



Probing the Phase Structure of Strongly Interacting Matter: Theory and Experiment

25-29 March 2019
GSI

Study QCD Phase Diagram by energy scan experiment at RHIC



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Outline

1. Introduction

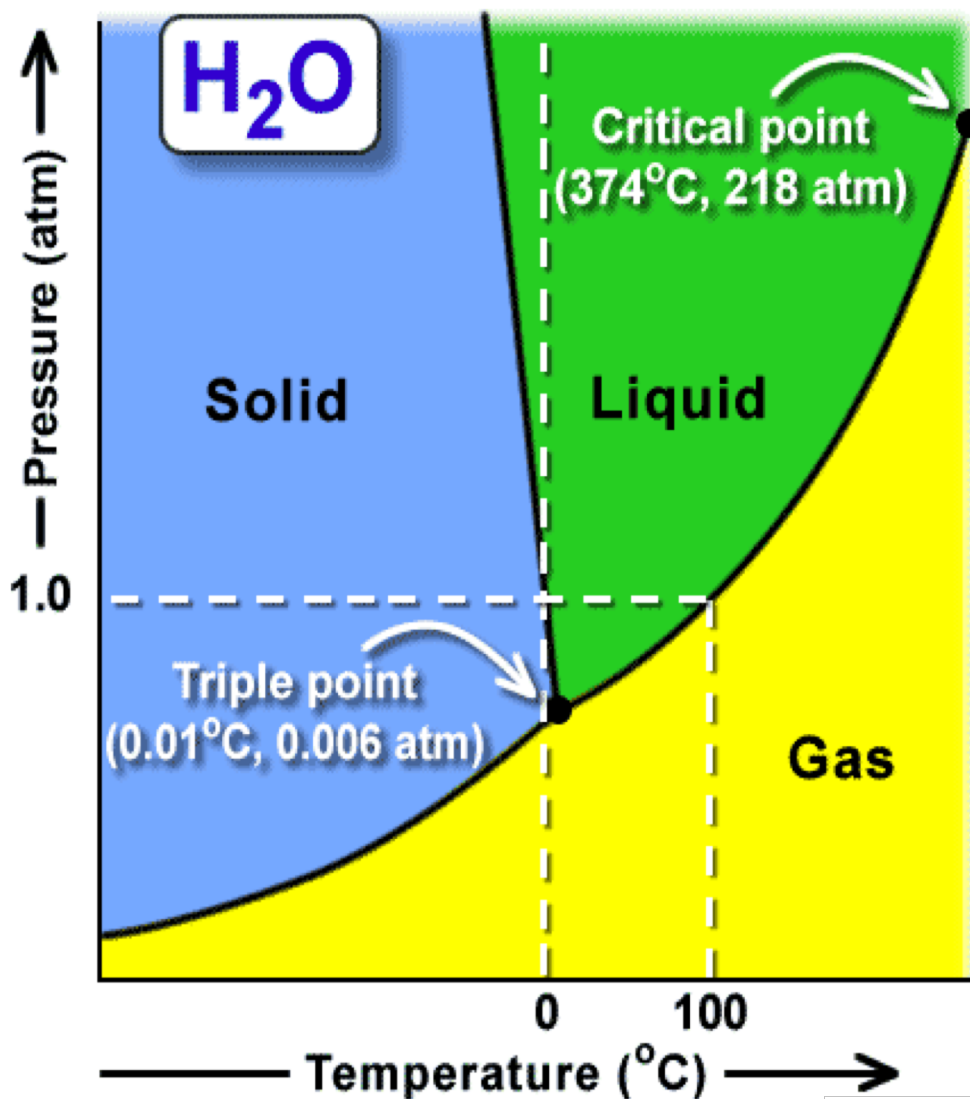
2. Results from energy scan at RHIC

- Freeze-out dynamics
- Collectivity : directed flow
- Criticality : higher moments and light nuclei

3. Summary and Future Plan



Phase Diagram of Water



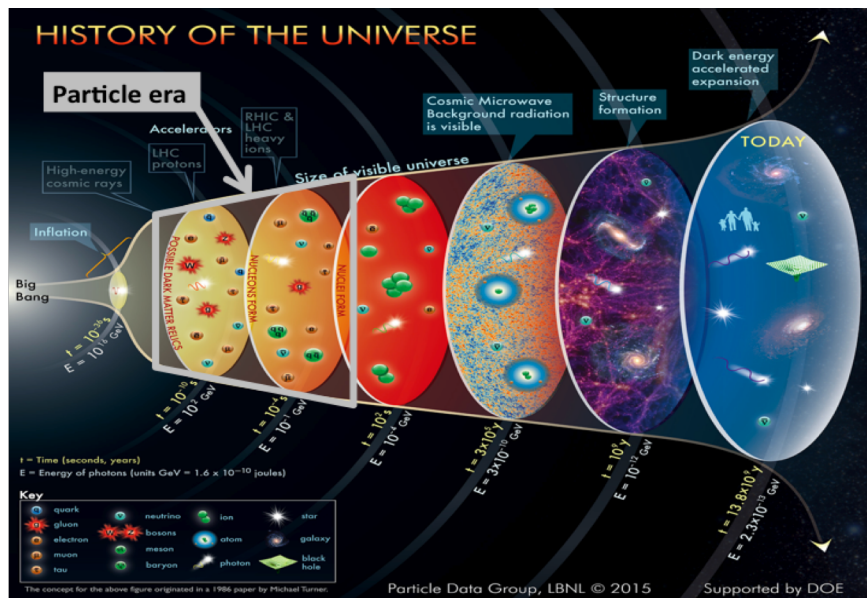
H_2O

EM Interaction



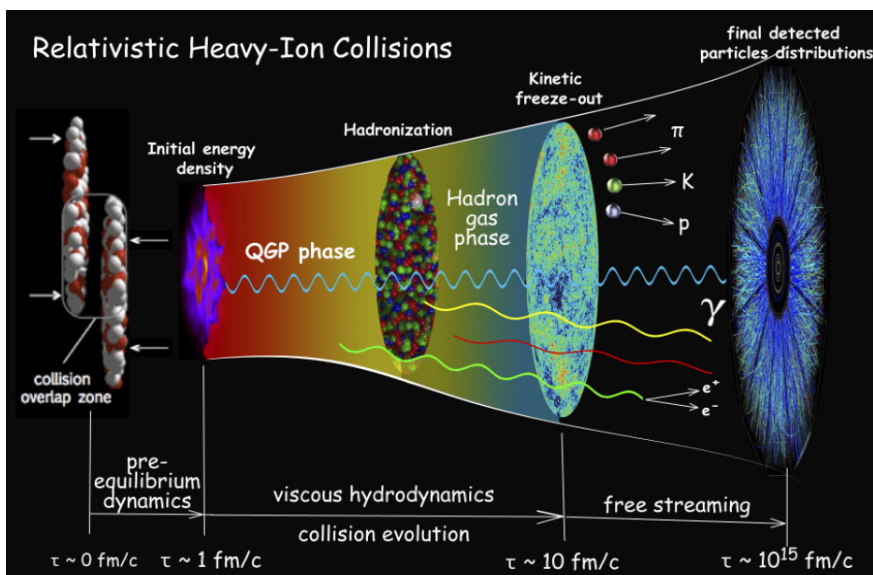
Heat matter to trillion (10^{12}) degree

Big Bang



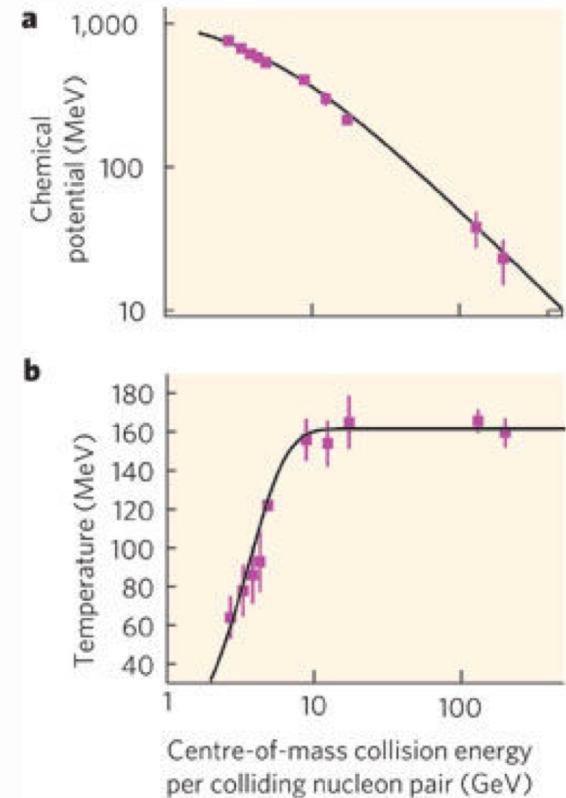
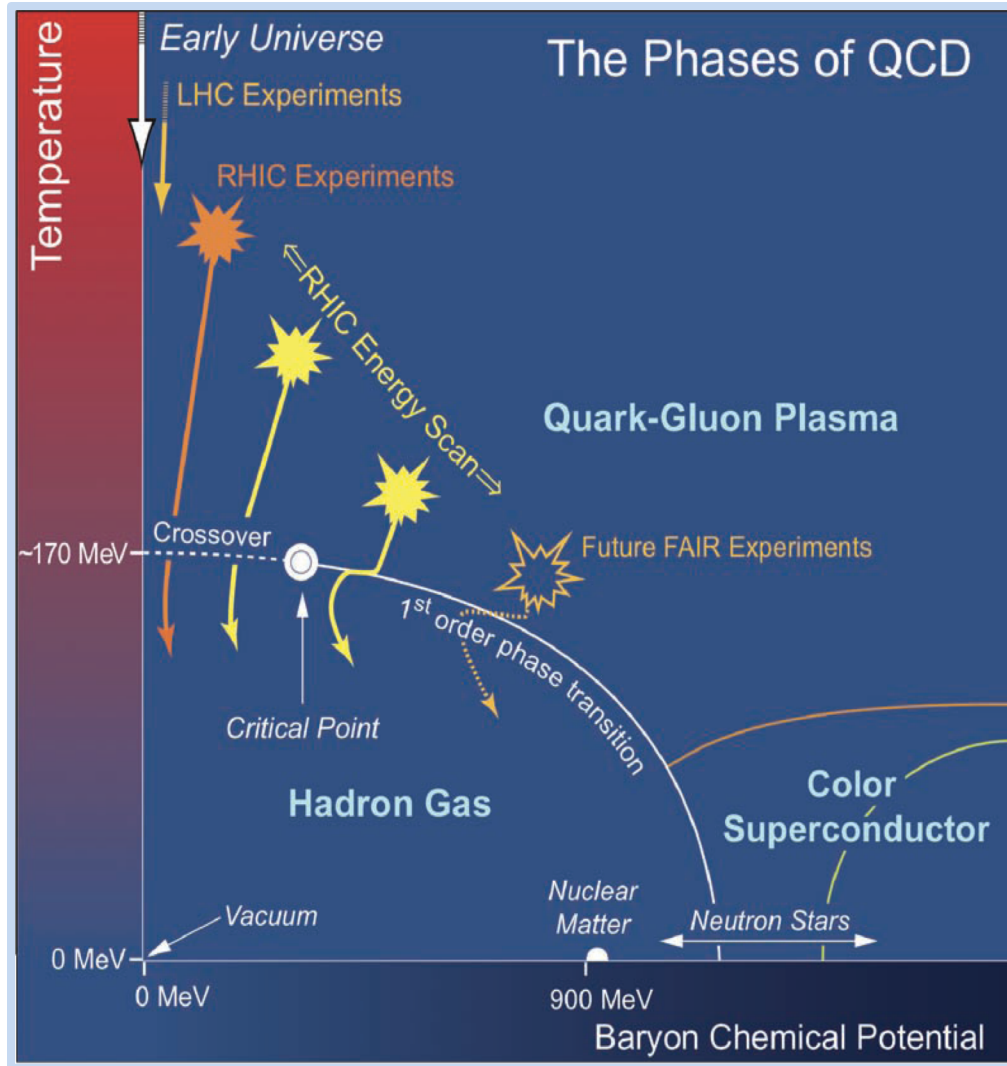
Quark-Gluon Plasma (**QGP**): a state of matter where the quarks and gluons are the relevant degrees of freedom, exist at few μ s after the Big-Bang

Little Bang



Relativistic heavy-ion collisions are a unique tool to create and study hot QCD matter and its phase transition under controlled conditions.

QCD Phase Diagram and Beam Energy Scan



PBM&Johanna, *Nature* **448**, 302-309 (2007)

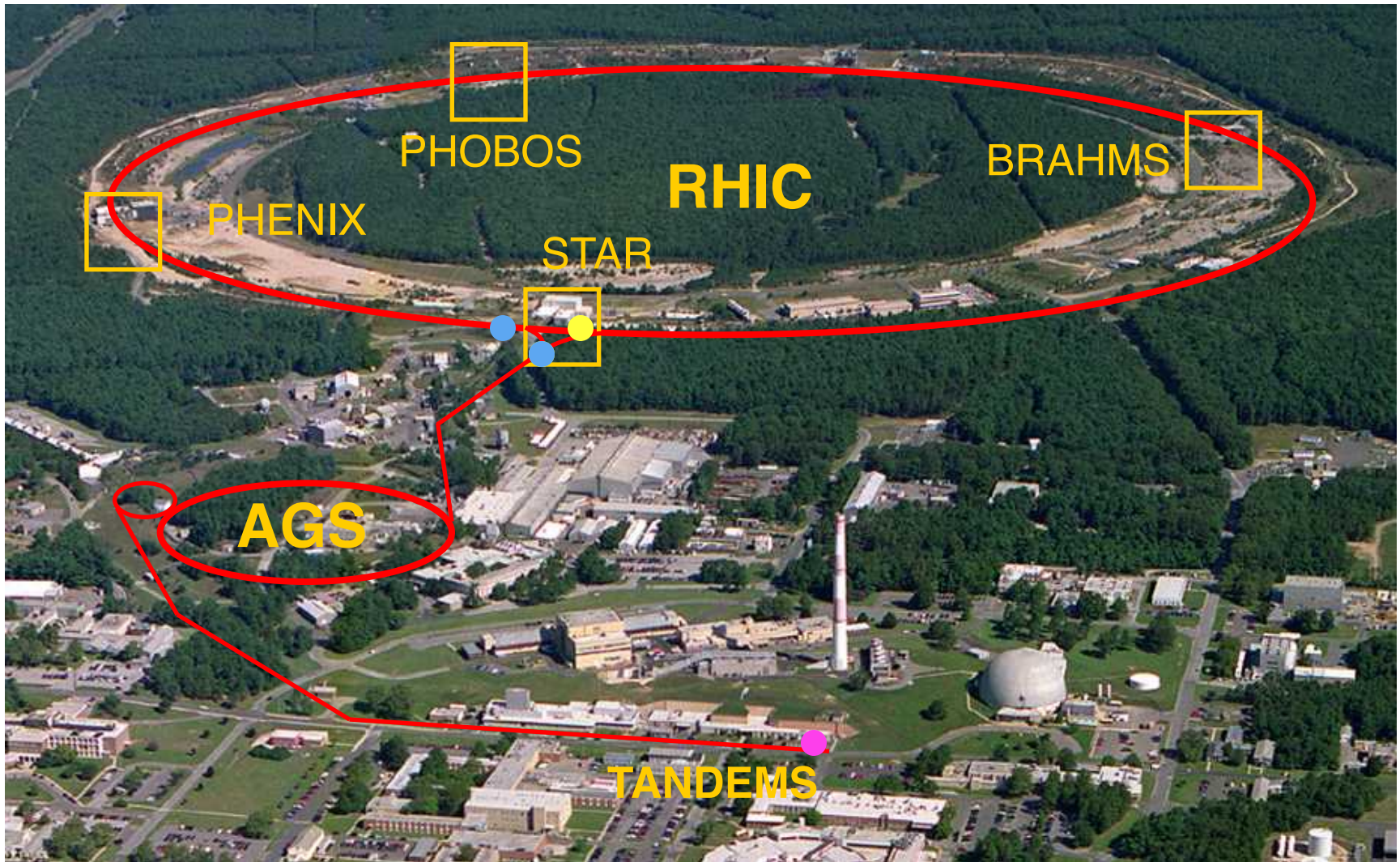
Search for signatures of QCD phase transition in heavy-ion collisions.

1. **Crossover**
2. **Critical point**
3. **1st order phase transition**



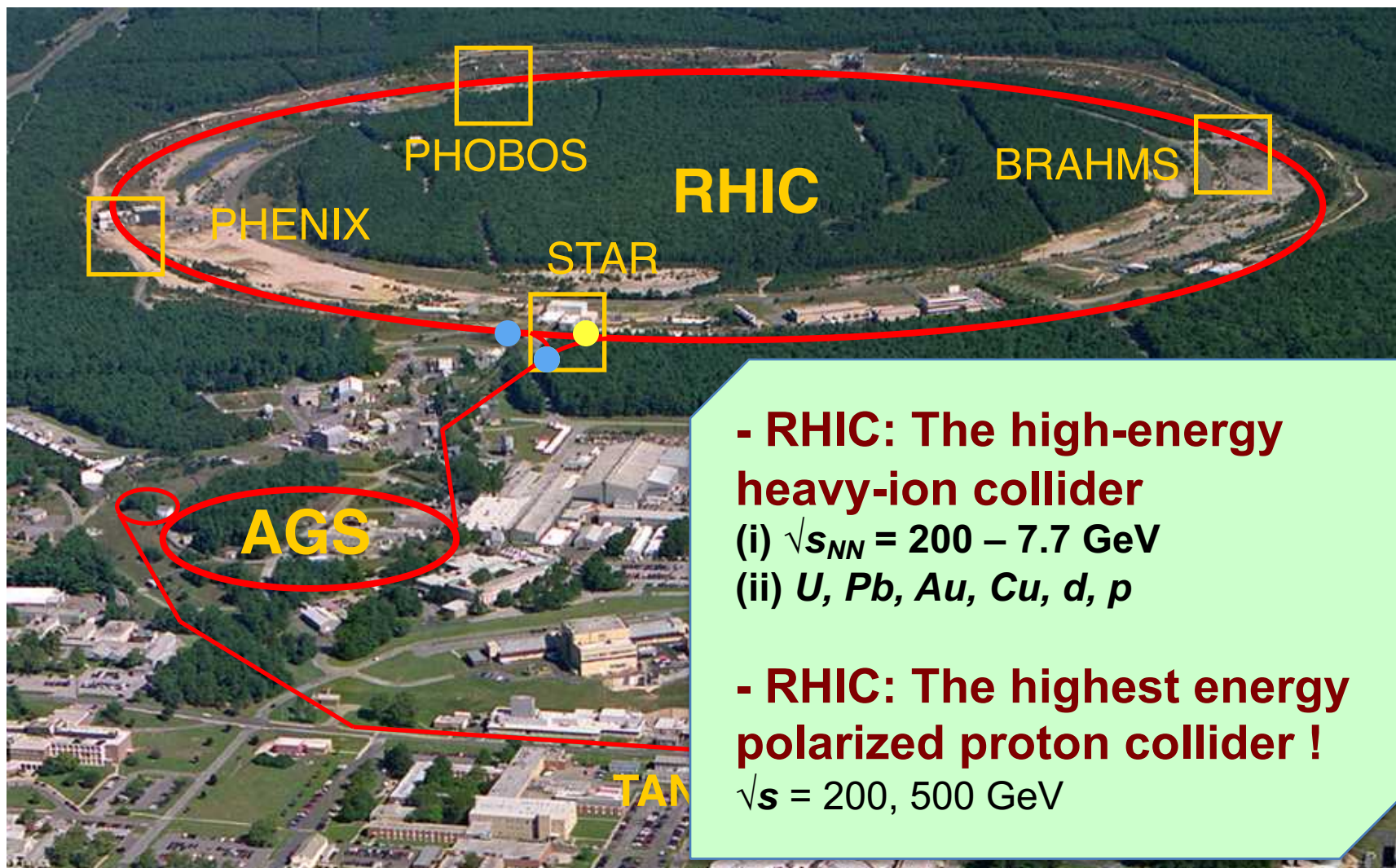
Relativistic Heavy Ion Collider (RHIC)

Brookhaven National Laboratory (BNL), Upton, NY

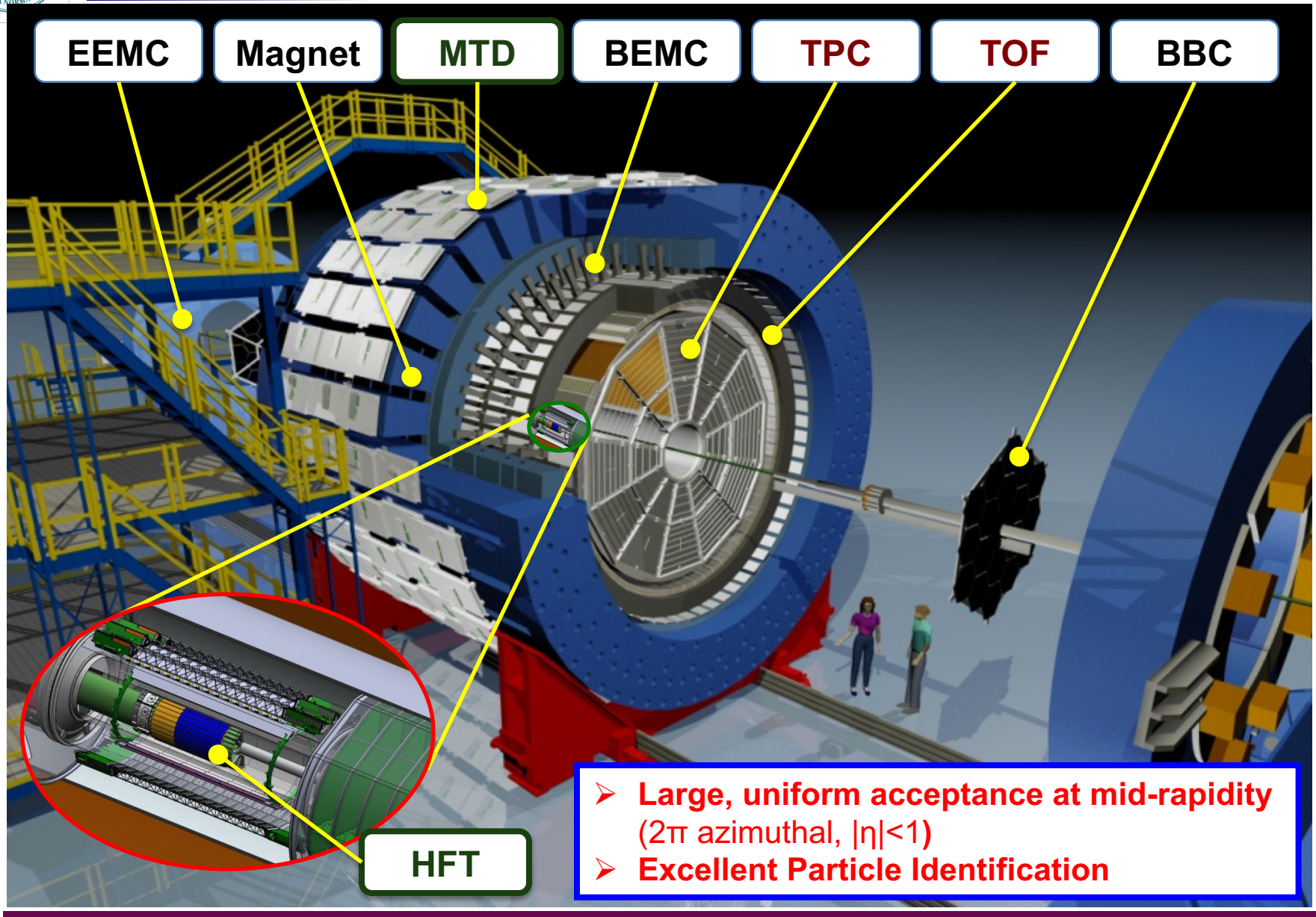


Relativistic Heavy Ion Collider (RHIC)

Brookhaven National Laboratory (BNL), Upton, NY



STAR Detector System



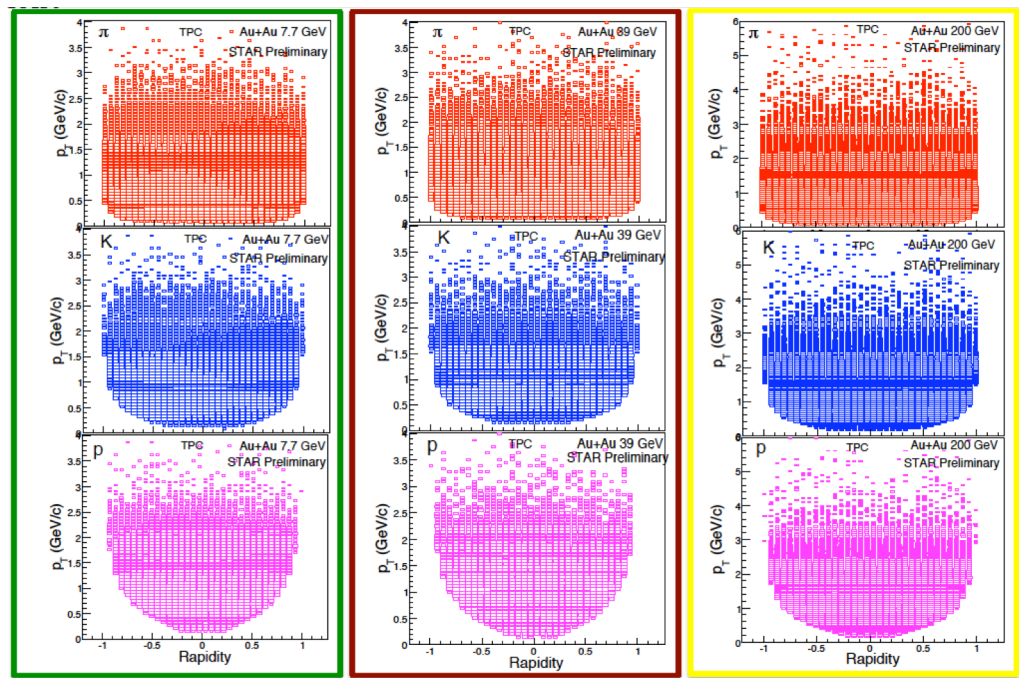


RHIC Beam Energy Scan-I (2010-2014)

Au+Au Collisions

$\sqrt{s_{NN}}$ (GeV)	Events (10^6)	$^*\mu_B$ (MeV)	$^*T_{CH}$ (MeV)
200	238	25	166
62.4	45	73	165
54.4	1200	83	165
39	86	112	164
27	32	156	162
19.6	15	206	160
14.5	13	264	156
11.5	7	316	152
7.7	3	422	140

Uniform acceptance at Mid-rapidity



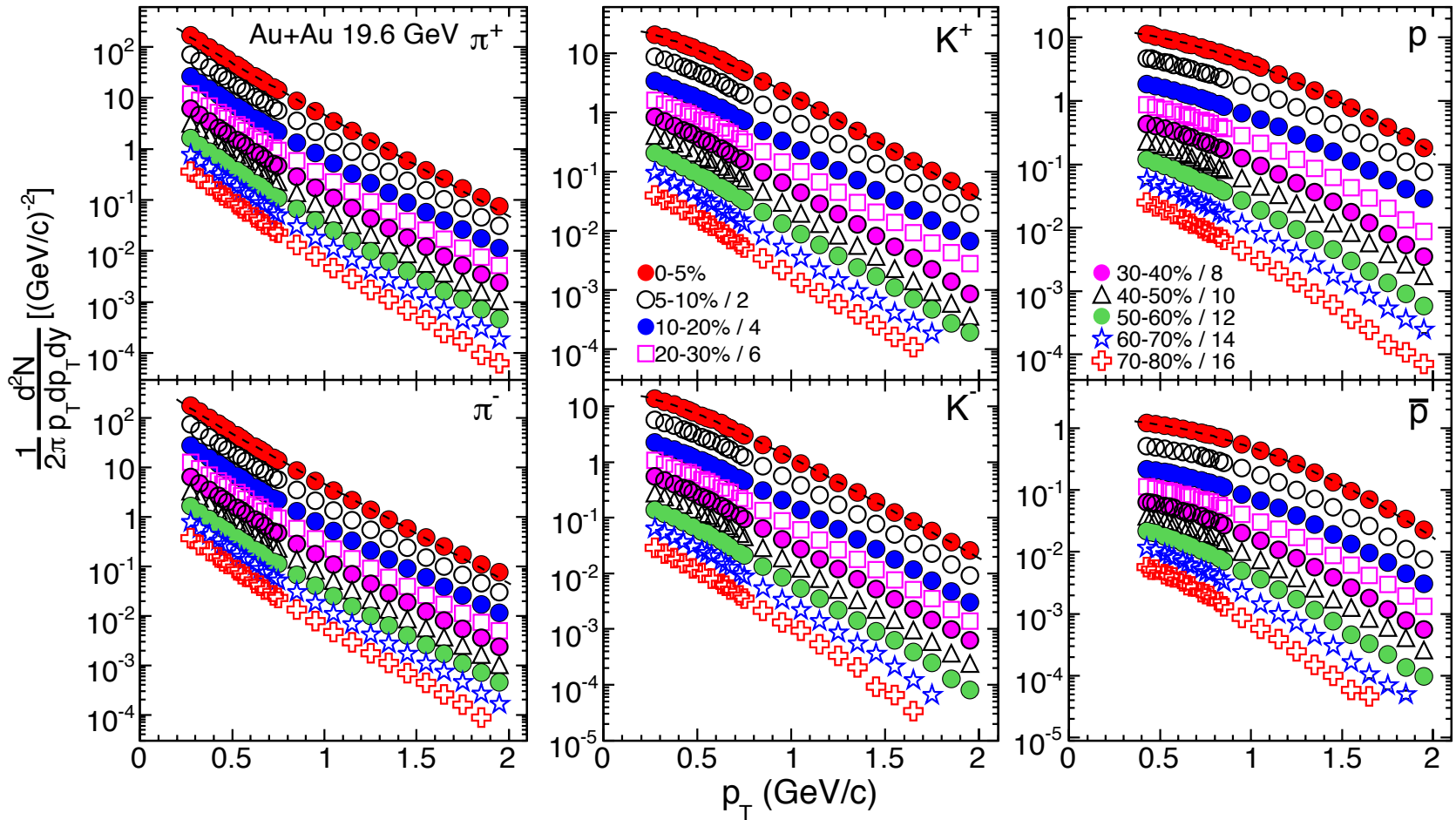
$^*(\mu_B, T_{CH})$: J. Cleymans et al., PRC 73, 034905 (2006)

➤ Access the QCD phase diagram: vary collision energies/centralities.

RHIC BES-I : $20 < \mu_B < 420$ MeV

Transverse Momentum Spectra

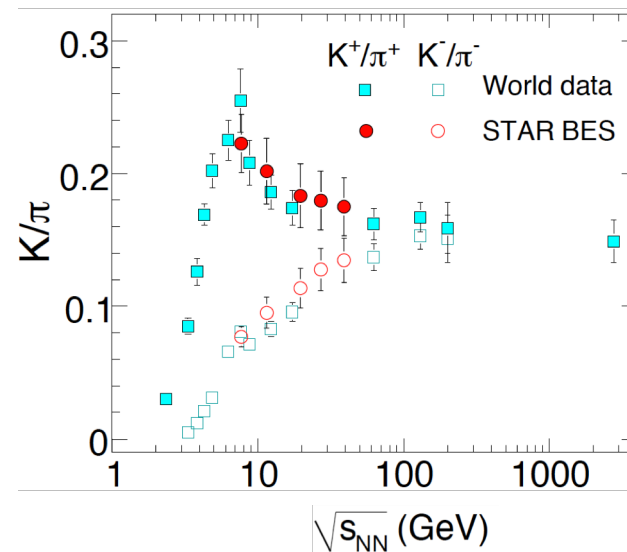
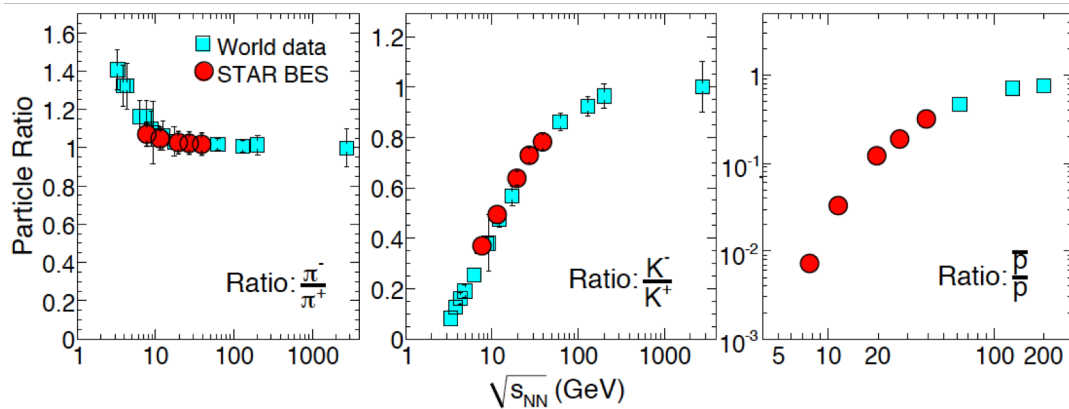
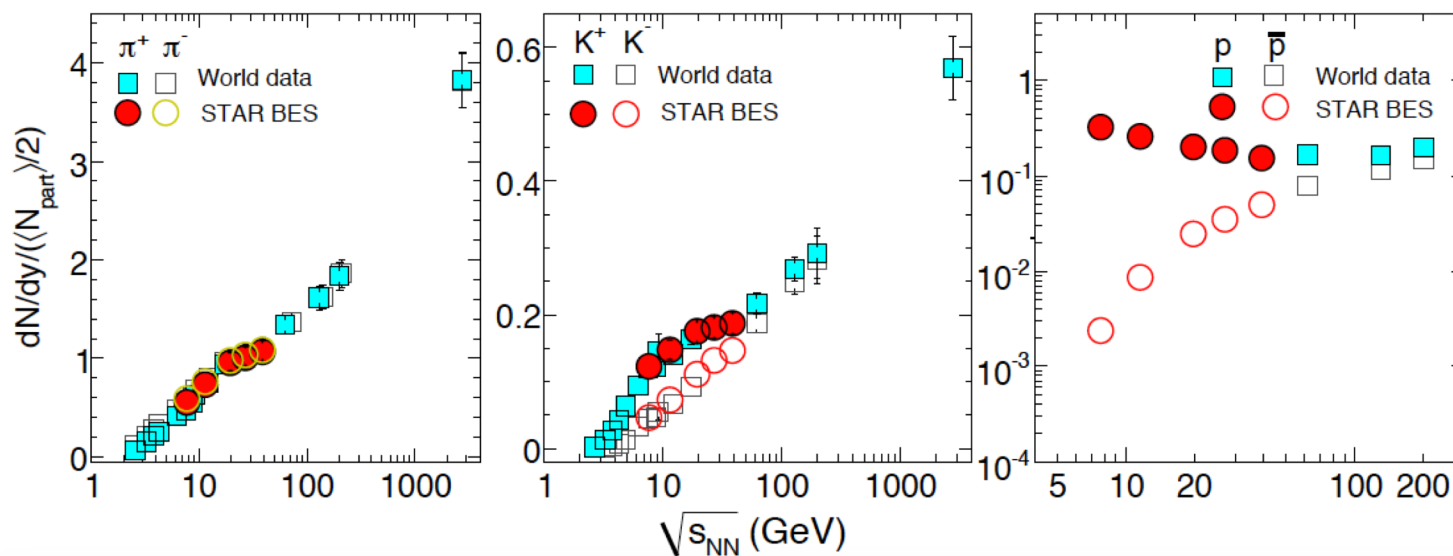
$\sqrt{s_{NN}} = 19.6$ GeV Au+Au Collisions



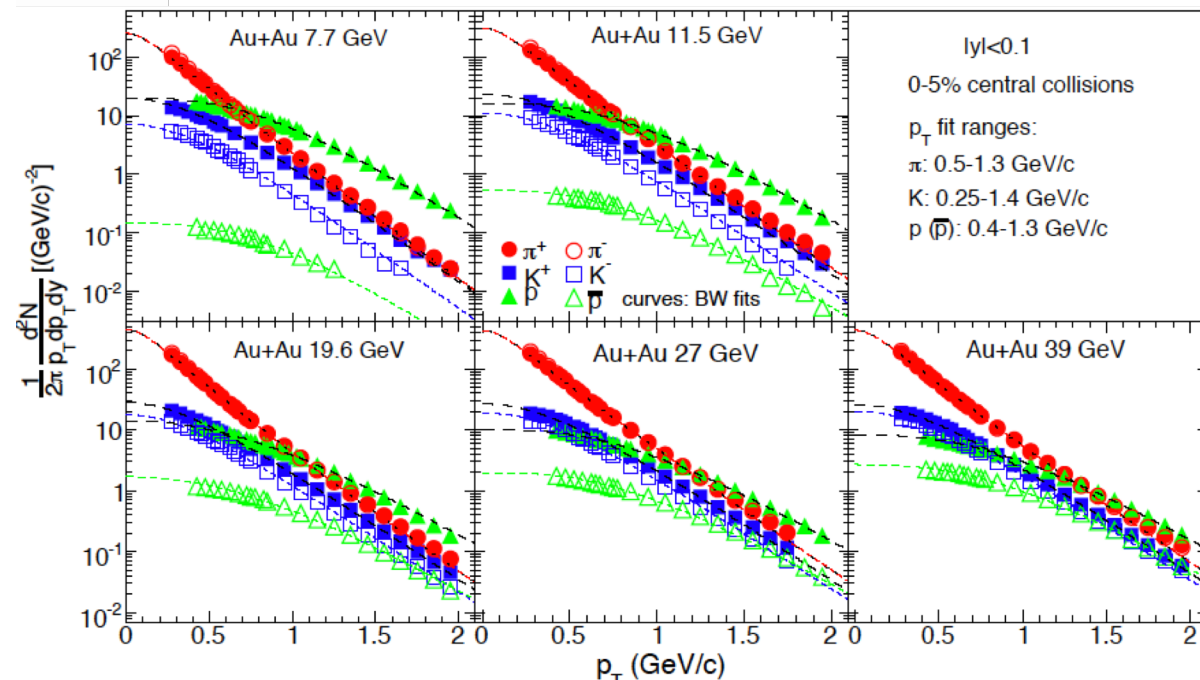
STAR: Phys. Rev. C 96, 044904 (2017).



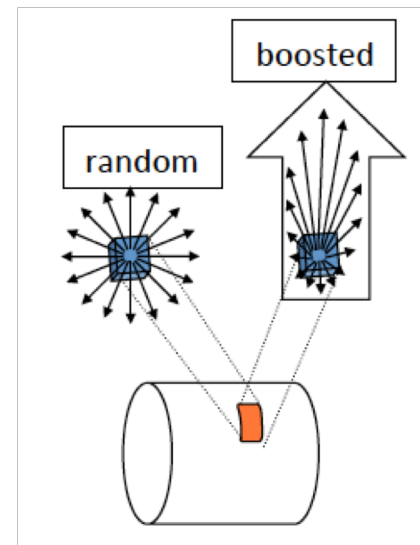
Particle Yields and Ratios



Kinetic Freeze-out Dynamics



Blast Wave Model



$$E \frac{d^3 N}{dp^3} \propto \int_{\sigma} e^{-(u^\mu p_\mu)/T_{fo}} p d\sigma_\mu \Rightarrow$$

$$\frac{dN}{m_T dm_T} \propto \int_0^R r dr m_T K_1 \left(\frac{m_T \cosh \rho}{T_{fo}} \right) I_0 \left(\frac{p_T \sinh \rho}{T_{fo}} \right)$$

$$\rho = \tanh^{-1} \beta_T \quad \beta_T = \beta_s \left(\frac{r}{R} \right)^\alpha \quad \alpha = 0.5, 1, 2$$

Expanding source is **assumed** to be:

1. Locally thermal equilibrated.
2. Boosted in radial direction.

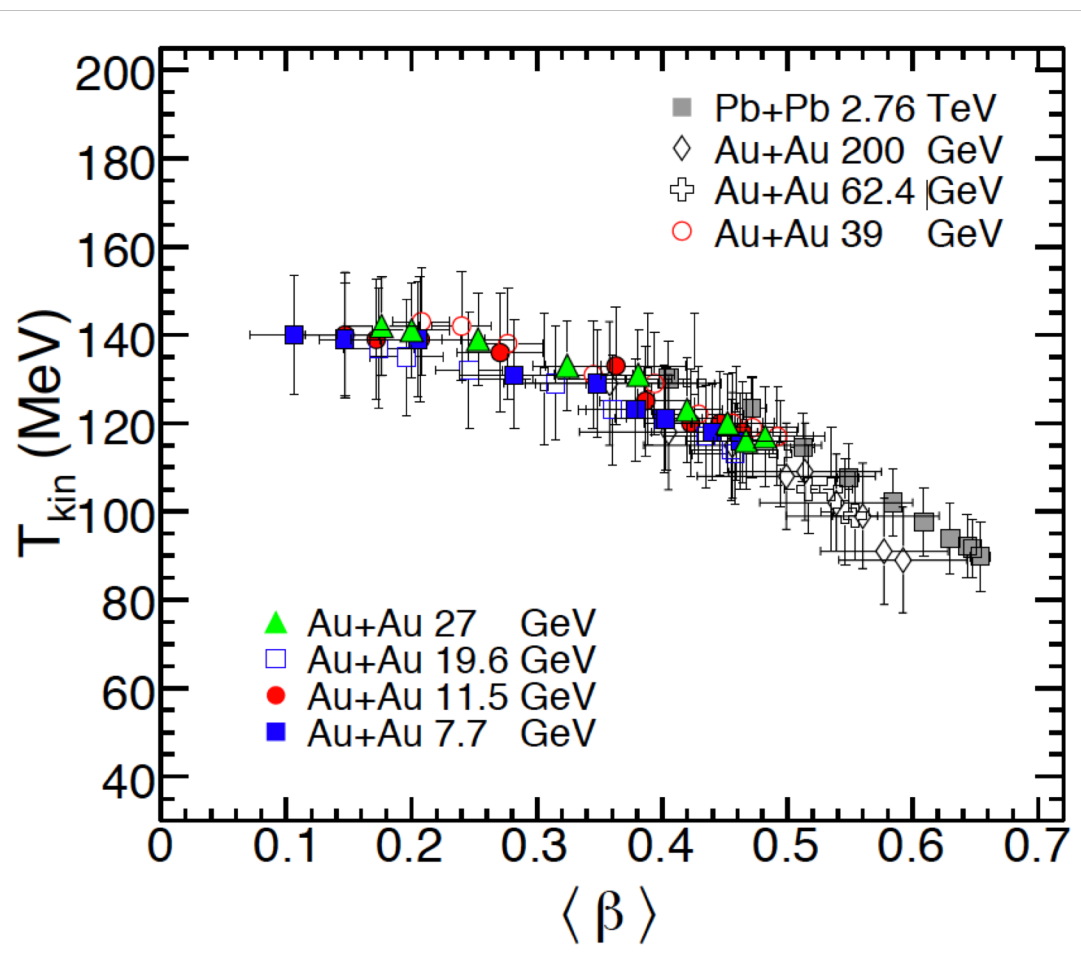
Matters flow with the same velocity.

Fit p_T spectra to obtain parameters:

- Thermal temperature : T_{fo} ,
- Transverse velocity parameter: $\langle \beta_T \rangle$



Kinetic Freeze-out Parameters : T_{f0} , $\langle\beta_T\rangle$



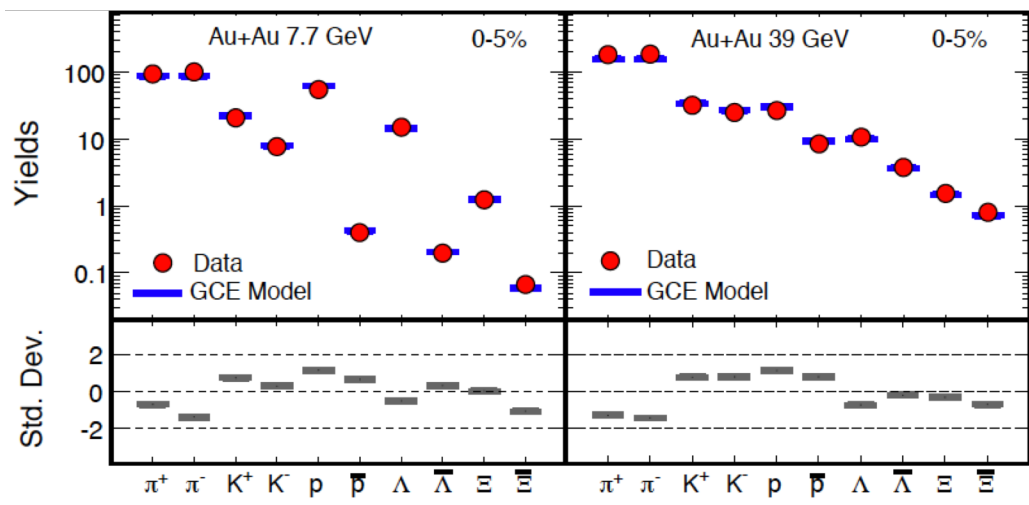
- Anti-correlation : T_{f0} Vs. $\langle\beta_T\rangle$
- Higher energies/central collisions: stronger collectivity and lower temperature.

ALICE: Phys. Rev. C 91, 024609 (2015).

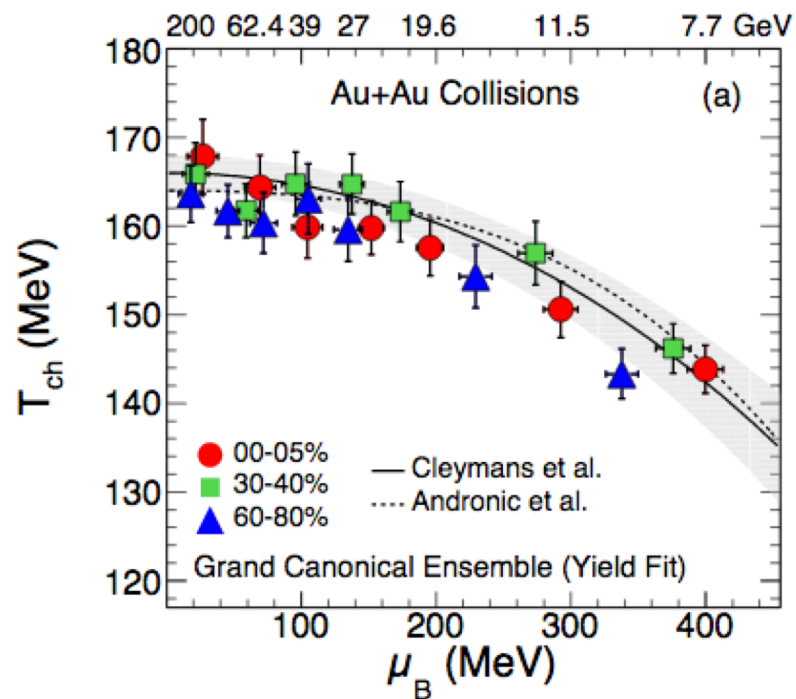
STAR: Phys. Rev. C 96, 044904 (2017).



Chemical Freeze-out : Data Vs. Thermal Model



Good agreement between data and thermal model



THERMUS: S. Wheaton, J. Cleymans Comp.
Phys. Comm. 180, 84 (2009)

$$\ln Z^{GC}(T, V, \{\mu_i\}) = \sum_{\text{species } i} \frac{g_i V}{(2\pi)^3} \int d^3p \ln(1 \pm e^{-\beta(E_i - \mu_i)}) \pm 1$$

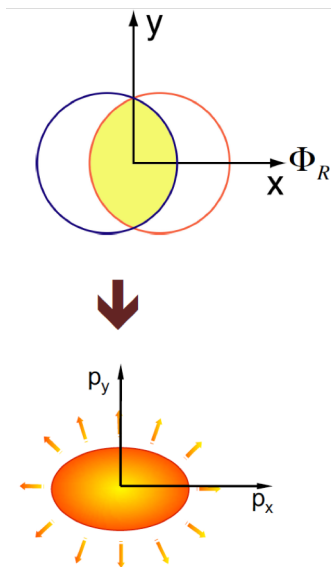
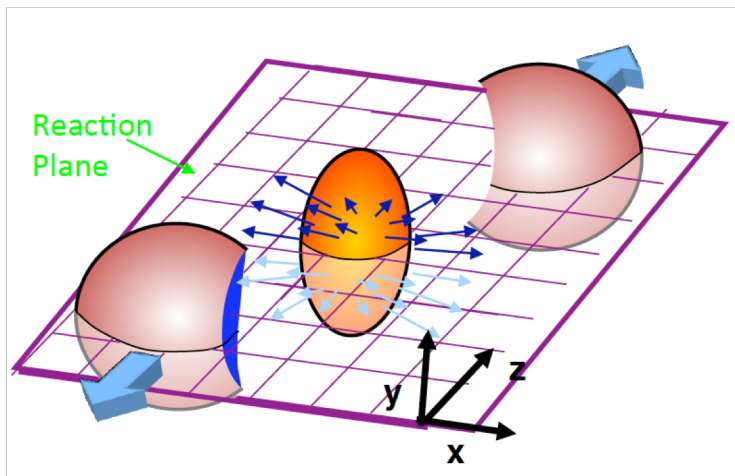
$$N_i^{GC} = T \frac{\partial \ln Z^{GC}}{\partial \mu_i} = \frac{g_i V}{2\pi^2} \sum_{k=1}^{\infty} (\mp 1)^{k+1} \frac{m_i^2 T}{k} K_2 \left(\frac{km_i}{T} \right) \times e^{\beta k \mu_i}$$

Model Features:

- Non-interacting hadrons and resonances
- Thermodynamically equilibrium system

Dynamics characterized by:
Temperature T_{ch} and
baryon chemical potential μ_B

Search for 1st order phase transition : Directed Flow



➤ Initial spatial anisotropy transfer to final momentum anisotropy via interactions

➤ Sensitive to EoS and the early stages of the system evolution.

Directed flow(v_1) :

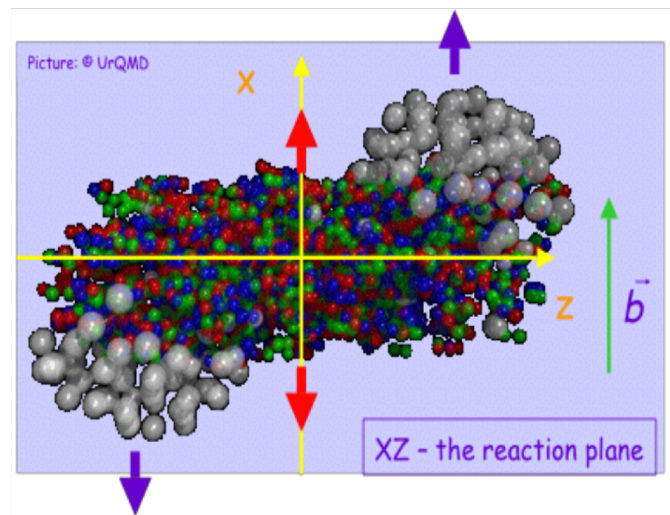
Spectators deflected from dense reaction zone
Very sensitive to EoS.

$$E \frac{dN^3}{d^3 p} = \frac{1}{2\pi} \frac{d^2 N}{p_t dp_t dy} \left(1 + \underset{\substack{\uparrow \\ \text{isotropic}}}{2v_1} \cos(\phi - \psi_R) + \underset{\substack{\uparrow \\ \text{directed}}}{2v_2} \cos 2(\phi - \psi_R) + \dots \right)$$

isotropic directed elliptic

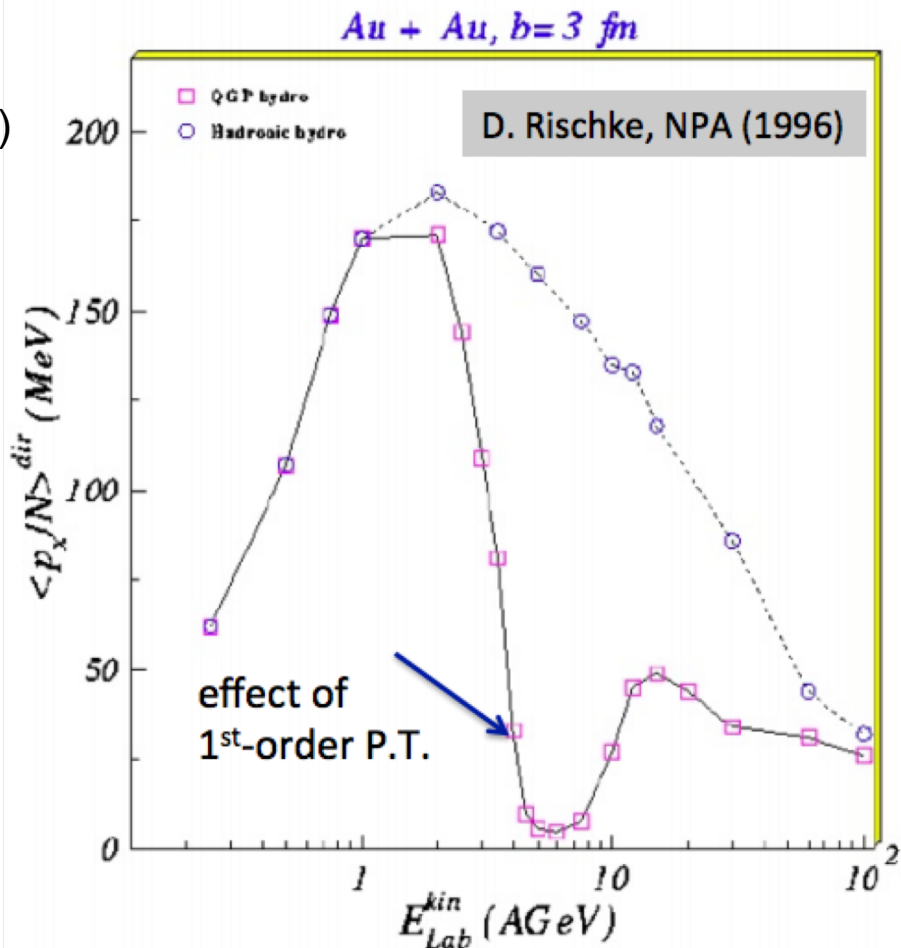
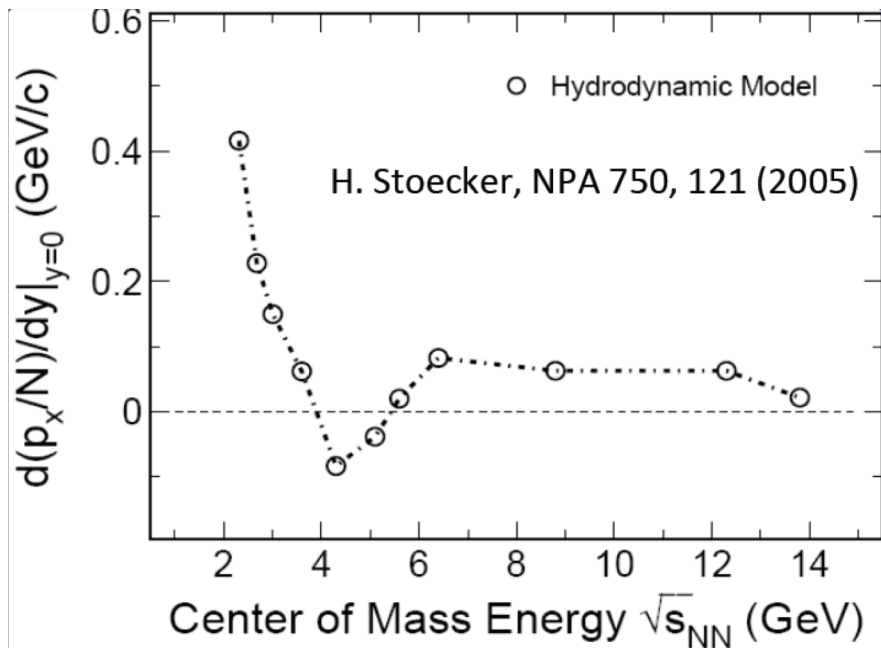
$$v_n = \left\langle \cos \left[n(\phi - \psi) \right] \right\rangle$$

Directed flow: predicted and observed (Plastic Ball) in 1980's



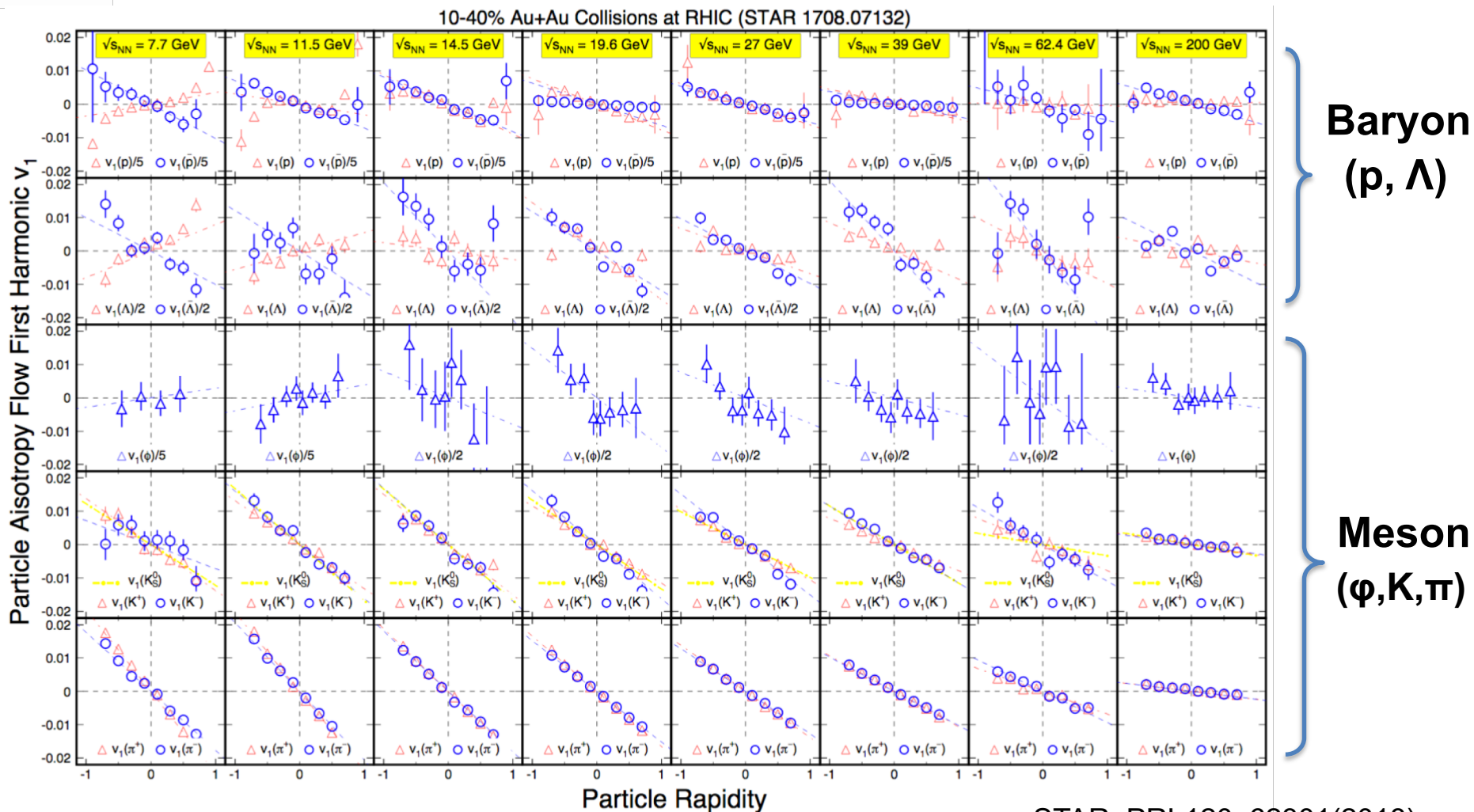
Directed flow- early pressure probe

Directed flow of baryon currents (“net-baryon”)



Long-standing probe for 1st order phase transition: reduction of system pressure (softening of EoS).

v_1 versus rapidity at RHIC-BES

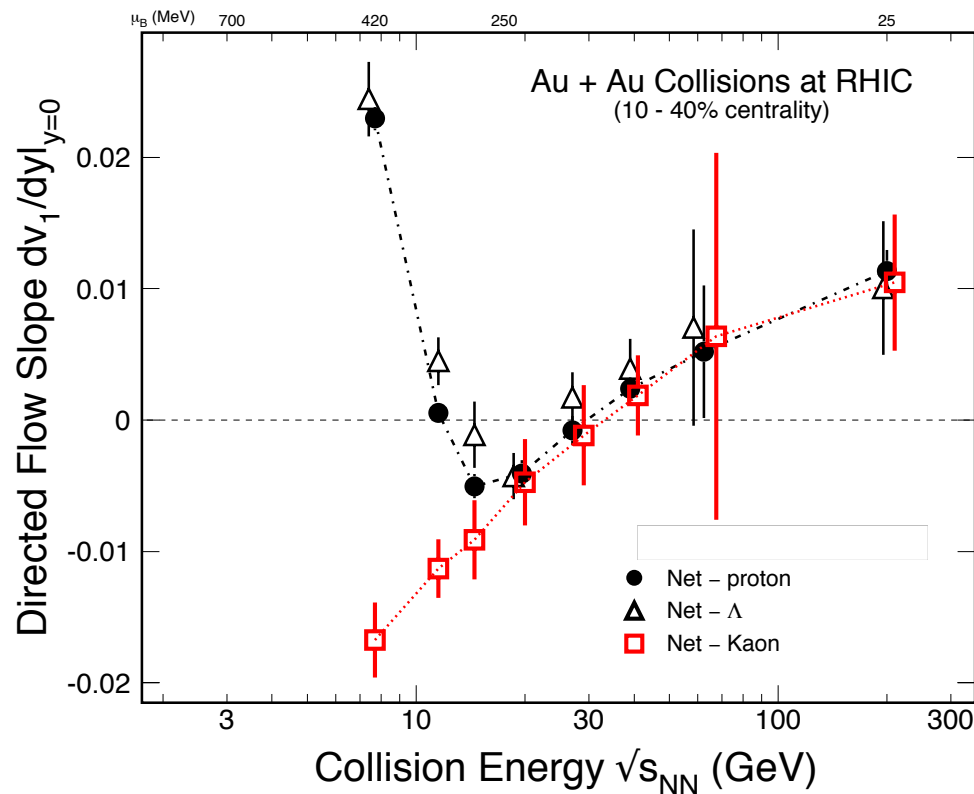


STAR: PRL120, 62301(2018)

- Meson and anti-baryons show negative slope except ϕ meson below 14.5 GeV.
- Directed flow slope of proton and Lambda change sign around 11.5 GeV.



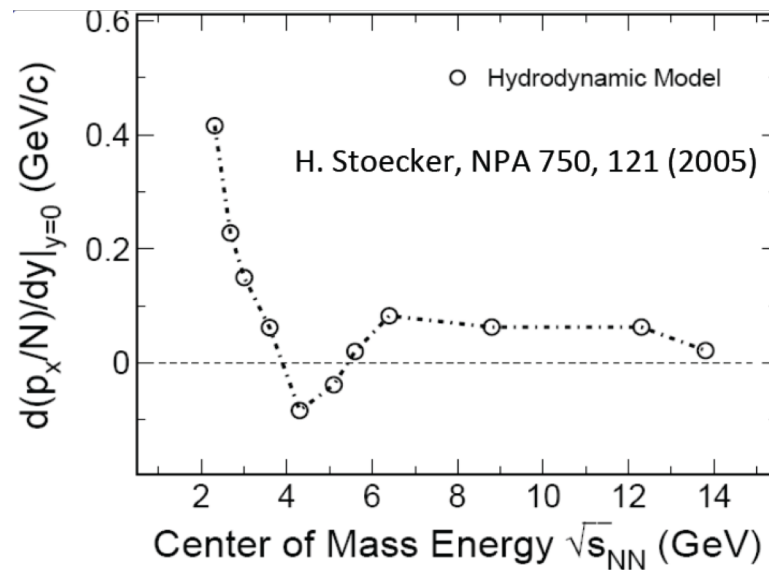
v_1 slope vs. Energy: Softest Point ?



$$v_{1p} = r(y) v_{1\bar{p}} + [1 - r(y)] v_{1\text{net } p},$$

$r(y)$ is the ratio of observed pbar to p yield.

STAR: Phys. Rev. Lett. 112, 162301(2014)
Phys. Rev. Lett. 120, 062301(2018)



- 1) Double sign changes and minimum around $\sqrt{s_{NN}} = 15\text{-}20$ GeV for net-p and net- Λ slope, but net-Kaon data are decreasing as energy decreases

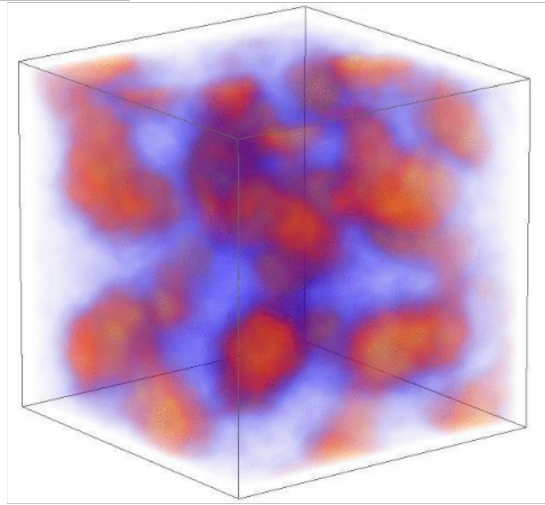
Softening of EoS ?

- 2) Need model calculations to explain.

- M. Isse, A. Ohnishi et al, PRC72, 064908(2005)
- Y. Nara, A. Ohnishi, H. Stoecker, PRC94, 034906(2016).
- C. Zhang, J. Chen, X. Luo et al. PRC97, 064913 (2018).



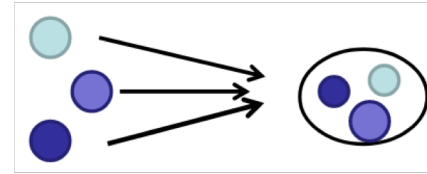
Search for the QCD critical point



In the vicinity of critical point



Large density fluctuations and long range corr.

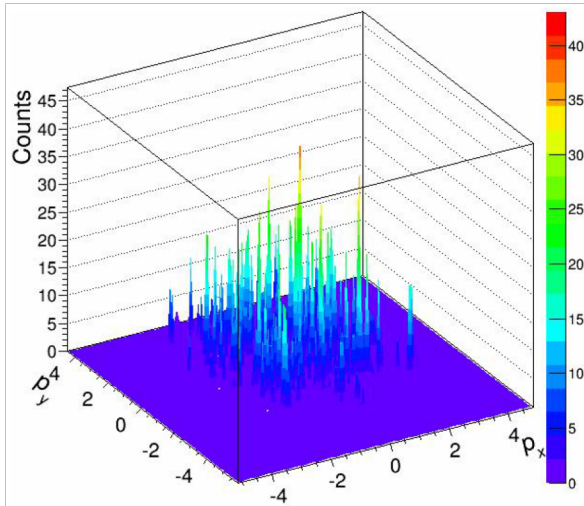


**E-by-E conserved charge
(B, Q, S) fluctuations**

**Baryon clustering:
light nuclei production**

Experimental Signatures:

**Non-monotonic variation as a function of
collision energy.**





Higher Moments of Conserved Quantities (B, Q, S)

- Higher order cumulants/moments: describe the shape of distributions and quantify fluctuations. (sensitive to the correlation length (ξ))

$$\langle \delta N \rangle = N - \langle N \rangle$$

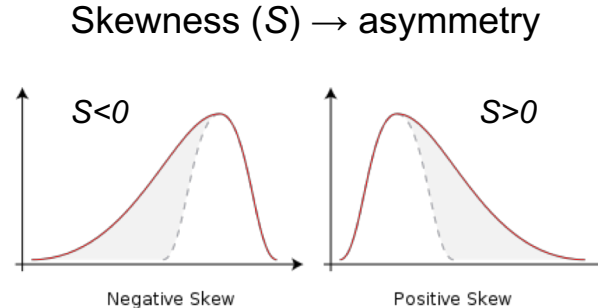
$$C_1 = M = \langle N \rangle$$

$$C_2 = \sigma^2 = \langle (\delta N)^2 \rangle$$

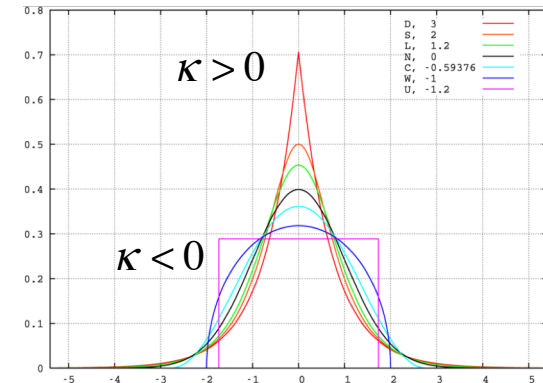
$$C_3 = S\sigma^3 = \langle (\delta N)^3 \rangle$$

$$C_4 = \kappa\sigma^4 = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2$$

$$\langle (\delta N)^3 \rangle_c \approx \xi^{4.5}, \quad \langle (\delta N)^4 \rangle_c \approx \xi^7$$



Kurtosis (κ) → Sharpness



M. A. Stephanov, *Phys. Rev. Lett.* 102, 032301 (2009); 107, 052301 (2011).

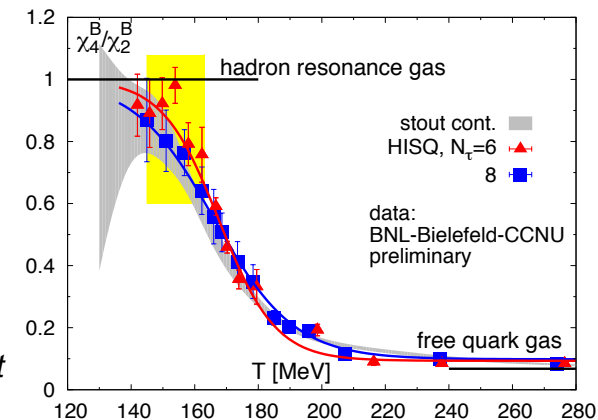
M. Asakawa, S. Ejiri and M. Kitazawa, *Phys. Rev. Lett.* 103, 262301 (2009).

- Direct connect to the susceptibility of the system.

$$\frac{\chi_q^4}{\chi_q^2} = \kappa\sigma^2 = \frac{C_{4,q}}{C_{2,q}}, \quad \frac{\chi_q^3}{\chi_q^2} = S\sigma = \frac{C_{3,q}}{C_{2,q}},$$

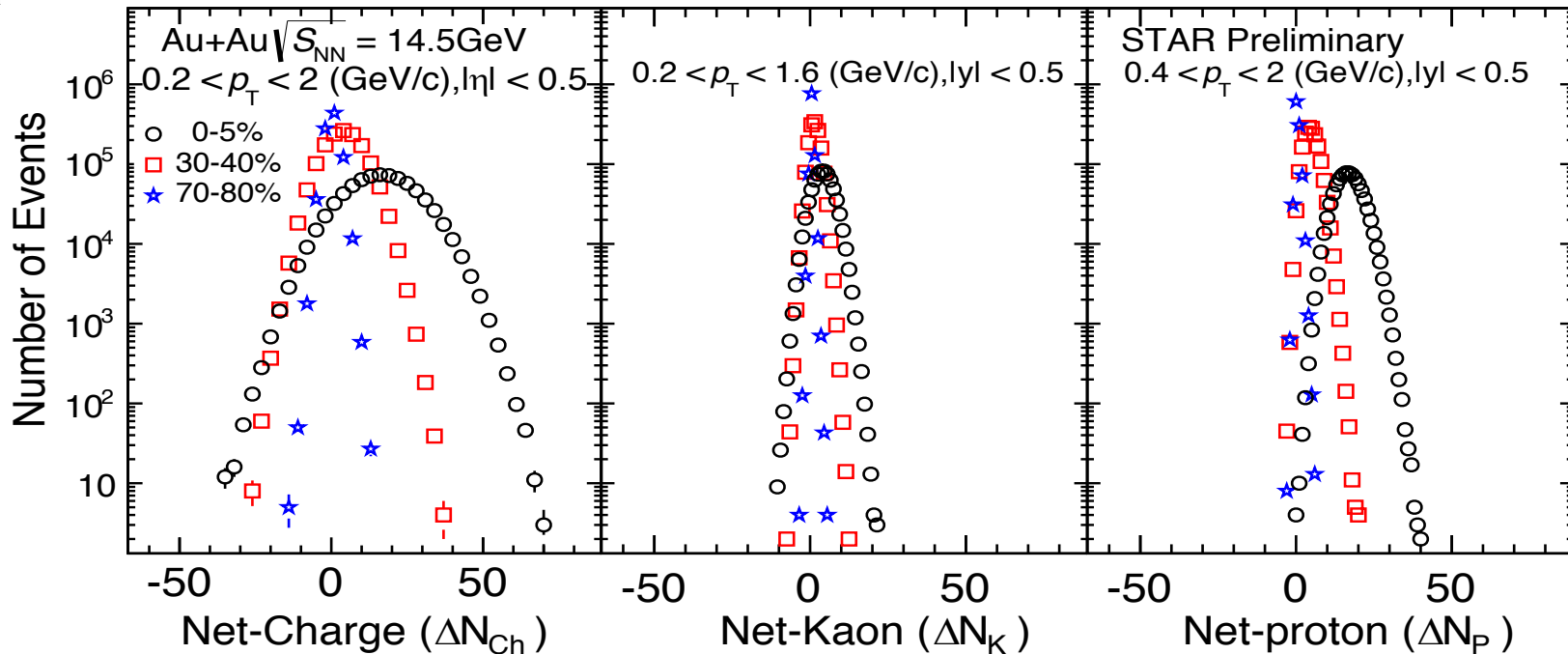
$$\chi_q^{(n)} = \frac{1}{VT^3} \times C_{n,q} = \frac{\partial^n (p/T^4)}{\partial (\mu_q)^n}, q = B, Q, S$$

S. Ejiri et al, *Phys. Lett. B* 633 (2006) 275. Cheng et al, *PRD* (2009) 074505. B. Friman et al., *EPJC* 71 (2011) 1694. F. Karsch and K. Redlich, *PLB* 695, 136 (2011). S. Gupta, et al., *Science*, 332, 1525(2012). A. Bazavov et al., *PRL* 109, 192302(12) // S. Borsanyi et al., *PRL* 111, 062005(13)





Data Analysis Methods



Effects needed to be addressed to get final moments/cumulants:

1. Statistical errors estimation : Delta theorem or bootstrap
2. Avoid auto-correlation effects: New centrality definition.
3. Suppress volume fluctuation: Centrality bin width correction
4. Finite detector efficiency correction (binomial response func.)

$$\text{error}(\kappa\sigma^2) \propto \frac{\sigma^2}{\varepsilon^2} \frac{1}{\sqrt{N_{\text{evts}}}}$$

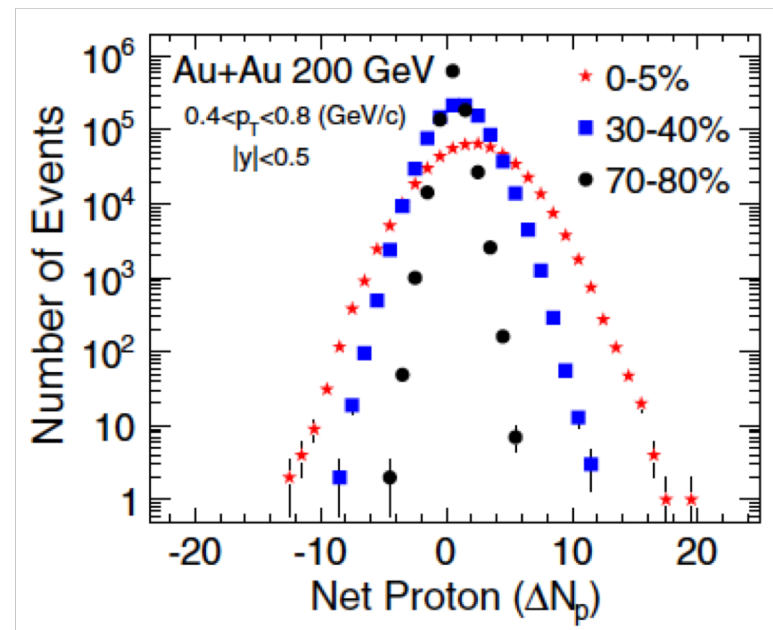
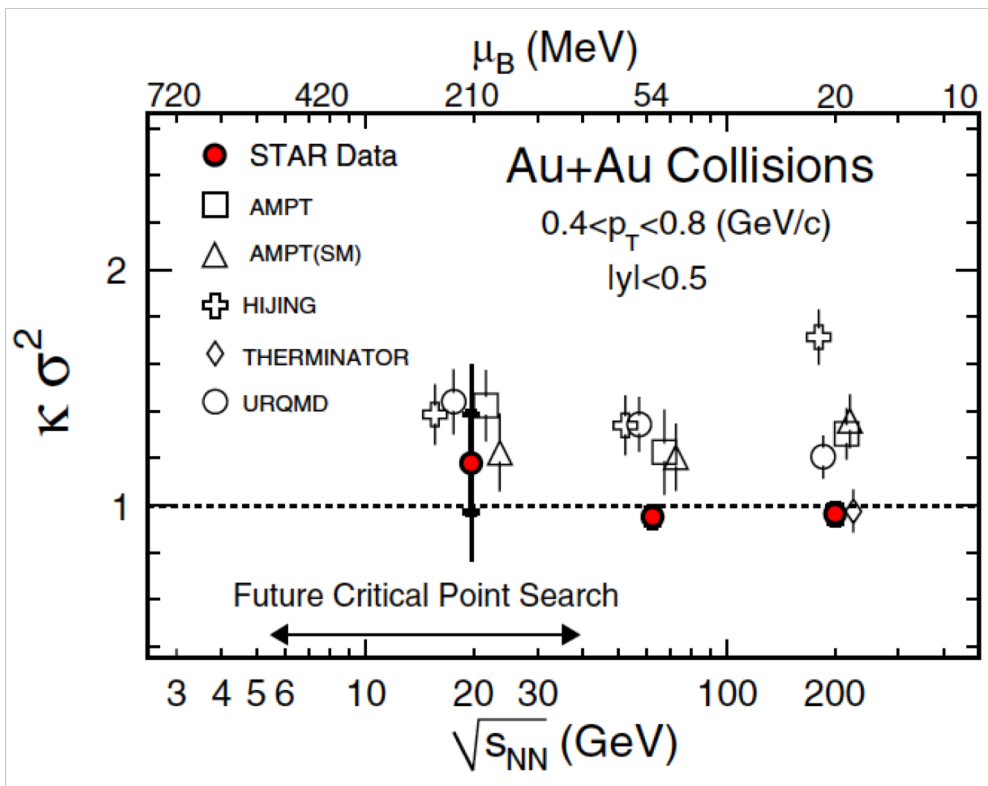
Review Article: X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017).

X.Luo, et al. J. Phys. G39, 025008 (2012); A. Bzdak and V. Koch, PRC86, 044904 (2012); X.Luo, et al. J. Phys. G40, 105104(2013); X.Luo, Phys. Rev. C 91, 034907 (2015); A. Bzdak and V. Koch, PRC91, 027901 (2015). T. Nonaka et al., PRC95, 064912 (2017). M. Kitazawa and X. Luo, PRC96, 024910 (2017). X. Luo, T. Nonaka, arXiv: 1812.10303

First measurement of high moments of net-proton distributions

Only TPC used for proton/anti-proton PID.

p_T coverage : $0.4 < p_T < 0.8$ GeV/c

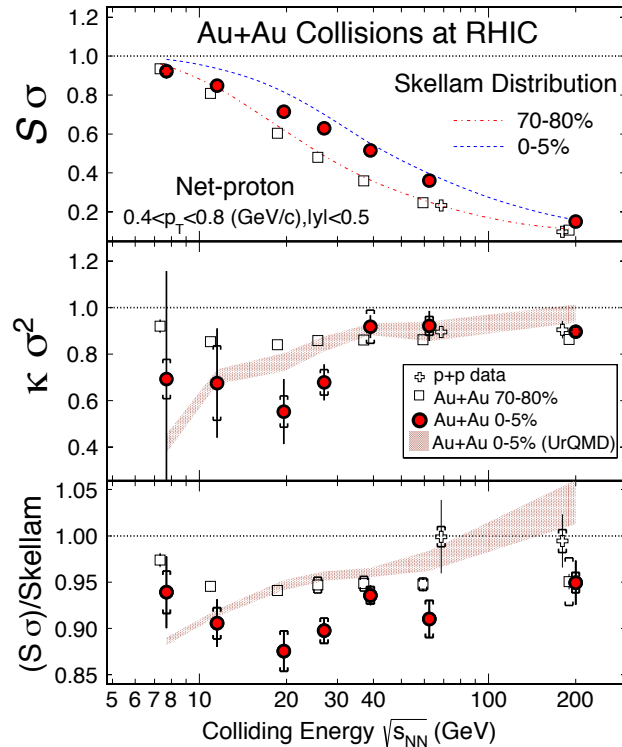


STAR: Phys. Rev. Lett. 105, 022302 (2010).

No critical point signature is observed at $\mu_B < 210$ MeV.

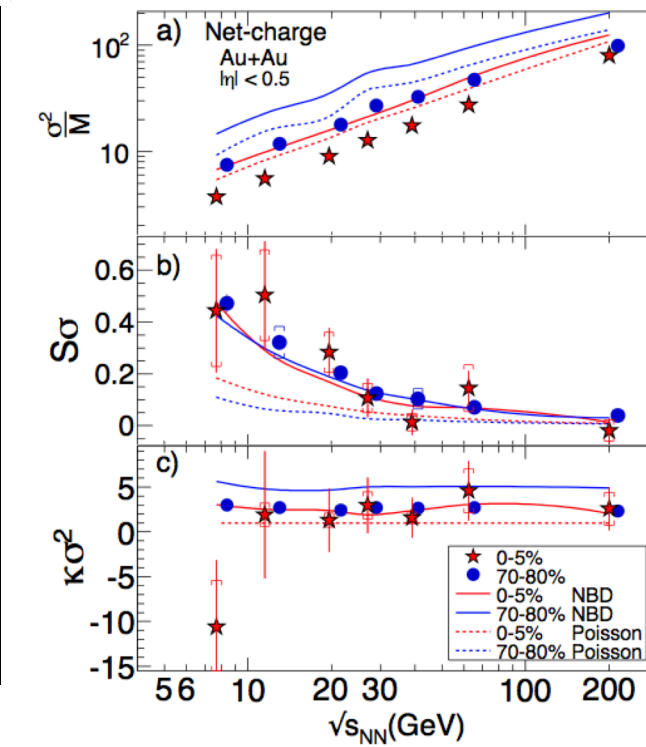
BES-I (2010-2014) : Net-Particle Fluctuation Measurements

Net-Proton



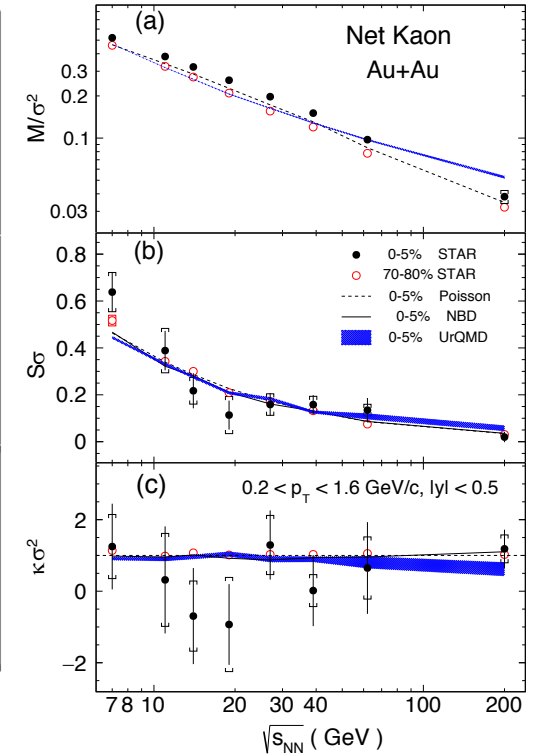
Phys. Rev. Lett. 112, 032302 (2014).

Net-Charge



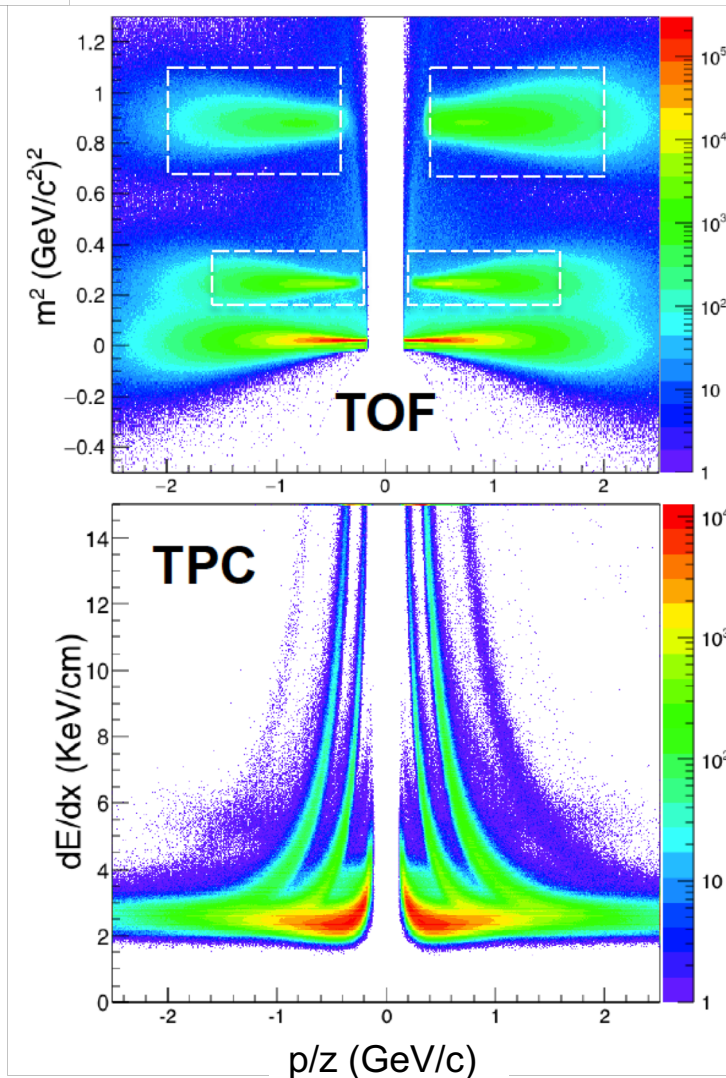
Phys. Rev. Lett. 113 092301 (2014).

Net-Kaon



Phys. Lett. B 785, 551 (2018).

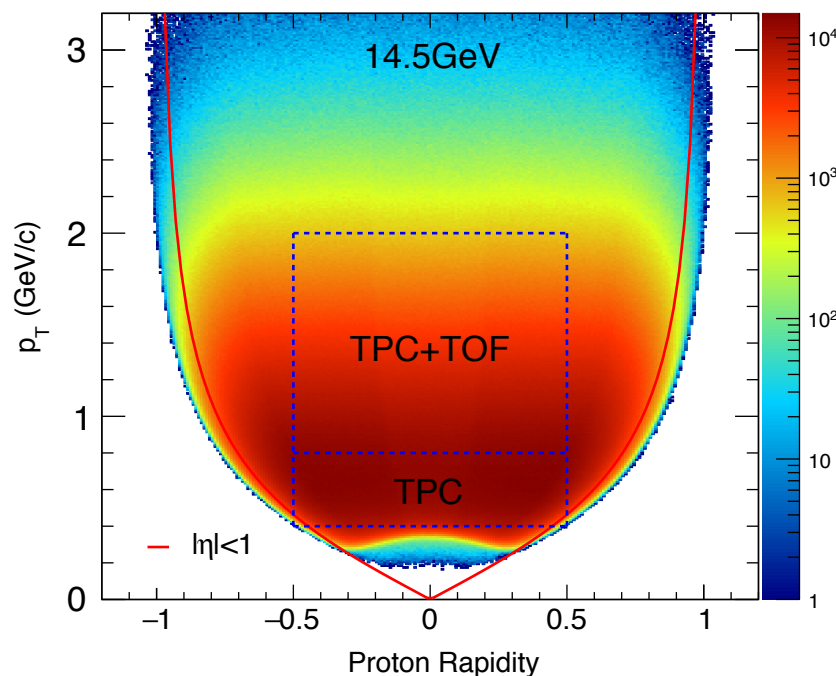
(Anti-) Proton PID and Acceptance



Extend the phase space coverage by TOF.
Doubled the accepted number of proton/anti-proton

$$|y| < 0.5, 0.4 < p_T < 0.8 \text{ (TPC PID)}$$

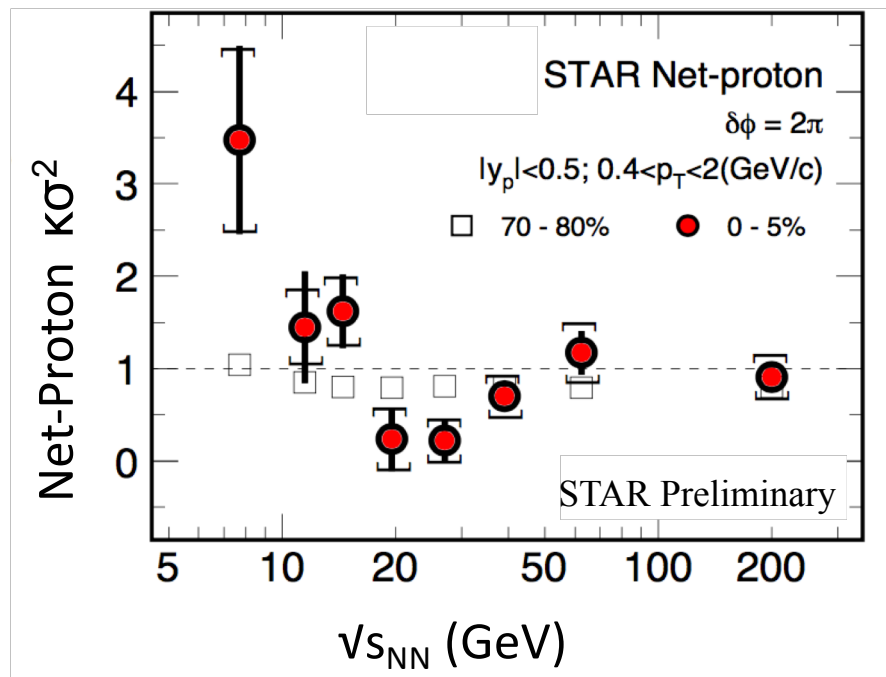
$$0.8 < p_T < 2 \text{ (TPC+TOF PID)}$$



➤ Sufficiently large acceptance is important for fluctuation analysis

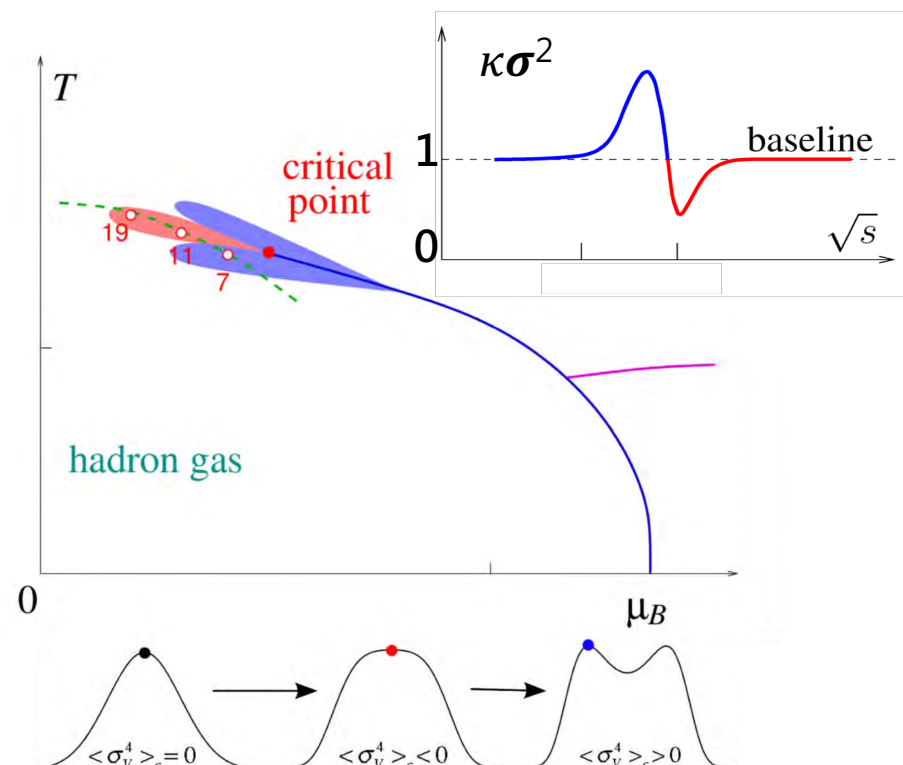
Net-Proton Fluctuations

Experimental Measure



STAR: Phys. Rev. Lett. 105, 022302 (2010).
 Phys. Rev. Lett. 112, 032302 (2014).
 PoS CPOD2014 (2015) 019.

Theoretical calculations



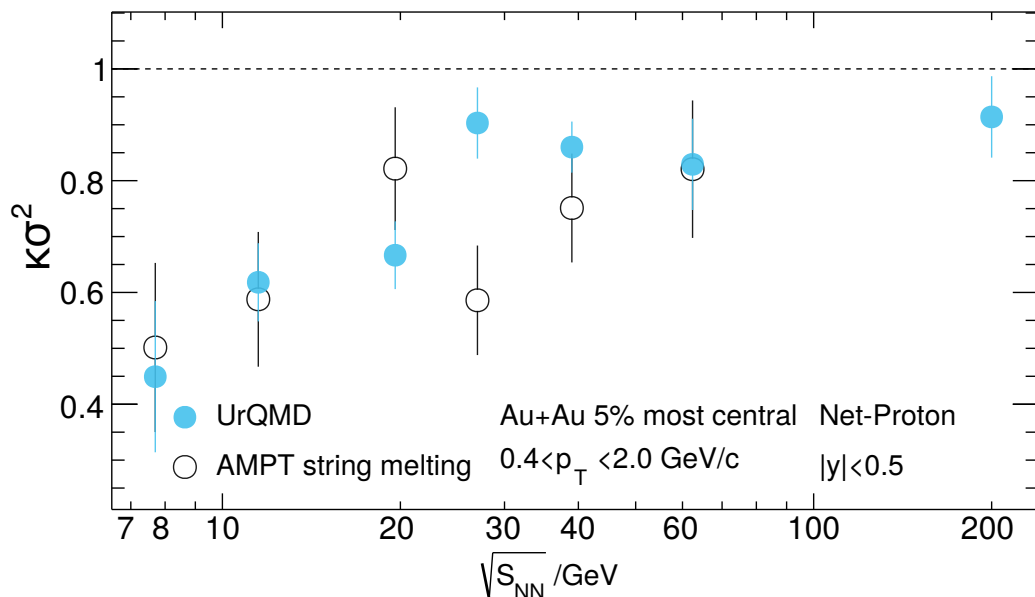
M. Stephanov, PRL107, 052301(2011)
 J. Phys. G: 38, 124147 (2011).

- First observation of the non-monotonic energy dependence of fourth order net-proton fluctuations. **Hint of entering Critical Region ?**

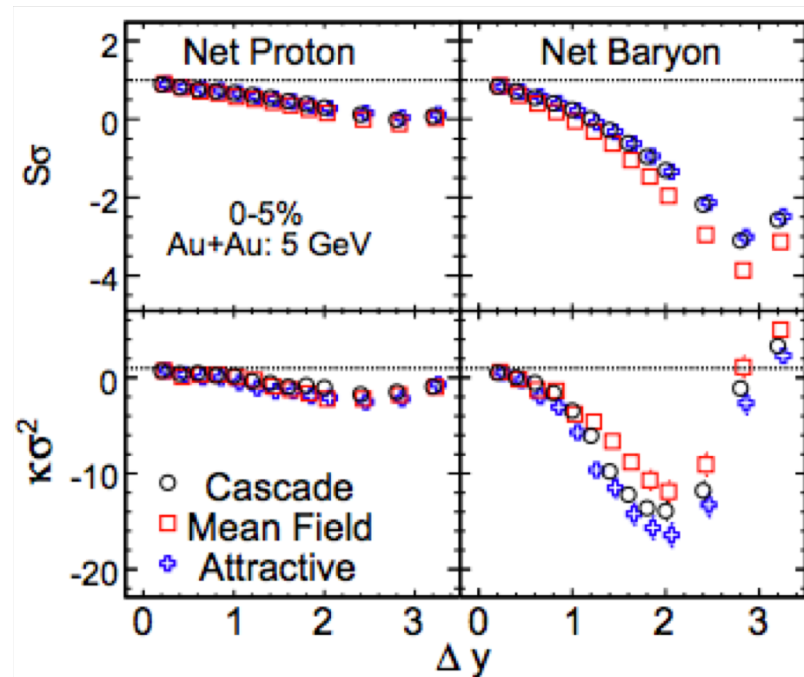


Non-critical Contributions: Transport Model Studies

UrQMD and AMPT models



JAM model

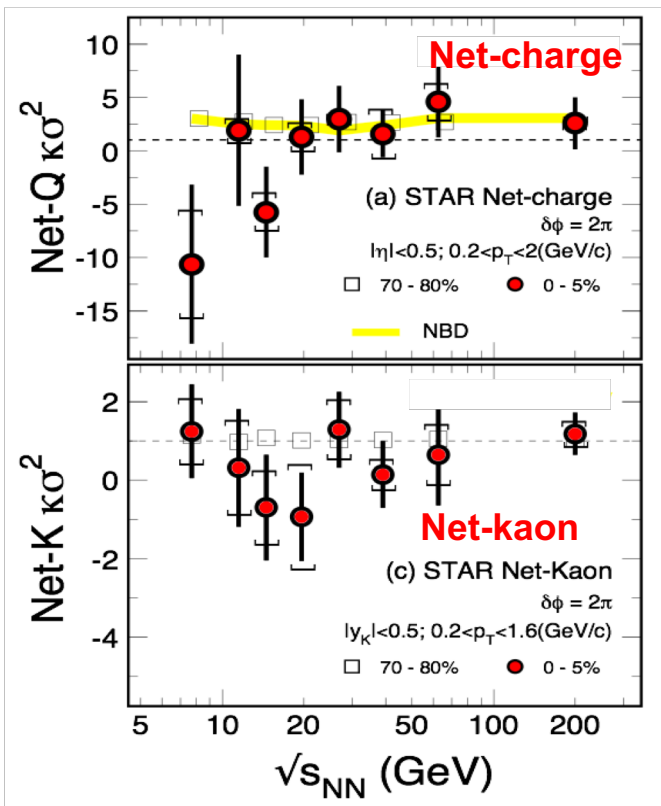


- Transport model (no CP physics) results show monotonic energy dependence: dominated by baryon number conservations
- Mean field potential can not explain the enhancement of $K\sigma^2$ at low energy.

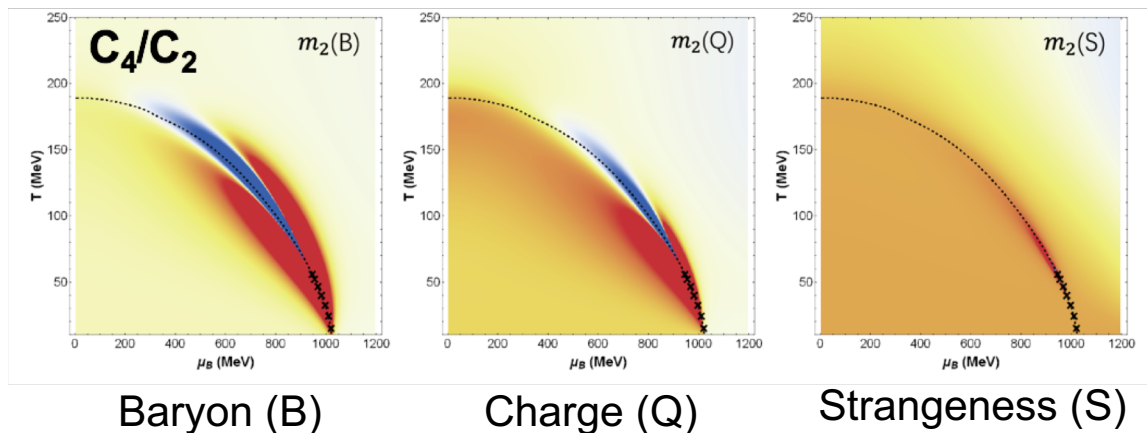
Z. Feckova, et al., PRC92, 064908(2015). J. Xu, et. al., PRC94, 024901(2016). X. Luo et al., NPA931, 808(14), P.K. Netrakanti et al. 1405.4617, NPA947, 248(2016), P. Garg et al. PLB 726, 691(2013). S. He, et. al., PLB762, 296 (2016). S. He, X. Luo, PLB 774, 623 (2017).

Net-charge and Net-kaon Fluctuations

STAR Data



NJL Model calculations



W. Fan, X. Luo, H.S. Zong, IJMPA 32, 1750061 (2017).

Critical signals: $B > Q > S$

(Due to the mass of strange quark is much larger than u,d quarks, $m_s \gg m_{u,d}$)

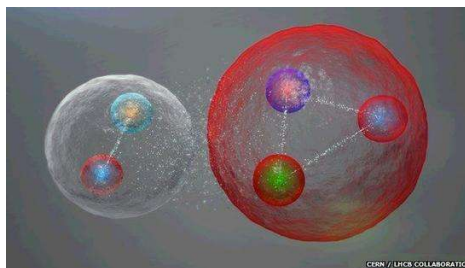
STAR : Phys. Rev. Lett. 113 092301 (2014).
 Phys. Lett. B 785, 551 (2018).

$$error(\kappa\sigma^2) \propto \frac{\sigma^2}{\varepsilon^2} \frac{1}{\sqrt{N_{evts}}}$$

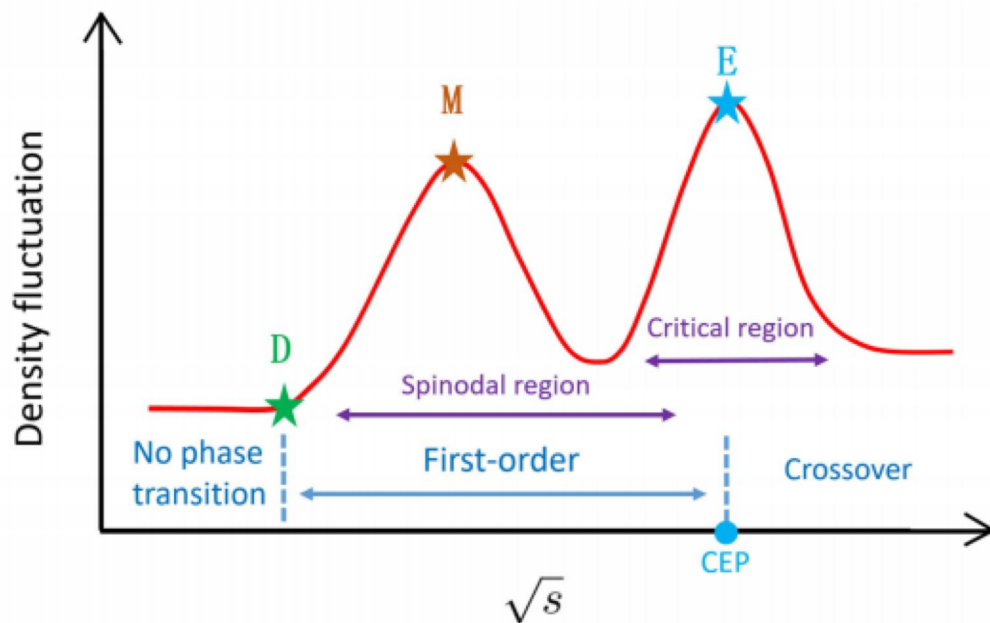
- 1) Within errors, the results of net-Q and net-Kaon show flat energy dependence.
- 2) More statistics are needed at low energies.

New Observable for CP: Light Nuclei Production

Near CP or 1st order phase transition, baryon density fluctuation become large.



Light nuclei production
(Baryon Clustering)



Coalescence + nucleon density flu.

$$N_d = \frac{3}{2^{1/2}} \left(\frac{2\pi}{m_0 T_{\text{eff}}} \right)^{3/2} N_p \langle n \rangle (1 + \alpha \Delta n),$$

$$N_{3H} = \frac{3^{3/2}}{4} \left(\frac{2\pi}{m_0 T_{\text{eff}}} \right)^3 N_p \langle n \rangle^2 [1 + (1 + 2\alpha) \Delta n],$$

$$N_t \cdot N_p / N_d^2 \approx g(1 + \Delta n)$$

Neutron density fluctuations:

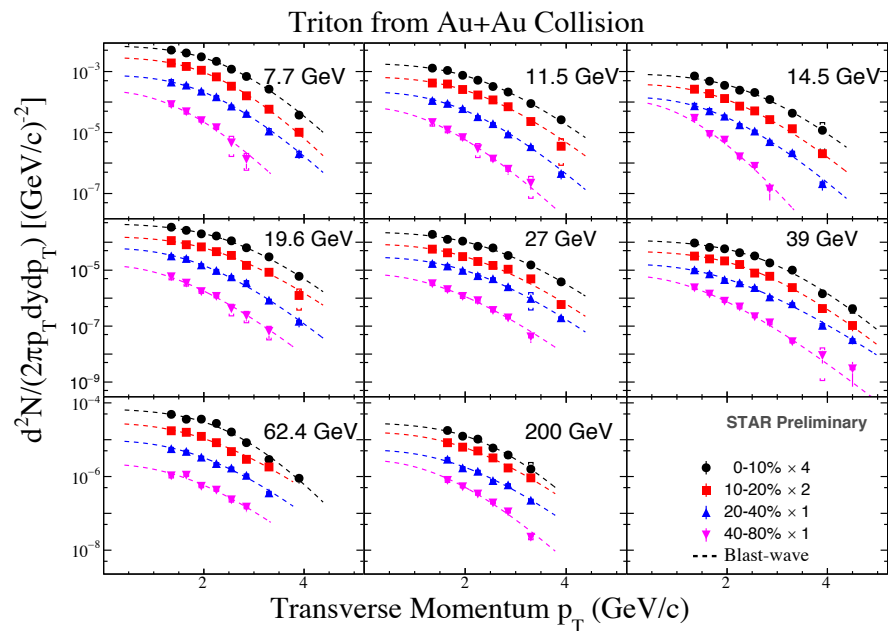
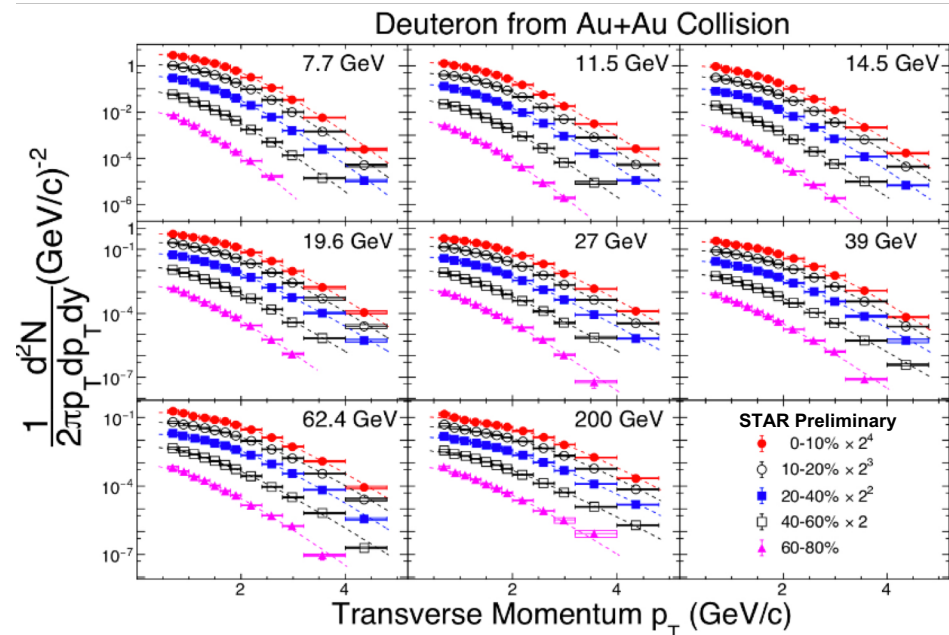
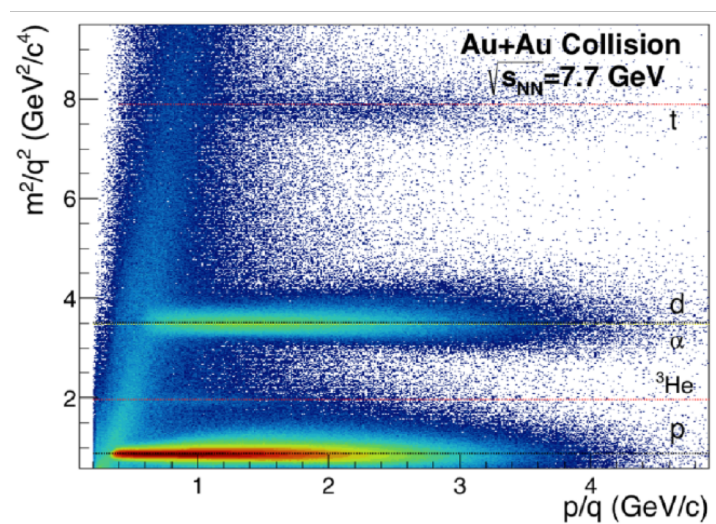
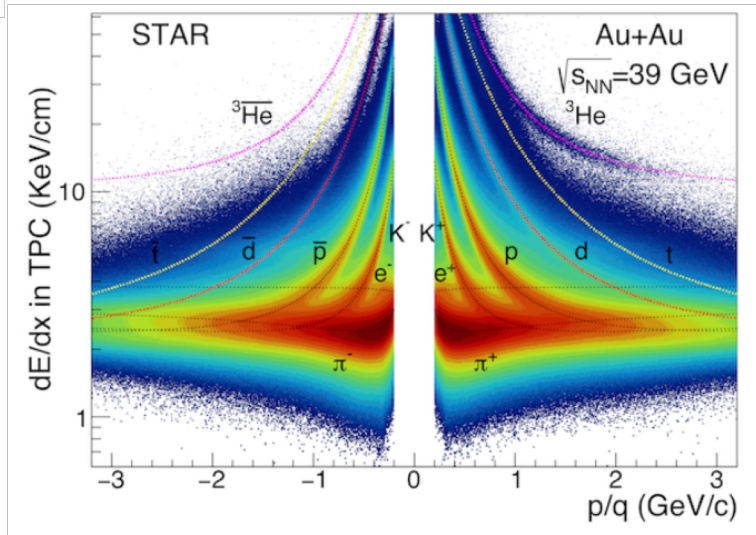
$$\Delta n = \langle (\delta n)^2 \rangle / \langle n \rangle^2$$

K. J. Sun, L. W. Chen, C. M. Ko, Z. Xu, Phys. Lett. B774, 103 (2017).

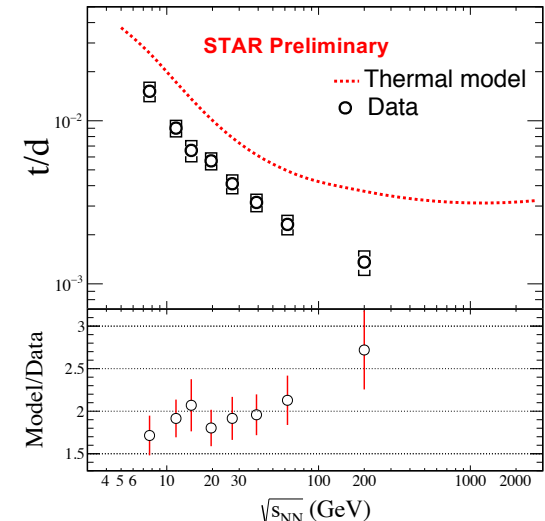
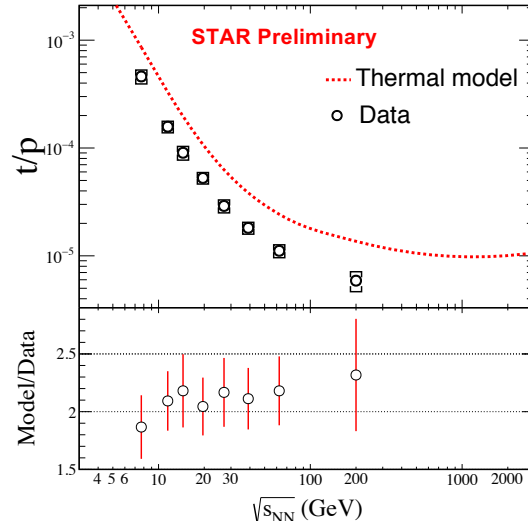
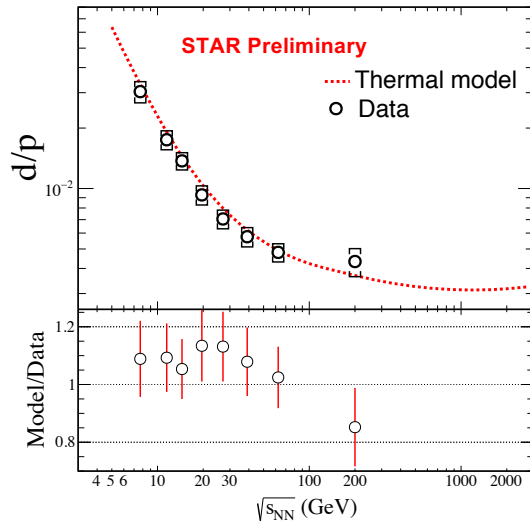
K. J. Sun, L. W. Chen, C. M. Ko, J. Pu, Z. Xu, Phys. Lett. B781, 499 (2018).

Edward Shuryak and Juan M. Torres-Rincon, arXiv:1805.04444

Deuteron and triton production from BES-I at RHIC



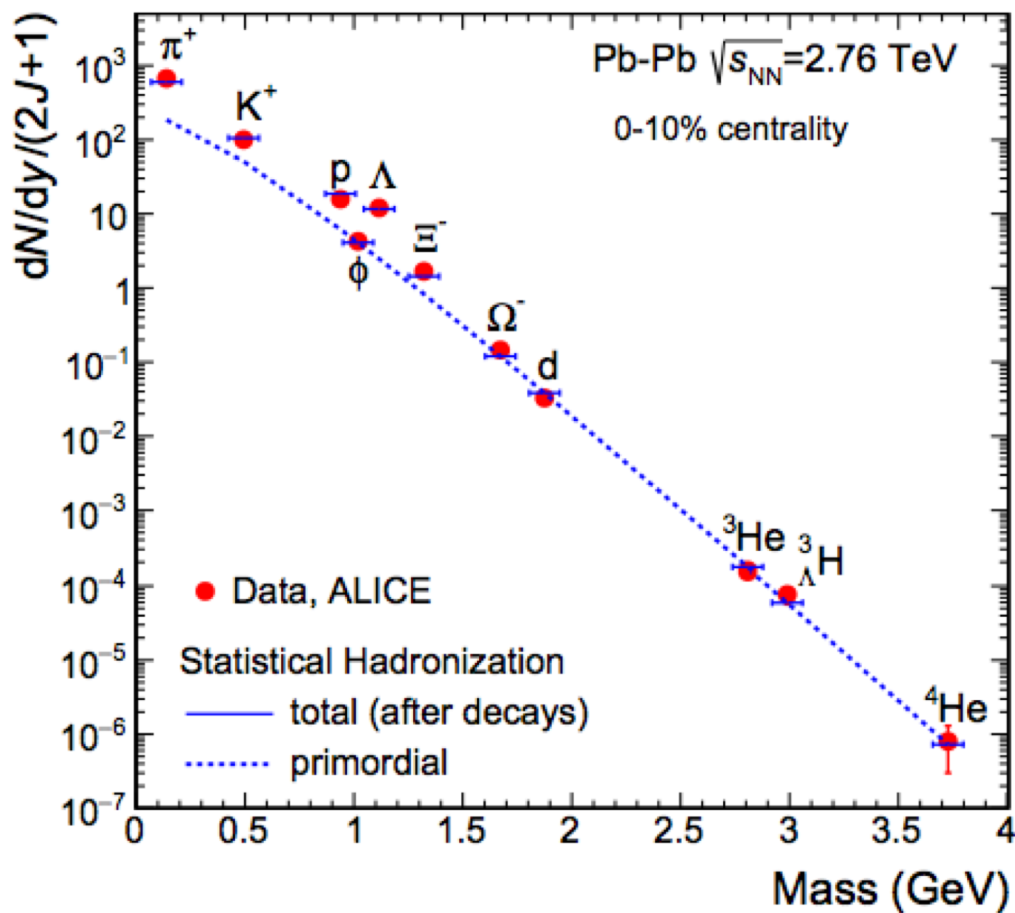
Light Nuclei Yield Ratio Vs. Thermal model



- At RHIC energies, thermal model can describe the d/p ratios, but can not describe the t/p , t/d ratios.
- If deuteron is formed at very late stage via nucleon cola., why it can be described by thermal model ?



ALICE Data Vs. Thermal Model



Why the yield of **triton** and even **alpha** can be well described by thermal model at LHC energies
But not at RHIC energies ?

Different production mechanism of light nuclei at RHIC and LHC energies ??

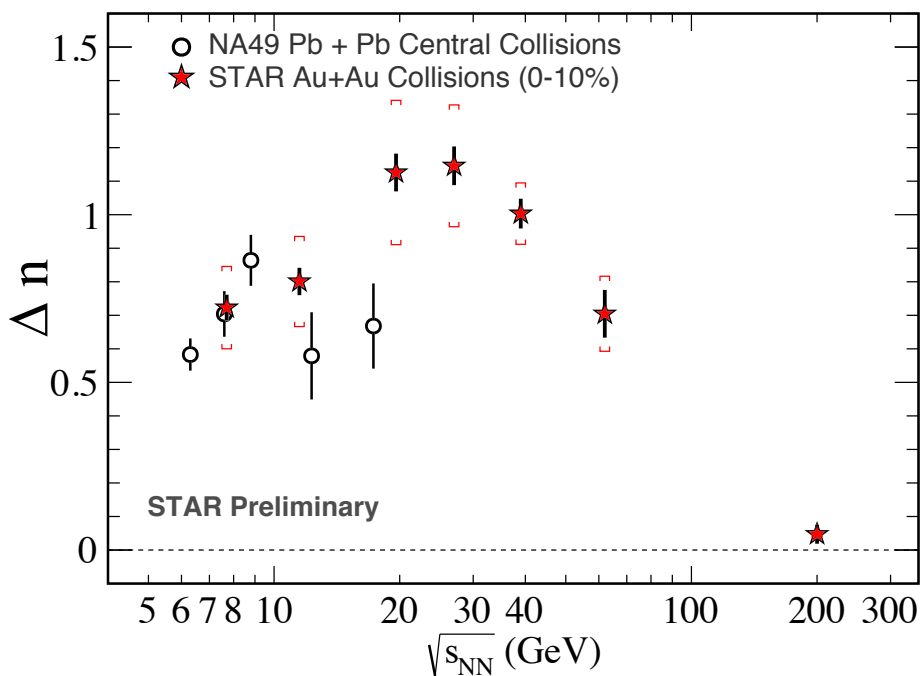
A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561, 321 (2018).



Nucleon Density Fluctuations

- The particle ratios of light nuclei is sensitive to the **nucleon density fluctuation** at kinetic freeze-out. **This conclusion is based on coalescence model.**

$$N_d = \frac{3}{2^{\frac{1}{2}}} \left(\frac{2\pi}{m_0 T_{eff}} \right)^{\frac{3}{2}} N_p \langle n \rangle (1 + \alpha \Delta n) \quad N_t = \frac{3^{\frac{3}{2}}}{4} \left(\frac{2\pi}{m_0 T_{eff}} \right)^3 N_p \langle n \rangle^2 [1 + (1 + 2\alpha) \Delta n]$$



$$\frac{\langle (\delta n)^2 \rangle}{\langle n \rangle^2} = \Delta n = \frac{1}{g} \frac{N_t N_p}{N_d^2} - 1$$

N_t : Triton yield, N_d : Deuteron yield
 N_p : Proton yield

Neutron density fluctuation Δn shows a **non-monotonic behavior** on collision energy.

Peak around 20 GeV.

Dingwei Zhang, QM2018

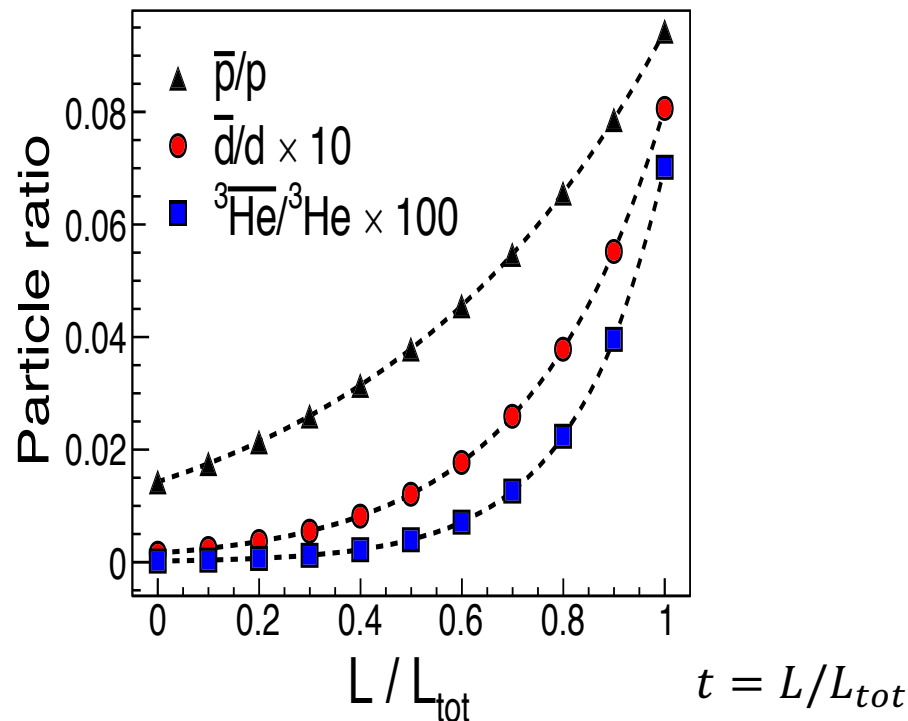
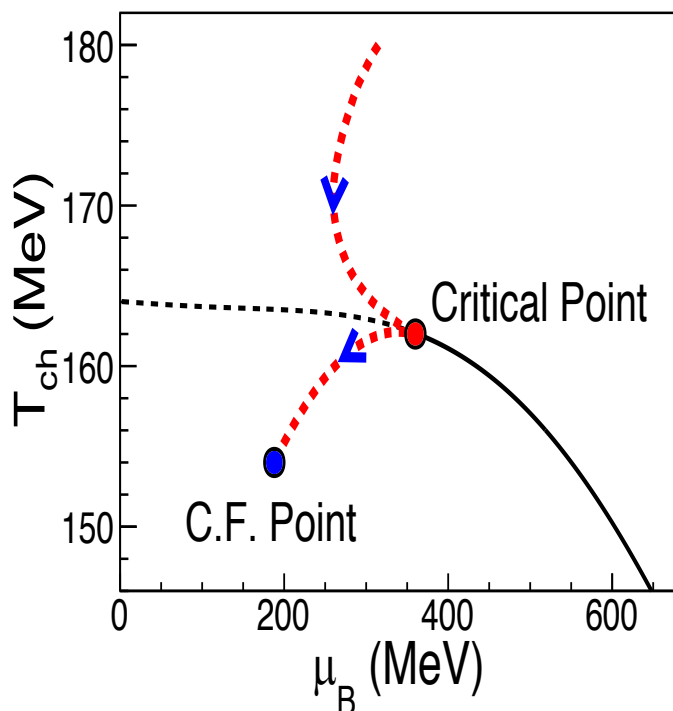
K. J. Sun, L. W. Chen, C. M. Ko, Z. Xu, Phys. Lett. B774, 103 (2017).
 K. J. Sun, L. W. Chen, C. M. Ko, J. Pu, Z. Xu, Phys. Lett. B781, 499 (2018).
 Edward Shuryak and Juan M. Torres-Rincon, NPA 982, 831 (2019)



QCD Critical Point Focusing Effects

QCD critical point focusing effect : critical point will serve as an attractor of the trajectory evolution in the $T - \mu_B$ plane.

M. Asakawa, S. A. Bass B. Muller, C. Nonaka
PRL 101, 122302 (2008)



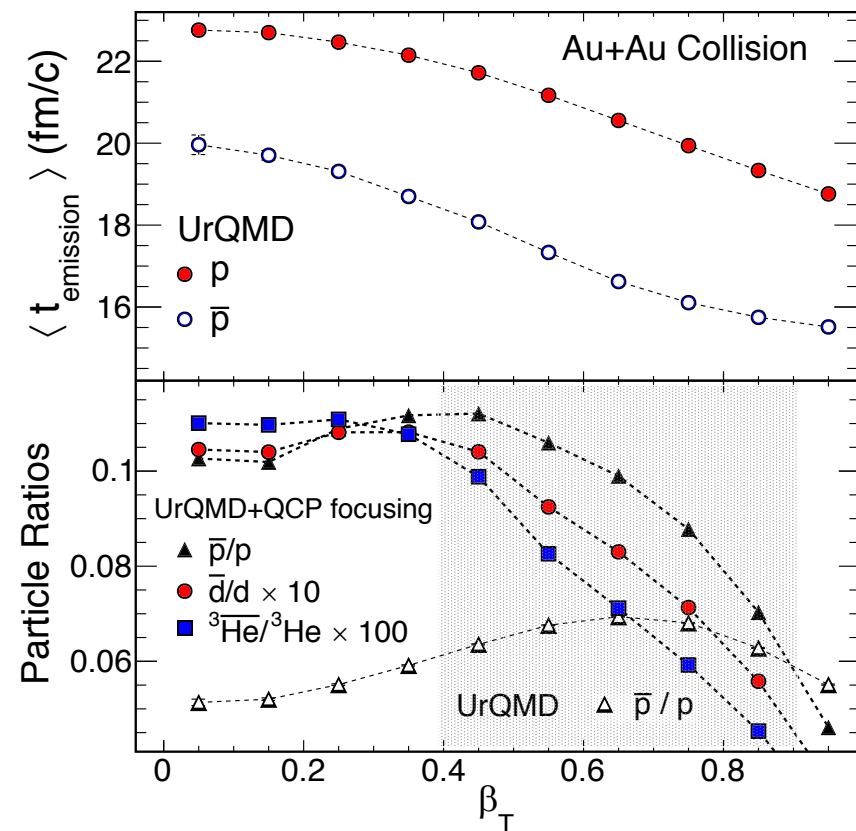
N. Yu, D. Zhang, and X. Luo arXiv:1812.04291

The particle emitting probability along the trajectory is assumed to be proportional to the particle density calculated from the thermal model with corresponding T and μ_B .



QCP Focusing Effects : Model Implementation

- $\beta_T - t$ distribution is from UrQMD central Au+Au collision at $\sqrt{s_{NN}} = 19.6$ GeV.



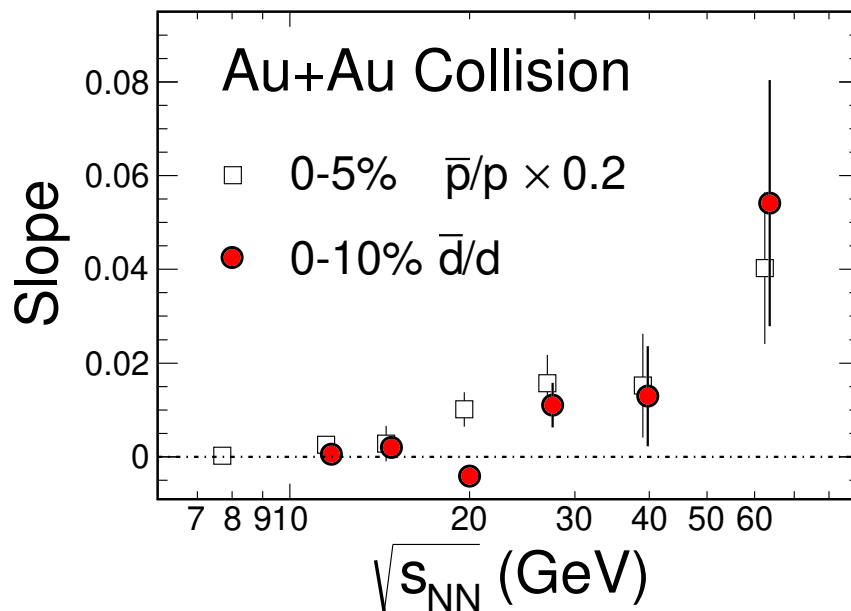
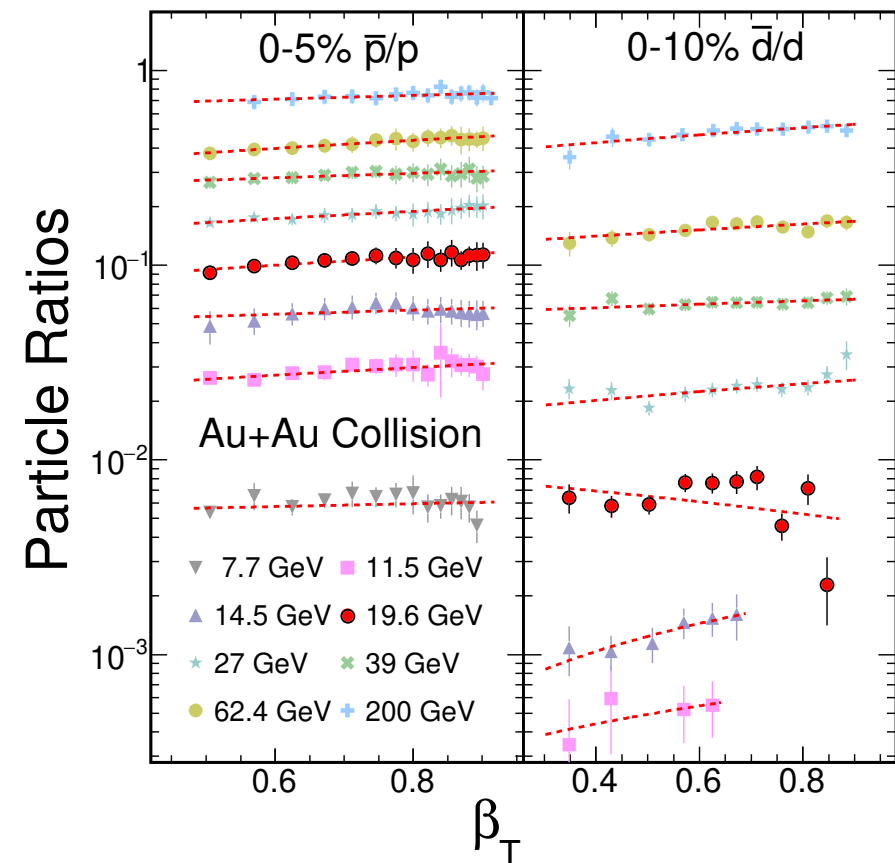
- The $\bar{p}/p$, $\bar{d}/d$, and ${}^3\text{He}/\text{He}$ **decrease with increasing β_T** with QCP focusing effect.
- This behavior is more easier to be observed for heavier particles

Can we observe the QCP focusing effects in HIC experiment ?

N. Yu, D. Zhang, and X. Luo arXiv:1812.04291



Transverse velocity dependence of dbar/d ratio

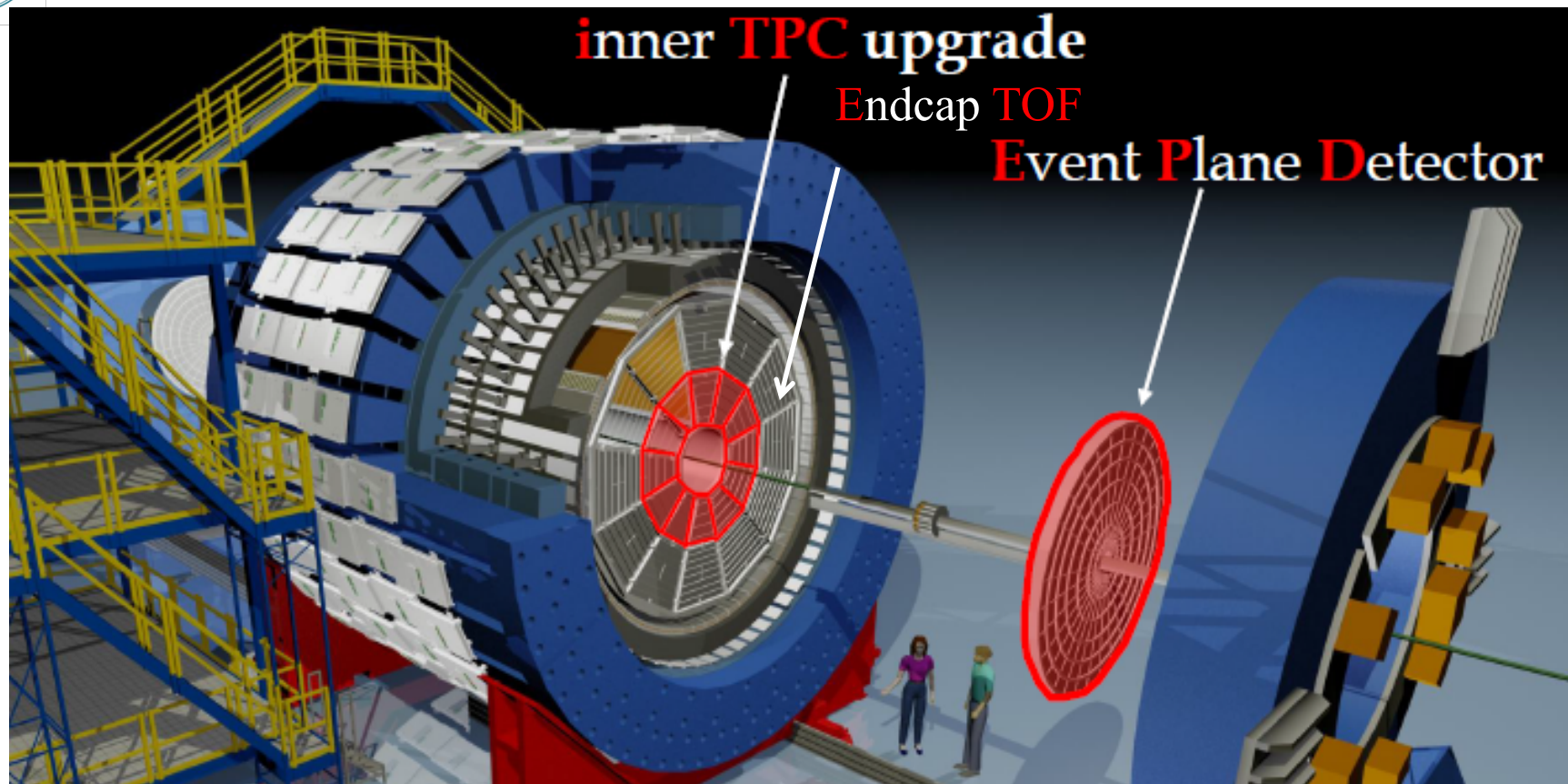


➤ The decreasing anti-particle to particle ratio only occur at 19.6 GeV for **deuteron**.

N. Yu, D. Zhang, and X. Luo arXiv:1812.04291

We observe anomalous and negative slope for dbar/d ratio at 19.6 GeV measured by the STAR experiment !

STAR Upgrades for BES Phase-II (2019-2021)



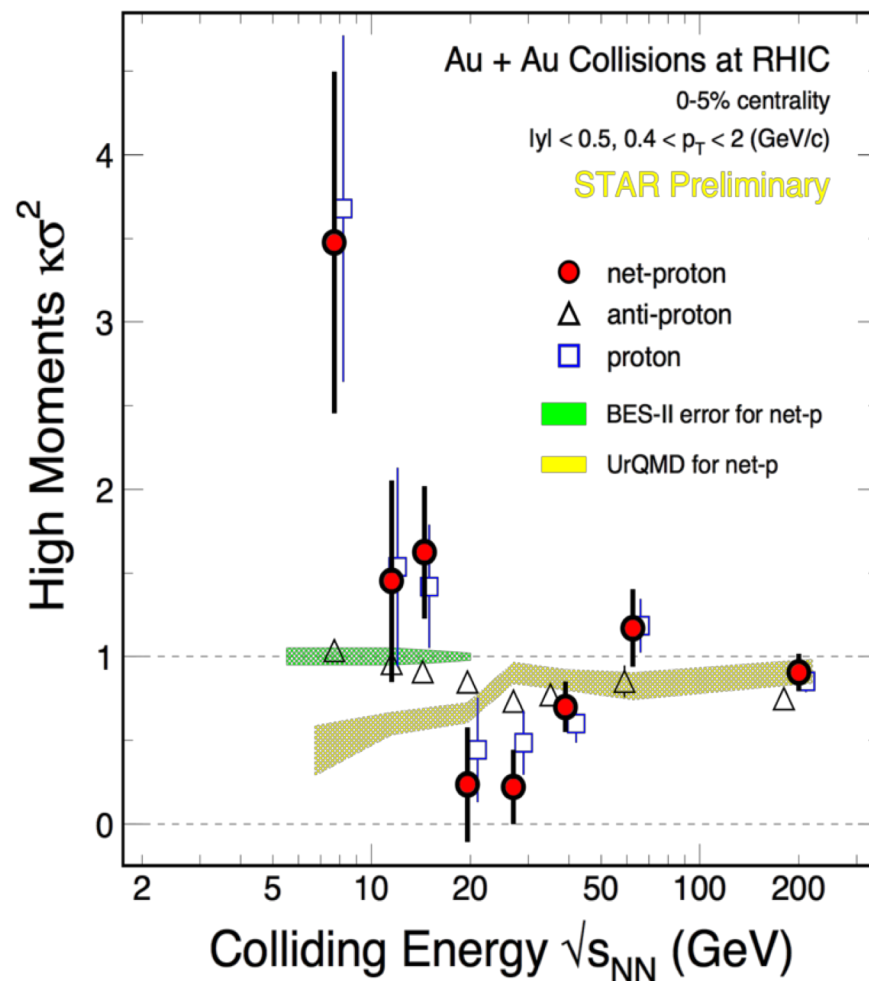
- **Enlarge Acceptance** : η coverage from 1.0 to 1.5
- **Improve dE/dx and forward PID**
- **Improve centrality/event plane determination**

**iTPC, EPD, eTOF
Upgrade complete
Dedicated runs at :
2019-2021**



BES-II at RHIC (2019-2021)

$\sqrt{s_{NN}}$ (GeV)	Events (10^6)	BES II / BES I
200	238	2010
62.4	45	2010
54.4	1200	2017
39	86	2010
27	32	2011
19.6	400 / 15	2019 / 2011
14.5	300 / 13	2019 / 2014
11.5	230 / 7	2020 / 2010
9.2	160 / 0.3	2020 / 2008
7.7	100 / 3	2020-202 / 2010

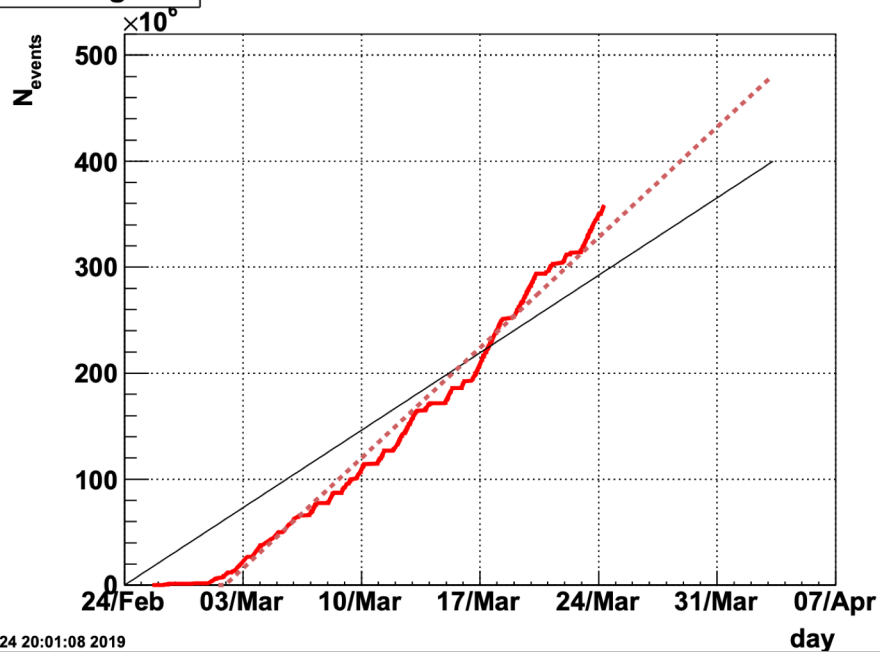


BES-II 19.6 GeV data taking is almost finished and will switch to 14.5 GeV at April. 3rd, 2019

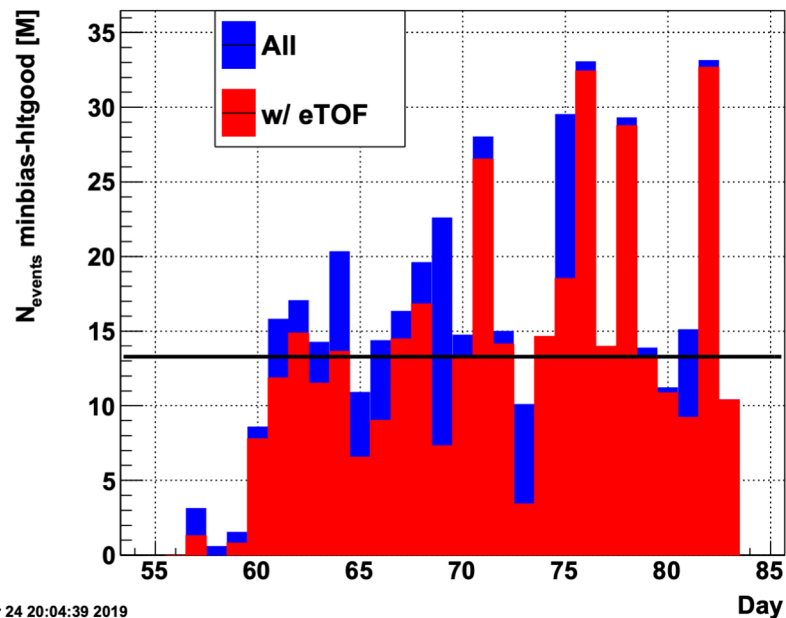


Status for BES-II 19.6 GeV

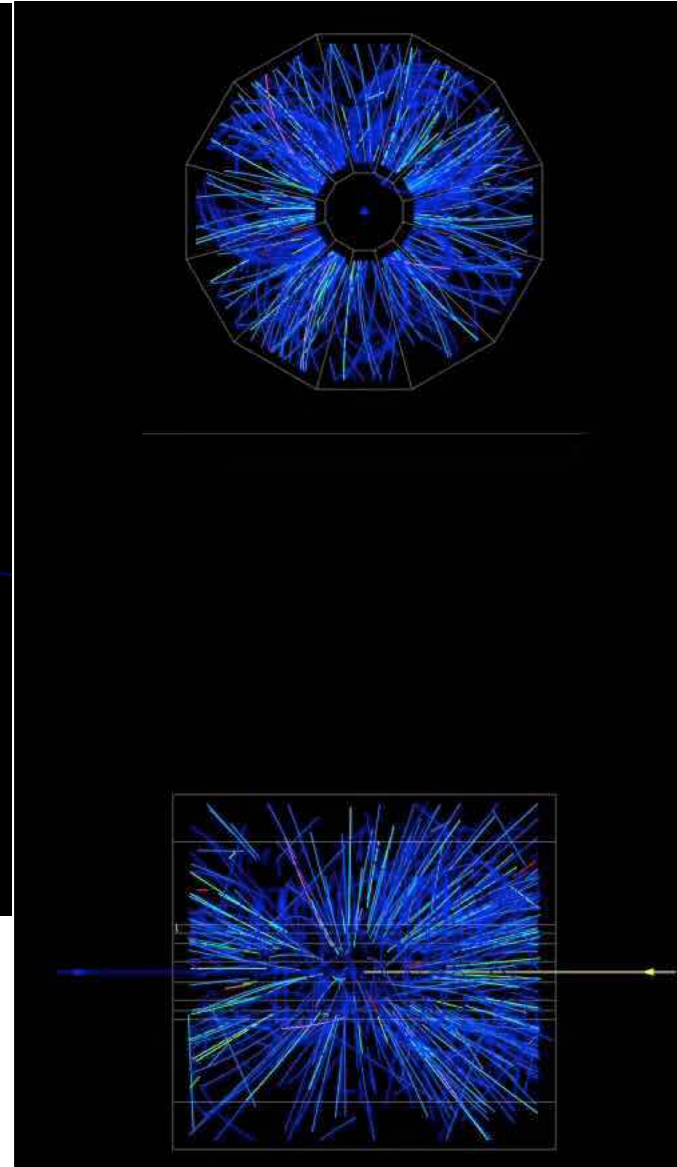
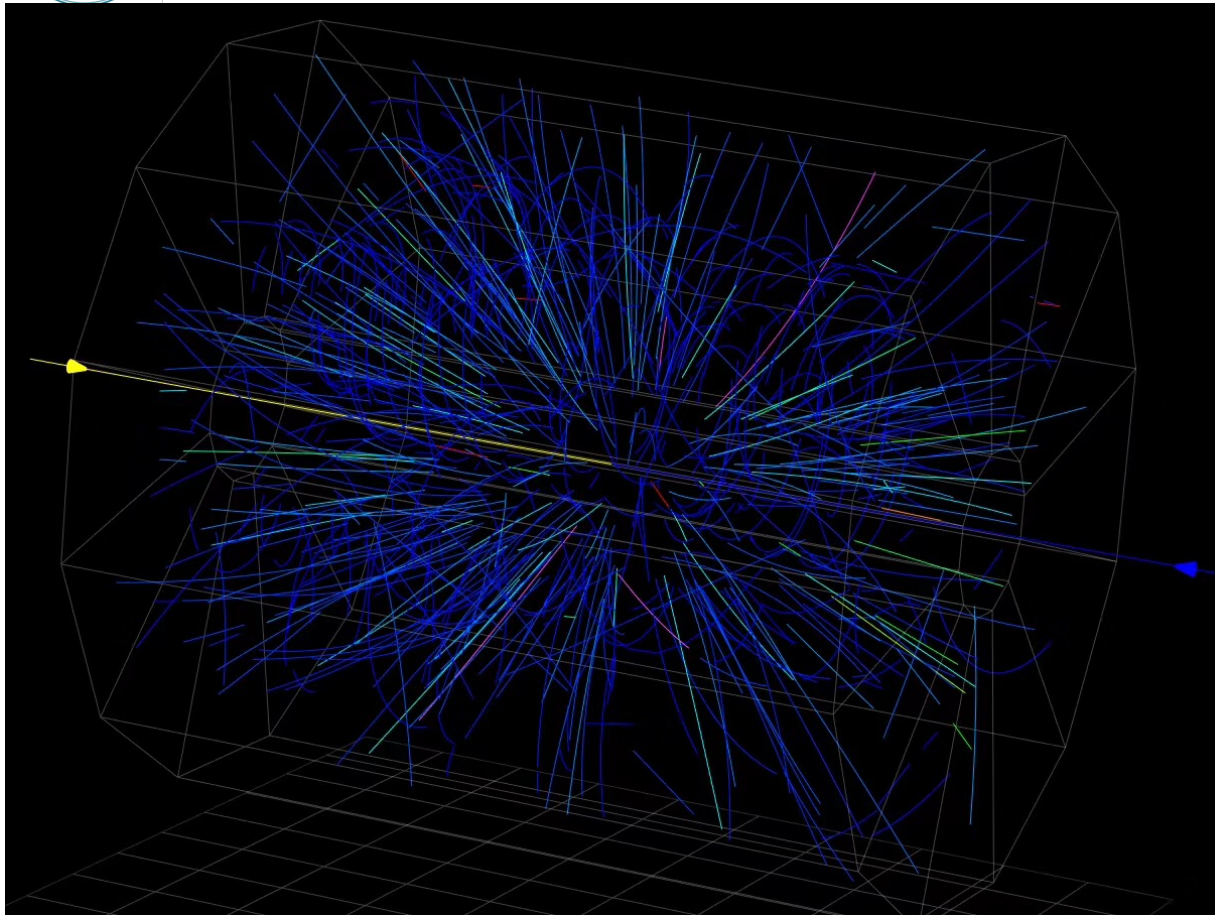
minbias-hltgood



../runningStateAu19GeV/nevents_perday_mb_hlt_good.txt

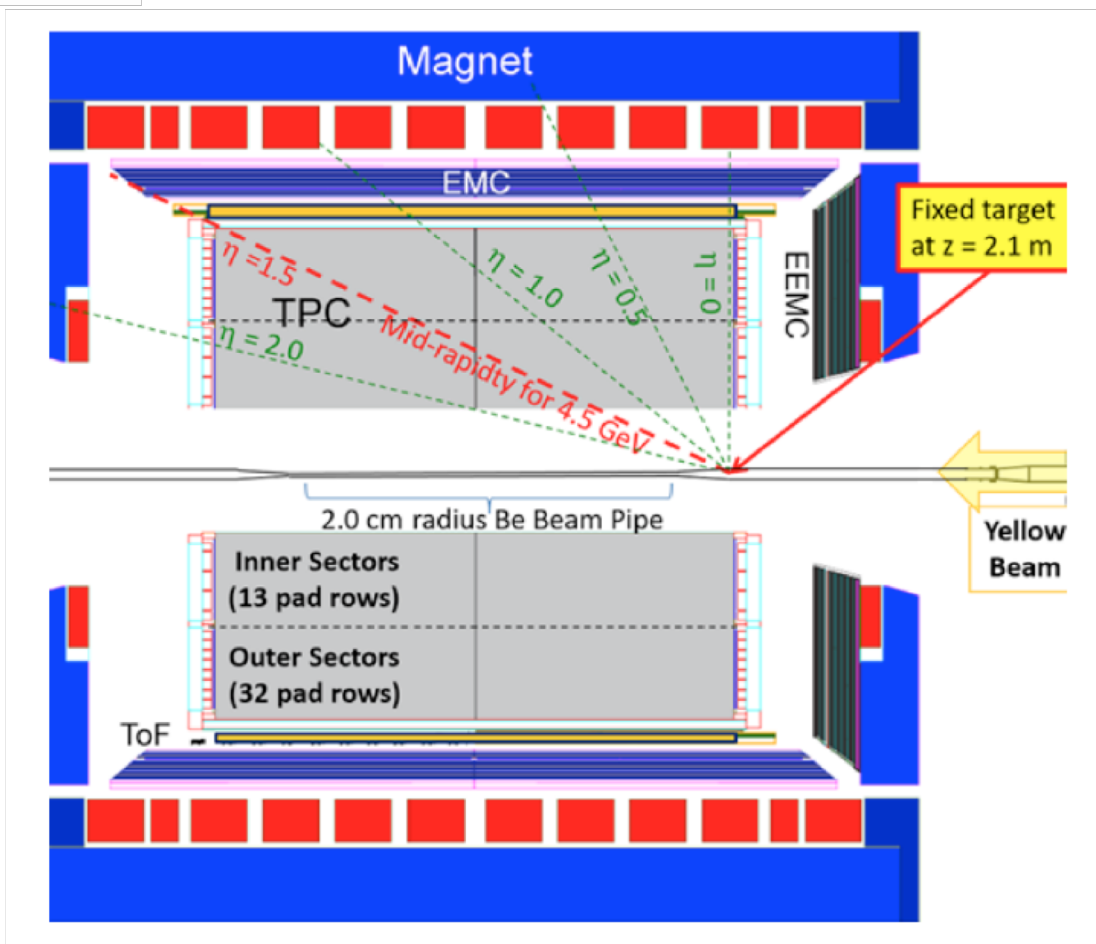


3D Event Display at STAR

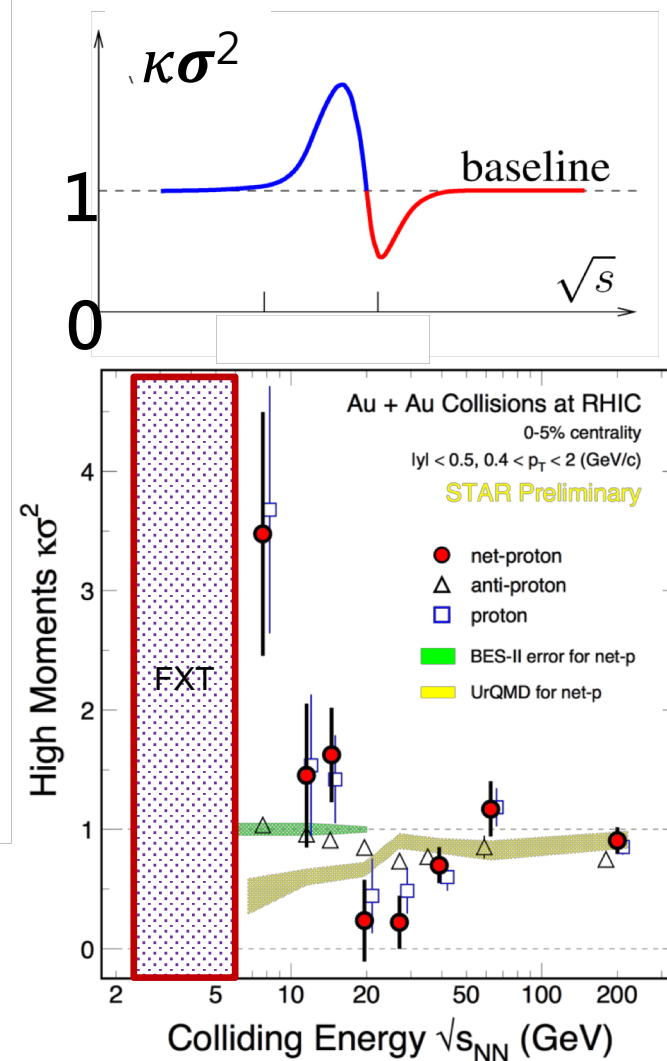


BES-II, Au+Au collisions at 19.6 GeV.

FXT Experiments at STAR (2018-2020)



FXT Data Taking Plan:
 2018: Au+Au :3 GeV (>100 million events)
 2019-2020: Au+Au: 6.2, 5, 4.5, 4, 3.5 GeV

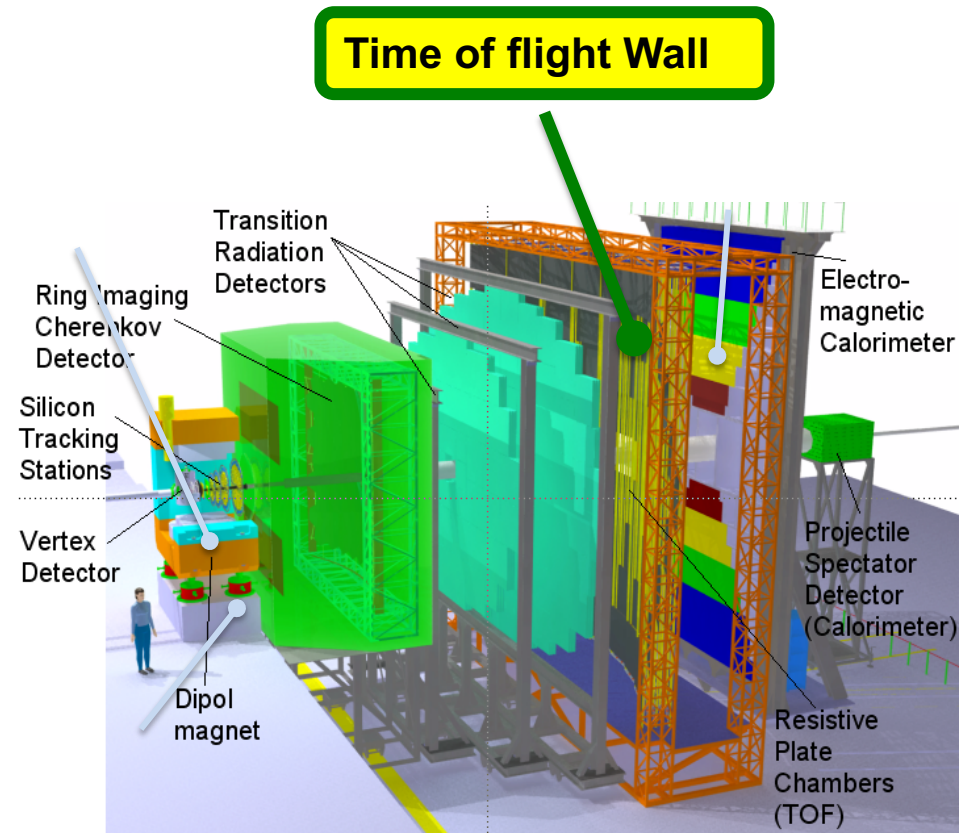
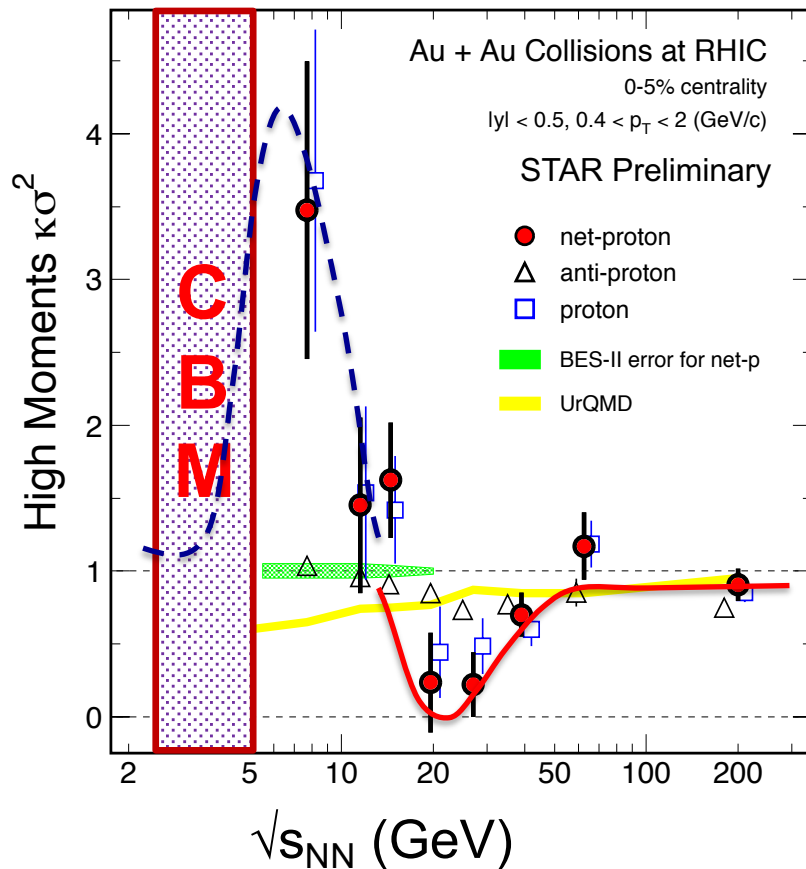


Future CBM experiment

FAIR/CBM Fix target experiment:

Energy: 2.7 – 5 GeV

Starting : 2025 –

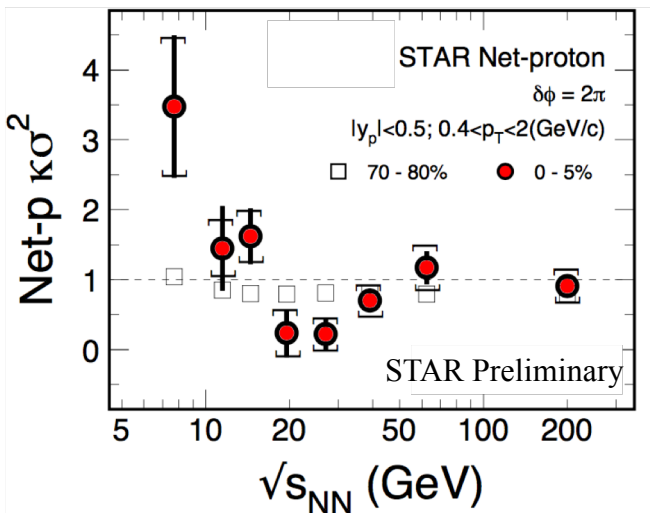


Explore QCD phase structure at higher baryon density region with high precision

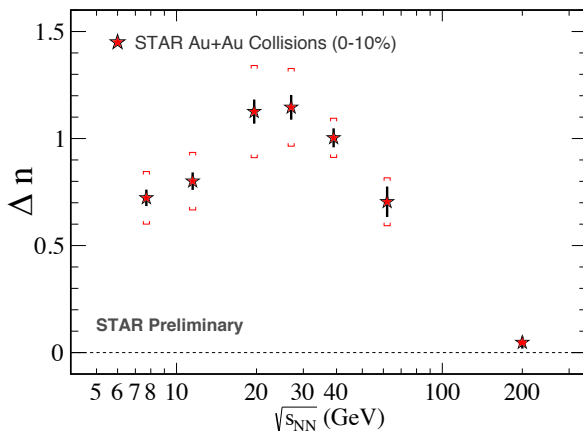


Non-monotonic structures observed by STAR experiment

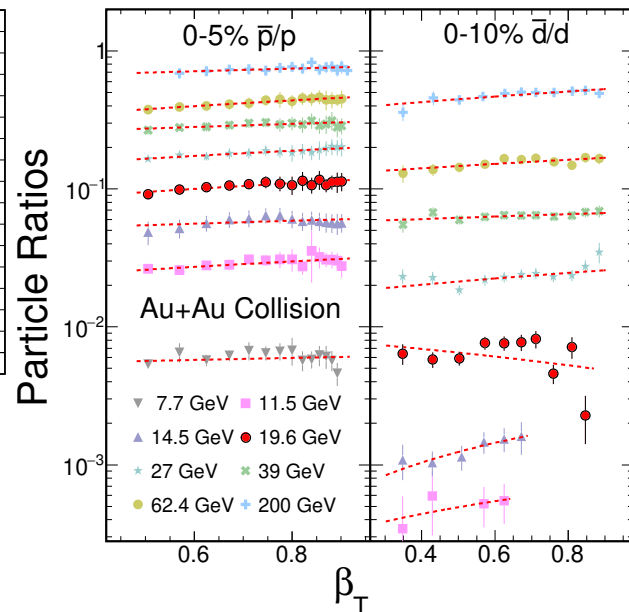
Net-p flu.



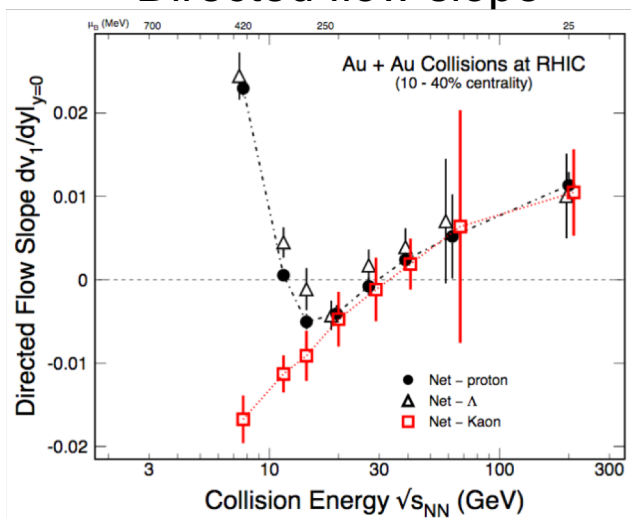
Neutron density flu.



Focusing Eff.



Directed flow slope



Peak and/or dip structures observed at common energy ranges : 20-30 GeV !!

What are the driven physics for those structures ?



Summary

Explore the QCD phase structure with Beam Energy Scan

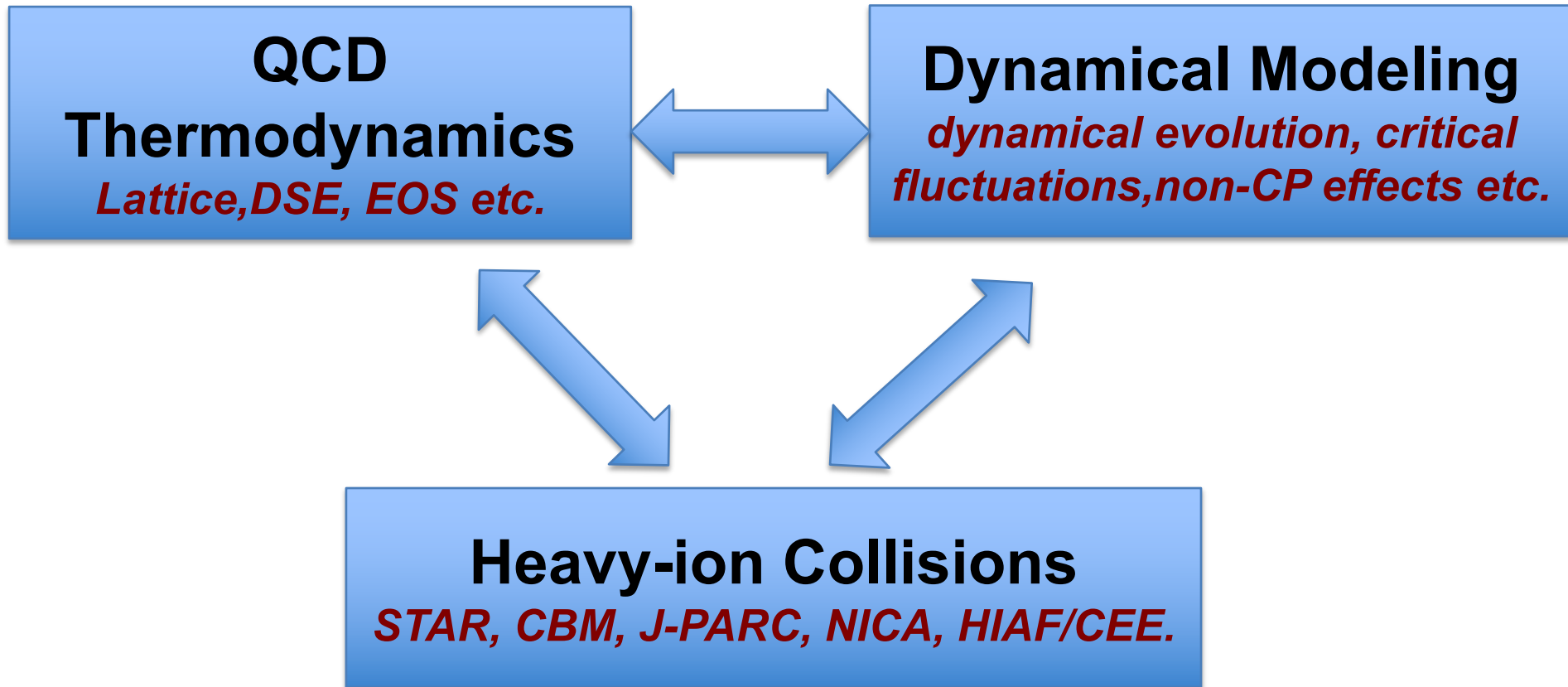
- Energy dependence of net-proton v_1 slope change sign twice and show a minimum around 15 GeV . **Possible signature of 1st order phase transition.**
- Fourth order net-proton fluctuations (C_4/C_2) in central Au+Au collisions shows non-monotonic energy dependence, with a minimum around 20-30 GeV. **Hint of entering the critical region.**
- Neutron density fluctuations in 0-10% central Au+Au collisions shows non-monotonic energy dependence with a peak around 20-30 GeV.
Hint of entering the critical region.
- In BES-II, we can study the QCD phase structure with high precision at $\sqrt{s_{NN}} = 7.7-19.6$ GeV (collider mode) and 3-7.7 GeV (Fix-target mode)

Stay tuned for RHIC BES-II !!



More is Different

Need efforts from both theorist and experimentalist !



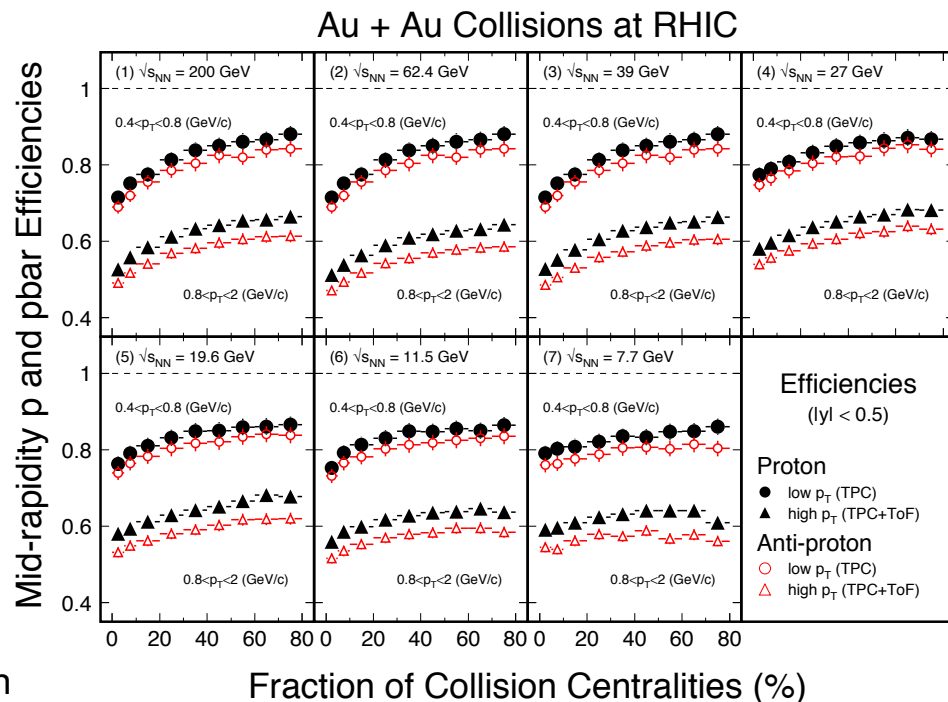
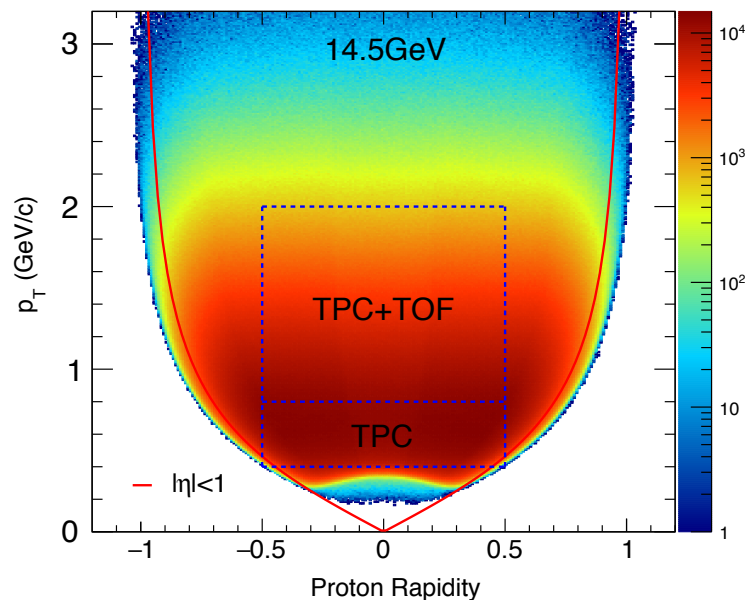


Thank you !

(Anti-) Proton Acceptance and Efficiencies

TOF PID can extend the phase space coverage

$|y| < 0.5$, $0.4 < p_T < 0.8$ (TPC PID only)
 $0.8 < p_T < 2$ (TPC+TOF PID)



Doubled the accepted number of proton/anti-proton

- Sufficiently large acceptance is important for fluctuation analysis and critical point search.
- Efficiency : Proton > Anti-proton, Low p_T > High p_T , low energy > High Energy, Peripheral > Central



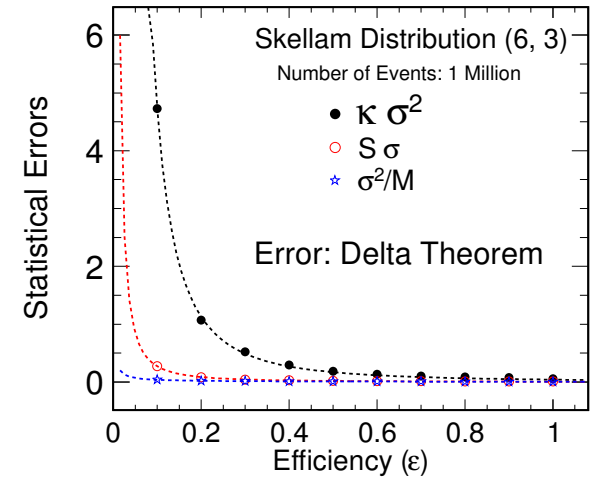
Efficiency Correlation and Error Estimation

We provide a unified description of efficiency correction and error estimation for higher moments analysis in heavy-ion collisions.

X. Luo, arXiv: 1410.3914

Error Estimation: MC simulation

$$\begin{aligned}
 F_{r_1, r_2}(N_p, N_{\bar{p}}) &= F_{r_1, r_2}(N_{p_1} + N_{p_2}, N_{\bar{p}_1} + N_{\bar{p}_2}) \\
 &= \sum_{i_1=0}^{r_1} \sum_{i_2=0}^{r_2} s_1(r_1, i_1) s_1(r_2, i_2) \langle (N_{p_1} + N_{p_2})^{i_1} (N_{\bar{p}_1} + N_{\bar{p}_2})^{i_2} \rangle \\
 &= \sum_{i_1=0}^{r_1} \sum_{i_2=0}^{r_2} s_1(r_1, i_1) s_1(r_2, i_2) \langle \sum_{s=0}^{i_1} \binom{i_1}{s} N_{p_1}^{i_1-s} N_{p_2}^s \sum_{t=0}^{i_2} \binom{i_2}{t} N_{\bar{p}_1}^{i_2-t} N_{\bar{p}_2}^t \rangle \\
 &= \sum_{i_1=0}^{r_1} \sum_{i_2=0}^{r_2} \sum_{s=0}^{i_1} \sum_{t=0}^{i_2} s_1(r_1, i_1) s_1(r_2, i_2) \binom{i_1}{s} \binom{i_2}{t} \langle N_{p_1}^{i_1-s} N_{p_2}^s N_{\bar{p}_1}^{i_2-t} N_{\bar{p}_2}^t \rangle \\
 &= \sum_{i_1=0}^{r_1} \sum_{i_2=0}^{r_2} \sum_{s=0}^{i_1} \sum_{t=0}^{i_2} \sum_{u=0}^{i_1-s} \sum_{v=0}^s \sum_{j=0}^{i_2-t} \sum_{k=0}^t s_1(r_1, i_1) s_1(r_2, i_2) \binom{i_1}{s} \binom{i_2}{t} \\
 &\quad \times s_2(i_1-s, u) s_2(s, v) s_2(i_2-t, j) s_2(t, k) \times F_{u, v, j, k}(N_{p_1}, N_{p_2}, N_{\bar{p}_1}, N_{\bar{p}_2})
 \end{aligned}$$



Fitting formula: $f(\epsilon) = \frac{1}{\sqrt{n}} \frac{a}{\epsilon^b}$

We can express the moments and cumulants in terms of the factorial moments, which can be easily efficiency corrected.

$$F_{u, v, j, k}(N_{p_1}, N_{p_2}, N_{\bar{p}_1}, N_{\bar{p}_2}) = \frac{f_{u, v, j, k}(n_{p_1}, n_{p_2}, n_{\bar{p}_1}, n_{\bar{p}_2})}{(\epsilon_{p_1})^u (\epsilon_{p_2})^v (\epsilon_{\bar{p}_1})^j (\epsilon_{\bar{p}_2})^k}$$

One can also see:

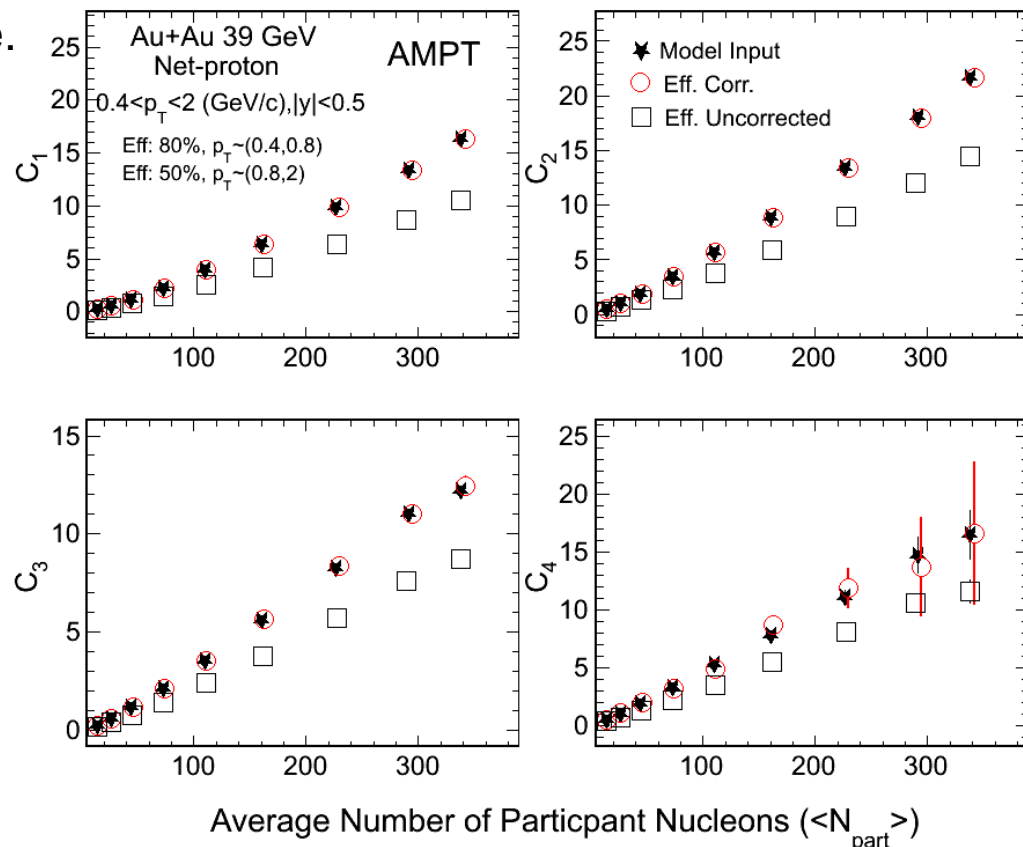
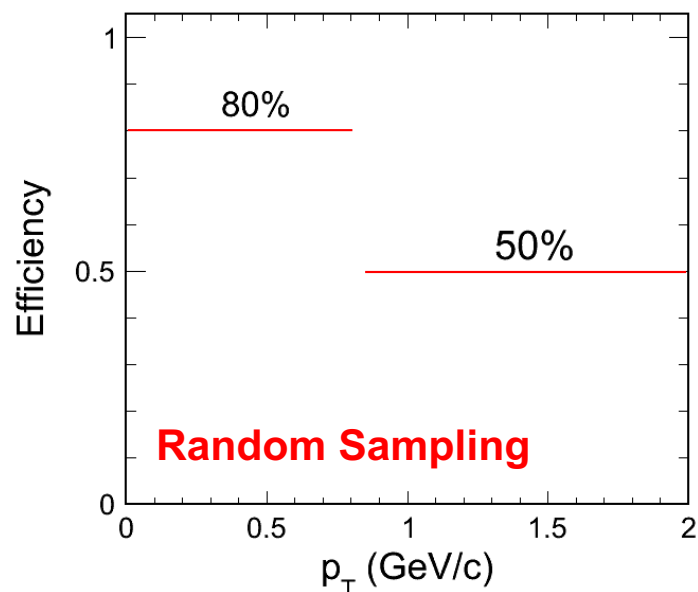
A. Bzdak and V. Koch,
PRC91, 027901(2015),
PRC86, 044904(2012).

For other analysis techniques, see: STAR, PRL112, 032302 (2014); PRL113, 092301 (2014).

Verification of the Eff. Correction with Model

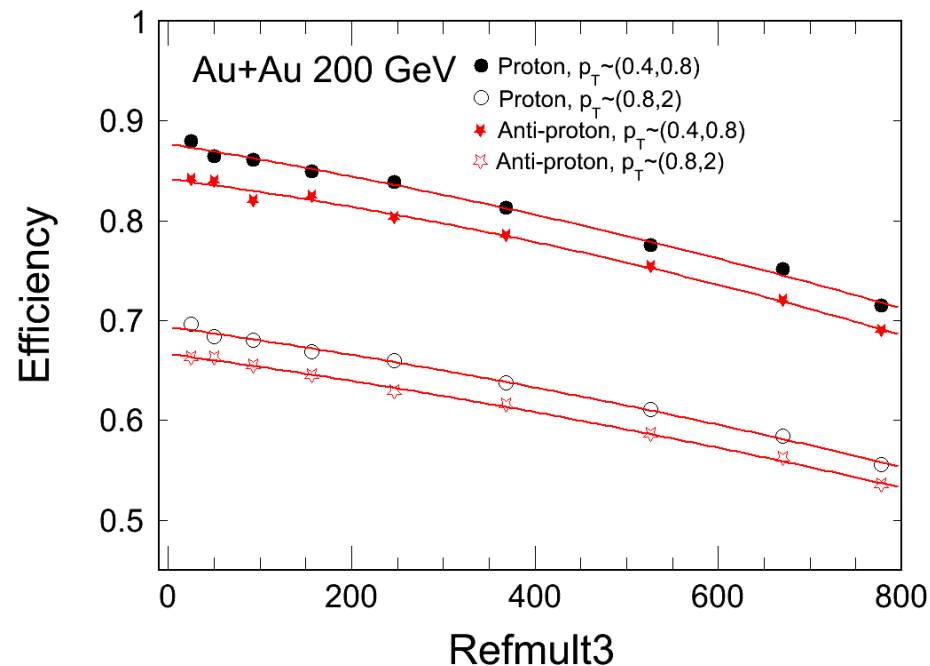
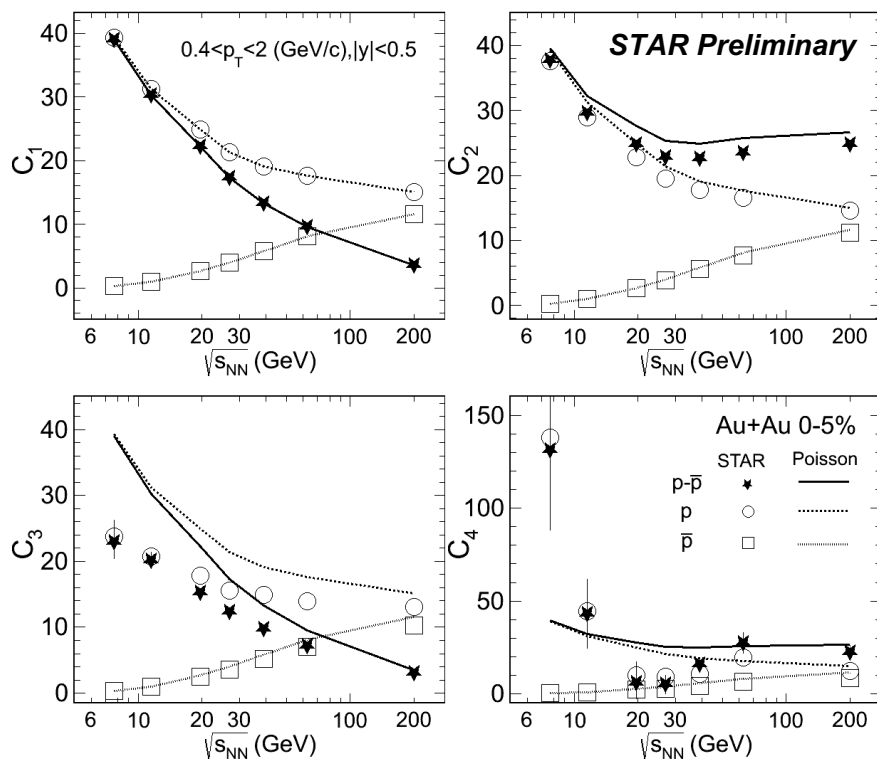
AMPT model: Au+Au 39 GeV.

Set different efficiency for two p_T range.
(0.4, 0.8): 80%, (0.8, 2): 50% for p and pbar.



1. The eff. corrected results match the model inputs very well, which indicate the efficiency correction method works well.
2. The error estimation for eff. corrected results are based on the Delta theorem.

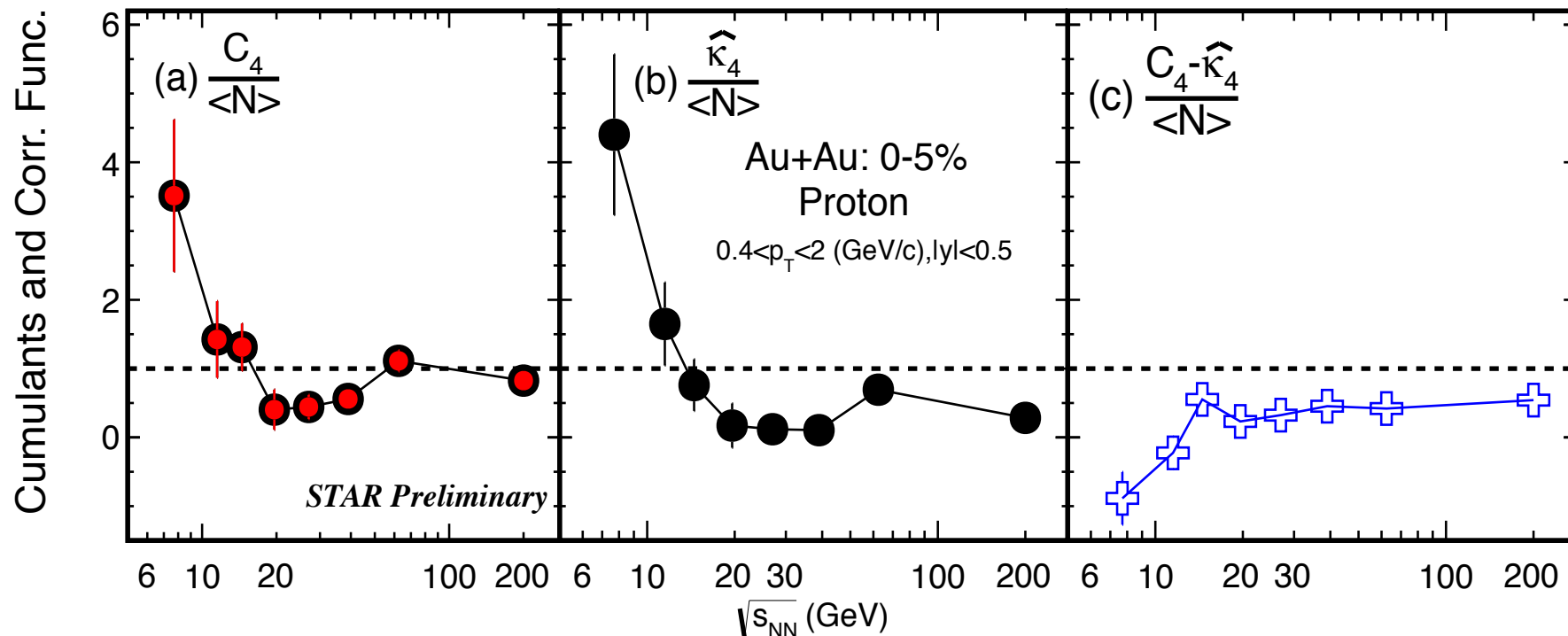
Net-Proton Cumulants vs. Collision Energy



- Net-proton number is dominated by protons at low energies and increases when energy decreases. (Interplay between baryon stopping and pair production)
- The higher the order of cumulants the larger deviations from Poisson expectations for net-proton and proton.



Proton Cumulants and Correlation Functions



Four-particle correlation dominated the non-monotonic behavior observed in fourth order net-proton fluctuations.

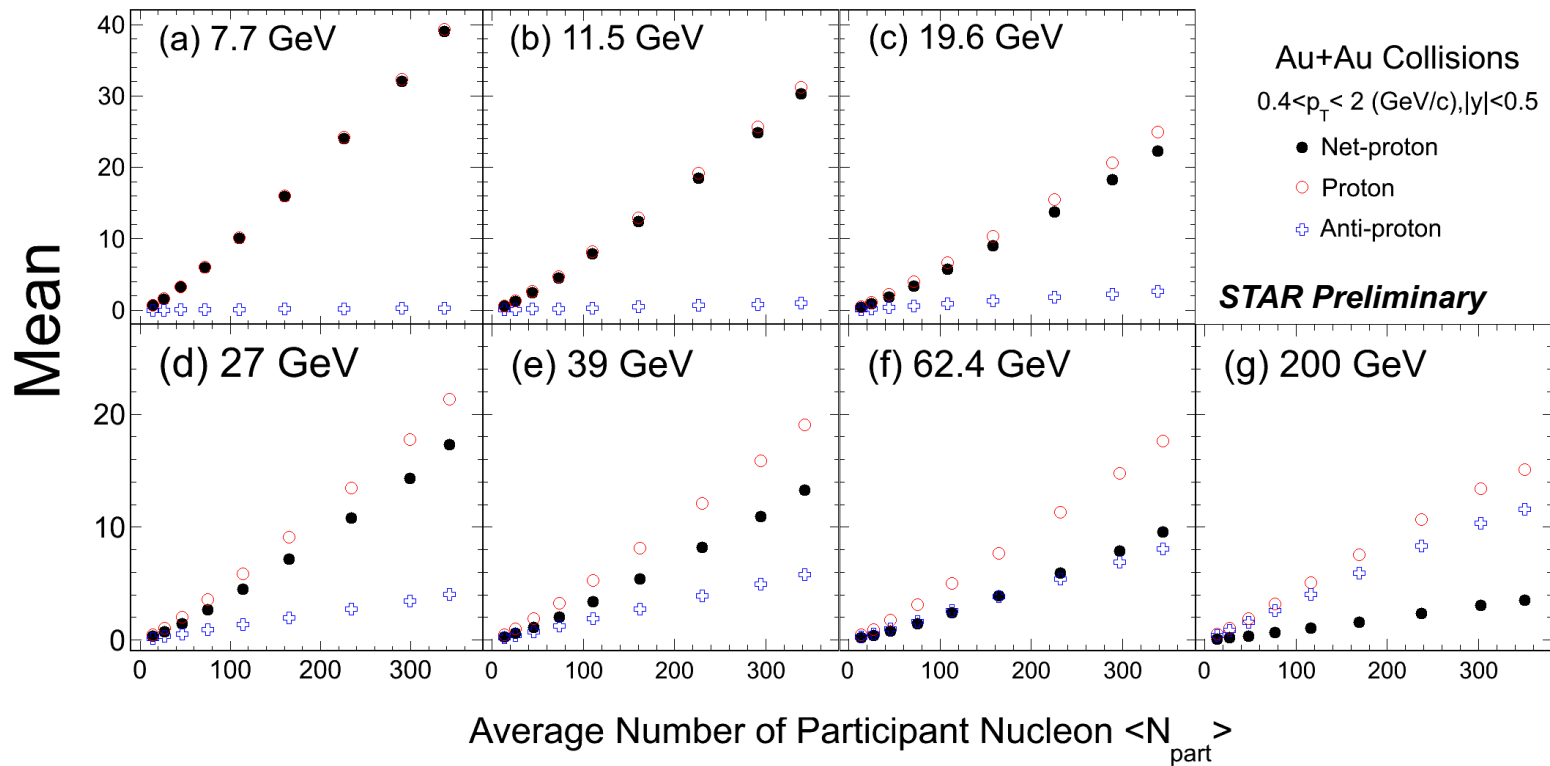
$$C_2 = \langle N \rangle + \hat{k}_2$$

$$C_3 = \langle N \rangle + 3\hat{k}_2 + \hat{k}_3$$

$$C_4 = \langle N \rangle + 7\hat{k}_2 + 6\hat{k}_3 + \hat{k}_4$$

$\hat{k}_2, \hat{k}_3, \hat{k}_4$: 2,3,4-particle correlation function

Results: Mean Net-p, p and pbar



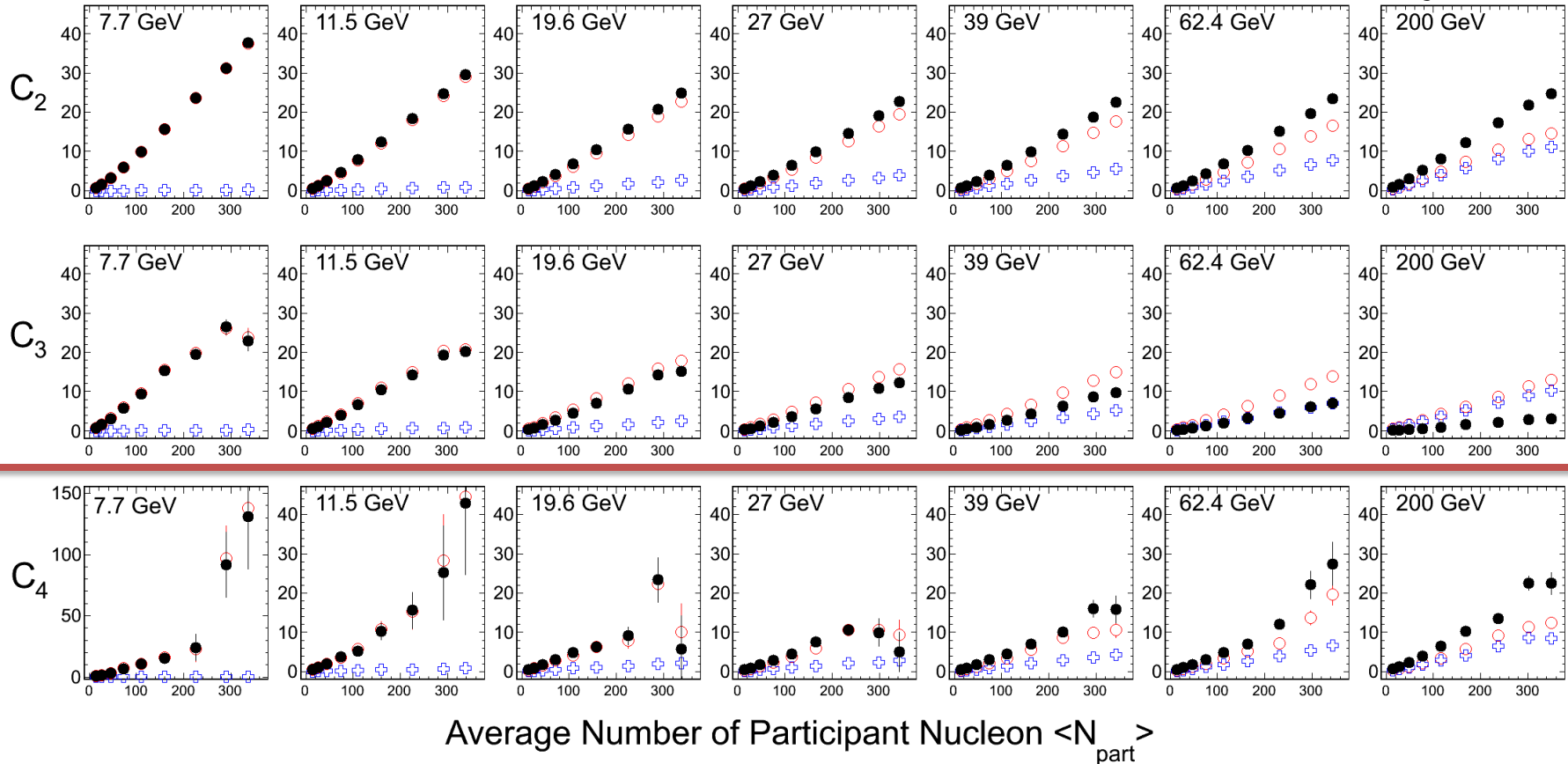
- Mean Net-proton, proton and anti-proton number increase with $\langle N_{part} \rangle$
- Net-proton number is dominated by protons at low energies and increases when energy decreases.
(Interplay between baryon stopping and pair production)

Higher Order Cumulants for Net-p, p, pbar

Au+Au Collisions $0.4 < p_T < 2$ (GeV/c), $|y| < 0.5$

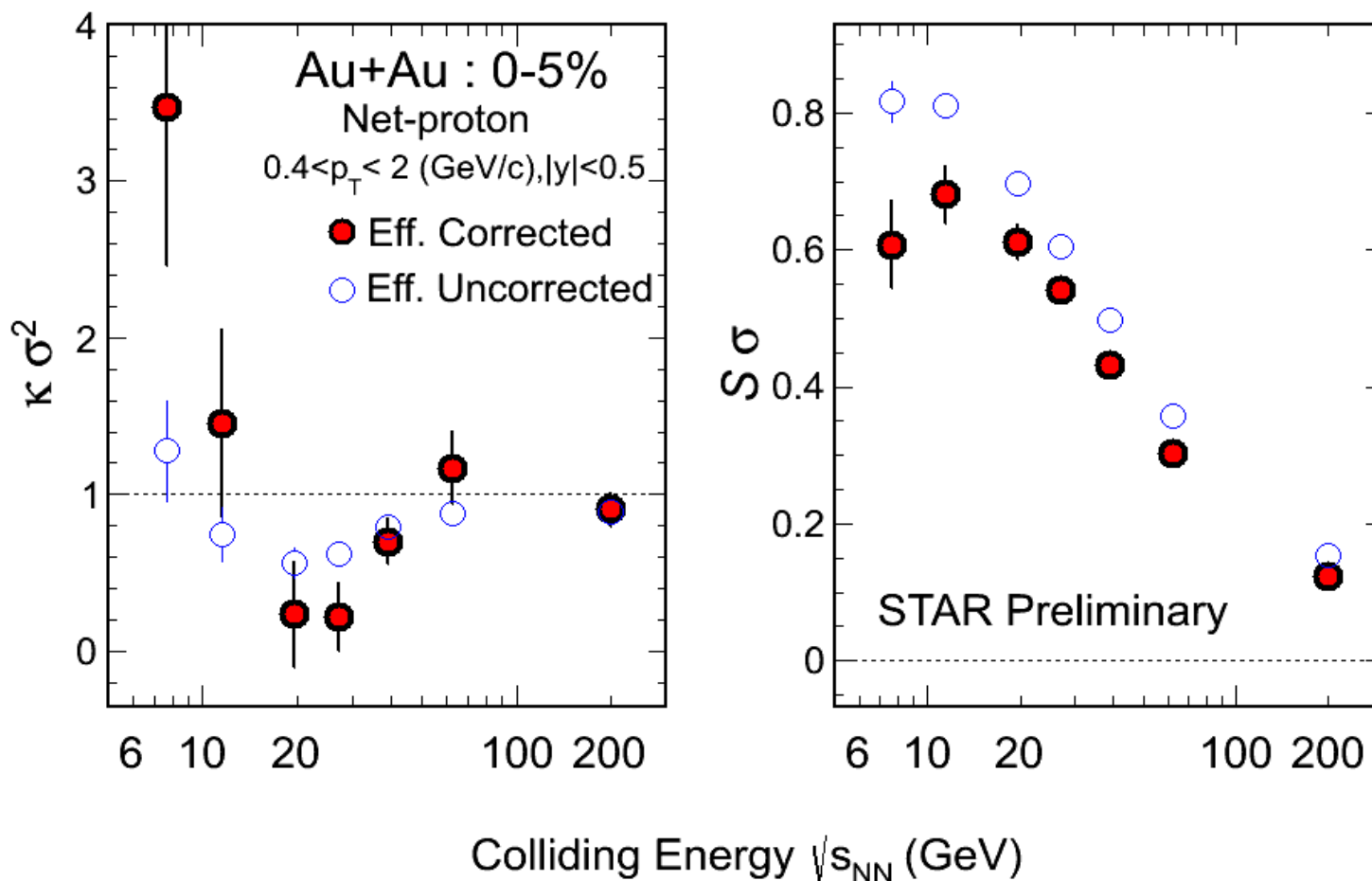
● Net-proton ○ Proton + Anti-proton

STAR Preliminary



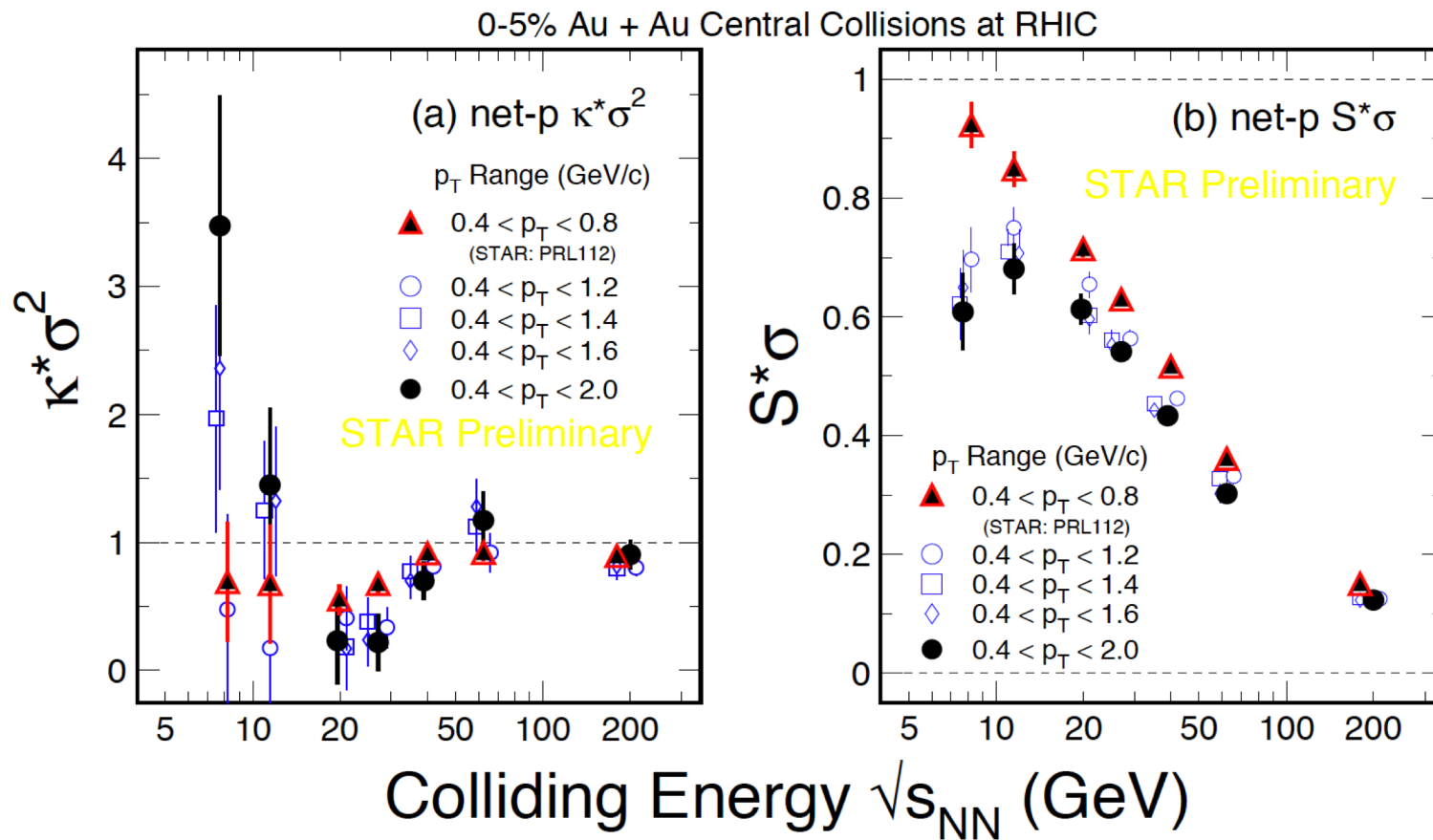
- In general, cumulants of Net-p, p and pbar are increasing with $\langle N_{part} \rangle$.
- The cumulants of net-proton distributions closely follow the proton cumulants when the colliding energy is decreasing.

Eff. Corrected and Eff. Uncorrected



Efficiency corrections are important not only for the values in the higher moments analysis, but also the statistical errors. $error \propto O(\sigma^n / \epsilon^\alpha)$

Acceptance Study (I): p_T

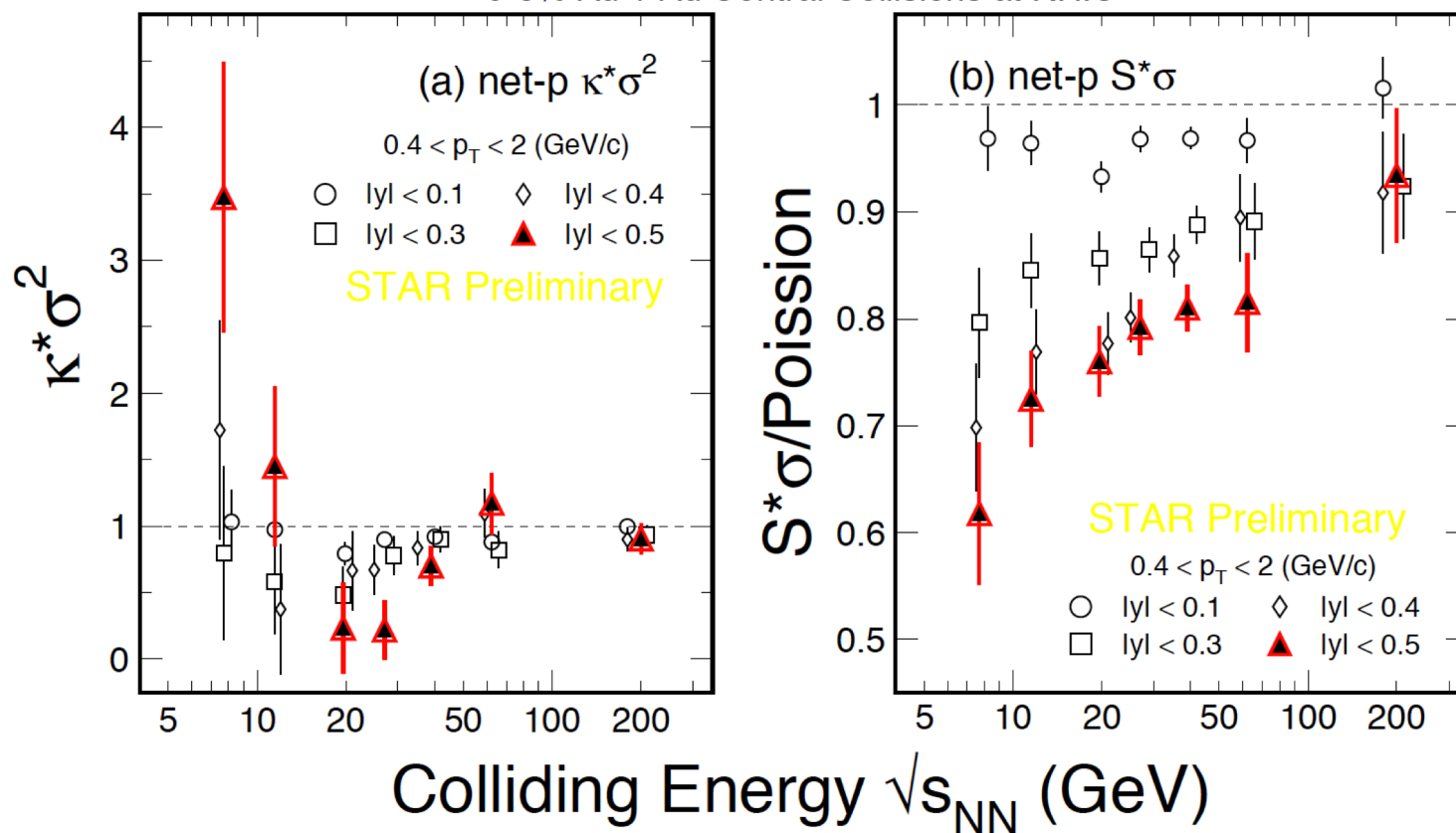


- $K^* \sigma^2$: the energy dependence tends to be more pronounced with wider p_T acceptance, relative to published results.
- $S^* \sigma$: the values are smaller for wider p_T acceptance.



Acceptance Study (II): Rapidity

0-5% Au + Au Central Collisions at RHIC



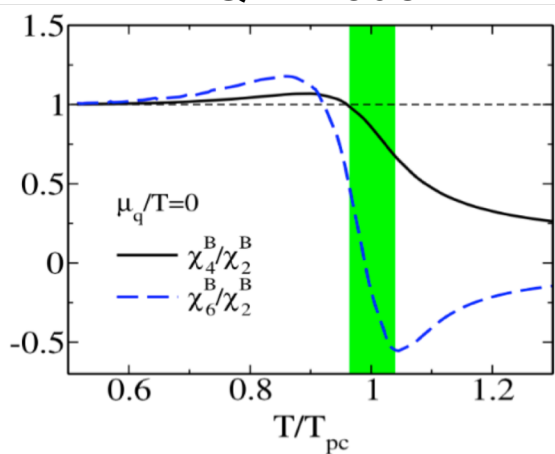
- The smaller the rapidity window the closer to the Poisson values.
- The acceptance needs to be large enough to capture the dynamical fluctuations. The related systematic errors should be carefully addressed.

The sixth-order (C_6) fluctuation measurement

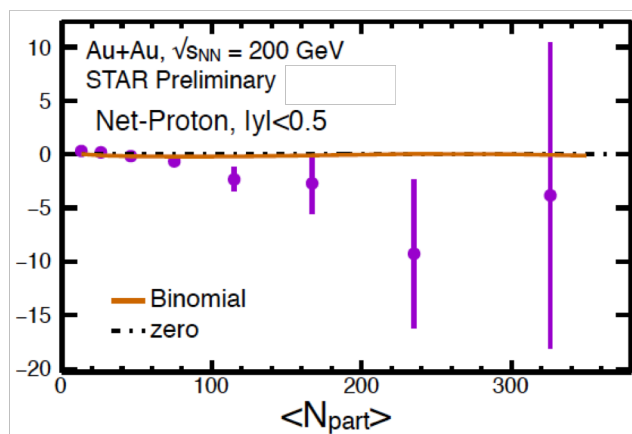
- The sixth-order cumulants of net-charge and net-baryon distributions are predicted to be negative if the chemical freeze-out is close enough to the phase transition.
- In Au+Au collisions at 200 GeV, negative values are observed in net-proton C_6/C_2 systematically from peripheral to central. Results of net-charge C_6/C_2 are consistent with zero within large statistical errors.

$$error(C_n / C_2) \propto \frac{\sigma^{n-2}}{\sqrt{N_{evts}}}$$

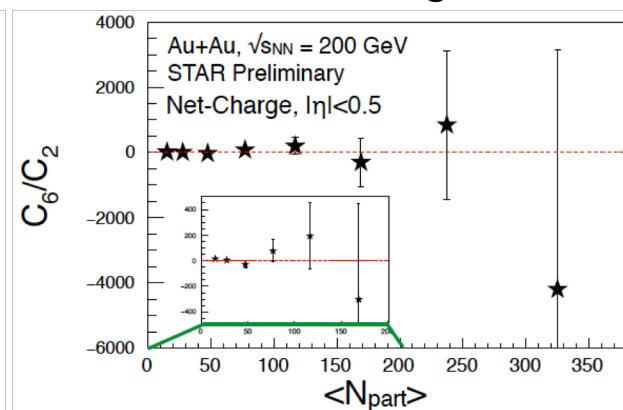
PQM model



Net-proton



Net-charge



B. Friman et al., Eur. Phys. J. C 71 (2011) 1694