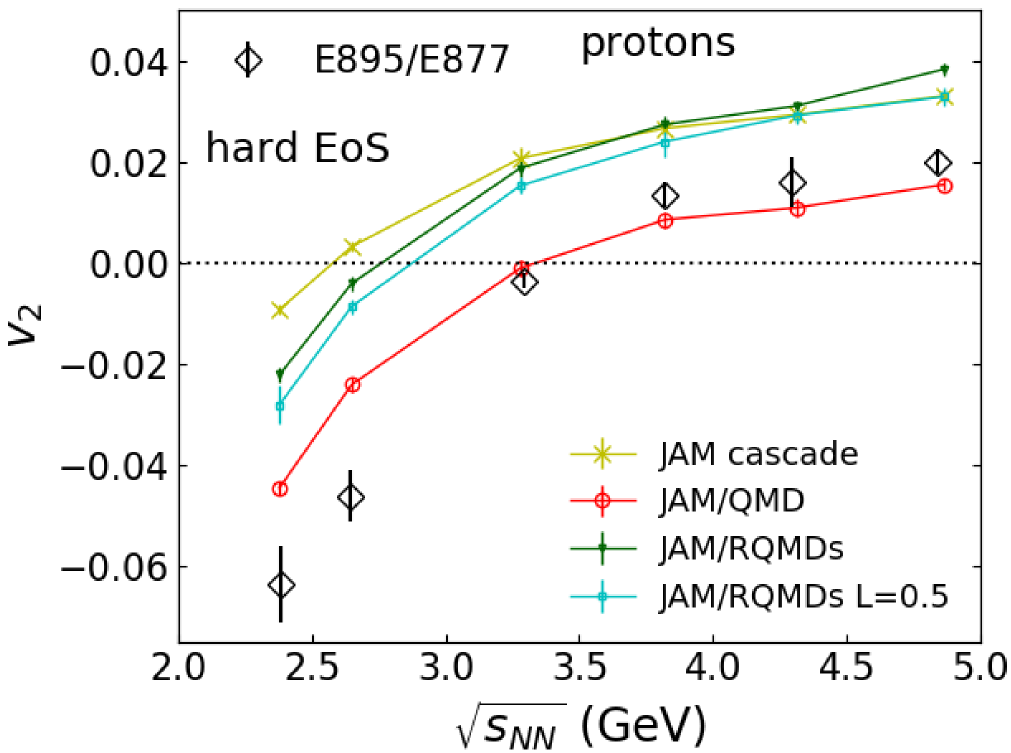
dN/dmt for p at AGS energies



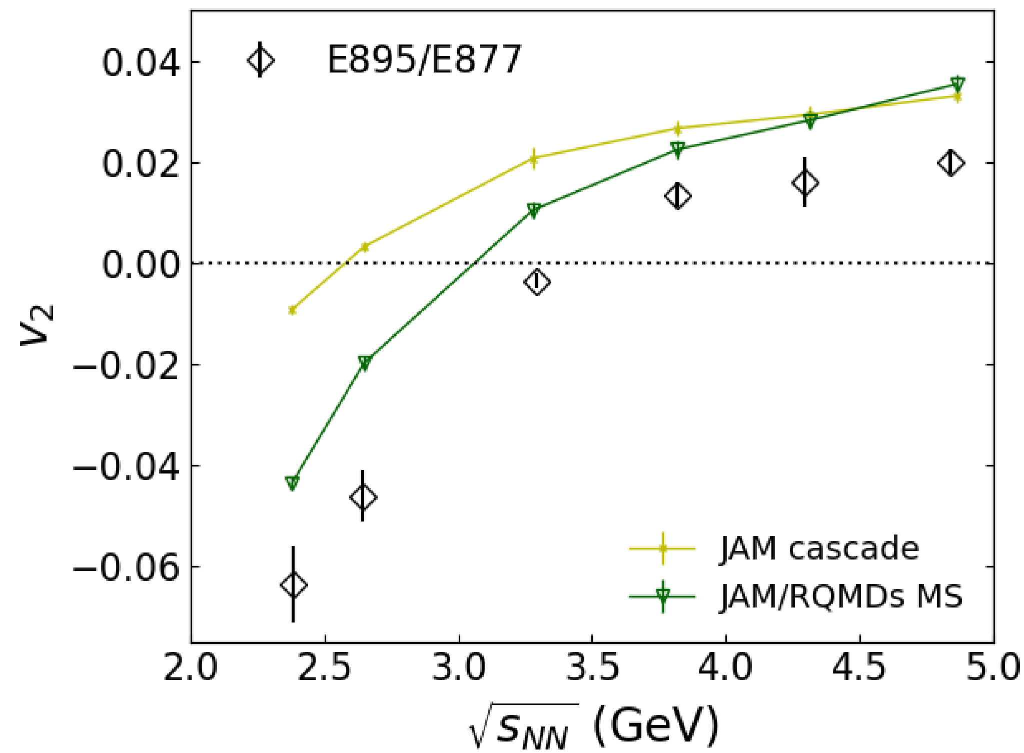
Nuclear cluster formation is imortant for low pt regaion.

V2 at AGS energies (Scalar)

Hard EoS (K=380MeV)



Mom. Dep. Soft (K=270MeV)

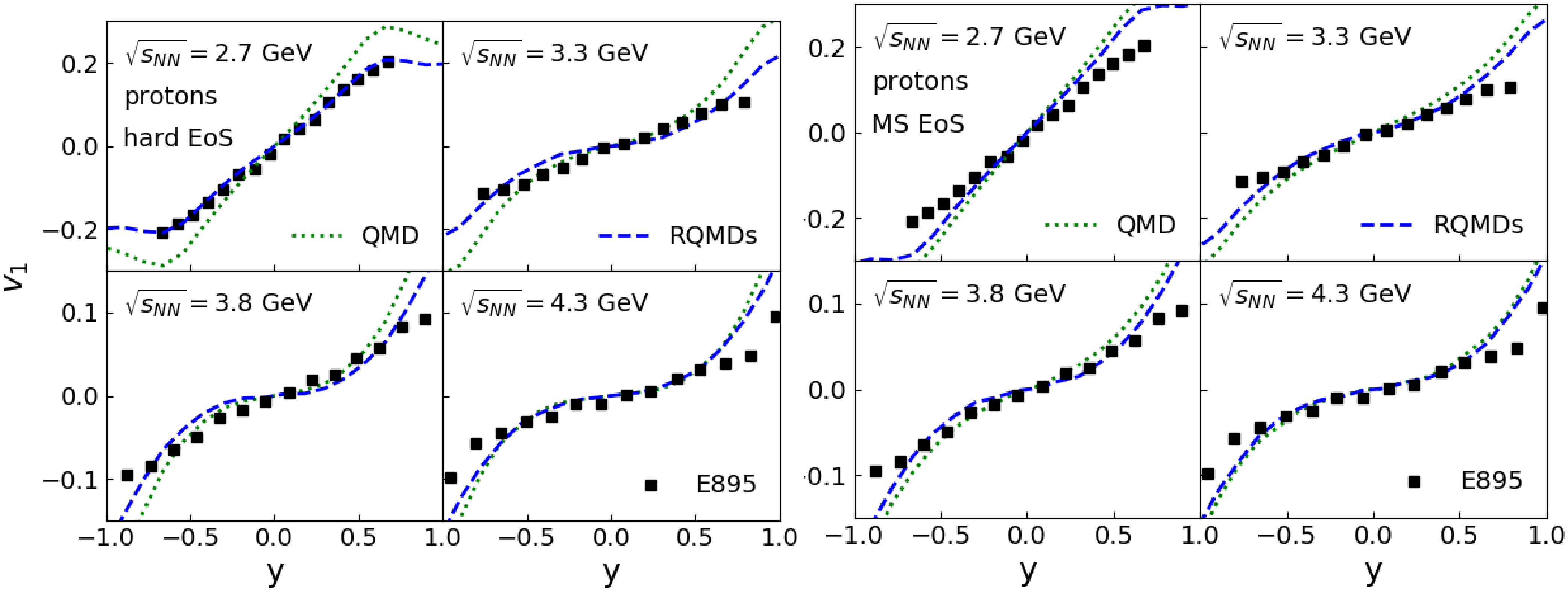


V2 from scalar potential is not sufficient to describe squeeze-out effect.

V1 at AGS energies (Scalar)

Hard EoS (K=380MeV)

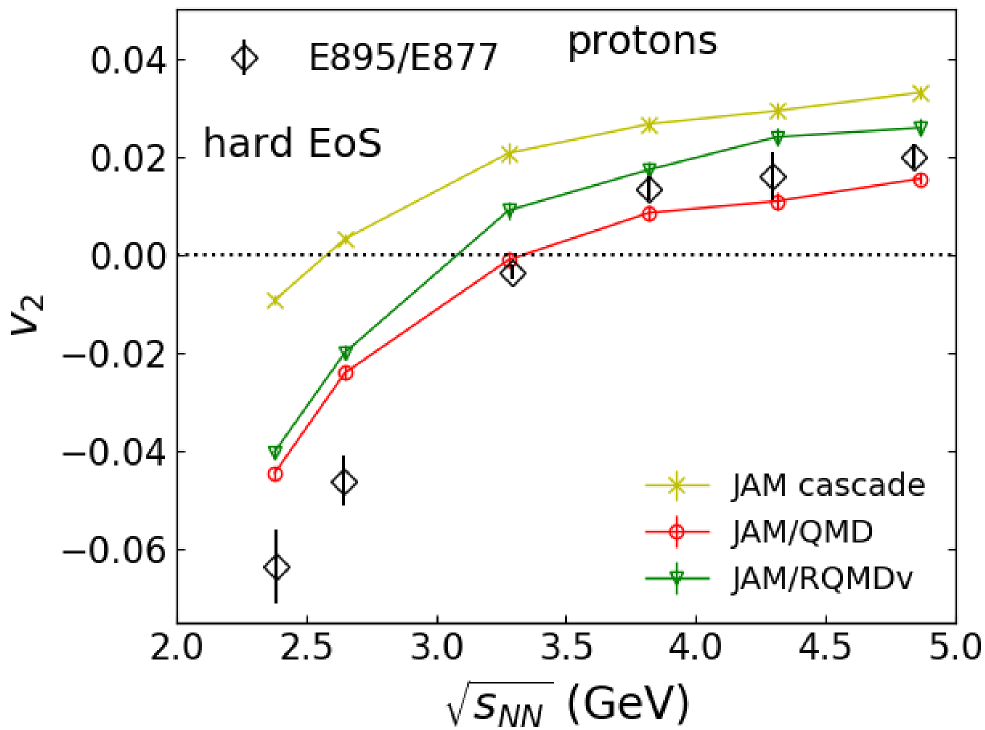
Mom. Dep. Soft EoS



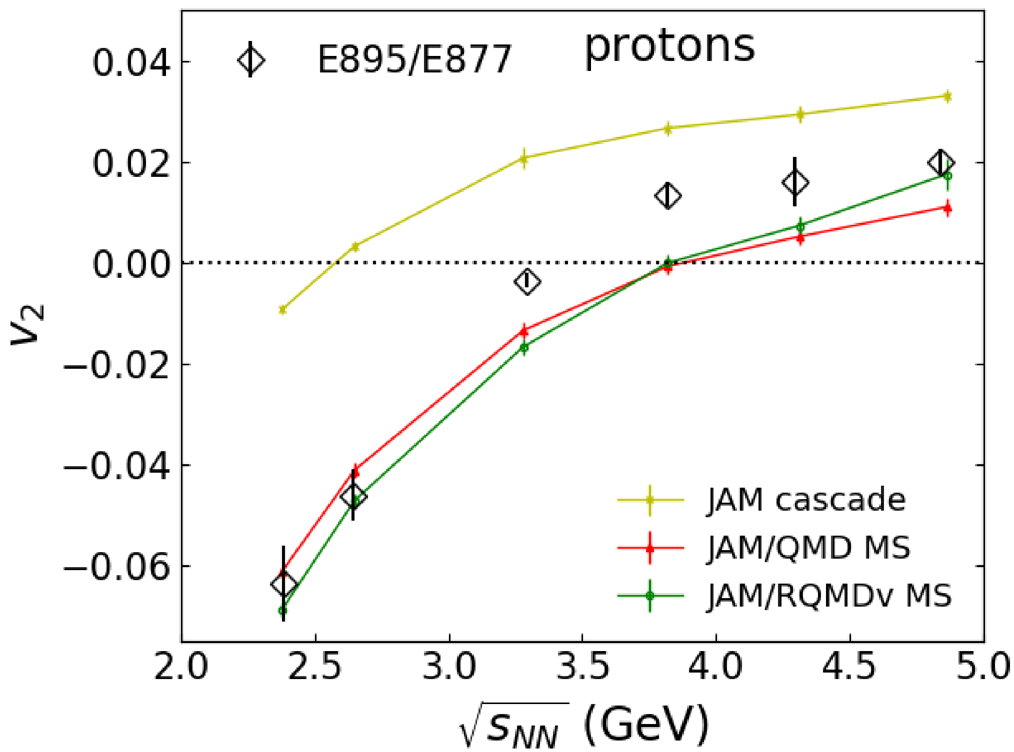
Potential effect in non-relativistic QMD is larger than in RQMDs.
As incident energy becomes higher, QMD and RQMDs yields the similar v1.
This is due to the stronger squeeze-out effect in QMD and shadowing reduces the v1.

V2 at AGS energies (Vector)

Hard EoS (K=380MeV)



Mom. Dep. Soft (K=270MeV)

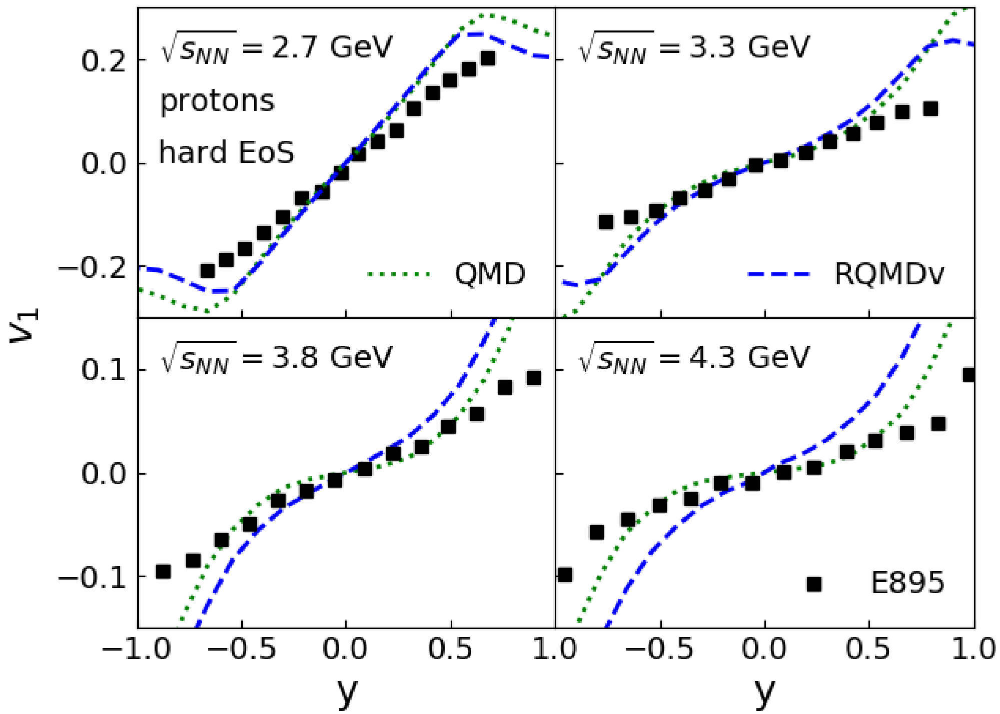


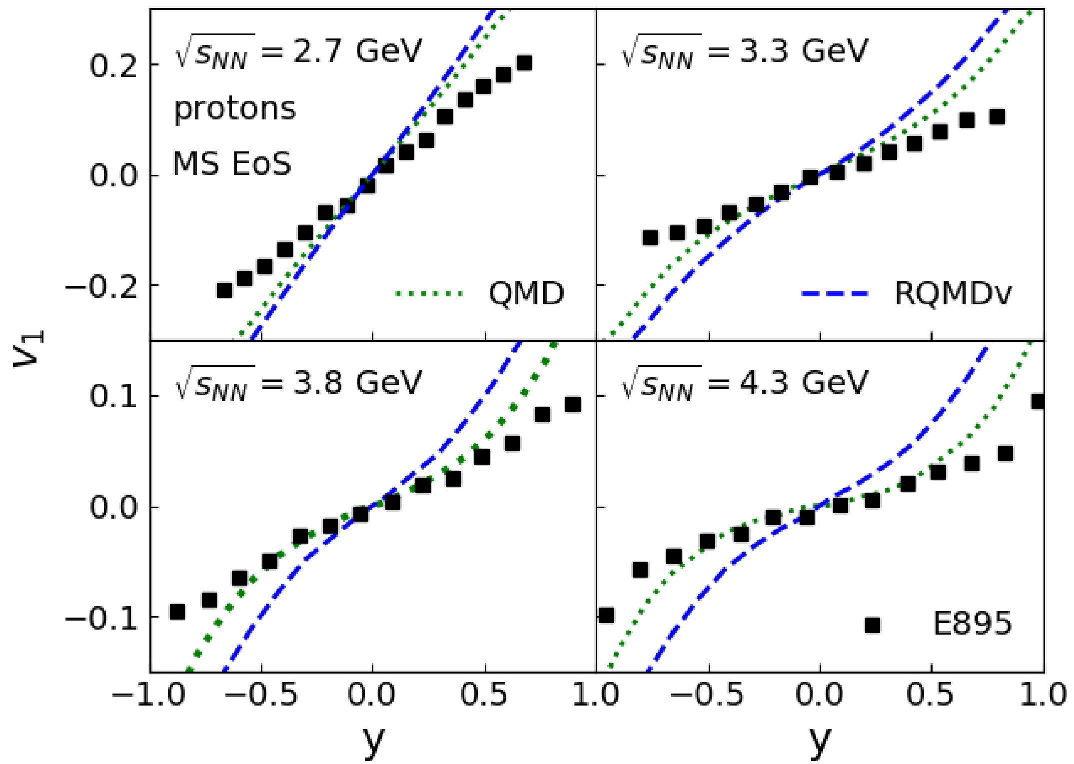
At Elab=1AGeV, v2 from RQMDv and non-rel QMD are very close.

V1 at AGS energies (Vector)

Hard EoS (K=380MeV)

Mom. Dep. Soft EoS

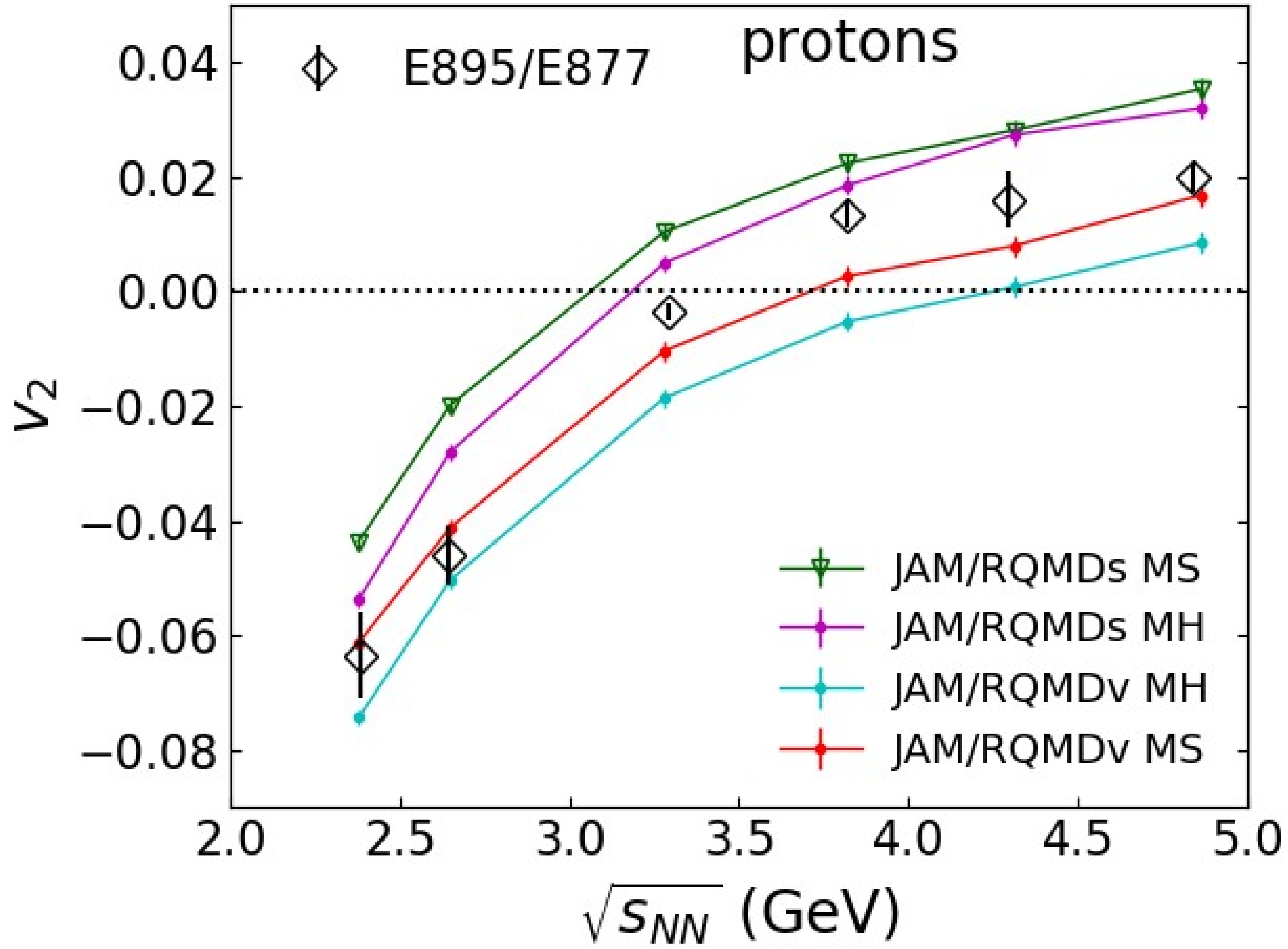




V1 in non-rel QMD yields less v1 than in RQMDv which is due to strong squeeze-out in non-rel QMD.

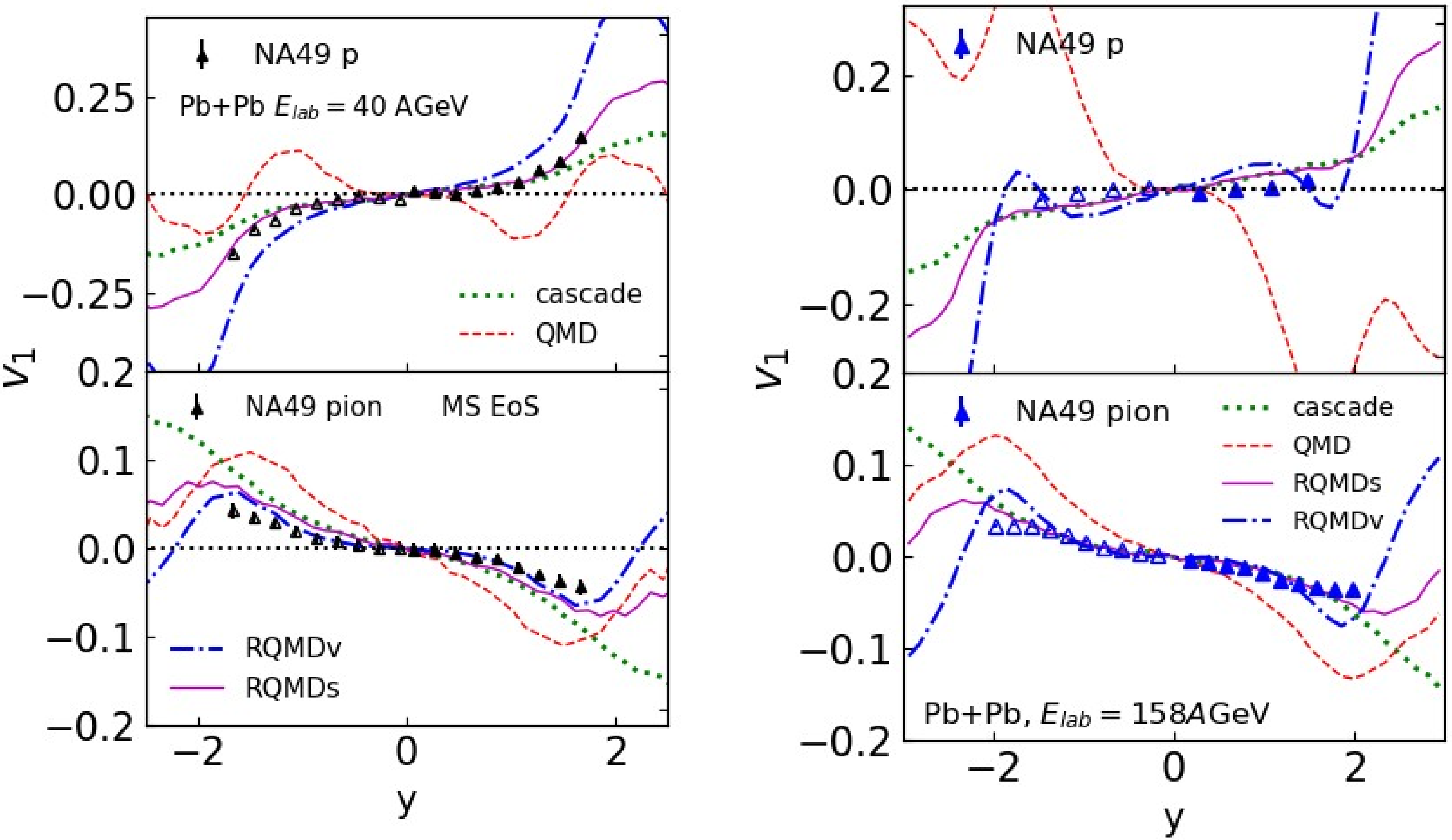
EoS dependence on v2

Mom. Dep. Soft EoS (K=270MeV), hard (K=370 MeV)



V1 at Elab=40 and 158AGeV

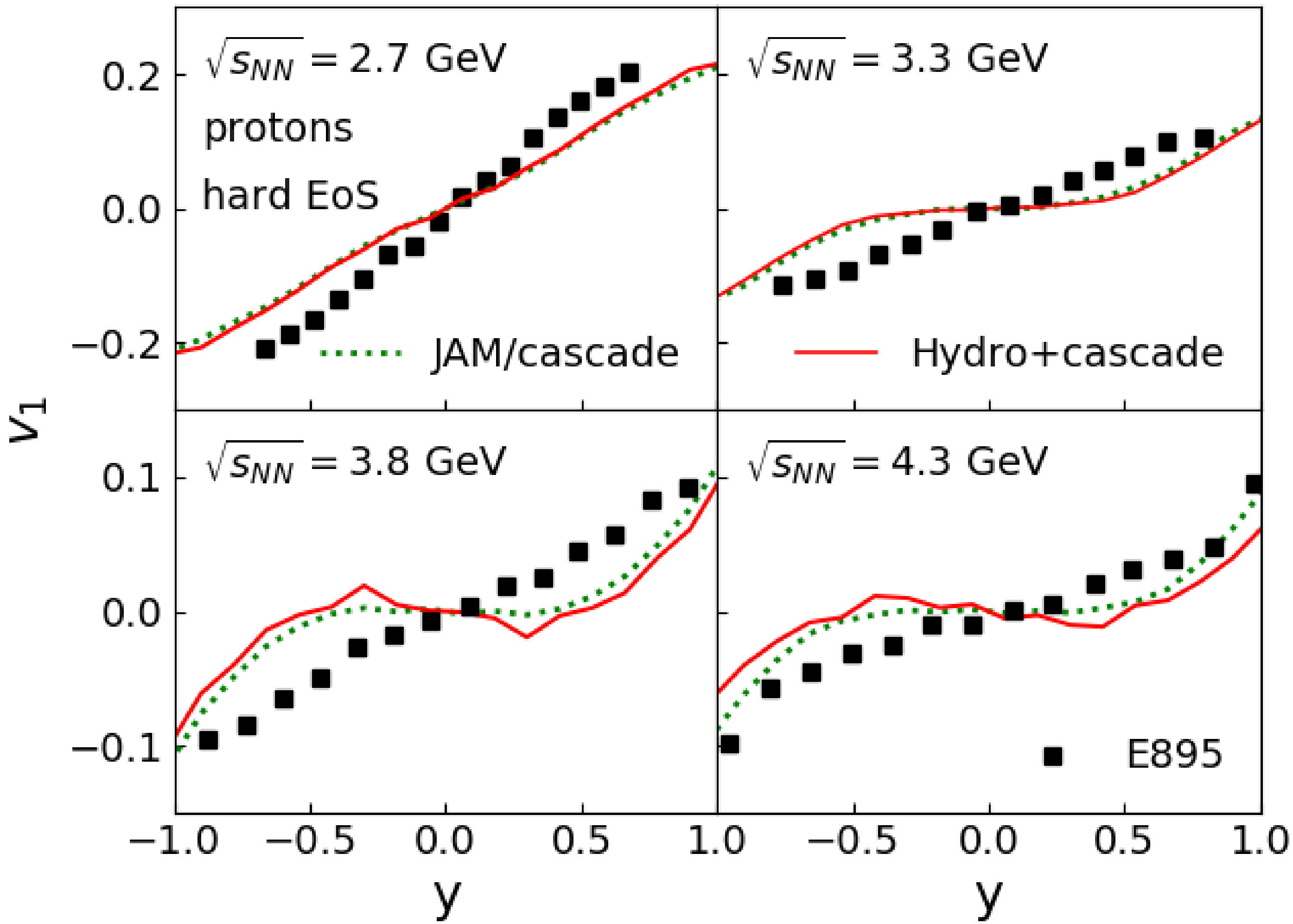
Mom. Dep. Soft (K=270MeV)



At SPS energies, vector potential is too strong, and scalar potential is better.

V1 from the Hydro + JAM/cascade model

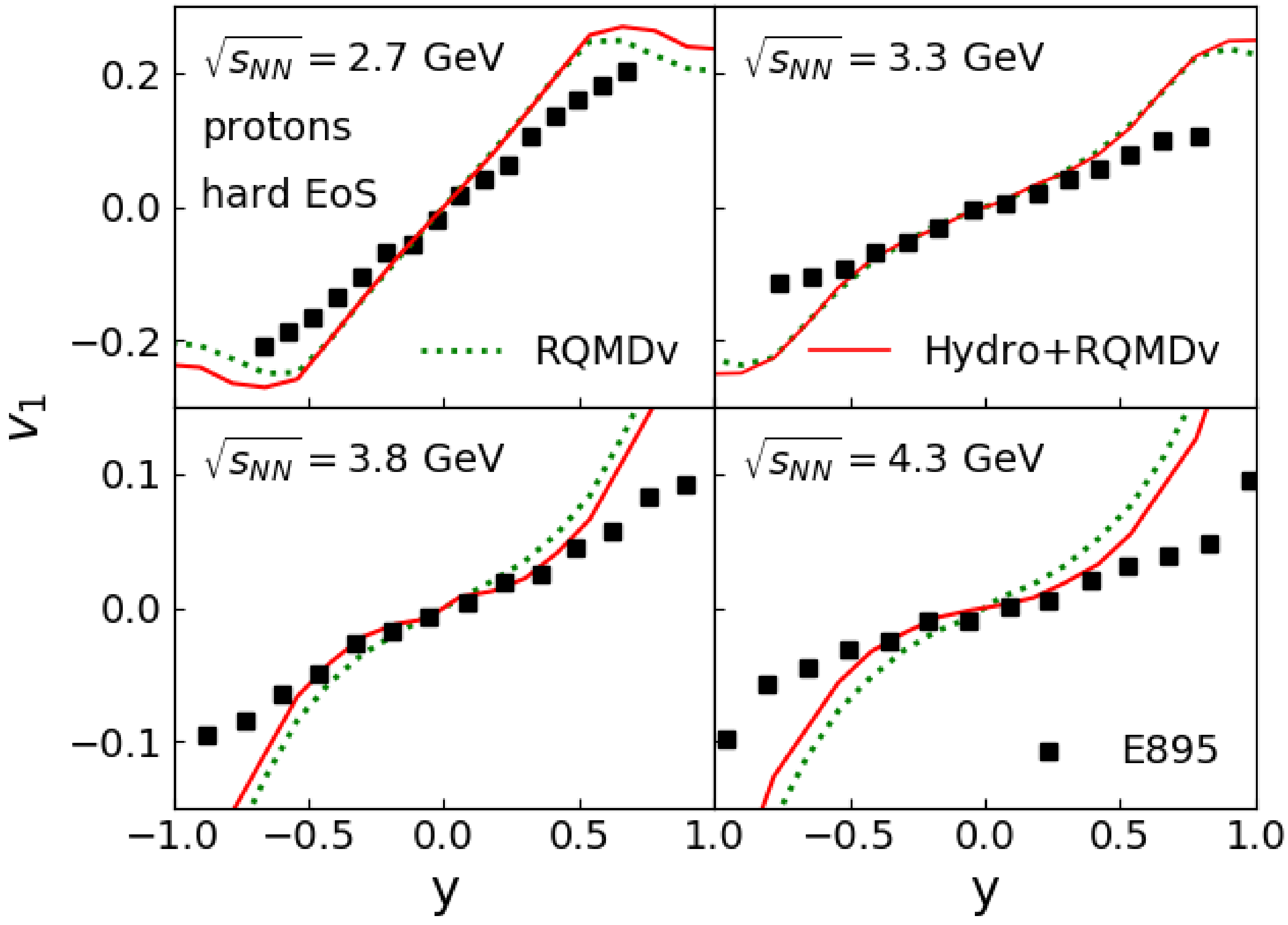
Hydro + cascade approach does not reproduce v1 slope at low energies.



Single particle potential: $V_i = \alpha \left(\frac{\rho}{\rho_0}\right) + \beta \left(\frac{\rho}{\rho_0}\right)^{\gamma}$ Hard EoS: K=380MeV

Hydro + RQMDv result

Results from RQMDv in which potentials are implemented as a vector.



Mean-field in the particle phase is very important for the flow.

Summary

- JAM is a microscopic transport model for high energy nuclear collisions based on strings and hadronic resonances.

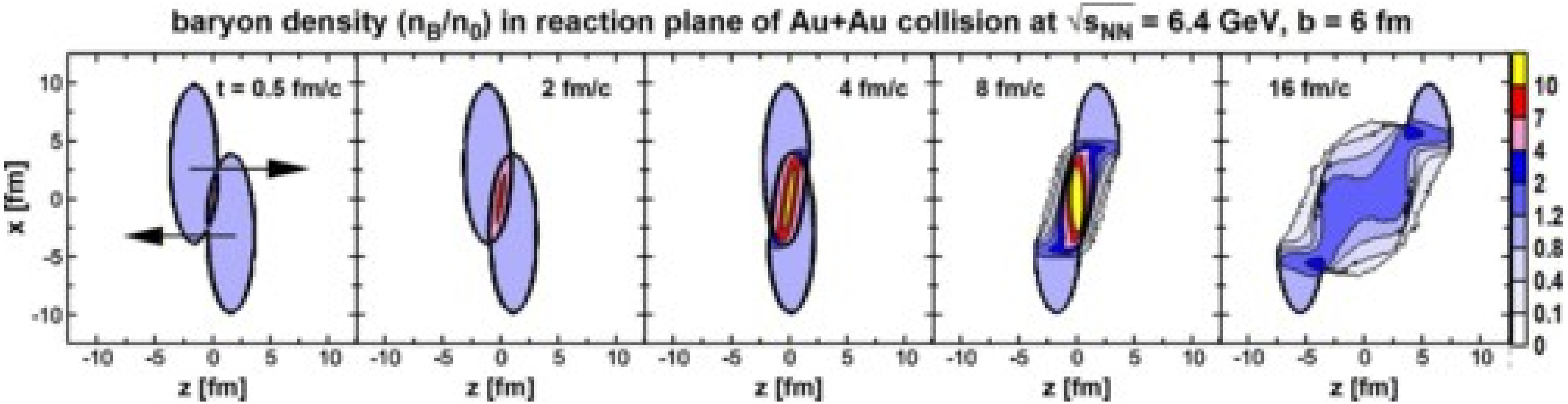
- We have investigate the effects of the scalar (RQMDs) and vector potential (RQMDv) effects on particle productions and flows.

- Both RQMDs and RQMDv cannot explain the excitation function of flows with the single parameter set.

- We extend the JAM+hydro approach by including the EoS effects in the particle phase within the QMD approach: Hydro + QMD.

New dynamically integrated transport model

Picture from 3FD model: P. Batyuk et.al. PRC94(2016)044817



low density part: hadronic transport model
high density part: hydrodynamics

We solve the dynamical evolution of hydro and particle part
at the same time.

Y. Akamatsu, M. Asakawa, T. Hirano, M. Kitazawa,K. Morita, K. Murase,
Y. Nara, C. Nonaka, A. Ohnishi, Phys.Rev. C98 (2018) no.2, 024909

Phys.Rev. C98 (2018) no.2, 024909

A new approach: JAM+hydro model

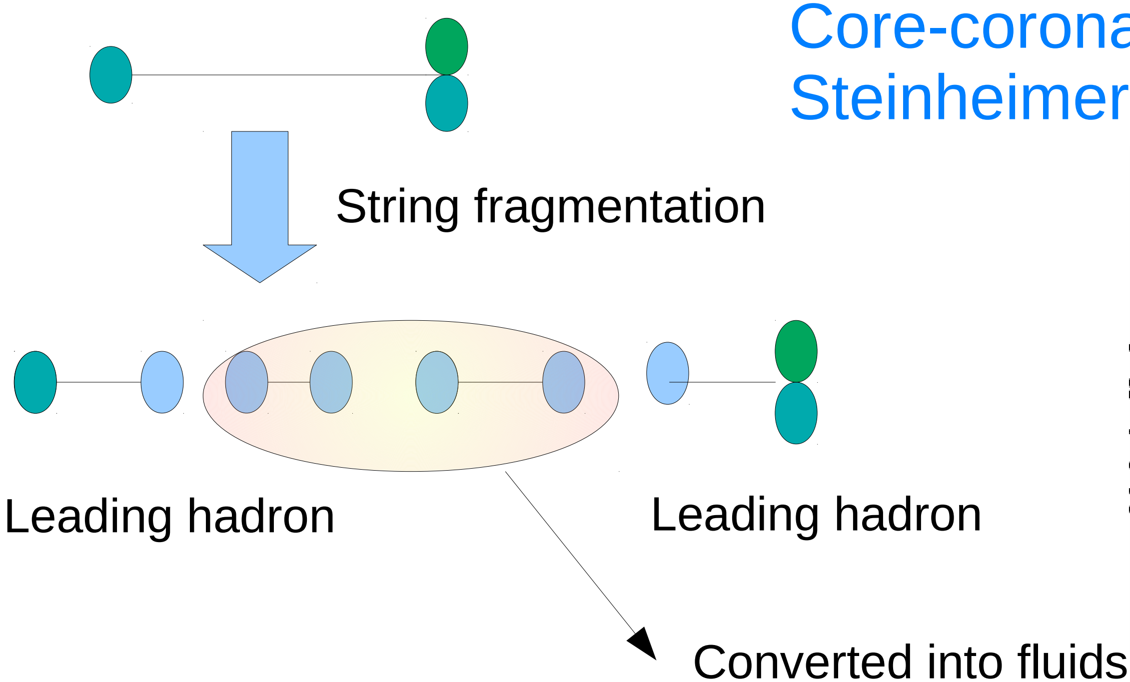
Dynamical coupling of fluids through source terms

$$\partial_\mu T_f^{\mu\nu} = J^\nu, \quad \partial_\mu N_B^\mu = \rho_B$$

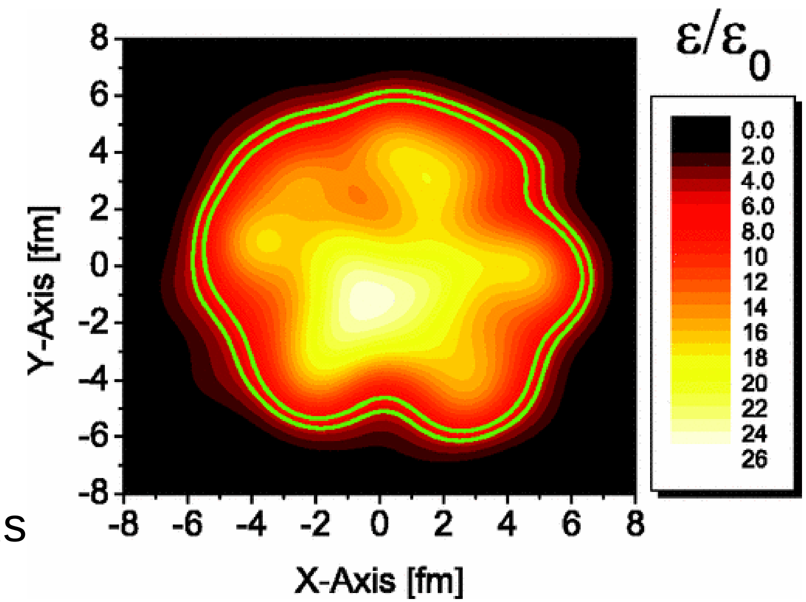
Dynamical initial condition for hydrodynamics M. Okai, et. al Phys. Rev C 95, 054914 (2017)

Time dependent Core-corona separation

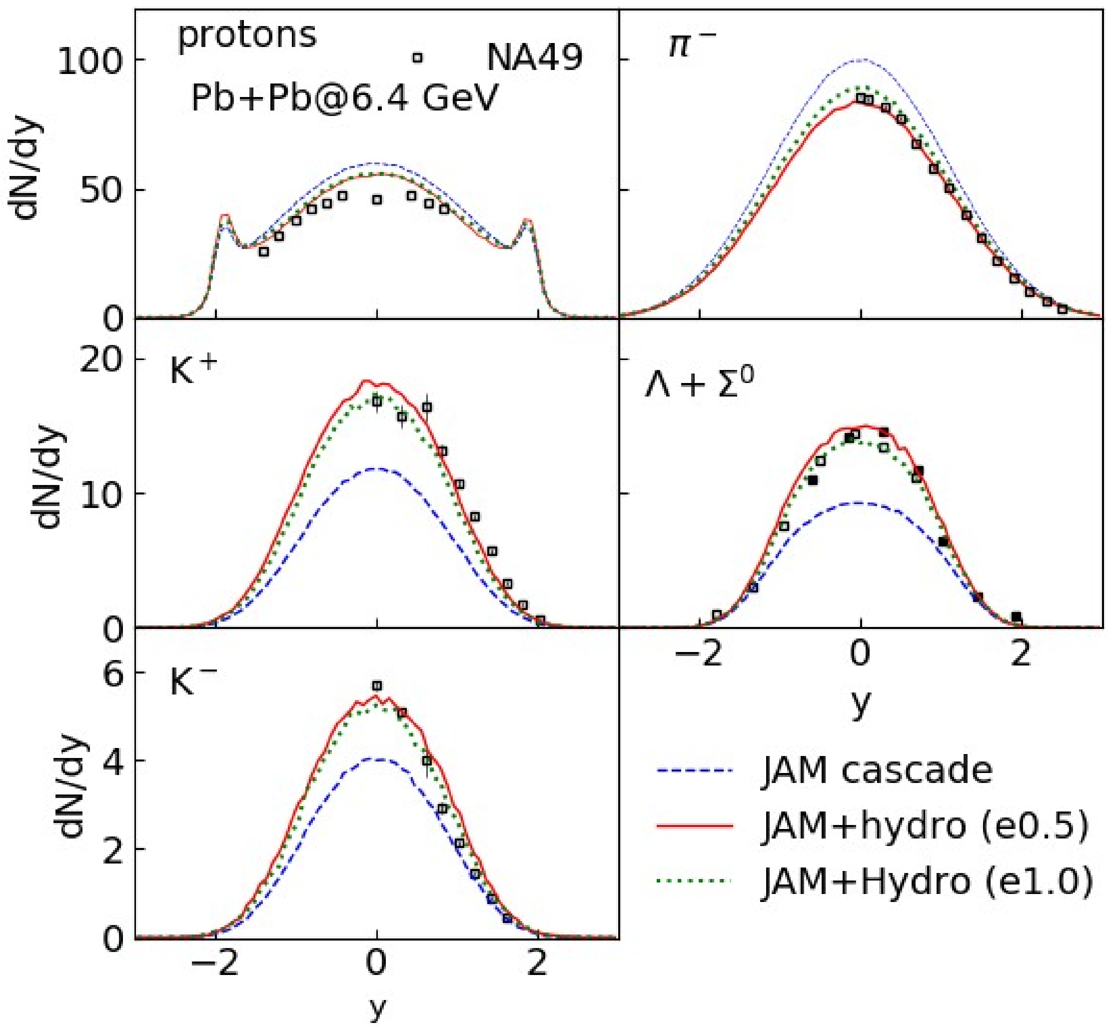
Put Hadrons from string or resonance decay into fluids after their formation time except leading hadrons when local energy density exceeds a hydronization energy density

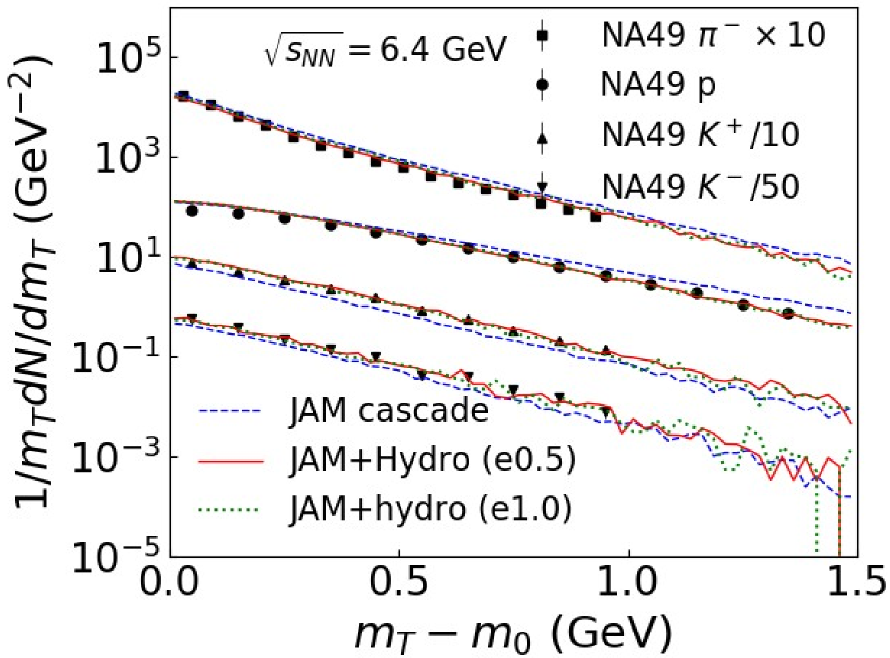


Core-corona separation (K. Werner, 2007)
Steinheimer and Bleicher (2011)



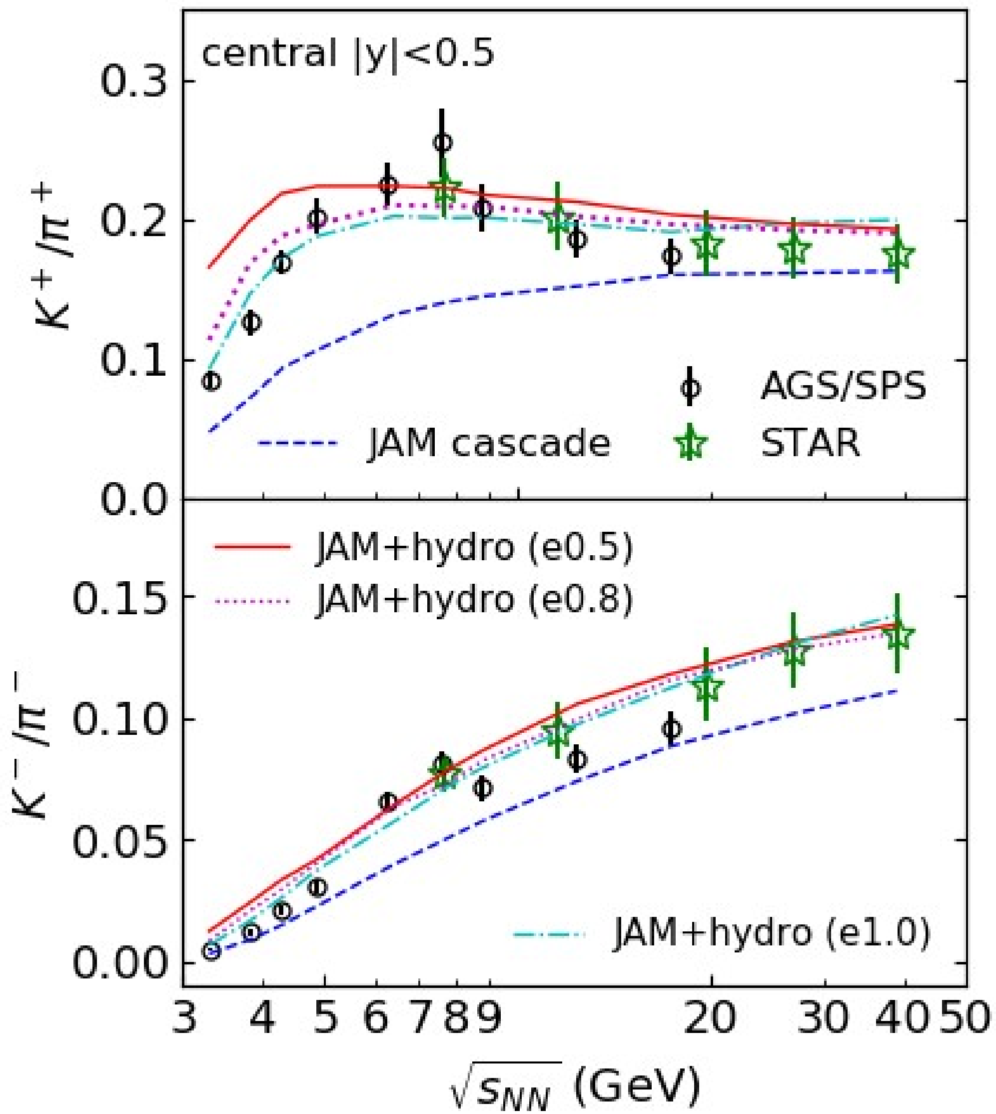
Particle spectra from a new hybrid model in Pb+Pb at Elab=20AGeV



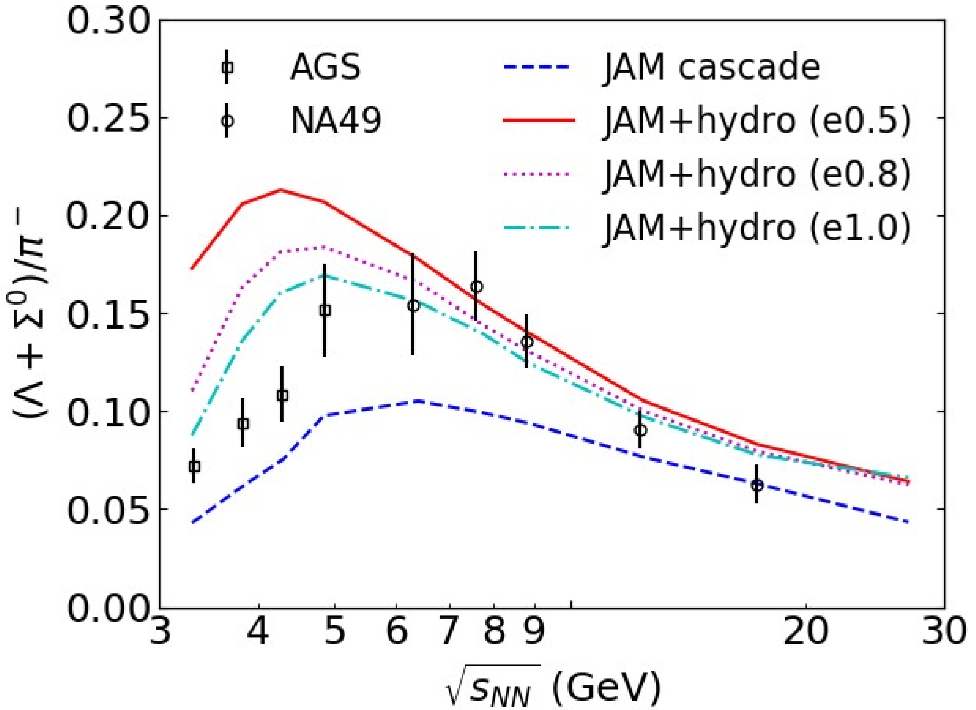


Fluidization energy density 0.5 or 1.0

Beam energy dependence of K/pi ratios from hybrid model



Incomplete thermalization of the sytem is important for the description of K/pi ratio.



Beam energy dependence of transverse mass and multiplicities from a new hybrid model