Conserved charge fluctuations and critical behavior from lattice QCD at $\mu_B = 0$

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- **Critical behavior** in the limit of vanishing light quark masses

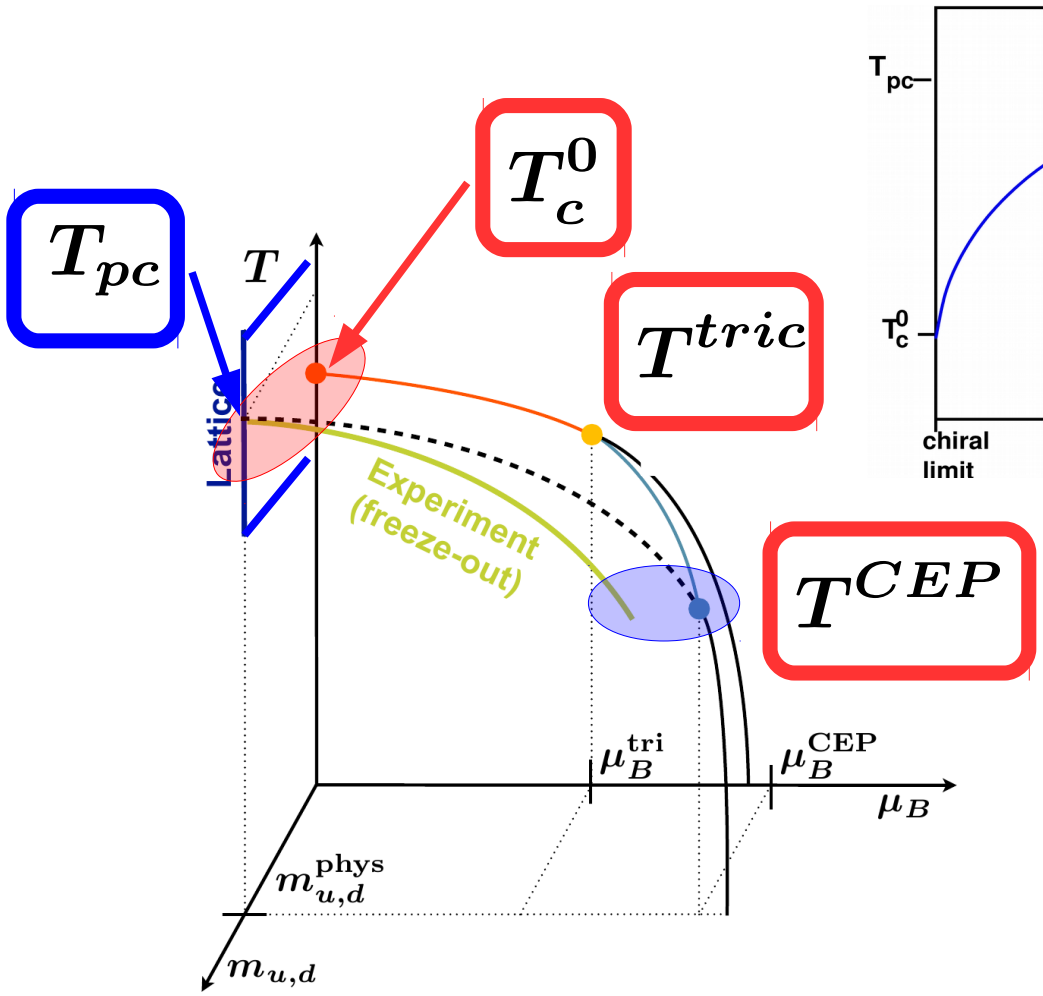
  --- Finite size scaling and chiral limit; **chiral PHASE TRANSITION**

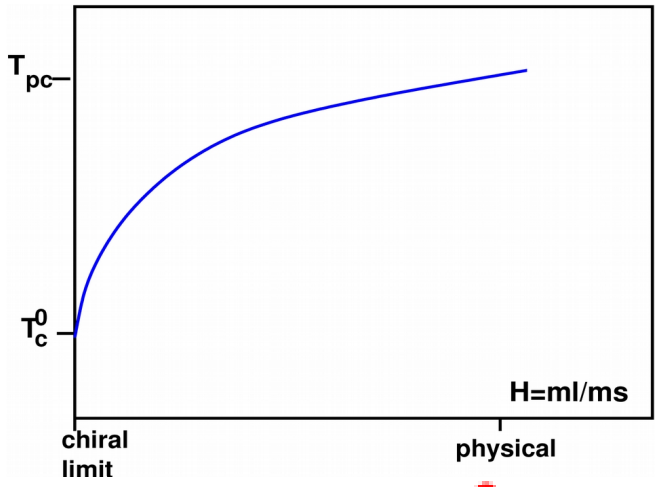
- Higher order cumulants of **conserved charge fluctuations**

  --- making contact to fluctuation data from **heavy ion experiments**

supported by BMBF-project ALICE-Germany

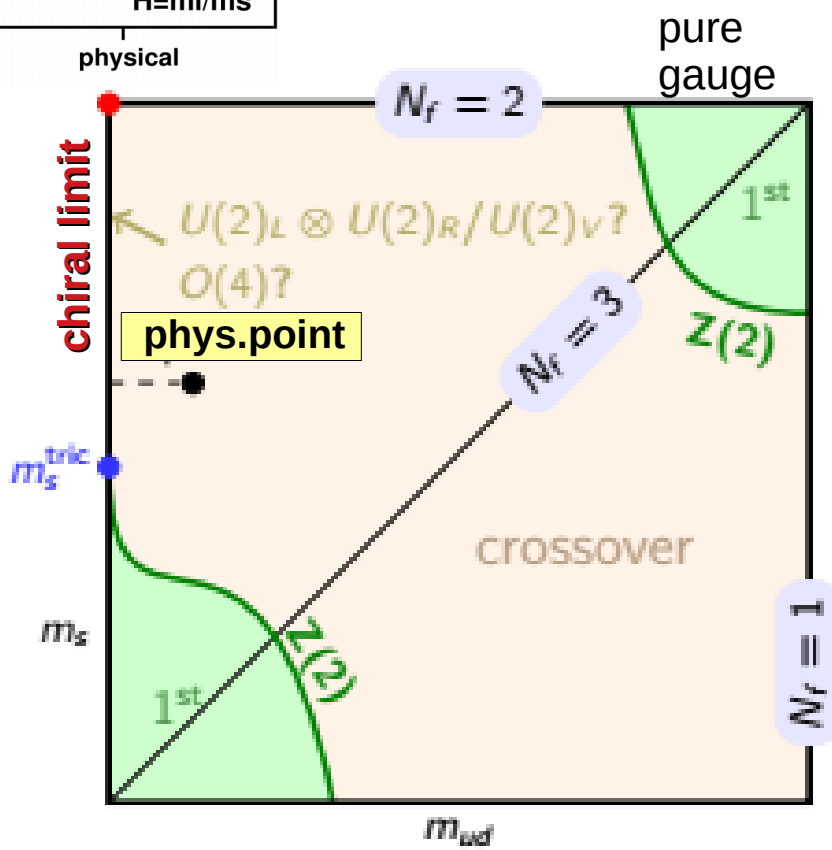
Phases of strong-interaction matter





this talk focuses on

$\mu_B = 0$



– determination of the critical temperature

(and the order of the transition)

→ see talk by Anirban Lahiri (tomorrow)

Critical behavior in QCD

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

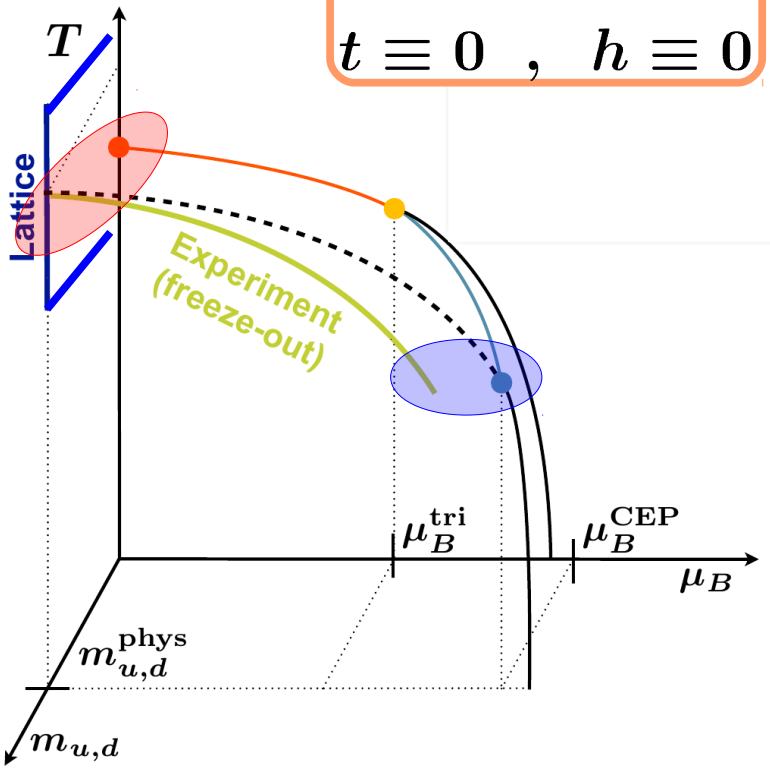
singular regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

critical line: $t \equiv 0 \;\;,\;\; h \equiv 0$

$$t \sim \frac{T - T_c}{T_c} + \kappa_2 \left(\frac{\mu_q}{T}\right)^2 \;\;,\;\; h \sim \frac{m_q}{T_c}$$

"thermal" coupling "magnetic" coupling



here only: $\mu_q \equiv 0$

question: Where is the chiral PHASE TRANSITION for $m_u = m_d = 0$ located? What is its influence on observables at the pseudo-critical temperature?

Critical behavior in QCD

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$$t \sim \frac{T - T_c}{T_c} + \kappa_2 \left(\frac{\mu_q}{T}\right)^2$$

$$h \sim \frac{m_q}{T_c}$$

Pseudo-critical temperatures

response functions
2nd order cumulants

- magnetic

$$\frac{\partial^2 \ln Z}{\partial h^2} \sim \left(\frac{m_l}{T_c}\right)^{1/\delta - 1} \sim -0.79$$

mixed

$$\frac{\partial^2 \ln Z}{\partial h\, \partial t} \sim \left(\frac{m_l}{T_c}\right)^{(\beta-1)/\beta\delta} \sim -0.34$$

thermal

$$\frac{\partial^2 \ln Z}{\partial t^2} \sim \left(\frac{m_l}{T_c}\right)^{-\alpha/\beta\delta} \sim +0.11$$

O(4) critical exponents

$\alpha = -0.21$

$\beta = 0.38$

$\delta = 4.82$

divergence: strong moderate none

Critical behavior in QCD

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

singular regular

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Pseudo-critical temperatures

response functions
2nd order cumulants

- magnetic

magnetic	mixed	thermal
$\frac{\partial^2 \ln Z}{\partial h^2}$	$\frac{\partial^2 \ln Z}{\partial h\, \partial t}$	~~$\frac{\partial^2 \ln Z}{\partial t^2}$~~ $\frac{\partial^3 \ln Z}{\partial t^3}$
$\sim \left(\frac{m_l}{T_c}\right)^{1/\delta - 1}$	$\sim \left(\frac{m_l}{T_c}\right)^{(\beta-1)/\beta\delta}$	$\sim \left(\frac{m_l}{T_c}\right)^{-(\alpha+1)/\beta\delta}$
~ -0.79	~ -0.34	~ -0.43
strong	moderate	moderate

divergence:

O(4) critical exponents

$\alpha = -0.21$

$\beta = 0.38$

$\delta = 4.82$

Murphy's law:

magnetic susceptibilities are best to establish
critical behavior in QCD – strong singularity,
small regular admixture from terms

thermal susceptibilities are accessible in heavy ion experiments

– no singularity in 2nd order response fctn
dominated by regular contributions

– need 3rd order thermal derivatives
~ 6th order derivatives in terms of
chemical potentials

Chiral observables in QCD

– chiral condensate: $\langle\bar{\psi}\psi\rangle_q = \dfrac{\partial P/T}{\partial m_q/T}$, $\langle\bar{\psi}\psi\rangle_l = (\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d)/2$

– chiral order parameter: $M = \dfrac{2}{f_K^4}\left[m_s\langle\bar{\psi}\psi\rangle_l - m_l\langle\bar{\psi}\psi\rangle_s\right]$

$$m_l = (m_u + m_d)/2$$

– chiral susceptibility: $\chi_M = m_s\left(\dfrac{\partial M}{\partial m_u} + \dfrac{\partial M}{\partial m_d}\right)$ **magnetic**

– mixed chiral susceptibility: $\chi_t = T\dfrac{\partial M}{\partial T}$ **mixed**

– conserved charge fluctuations: $\chi_X = T^2\dfrac{\partial^2 P/T^4}{\partial\mu_X^2}\Big|_{\mu_X=0}$ **thermal**

$$X = B,\ S, \ldots$$

Scaling in the thermodynamic (infinite volume) limit
– approaching the chiral limit –

– order parameter M and its susceptibility

$$M = h^{1/\delta} f_G(z) + f_{sub}(T,H)$$
$$\chi_M = h_0^{-1} h^{1/\delta - 1} f_\chi(z) + \tilde{f}_{sub}(T,H)$$

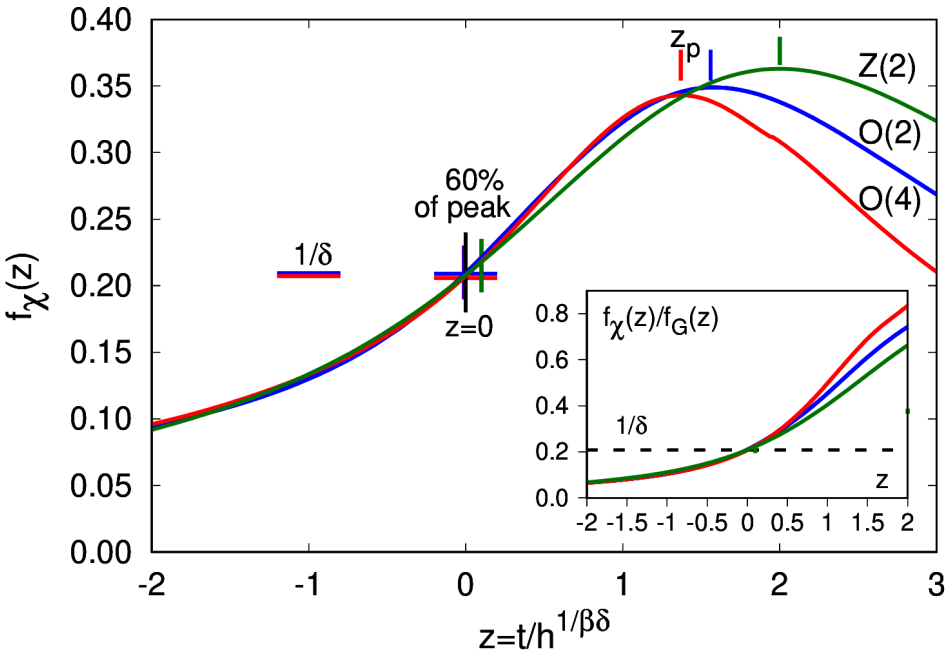
for ANY fixed z:

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z}{z_0} H^{1/\beta\delta}\right) + \text{sub leading}$$

– corrections-to-scaling
– regular terms

some definitions

$$z = \frac{t}{h^{1/\beta\delta}}$$
$$t \equiv \frac{1}{t_0}\frac{T - T_c}{T_c}$$
$$h = \frac{1}{h_0} H$$
$$H \equiv \frac{m_l}{T_c}$$
$$z_0 = h_0^{1/\beta\delta}/t_0$$



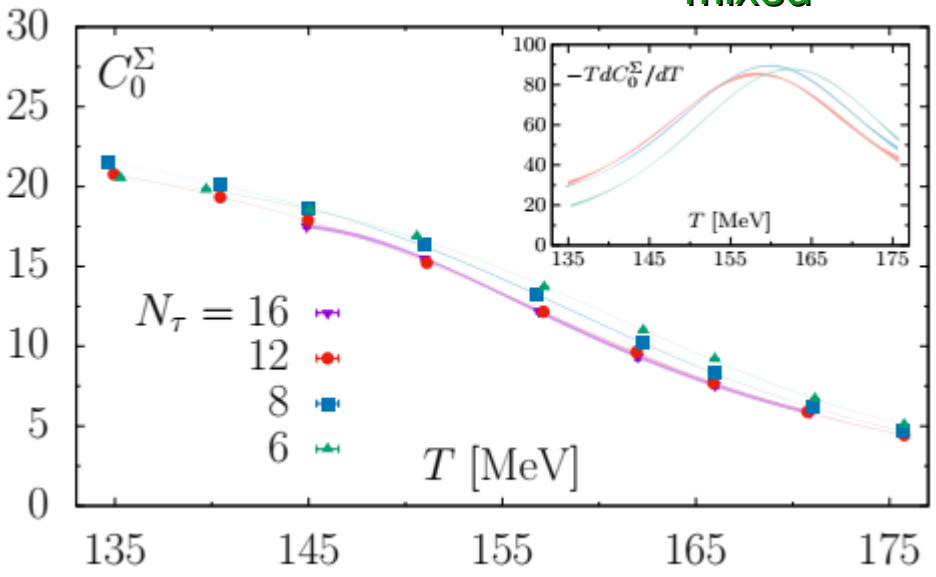
scaling functions $f_\chi(z)$ for some 3-d universality classes:

	δ	$1/\beta\delta$	z_p	z_{60}	z_δ
Z(2)	4.805	0.640	2.00(5)	0.10(1)	0
O(2)	4.780	0.599	1.58(4)	-0.005(9)	0
O(4)	4.824	0.545	1.37(3)	-0.013(7)	0

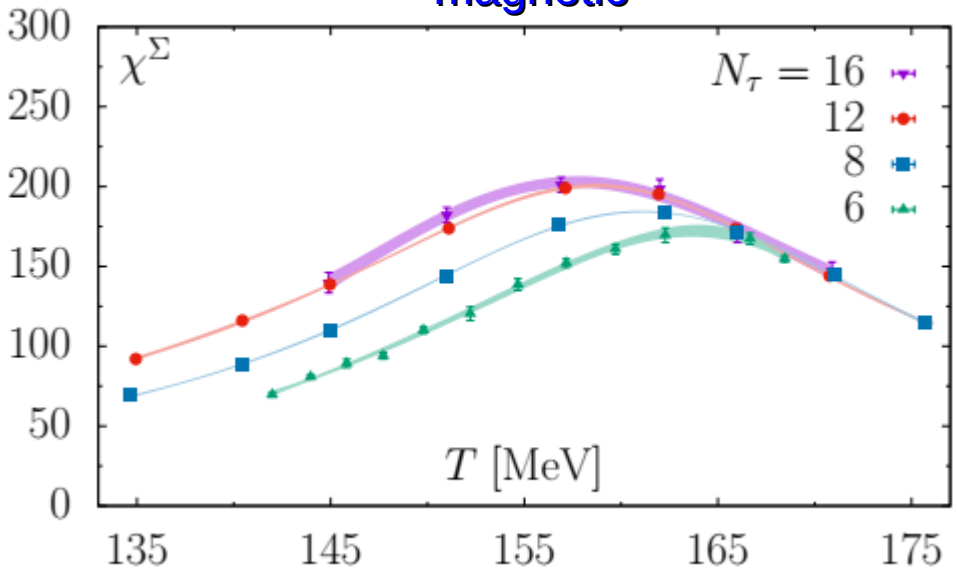
characteristic points on the scaling function $f_\chi(z)$

Pseudo-critical temperatures from chiral observables

mixed

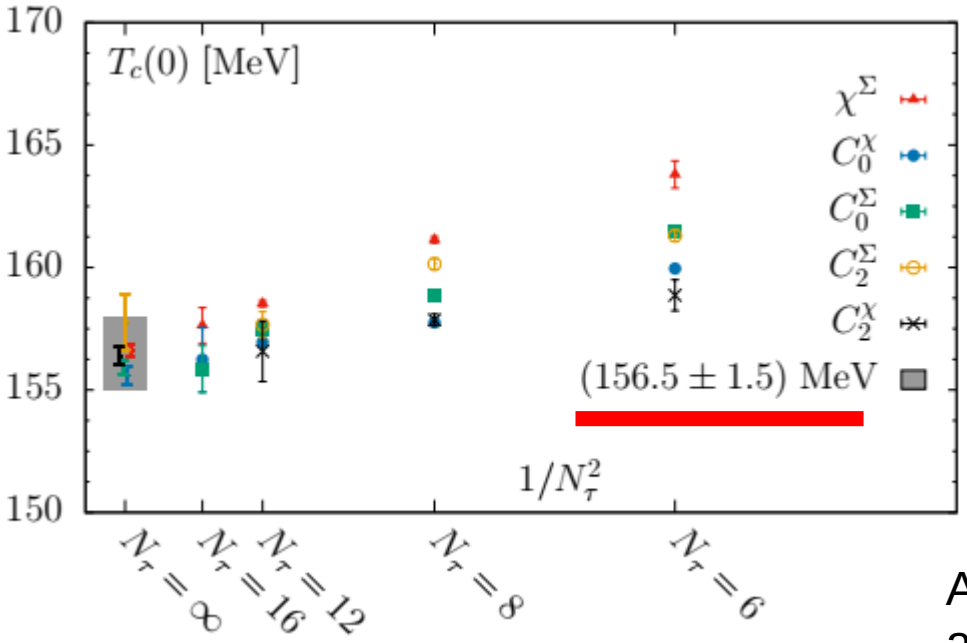


magnetic



physical
light & strange
quark masses;

continuum
extrapolated



$\chi^\Sigma \Leftrightarrow \chi_M$

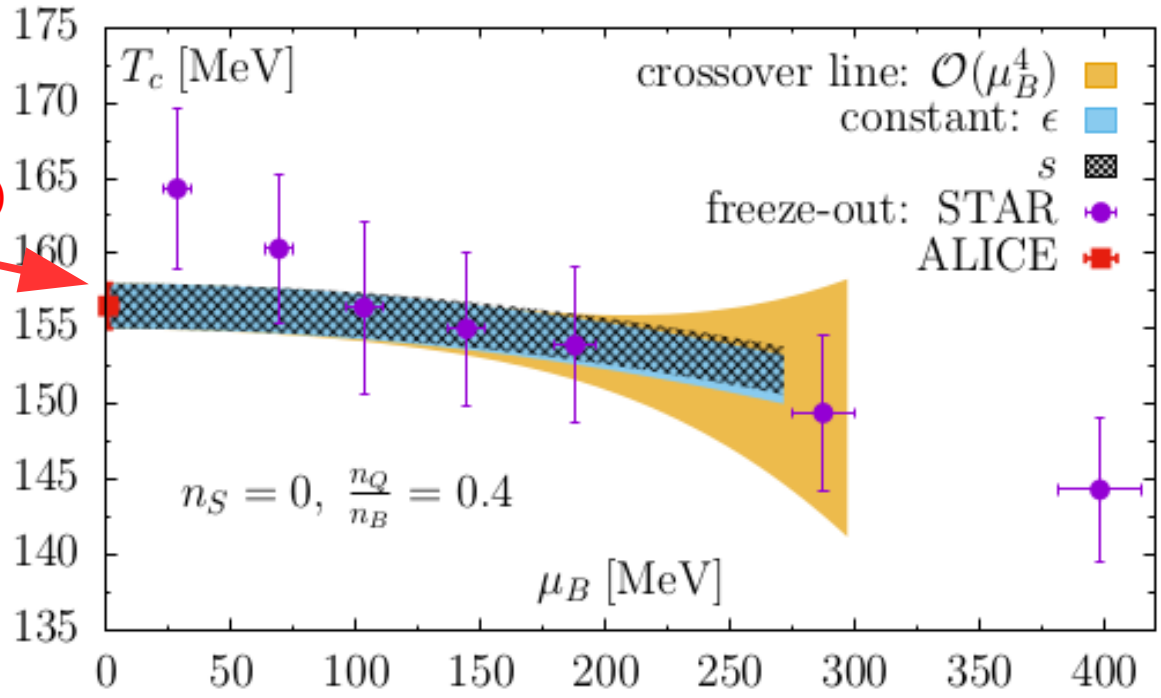
$C_0^\Sigma \Leftrightarrow T(\partial M/\partial T)$

A. Bazavov et al [HotQCD], arXiv:1812.08235

Pseudo-critical (crossover) temperature

$$T_{pc}(\mu_B) = T_{pc}\left(1 + \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 + \ldots\right)$$

A. Andronic et al., Nature 561 (2018) 321



more on

$$\mu_B/T > 0$$

see talk by S. Mukherjee (Wednesday)

$$T_{pc} = (156.5 \pm 1.5)\ \text{MeV}$$

$$\kappa_2 = 0.012(4)$$

A. Bazavov et al [HotQCD], arXiv:1812.08235

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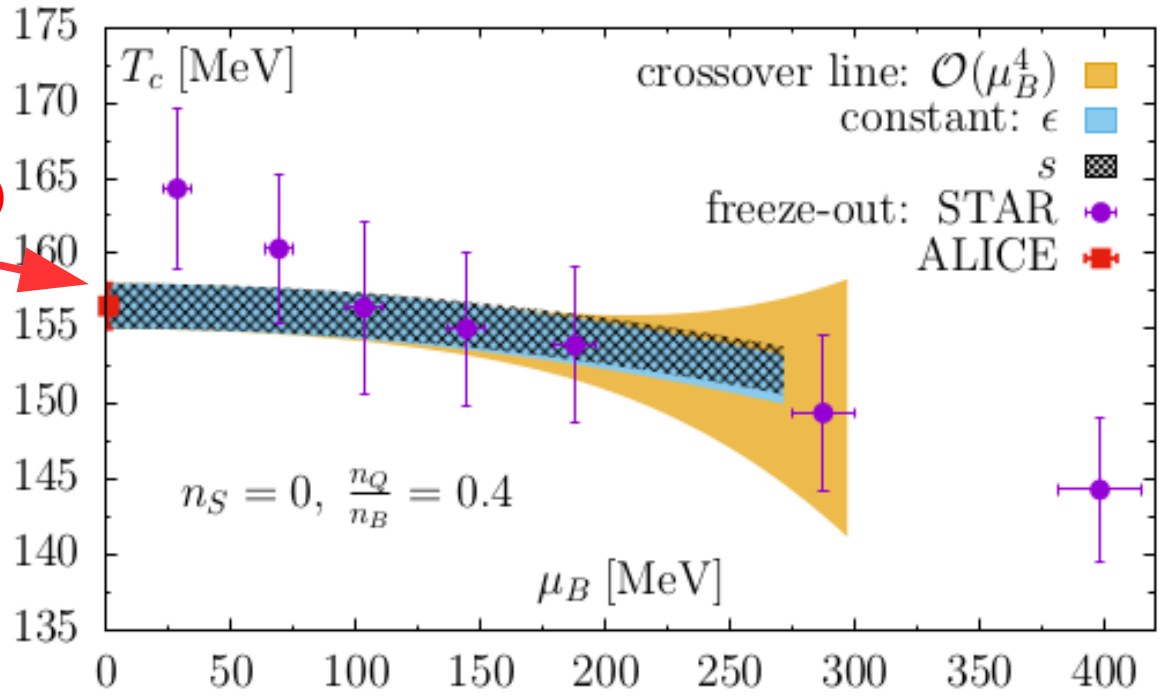

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$$\kappa_2 = 0.012(4)$$

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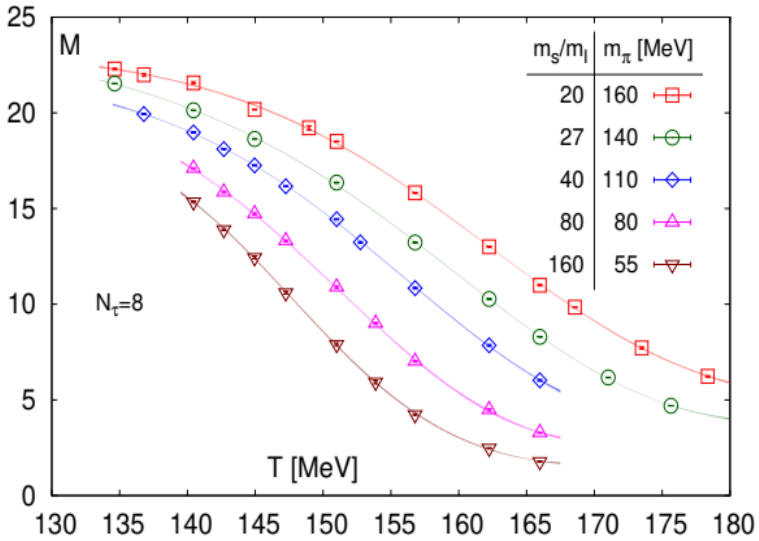
my wish: EMMI task force
ALICE and STAR should come up with a statement on the reason for differences in the freeze-out temperature at

$$\mu_B \simeq 0$$

The Chiral PHASE TRANSITION in (2+1)-flavor QCD

$$M \sim m_s \frac{\partial \ln Z}{\partial m_l}$$



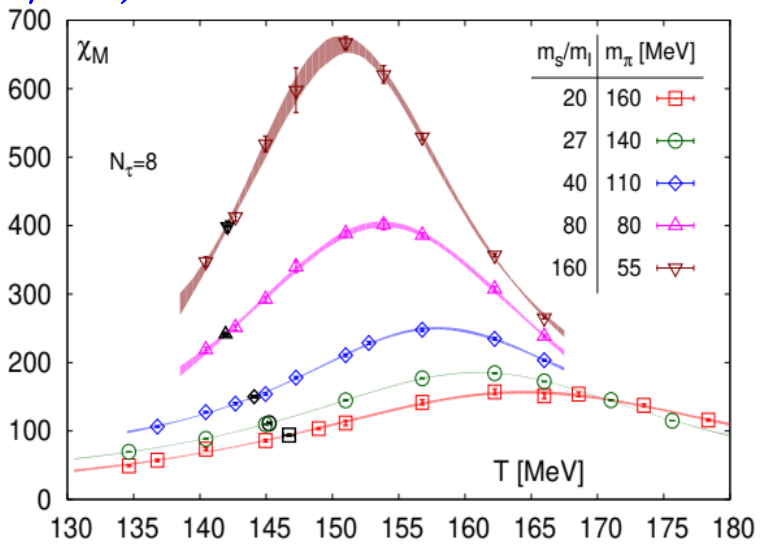
$$m_l = (m_u + m_d)/2 \Rightarrow 0$$

m_s fixed, physical

"magnetic" susceptibility

$$\sim (m_s/m_l)^{0.79}$$

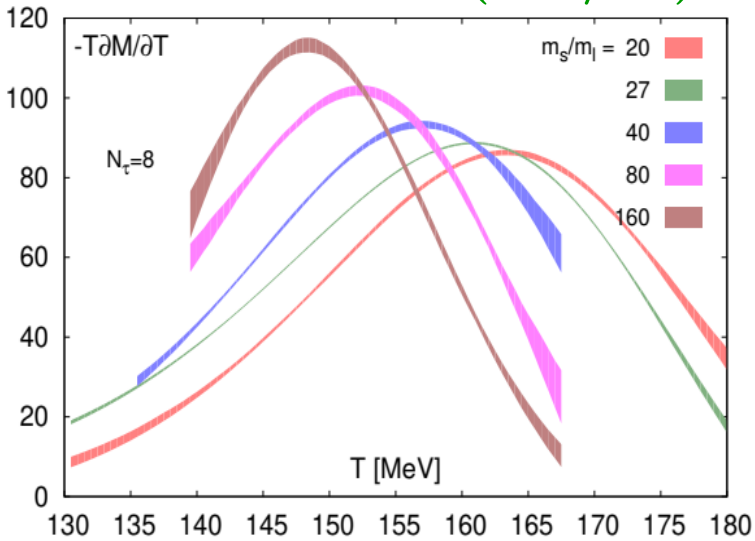
$$(160/27)^{0.79} \sim 4$$



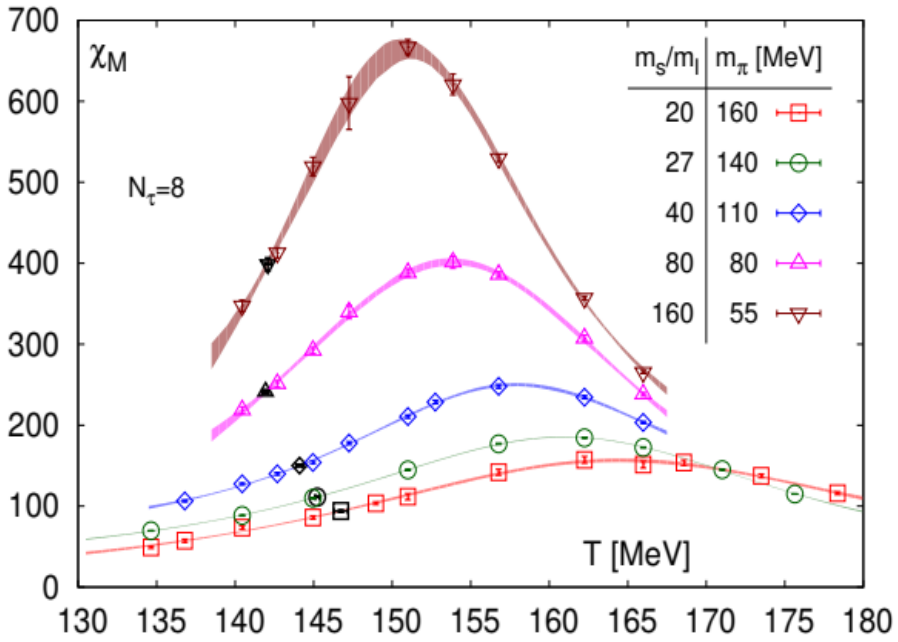
"mixed" susceptibility

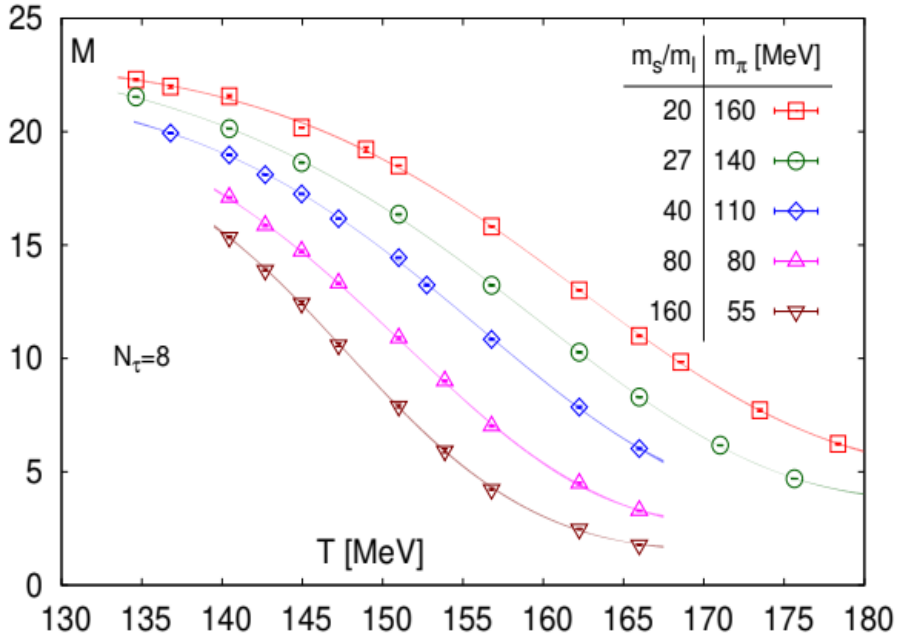
$$\sim (m_s/m_l)^{0.34}$$

$$(160/27)^{0.34} \sim 1.8$$



Chiral PHASE TRANSITION





black symbols: 60% of peak height

a universal ratio at any fixed z

$$\frac{H\chi_M}{M} = \frac{f_\chi(z)}{f_G(z)} + \text{regular}$$

$$= \begin{cases} 1/\delta & , \; z = 0 \\ \sim 0.5 & , \; z = z_p \end{cases} \Rightarrow T_\delta$$

!! **holds approx. for** $m_s/m_l = 27 - 160$

Chiral PHASE TRANSITION

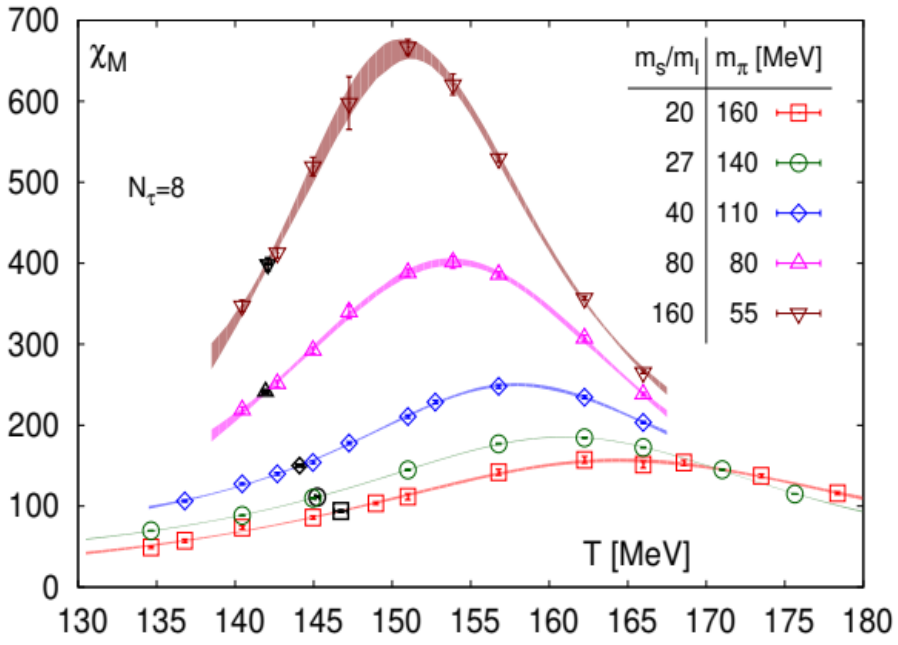
– "**magnetic**" (chiral) susceptibilities provide a strong signal for the chiral PHASE TRANSITION

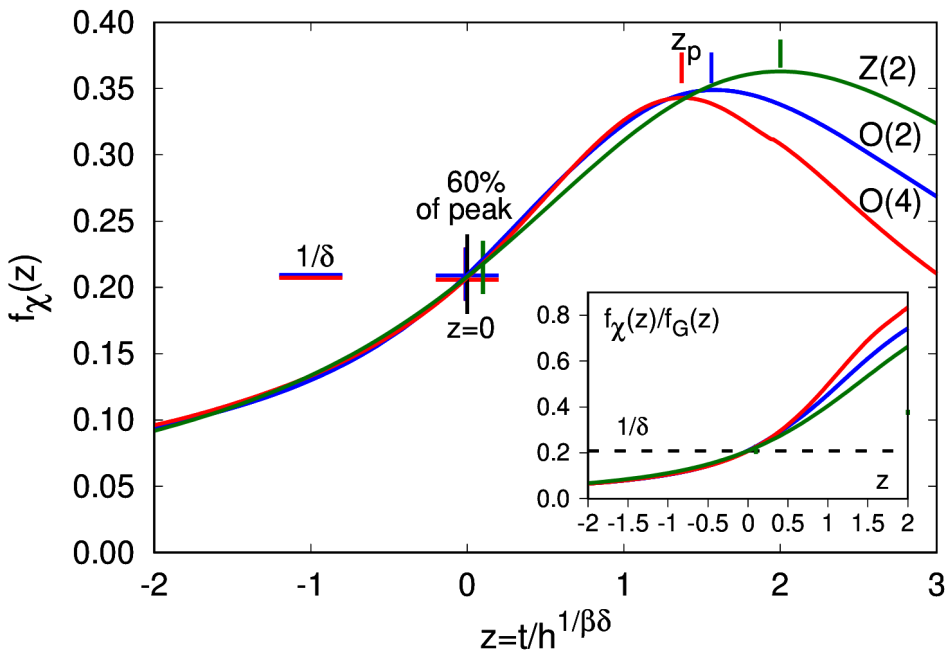
– ratios such as $H\chi_M/M$ are close to the universal value predicted by ratios of scaling functions, even for physical values of the quark masses

– at least in the chiral sector it thus is reasonable to claim

QCD with physical values of the quark masses is sensitive to universal behavior enforced by the chiral PHASE TRANSITION

Chiral PHASE TRANSITION temperature





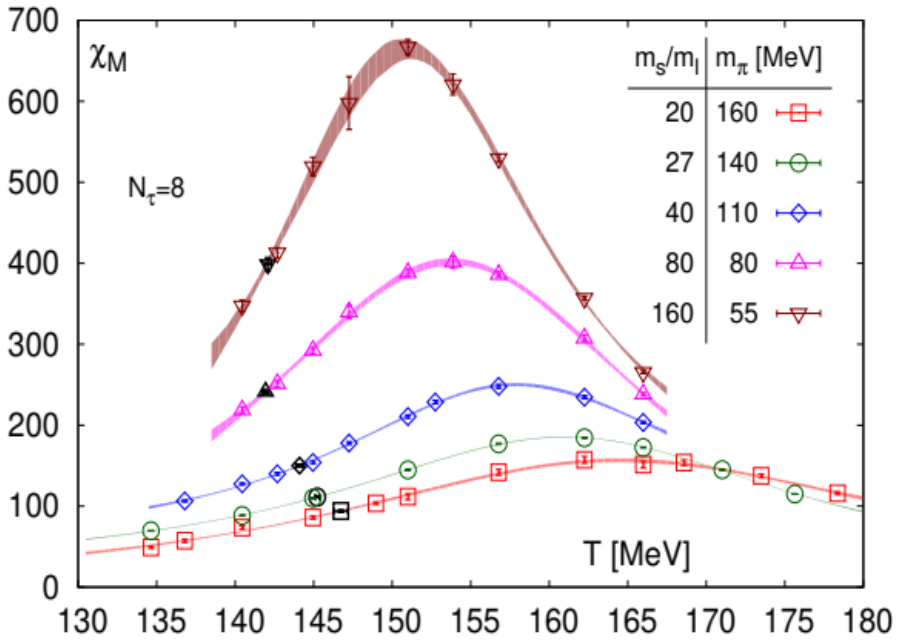
T_{60} temperature at which χ_M reaches 60% of its maximal value

estimators for $\sim T_c^0$

$$\frac{H\chi_M}{M} = \frac{f_\chi(z)}{f_G(z)} + \text{regular}$$

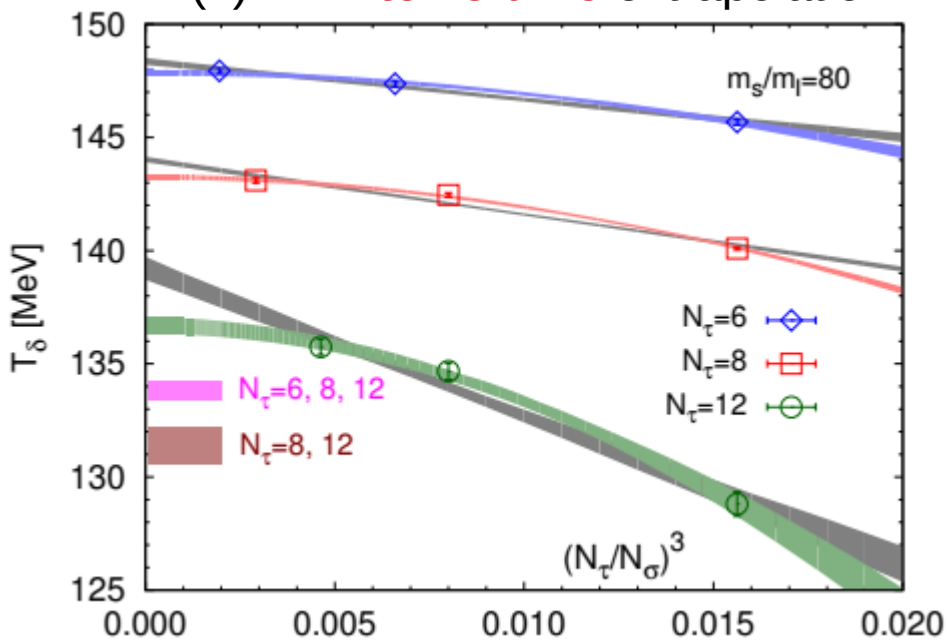
$$= \begin{cases} 1/\delta & , \ z = 0 \\ \sim 0.5 & , \ z = z_p \end{cases} \Rightarrow T_\delta$$

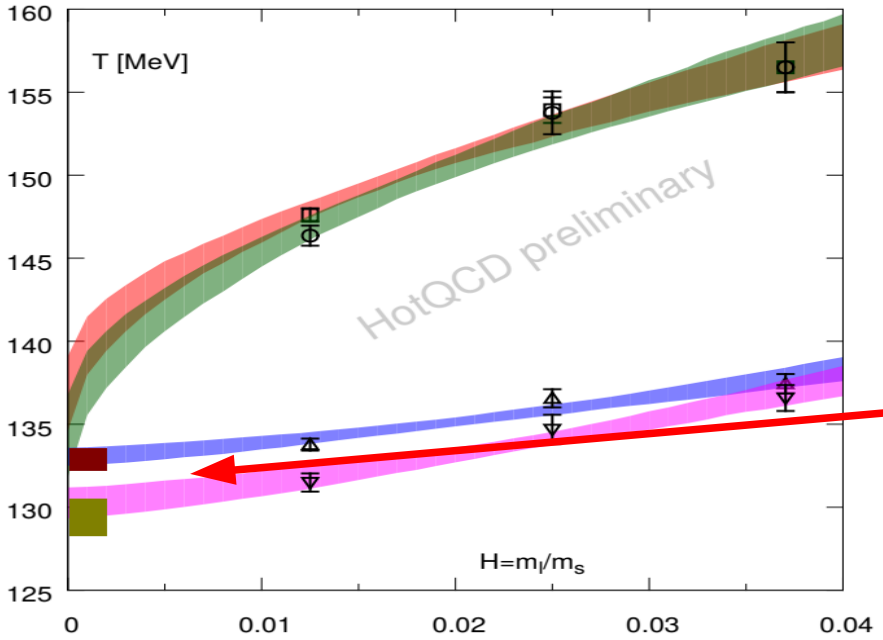
Chiral PHASE TRANSITION temperature



(1) **infinite volume** extrapolation

(2) **continuum limit** extrapolation





$$T_{pc}^{phys} = (156.5 \pm 1.5)\ \mathrm{MeV}$$

A. Bazavov et al [HotQCD], arXiv:1812.08235

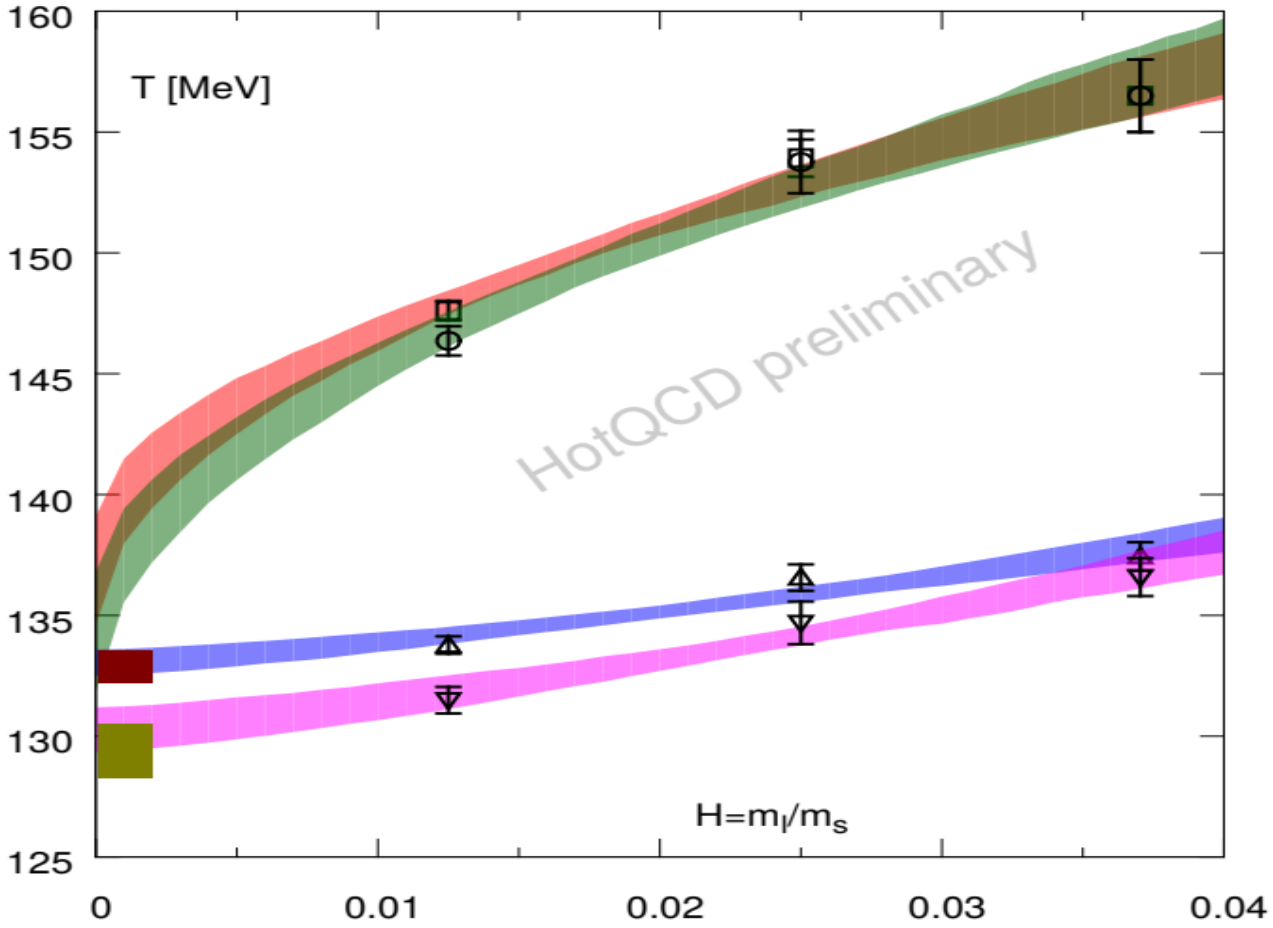
(3) **chiral limit** extrapolation

$$T_c^0 = 132^{+3}_{-6}\ \mathrm{MeV}$$

H.-T. Ding et al [HotQCD], arXiv:1903.04801

more on T_c^0 Anirban Lahiri (tomorrow)

Chiral PHASE TRANSITION temperature



$$T_{pc}^{phys}$$

$$(156.5 \pm 1.5)\ \mathrm{MeV}$$

$$\Delta T \simeq 25\ \mathrm{MeV}$$

$$T_c^0 = 132^{+3}_{-6}\ \mathrm{MeV}$$

Critical behavior and higher order cumulants
– Taylor expansion and universality –

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu_{B,Q,S}=0} , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

Critical behavior and higher order cumulants

– the breakdown of the HRG model description in the “vicinity of Tc” becomes obvious in properties of higher order cumulants,

pressure: $$\frac{p}{T^4} = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

$$t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_q}{T}\right)^2 \qquad h \sim \frac{m_q}{T_c}$$

	alpha
O(4)	-0.213
Z(2)	+0.107

$$\frac{\partial}{\partial T} \simeq \frac{\partial^2}{\partial(\mu_B/T)^2}$$

chemical potentials are “**thermal couplings**”

T-derivative ⟺ two μ-derivatives

FK et al., arXiv:1009.5211

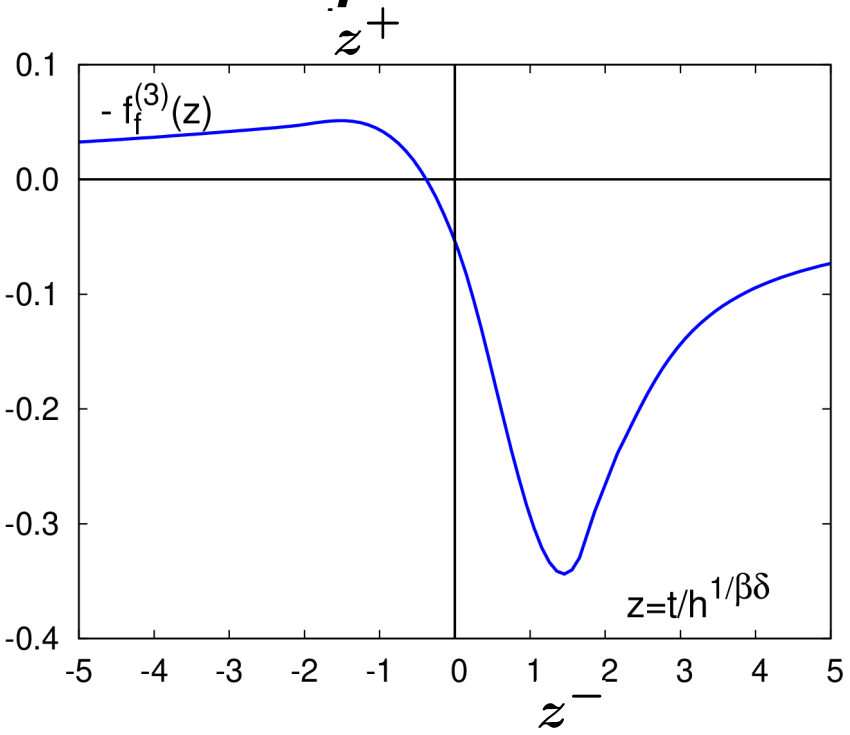
cumulants:

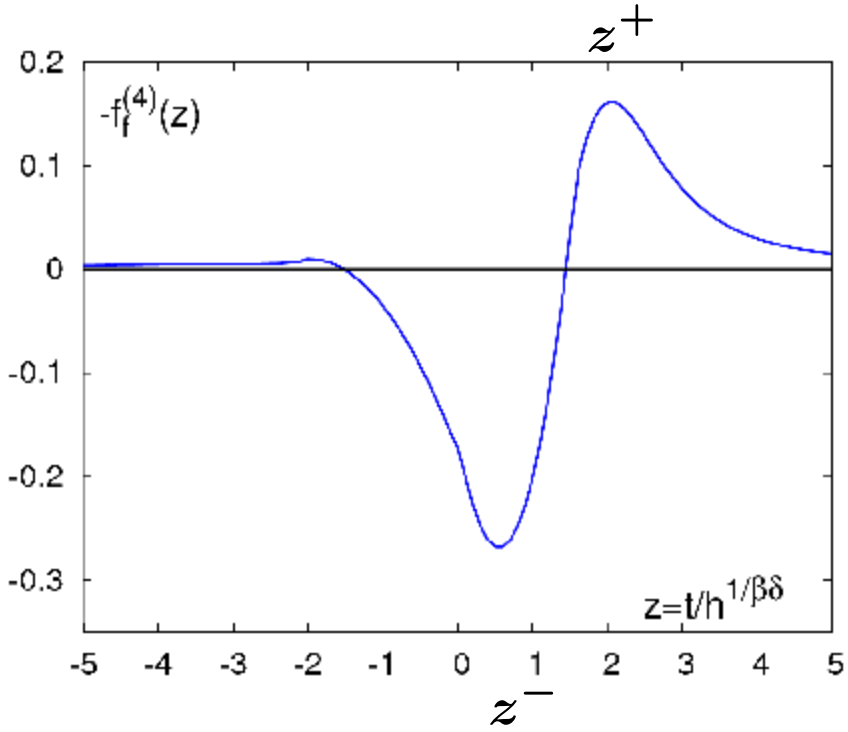
$$\chi_X^{(2n)} = \left.\frac{\partial^{2n} p/T^4}{\partial(\mu_X/T)^{2n}}\right|_{\mu_X=0} = m_q^{(2-\alpha-n)/\beta\delta} f_f^{(n)}(t/h^{1/\beta\delta}) \;,\; X = B, S, \ldots$$

O(4): singular terms dominate only for $2n \geq 6$

Critical behavior and higher order cumulants

pressure: $\frac{p}{T^4} = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$





universal ratios:

$$\frac{\chi_6^B(T^-)}{\chi_6^B(T^+)} = \frac{f_f^{(3)}(z^-)}{f_f^{(3)}(z^+)} \simeq -5.9$$

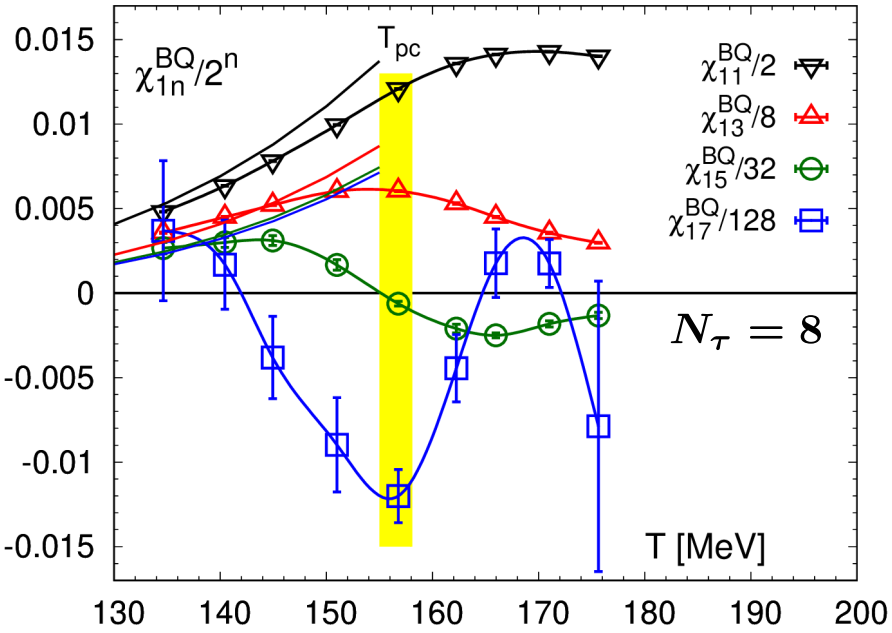
$$\frac{\chi_8^B(T^-)}{\chi_8^B(T^+)} = \frac{f_f^{(4)}(z^-)}{f_f^{(4)}(z^+)} \simeq -1.76$$

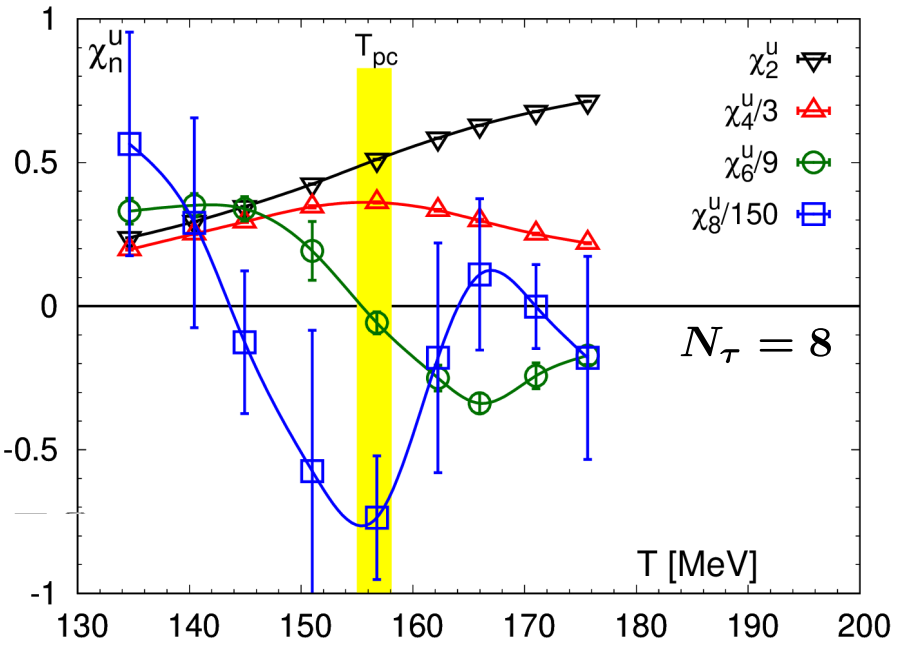
FK, J. Engels, arXiv:1105.0584

Critical behavior and higher order cumulants

$$\chi_{1n}^{BQ} = \left. \frac{\partial^{n+1} P/T^4}{\partial \hat{\mu}_B \partial \hat{\mu}_Q^n} \right|_{\mu_{B,Q,S}=0}$$

$$\chi_n^u = \left. \frac{\partial^n P/T^4}{\partial \hat{\mu}_u^n} \right|_{\mu_{u,d,s}=0}$$





lines are drawn to guide the eye (sorry – it still looks messy)

$$\frac{\partial^2}{\partial(\mu_X/T)^2} \simeq \frac{\partial}{\partial T}$$

– expected from structure of O(N) scaling fields

$$t \sim \frac{T - T_c}{T_c} + \kappa_q \left(\frac{\mu_q}{T}\right)^2$$

universal ratios receive large regular contributions

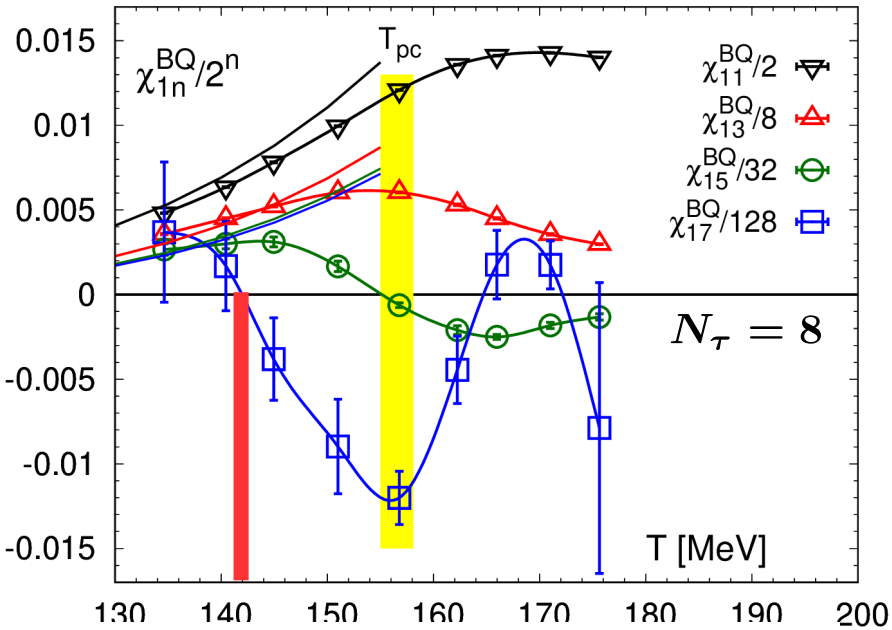
many 8th order cumulants turn negative for

$$T^- \gtrsim (140 - 145)\ \mathrm{MeV}$$

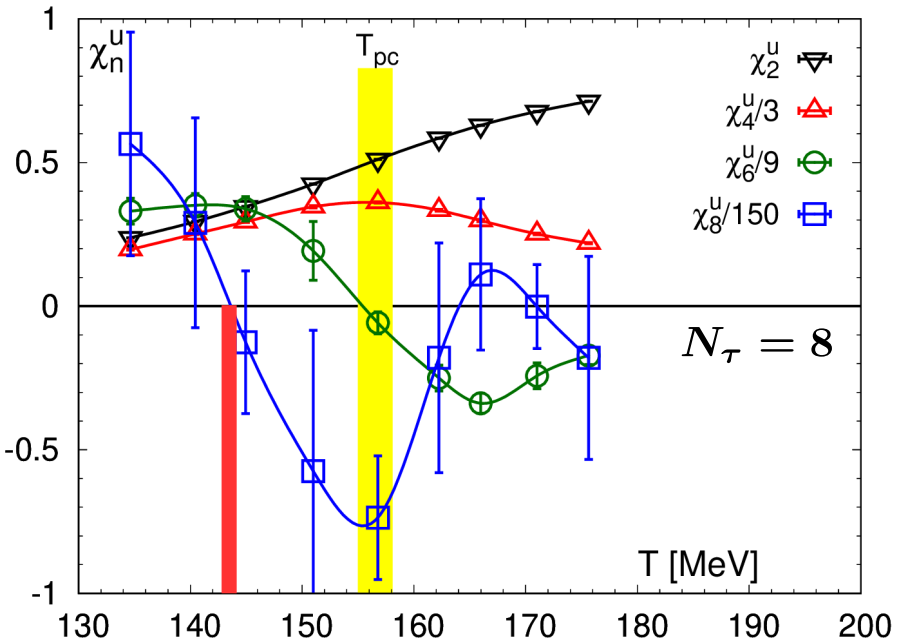
suggests zeroes in complex plane → no phase tr.

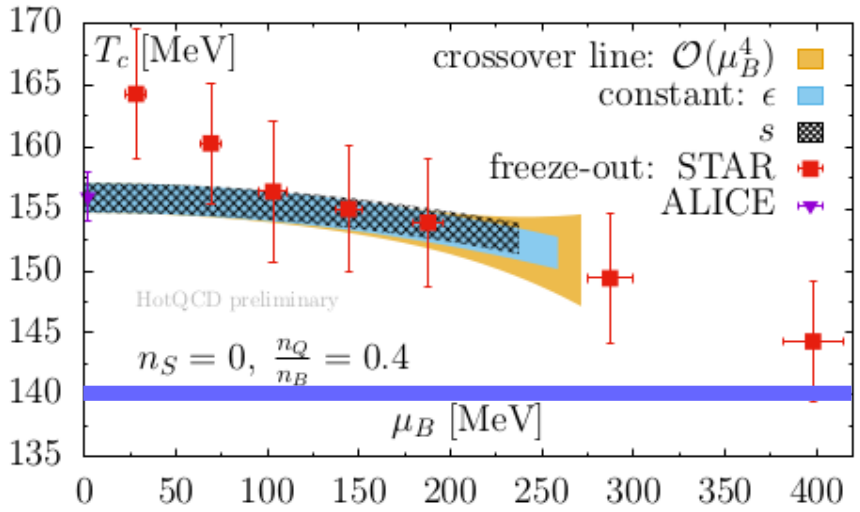
Critical behavior and higher order cumulants

$$\chi^{BQ}_{1n} = \left. \frac{\partial^{n+1} P/T^4}{\partial \hat{\mu}_B \partial \hat{\mu}_Q^n} \right|_{\mu_{B,Q,S}=0}$$



$$\chi^{u}_{n} = \left. \frac{\partial^{n} P/T^4}{\partial \hat{\mu}_u^n} \right|_{\mu_{u,d,s}=0}$$





many 8th order cumulants turn negative for

$$T^- \gtrsim (140 - 145)\ \mathrm{MeV}$$

suggests zeroes in complex plane → no phase tr.

→ plausible scenario:

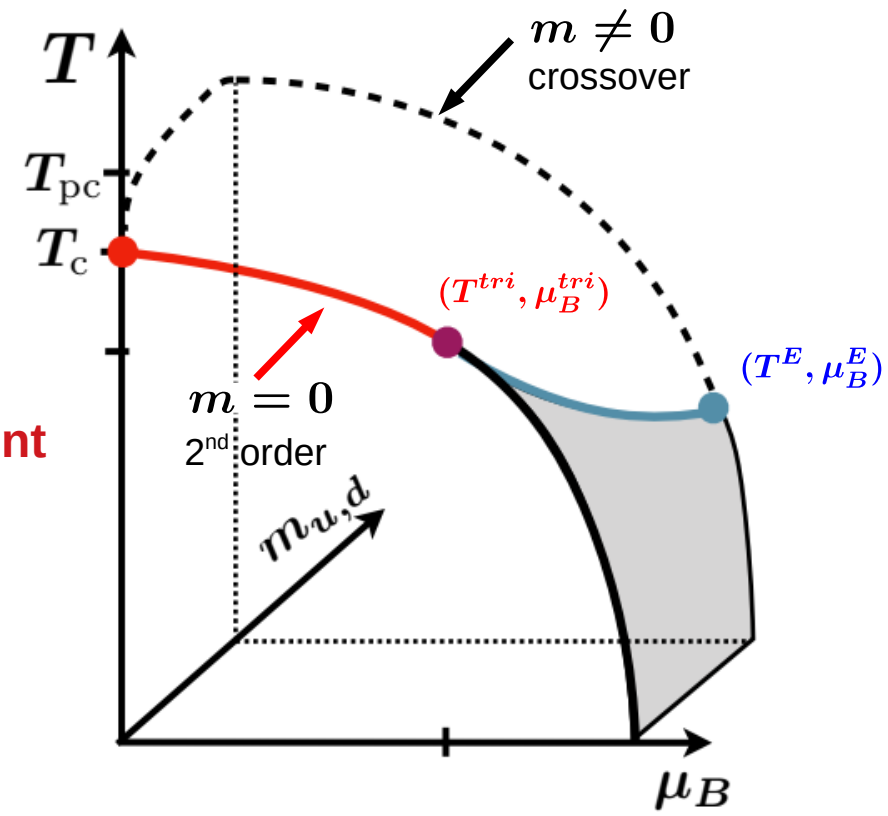
$$T_{cp} < 140\mathrm{MeV}\ ,\ \mu_B^{cp} > 400\mathrm{MeV}$$

consistent with $T_c^0 = 132^{+3}_{-6}\ \mathrm{MeV}$

Crossover, chiral phase transition at $\mu_B = 0$ and the (tri)-critical point at $\mu_B > 0$

(156.5 ± 1.5) MeV

132^{+3}_{-6} MeV



a possible tri-critical point will show up only at

$T < 135$ MeV

the critical point is likely to be located at

$T < 130$ MeV

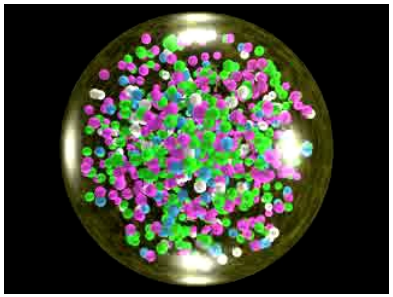
Random Matrix Model	A. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov, J.J.,M. Verbaarschot, Phys. Rev. D58 (1998) 096007
QCD	M. Stephanov, Phys. Rev. D73 (2006) 094508
NJL	M. Buballa, S. Carignano, Phys. Lett. B791 (2019) 361

Probing the properties of matter through the analysis of conserved charge fluctuations

baryon number, strangeness, electric charge

High T: ideal gas

ideal quark (fermi) gas, m=0



fractional charges

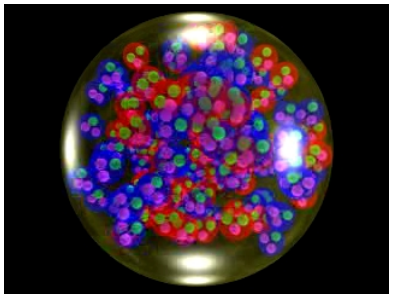
baryon number: B= +/- 1/3

electric charge: Q= +/- 1/3, +/- 2/3

strangeness: S= 0, +/- 1

Low T: HRG

hadron resonance gas



integer charges

baryon number: B= +/-1

electric charge: Q= 0 =+/- 1, +/- 2

strangeness: S= 0, +/- 1, +/- 2, +/- 3

Baryon-Strangeness Correlations: A Diagnostic of Strongly Interacting Matter

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(Received 23 May 2005; published 24 October 2005)

The correlation between baryon number and strangeness elucidates the nature of strongly interacting matter, such as that formed transiently in high-energy nuclear collisions. This diagnostic can be extracted theoretically from lattice QCD calculations and experimentally from event-by-event fluctuations. The analysis of present lattice results above the critical temperature severely limits the presence of $q\bar{q}$ bound states, thus supporting a picture of independent (quasi)quarks.

DOI: 10.1103/PhysRevLett.95.182301 PACS numbers: 25.75.Gz, 12.38.Gc, 12.38.Mh, 25.75.Nq

The principal goal of high-energy heavy-ion collisions is the creation and exploration of a novel state of matter in which the quarks and gluons are deconfined over distances considerably larger than that of a hadron [1]. It had originally been assumed that asymptotic freedom would cause such matter to behave as a plasma of massless quarks and gluons interacting with a relatively weak screened chromodynamic Coulomb force. While this picture is supported qualitatively by the rapid rise in the entropy density at a temperature of about $T_c \simeq 170$ MeV, as obtained by lattice QCD calculations, the fact that the high-T behavior falls somewhat below that of an ideal gas of massless quarks and gluons [2] indicates that the chromodynamic plasma has a more complex structure.

Indeed, recent results from lattice QCD calculation on spectral functions [3–6] suggest the presence of bound, color-neutral states well above T_c. This has led to the suggestion that at moderate temperatures, $T \simeq 1 - 2T_c$, the system is composed of medium-modified (massive) quarks and gluons together with their (many and possibly colored) bound states [7–10].

Furthermore, jet quenching observed at the Relativistic Heavy-Ion Collider (RHIC) suggests that the matter formed is not hadronic [11]. Moreover, the RHIC collisions exhibit strong elliptic and radial collective flows that are consistent with predictions by ideal fluid dynamics [12,13]. If ideal hydrodynamics is indeed the correct framework, rapid thermalization must occur and this in turn would seem to rule out a weakly interacting plasma. Thus, the nature of the matter being created at RHIC is still not clarified and the purpose of this Letter is to offer a novel diagnostic tool for elucidating this issue by probing the relevant degrees of freedom and their correlations.

In particular, we will argue that the correlation between the strangeness S and the baryon number B provides a useful diagnostic for the presence of strong correlations between quarks and antiquarks. In order to understand this, consider first a situation in which the basic degrees of freedom are weakly interacting quarks and gluons. Then strangeness is carried exclusively by the s and $\bar{s}$ quarks which in turn carry baryon number in strict proportion to their strangeness, $B_s = -\frac{1}{3}S_s$, thus rendering strangeness and baryon number strongly correlated. This feature is in stark contrast to a hadron gas in which the relation between strangeness and baryon number is less intimate. For example, at small baryon chemical potential the strangeness is carried primarily by kaons, which have no baryon number.

These elementary considerations lead us to introduce the following correlation coefficient,

$$C_{BS} \equiv -3\frac{\sigma_{BS}}{\sigma_S^2} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} = -3\frac{\langle BS\rangle}{\langle S^2\rangle}. \quad (1)$$

The average $\langle\cdot\rangle$ is taken over a suitable ensemble of events and the last expression uses the fact that $\langle S\rangle$ vanishes. When the active degrees of freedom are individual quarks, the total strangeness is $S = \nu_{\bar{s}} - \nu_s$, while the baryon number can be expressed as $B = \frac{1}{3}(U + D - S)$, where $U = \nu_u - \nu_{\bar{u}}$ is the upness and $D = \nu_d - \nu_{\bar{d}}$ is the downness. Thus, if the flavors are uncorrelated, we have $\sigma_{BS} = -\frac{1}{3}\sigma_S^2$ and C_{BS} is unity.

In a gas of hadron resonances, the total baryon number is $B = \sum_k n_k B_k$ and its total strangeness is $S = \sum_k n_k S_k$, where the species k has baryon number B_k and strangeness S_k. If the multiplicities $\{n_k\}$ are uncorrelated, as is generally assumed in the standard statistical treatment [14], C_{BS} may be expressed in terms of the associated multiplicity variances $\sigma_k^2 \equiv \langle n_k^2\rangle - \langle n_k\rangle^2 \approx \langle n_k\rangle$,

$$C_{BS} = -3\frac{\sum_k \sigma_k^2 B_k S_k}{\sum_k \sigma_k^2 S_k^2} \approx -3\frac{\sum_k \langle n_k\rangle B_k S_k}{\sum_k \langle n_k\rangle S_k^2}, \quad (2)$$

where the approximate expression holds for Poisson statistics. Thus, in a gas of uncorrelated hadrons, the numerator receives contributions from only (strange) baryons (and antibaryons), while the denominator receives contributions also from (strange) mesons,

$$C_{BS} \approx 3\frac{\langle\Lambda\rangle + \langle\bar{\Lambda}\rangle + \cdots + 3\langle\Omega^-\rangle + 3\langle\bar{\Omega}^+\rangle}{\langle K^0\rangle + \langle\bar{K}^0\rangle + \cdots + 9\langle\Omega^-\rangle + 9\langle\bar{\Omega}^+\rangle}. \quad (3)$$

While Eq. (1) [or its more explicit version (6)] holds

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Conserved charge correlations

baryon number – strangeness correlations

$$\frac{\chi_{11}^{BS}(T,\mu_B)}{\chi_2^S(T,\mu_B)} \equiv \frac{\langle BS\rangle}{\langle S^2\rangle}$$

$$C_{BS} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} = -3\frac{\langle BS\rangle}{\langle S^2\rangle}$$

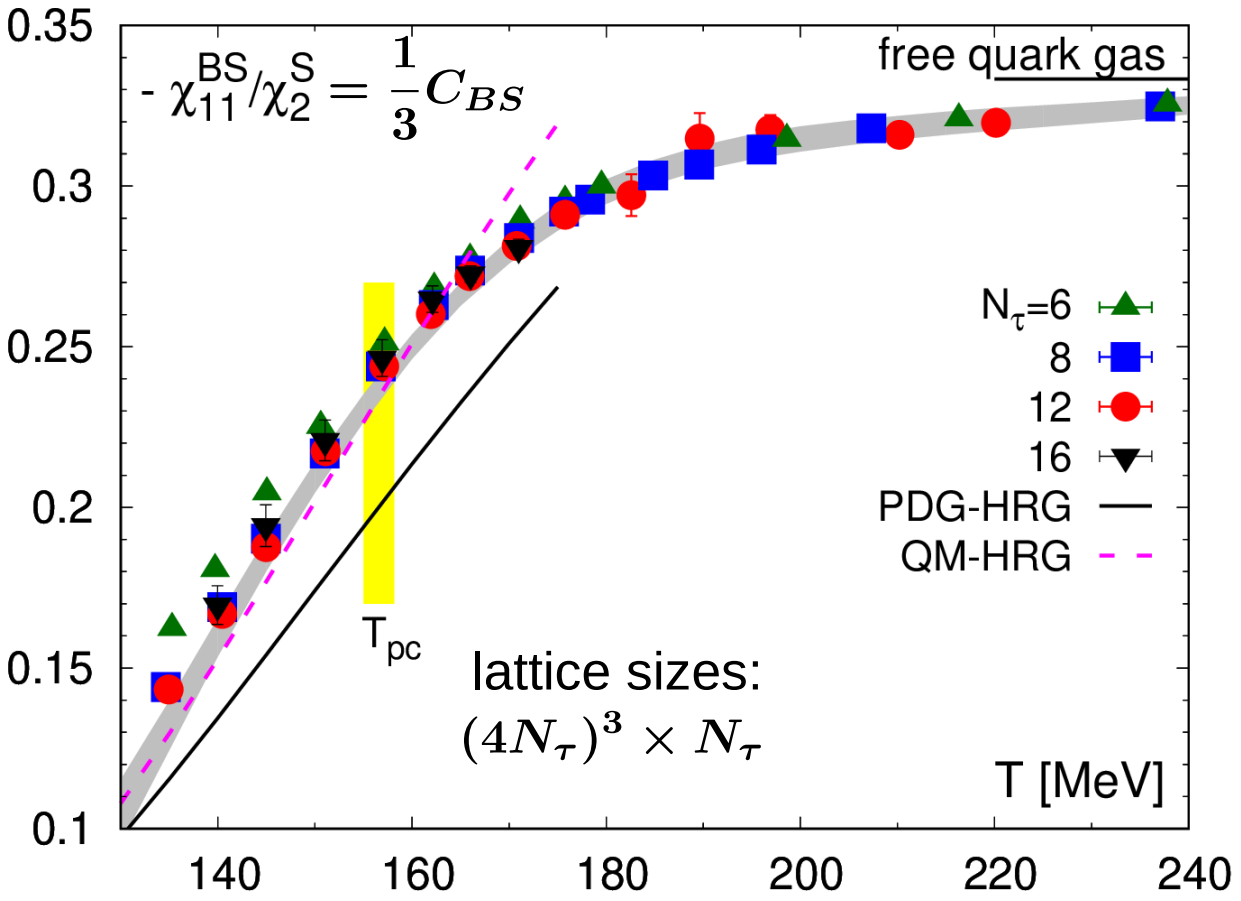
Free Quark Gas

$$C_{BS} = -3\frac{\langle BS\rangle}{\langle S^2\rangle} = -3\frac{\frac{\pm 1}{3}\cdot(\mp 1)}{(\pm 1)^2} = 1$$

Hadron Resonance Gas (HRG)

$$C_{BS} \approx 3\,\frac{\langle\Lambda\rangle + \langle\bar{\Lambda}\rangle + \ldots + 3\langle\Omega^-\rangle + 3\langle\bar{\Omega}^+\rangle}{K^0 + \bar{K}^0 + \ldots + 9\langle\Omega^-\rangle + 9\langle\bar{\Omega}^+\rangle}$$

Ratio of baryon number – strangeness correlation and net strangeness fluctuations



- evidence for experimentally not yet observed strange baryons?

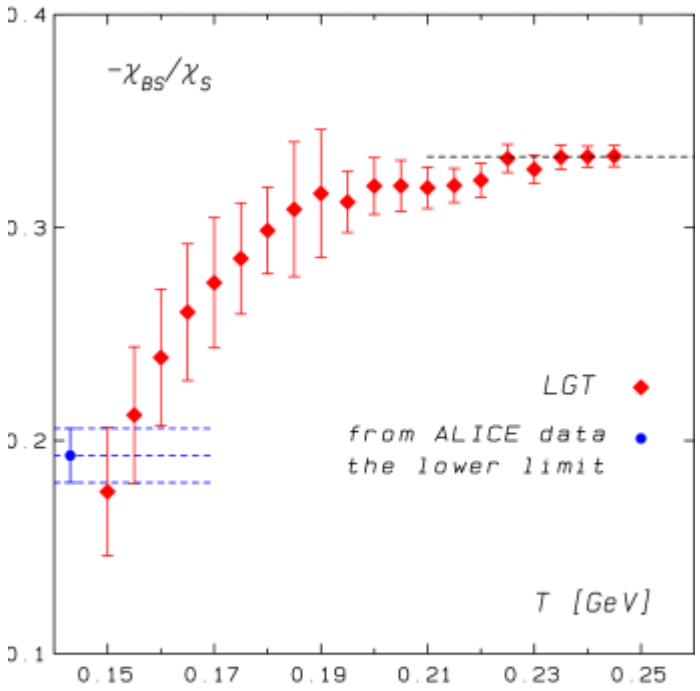
$$\text{at } T_{pc}$$

$$\chi_{11}^{BS}/\chi_2^S = -0.235(15)$$

A. Bazavov et al., arXiV:1404.6511

PDG-HRG: uses experimentally known hadron spectrum listed by the Particle Data Group
QM-HRG: uses additional hadrons predicted to exist in Quark Model calculations

baryon number – strangeness correlation from HIC

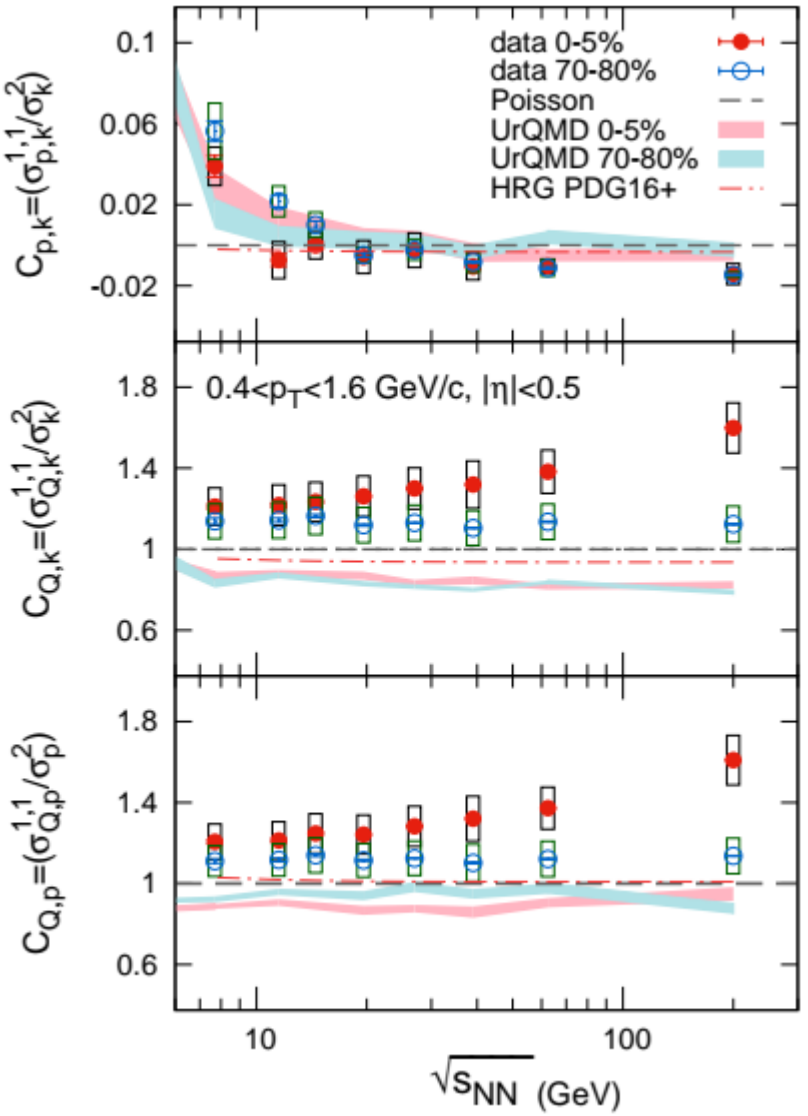


HRG motivated analysis:

$$-\frac{\chi_{BS}}{T^2} > \frac{1}{VT^3}[2\langle \Lambda + \Sigma^0\rangle + 4\langle \Sigma^+\rangle + 8\langle \Xi\rangle + 6\langle \Omega^-\rangle] = 97.4 \pm 5.8.$$

$$\frac{\chi_S}{T^2} \simeq \frac{1}{VT^3}[(\langle K^+\rangle + \langle K^0\rangle + \langle \Lambda + \Sigma^0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + 4\langle \Xi^-\rangle + 4\langle \Xi^0\rangle + 9\langle \Omega^-\rangle + \text{antiparticles}) - (\Gamma_{\phi\to K^+} + \Gamma_{\phi\to K^-} + \Gamma_{\phi\to K^0} + \Gamma_{\phi\to \bar{K}^0})\langle\phi\rangle].$$

P. Braun-Munzinger, A. Kalweit, K. Redlich, J. Stachel, Phys. Lett. B747 (2015) 292, arXiv:1412.8614



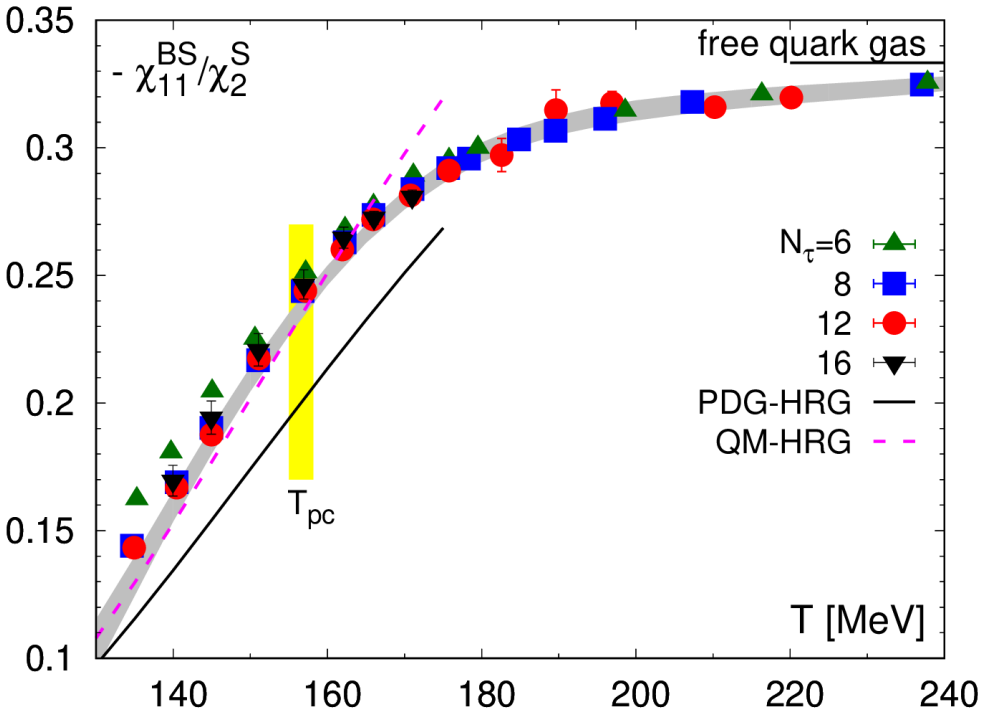
kaon ⇨ net-strangeness proxy
proton ⇨ net baryon-number proxy

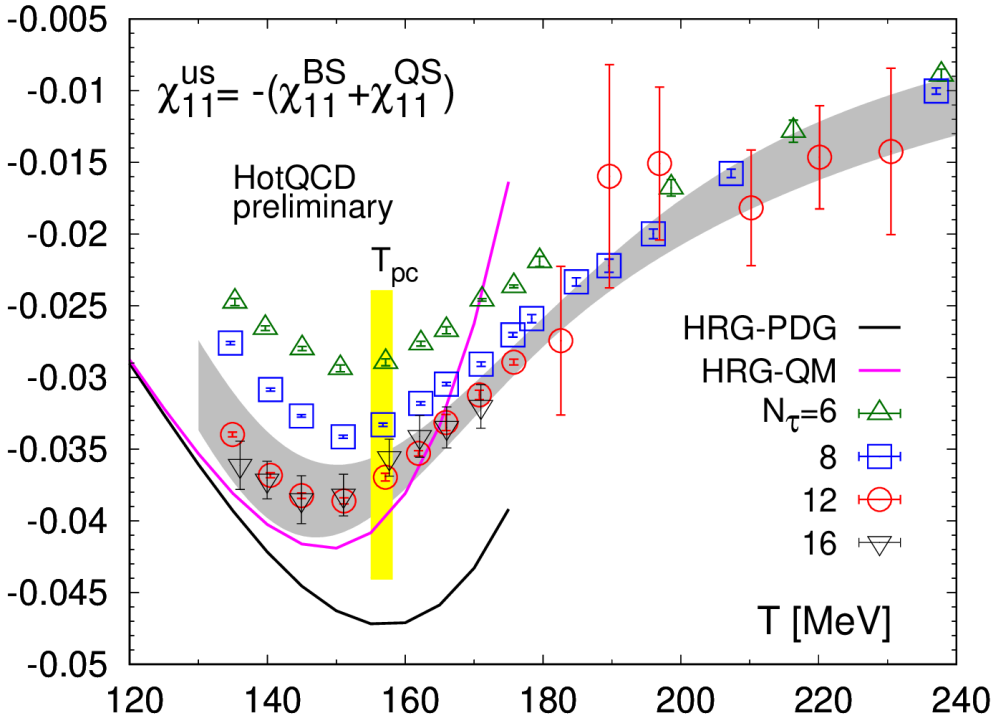
J. Adam et al (STAR Collaboration), arXiv:1903.05370

Ratio of baryon number – strangeness correlation and net strangeness fluctuations

2nd order cumulants

BS ratios probe flavor-correlations





conserved charge ⇔ quark number fluctuations:

$$\chi_{11}^{BS} = -\tfrac{1}{3}\chi_{11}^{us} - \tfrac{1}{3}\chi_{11}^{ds} - \tfrac{1}{3}\chi_2^s$$

$$-\frac{\chi_{11}^{BS}}{\chi_2^S} = \frac{1}{3} + \frac{2}{3}\frac{\chi_{11}^{us}}{\chi_2^s}$$

$\mathcal{O}(g^6 \ln g^2)$

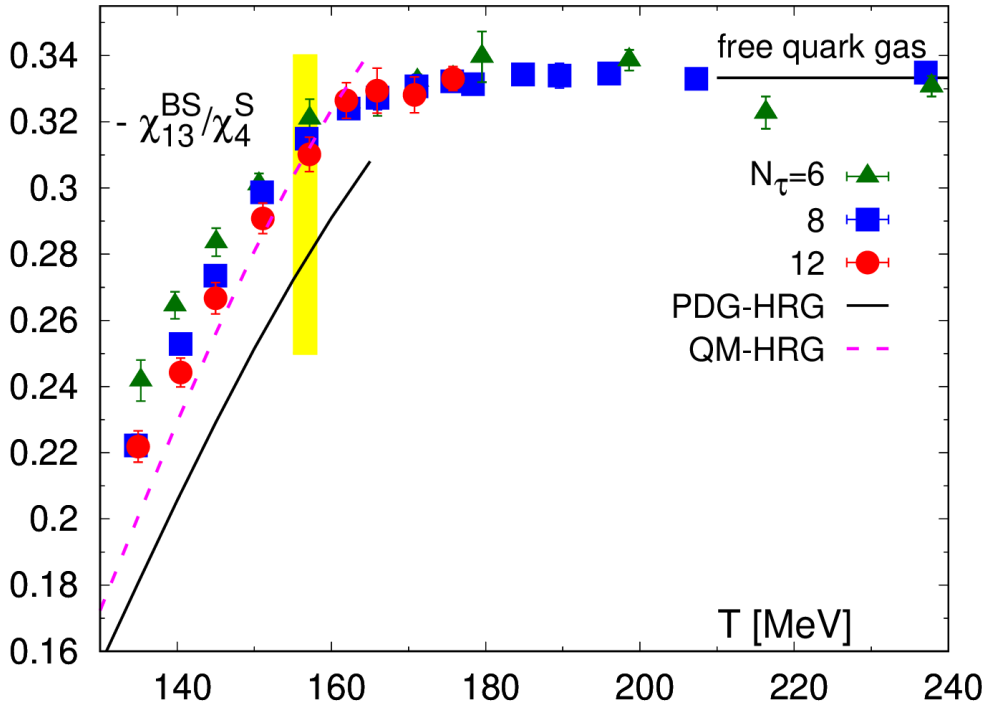
J.-P. Blaizot, E. Iancu, A. Rebhan, Phys. Lett. B 523 (2001) 143

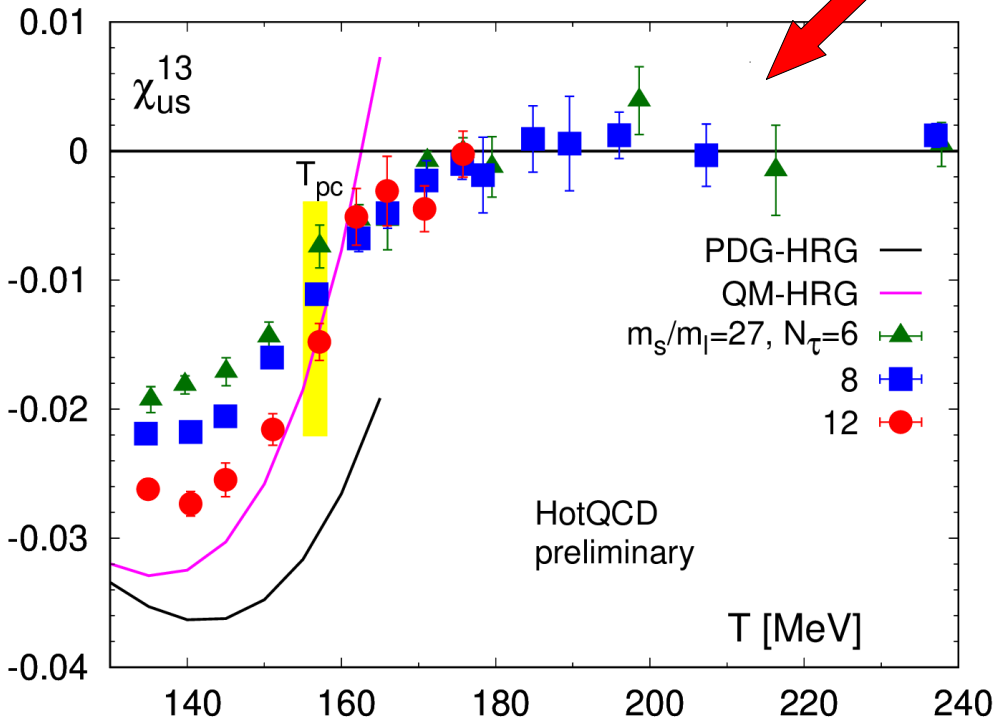
Ratio of baryon number – strangeness correlation and net strangeness fluctuations

4^{th} order cumulants

rapid approach to free quark gas limit

BS ratios probe flavor-correlations





conserved charge ⇔ quark number fluctuations:

$$\chi_{13}^{BS} = -\tfrac{1}{3}\chi_{13}^{us} - \tfrac{1}{3}\chi_{13}^{ds} - \tfrac{1}{3}\chi_4^s$$

$$-\frac{\chi_{13}^{BS}}{\chi_4^S} = \frac{1}{3} + \frac{2}{3}\frac{\chi_{13}^{us}}{\chi_4^s}$$

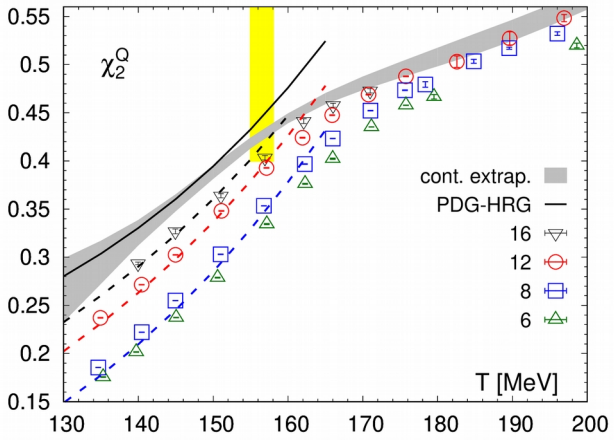
$\mathcal{O}(g^6 \ln g^2)$

J.-P. Blaizot, E. Iancu, A. Rebhan, Phys. Lett. B 523 (2001) 143

2nd order cumulants: fluctuations and correlations

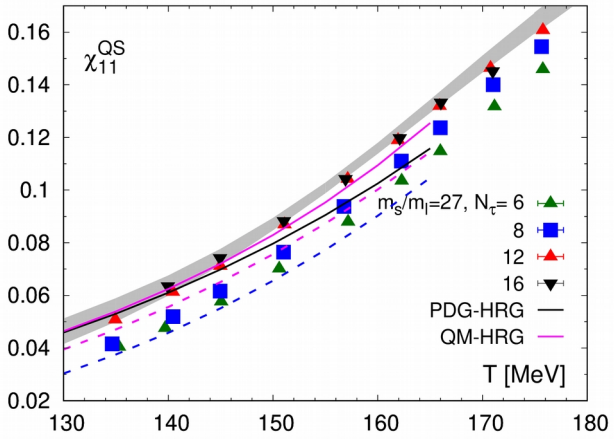
$m_u \simeq m_d$ → two constraints: only 4 out of 6 2nd order observables are independent

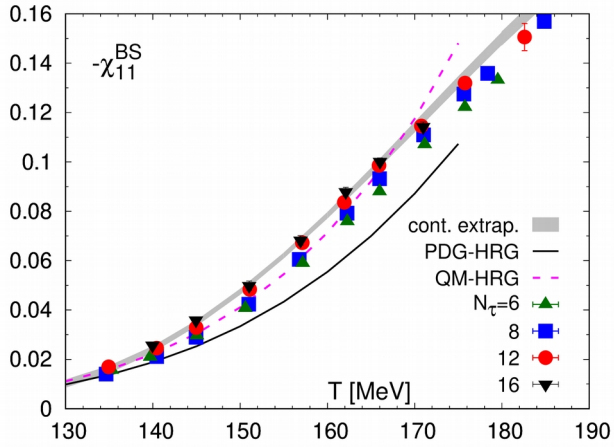
$T_{pc} = (156.5 \pm 1.5)\mathrm{MeV}$

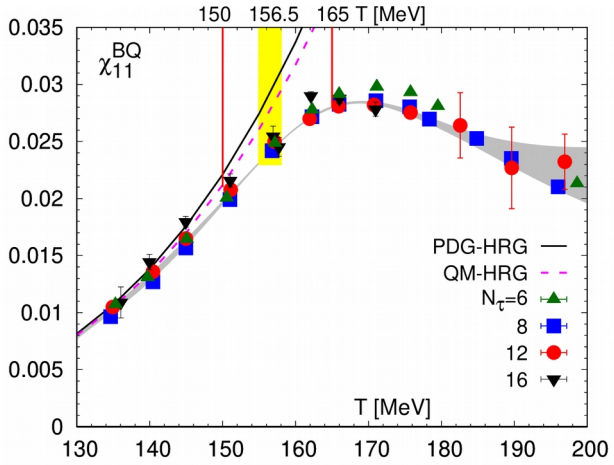


$$\chi_2^S = 2\chi_{11}^{QS} - \chi_{11}^{BS}$$
$$\chi_2^B = 2\chi_{11}^{BQ} - \chi_{11}^{BS}$$

– exact relations in QCD for $m_u = m_d$
– satisfied in HRG models to better than 1%

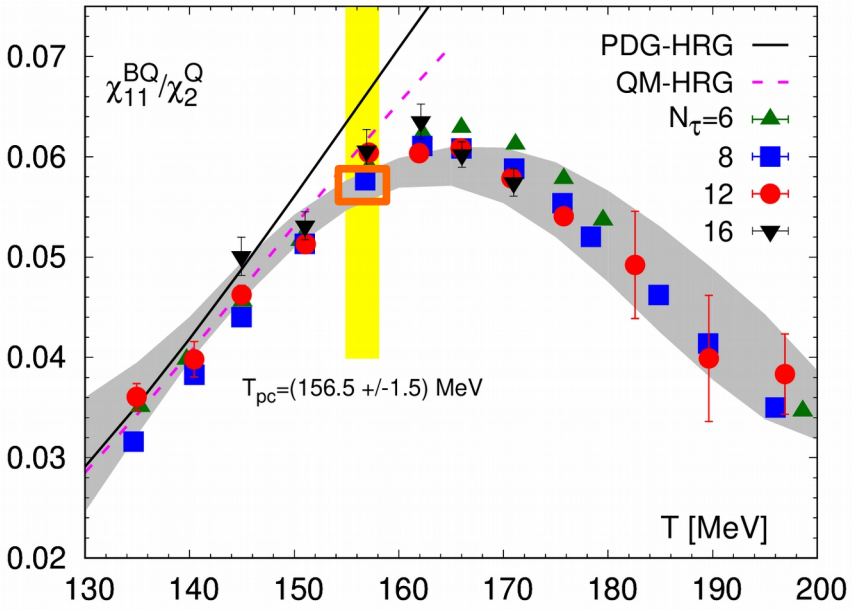


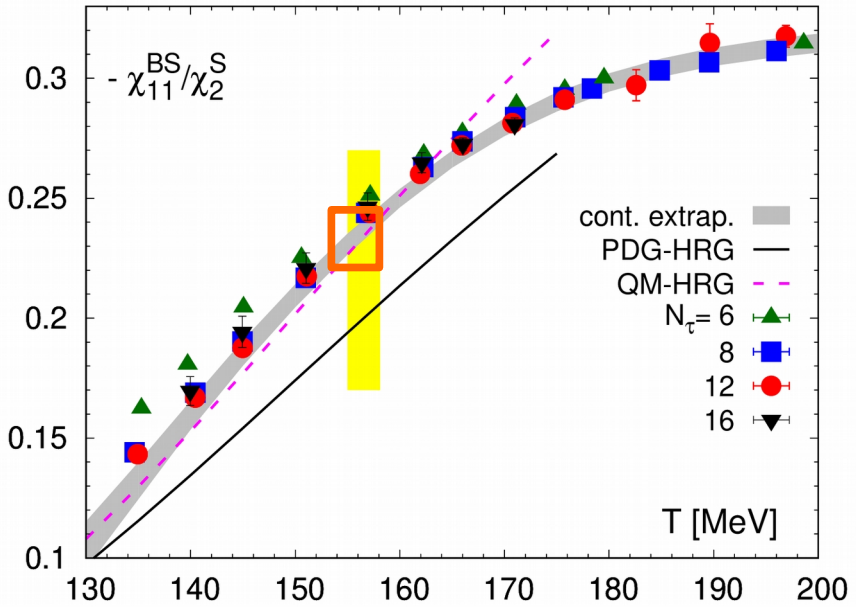




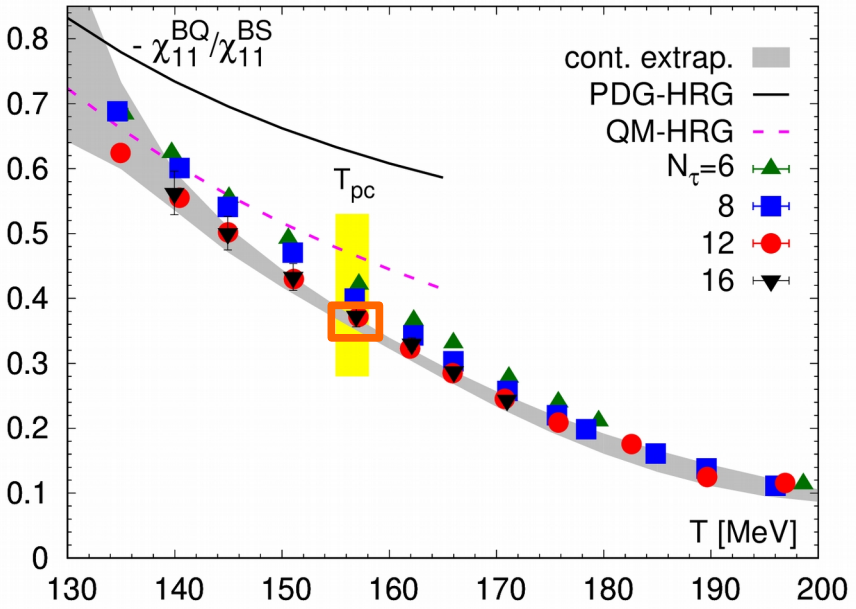
– continuum extrapolated results for all 2nd order cumulants in (2+1)-flavor QCD

Ratios of Taylor expansion coefficients at $\mu_B = 0$
– observables at the LHC ? –





3 independent ratios; arbitrarily chosen



at ALICE freeze-out temperature
== QCD crossover temperature

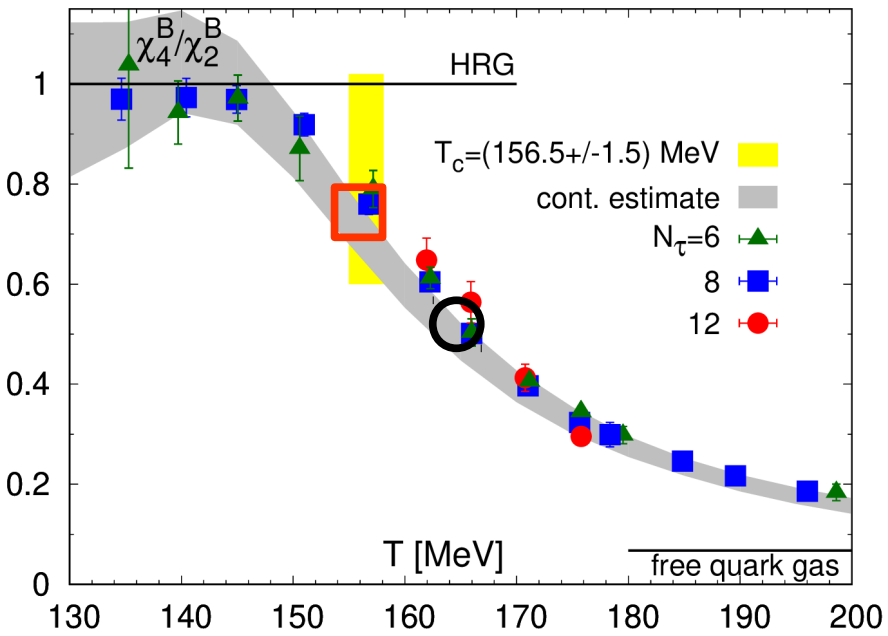
$$T_{fo} = 156.5(1.5)\mathrm{MeV}$$

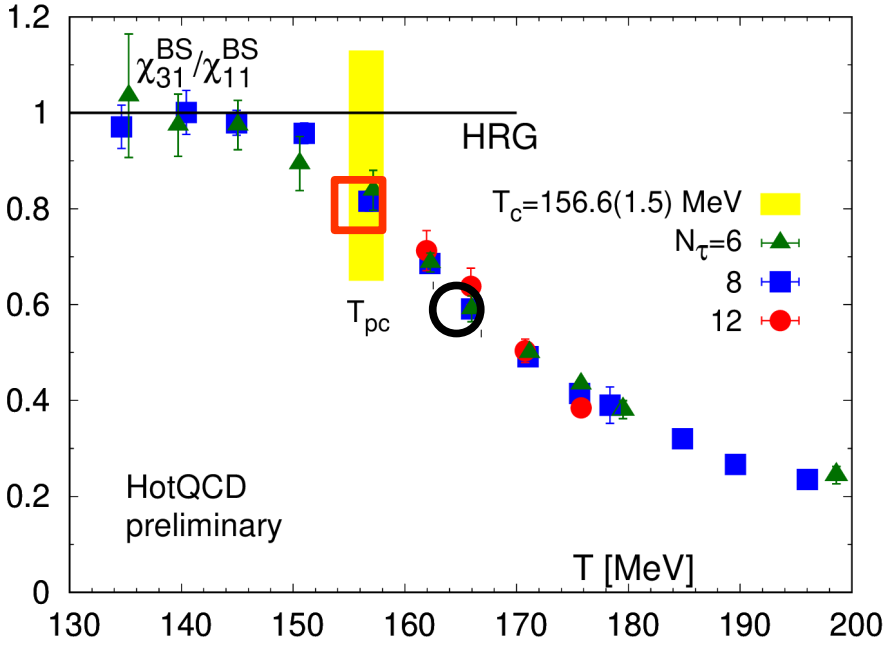
$$\chi_{11}^{BQ}/\chi_2^Q = 0.058(2)$$

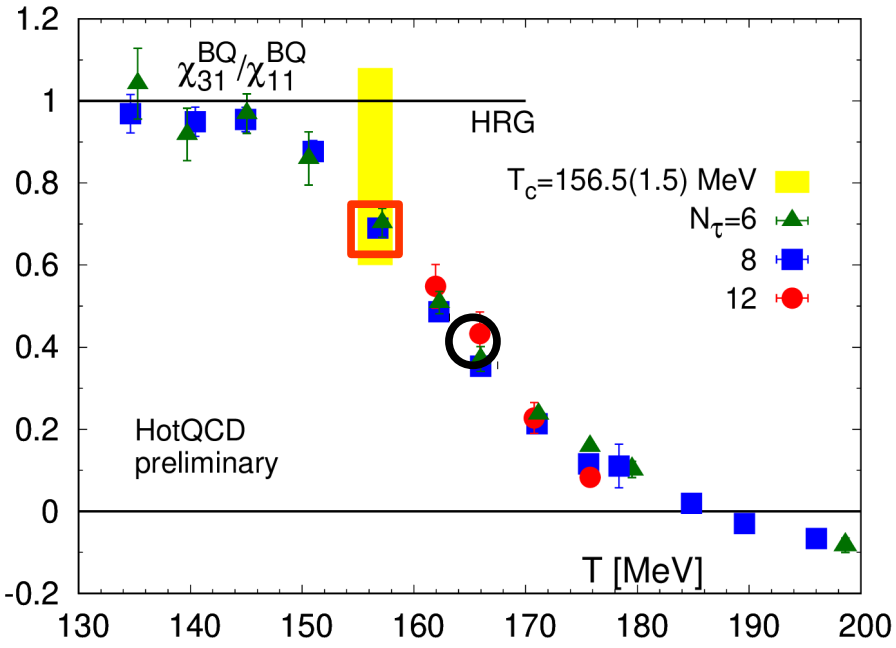
$$\chi_{11}^{BS}/\chi_2^S = -0.235(15)$$

$$\chi_{11}^{BQ}/\chi_{11}^{BS} = -0.37(3)$$

Ratios of 4th and 2nd order cumulants







– ratios of 4th and 2nd order cumulants differ from non-inter. HRG for T>145 MeV

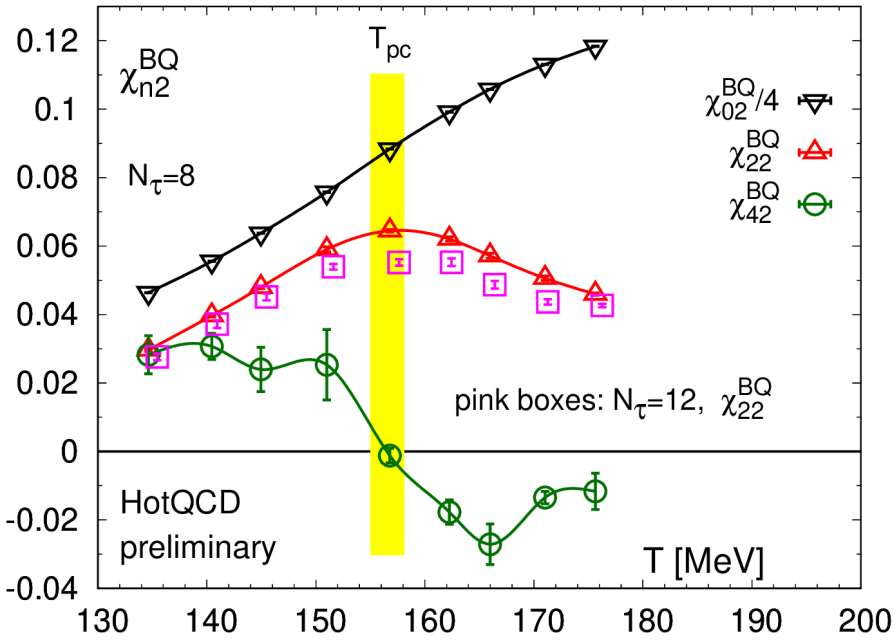
– they change by ~(20-40)% in the crossover region

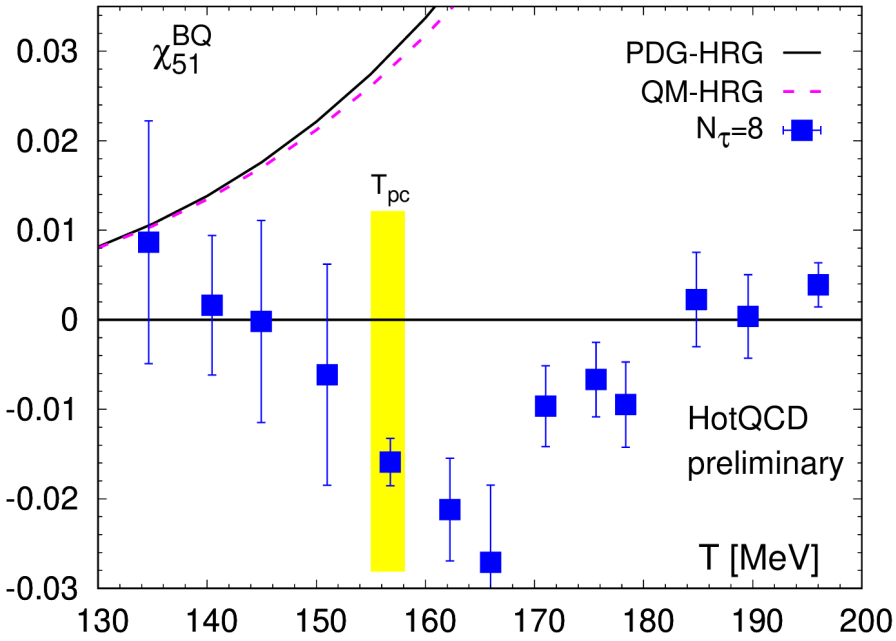
sensitive probes for freeze-out

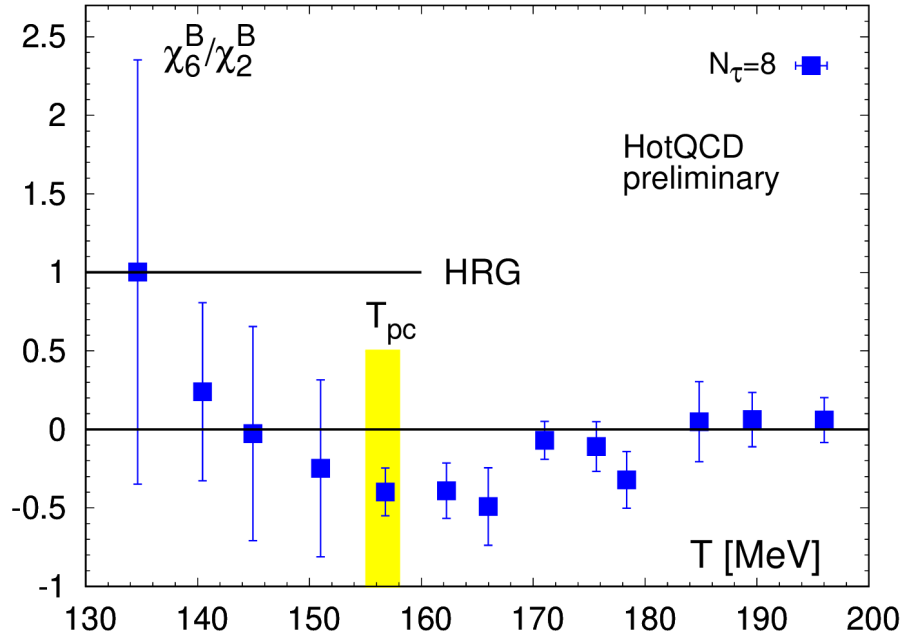
at ALICE freeze-out temperature $T_{fo} = 156.5(1.5)\mathrm{MeV}$

STAR at 200 GeV: $T_{fo} = 165(3)\mathrm{MeV}$

some 6th order cumulants







- 6^{th} order cumulants of baryon number fluctuations and their correlations with electric charge are negative at the pseudo-critical temperature

- large deviations from the non-interacting HRG model

- still largely influenced by regular terms

Conclusions

– the chiral phase transition is located at a temperature T_c^0 about 25 MeV lower than the pseudo-critical temperature T_{pc} at physical values of the quark masses

– for $T \simeq T_{pc}$ chiral (magnetic) observables at physical values of the quark masses are sensitive to critical behavior in the chiral limit.

– higher order cumulants show large deviations from non-interacting (point-like) HRG model calculations even below T_{pc}

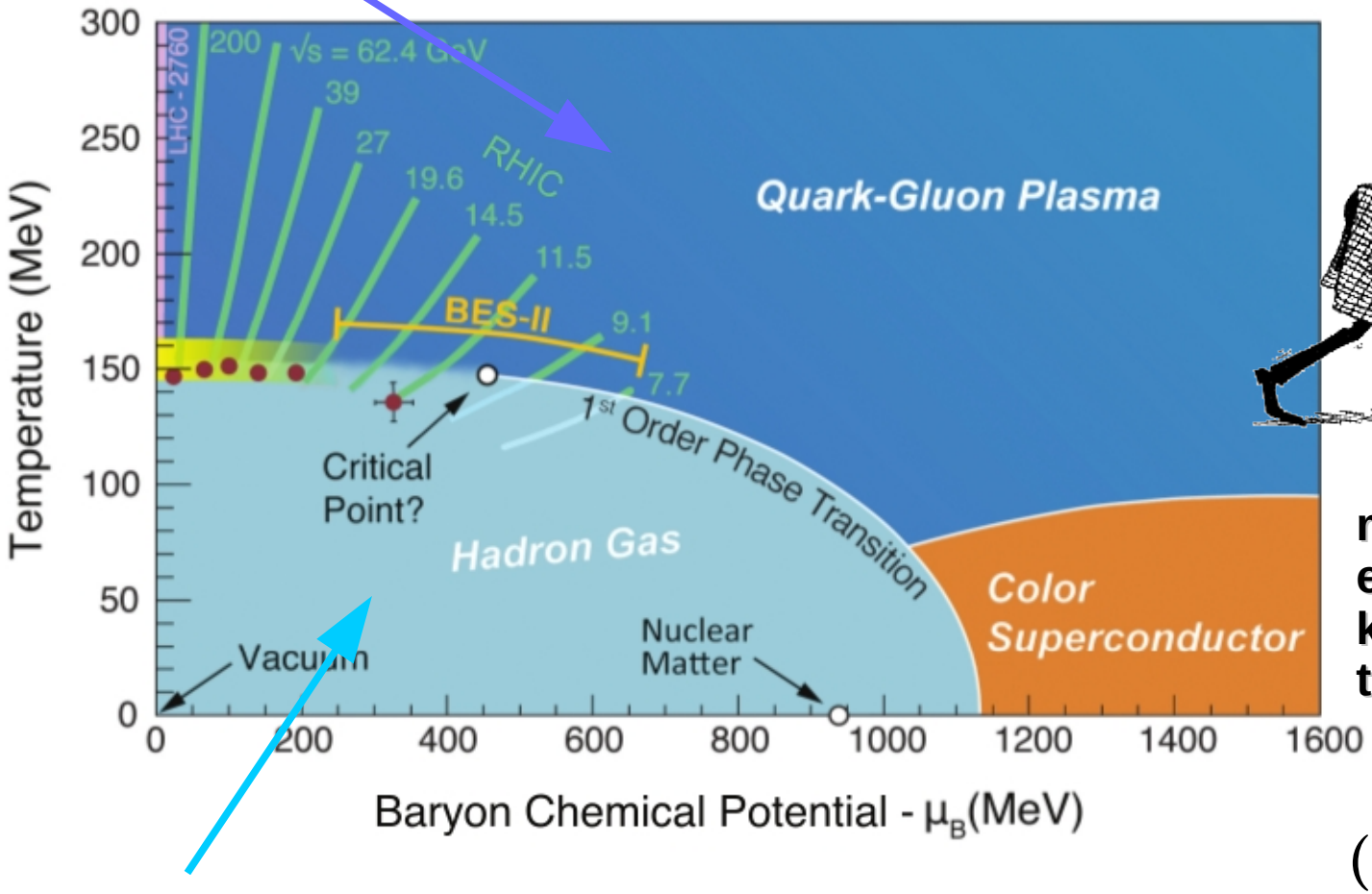
negative 6th & 8th order cumulants as well as the low chiral phase transition temperature suggest that a possibly existing critical point may be found only for

$$\mu_B^{cp} > 400\mathrm{MeV}\ ,\ T_{cp} < (130-140)\mathrm{MeV}$$

many thanks to Jishnu Goswami, Anirban Lahiri, Patrick Steinbrecher, Christian Schmidt for their help with the incorporation of recent HotQCD results in this talk

Phases of strong-interaction matter

thermodynamics of quarks and gluons:
high-T – QCD perturbation theory



Where is the critical point?

most relevant for current experiments is to reach knowledge on the QCD thermodynamics for

$$0 \leq \mu_B/T \leq 3$$

$$(T \simeq T_{pc} \simeq 155\ \mathrm{MeV})$$

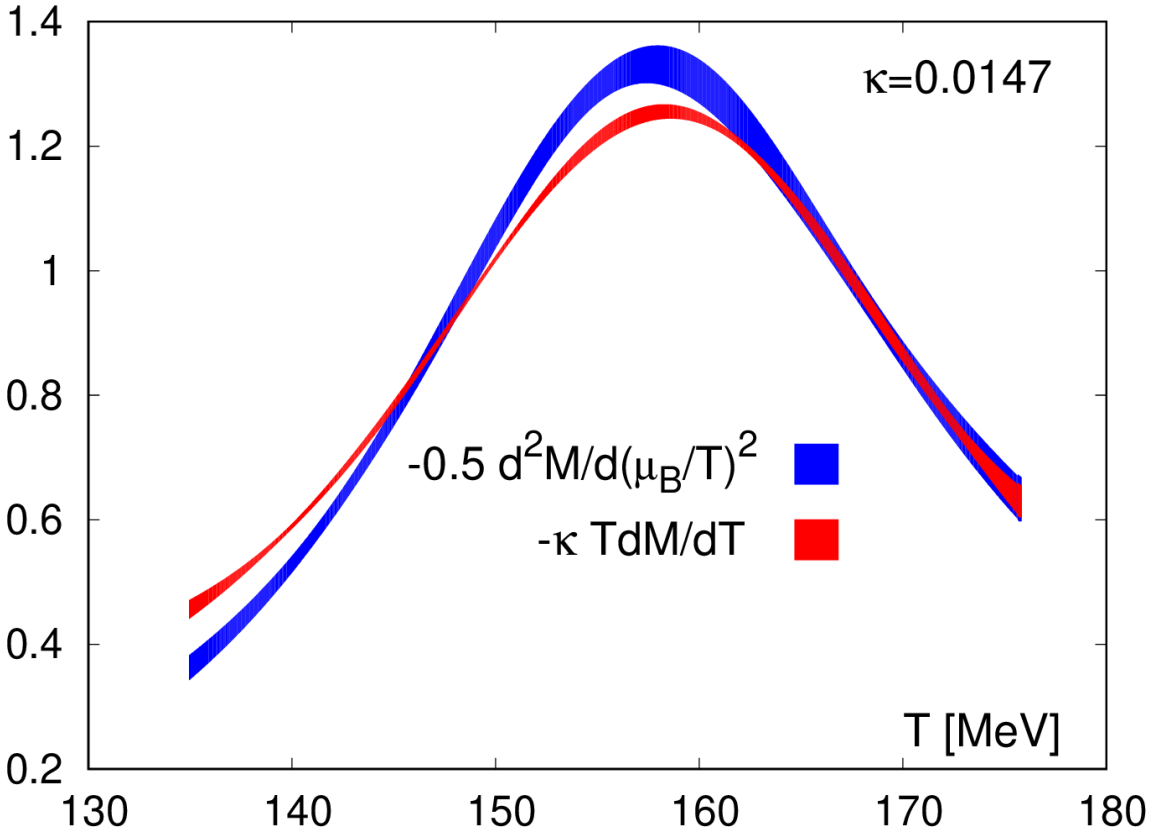
thermodynamics of a hadron gas

Critical behavior and higher order cumulants

critical behavior in chiral observables: the T-derivative of the chiral condensate

a mixed susceptibility

$$\Delta_{ls}(T,\mu_B) = \Delta_{ls}(T,0) + \frac{1}{2}\left.\frac{\partial^2 \Delta_{ls}}{\partial(\mu_B/T)^2}\right|_{\mu_B=0}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$



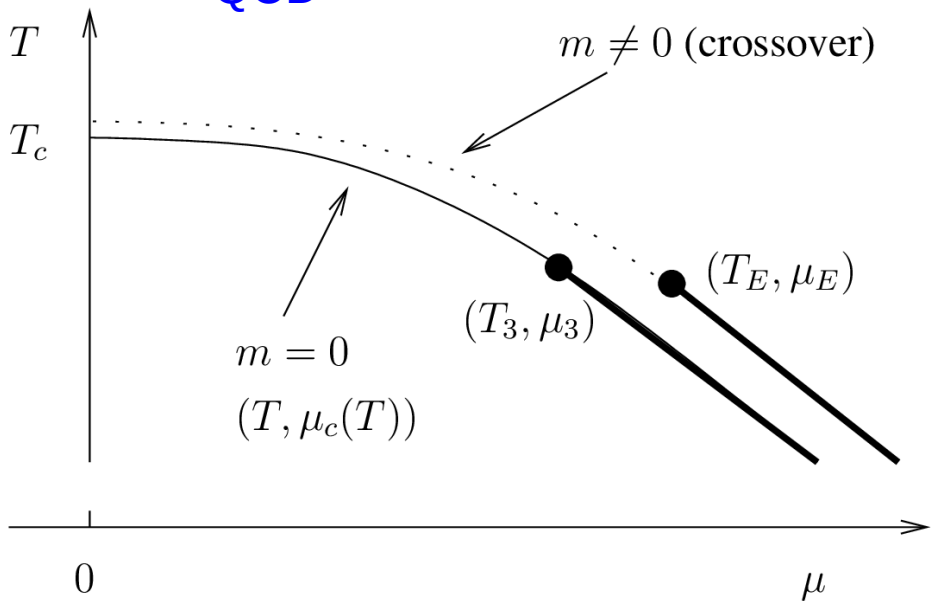
$$\frac{\partial^2}{\partial(\mu_B/T)^2} \simeq \frac{\partial}{\partial T}$$

$$t \sim \frac{T - T_c}{T_c} + \kappa_2 \left(\frac{\mu_B}{T}\right)^2$$

curvature of crossover line

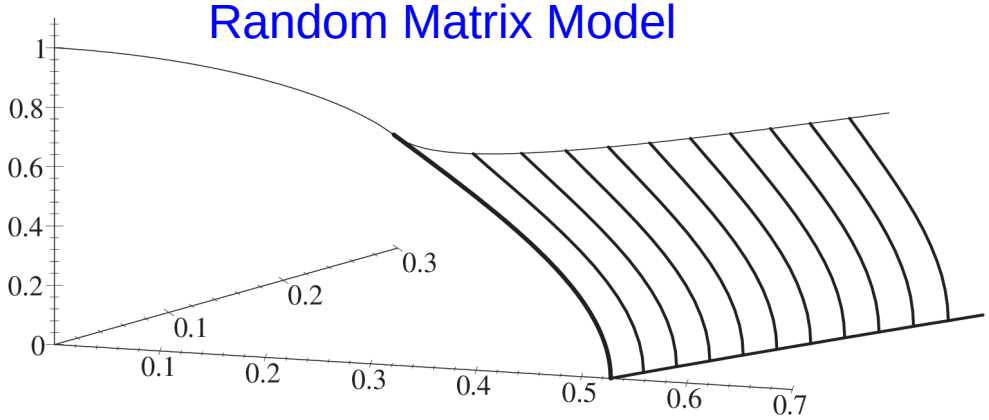
Crossover, chiral phase transition at $\mu_B = 0$ and the (tri)-critical point at $\mu_B > 0$

QCD



M. Stephanov, Phys. Rev. D73 (2006) 094508

Random Matrix Model



A. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov, J.J.,M. Verbaarschot, Phys. Rev. D58 (1998) 096007

Baryon-Strangeness Correlations: A Diagnostic of Strongly Interacting Matter

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The correlation between baryon number and strangeness elucidates the nature of strongly interacting matter, such as that formed transiently in high-energy nuclear collisions. This diagnostic can be extracted theoretically from lattice QCD calculations and experimentally from event-by-event fluctuations. The analysis of present lattice results above the critical temperature severely limits the presence of $q\bar{q}$ bound states, thus supporting a picture of independent (quasi)quarks.

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The principal goal of high-energy heavy-ion collisions is the creation and exploration of a novel state of matter in which the quarks and gluons are deconfined over distances considerably larger than that of a hadron [1]. It had originally been assumed that asymptotic freedom would cause such matter to behave as a plasma of massless quarks and gluons interacting with a relatively weak screened chromodynamic Coulomb force. While this picture is supported qualitatively by the rapid rise in the entropy density at a temperature of about $T_c \simeq 170$ MeV, as obtained by lattice QCD calculations, the fact that the high-T behavior falls somewhat below that of an ideal gas of massless quarks and gluons [2] indicates that the chromodynamic plasma has a more complex structure.

Indeed, recent results from lattice QCD calculation on spectral functions [3–6] suggest the presence of bound, color-neutral states well above T_c. This has led to the suggestion that at moderate temperatures, $T \simeq 1 - 2T_c$, the system is composed of medium-modified (massive) quarks and gluons together with their (many and possibly colored) bound states [7–10].

Furthermore, jet quenching observed at the Relativistic Heavy-Ion Collider (RHIC) suggests that the matter formed is not hadronic [11]. Moreover, the RHIC collisions exhibit strong elliptic and radial collective flows that are consistent with predictions by ideal fluid dynamics [12,13]. If ideal hydrodynamics is indeed the correct framework, rapid thermalization must occur and this in turn would seem to rule out a weakly interacting plasma. Thus, the nature of the matter being created at RHIC is still not clarified and the purpose of this Letter is to offer a novel diagnostic tool for elucidating this issue by probing the relevant degrees of freedom and their correlations.

In particular, we will argue that the correlation between the strangeness S and the baryon number B provides a useful diagnostic for the presence of strong correlations between quarks and antiquarks. In order to understand this, consider first a situation in which the basic degrees of freedom are weakly interacting quarks and gluons. Then strangeness is carried exclusively by the s and $\bar{s}$ quarks which in turn carry baryon number in strict proportion to their strangeness, $B_s = -\frac{1}{3}S_s$, thus rendering strangeness and baryon number strongly correlated. This feature is in stark contrast to a hadron gas in which the relation between strangeness and baryon number is less intimate. For example, at small baryon chemical potential the strangeness is carried primarily by kaons, which have no baryon number.

These elementary considerations lead us to introduce the following correlation coefficient,

$$C_{BS} \equiv -3\frac{\sigma_{BS}}{\sigma_S^2} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} = -3\frac{\langle BS\rangle}{\langle S^2\rangle}. \quad (1)$$

The average $\langle\cdot\rangle$ is taken over a suitable ensemble of events and the last expression uses the fact that $\langle S\rangle$ vanishes. When the active degrees of freedom are individual quarks, the total strangeness is $S = \nu_{\bar{s}} - \nu_s$, while the baryon number can be expressed as $B = \frac{1}{3}(U + D - S)$, where $U = \nu_u - \nu_{\bar{u}}$ is the upness and $D = \nu_d - \nu_{\bar{d}}$ is the downness. Thus, if the flavors are uncorrelated, we have $\sigma_{BS} = -\frac{1}{3}\sigma_S^2$ and C_{BS} is unity.

In a gas of hadron resonances, the total baryon number is $B = \sum_k n_k B_k$ and its total strangeness is $S = \sum_k n_k S_k$, where the species k has baryon number B_k and strangeness S_k. If the multiplicities $\{n_k\}$ are uncorrelated, as is generally assumed in the standard statistical treatment [14], C_{BS} may be expressed in terms of the associated multiplicity variances $\sigma_k^2 \equiv \langle n_k^2\rangle - \langle n_k\rangle^2 \approx \langle n_k\rangle$,

$$C_{BS} = -3\frac{\sum_k \sigma_k^2 B_k S_k}{\sum_k \sigma_k^2 S_k^2} \approx -3\frac{\sum_k \langle n_k\rangle B_k S_k}{\sum_k \langle n_k\rangle S_k^2}, \quad (2)$$

where the approximate expression holds for Poisson statistics. Thus, in a gas of uncorrelated hadrons, the numerator receives contributions from only (strange) baryons (and antibaryons), while the denominator receives contributions also from (strange) mesons,

$$C_{BS} \approx 3\frac{\langle\Lambda\rangle + \langle\bar{\Lambda}\rangle + \cdots + 3\langle\Omega^-\rangle + 3\langle\bar{\Omega}^+\rangle}{\langle K^0\rangle + \langle\bar{K}^0\rangle + \cdots + 9\langle\Omega^-\rangle + 9\langle\bar{\Omega}^+\rangle}. \quad (3)$$

While Eq. (1) [or its more explicit version (6)] holds

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Conserved charge correlations

baryon number – strangeness correlations

$$\frac{\chi_{11}^{BS}(T,\mu_B)}{\chi_2^S(T,\mu_B)} \equiv \frac{\langle BS\rangle}{\langle S^2\rangle}$$

$$C_{BS} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} = -3\frac{\langle BS\rangle}{\langle S^2\rangle}$$

Free Quark Gas

$$C_{BS} = -3\frac{\langle BS\rangle}{\langle S^2\rangle} = -3\frac{\frac{\pm 1}{3}\cdot(\mp 1)}{(\pm 1)^2} = 1$$

Hadron Resonance Gas (HRG)

$$C_{BS} \approx 3\,\frac{\langle\Lambda\rangle + \langle\bar{\Lambda}\rangle + \ldots + 3\langle\Omega^-\rangle + 3\langle\bar{\Omega}^+\rangle}{K^0 + \bar{K}^0 + \ldots + 9\langle\Omega^-\rangle + 9\langle\bar{\Omega}^+\rangle}$$

Net quark-number fluctuations

C.R.Allton et al, PRD71,054508 (2005)

perturbation theory: $\frac{\chi_{ff}(T,\mu)}{T^2} = \frac{\partial^2\Omega(T,\mu)}{\partial(\mu_f/T)^2}$, $\frac{\chi_{fk}(T,\mu)}{T^2} = \frac{\partial^2\Omega(T,\mu)}{\partial(\mu_f/T)\partial(\mu_k/T)}$

$$\frac{p}{T^4}(T,\mu) = \Omega^{(0)}(T,\mu) + g^2\,\Omega^{(2)}(T,\mu) + g^3\,\Omega^{(3)}(T,\mu) + \mathcal{O}(g^4)$$

A. Vuorinen, PRD68, 054017 (2003)

$$\frac{p_{SB}}{T^4} = \Omega^{(0)}(T,\mu) = \frac{8\pi^2}{45} + \sum_{f=u,d,..}\left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_f}{T}\right)^4\right] ,$$

$$\Omega^{(2)}(T,\mu) = -\left(\frac{1}{6} + \frac{5n_f}{72} + \frac{1}{4\pi^2} + \frac{1}{8\pi^4}\sum_{f=u,d,..}\left(\frac{\mu_f}{T}\right)^4\right) ,$$

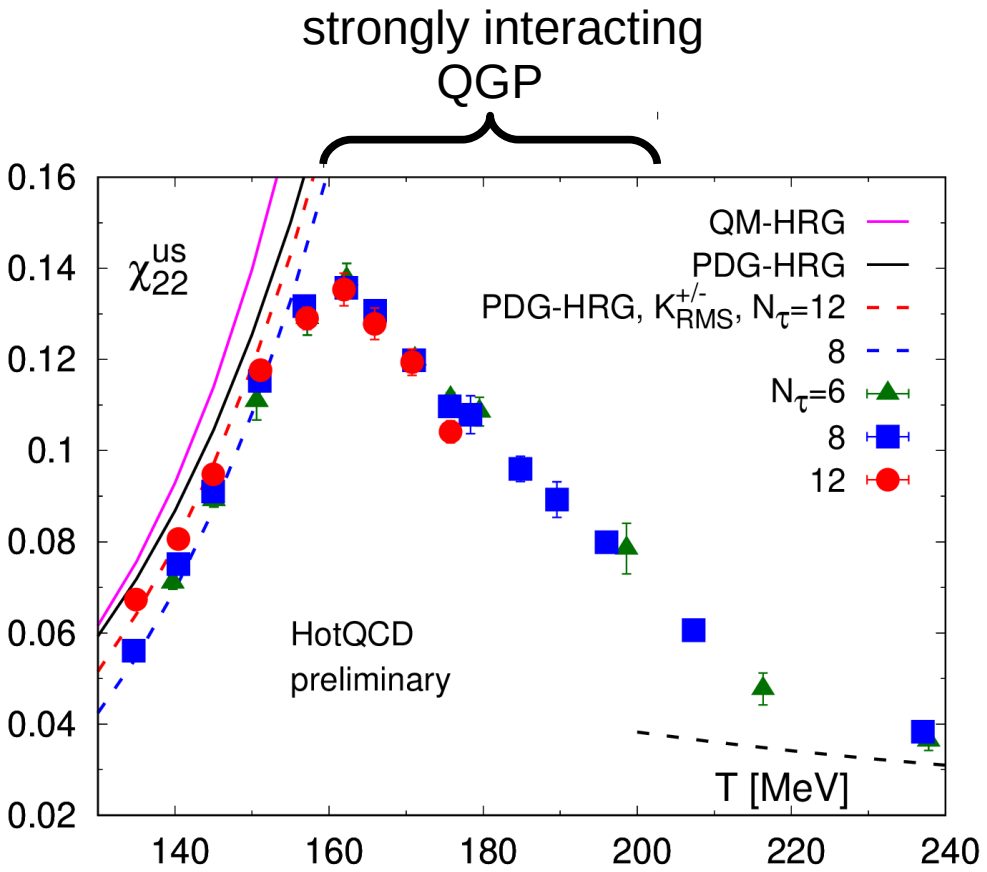
$$\Omega^{(3)}(T,\mu) = \frac{1}{6\pi}\left(\frac{m_E}{gT}\right)^3 = \frac{1}{6\pi}\left(1 + \frac{n_f}{6} + \frac{1}{2\pi^2}\sum_{f=u,d,...}\left(\frac{\mu_f}{T}\right)^2\right)^{3/2}$$

$$\frac{\chi_{ff}(T,\mu)}{T^2} = 1 + \frac{3}{\pi^2}\left(\frac{\mu_f}{T}\right)^2 + \mathcal{O}(g^2) \qquad \chi_4^f - \chi_4^{f,ideal} \sim \mathcal{O}(g^2)$$

$$\frac{\chi_{fk}(T,\mu)}{T^2} = \frac{g^3}{2\pi^5}\left(1 + \frac{n_f}{6} + \frac{1}{2\pi^2}\sum_{f=u,d,...}\left(\frac{\mu_f}{T}\right)^2\right)^{-1/2}\frac{\mu_f}{T}\frac{\mu_k}{T} \qquad \chi_{22}^{fk} \sim \mathcal{O}(g^3)$$

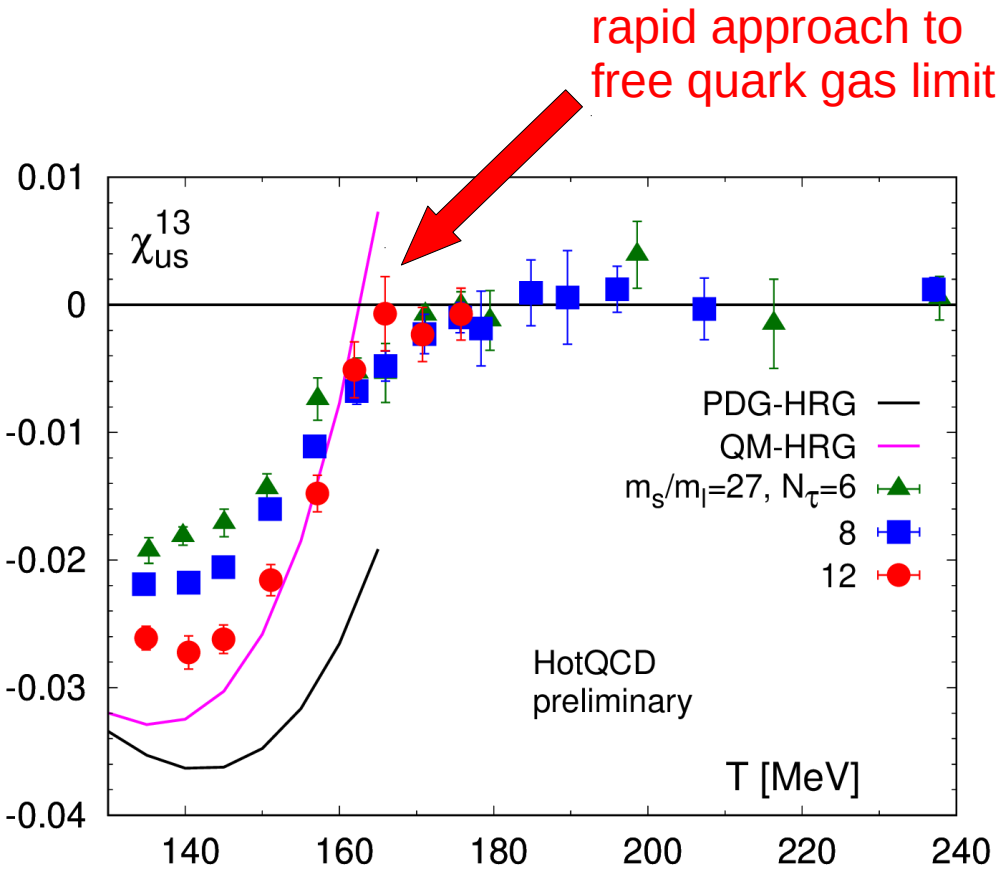
$$\frac{\chi_{fk}(T,0)}{T^2} \simeq -\frac{5}{144\pi^6}\,g^6\ln 1/g \qquad \chi_{13}^{fk} \sim \mathcal{O}(g^6\ln g)$$

Net quark-number fluctuations



$\mathcal{O}(g^3)$

(Debye mass)



$\mathcal{O}(g^6 \ln g)$

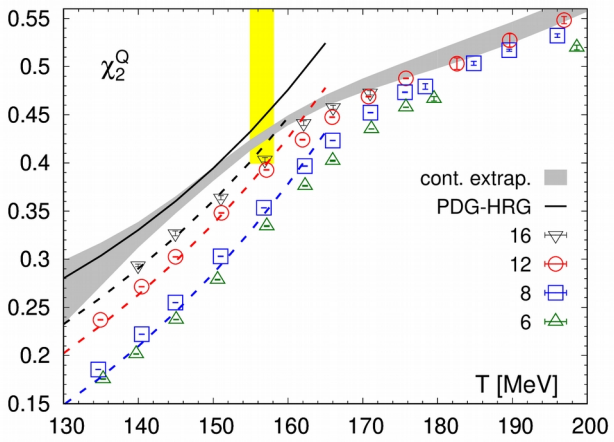
2nd order cumulants: fluctuations and correlations

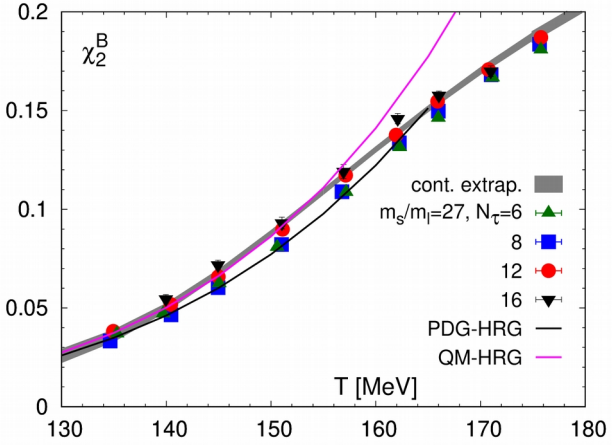
$m_u \simeq m_d$ ⇒ two constraints: only 4 out of 6 2nd order observables are independent

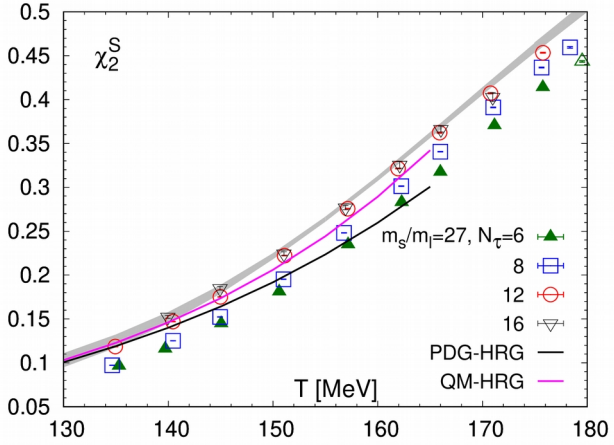
$T_c = (156.5 \pm 1.5)\mathrm{MeV}$

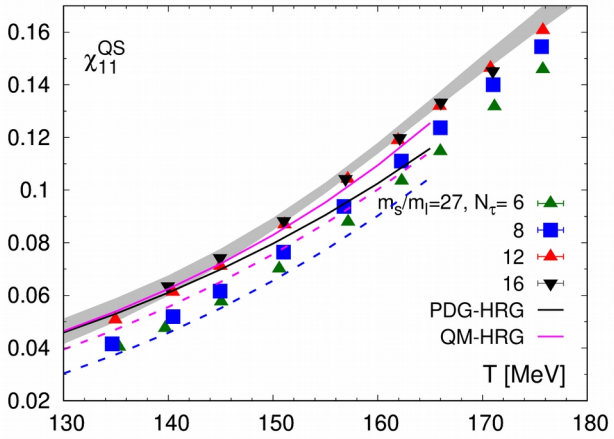
$\chi_2^B = 2\chi_{11}^{BQ} - \chi_{11}^{BS}$

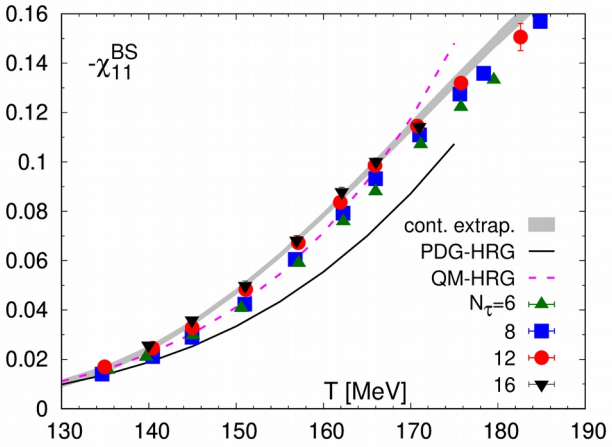
$\chi_2^S = 2\chi_{11}^{QS} - \chi_{11}^{BS}$

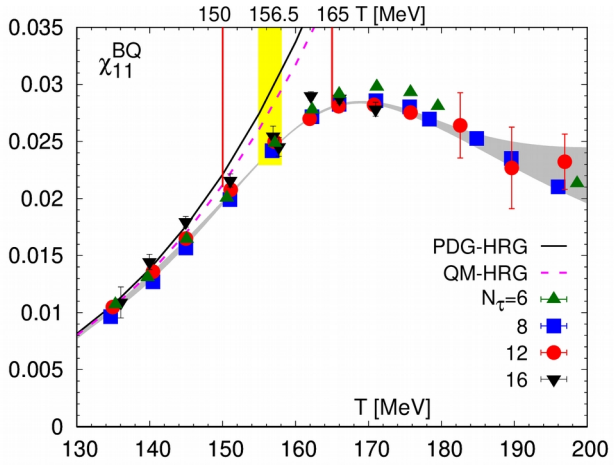












– continuum extrapolated results for all 2nd order cumulants in (2+1)-flavor QCD

A. Bazavov et al. (Bielefeld-BNL-CCNU)
arXiv:1701.04325 + preliminary HotQCD updates