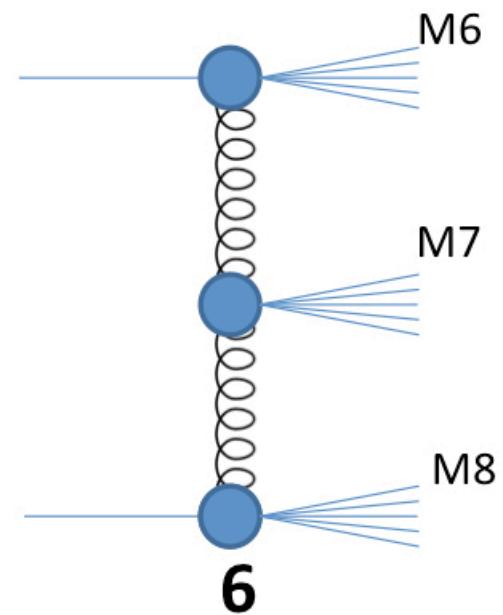
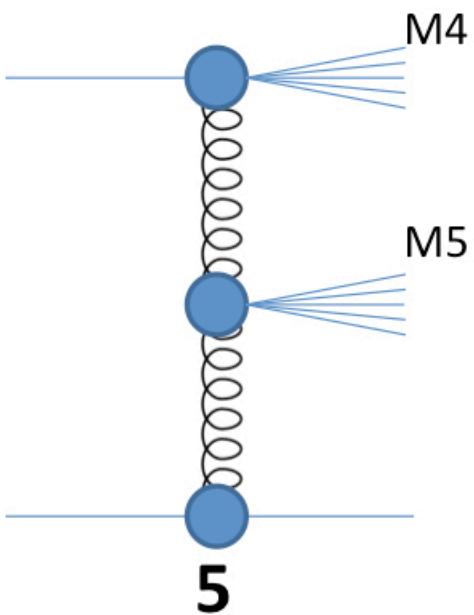
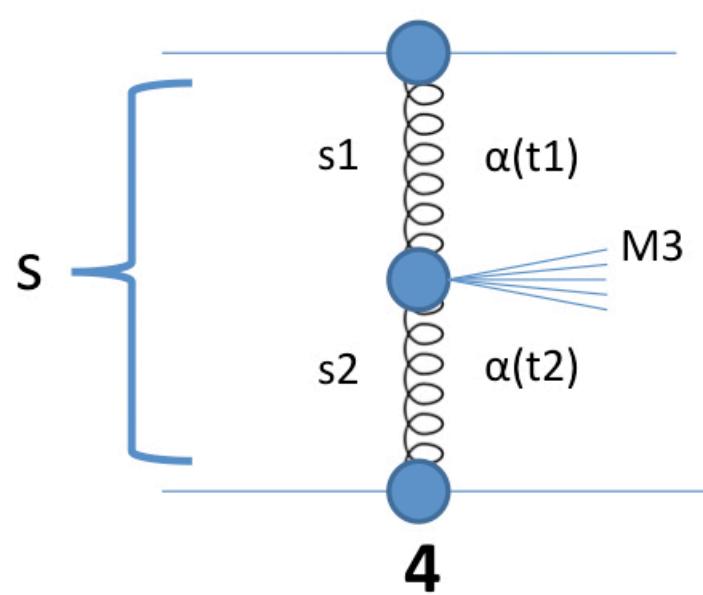
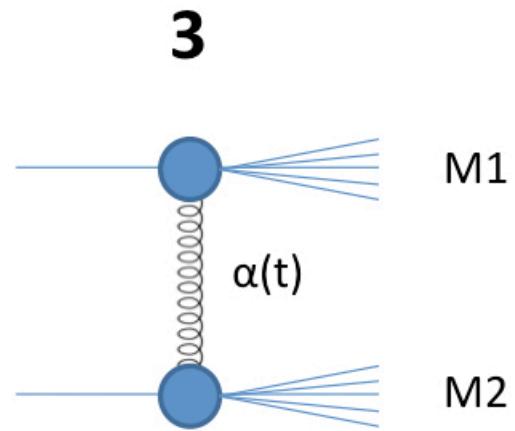
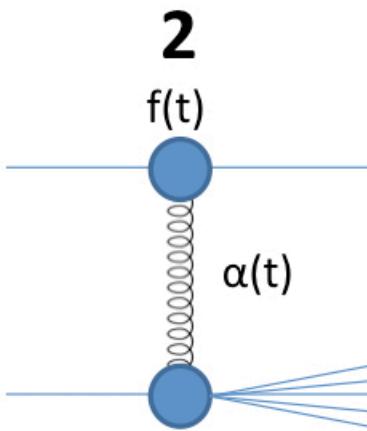
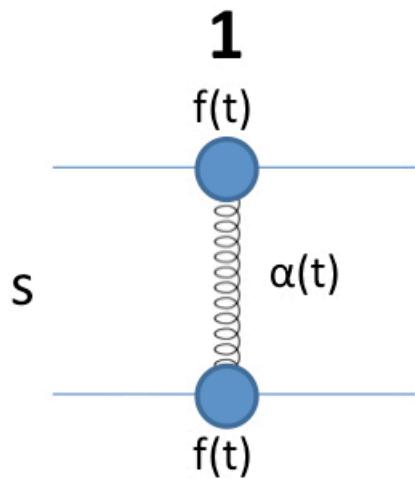


Central exclusive production at the LHC  
Heidelberg University, February 6, 2019

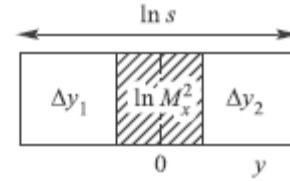
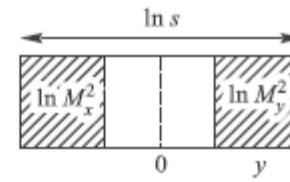
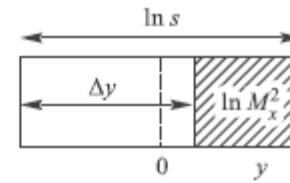
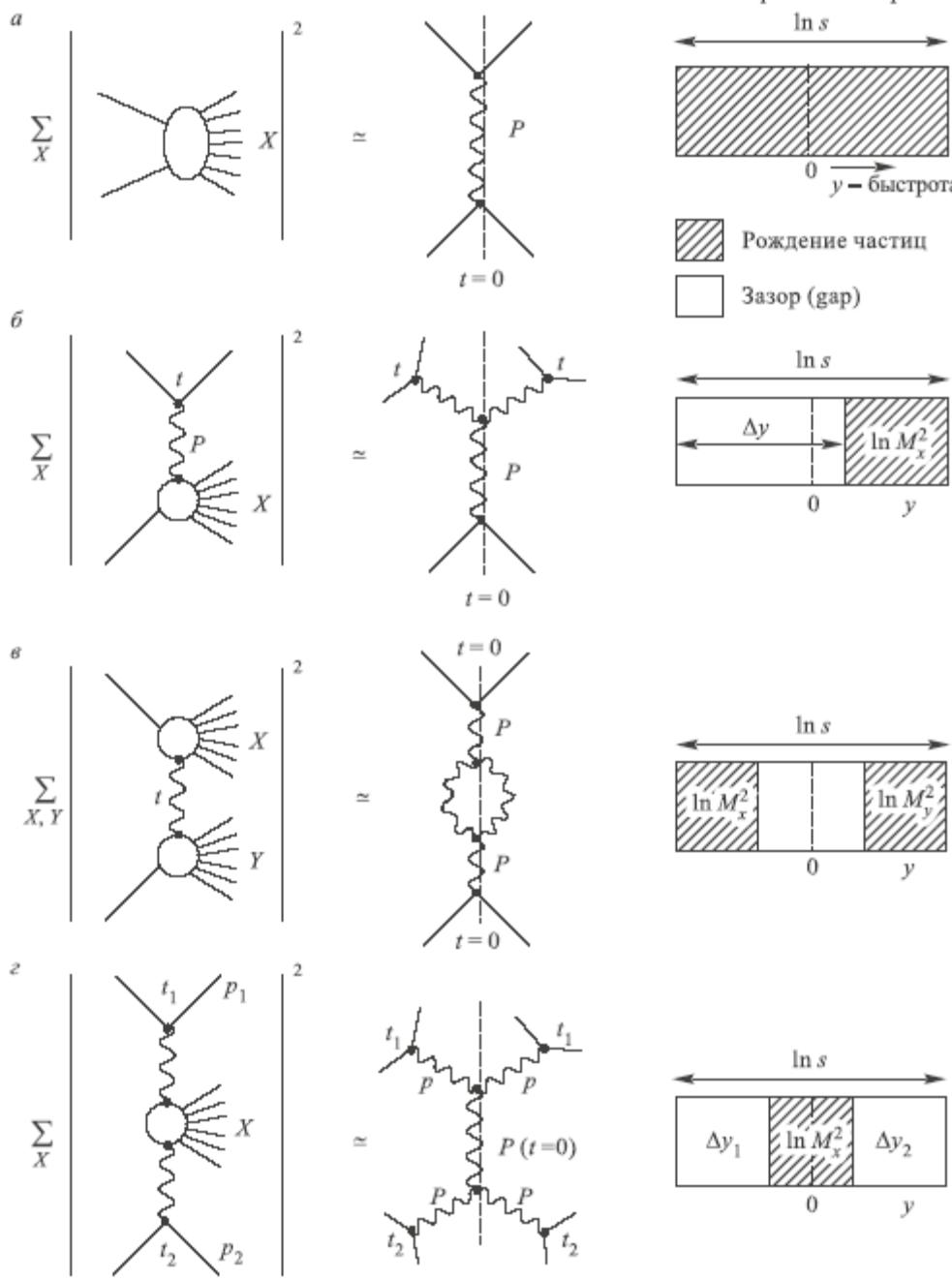
# CEP in a Veneziano-type model

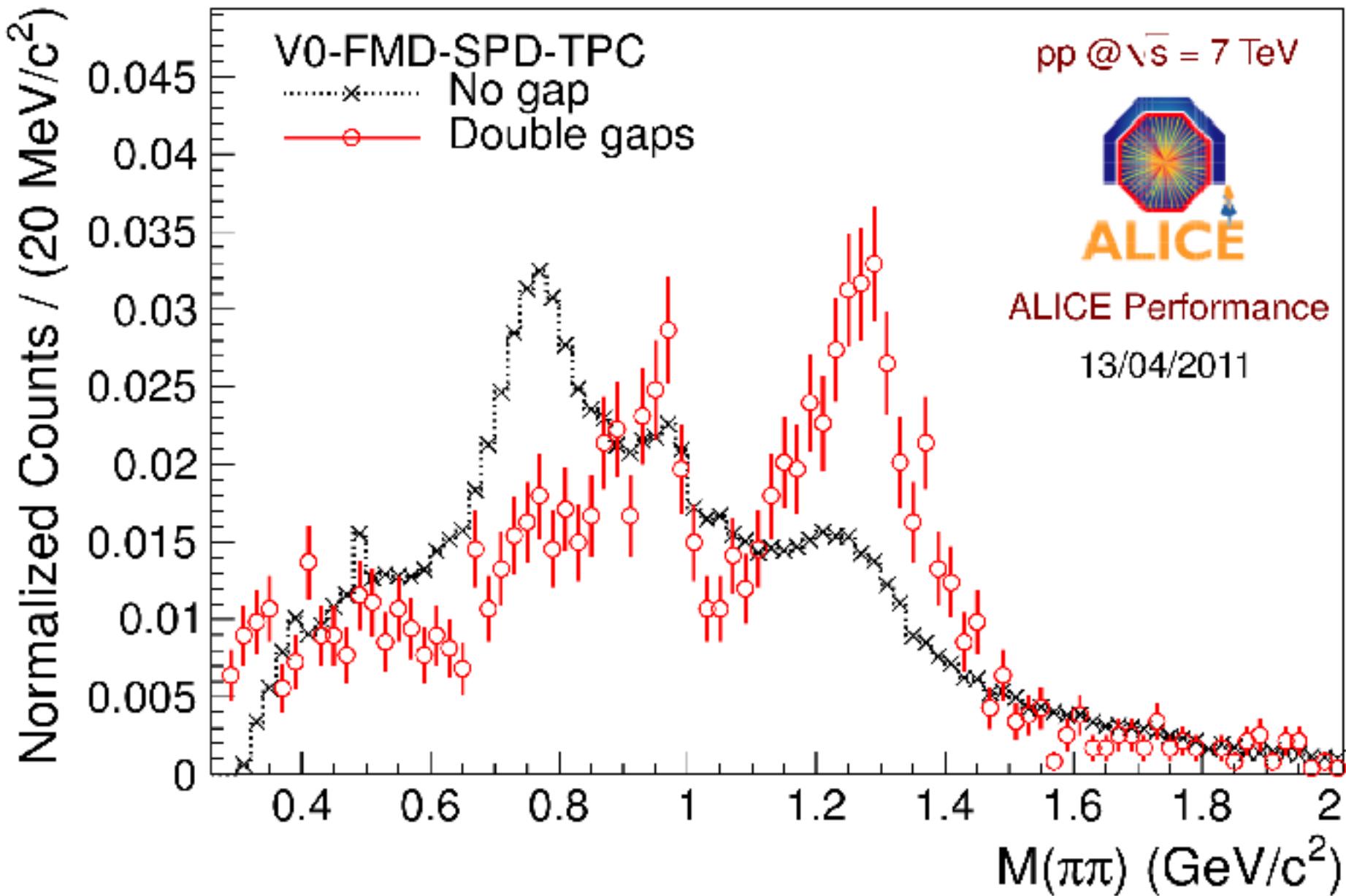
László L. Jenkovszky  
[jenk@bitp.kiev.ua](mailto:jenk@bitp.kiev.ua)

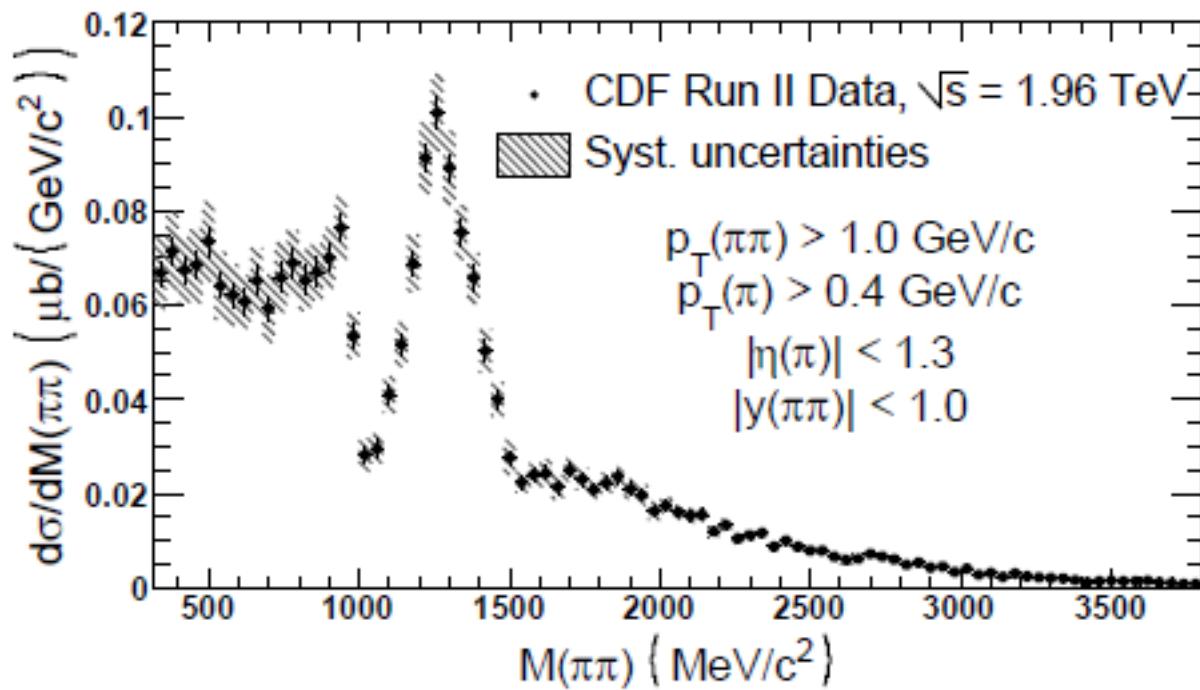


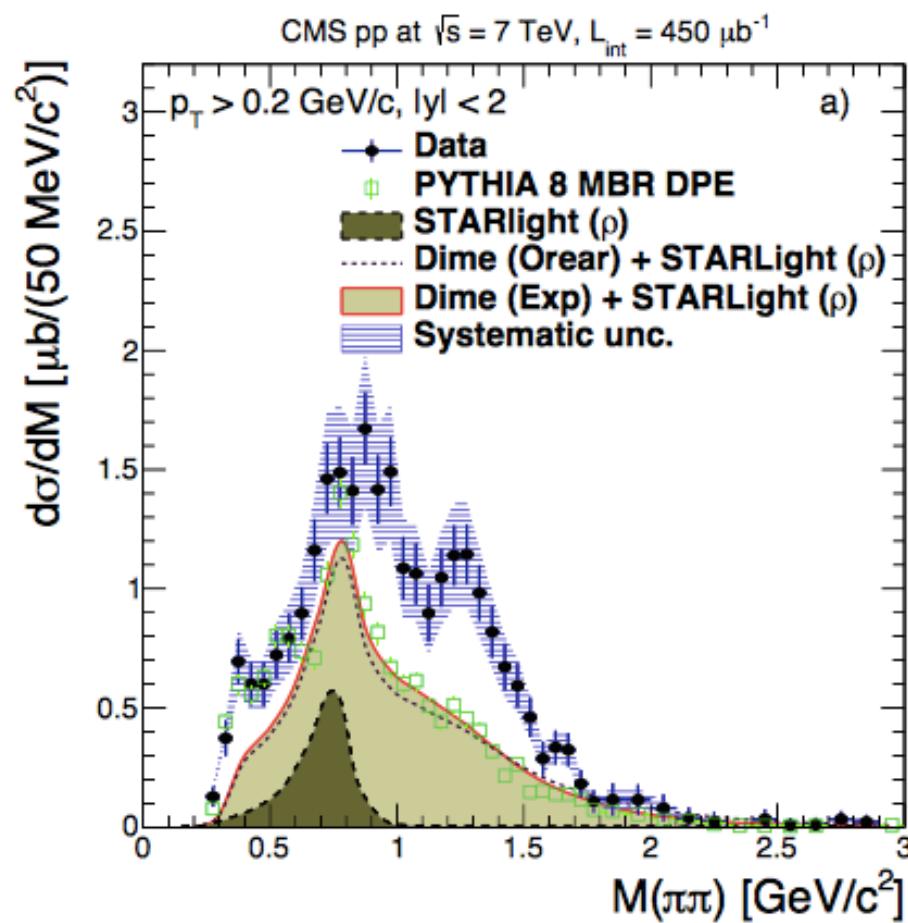
- A. Donnachie, H. G. Dosch, P. V. Landshoff and O. Nachtmann, Pomeron physics and QCD, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 19 (2002) 1;
- [A. Kirk](#), **Resonance production in central pp collisions at the CERN Omega Spectrometer**, Phys.Lett.B489:237,2000, [arXiv:hep-ph/0008053](#);
- [Frank Close](#), [Andrew Kirk](#), **A Glueball-  $q\bar{q}$  Filter in Central Hadron Production**, [arXiv:hep-ph/9701222](#);
- [Felipe J. Llanes-Estrada](#), [Stephen R. Cotanch](#), [Pedro J. de A. Bicudo](#), [J. Emilio F. T. Ribeiro](#), [Adam P. Szczepaniak](#), **QCD Glueball Regge Trajectories and the Pomeron**, [arXiv:hep-ph/0008212](#).
- [Daniel Britzger](#), [Carlo Ewerz](#), [Sasha Glazov](#), [Otto Nachtmann](#), [Stefan Schmit](#), **The Tensor Pomeron and Low-x Deep Inelastic Scattering**, [arXiv:1901.08524](#);

- R. Fiore, L. Jenkovszky, R. Schicker, Exclusive diffractive resonance production in proton-proton collisions at high energies, Eur. Phys. J. C (2018) 78: 468; [arXiv:1711.08353](#);
- Roberto Fiore, Laszlo Jenkovszky, Rainer Schicker, Exclusive Diffractive Resonance Production in Proton-Proton Collisions at the LHC, Proceedings International Workshop on Diffraction in High-Energy Physics, DIFFRACTION 2016, Sept 2-8, 2016, Acireale, [arXiv:1612.06379](#);
- R.Schicker, R.Fiore, L.Jenkovszky, Resonance production in Pomeron-Pomeron collisions at the LHC, Proceedings XXIV Int. Workshop on Deep-Inelastic Scattering and Related Subjects (DIS16), DESY, 11-15 April 2016, [arXiv:1607.04496](#);
- R. Fiore. L. Jenkovszky, R. Schicker, Resonance production in Pomeron-Pomeron collisions at the LHC, The European Physical Journal C, 76(1), 2016, 1-10; [arXiv:1512.04977](#).

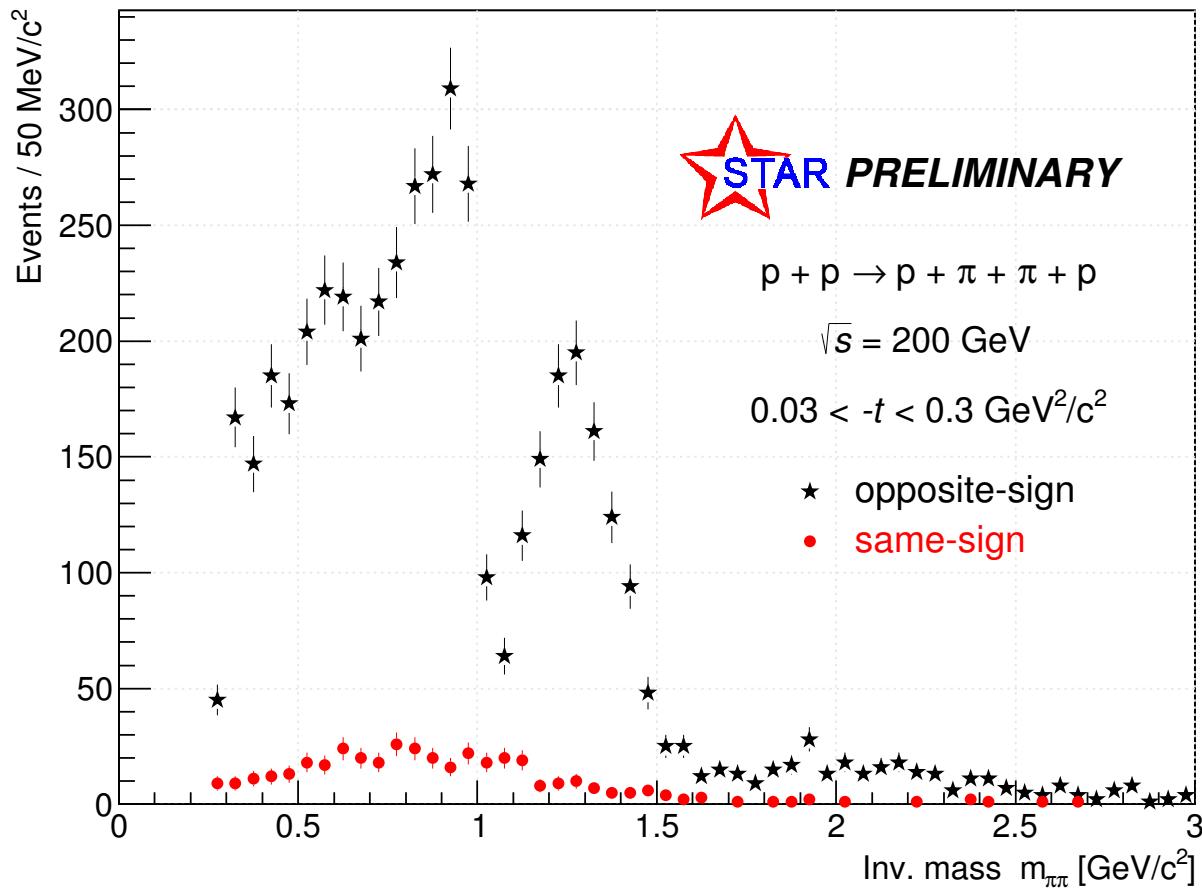








Invariant mass of  $\pi\pi$ ,  $p_T^{\text{miss}} < 0.1 \text{ GeV}/c$ , not acceptance-corrected, statistical errors only



$$\sigma_t(s) = \frac{4\pi}{s} \text{Im} A(s, t=0); \quad \frac{d\sigma}{dt} = \frac{\pi}{s^2} |A(s, t)|^2; \quad n(s);$$

$$\sigma_{el} = \int_{t_{min} \approx -s/2 \approx \infty}^{t_{thr. \approx 0}} \frac{d\sigma}{dt} dt; \quad \sigma_{in} = \sigma_t - \sigma_{el}; \quad B(s, t) = \frac{d}{dt} \ln\left(\frac{d\sigma}{dt}\right);$$

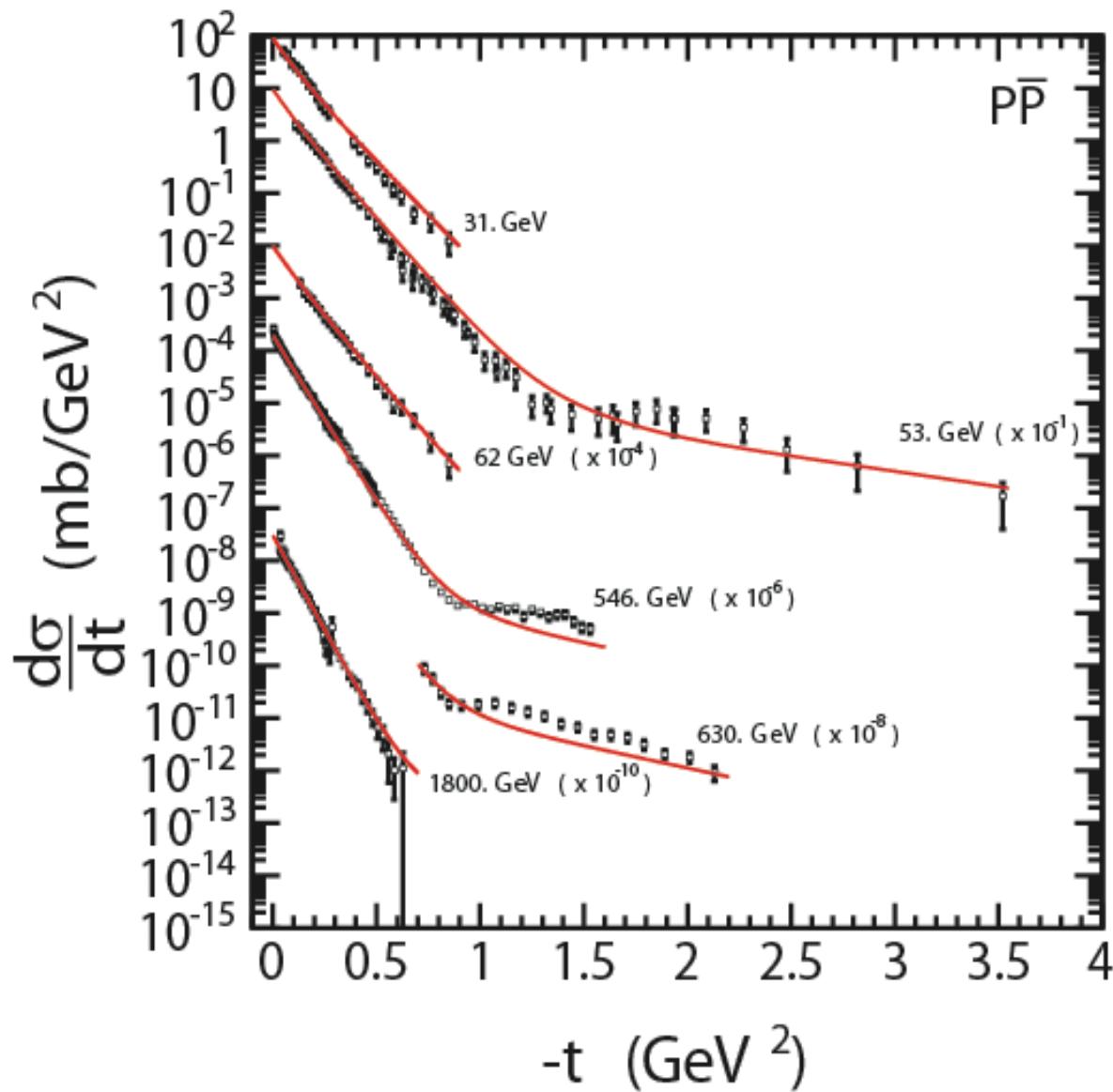
$$A_{pp}^{p\bar{p}}(s, t) = P(s, t) \pm O(s, t) + f(s, t) \pm \omega(s, t) \rightarrow_{LHC} P(s, t) \pm O(s, t),$$

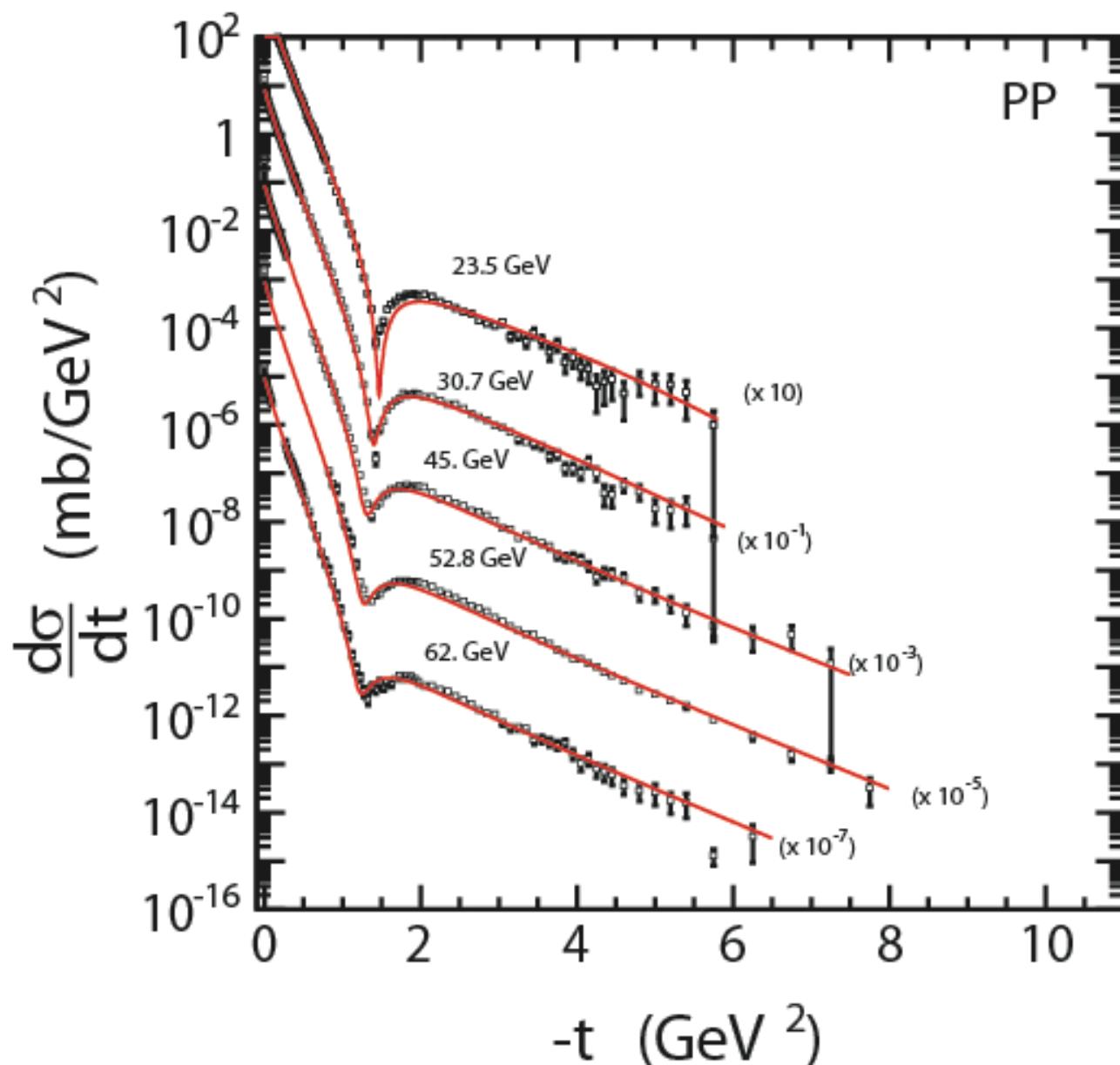
where  $P$ ,  $O$ ,  $f$ .  $\omega$  are the Pomeron, odderon and non-leading Reggeon contributions.

<b>a(0)\C</b>	<b>+</b>	<b>-</b>
<b>1</b>	<b>P</b>	<b>O</b>
<b>1/2</b>	<b>f</b>	<b>ω</b>

**NB: The S-matrix theory (including Regge pole) is applicable to asymptotically free states only (not to quarks and gluons)!**

## The ODDERON





## Pomerons (diffraction's) fraction

Energy variation of the relative importance of the Pomeron with respect to contributions from the secondary trajectories and the Odderon:

$$R(s, t = 0) = \frac{\Im m(A(s, t) - A_P(s, t))}{\Im A(s, t)}, \quad (1)$$

where the total scattering amplitude  $A$  includes the Pomeron contribution  $A_P$  plus the contribution from the secondary Reggeons and the Odderon.

Starting from the Tevatron energy region, the relative contribution of the non-Pomeron terms to the total cross-section becomes smaller than the experimental uncertainty and hence at higher energies they may be completely neglected, irrespective of the model used.

$$R(s, t) = \frac{|(A(s, t) - A_P(s, t)|^2}{|A(s, t)|^2}. \quad (2)$$

## The Pomeron trajectory

The Pomeron trajectory has threshold singularities, the lowest one being due to the two-pion exchange, required by the  $t$ -channel unitarity. There is a constrain (Barut, Zwanziger; Gribov) from the  $t$ -channel unitarity, by which

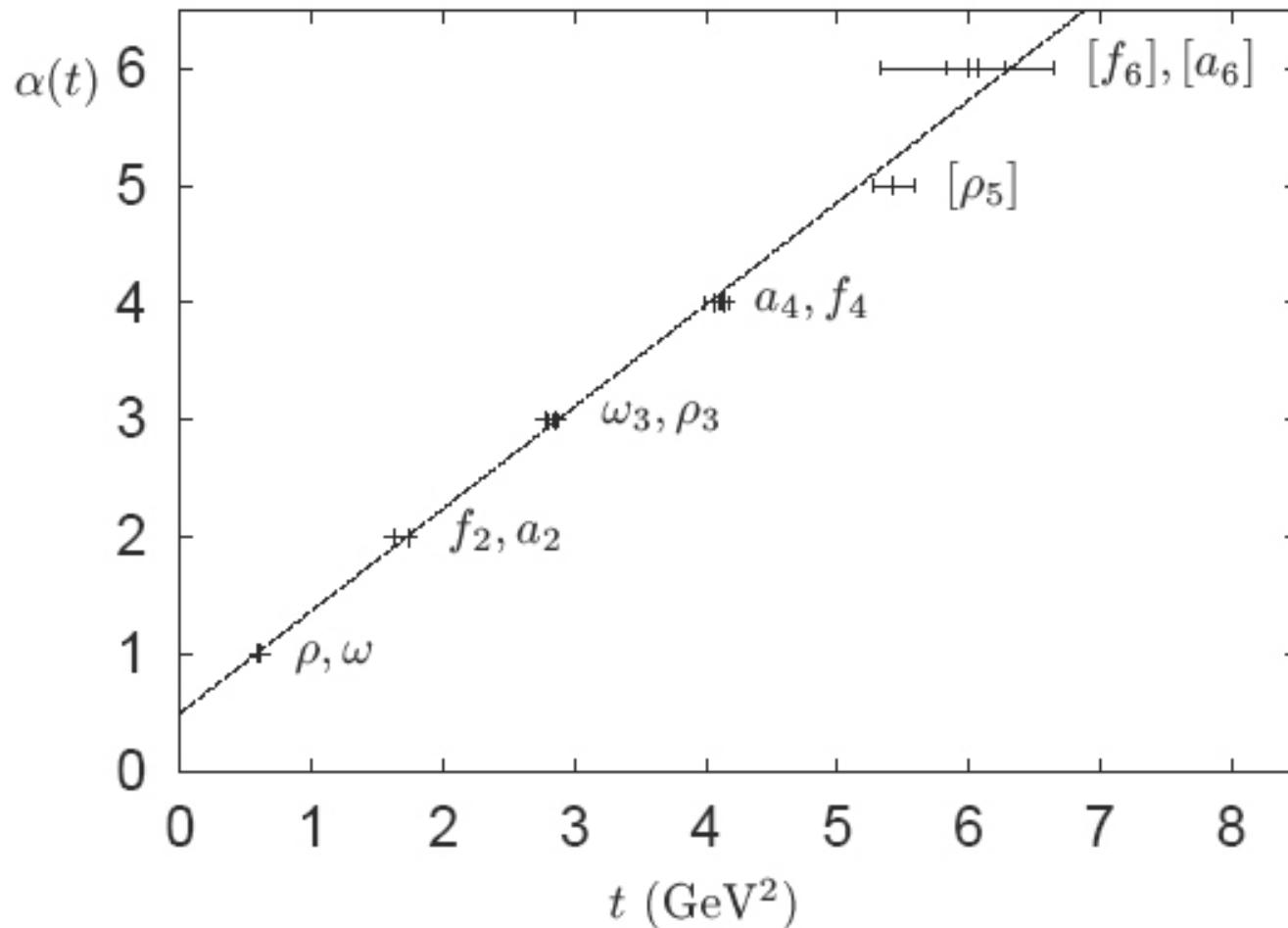
$$\Im\alpha(t) \sim (t - t_0)^{\Re\alpha(t_0) + 1/2}, \quad t \rightarrow t_0,$$

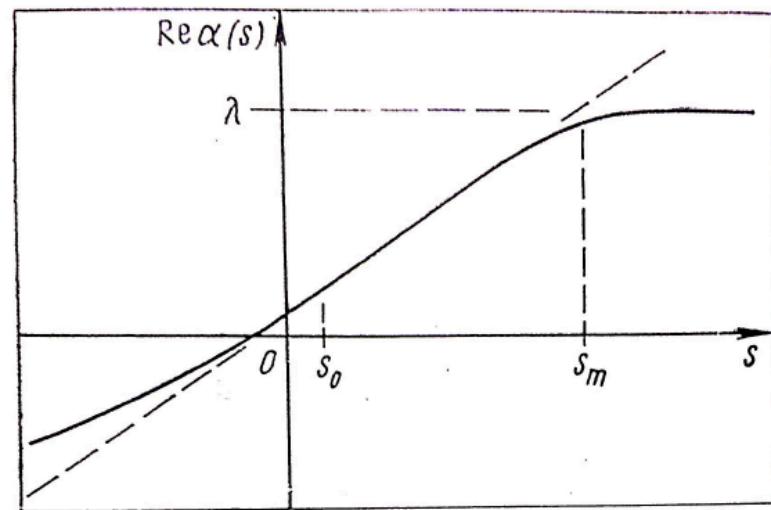
where  $t_0$  is the lightest threshold. For the Pomeron trajectory it is  $t_0 = 4m_\pi^2$ , and near the threshold:

$$\alpha(t) \sim \sqrt{4m_\pi^2 - t}. \tag{1}$$

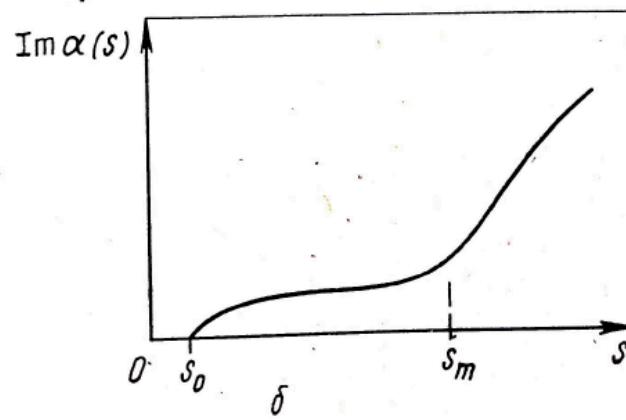
# Linear particle trajectories

Plot of spins of families of particles against their squared masses:





$\alpha$

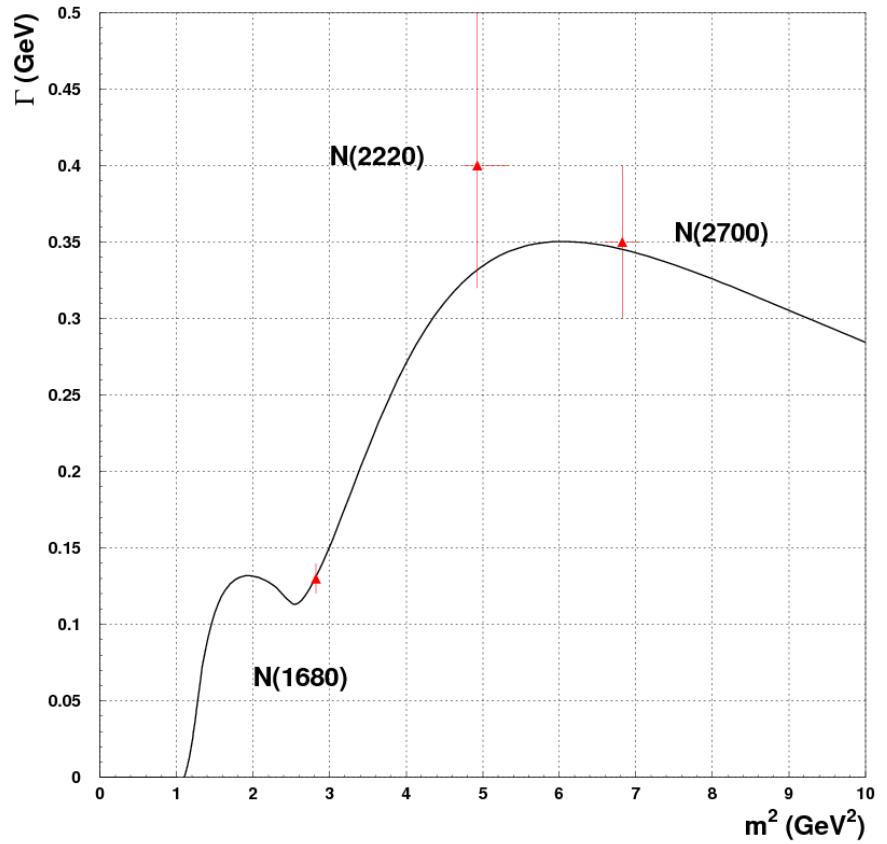
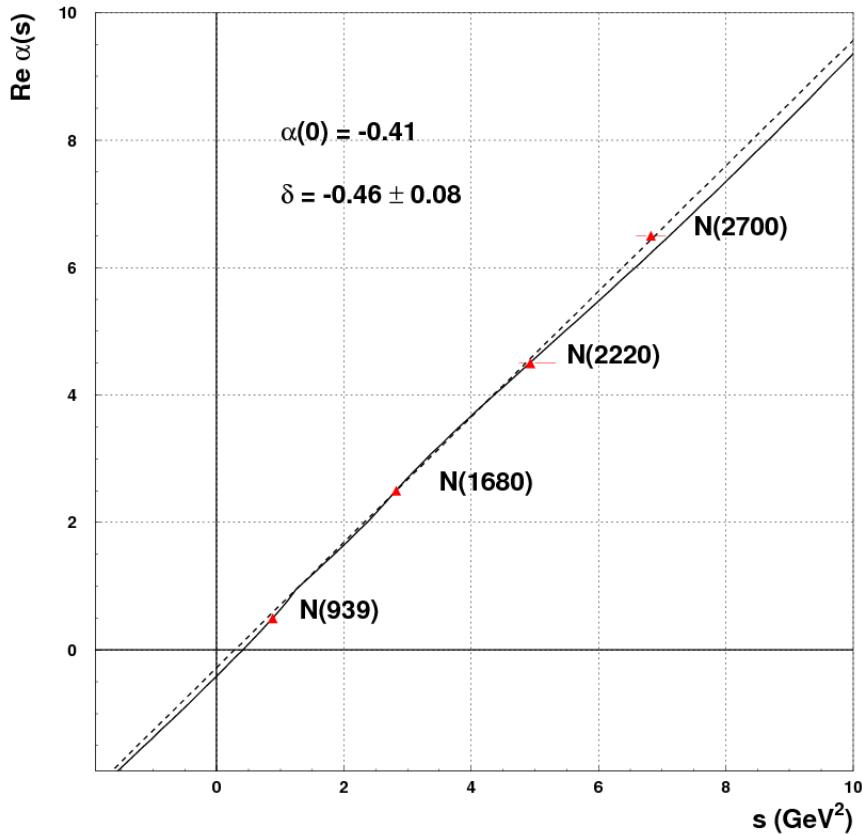


The real part of the proton trajectory is given by

$$\mathcal{R}e \alpha(s) = \alpha(0) + \frac{s}{\pi} \sum_n c_n \mathcal{A}_n(s) , \quad (1)$$

where

$$\begin{aligned} \mathcal{A}_n(s) = & \frac{\Gamma(1-\delta)\Gamma(\lambda_n+1)}{\Gamma(\lambda_n-\delta+2)s_n^{1-\delta}} {}_2F_1 \left( 1, 1-\delta; \lambda_n - \delta + 2; \frac{s}{s_n} \right) \theta(s_n - s) + \\ & \left\{ \pi s^{\delta-1} \left( \frac{s-s_n}{s} \right)^{\lambda_n} \cot[\pi(1-\delta)] - \right. \\ & \left. \frac{\Gamma(-\delta)\Gamma(\lambda_n+1)s_n^\delta}{s\Gamma(\lambda_n-\delta+1)} {}_2F_1 \left( \delta - \lambda_n, 1; \delta + 1; \frac{s_n}{s} \right) \right\} \theta(s - s_n) . \end{aligned}$$



The imaginary part of the trajectory can be written in the following way:

$$\text{Im } \alpha(s) = s^\delta \sum_n c_n \left( \frac{s - s_n}{s} \right)^{\lambda_n} \cdot \theta(s - s_n), \quad (1)$$

where  $\lambda_n = \text{Re } \alpha(s_n)$ .

# The optical (generalised optical (Müller) theorem and triple-Regge limit (for high M only!)

The diagram illustrates the optical (generalised optical (Müller) theorem and triple-Regge limit (for high M only!).

**Top Left:** A Feynman diagram showing two horizontal lines, each with a vertex and a wavy gluon exchange. The top line is labeled  $h \rightarrow h$  and the bottom line is labeled  $A \rightarrow A$ . The vertices are connected by a wavy line labeled  $t$ , with a label  $g_{hh}(t)$  above the top vertex and  $g_{AA}(t)$  below the bottom vertex.

**Top Right:** A Feynman diagram showing a horizontal line  $h \rightarrow X$  with a vertex and a wavy gluon exchange. The exchange is labeled  $t$  and the vertex is connected to another horizontal line  $A \rightarrow A$  with a label  $g_{hx}(t)$  above the vertex and  $g_{AA}(t)$  below the line.

**Bottom:** Two sets of Feynman diagrams illustrating the derivation of the optical theorem and the triple-Regge limit.

**Left Set:**

$$\frac{d^2\sigma}{dt dx} = \left| \begin{array}{c} h \\ | \\ p \end{array} \right. \left. \begin{array}{c} X \\ | \\ t \\ | \\ p \end{array} \right|^2 = \text{Diagram with } t=0 \text{ (two gluons merging)} = \begin{array}{c} h \quad h \\ | \quad | \\ t=0 \\ | \quad | \\ p \quad p \end{array}$$

**Right Set:**

$$\sigma_{\text{tot}} = \left| \begin{array}{c} h \\ | \\ p \end{array} \right. \left. \begin{array}{c} h \\ | \\ t=0 \\ | \\ p \end{array} \right|^2 = \text{Diagram with } t=0 \text{ (three gluons merging)}$$

The differential cross section for  $1 + 2 \rightarrow X$  is

$$\frac{d^2\sigma}{dt dM^2} = \frac{G(t)}{16\pi^2 s_0^2} \left(\frac{s}{s_0}\right)^{2\alpha(t)-2} \left(\frac{M^2}{s_0}\right)^{\alpha(0)-2\alpha(t)}, \quad (1)$$

where  $G(t)$  is the triple Pomeron vertex,  $G(t) = Ge^{at}$  for simplicity, and  $\alpha(t) = \alpha^0 + \alpha'(t)$  is the (linear for the moment) Pomeron trajectory.

For a critical Pomeron,  $\alpha^0 = 1$ , one can use the formula

$$\int \frac{dx}{x \ln x} = \ln(\ln x) \quad (2)$$

to get

$$\sigma^{SD}(s) \sim (2\alpha')^{-1} \ln\left(1 + \frac{2\alpha'}{a} \ln s\right) \sim \ln(\ln s), \quad (3)$$

while the total cross section

$$\sigma^{tot}(s) \rightarrow \text{const.} \quad (4)$$

It contradicts unitarity since e.g. for critical Pomeron,  $\alpha^0 = 1$ , the partial (SD) cross section overshoots the total cross section  $\sigma^{SD} > \sigma^{tot}$ .

A trivial trick to avoid violation of unitarity is to assume the triple Pomeron vertex  $G(t)$  vanishing at  $t = 0$ . Huge literature (Kaidalov, Brower, Ganguli, Kopeliovich,...) exists reflecting the efforts along this direction. The main conclusion is that decoupling (vanishing of the triple Pomeron vertex at  $t = 0$ ) is incompatible with the data.

To remedy this difficulty, Dino Goulian suggested a renormalization procedure, by which the Pomeron flux is multiplied by a factor  $N(s)$  moderating the rise of inelastic diffraction starting from a certain threshold. The appearance of a threshold, however may violate analyticity.

Violation of unitarity in DD:  $\sigma_{SD}(s) > \sigma_t(s)$ ,  $s \rightarrow \infty$ ??)

$$\frac{d^2\sigma}{dt dM^2} = \frac{G^{PPP}(t)}{16\pi^2 s_0^2} (s/s_0)^{2\alpha(t)-2} (M^2/s_0)^{\alpha(0)-2\alpha(t)}.$$

Let, for simplicity,  $\alpha(t) = \alpha^0 + \alpha' t$ , then

$$\sigma^{SD}(s) = \frac{s^{2\alpha^0-2}}{16\pi^2(s_0)^{\alpha^0}} \int_{\epsilon}^s \frac{dM^2}{M^{2\alpha^0}} \int_{-\infty}^0 dt G(t) \exp[2\alpha' t \ln(s/M^2)].$$

Let  $\alpha(t) = 1 + \alpha' t$  and  $G(t) \sim e^{b_0 t}$ , then, due to the relation  $\int \frac{dx}{x \ln x} = \ln \ln x$ , one gets

$$\sigma^{SD} \sim \frac{1}{2\alpha'} \ln(\ln s), \quad s \rightarrow \infty,$$

while  $\sigma^t = \text{const.}$

The Pomeron is a dipole in the  $j$ -plane

$$A_P(s, t) = \frac{d}{d\alpha_P} \left[ e^{-i\pi\alpha_P/2} G(\alpha_P) \left( s/s_0 \right)^{\alpha_P} \right] = \quad (1)$$

$$e^{-i\pi\alpha_P(t)/2} \left( s/s_0 \right)^{\alpha_P(t)} \left[ G'(\alpha_P) + \left( L - i\pi/2 \right) G(\alpha_P) \right].$$

Since the first term in squared brackets determines the shape of the cone, one fixes

$$G'(\alpha_P) = -a_P e^{b_P [\alpha_P - 1]}, \quad (2)$$

where  $G(\alpha_P)$  is recovered by integration, and, as a consequence, the Pomeron amplitude can be rewritten in the following “geometrical” form

$$A_P(s, t) = i \frac{a_P}{b_P} \frac{s}{s_0} [r_1^2(s) e^{r_1(s)[\alpha_P - 1]} - \varepsilon_P r_2^2(s) e^{r_2(s)[\alpha_P - 1]}], \quad (3)$$

where  $r_1^2(s) = b_P + L - i\pi/2$ ,  $r_2^2(s) = L - i\pi/2$ ,  $L \equiv \ln(s/s_0)$ .

A Regge dipole:

$$a(j, t) = \frac{\beta(j)}{[j - \alpha(t)]^2} = \frac{d}{d\alpha} \frac{\beta(j)}{j - \alpha(t)}.$$

Performing a Sommerfeld-Watson transform,

$$T(s, t) = \frac{d}{d\alpha(t)} \frac{1 + e^{i\pi j}}{\sin(\pi j)} \frac{\beta(j)}{j - \alpha(t)} (s/s_0)^j (2j + 1) dj,$$

one gets:

$$T(s, t) = \frac{d}{d\alpha} \left[ e^{-i\pi\alpha/2} G(\alpha) (s/s_0)^\alpha \right] = e^{-i\pi/2} (s/s_0)^\alpha \left[ G' + (L - i\pi/2) G \right], \quad L = \ln(s/s_0).$$

$$\sigma^t(s) = \sigma_0(1 + \lambda L), \quad \lambda = \frac{G(0)}{G'(0)};$$

$$t_{min} = \frac{1}{\alpha'b} \ln \frac{b\gamma L}{b + L}, \quad t_{max} = \frac{1}{\alpha'b} \ln \frac{\gamma b(4L^2 + \pi^2)}{4(b + L)^2 + \pi^2}, \quad \left( \frac{d\sigma}{dt} \right)_{max} / \left( \frac{d\sigma}{dt} \right)_{min} \approx L^2.$$

## DP SD compatible with unitarity

(hep-ph/9608386 and in the Proc. of the Crimean Conf., 1996)

From the generalized optical theorem, we have for the single diffractive cross-section

$$M^2 \frac{d\sigma}{dt dM^2} = \frac{1}{16\pi} \frac{d}{d\alpha_0} \frac{d}{d\alpha_1} \frac{d}{d\alpha_2} \times \beta(t_0, \alpha_0) \beta(t_1, \alpha_1) \beta(t_2, \alpha_2) \\ G_{3IP}(t_0, t_1, t_2; \alpha_0, \alpha_1, \alpha_2) \left(\frac{s}{M^2}\right)^{\alpha_1 + \alpha_2 - 2} \left(\frac{M^2}{s_0}\right)^{\alpha_0 - 1} |_{t_0=0, t_1=t_2=t; \alpha_i=\alpha} IP^{(t_i)},$$

where  $G_{3IP}(t_0, t_1, t_2; \alpha_0, \alpha_1, \alpha_2)$  is the generalization of the usual triple pomeron,  $3IP$ -vertex. Evidently,  $G_{3IP}$  can not be constant, since in this case the integrated (over  $M^2$  and  $t$ ) cross-section

$$\sigma^{SD} = \int_{\xi_0}^{\xi-\xi_0} d\xi_1 \int_{-\infty}^{-|t|_{min}} dt \frac{d\sigma}{dt d\xi_1} \propto \xi^3 = \ln^3(s/s_0),$$

would violate the unitary inequality  $\sigma^{SD} \leq \sigma^{tot}$ . We remind that in the dipole Pomeron model  $\sigma^{tot} \propto \ln(s/s_0)$ .

Here  $\xi_1 = \ln(s/M^2)$ ;  $\xi_0$  is a large constant restricting the region where Regge behaviour is valid. The upper limit of integration over  $t$ ,  $(-|t|_{min})$ , generally speaking depends on  $M^2/s$ , but  $|t|_{min} \sim m^2(M^2/s)^2 \ll 1$  in the region under consideration ( $m$  is the proton mass), so it can be set zero.

One easily obtains a general expression for the differential cross-section:

$$\frac{d\sigma}{dt d\xi_1} = \frac{1}{16\pi} \beta^3(0) G_3 \not{P} \exp[(B + 2\alpha' \not{P} \xi_1)t] \\ \times \{G_1(\xi - \xi_1)\xi_1^2(2\alpha' \not{P} t)^2 + G_2(\xi - \xi_1)\xi_1(2\alpha' \not{P} t) + G_3\xi_1^2(2\alpha' \not{P} t) + G_4\xi_1 + G_5(\xi - \xi_1)\}, \quad (5)$$

where

$$G_1 = (g_1 + g_2)(\tilde{g}_1 + \tilde{g}_2), \quad G_2 = 2[g_1(\tilde{g}_1 + \tilde{g}_2) + \tilde{g}_1(g_1 + g_2)], \\ G_3 = (g_1 + g_2 + \tilde{g}_1 + \tilde{g}_2), \quad G_4 = 2(g_1 + \tilde{g}_1), \quad G_5 = 2g_1\tilde{g}_1. \\ B = 2b + 2\bar{b} \quad \text{if} \quad \beta(t) = \beta(0)e^{bt}.$$

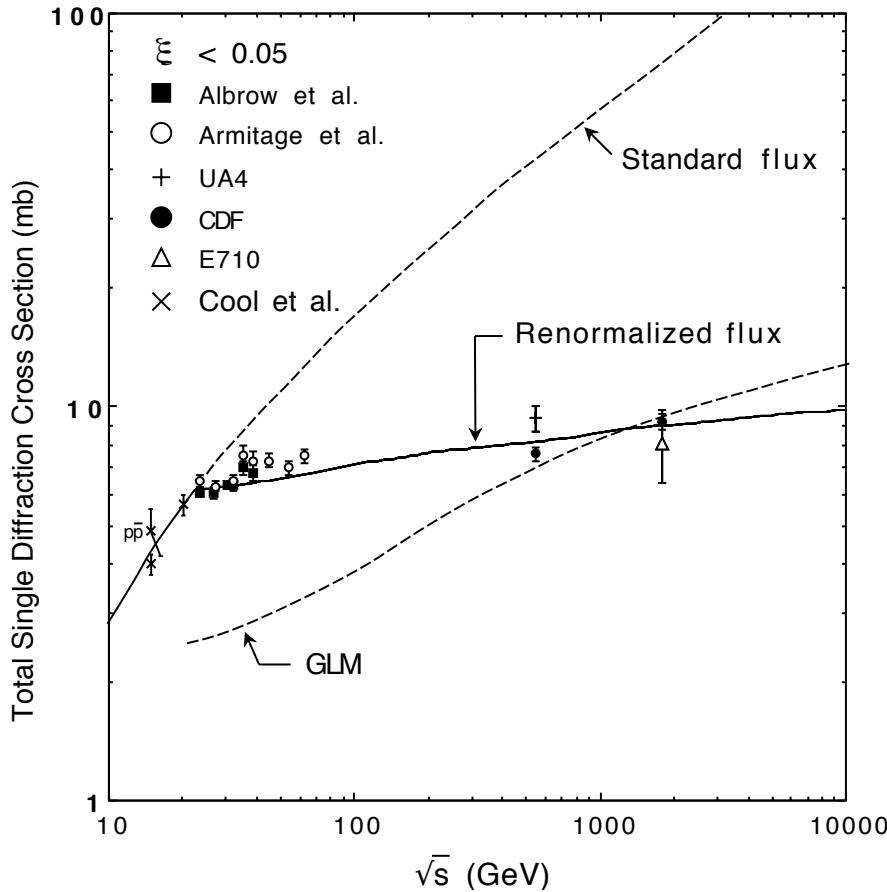
After integration in  $t$  and  $\xi_1$  we find that the term violating the inequality  $\sigma^{SD} \leq \sigma^{tot}$  (it behaves like  $\xi \ln \xi$  for  $\xi \rightarrow \infty$ ) has a factor  $2G_1 - G_2 + G_5$ . Hence, by setting

$$2G_1 - G_2 + G_5 = 0, \quad (6)$$

we obtain

$$\sigma^{SD} = C_1 \ln(s/s_0) + C_2 \ln(\ln(s/s_0)) + C_3 + \dots \quad (7)$$

(*Further details by V. Zachezhuhruva at this School.*)

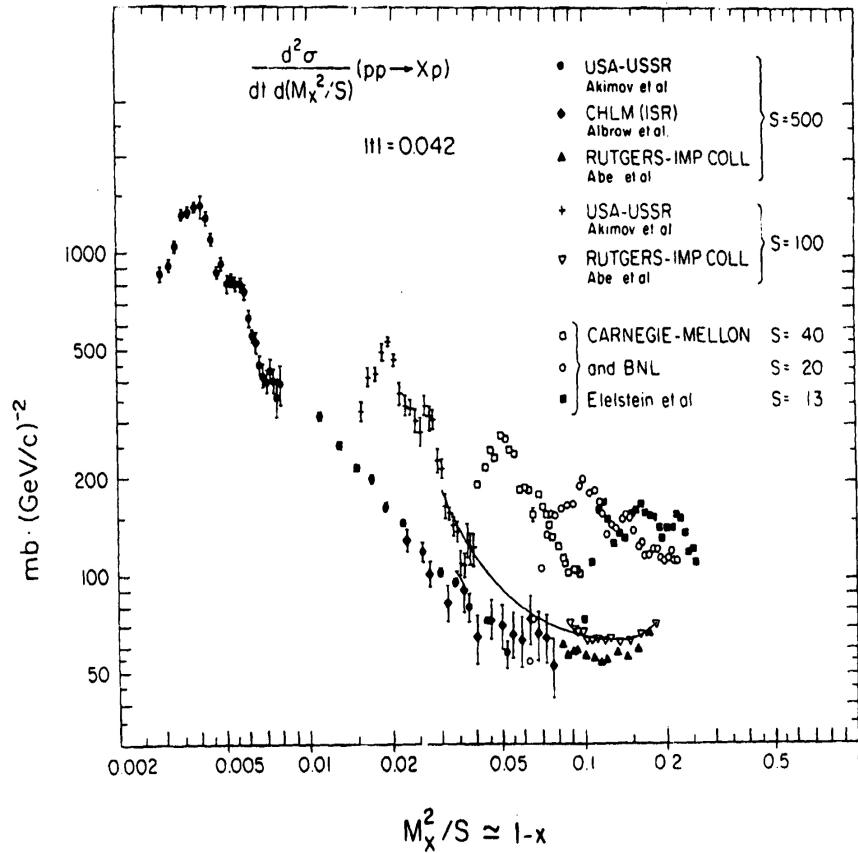
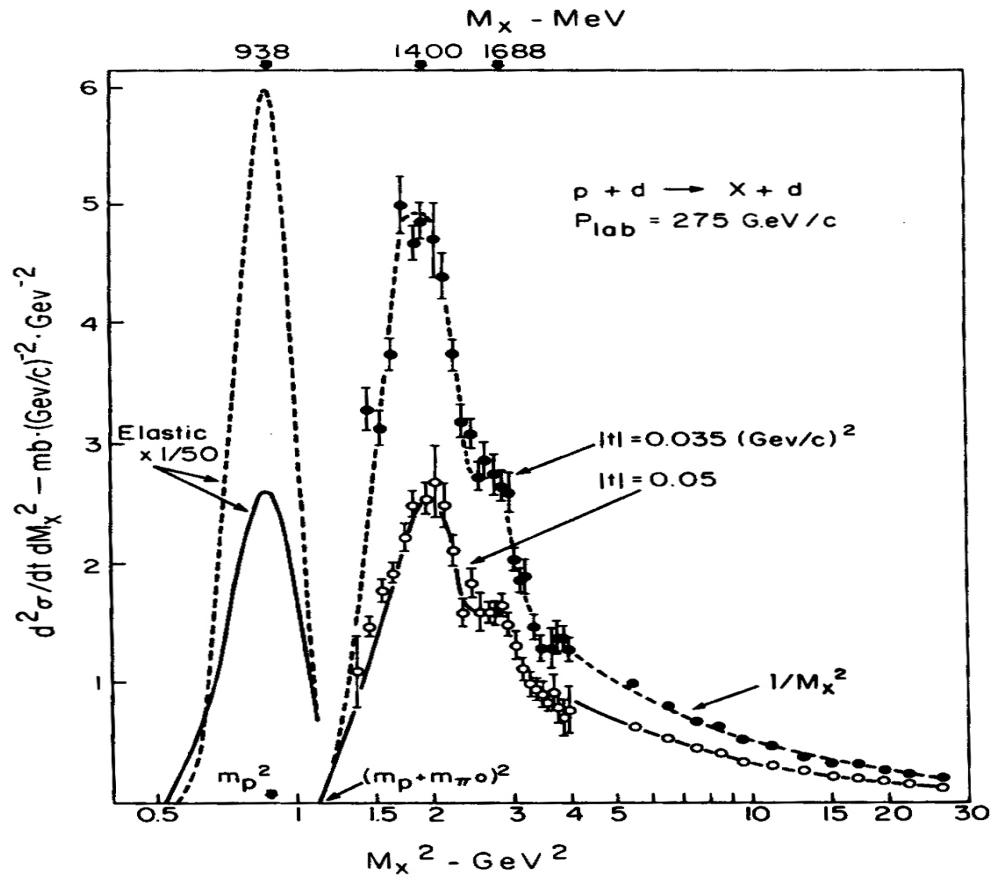


The function  $N(s)$  is the so-called renormalization factor, introduced by K. Goulianios,

$$N_s \equiv \int_{\xi(\min)}^{\xi(\max)} \int_{t=0}^{-\infty} dt f_{P/p}(\xi, t) \sim s^{2\epsilon},$$

where  $\xi(\min) = 1.4/s$  and  $\xi(\max) = 0.1$ . This factor secures unitarity.

# FNAL



$$\nu \frac{d^2\sigma}{dt dM_X^2} \Big|_{|t|=0.035} (p+d \rightarrow X+d) / F_d$$

( $p_{LAB} = 275 \text{ GeV}/c$ )

Duality in missing mass: finite mass sum rule (FMSR),

$$|t| \frac{d\sigma_{el}}{dt} + \int_0^{\nu_1} \nu \frac{d^2\sigma}{dtd\nu} d\nu = \int_{\nu_1}^{\nu_{as}} \frac{d^2\sigma}{dtd\nu} \Big|_{\nu_{as}} d\nu, \quad \nu = M_X^2 - M_p^2 - t.$$

elastic

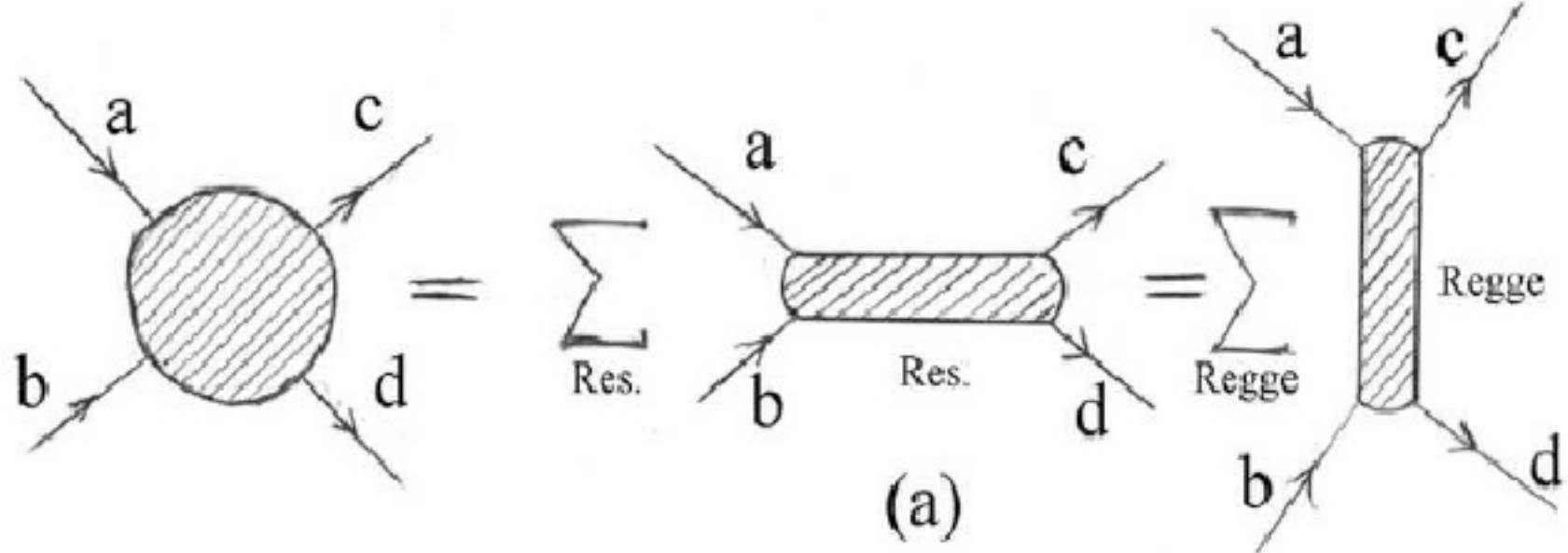
$M_X^2$

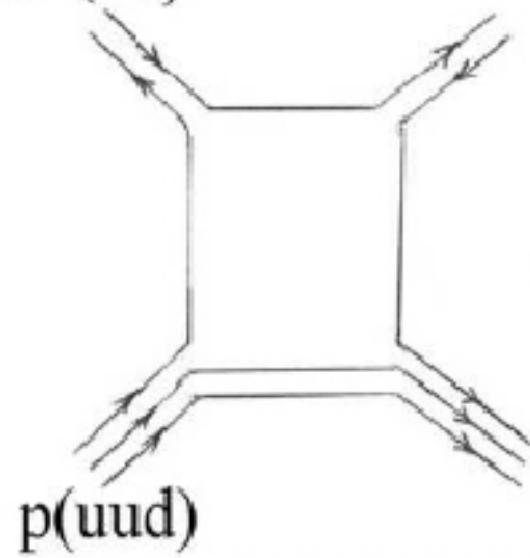
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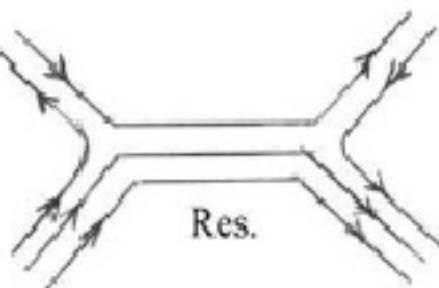
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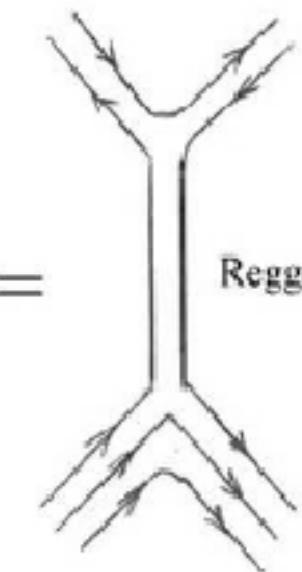


$\pi^- (\bar{u}d)$ 

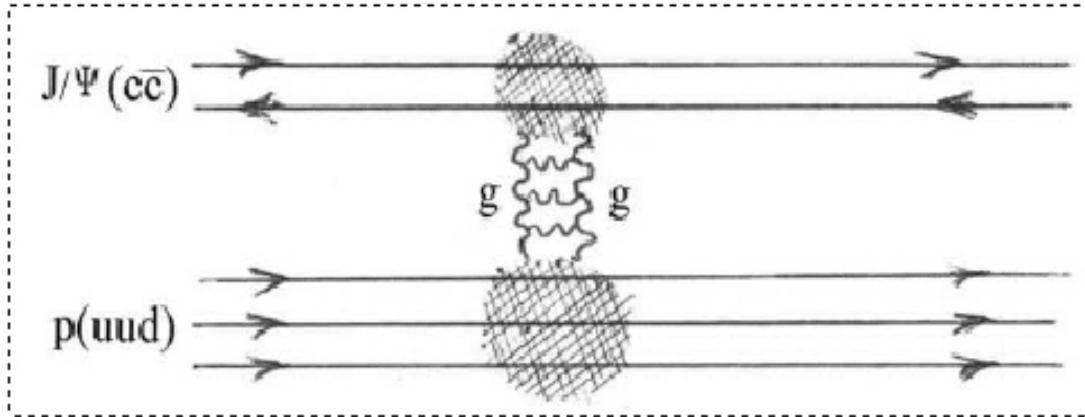
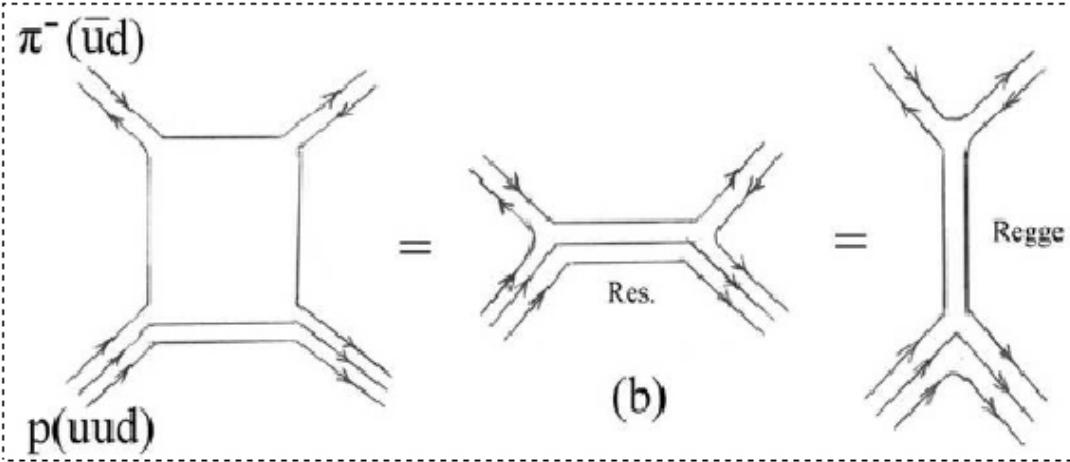
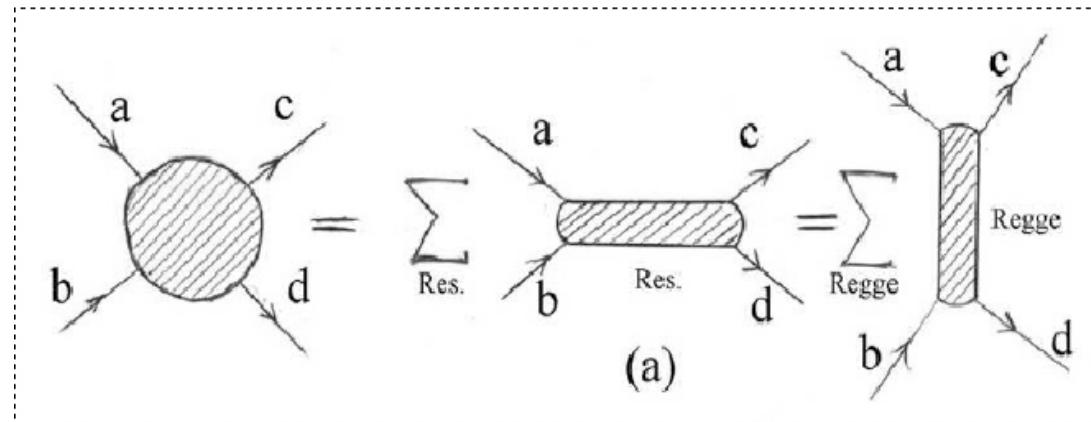
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(b)



Regge



Direct-channel poles + Regge asymptotic + crossing symmetry = Veneziano-duality

Create an infinite number of poles:

$$A(s, t) = \int_0^1 dx x^{-\alpha(s)-1} f(s, t, x) = \int_0^1 dx x^{-\alpha(s)-1} \sum_{k=0}^{\infty} a_k(t, s) x^k = \sum_{k=0}^{\infty} \frac{a_k(t, s)}{k - \alpha(s)}.$$

To make it crossing symmetric, set  $f(s, t, x) = (1-x)^{-\alpha(t)-1}$ . Then:

$$A(s, t) \rightarrow V(s, t) = \int_0^1 dx x^{-\alpha(s)-1} (1-x)^{-\alpha(t)-1} = B(-\alpha(s), -\alpha(t)) = \frac{\Gamma(-\alpha(s)\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))}.$$

Direct-channel (B-W "resonance") poles:

$$V(s, t) = \sum_{n=0}^{\infty} \frac{1}{n - \alpha(s)} \frac{\Gamma(n + \alpha(t) + 1)}{n! \Gamma(\alpha(t) + 1)} = \sum_{n=0}^{\infty} \frac{1}{n - \alpha(t)} \frac{\Gamma(n + \alpha(s) + 1)}{n! \Gamma(\alpha(s) + 1)},$$

Regge-pole asymptotic (Stirling formula) behaviour:

$$V(s, t)|_{|\alpha(s)| \rightarrow \infty} = |-\alpha(s)|^{\alpha(t)} \Gamma(-\alpha(t)) \left[ \sum_{n=0}^N \frac{a_n(t)}{|-\alpha(s)|^n} + O\left(\frac{1}{|-\alpha(s)|^{N+1}}\right) \right].$$

## DA with MA

$$V(s, t) \rightarrow D(s, t) = \int_0^1 dx (x)^{-\alpha(s')-1} (1-x)^{-\alpha(t')-1},$$

where  $s' = sx$ ,  $t' = t(1-x)$  are "homotopies", mapping physical trajectories  $\alpha(s, 0) = \alpha(s)$ ,  $\alpha(t, 0) = \alpha(t)$  onto linear functions,  $\alpha(s, 1) = \alpha_0 + \alpha's$ ,  $\alpha(t, 1) = \alpha_0 + \alpha't$ .

*Basic properties:*

1. Threshold singularities, entering via the complex, non-linear trajectories; finite widths of resonances;
2. Double spectral function (Mandelstam analyticity). Singularities required by unitarity;
3. Regge asymptotic behaviour (polynomial boundedness) imposes bounds on the trajectories (termination of their real part)  $\rightarrow$  finite number of resonances;
4.  $N$ -particle amplitudes.

The  $(s, t)$  term of a dual amplitude is

$$D(s, t) = c \int_0^1 dx \left(\frac{x}{g_1}\right)^{-\alpha(s')-1} \left(\frac{1-x}{g_2}\right)^{-\alpha(t')-1},$$

where  $s$  and  $t$  are the Mandelstam variables, and  $g_1, g_2$  are parameters,  $g_1, g_2 > 1$ . For simplicity, we set  $g_1 = g_2 = g_0$ .

1. Regge behavior,  $s \rightarrow \infty$ ,  $t = \text{const}$  :  $D(s, t) \sim s^{\alpha(t)-1}$ ;
2. Threshold behavior,  $s \rightarrow s_0$  :  $D(s, t) \sim \sqrt{s_0 - s} [\text{const} + \ln(1 - s_0/s)]$ ;

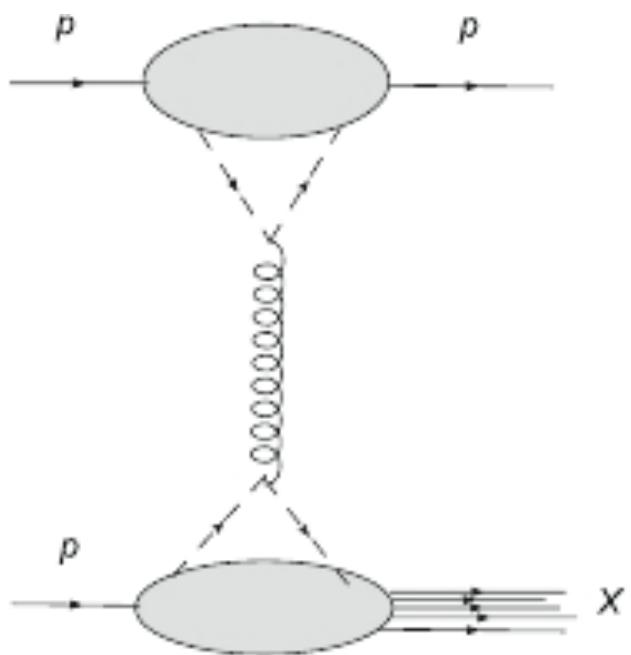
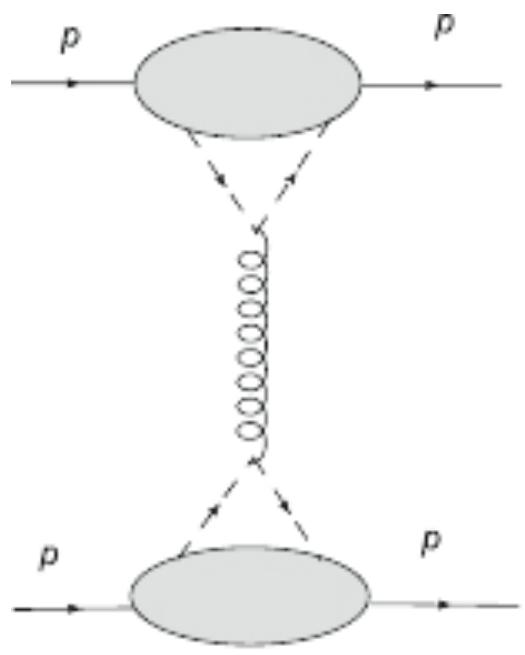
### 3. Direct-channel poles:

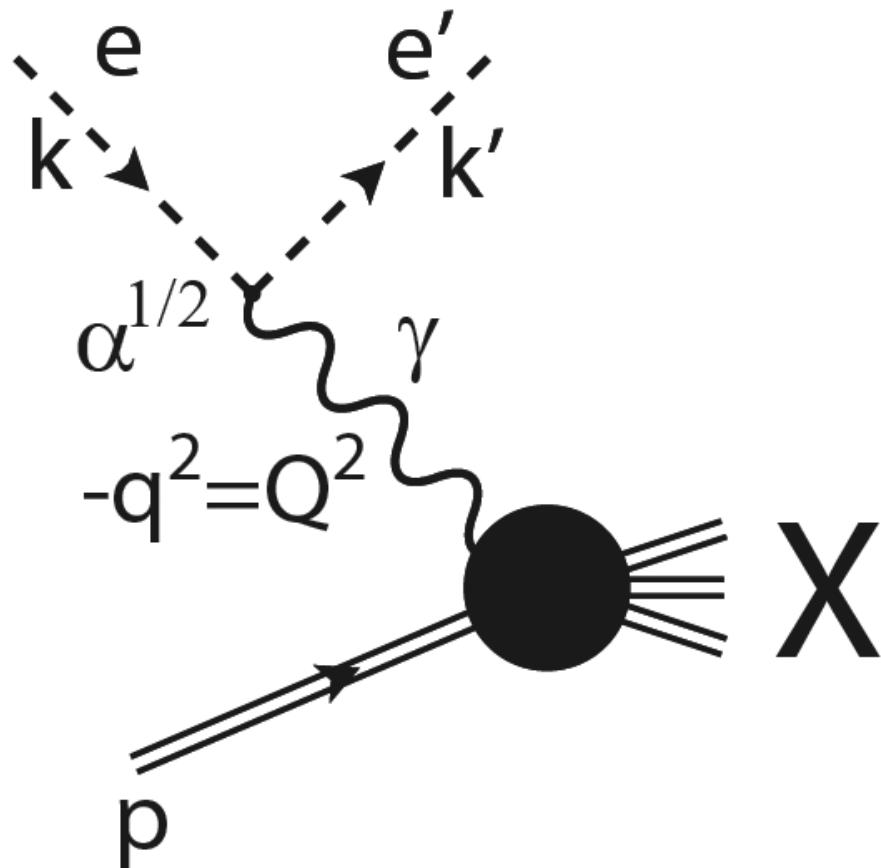
$$D(s, t) = \sum_{n=0}^{\infty} g^{n+1} \sum_{l=o}^n \frac{[-s\alpha'(s)]^l C_{n-l}(t)}{[n - \alpha(s)]^{l+1}}.$$

Exotic direct-channel trajectory:  $\alpha(s) = \alpha(0) + \alpha_1(\sqrt{s_0} - \sqrt{s_0 - s})$ .

"GOLDEN" diffraction reaction:  $J/\Psi p -$  scattering: By VMD, photoproduction is reduced to elastic hadron scattering:

$$D(\gamma p - Vp) = \sum \frac{e}{f_V} D(Vp - Vp).$$





$$\left| \text{Diagram} \right|^2 = \sum_{X} \text{Diagram}_X = \text{Unitarity}_{t=0} = \sum_{R} \text{Diagram}_R = \sum_{\text{Res}} \text{Diagram}_{\text{Res}}$$

The equation shows the decomposition of the total cross-section of the process into various contributions. The first term is the sum over all intermediate states  $X$ . The second term is labeled "Unitarity  $t=0$ ". The third term is the sum over all resonance states  $R$ . The fourth term is labeled "Veneziano duality".

Similar to the case of elastic scattering, the double differential cross section for the SDD reaction, by Regge factorization, can be written as

$$\frac{d^2\sigma}{dt dM_X^2} = \frac{9\beta^4 [F^p(t)]^2}{4\pi \sin^2[\pi\alpha_P(t)/2]} (s/M_X^2)^{2\alpha_P(t)-2} \times \\ \left[ \frac{W_2}{2m} \left( 1 - M_X^2/s \right) - mW_1(t+2m^2)/s^2 \right], \quad (1)$$

where  $W_i$ ,  $i = 1, 2$  are related to the structure functions of the nucleon and  $W_2 \gg W_1$ . For high  $M_X^2$ , the  $W_{1,2}$  are Regge-behaved, while for small  $M_X^2$  their behavior is dominated by nucleon resonances. The knowledge of the inelastic form factors (or transition amplitudes) is crucial for the calculation of low-mass diffraction dissociation.

The  $pp$  scattering amplitude

$$A(s, t)_P = -\beta^2 [f^u(t) + f^d(t)]^2 \left(\frac{s}{s_0}\right)^{\alpha_P(t)-1} \frac{1 + e^{-i\pi\alpha_P(t)}}{\sin \pi\alpha_P(t)}, \quad (1)$$

where  $f^u(t)$  and  $f^d(t)$  are the amplitudes for the emission of  $u$  and  $d$  valence quarks by the nucleon,  $\beta$  is the quark-Pomeron coupling, to be determined below;  $\alpha_P(t)$  is a vacuum Regge trajectory. It is assumed that the Pomeron couples to the proton via quarks like a scalar photon.

A single-Pomeron exchange is valid at the LHC energies, however at lower energies (e.g. those of the ISR or the SPS) the contribution of non-leading Regge exchanges should be accounted for as well.

Thus, the unpolarized elastic  $pp$  differential cross section is

$$\frac{d\sigma}{dt} = \frac{[3\beta F^p(t)]^4}{4\pi \sin^2[\pi\alpha_P(t)/2]} (s/s_0)^{2\alpha_P(t)-2}. \quad (2)$$

The final expression for the double differential cross section reads:

$$\begin{aligned}
& \frac{d^2\sigma}{dt dM_X^2} = \\
& A_0 \left( \frac{s}{M_X^2} \right)^{2\alpha_P(t)-2} \frac{x(1-x)^2 [F^p(t)]^2}{(M_x^2 - m^2) \left( 1 + \frac{4m^2x^2}{-t} \right)^{3/2}} \times \\
& \sum_{n=1,3} \frac{[f(t)]^{2(n+1)} \operatorname{Im} \alpha(M_X^2)}{(2n + 0.5 - \operatorname{Re} \alpha(M_X^2))^2 + (\operatorname{Im} \alpha(M_X^2))^2}. \tag{1}
\end{aligned}$$

# SD and DD cross sections

$$\frac{d^2\sigma_{SD}}{dt dM_x^2} = F_p^2(t) F(x_B, t) \frac{\sigma_T^{Pp}(M_x^2, t)}{2m_p} \left( \frac{s}{M_x^2} \right)^{2(\alpha(t)-1)} \ln \left( \frac{s}{M_x^2} \right)$$

$$\frac{d^3\sigma_{DD}}{dt dM_1^2 dM_2^2} = C_n F^2(x_B, t) \frac{\sigma_T^{Pp}(M_1^2, t)}{2m_p} \frac{\sigma_T^{Pp}(M_2^2, t)}{2m_p}$$

$$\times \left( \frac{s}{(M_1 + M_2)^2} \right)^{2(\alpha(t)-1)} \ln \left( \frac{s}{(M_1 + M_2)^2} \right)$$

## “Reggeized (dual) Breit-Wigner” formula:

$$\sigma_T^{Pp}(M_x^2, t) = \text{Im } A(M_x^2, t) = \frac{A_{N^*}}{\sum_n n - \alpha_{N^*}(M_x^2)} + Bg(t, M_x^2) =$$

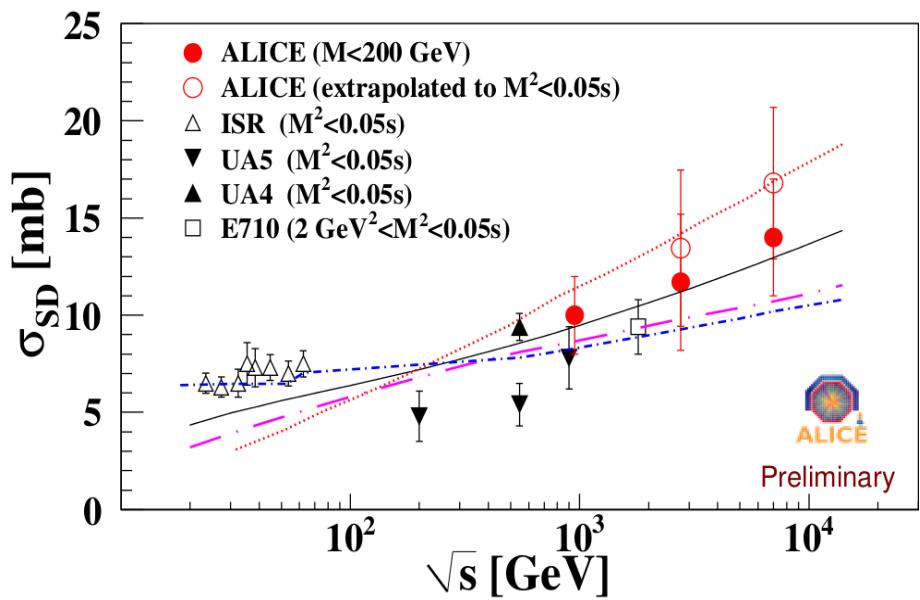
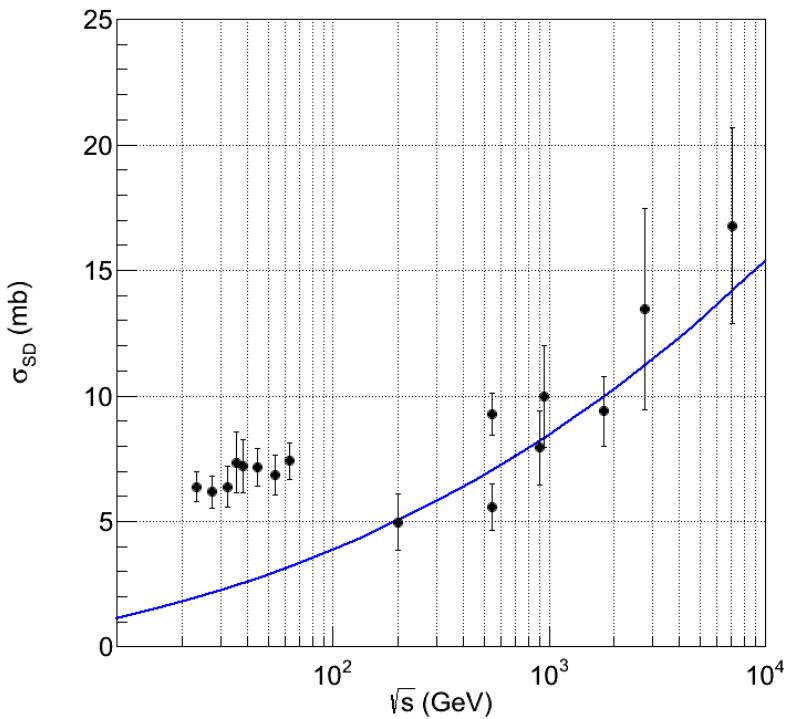
$$= A_n \sum_{n=0,1,\dots} \frac{[f(t)]^{2(n+1)} \text{Im } \alpha(M_x^2)}{(2n + 0.5 - \text{Re } \alpha(M_x^2))^2 + (\text{Im } \alpha(M_x^2))^2} + B_n e^{b_{in}^{bg} t} (M_x^2 - M_{p+\pi}^2)^\epsilon$$

$$F(x_B, t) = \frac{x_B(1 - x_B)}{(M_x^2 - m_p^2) (1 + 4m_p^2 x_B^2 / (-t))^{3/2}}, \quad x_B = \frac{-t}{M_x^2 - m_p^2 - t}$$

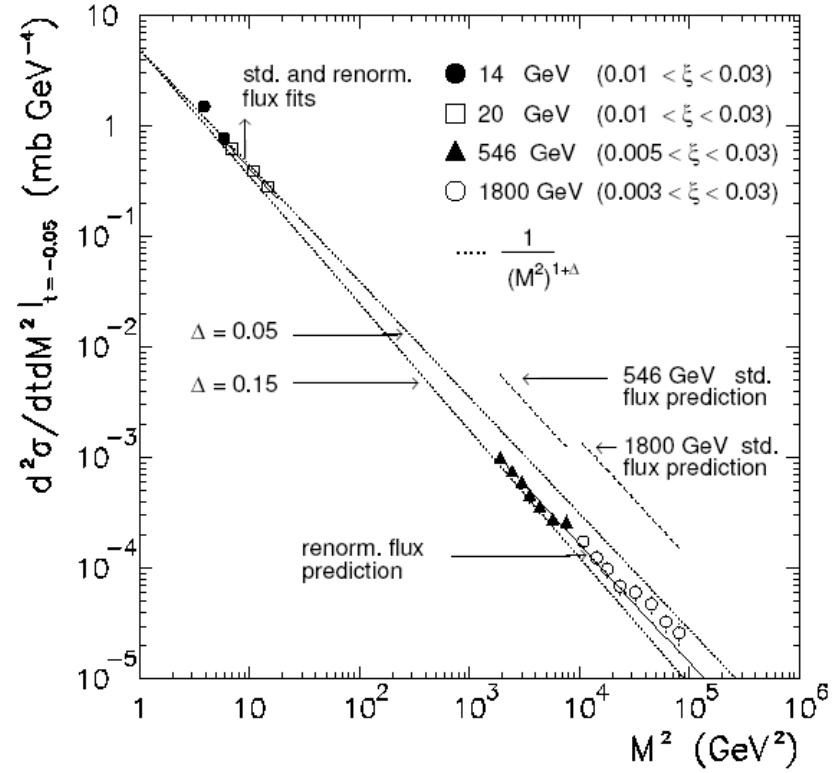
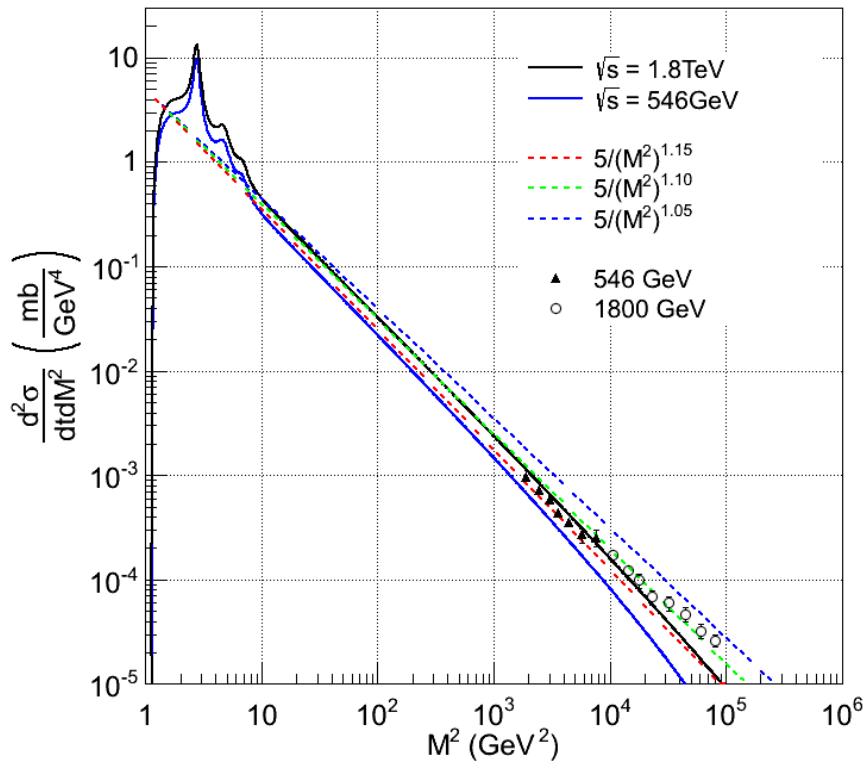
$$F_p(t) = \frac{1}{1 - \frac{t}{0.71}}, \quad f(t) = e^{b_{in} t}$$

$$\alpha(t) = \alpha(0) + \alpha' t = 1.04 + 0.25t$$

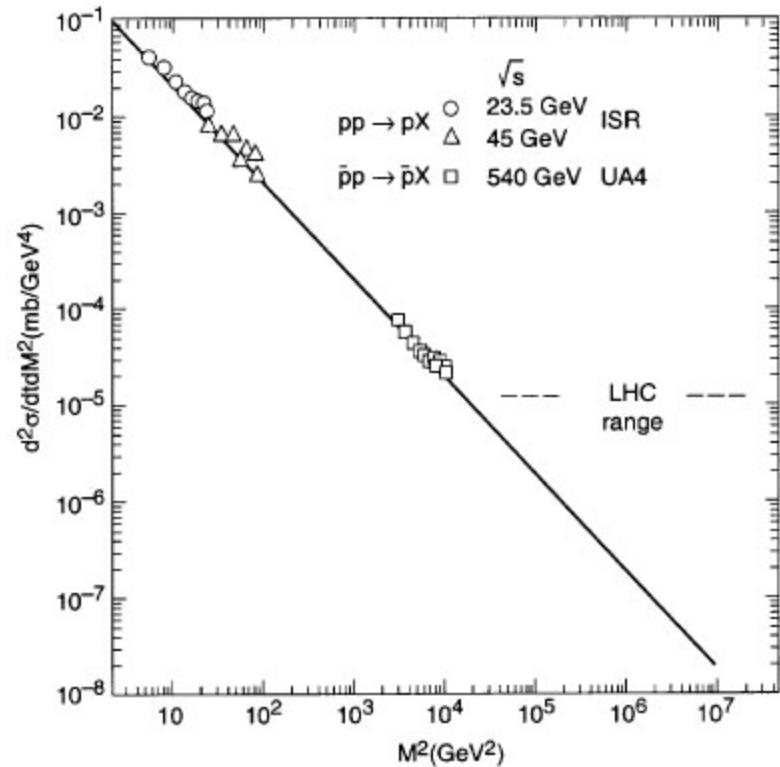
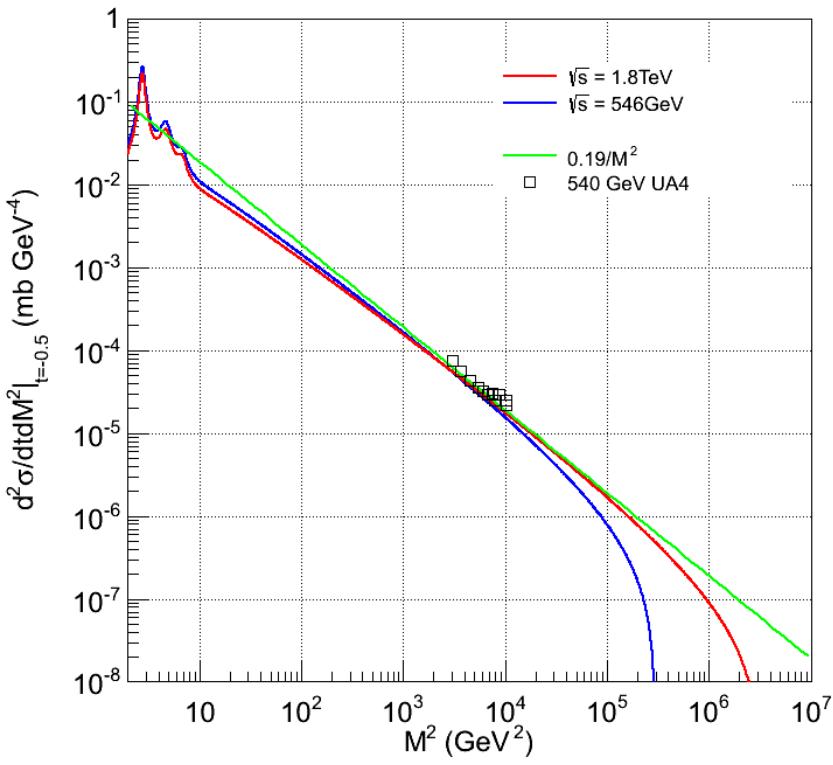
# SDD cross sections vs. energy.



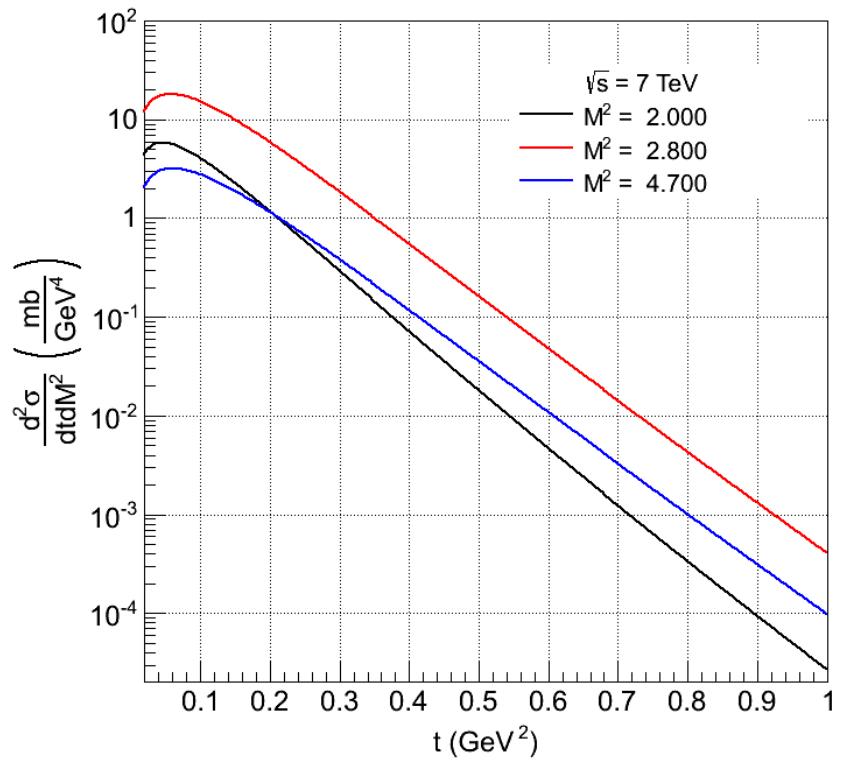
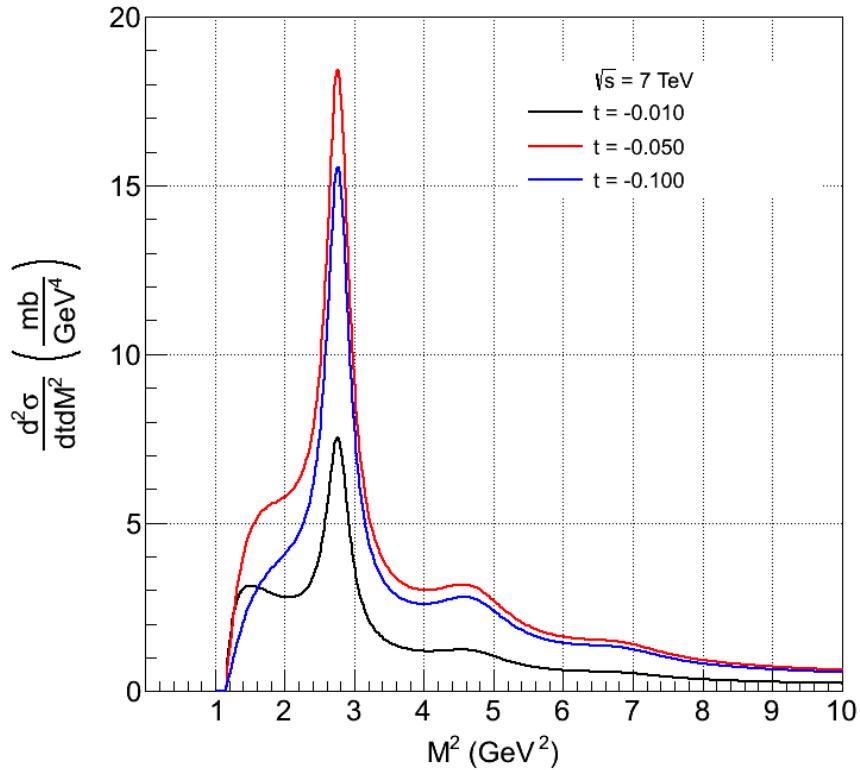
# Approximation of background to reference points (t=-0.05)



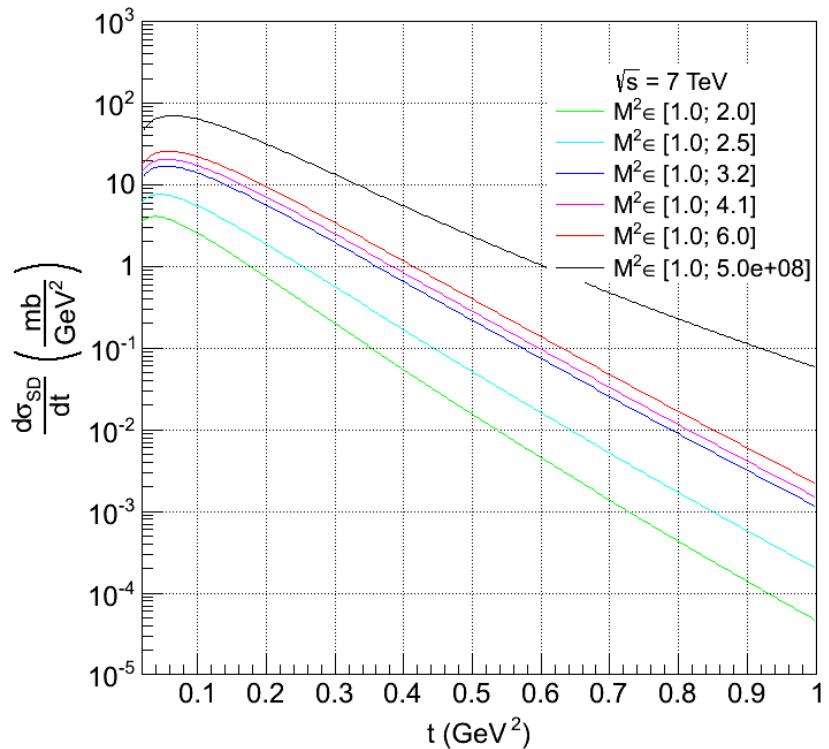
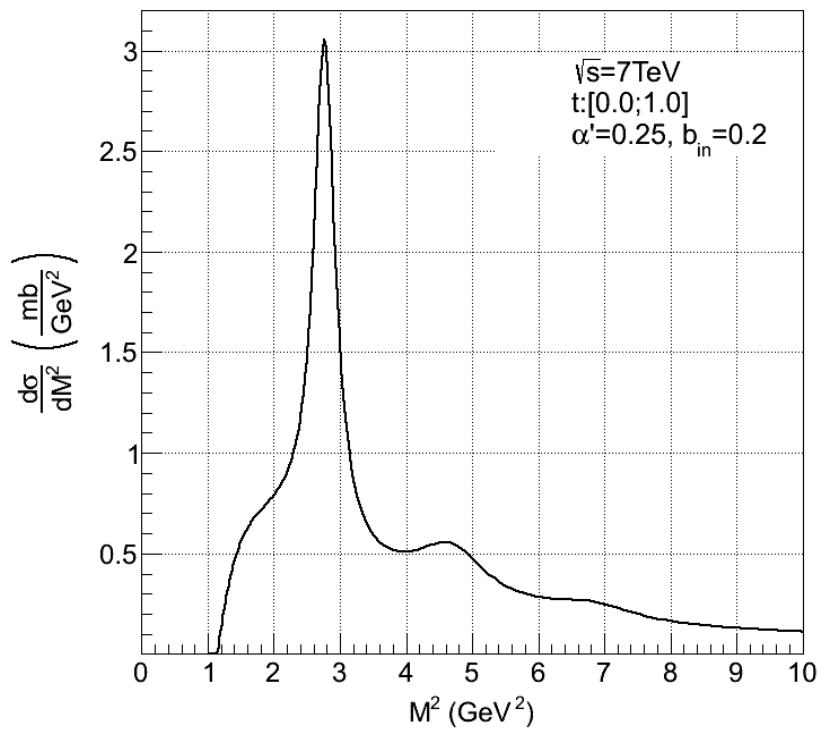
# Approximation of background to reference points ( $t=-0.5$ )



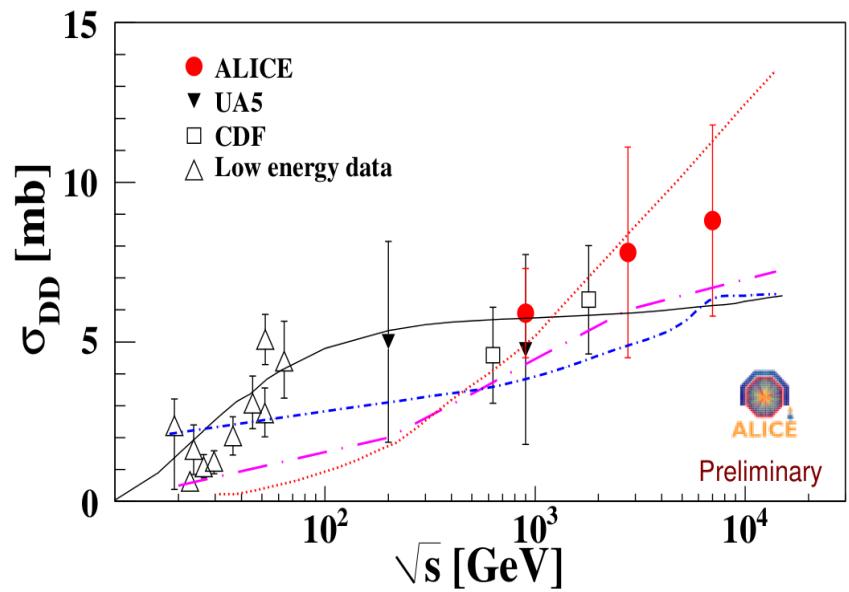
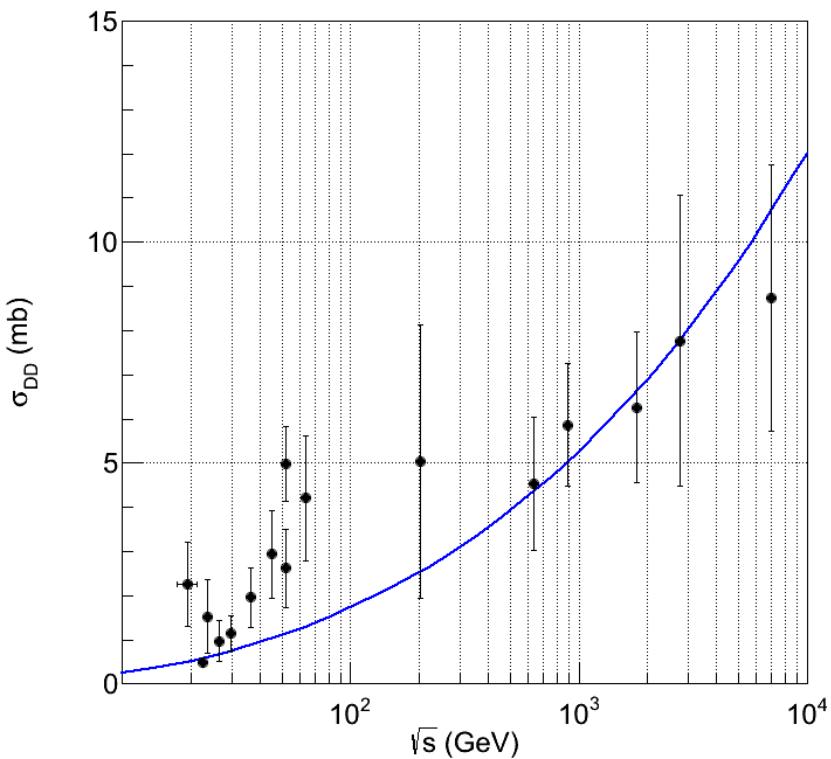
# Double differential SD cross sections



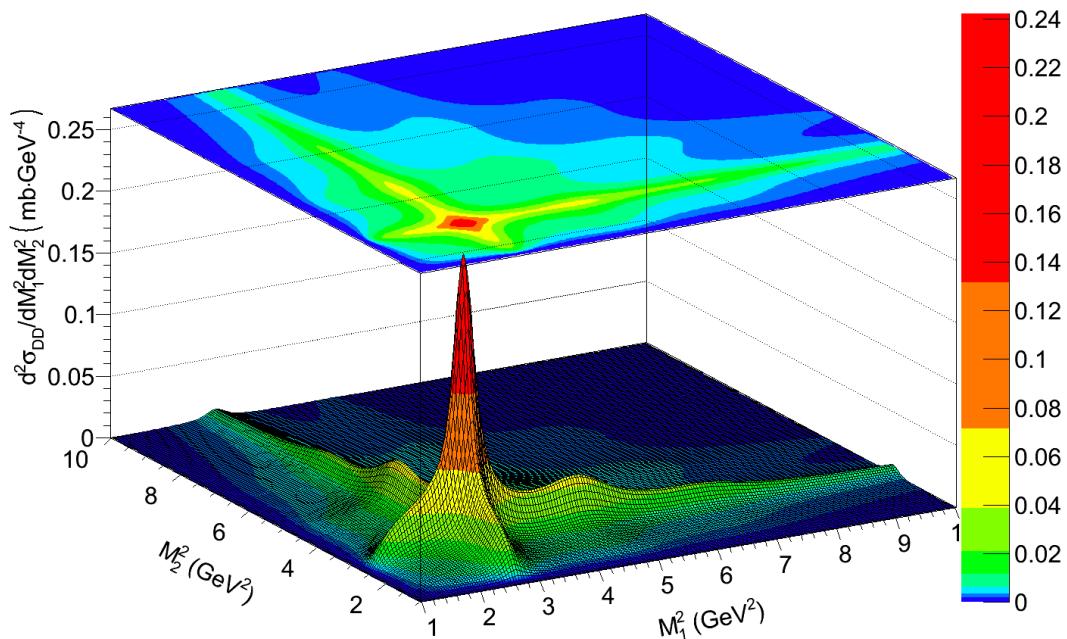
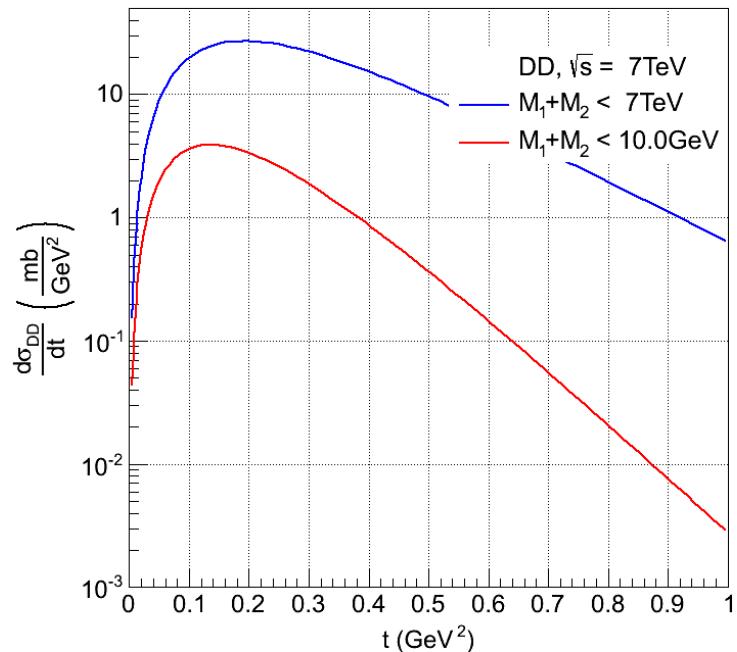
# Single differential integrated SD cross sections



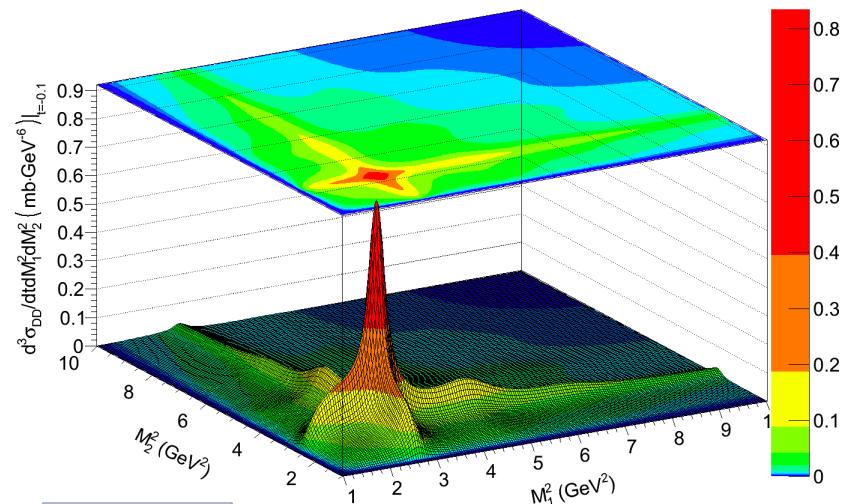
# DDD cross sections vs. energy.



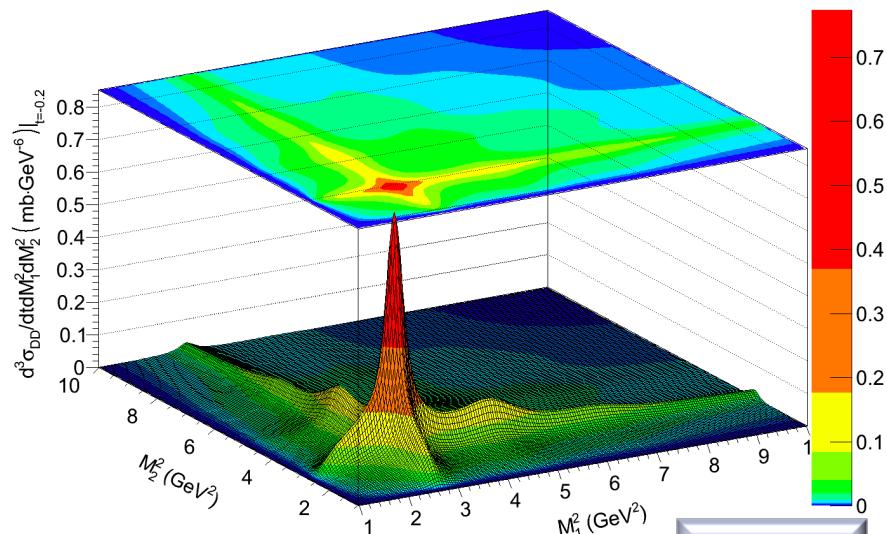
# Integrated DD cross sections



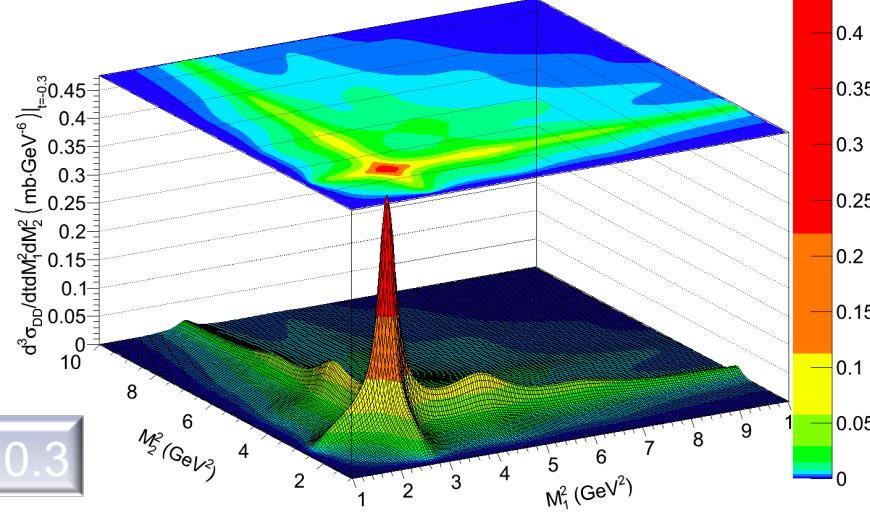
# Triple differential DD cross sections



$t = -0.1$



$t = -0.2$



$t = -0.3$

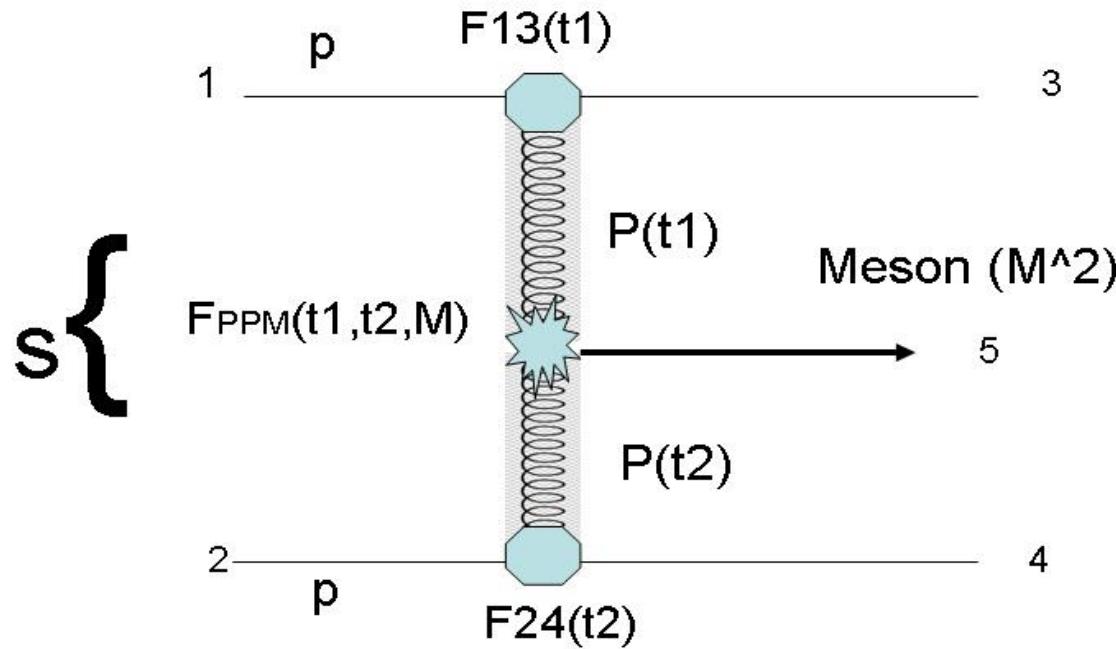
# The parameters and results

$b_{in}$ ( $GeV^{-2}$ )	0.2
$b_{in}^{bg}$ ( $GeV^{-2}$ )	3
$\alpha'$ ( $GeV^{-2}$ )	0.25
$\alpha(0)$	1.04
$\epsilon$	1.03
$A_n$	18.7
$B_n$	8.8
$C_n$	3.79e-2

$\sigma_{SD}$ ( $mb$ )	14.13
$\sigma_{SD}(M < 3.5GeV)$ ( $mb$ )	4.68
$\sigma_{SD}(M > 3.5GeV)$ ( $mb$ )	9.45
$\sigma_{Res}^{SD}$ ( $mb$ )	2.48
$\sigma_{Bg}^{SD}$ ( $mb$ )	9.45
$\sigma_{DD}$ ( $mb$ )	10.68
$\sigma_{DD}(M < 10GeV)$ ( $mb$ )	1.05
$\sigma_{DD}(M > 10GeV)$ ( $mb$ )	9.63

## *Prospects (future plans):*

### 1. Central diffractive meson production (double Pomeron exchange);



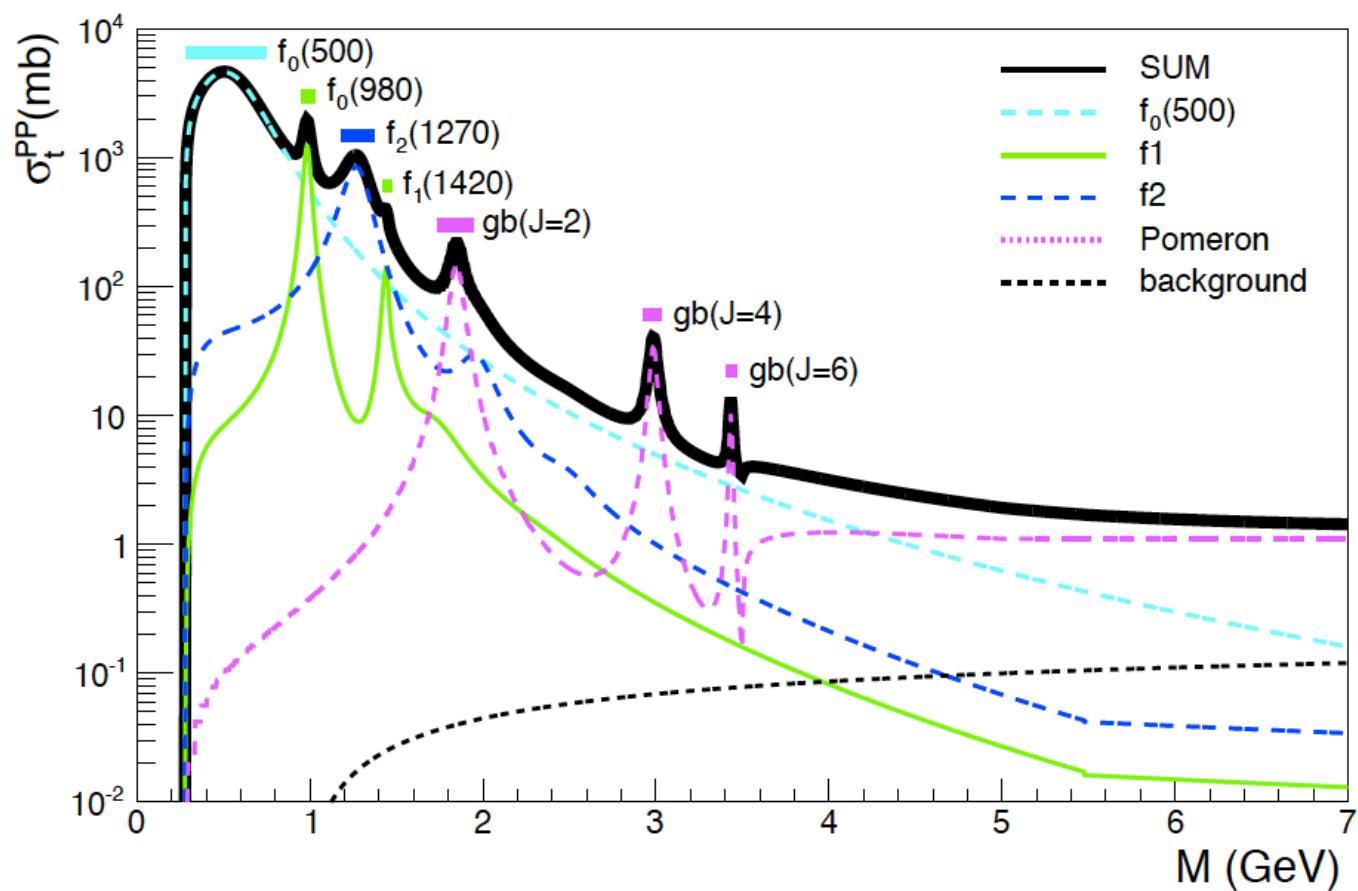
### 2. Charge exchange reactions at the LHC (single Reggeon exchange), e.g. $pp \rightarrow n\Delta$ (in collaboration with Oleg Kuprash and Rainer Schicker)

Representative examples of the Pomeron trajectories: 1) Linear; 2) With a square-root threshold, required by  $t$ -channel unitarity and accounting for the small- $t$  “break” as well as the possible “Orear”,  $e^{\sqrt{-t}}$  behavior in the second cone; and 3) A logarithmic one, anticipating possible “hard effects” at large  $|t|$   $|t| < 8 \text{ GeV}^2$ .

$$\alpha_P \equiv \alpha_P(t) = 1 + \delta_P + \alpha_{1P} t, \quad (\text{TR.1})$$

$$\alpha_P \equiv \alpha_P(t) = 1 + \delta_P + \alpha_{1P} t - \alpha_{2P} \left( \sqrt{4\alpha_{3P}^2 - t} - 2\alpha_{3P} \right), \quad (\text{TR.2})$$

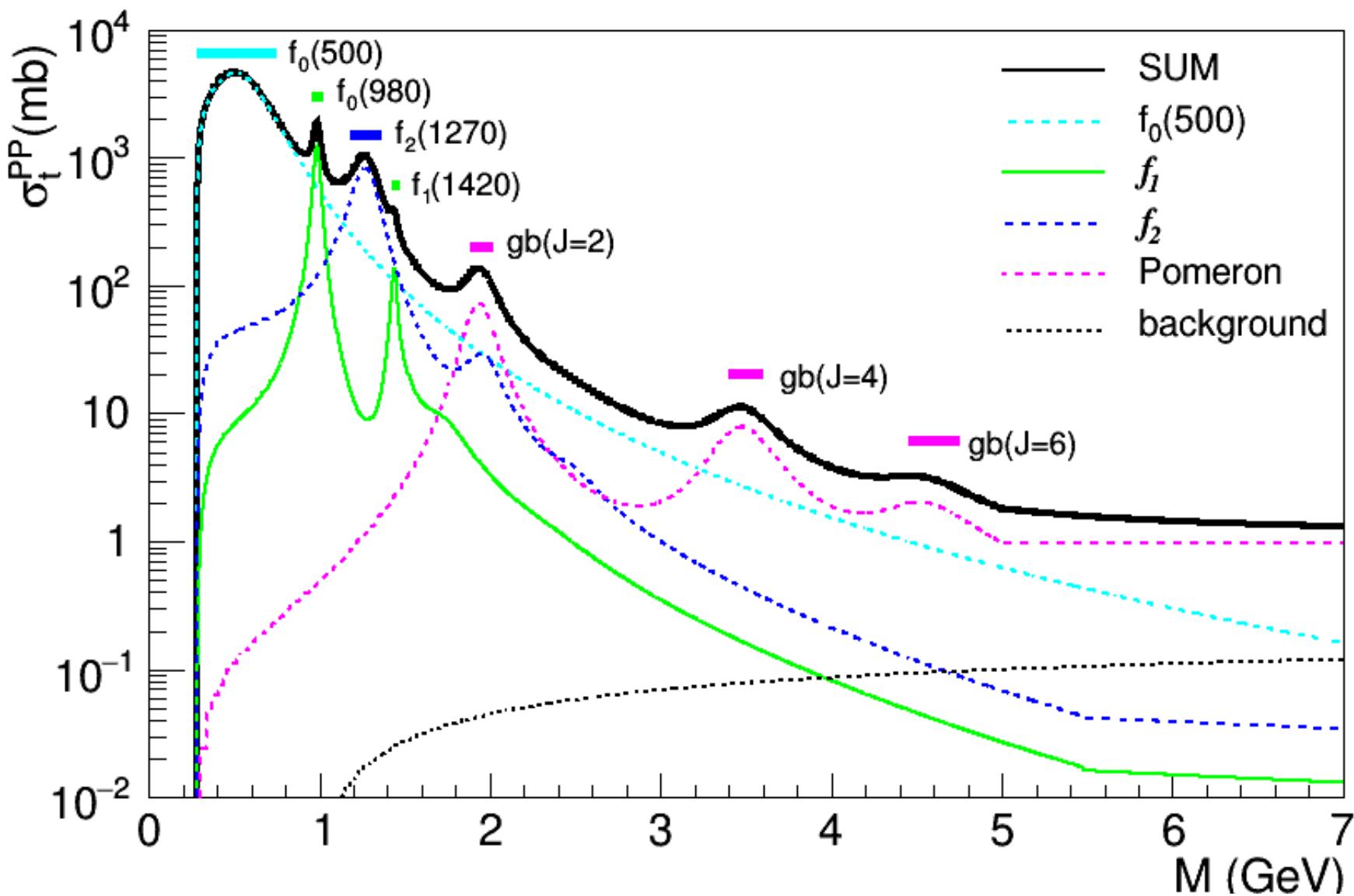
$$\alpha_P \equiv \alpha_P(t) = 1 + \delta_P - \alpha_{1P} \ln(1 - \alpha_{2P} t). \quad (\text{TR.3})$$



$$\alpha(s) = \alpha_0 + \alpha' s + \alpha_1 \sqrt{s_0 - s};$$

$$\alpha(s) = \alpha_0 + \alpha_2 \sqrt{s_0 - s} + \alpha_3 \sqrt{s_1 - s};$$

$$\alpha(s) = \frac{\alpha_0 + \alpha' s}{1 + \alpha_4 \sqrt{s_0 - s}},$$



A recent review:

László Jenkovszky, Rainer Schicker, and István Szanyi:  
**Elastic and diffractive scatterings in the LHC era,**  
**International Journal of Modern Physics E, Vol. 27, No. 08,**  
**1830005 (2018),** <https://doi.org/10.1142/S0218301318300059>

**Thank you !**