Particlization of stochastic hydrodynamics

Dmytro (Dima) Oliinychenko

LBNL with Volker Koch

arXiv:1902.09775

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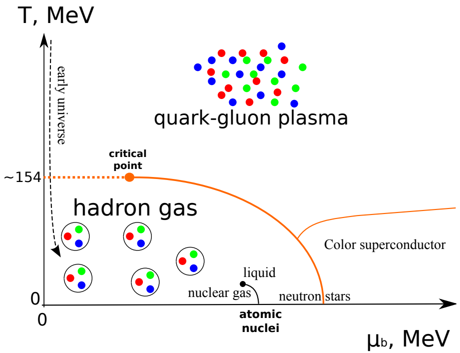
Where is phase transition/critical point?



Generic critical point features:

- Correlation length increases: critical opalescence
- Fluctuations increase

In heavy ion collisions: do they survive until measurement?
Need dynamical modelling.

Dynamical approaches with phase transition

- Transport: which degrees of freedom?
  - PHSD or AMPT with potentials generating phase transition?
  - RSP code with NJL potentials Marty, PRC 92, 015201 (2015), arXiv:1412.5375 only mesons, code abandoned(?)
  - Danielewicz transport Fundam.Theor.Phys. 95 (1999) 69-84, arXiv:nucl-th/9808013 selected hadrons with $m(S)$, $m = 0$ matches QGP entropy; no follow-up?
- Hydrodynamics with
  - EoS with phase transition (Maxwell construction)
  - Surface tension terms
    Steinheimer, Randrup Eur. Phys. J. A52 (2016) no.8, 239
    Pratt PRC 96 (2017) no.4, 044903
  - Stochastic terms
    Kapusta et al, PRC 85, 054906 (2012)
    Kumar et al., Nucl. Phys. A **925**, 199 (2014)
    Nahrgang et al., arXiv:1804.05728
  - Fields with stochastic terms
    Nahrgang et al., PRC **84**, 024912 (2011)
    Herold et al., J. Phys. G **41**, no. 11, 115106 (2014)

Important: not to lose fluctuations at particlization

Standard particlization in hydro + transport hybrids

On average by events:

How many particles cross a moving surface $\equiv$
are produced from a hypersurface element with a normal $d\sigma_\mu$?
Cooper-Frye formula:

$$dN = \frac{g}{(2\pi\hbar c)^3}\frac{p^\mu}{p^0} f(p^\alpha u_\alpha, T, \mu) d^3p \, d\sigma_\mu = j^\mu d\sigma_\mu$$

- Cooper-Frye formula does not specify multiplicity distribution
- Standard choice $P(N) = Poisson(\bar{N})$
  motivated by grand-canonical ensemble + classical statistics
- Particles in different cells sampled independently

Correlations and fluctuations: from hydro to particles

$B_H(i)$ – baryon number from hydro cell i

$B_S(i)$ – baryon number sampled from cell i

typically $B_S(i) = Poisson(B_H(i))$

$\delta B(i) = B_S(i) - B_H(i)$

$\langle \delta B(i) \rangle = 0$

$\langle \dots \rangle$ – average over samples

$\overline{\dots}$ – average over hydro events

$\langle\langle \dots \rangle\rangle$ – average over samples and hydro events

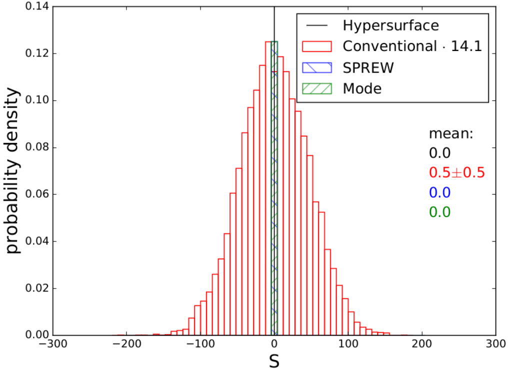
$$\langle\langle B_S(i) B_S(j) \rangle\rangle - \langle\langle B_S(i) \rangle\rangle \langle\langle B_S(j) \rangle\rangle = \overline{B_H(i) B_H(j)} + \langle\langle \delta B(i) \delta B(j) \rangle\rangle - \overline{B_H(i)}\, \overline{B_H(j)}$$

$$\langle\langle \delta B(i) \delta B(j) \rangle\rangle = \delta_{ij} \overline{B_H(i)}$$

- Standard sampling
  - Preserves correlations
  - Increases fluctuations
- Sampling with local conservation laws: $\langle\langle \delta B(i) \delta B(j) \rangle\rangle = 0$

Conserved quantities from standard Cooper-Frye sampling

Usual particlization assumes independent particles
But particles should be correlated due to conservation laws



C. Schwarz, DO, L.-G. Pang, S. Ryu, H. Petersen, J Phys G 45 (2018), 015001

E-by-e conservation laws are necessary to study fluctuations

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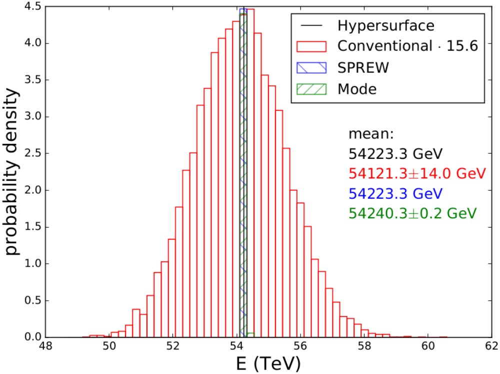

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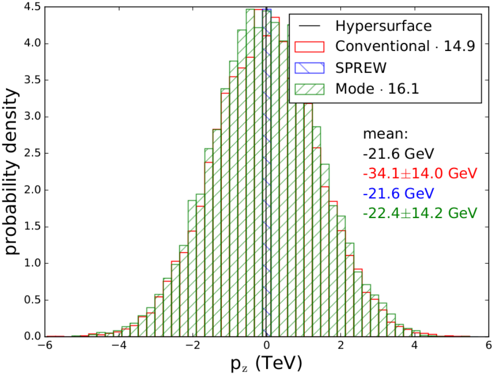

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E-by-e conservation laws are necessary to study fluctuations

Correlations and fluctuations

Cooper-Frye formula tells nothing about

- correlations between charges, momenta, energies, ...
- fluctuations of B, S, Q, $< p_T >$, ...

They are determined by a sampling algorithm:

- Standard choice: all particles independent
- UrQMD hybrid: attempt to account for conservation laws
  "mode sampling" Huovinen, Petersen Eur.Phys.J. A48 (2012) 171
- Bozek/Broniowski: always sample particle and antiparticle
  relies on $\mu = 0$, Phys.Rev.Lett. 109 (2012) 062301
- SPREW: reject particles driving conserved quantities in wrong direction
  SER: canonical rejection

  C. Schwarz, DO, L.-G. Pang, S. Ryu, H. Petersen, J Phys G 45 (2018), 015001

First formulate mathematical problem, then algorithm!

Systematically taking conservation laws into account

Quantities to conserve:

$$\begin{pmatrix} P^{\mu}_{tot} \\ B_{tot} \\ S_{tot} \\ Q_{tot} \end{pmatrix} = \sum_{\substack{\text{cells} \\ i}} \int \begin{pmatrix} p^{\mu}_{i} \\ B_i \\ S_i \\ Q_i \end{pmatrix} \frac{p^{\nu} d\sigma_{\nu}}{p^0} f_i(p^{\alpha} u_{\alpha}, T, \mu_i) \frac{g_i d^3 p}{(2\pi\hbar)^3}$$

Conservation laws applied to parts of the hypersurface: patches
Splitting into patches discussed further

Systematically taking conservation laws into account

Distribution to sample:

$$P(N, \{N_s\}^{\text{species}}, \{x_i\}_{i=1}^N, \{p_i\}_{i=1}^N) = \mathcal{N}$$

$$\left(\prod_s \frac{1}{N_s!}\right) \prod_{i=1}^N \frac{g_i}{(2\pi\hbar)^3} \frac{d^3 p_i}{p_i^0} p_i^\mu d\sigma_\mu \, f_i(p_i^\nu u_\nu, T, \mu_i) \times$$

$$\delta^{(4)}(\sum_i p^\mu - P^\mu_{tot}) \, \delta^{B_{tot}}_{\sum_i B_i} \, \delta^{S_{tot}}_{\sum_i S_i} \, \delta^{Q_{tot}}_{\sum_i Q_i}$$

- Total quantities conserved
- Local variations of T, μ, u allowed
- Turns into standard microcanonical sampling in case of one cell
- Sampled with Metropolis algorithm

Metropolis algorithm: general

- Random walk (Markov chain) with many steps
- One step t:
  - in state ξ propose new state ξ', probability $T(\xi \to \xi')$
  - Accept this proposal with probability $A(\xi \to \xi')$
  - $w(\xi \to \xi') = T(\xi \to \xi') A(\xi \to \xi')$
- After many steps reach stationary distribution $P(\xi)$
- $P(\xi)$ should be the desired distribution

$$P^{t+1}(\xi) - P^t(\xi) = \sum_{\xi'} [w(\xi' \to \xi) P^t(\xi') - w(\xi \to \xi') P^t(\xi)]$$

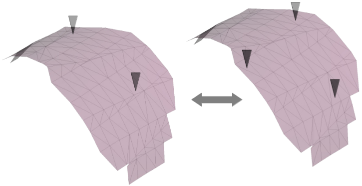
Sufficient condition for stationary distribution (detailed balance):

$$\frac{P(\xi')}{P(\xi)} = \frac{w(\xi \to \xi')}{w(\xi' \to \xi)} \implies \frac{A(\xi \to \xi')}{A(\xi' \to \xi)} = \frac{P(\xi')\, T(\xi' \to \xi)}{P(\xi)\, T(\xi \to \xi')}$$

Common choice:

$$a \equiv A(\xi \to \xi') = \min \left(1, \frac{P(\xi')\, T(\xi' \to \xi)}{P(\xi)\, T(\xi \to \xi')} \right)$$

Proposal function



1. With 50% probability choose a $2 \to 3$ or $3 \to 2$ transition.
2. Select the "incoming" particles by uniformly picking one of all possible pairs or triples.
3. Select the outgoing channel democratically with probability $1/N^{ch}$, N^{ch} – number of possible channels, satisfying quantum number and energy-momentum conservation.
4. For the selected channel sample the "collision" kinematics uniformly from the available phase space with probability $\frac{dR_n}{R_n}$, $n = 2$ or 3.
   $dR_n(\sqrt{s}, m_1, m_2, \dots, m_n) = \frac{(2\pi)^4}{(2\pi)^{3n}} \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \dots \frac{d^3p_n}{2E_n} \delta^{(4)}(P^\mu_{tot} - \sum P^\mu_i)$
5. Choose a cell for each of the outgoing particles uniformly from all cells in the patch.

Properties of proposal function

- Never changes total energy, momentum, or quantum numbers
- Generates proposal probabilities:

$$T(2 \to 3) = \frac{1}{2} \frac{G_2^{ch}}{G_2} \frac{1}{N_3^{ch}} \frac{dR_3^{ch}}{R_3^{ch}} \frac{1}{N_{cells}^3}$$

$$T(3 \to 2) = \frac{1}{2} \frac{G_3^{ch}}{G_3} \frac{1}{N_2^{ch}} \frac{dR_2^{ch}}{R_2^{ch}} \frac{1}{N_{cells}^2}$$

$G_2 = \frac{N(N-1)}{2!}$, $G_3 = \frac{N(N-1)(N-2)}{3!}$
total numbers of incoming pairs/triplets of any species
G_2^{ch}, G_3^{ch} – numbers of ways to select given incoming species
N_2^{ch}, N_3^{ch} – numbers of channels with necessary quantum numbers

Acceptance probability

$$a_{n\to m} = \frac{N_m^{ch} R_m}{N_n^{ch} R_n} \frac{N!}{(N+m-n)!} \frac{m!}{n!} \frac{k_m^{id}!}{k_n^{id}!} \times$$

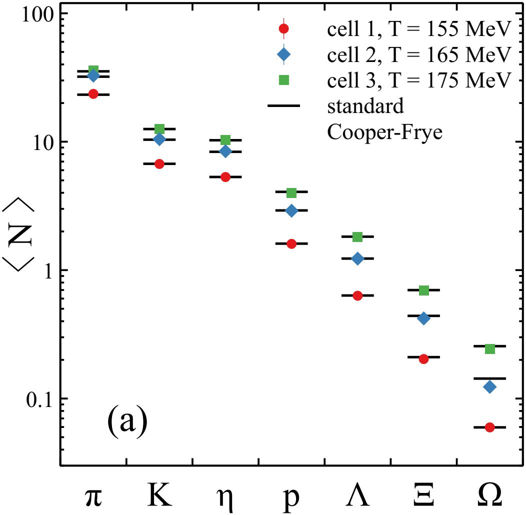
$$\left(\frac{2N_{cells}}{\hbar^3}\right)^{m-n} \frac{\displaystyle\prod_{i=1}^{m} g_i\, f_i(\mu_i - p_i^{\alpha} u_{\alpha}, T)\, p_i^{\mu} d\sigma_{\mu}}{\displaystyle\prod_{j=1}^{n} g_j\, f_j(\mu_j - p_j^{\alpha} u_{\alpha}, T)\, p_j^{\mu} d\sigma_{\mu}}$$

T, μ, u are taken at positions of the incoming/outgoing particles

Testing the sampling

- Patch consisting of 3 cells:
  - $d\sigma_1^\mu = (500.0, 50.0, 20.0, 30.0)\,\mathrm{fm}^3$,
    $d\sigma_2^\mu = (500.0, 40.0, 80.0, 30.0)\,\mathrm{fm}^3$,
    $d\sigma_3^\mu = (500.0, 20.0, 20.0, 20.0)\,\mathrm{fm}^3$
  - $\vec{v}_1 = (0.2, 0.3, 0.4)$, $\vec{v}_2 = (0.1, 0.5, 0.5)$, $\vec{v}_3 = (0.3, 0.4, 0.2)$
  - $T_1 = 0.155$ GeV, $T_2 = 0.165$ GeV, $T_3 = 0.175$ GeV
- Total energy of the patch 1268.2 GeV
- 416 different hadronic species generated ($m < 2.5$ GeV)
- Total energy, momentum, B, S, Q conserved
- Preserving local variations of T, μ, u
- Check local means, scaled variance $\omega \equiv \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$ of total multiplicities

Testing the sampling



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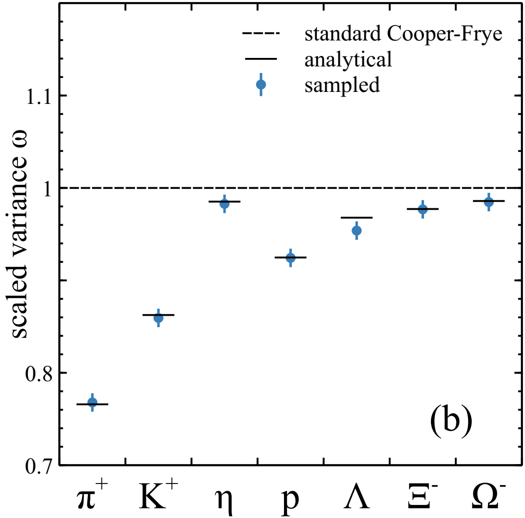
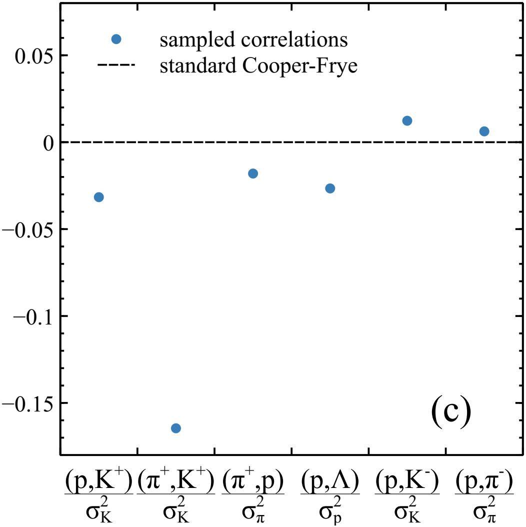


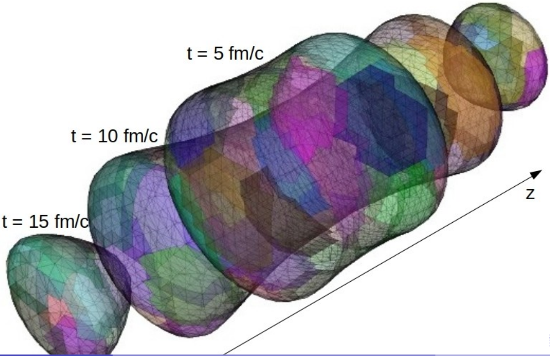
Analytical: M. Hauer, V. V. Begun and M. I. Gorenstein, Eur. Phys. J. C **58**, 83 (2008)

Testing the sampling



Local conservation laws

- How big should the patch be?
  - Not too small
  - Contain > 1 particle $\implies$ $> 100 - 1000$ cells per patch
  - Not too large: conservation laws should be local
- My choice: use patches of equal energies E_p
  Treat E_p as a parameter.
- Apply sampling on each patch



Summary

- Standard sampling neglects event-by-event conservation laws
- This obfuscates fluctuations
- Simple and fast method exists to include conservation laws
  - Split hypersurface into patches and conserve on every patch
  - Method uses Metropolis algorithm reminiscent of $2 \leftrightarrow 3$ stochastic collisions to thermalize
  - Passes non-trivial test cases
  - Code publically available
    github.com/doliinychenko/microcanonical_cooper_frye