

The semiclassical theory of deconfinement, chiral and Roberge-Weiss-like phase transitions

Edward Shuryak
Stony Brook University



**“Probing the phase structure of strongly interacting matter:
theory and experiment” GSI, March 2019**

outline

- **VEV of Polyakov line and the instanton-dyons**
- **Deconfinement transition**
- **Chiral symmetry breaking**
- **Generalized quark periodicity phases**
- **New set of phase transitions**
- **Instanton-dyons on the lattice**

**Instanton-dyons \Leftrightarrow Monopoles relation:
examples of the Poisson duality**

Semiclassics at finite T and pre-clustering at freeze out

**the semiclassical theory
Of gauge field solitons at finite T**

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Confinement \Rightarrow BEC of monopoles, flux tubes (1970's)

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Chiral symmetry breaking

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=> collectivization of instanton zero modes (1980's)**

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They are not quite instantons (?)**

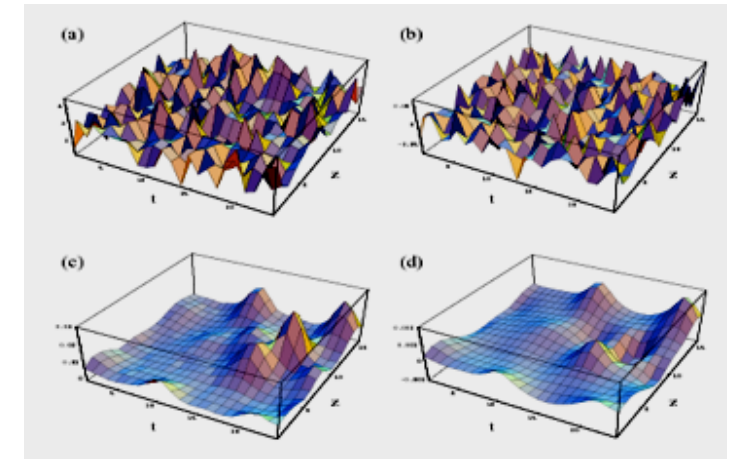
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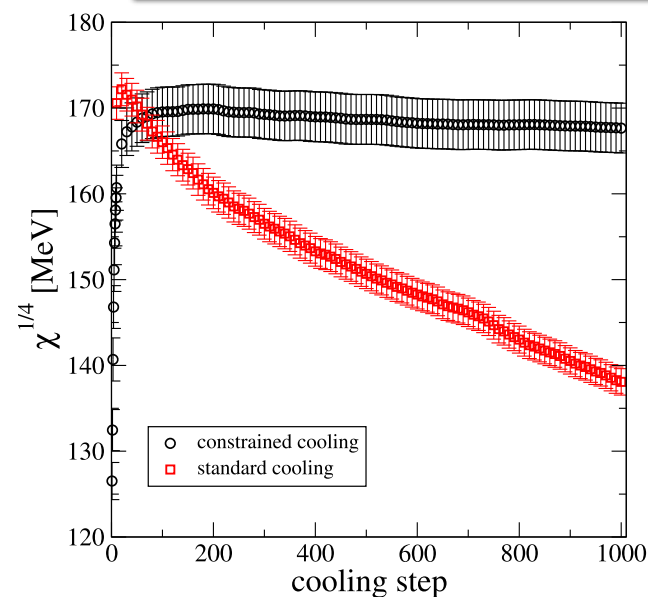
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“action cooling” is known to eliminate gluons and lead to instantons

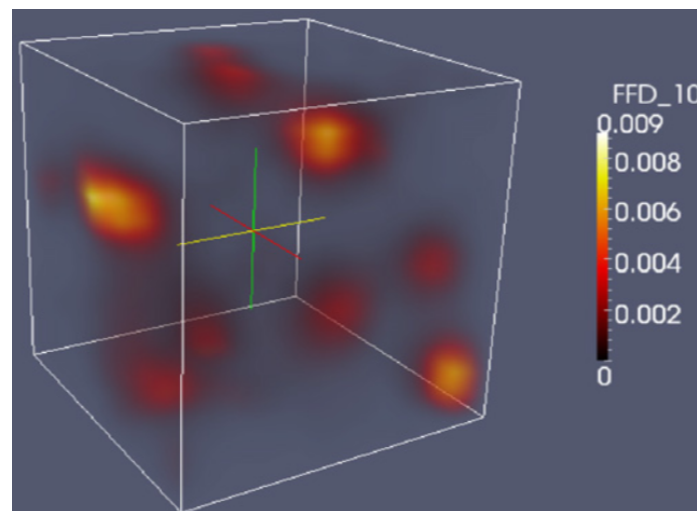


Negele et al

perhaps dyons were first observed in “constrained cooling” preserving local L



Langfeld and Ilgenfritz, 2011



**while the total top.charge
of the box is always integer,
local bumps are not!
They are all (anti)selfdual
But top charge and actions
Were not integers!**

a lot of work on finding instanton-dyons was done by
C.Gattringer et al, Ilgenfritz et al

Instantons have top.charges $Q = \pm 1$

and thus are elementary quanta of topology:

How can they have a substructure?

What about topological classification of fields?

**Like a nucleon is made of 3 quarks,
an instanton is made
of N_c instanton-dyons**

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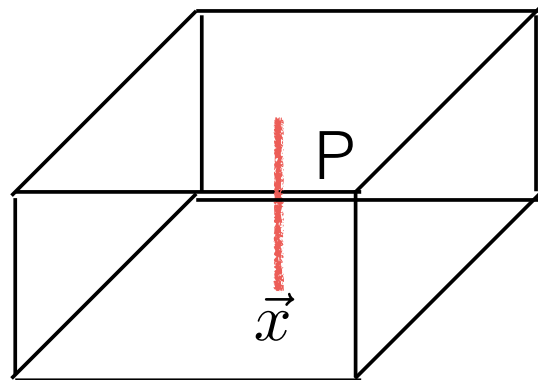
Because they have magnetic charges

They are connected by (invisible!)

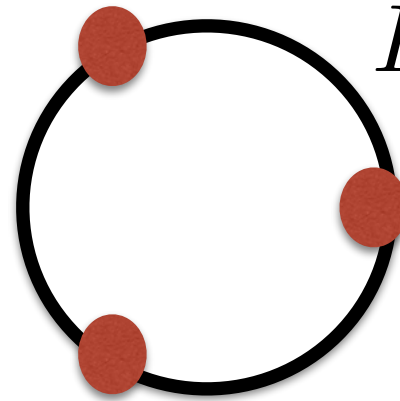
Dirac strings

Thus they avoid topological charge quantization

The Polyakov line is used as order parameter for deconfinement



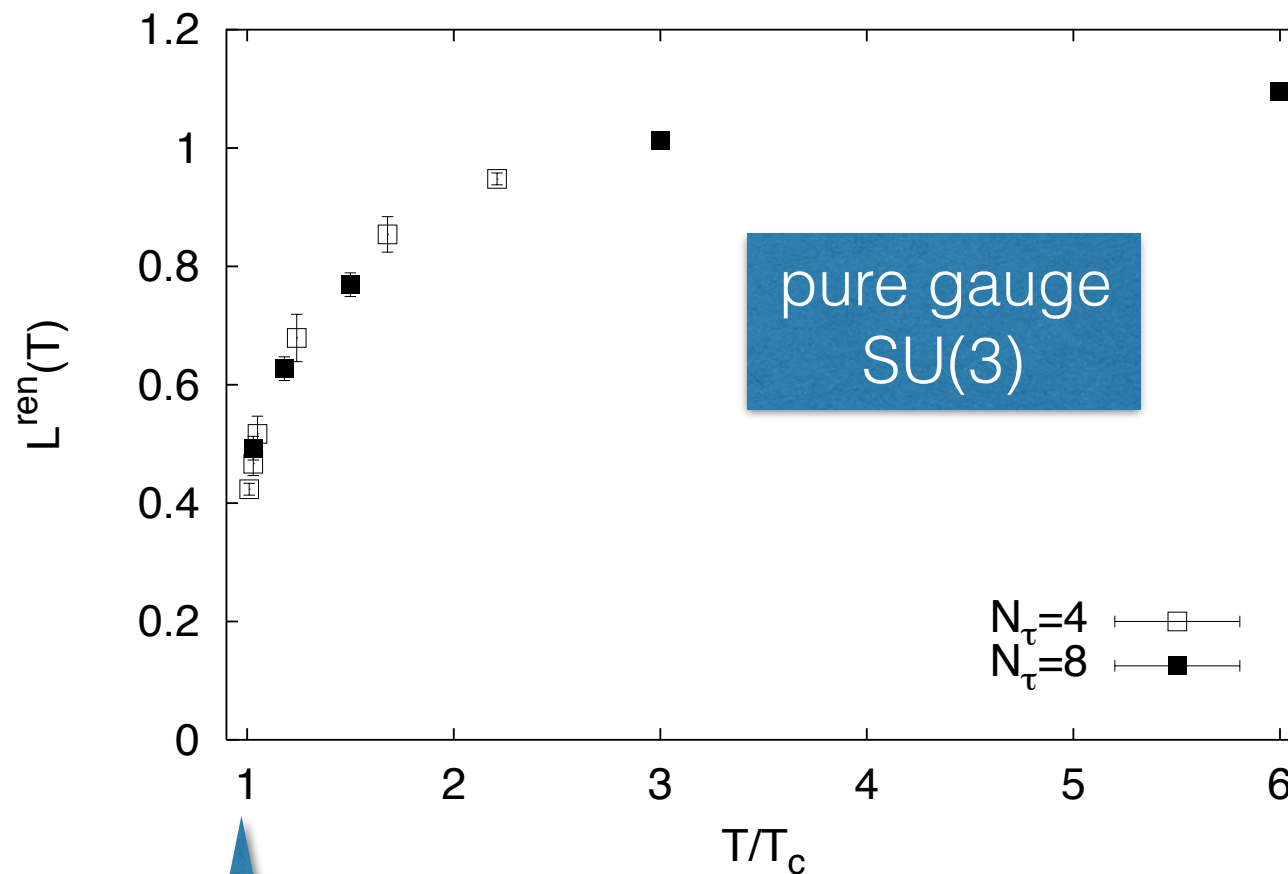
$\tau = x_4$
 $\in [0, \hbar/T]$



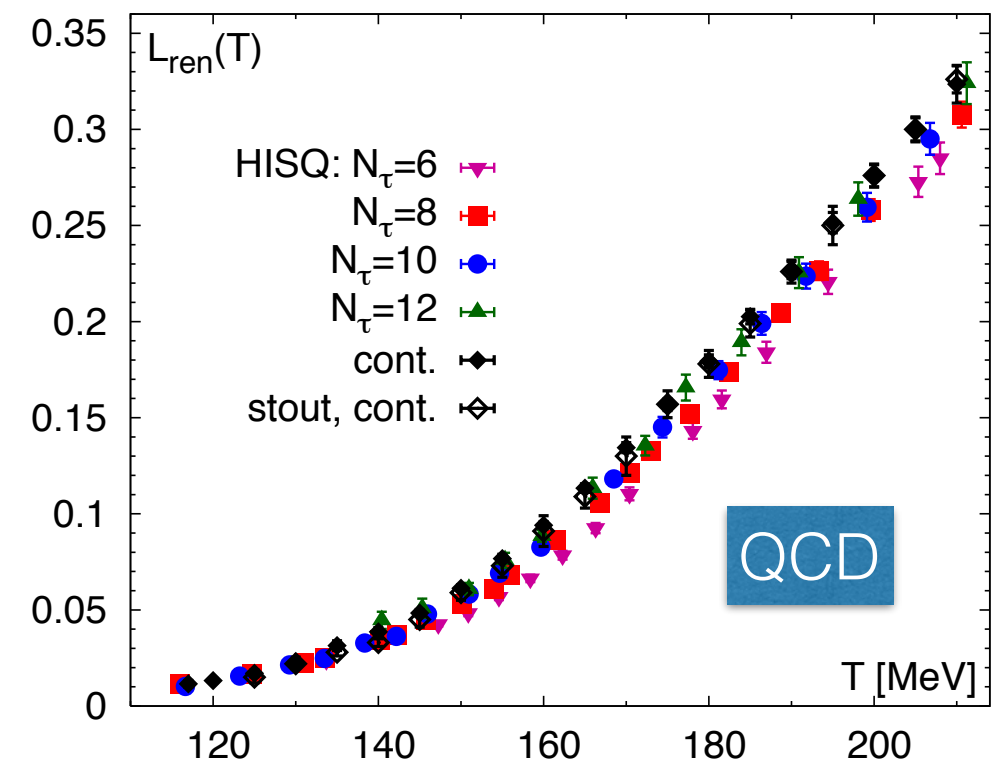
$$L = P = P \exp(i \oint A_\mu^a T^a dx_\mu)$$

$$L = \text{diag}(e^{i\mu_1}, e^{i\mu_2}, \dots, e^{i\mu_{N_c}})$$

$$\frac{1}{N_c} \text{Tr}(L) \sim e^{-F_q/T}$$



**L jumps to zero
the first order transition**



**Kaczmarek et al 2002
Bazavov et al 2016**

**Pisarski "semi-QGP" paradigm,
PNJL model**

Non-zero Polyakov line splits instantons
into **Nc** instanton-dyons
(Kraan, van Baal, Lee, Lu 1998)

Explained mismatch of quark condensate in SUSY QCD

V. Khoze (jr) et al 2001

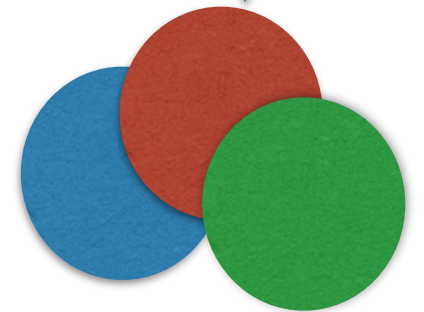
Explained confinement by back reaction to free energy

D. Diakonov 2012, Larsen+ES, Liu, Zahed+ES 2016

Explain chiral symmetry breaking in QCD
and in setting with **modified fermion periodicities**

R. Larsen+ES 2017, Unsal et al 2017

BPST



Pierre van Baal

general $SU(N_c)$

$$A_4 = 2\pi T \text{diag}(\mu_1, \mu_2, \dots, \mu_{N_c})$$

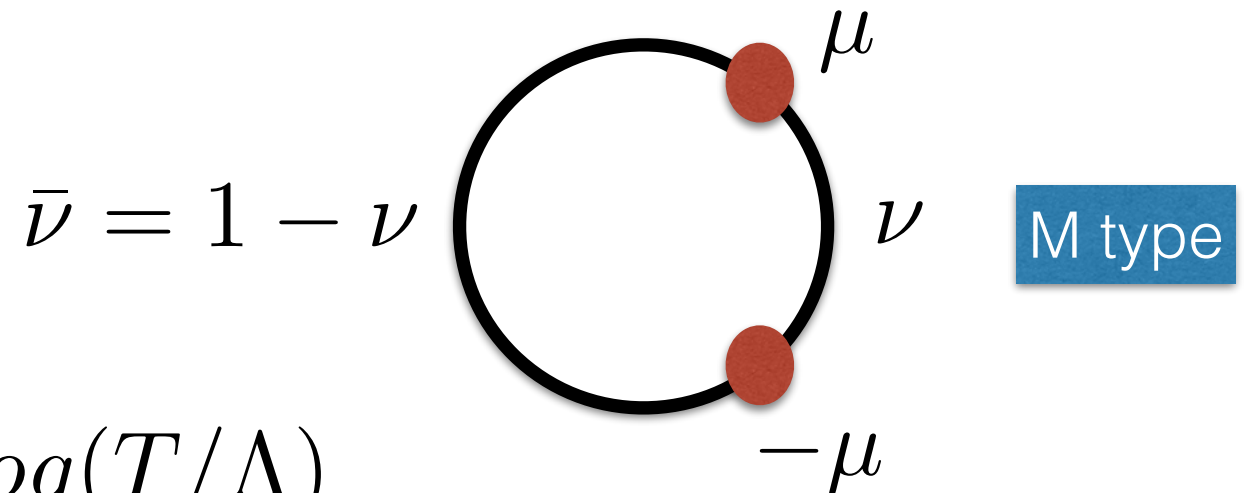
$$\nu_m = \mu_{m+1} - \mu_m$$

$$\sum_i \nu_i = 1$$

$$S_i = \nu_i \frac{8\pi^2}{g^2} = \nu_i \left(\frac{11N_c}{3} - \frac{2N_f}{3} \right) \log(T/\Lambda)$$

L type

the $SU(2)$ case is simpler



$$\bar{\nu} = 1 - \nu$$

$$A_4^a = \mp n_a v \Phi(vr)$$

together they make one instanton
instanton-dyons
=selfdual BPS mono

$$A_i^a = \epsilon_{aij} n_j \frac{1 - R(vr)}{r}$$

$$\vec{E} = \vec{B}$$

In $SU(2)$ there are 4 types of dyons,
Electric and magnetic charges = +1,-1

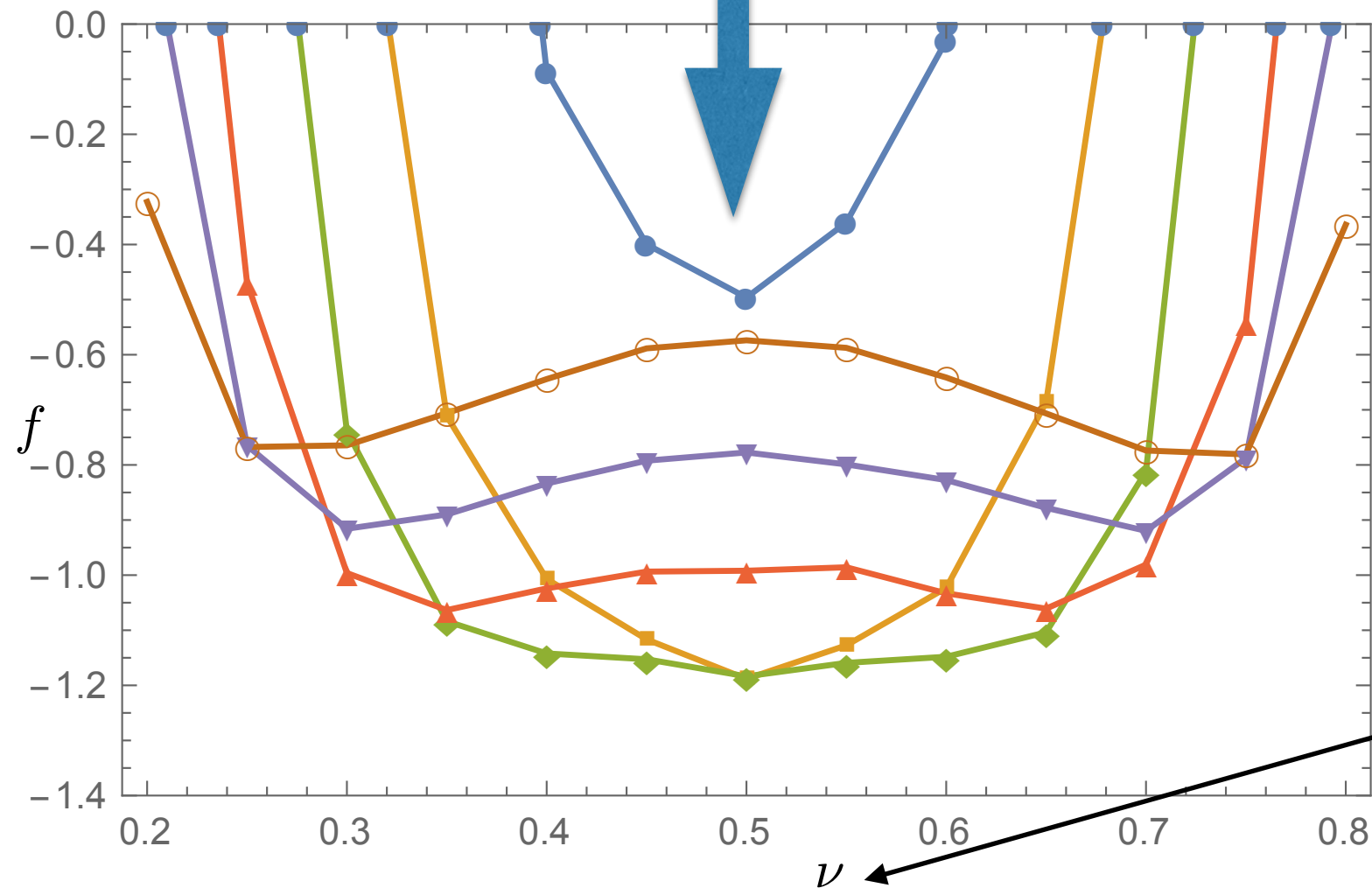
$$M, \bar{M}, L, \bar{L}$$

all one needs to do is to study their ensemble

(with Rasmus Larsen)

confined

free energy vs holonomy



$$\langle A_4^3 \rangle = v \frac{\tau^3}{2} = 2\pi T \nu \frac{\tau^3}{2}$$
$$\langle P \rangle = \cos(\pi\nu) \rightarrow 0$$

if $\nu = 1/2$

$\nu = 0$ is the trivial case
 $\nu = 1/2$ confining

So, as a function of the dyon density
the potential changes its shape
and confinement takes place

show only the “selfconsistent” input set.

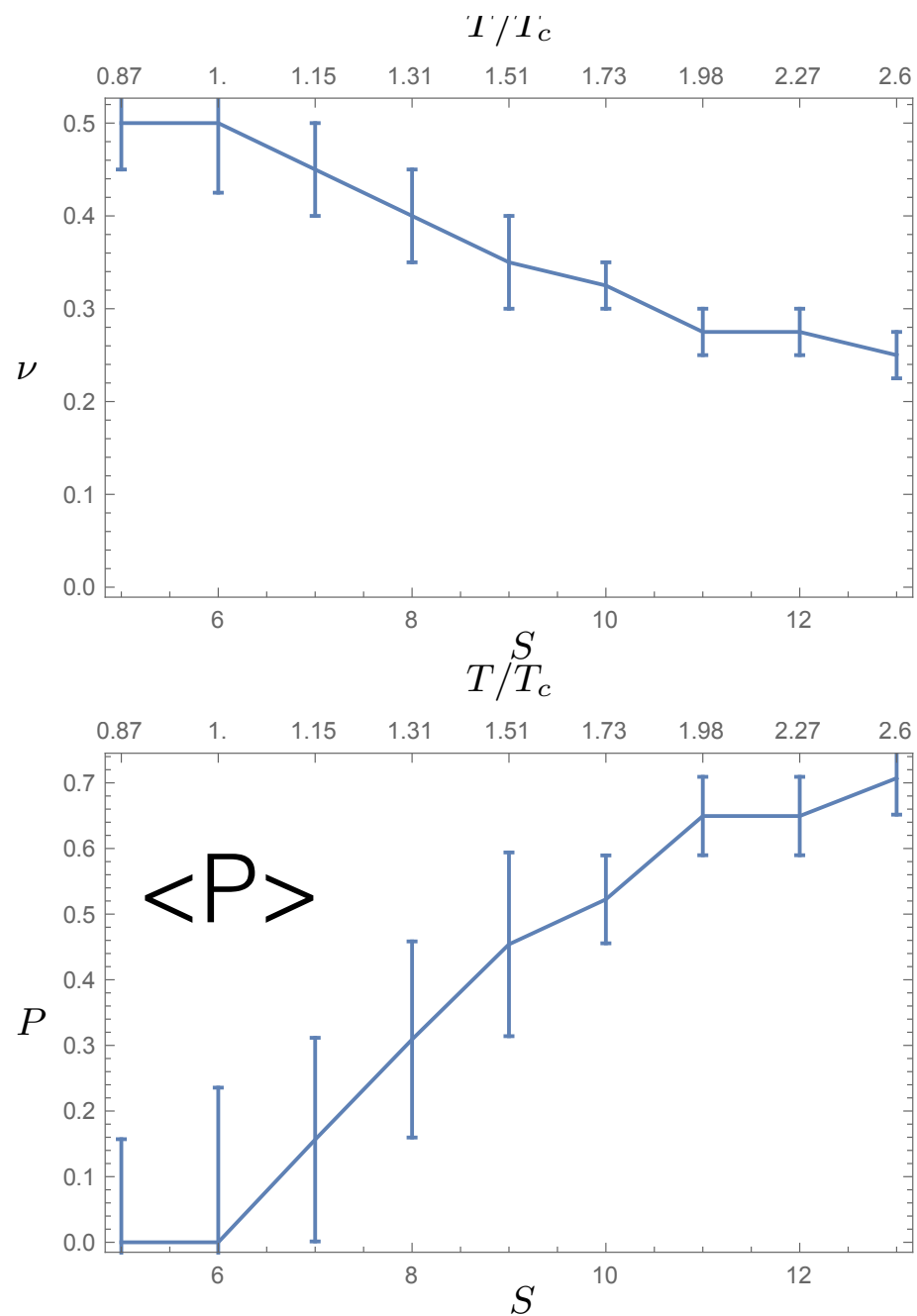


FIG. 6: Self-consistent value of the holonomy ν (upper plot) and Polyakov line (lower plot) as a function of action S (lower scales), which is related to T/T_c (upper scales). The error bars are estimates based on the fluctuations of the numerical data.

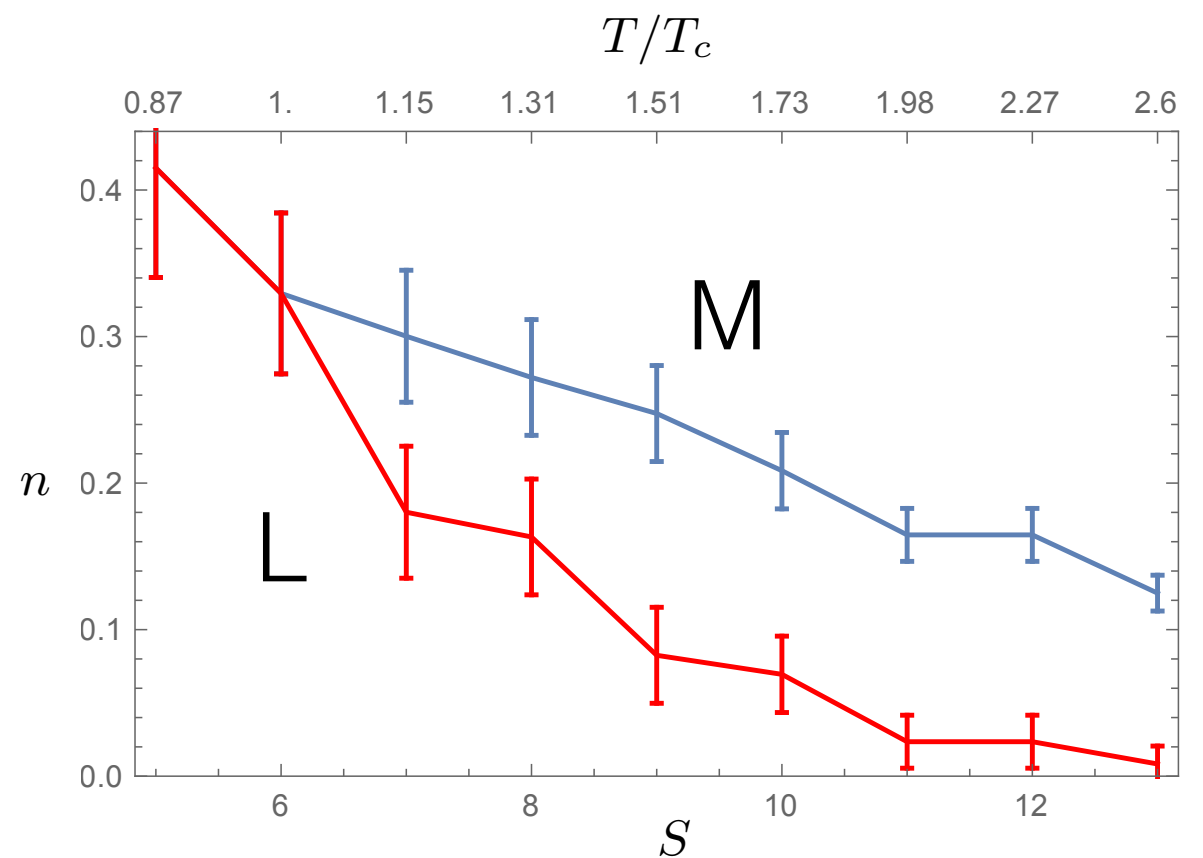


FIG. 8: (Color online). Density n (of an individual kind of dyons) as a function of action S (lower scale) which is related to T/T_c (upper scale) for M dyons (higher line) and L dyons (lower line). The error bars are estimates based on the density of points and the fluctuations of the numerical data.

confining phase is symmetric
 $n_L = n_M$

$$S = \left(\frac{11N_c}{3} - \frac{2N_f}{3} \right) \log\left(\frac{T}{\Lambda_T} \right).$$

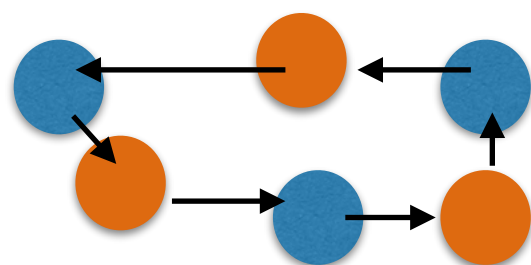
Instanton-dyon Ensemble with two Dynamical Quarks: the Chiral Symmetry Breaking

Rasmus Larsen and Edward Shuryak

Department of Physics and Astronomy, Stony Brook University, Stony Brook NY 11794-3800, USA

This is the second paper of the series aimed at understanding of the ensemble of the instanton-dyons, now with two flavors of light dynamical quarks. The partition function is appended by the fermionic factor, $(\det T)^{N_f}$ and Dirac eigenvalue spectra at small values are derived from the numerical simulation of 64 dyons. Those spectra show clear chiral symmetry breaking pattern at high dyon density. Within current accuracy, the confinement and chiral transitions occur at very similar densities.

$$|\langle \bar{\psi}\psi \rangle| = \pi \rho(\lambda)_{\lambda \rightarrow 0, m \rightarrow 0, V \rightarrow \infty}$$



**collectivized
zero mode zone**

**dip near zero is
a finite size effect**



**low density
unbroken chiral sum**

extracting condensate
is far from trivial...

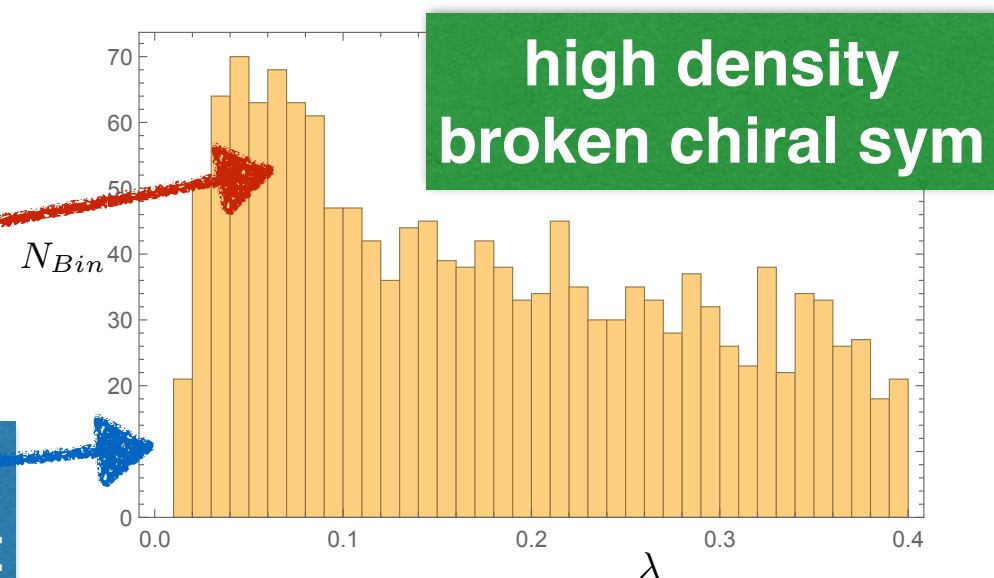


FIG. 1: Eigenvalue distribution for $n_M = n_L = 0.47$, $N_F = 2$ massless fermions.

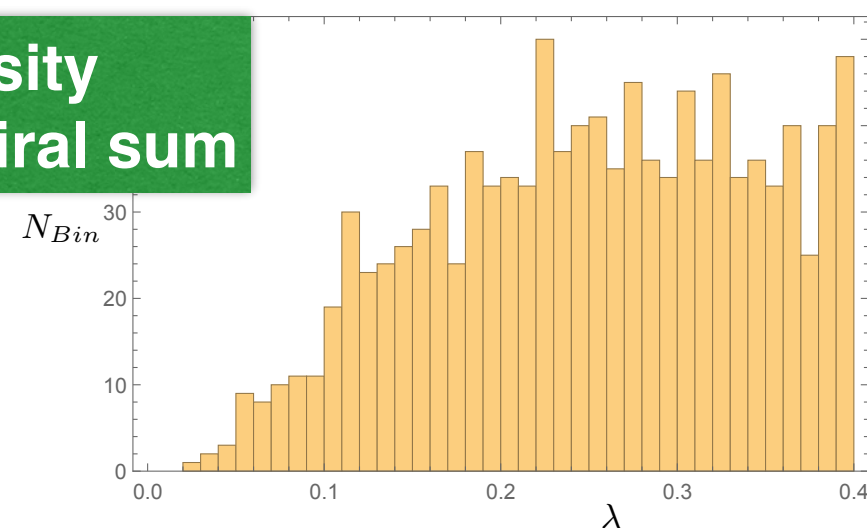


FIG. 2: Eigenvalue distribution for $n_M = n_L = 0.08$, $N_F = 2$ massless fermions.

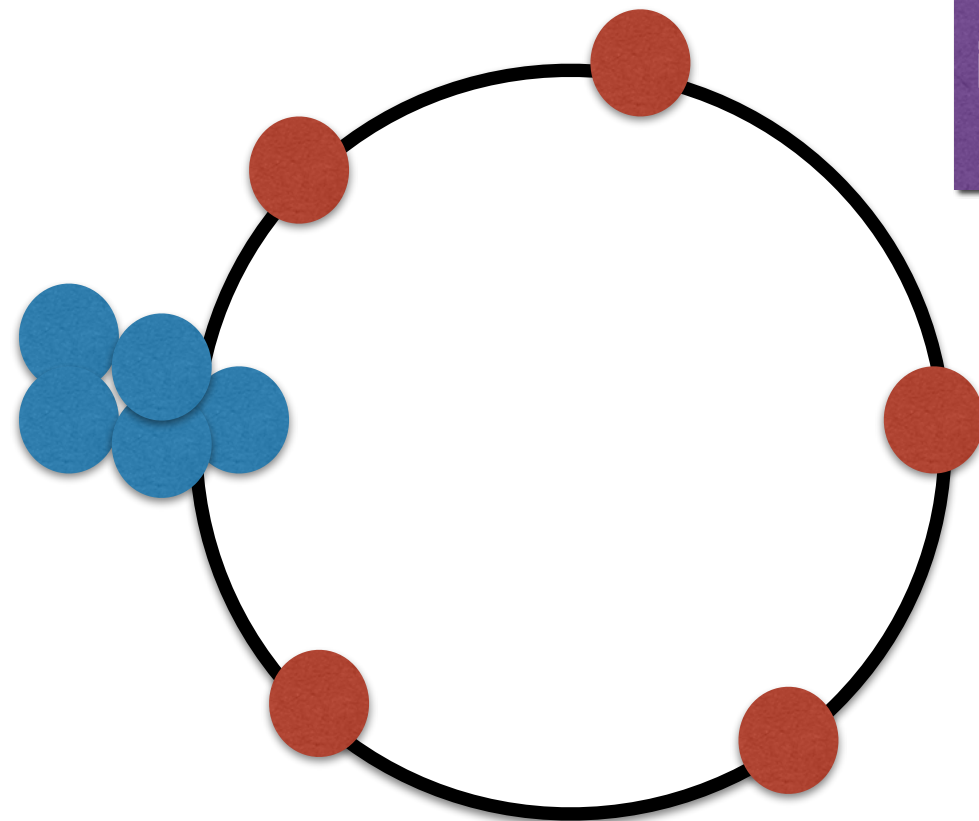
QCD with quarks having arbitrary periodicity phases (over the Matsubara time)

$$\psi(\tau + \hbar/T) = e^{2\pi i z_f} \psi(\tau)$$

$$z_f = 0 \text{ bosons}$$

$$z_f = -1/2 \text{ fermions}$$

Ordinary $N_c=N_f=5$ QCD

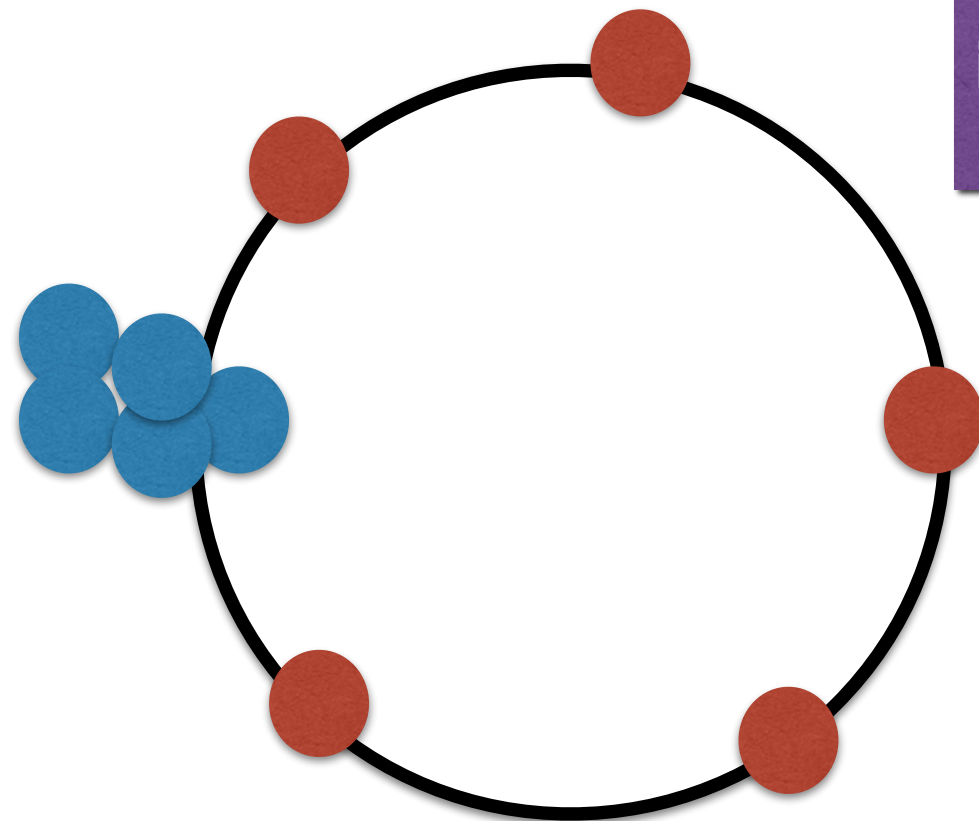


P without a trace
is a diagonal unitary matrix
 $\Rightarrow N_c$ phases (red dots)

quark periodicity
phases $\Rightarrow N_f$ blue dots
are in this case all $=\pi$
quarks are fermions

**as a consequence,
out of 5 types of instanton-dyons
only one L has zero modes**

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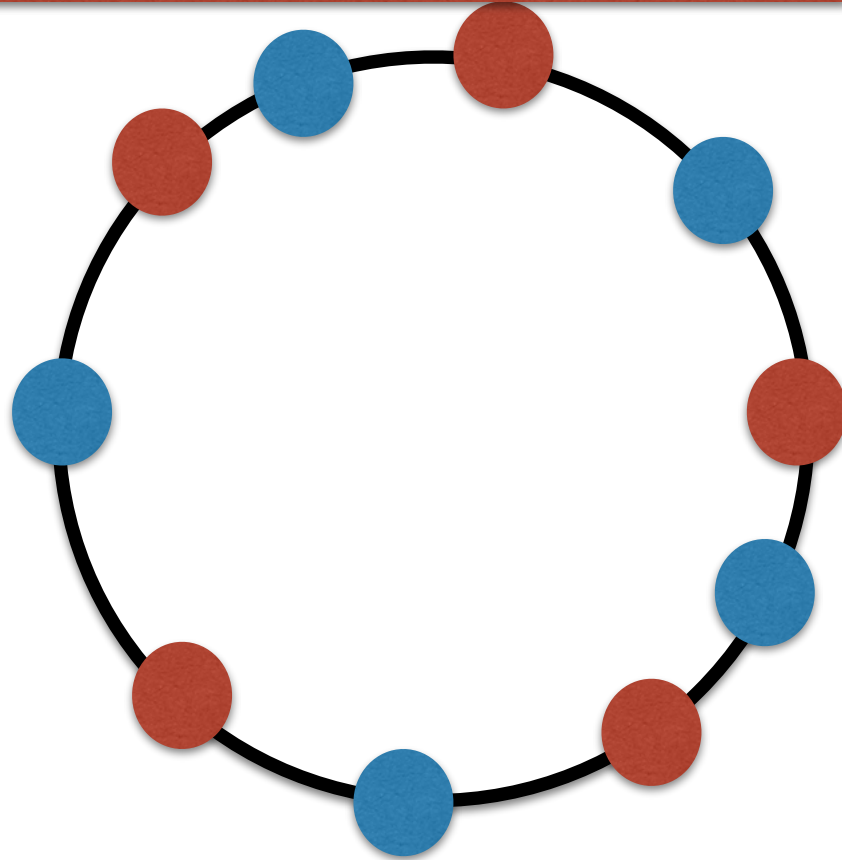
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But one can deform QCD moving fermion phases (blue dots) as we like!

**still $N_c=N_f=5$ but with
“most democratic” arrangement
ZN-symmetric QCD**

**H. Kouno, Y. Sakai, T. Makiyama, K. Tokunaga, T.
Sasaki and M. Yahiro, J. Phys. G 39, 085010 (2012).**



**quark periodicity
phases \Rightarrow N_f blue dots
are in this case
flavor-dependent**

**In this case each dyon type has
one zero mode
with one quark flavor
 $\Rightarrow N$ independent topological ZMZ's!**

Both transitions are dramatically different!

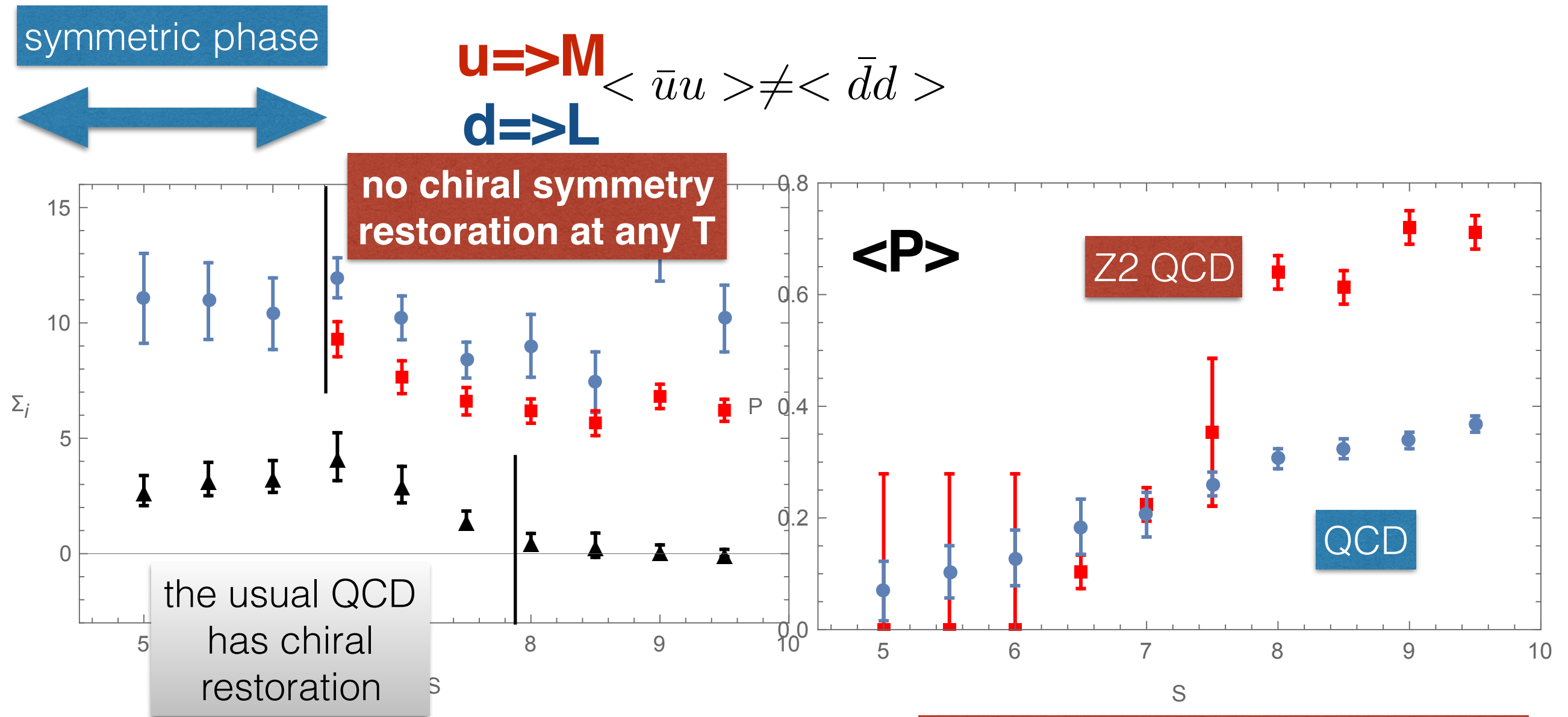


FIG. 6: Chiral condensate generated by u quarks and L dyons (red squares) and d quarks interacting with M dyons (blue circles) as a function of action S , for the Z_2 -symmetric model. For comparison we also show the results from II for the usual QCD-like model with $N_c = N_f = 2$ by black triangles.

why is condensate much larger for Z2?

confining phase gets much more robust: strong first order mixed phase (flat F) is observed at medium densities

lattice study of Z3 QCD

Lattice study on QCD-like theory with exact center symmetry

Takumi Iritani*

Yukawa Institute for Theoretical Physics, Kyoto 606-8502, Japan

Etsuko Itou†

High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan

Tatsuhiro Misumi‡

Department of Mathematical Science, Akita University,

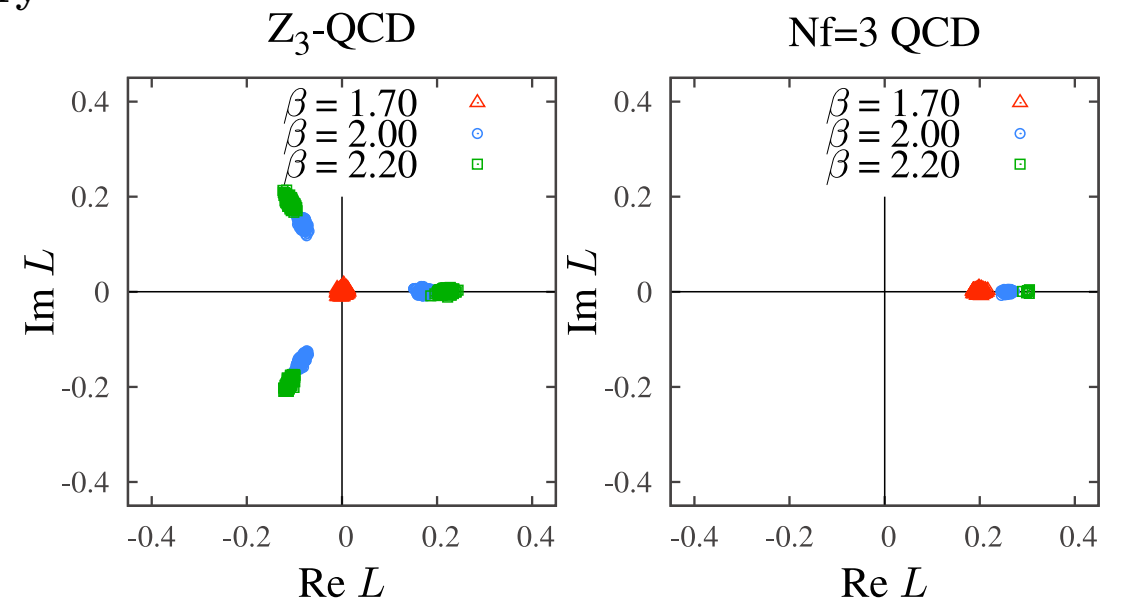
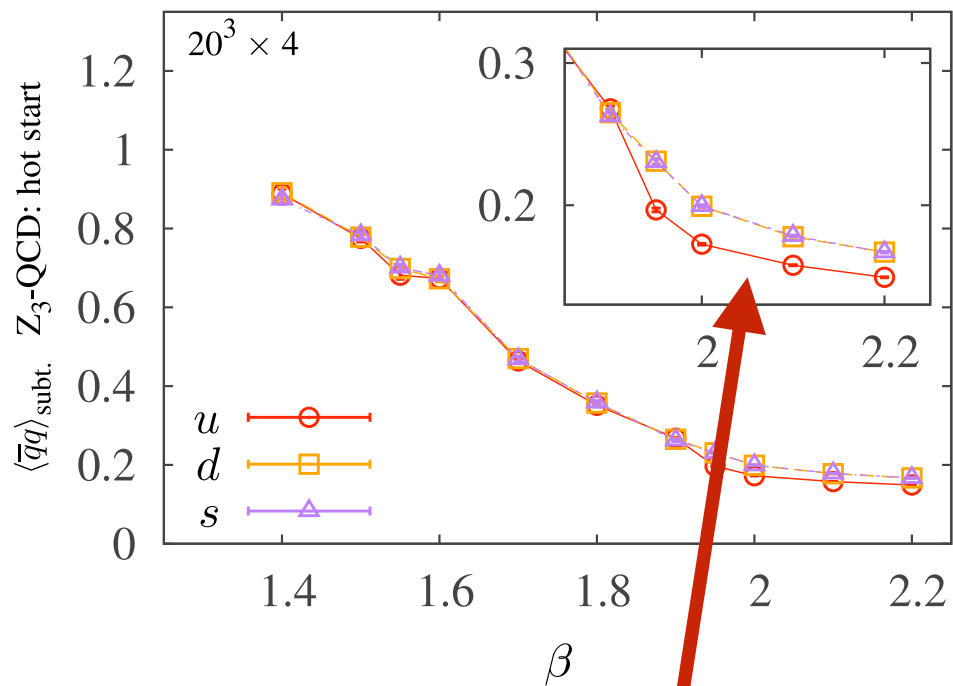
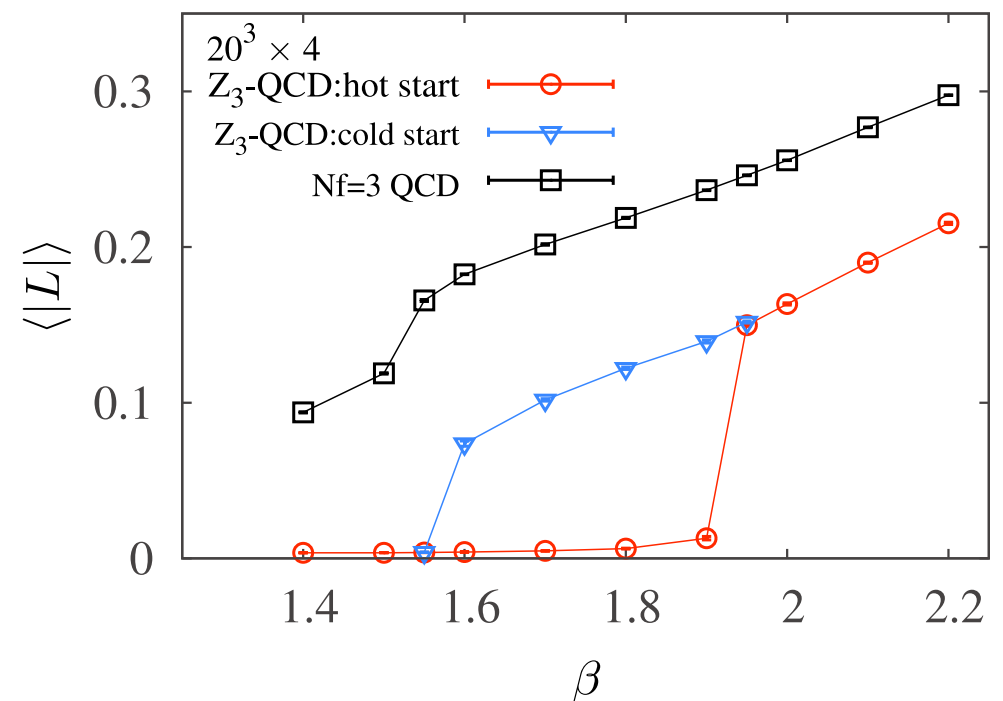


FIG. 1: Polyakov loop distribution plot in Z_3 -QCD (left) and the standard three-flavor QCD (right). Based on $16^3 \times 4$ lattice for $\beta = 1.70, 2.00, 2.20$ with the same values of κ in both panels.



lattice study of Z3 QCD

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Takumi Iritani*

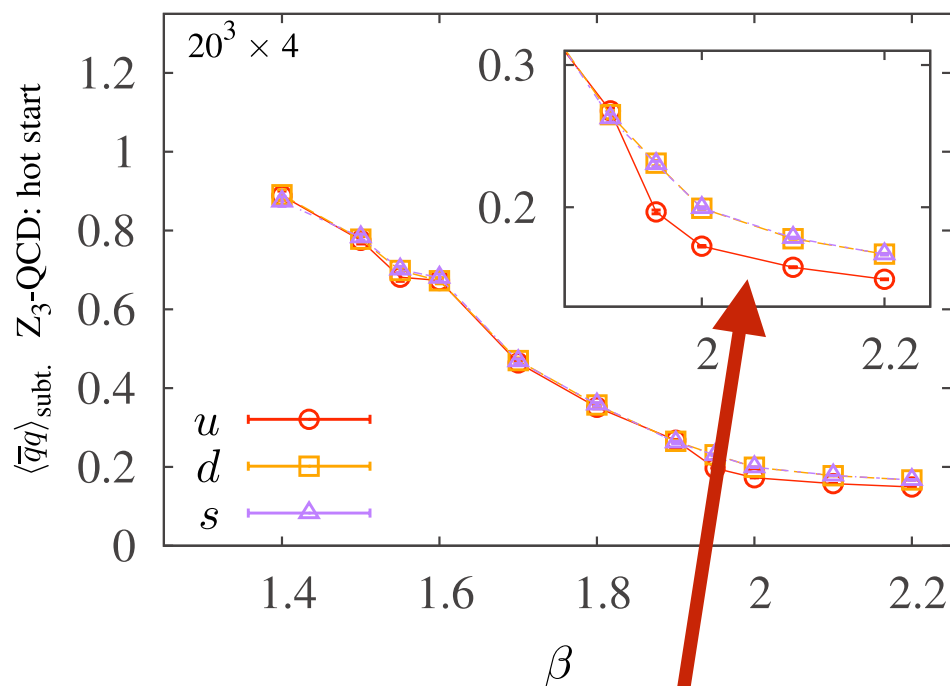
Yukawa Institute for Theoretical Physics, Kyoto 606-8502, Japan

Etsuko Itou†

High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan

Tatsuhiro Misumi‡

Department of Mathematical Science, Akita University,



explanation: three flavors of quarks interact with three different “liquids” of M1,M2,L instanton-dyons!

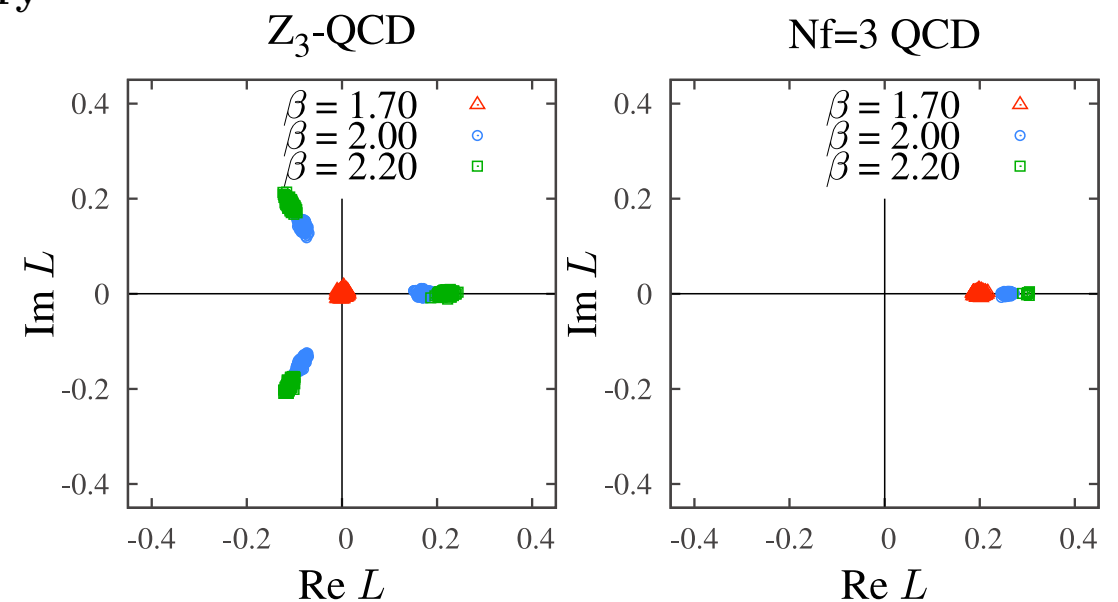
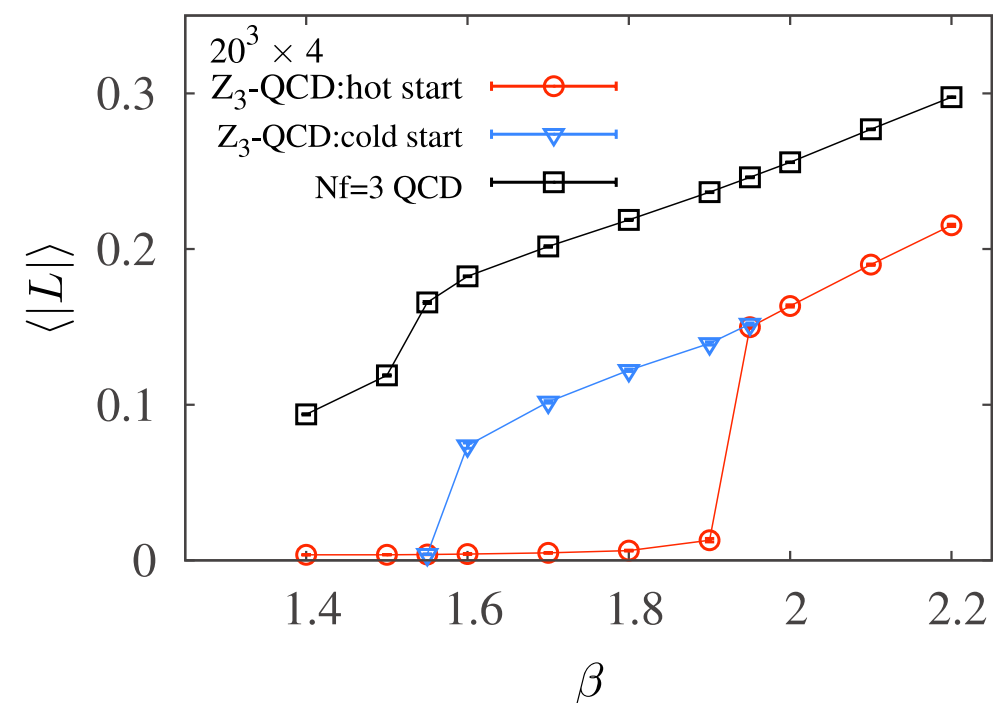
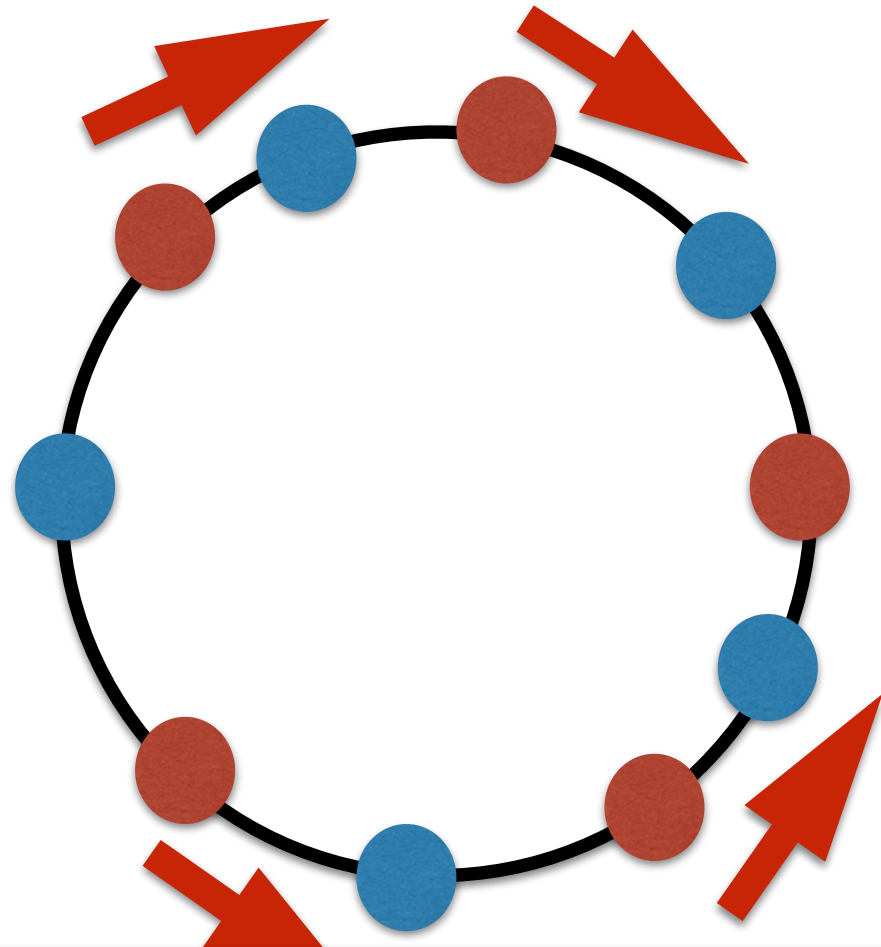


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still $N_c=N_f=5$

with growing T all red dots move toward zero
(Polyakov line $L \Rightarrow 1$)



each time
any L phase (red dot)
passes the
quark phase (blue dot),
the quark zero mode jump
to another dyon type

This leads to series of phase transitions related to
Roberge-Weiss transitions,
observed on the lattice at imaginary chem. potential

Dyons and Roberge - Weiss transition in lattice QCD

V.G. Bornyakov, D.L. Boyda, V.A. Goy, E. -M. Ilgenfritz, B.V. Martemyanov,
A.V. Molochkov, Atsushi Nakamura, A.A. Nikolaev, V.I. Zakharov
EPJ Web Conf. 137 (2017) 03002
[arXiv:1611.07789](https://arxiv.org/abs/1611.07789)

I suggest to call them
Van Baal transitions

instanton-dyons on the lattice

(with S.Sharma and R.Larsen)

**the cleanness case:
domain wall fermions
Q=1 configurations
Nt=8,Nx=32, T/Tc=1,1.08**

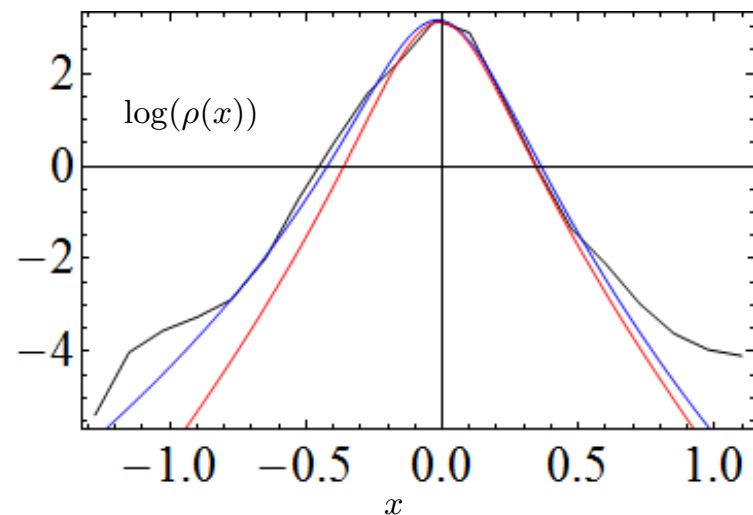


FIG. 8: $\log(\rho(x))$ of the zero mode of conf. 2960 at $\phi = \pi$ (black) and the log of the analytic formula for $P = 0.4$ and $P = 1$ though the maximum. $T = 1.08T_c$. Red peak only has been scaled to fit in height, while blue peak uses the found normalization.

**excellent agreement of the shape
with analytic formulae**

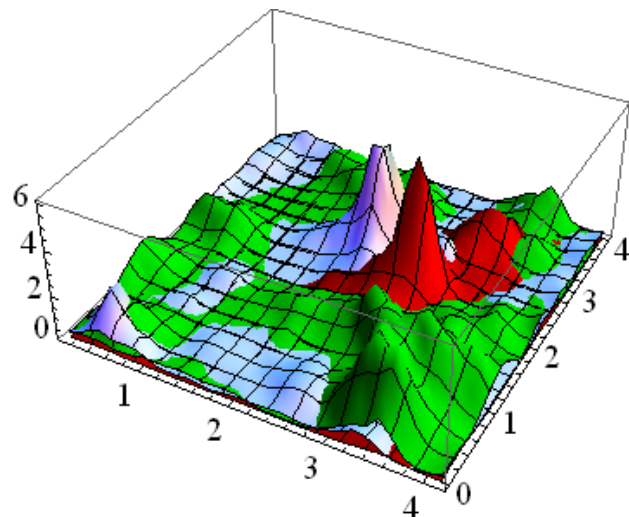


FIG. 17: $\rho(x, y)$ of the zero mode of conf. 2660 at $T = T_c$. $\phi = \pi$ (red), $\phi = \pi/3$ (blue), $\phi = -\pi/3$ (green). Peak height has been scaled to be similar to that of $\phi = \pi$.

Fermionic Zero modes and topology around T_c

Rasmus Larsen, Sayantan Sharma and Edward Shuryak

We study the vacuum of 2+1 flavor QCD immediately above the chiral crossover transition temperature. Since the overlap fermions have an index theorem on a finite lattice we use the zero modes of the valence overlap quarks to probe the topological structures of the sea domain wall fermion configurations on lattices of size $32^3 \times 8$. The change in the properties of the zero modes are studied in detail by changing the boundary conditions of the overlap Dirac operator along the temporal direction. Our studies show that the zero modes have strong similarity to the instanton-dyons. We further provide evidence for this by studying in detail the properties of the near-zero eigenvectors of the overlap operator.

**extracting the shape of
the fermionic zero mode
and modifying the phase
one can find all 3 dyons**

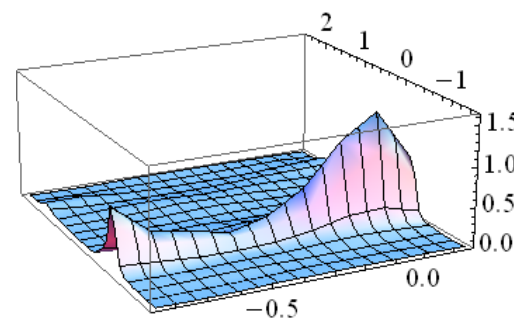


FIG. 3: $\rho(x, t)$ of the zero mode of conf. 2000 at $\phi = \pi/3$.

T_{T_c}

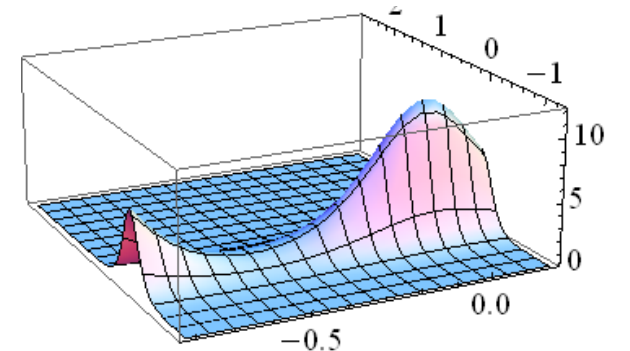


FIG. 4: Analytic zero mode density $\rho(x, t)$ at $\phi = \pi/3$. Main centered at the origin. Two other dyons at $(0.2, 0.0, 0.0)$ and $(-0.2, 0.0, 0.0)$.

**We found that their fields
interfere with each other
the interaction between them
is in excellent agreement with
van Baal analytic formulae**

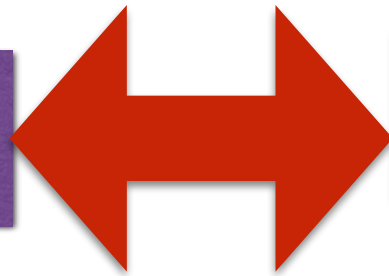
Semiclassical theory of instanton-dyon ensemble Is in Poisson duality with monopoles

One can start in the theory
in which there is a complete theoretical control
on both and **compare two approaches directly**

N.Dorey and A.Parnachev
JHEP 0108, 59 (2001)
hep-th/0011202]

N=4 extended supersymmetry
with Higgled scalar
compactified on a circle

Partition function calculated in
terms of **monopoles**

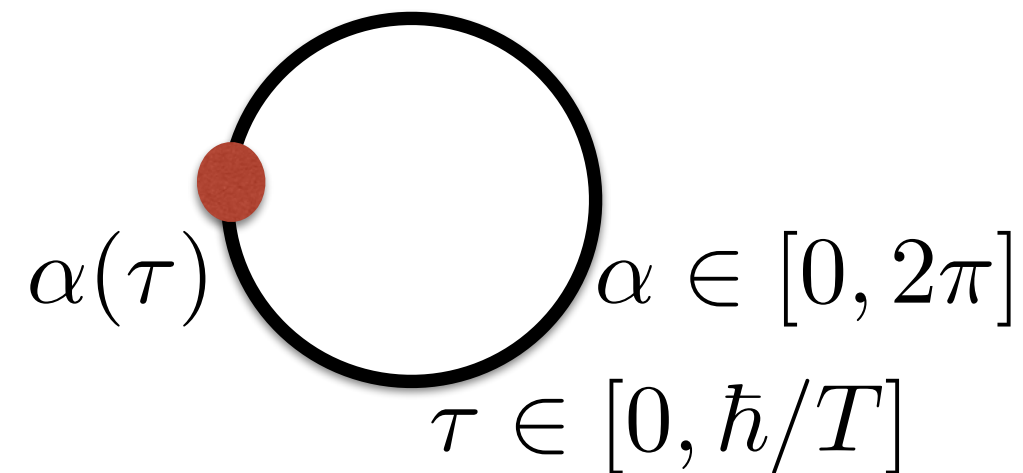


Partition function calculated in
terms of **instanton-dyons**

Configurations are obviously very different
Zs also look different,
and yet they are related
by the **Poisson summation formula**
and thus are the same!!!

Is there any relation between the semiclassical instanton-dyons and QCD monopoles?

Adith Ramamurti,^{*} Edward Shuryak,[†] and Ismail Zahed[‡]



**The same phenomenon in much simpler setting:
 quantum particle on a circle at finite T**

A Hamiltonian vs Lagrangian approaches

$$Z_1 = \sum_{l=-\infty}^{\infty} \exp \left(-\frac{l^2}{2\Lambda T} + il\omega \right)$$

**moment
of inertia**

**Aharonov-Bohm
phase**

$$Z_2 = \sum_{n=-\infty}^{\infty} \sqrt{2\pi\Lambda T} \exp \left(-\frac{T\Lambda}{2} (2\pi n - \omega)^2 \right).$$

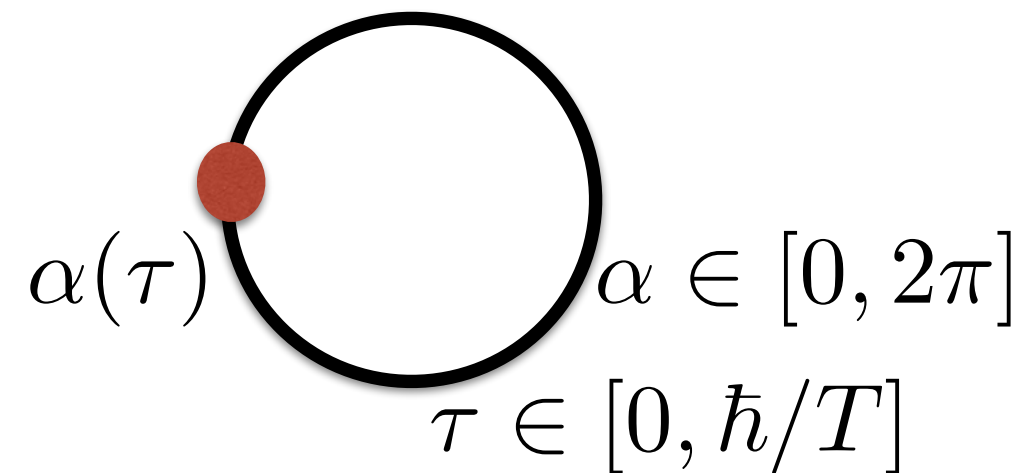
**Matsubara
winding number**

based on classical paths

$$\alpha_n(\tau) = 2\pi n \frac{\tau}{\beta},$$

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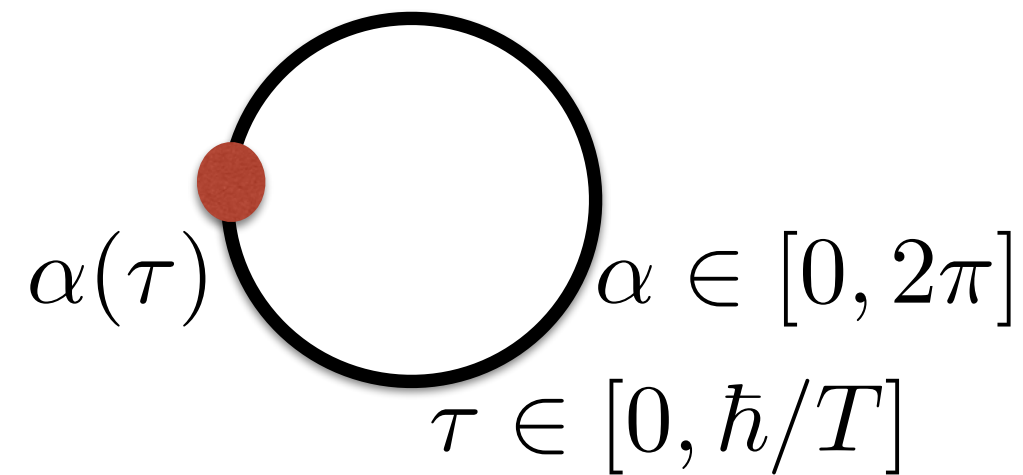
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on T and holonomy omega**

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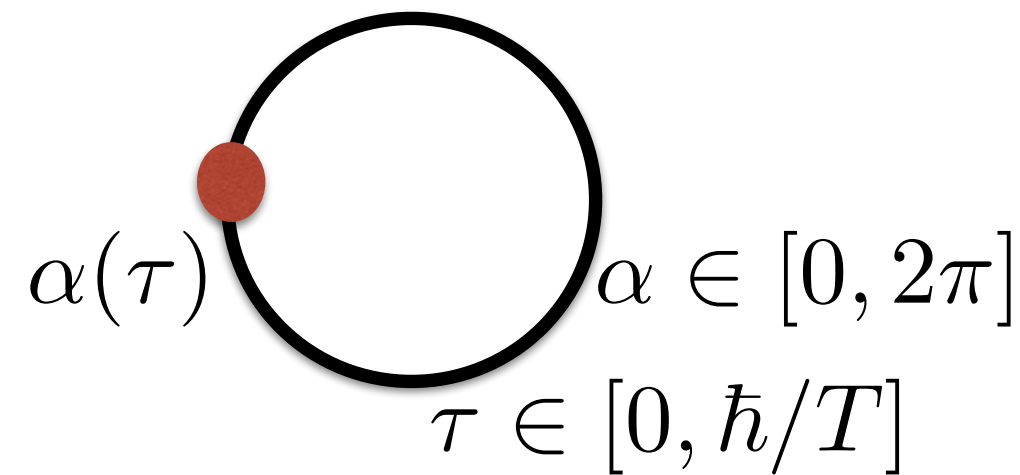
**Note completely different dependence
on T and holonomy omega**

**And yet, they are the same!
(elliptic theta function of the 3 type)**

$$Z_1 = Z_2 = \theta_3 \left(-\frac{\omega}{2}, \exp \left(-\frac{1}{2\Lambda T} \right) \right),$$

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**Matsubara
winding number**

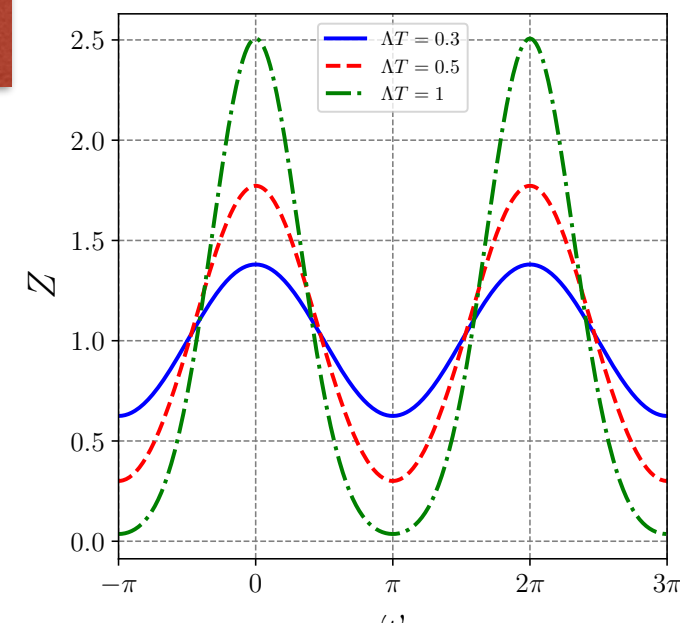
based on classical paths

$$\alpha_n(\tau) = 2\pi n \frac{\tau}{\beta},$$

**Note completely different dependence
on T and holonomy omega**

**And yet, they are the same!
(elliptic theta function of the 3 type)**

$$Z_1 = Z_2 = \theta_3 \left(-\frac{\omega}{2}, \exp \left(-\frac{1}{2\Lambda T} \right) \right),$$



Is there any relation between the semiclassical instanton-dyons and QCD monopoles?

Adith Ramamurti,^{*} Edward Shuryak,[†] and Ismail Zahed[‡]

instanton-dyons with winding number n

The twisted solution is obtained in two steps. The first is the substitution

$$v \rightarrow n(2\pi/\beta) - v, \quad (13)$$

and the second is the gauge transformation with the gauge matrix

$$\hat{\Omega} = \exp\left(-\frac{i}{\beta}n\pi\tau\hat{\sigma}^3\right), \quad (14)$$

where we recall that $\tau = x^4 \in [0, \beta]$ is the Matsubara time. The derivative term in the gauge transformation adds a constant to A_4 which cancels out the unwanted $n(2\pi/\beta)$ term, leaving v , the same as for the original static monopole. After “gauge combing” of v into the same direction, this configuration – we will call L_n – can be combined with any other one. The solutions are all

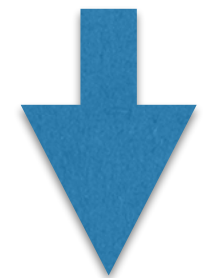
$$S_n = (4\pi/g^2)|2\pi n/\beta - v|$$

$$\sum_{n=-\infty}^{\infty} f(\omega + nP) = \sum_{l=-\infty}^{\infty} \frac{1}{P} \tilde{f}\left(\frac{l}{P}\right) e^{i2\pi l\omega/P}$$

Poisson summation formula can be used to derive the monopole Z

$$Z_{\text{inst}} = \sum_n e^{-\left(\frac{4\pi}{g_0^2}\right)|2\pi n - \omega|}$$

$$Z_{\text{mono}} \sim \sum_{q=-\infty}^{\infty} e^{iq\omega - S(q)}$$



$$S(q) = \log\left(\left(\frac{4\pi}{g_0^2}\right)^2 + q^2\right)$$

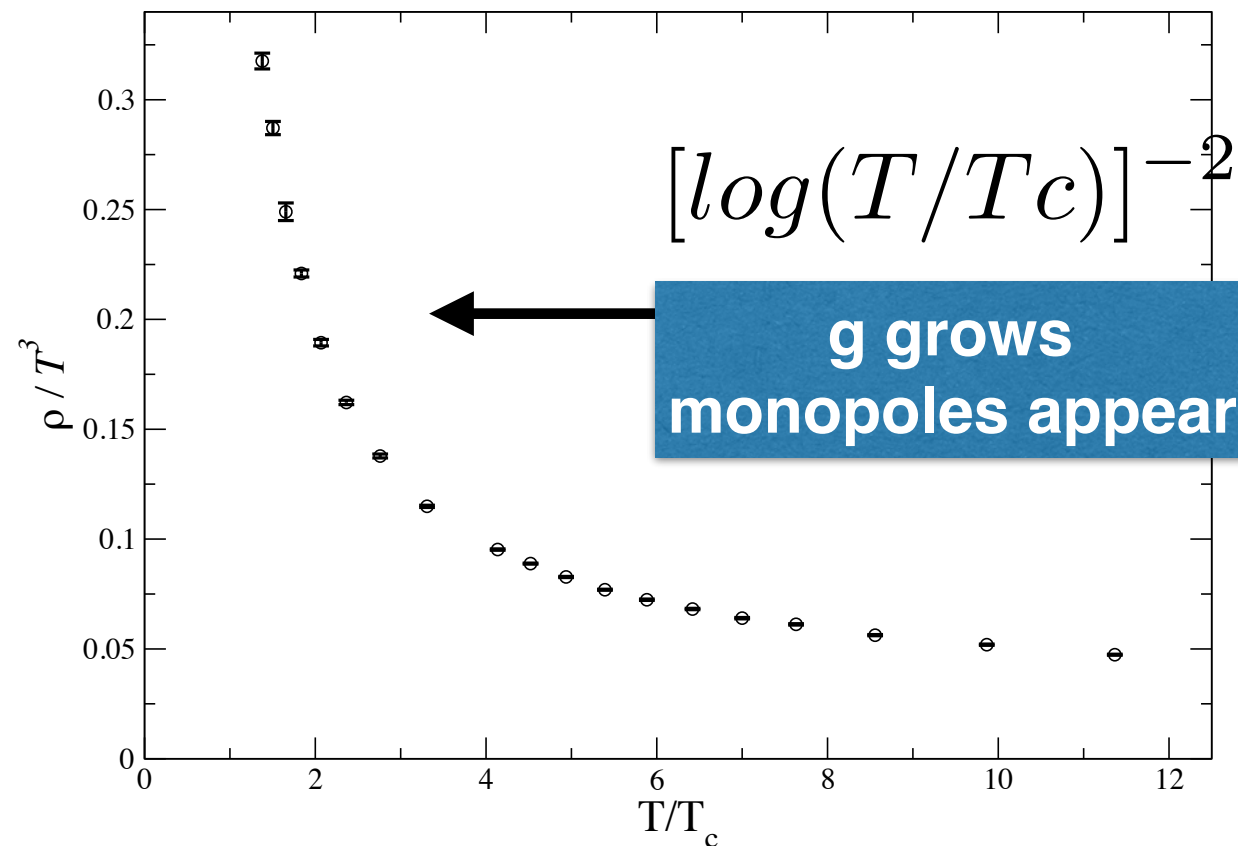
$$\approx 2\log\left(\frac{4\pi}{g_0^2}\right) + q^2\left(\frac{g_0^2}{4\pi}\right)^2 + \dots$$

q is angular momentum of rotating monopole, so it is electric charge

Therefore we now understand why
 The density of monopoles is well fitted by an inverse power of
 $\log(T)$, not power of $T \Rightarrow$

It is because they are not really semiclassical objects!

$$S_{mono} \sim \log(const/g^2) = \log(\log(T/T_c))$$



D'Alessandro, A. and D'Elia, M. (2008).
 Magnetic monopoles in the high temperature
 phase of Yang-Mills theories.
 Nucl. Phys., B799:241–254. 0711.1266.

For instantons and
 dyons it is different

Fig. 2.6 The normalized monopole density ρ/T^3 for the $SU(2)$ pure gauge theory as a function of the temperature, in units of the critical temperature T/T_c , above the deconfinement transition.

$$\exp(-S) \sim \exp(-const/g^2) = \exp(-const' * \log(T)) = 1/T^{power}$$

**Semiclassical theory at finite T
And few-nucleon clusters at freeze out
(with Juan Torres-Rincon)**

Standard textbook definition of the density matrix

$$P(x_0) = \sum |\psi(x_0)|^2 e^{-E_i/T} \quad ($$

As shown by Feynman, the density matrix for any quantum system can be expressed by the path integrals, over paths passing through the point x_0 . Analytic continuation to Euclidean (Matsubara) time defined on a circle $\tau \in [0, \beta = \hbar/T]$ lead to its finite temperature generalization

$$P(x_0) = \int Dx(t) e^{-S_E(x(\tau))} \quad (2)$$

taken over the periodic paths which starts and ends at x_0 . This expression has led to multiple applications, perturbative (using Feynman diagrams) or numerical (e.g. lattice gauge theory). This is so well known that any references are not needed.

**A novel semiclassical theory:
the path integral is dominated by minimal action (classical) path, called “flucton”.
The idea was introduced by me in 1988.
Unlike WKB, this approach works for multidimensional and QFT settings.
It leads to systematic perturbative series based
on Feynman diagrams, with clear rules for each order.**

An example at $T \neq 0$

anharmonic oscillator, defined by

$$S_E = \oint d\tau \left(\frac{\dot{x}^2}{2} + \frac{x^2}{2} + \frac{g}{2} x^4 \right)$$

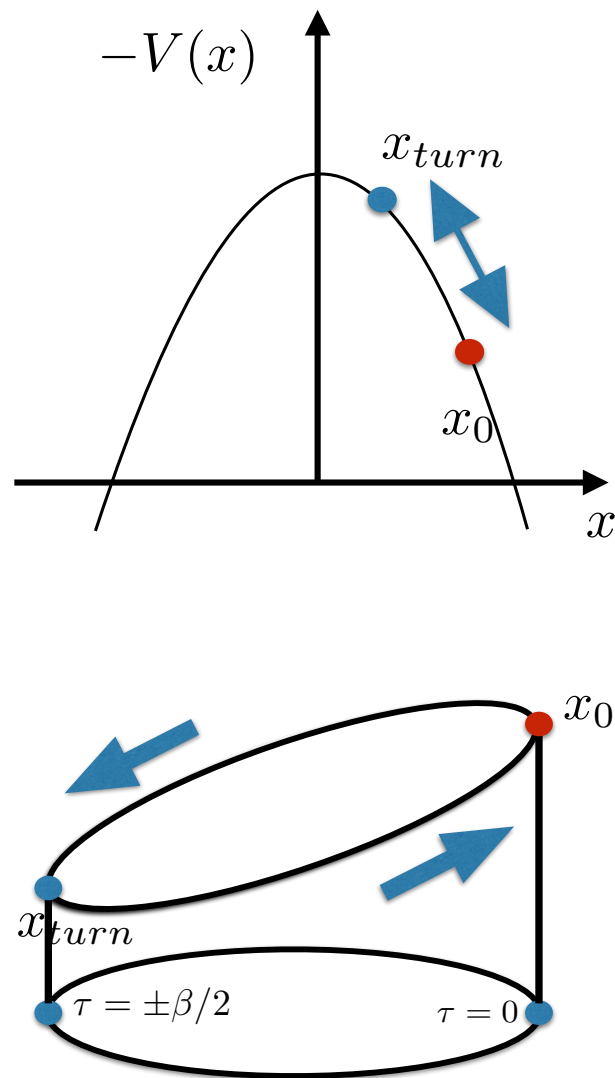


FIG. 2: Two sketches explaining properties of the flucton classical paths. The upper one shows the (flipped) potential $-V(x)$ versus its coordinate. The needed path starts from arbitrary observation point x_0 (red dot), goes uphill, turns back at the turning point x_{turn} (blue dot), and returns to x_0 during the required period $\beta = \hbar/T$. The lower plot illustrate the same path as a function of Euclidean time τ defined on a “Matsubara circle” with circumference β .

At $T=0$ period is infinite

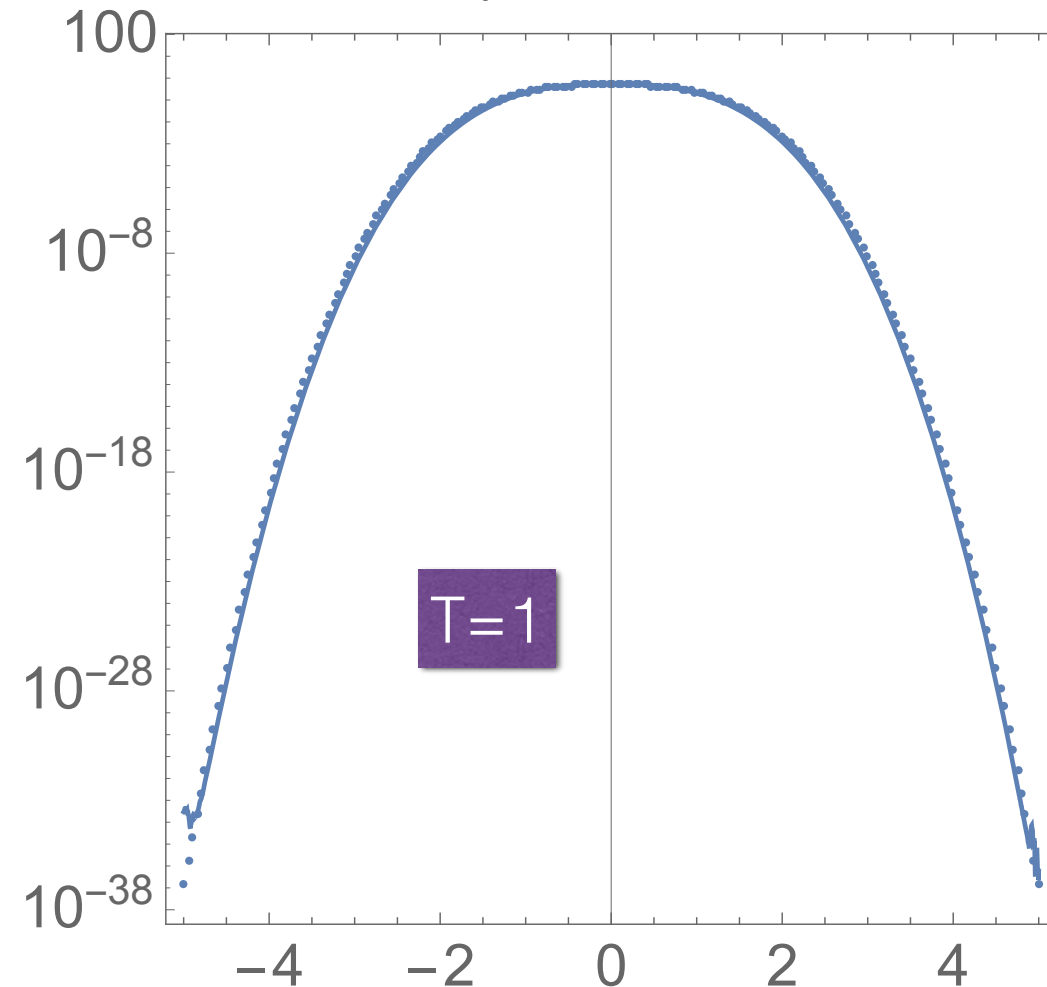
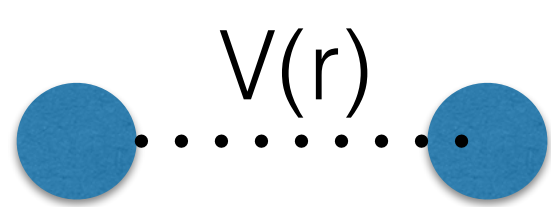
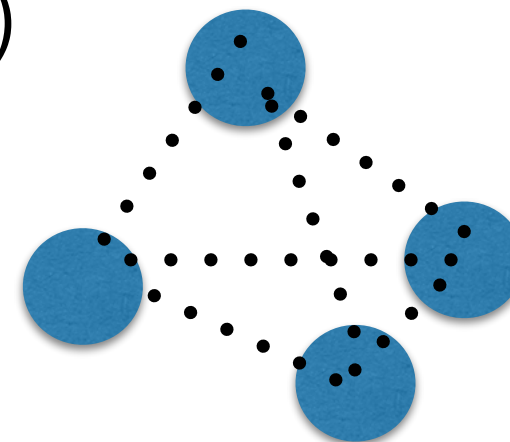
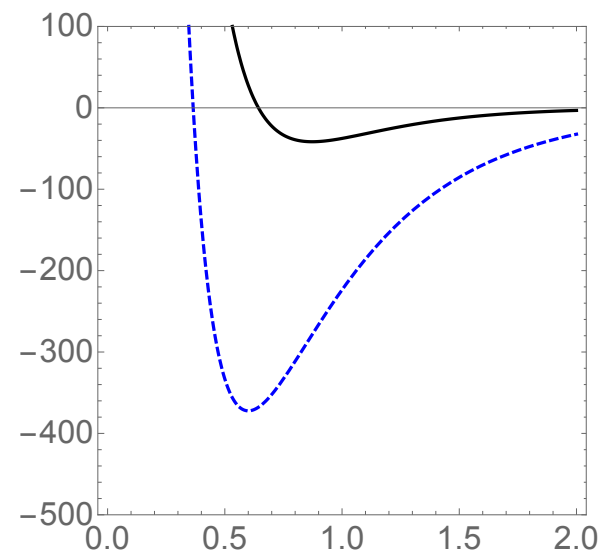


FIG. 3: Density matrix $P(x_0)$ vs x_0 for anharmonic oscillator with the coupling $g = 1$, calculated via the definition (1) (line) and the flucton method (points). The line is based on 60 wave functions found numerically: one can notice that finite number of them leads to deviations, which however happen at very distant tails, with the probability $P \sim 10^{-30}$.

semiclassical clustering of nucleons at Tf (about 100 MeV)

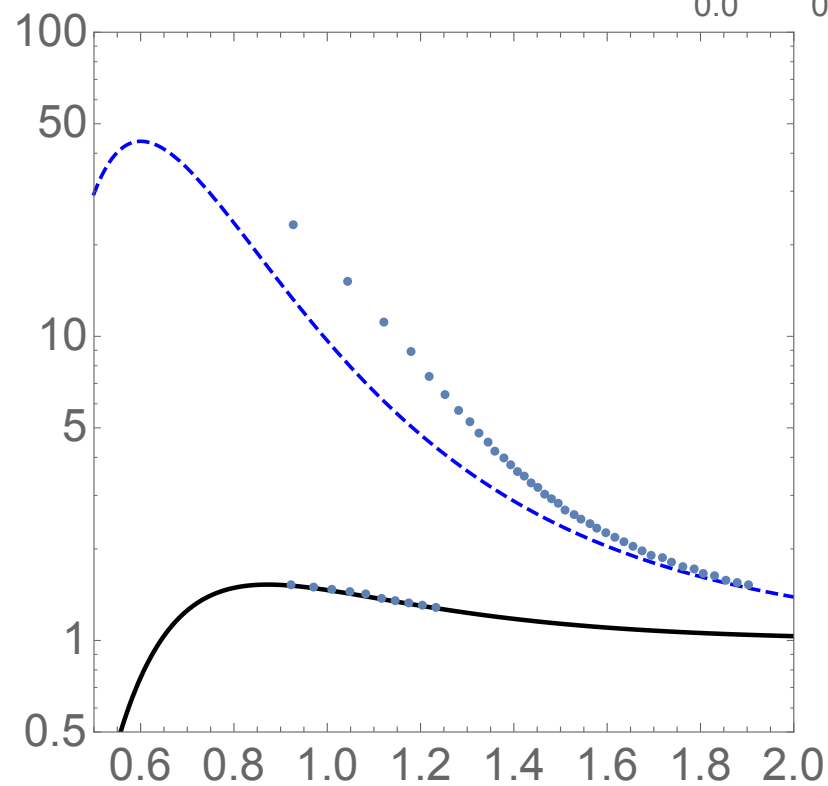


$$\ddot{R} = \left(\frac{2}{m}\right) \frac{\partial V(R)}{\partial R}$$



$A=4$

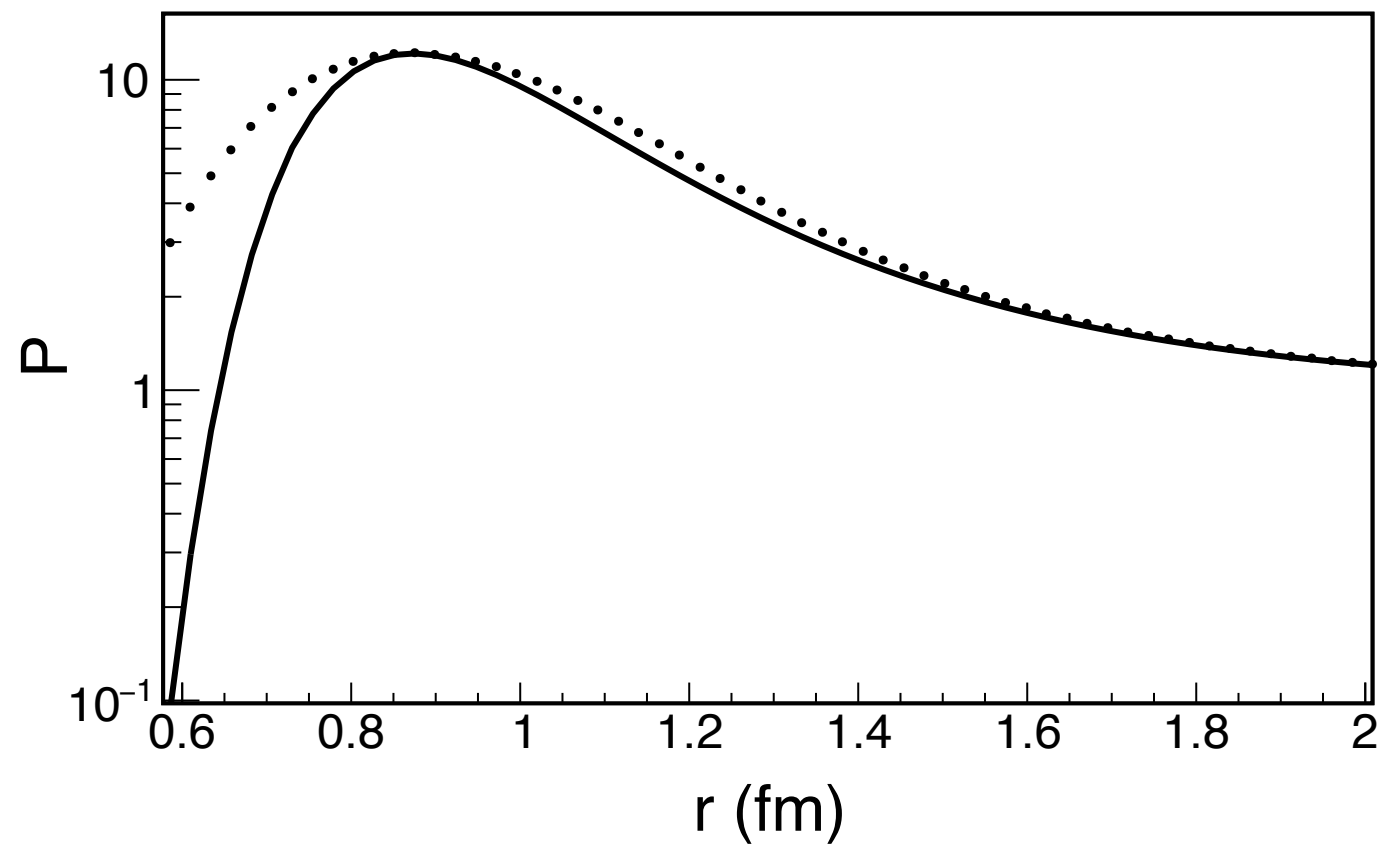
$$S = \oint d\tau \left[\frac{3m}{4} \dot{r}^2 + 6V(r) \right]$$



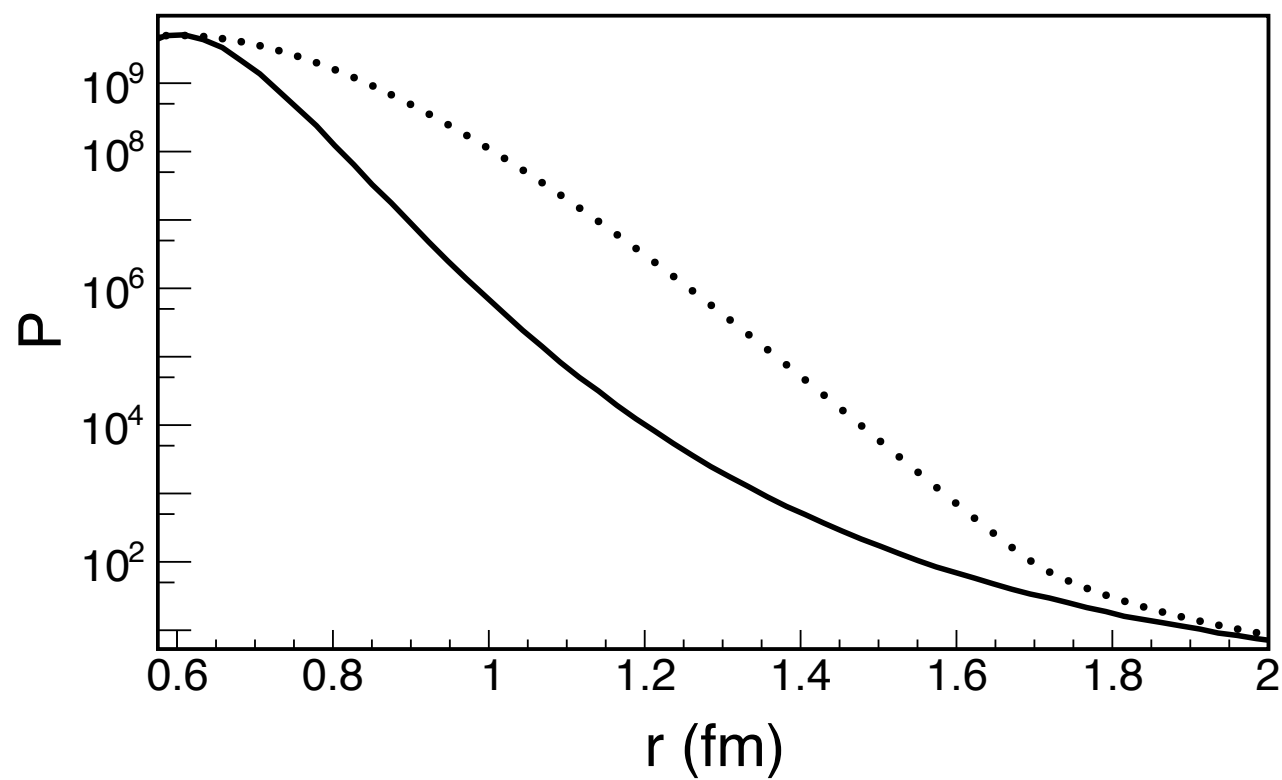
**Two potentials:
Unmodified and
strongly modified
(from Wambach et al spectral
density of sigma)**

$$\ddot{R} = \left(\frac{4}{m}\right) \frac{\partial V(R)}{\partial R}$$

FIG. 4: The probability for two nucleons being at distance r (fm) from each other at the temperature $T = 100 \text{ MeV}$. The lower and upper curves are Boltzmann factors for Walecka potential, with sigma masses $m_\sigma = 500$ and 285 MeV , respectively. The dots indicate the semiclassical probability distribution calculated via flucton method.



A=4 nucleons
line= $\exp(-V/T)$
dots=flucton
Upper plot -ordinary V
Lower is Modified V



He4 and K-harmonics

Jacobi coordinates

$$\vec{\xi}[1] = \frac{\vec{x}[1] - \vec{x}[2]}{\sqrt{2}}, \quad \vec{\xi}[2] = \frac{\vec{x}[1] + \vec{x}[2] - 2\vec{x}[3]}{\sqrt{6}},$$

$$\vec{\xi}[3] = \frac{\vec{x}[1] + \vec{x}[2] + \vec{x}[3] - 3\vec{x}[4]}{2\sqrt{3}}$$

The radial coordinate, or *hyperdistance*, is defined as

$$\rho^2 = \sum_{m=1}^3 \vec{\xi}[m]^2 = \frac{1}{4} \left(\sum_{i \neq j} (\vec{x}[i] - \vec{x}[j])^2 \right) \quad (\text{A1})$$

The radial part of the Laplacian in these Jacobi coordinates is $\psi''(\rho) + 8\psi'(\rho)/\rho$, and using substitution $\psi(\rho) = \chi(\rho)/\rho^4$ one arrives to conventional-looking Schrodinger eqn for $K = 0$ harmonics

$$\frac{d^2\chi}{d\rho^2} - \frac{12}{\rho^2}\chi - \frac{2M}{\hbar^2}(W(\rho) - E)\chi = 0 \quad (\text{A2})$$

**The main bound state at -28 MeV
is reproduced in literature**
Unexpectedly, we found another bound state
At -8 MeV
It corresponds to known resonance!

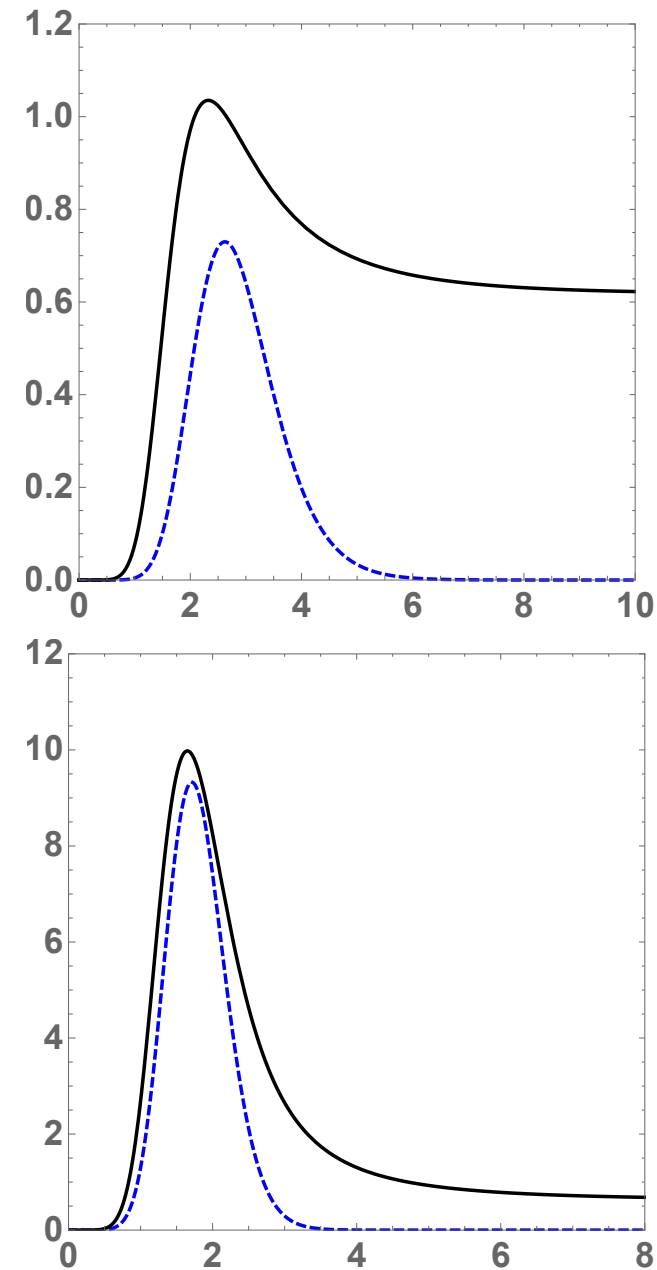


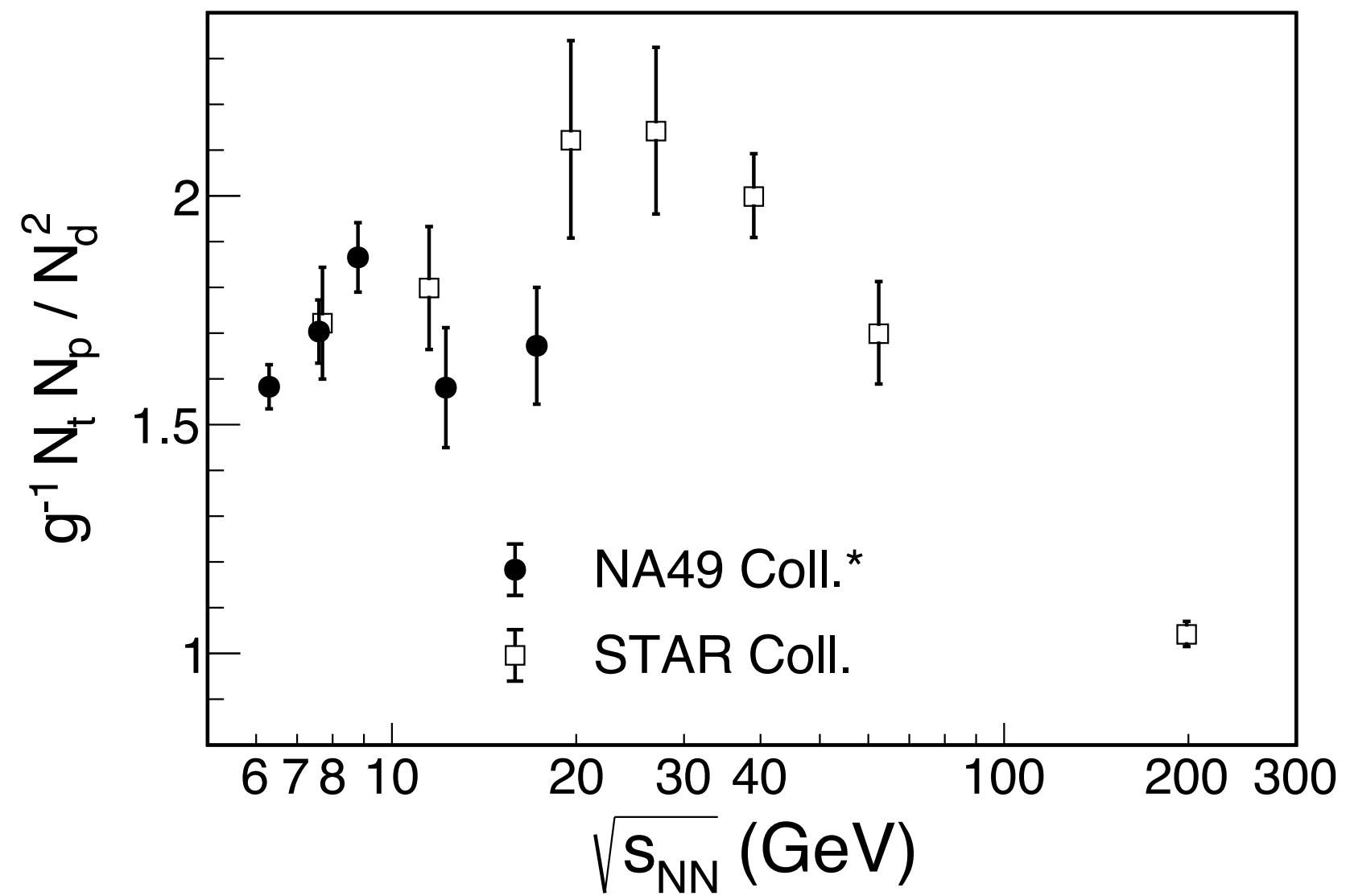
FIG. 8: The solid lines are Boltzmann-weighted density matrix, at $T = 100 \text{ MeV}$, using 40 lowest states of the K-harmonics radial equation, for the usual nuclear potential (upper plot) and the modified one (lower plot). In both case the blue dashed line show the contribution of the deepest bound state.

TABLE I: Low-lying resonances of He^4 system, from BNL properties of nuclides listed in *nndc.bnl.gov* web page. JP is total angular momentum and parity, Γ is the width. The last column is the decay channel branching ratios, in percents. p, n, d correspond to emission of proton, neutron or deuterons.

$E (MeV)$	JP	$\Gamma (MeV)$	decay modes, in %
20.21	0 +	0.50	p =100
21.01	0 -	0.84	n =24, p =76
21.84	2-	2.01	n = 37, p = 63
23.33	2-	5.01	n = 47, p = 53
23.64	1-	6.20	n = 45, p = 55
24.25	1-	6.10	n = 47, p = 50 , d=3
25.28	0-	7.97	n = 48 , p = 52
25.95	1-	12.66	n = 48 ,p = 52
27.42	2+	8.69	n = 3 , p = 3 ,d = 94
28.31	1+	9.89	n = 47 , p = 48 , d = 5
28.37	1-	3.92	n = 2, p = 2, d = 96
28.39	2-	8.75	n = 0.2, p = 0.2 , d = 99.6
28.64	0-	4.89	d=100
28.67	2+	3.78	d=100
29.89	2+	9.72	n = 0.4 , p = 0.4, d = 99.2

← Newly found radial excitation

**Statistical model implies that all thee resonances
Are also produced and decay, some
In observable channels like d+d,p+t**



Extra source for t: from pre-cluster decays?

Summary

the semiclassical theory based on instanton-dyons reproduces
(i) the deconfinement; (ii) chiral symmetry transitions;
(iii) not just in QCD (where quasicritical T_{dec} and T_{chir} are about the same) but with
arbitrary quark periodicity phases, where there are more phase transitions
=>

Path integral semiclassics at nonzero T
Applied to few-nucleon
clusters at freezeout
=> very sensitive to inter-N potential

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Instanton-dyon theory is Poisson-dual to
monopole theory
pro: simpler to use
con: restricted to Euclidean time
and cannot be used for kinetics

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There are **two** theories of non-perturbative phenomena,
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How are they related?

Should we sum up their effects?

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instanton-dyons and monopoles correspond
to two **different approaches to dynamics**,
but they result in **the same partition function**