CHIRAL SYMMETRY in NEUTRON STARS

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- Introductory notes about scales: inside the nucleon
- Nuclear chiral dynamics and neutron stars
  - In-medium chiral effective field theory
  - Chiral nucleon-meson field theory and functional renormalization group (with Matthias Drews)
- Neutron star matter as a relativistic Fermi liquid
  - Equation-of-state and Landau parameters (with Bengt Friman)

CHIRAL SYMMETRY

- (almost) **massless u- and d-quarks :**

$$\mathbf{SU}(2)_R \times \mathbf{SU}(2)_L$$

- Low energy : **spontaneous chiral symmetry breaking**
- **PIONS** as (almost) massless **Nambu-Goldstone bosons**

- **Symmetry breaking scale :**

$$\mathbf{\Lambda}_\chi = 4\pi\, \mathbf{f}_\pi \sim \mathbf{1\,GeV}$$

- **Pion decay constant :**

$\mathbf{f}_\pi^{(0)} \simeq \mathbf{86\,MeV}$ (chiral limit) $\mathbf{f}_\pi \simeq \mathbf{92\,MeV}$ (empirical)

- **Order parameter :**

- **Chiral condensate and sigma field**

$$\mathbf{f}_\pi^2 = -\frac{\mathbf{m_u + m_d}}{\mathbf{2\,m_\pi^2}} \langle \bar{\mathbf{q}}\mathbf{q} \rangle = \langle \sigma \rangle^2$$

CHIRAL SYMMETRY RESTORATION

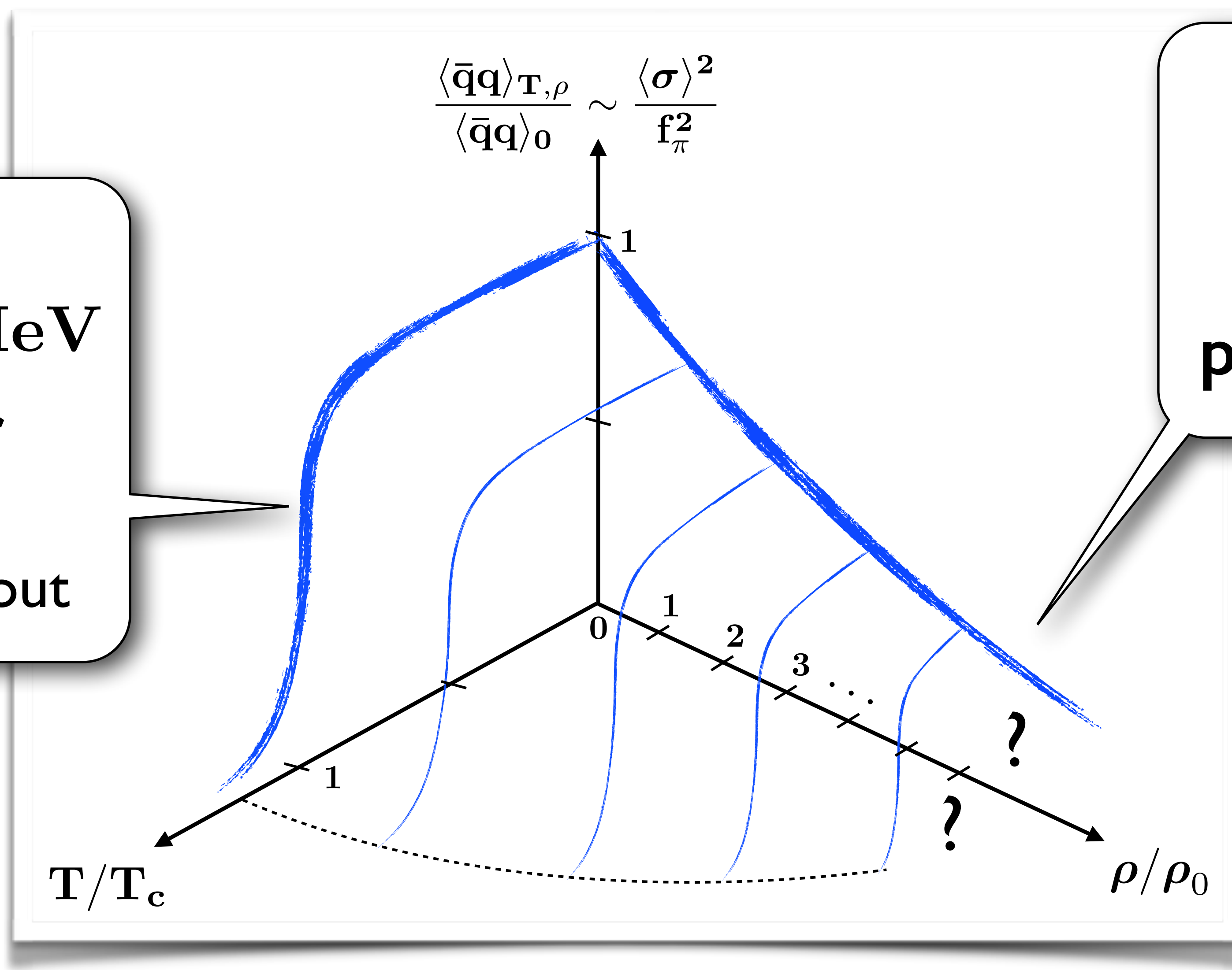
- **Thermodynamics of chiral order parameter :** dependence on **temperature and baryon chemical potential / baryon density**
- From **Nambu-Goldstone** to **Wigner-Weyl** realisations of chiral symmetry

$\mu_B = 0$

$T_c \simeq 156$ MeV

crossover

Lattice QCD
HIC Freeze-out



$\mu_B \sim M_N$

low $-$ T

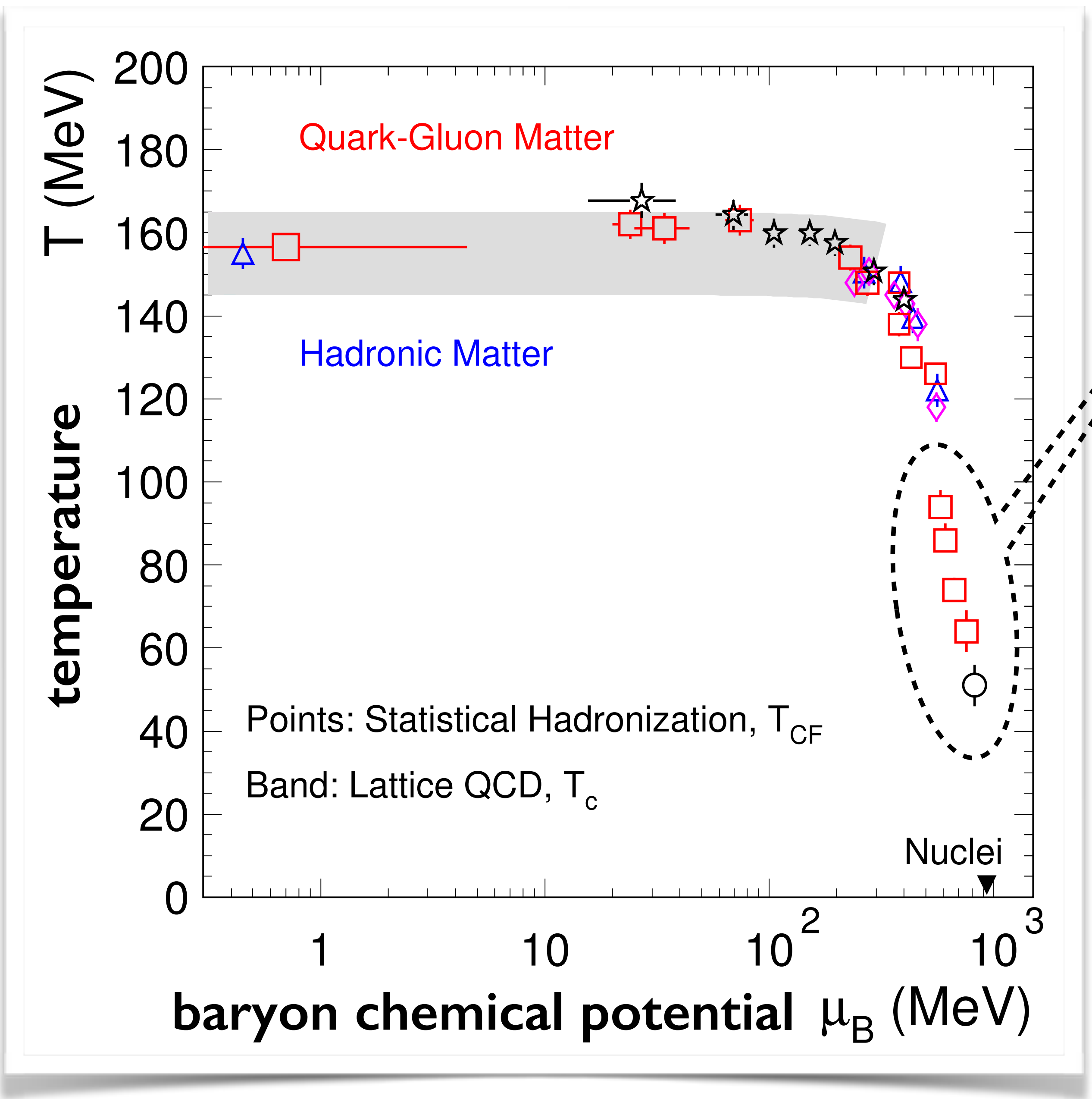
crossover ?

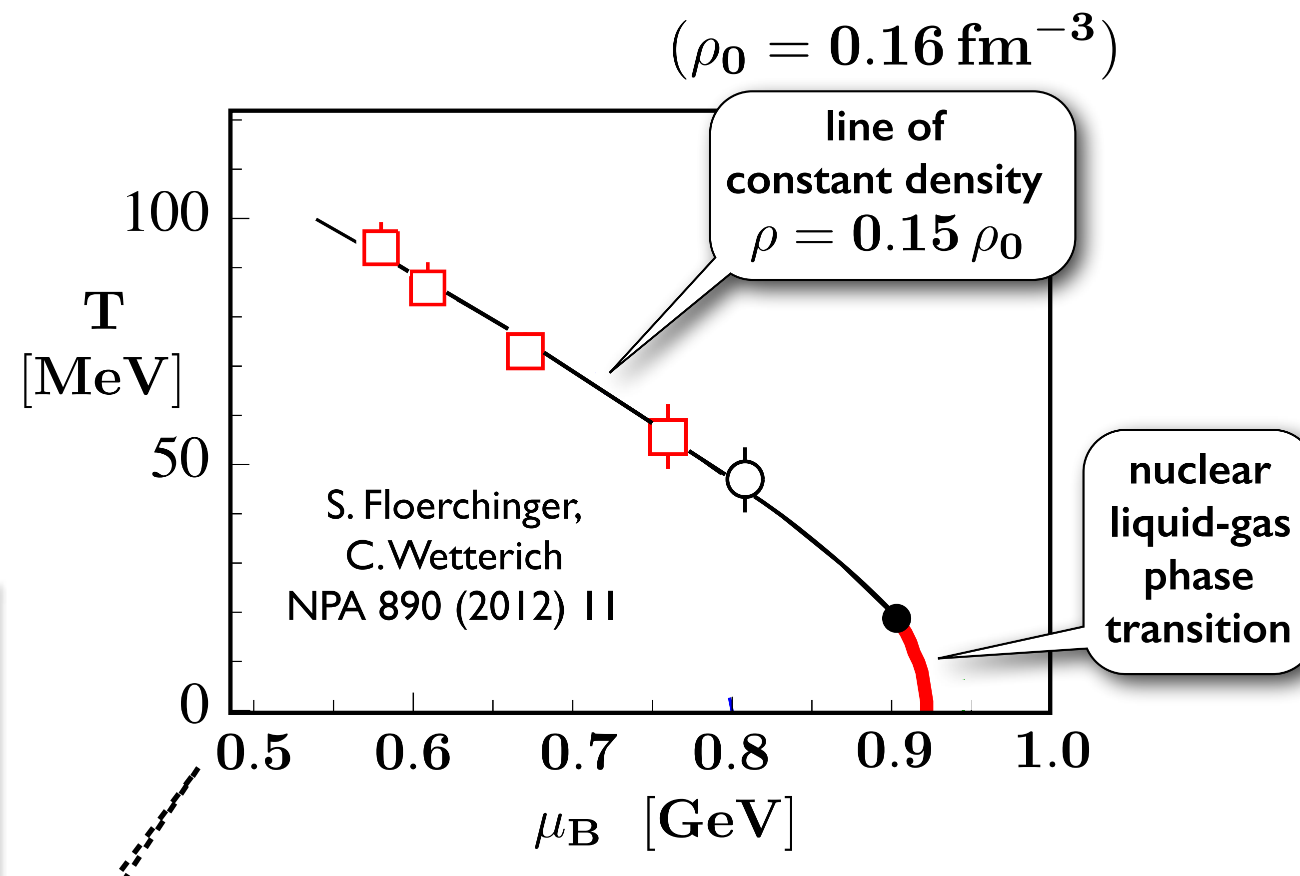
phase transition ?

**(P)NJL type models,
Quark-meson models,
...
suggest 1st order
chiral phase transition,
BUT ...**

CHEMICAL FREEZE-OUT

A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel
Nature 561 (2018) 321





- Freeze-out temperature

  $T_{CF} \simeq 156$ MeV

  at $\mu_B \simeq 0$

- Low-temperature freeze-out trajectory : NOT a first-order phase transition line

CHIRAL SYMMETRY RESTORATION in the CORE of NEUTRON STARS ?

- Chiral Nucleon-Meson field theory and **Functional Renormalization Group**
- Chiral order parameter : $\langle\sigma\rangle_{T,\mu} = f_\pi^*(T,\mu)$

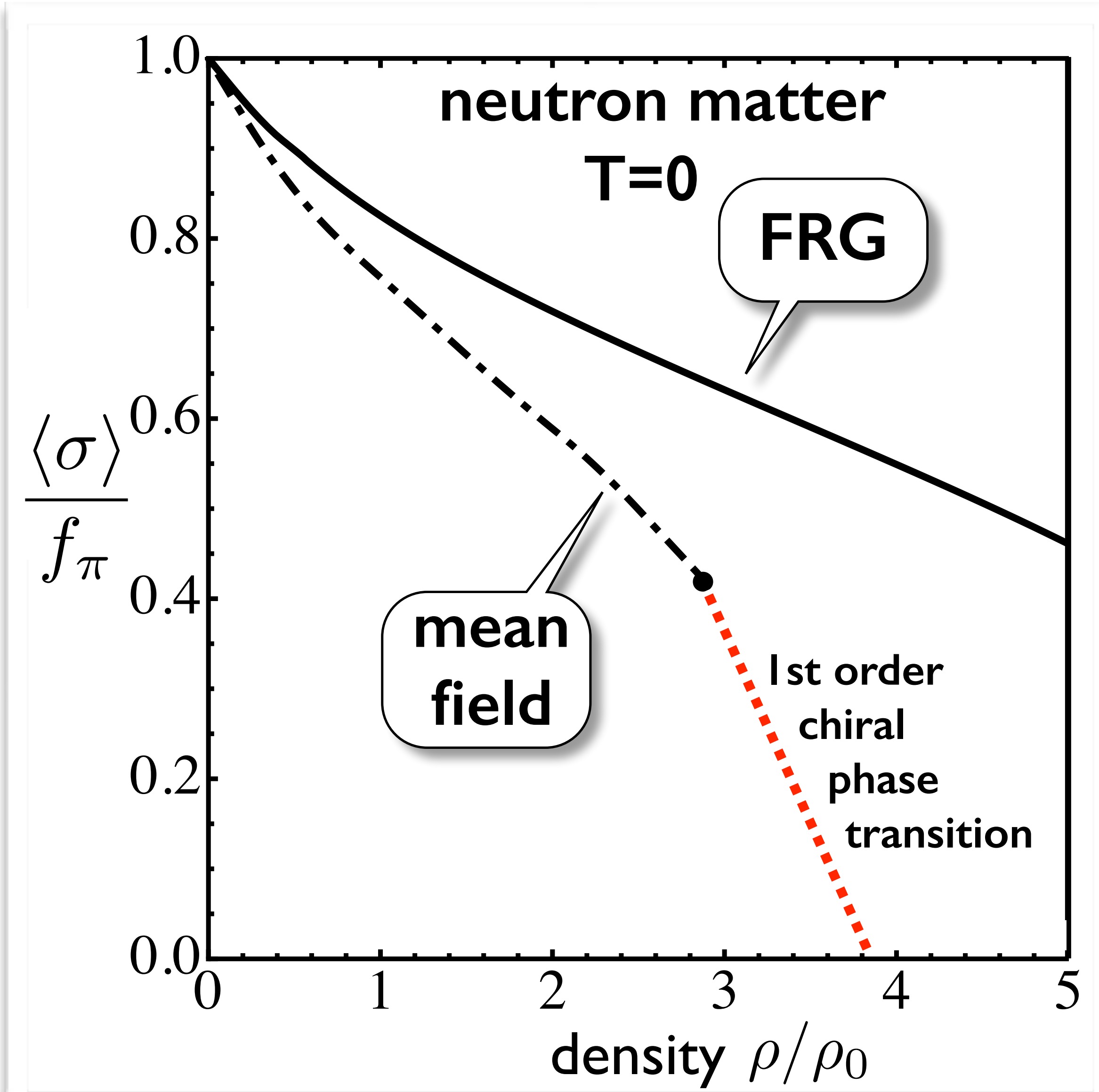
Important role of

fluctuations beyond mean-field

multi-pion loops

nucleon-hole excitations

many-particle correlations



NO chiral phase transition

$\rho \lesssim 6\,\rho_0$

1.
Introductory Notes about Scales: the Nucleon

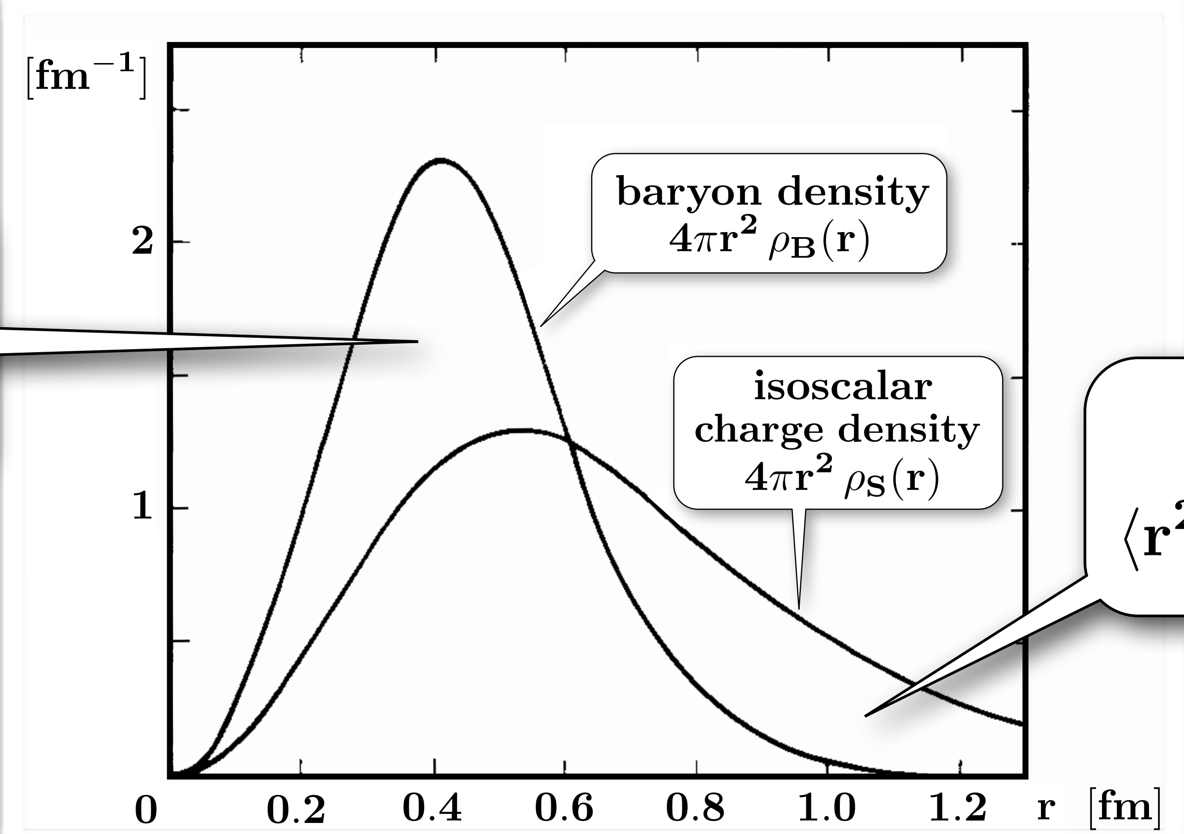
Reminder of **SIZES** : **CHIRAL SOLITON MODEL** of the **NUCLEON**

Spontaneously broken chiral symmetry + localisation (confinement)

- **NUCLEON** : compact valence quark core + mesonic cloud

baryonic core

$$\langle r^2 \rangle_B^{1/2} \simeq 0.5 \text{ fm}$$



N. Kaiser, U.-G. Meißner, W.W.

Nucl. Phys. A466 (1987) 685

mesonic cloud

$$\langle r^2 \rangle_{E,isoscalar}^{1/2} \simeq 0.8 \text{ fm}$$

- **Separation of scales** between **compact baryonic core** and (multi-)**pion cloud**

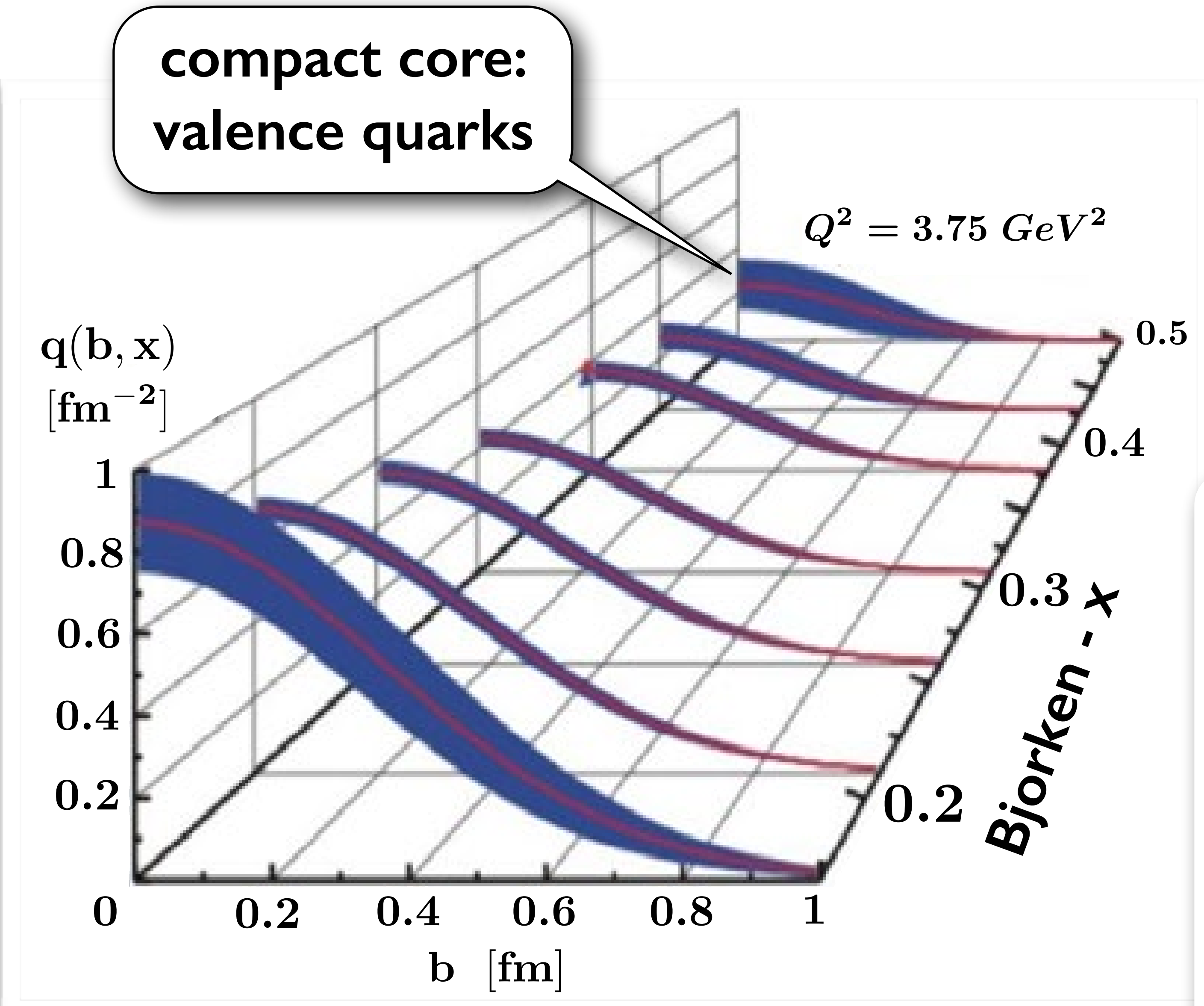
$$\left(\frac{R_{core}}{R_{cloud}}\right)^3 \sim 0.2$$

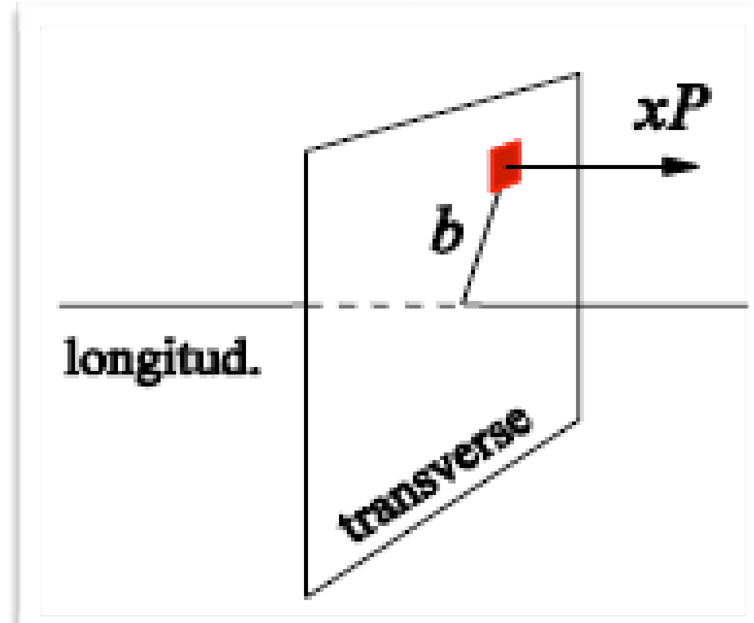
Technische Universität München

Transverse distributions of quarks in the proton

Deeply Virtual Compton Scattering @ JLab

R. Dupré, M. Guidal, M.Vanderhaeghen
Phys. Rev. D95 (2017) 011501
Rep. Prog. Phys. 76 (2013) 066202





$$\langle \mathrm{b}^2 \rangle \simeq 0.16\,\mathrm{fm}^2 \cdot \ln(1/\mathrm{x_B})$$

- **Valence quark region:**

$$\frac{1}{3} < \mathrm{x_B} < \frac{1}{2}$$

- **Core size:**

$$\mathrm{R}_{core} = \sqrt{\frac{3}{2}\langle \mathrm{b}^2 \rangle} \sim 0.4 - 0.5\ \mathrm{fm}$$

New developments:

Form factors of the **Energy-Momentum Tensor** of the **Nucleon**

Energy Density Momentum Density

$$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}$$

shear forces

pressure

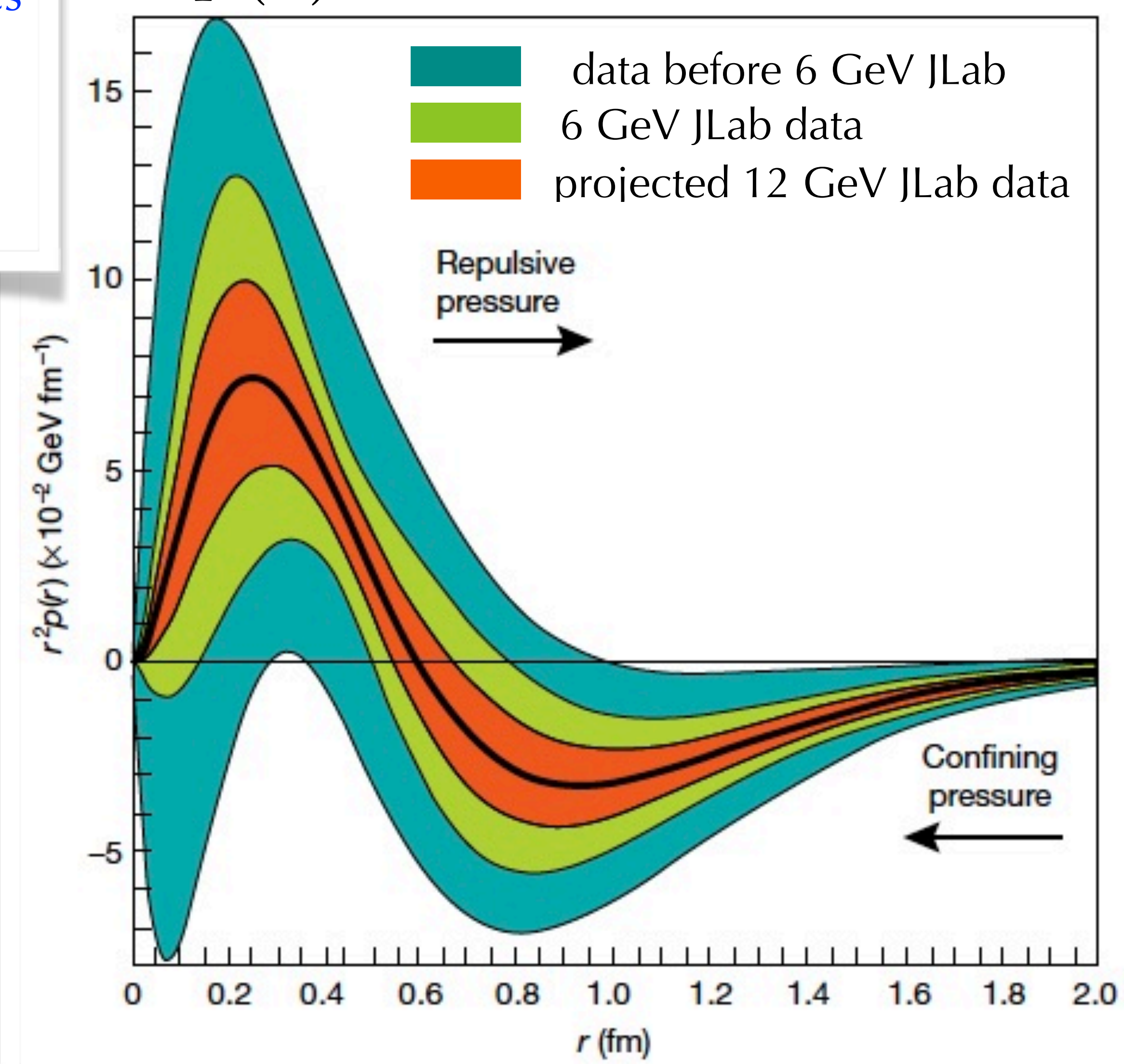
Energy Flux Momentum Flux

Deeply Virtual Compton Scattering @ JLab

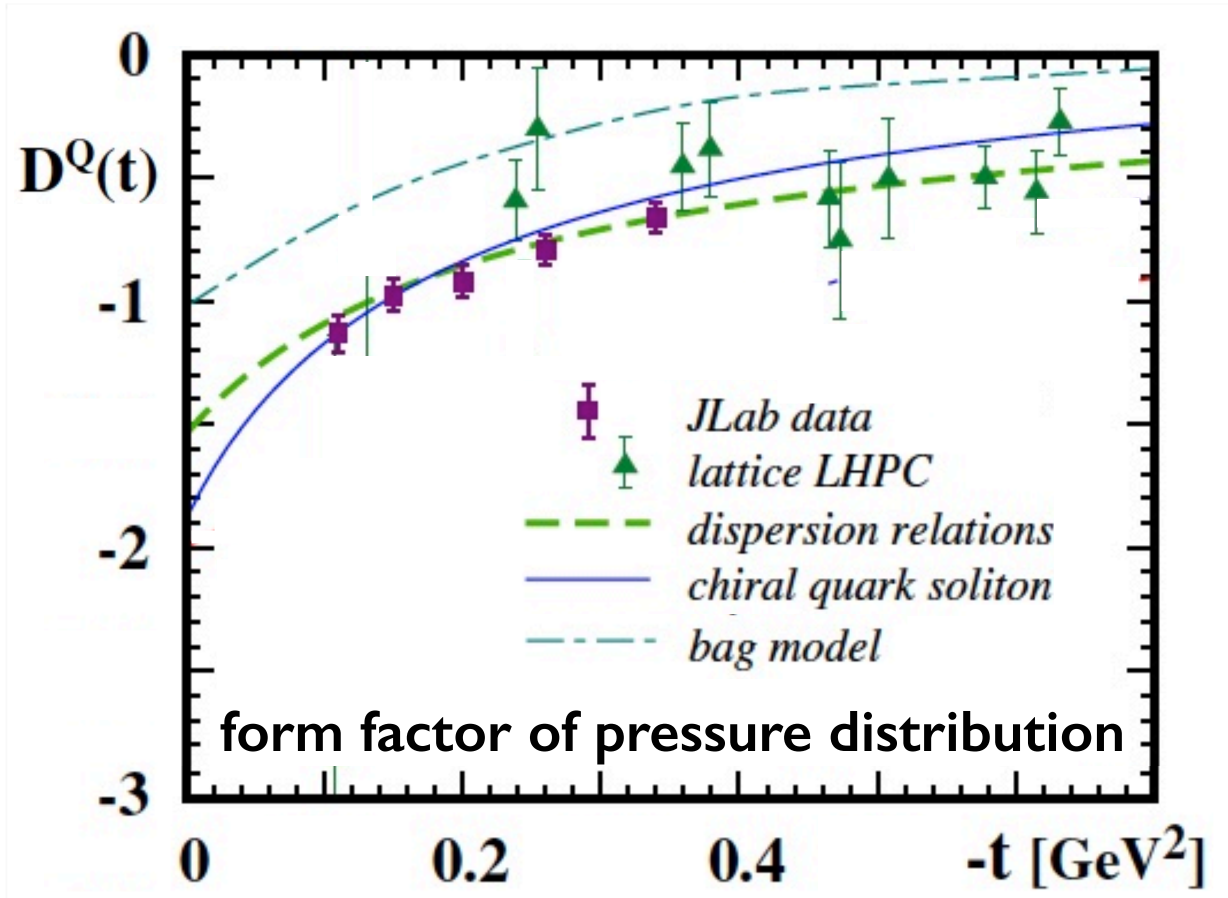
pressure distribution in the proton

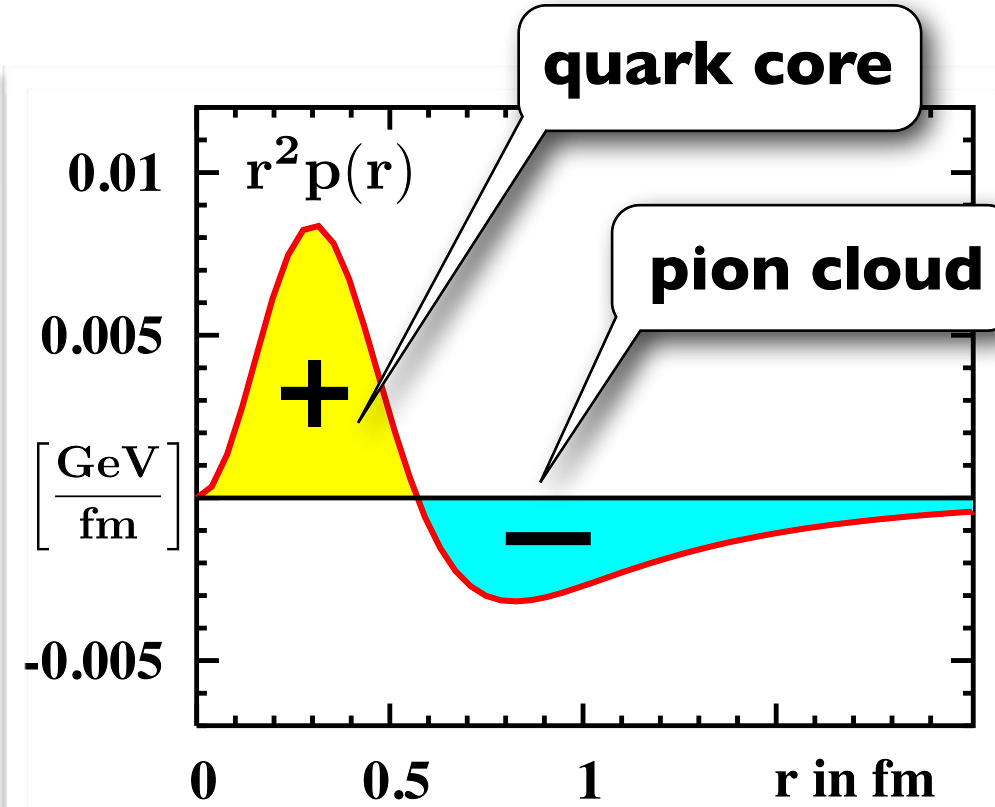
V.D. Burkert, L. Elouadrhiri, F.X. Girod
Nature 557 (2018) 7705

$r^2p(r)$ radial pressure distribution



Pressure Distribution in the **Nucleon : Chiral Quark-Soliton**





K. Goeke et al.
Phys. Rev. D75 (2007) 094021

- deep interior of the nucleon:
  a piece of "quark matter"
- baryon density, pressure and energy density in the nucleon core:

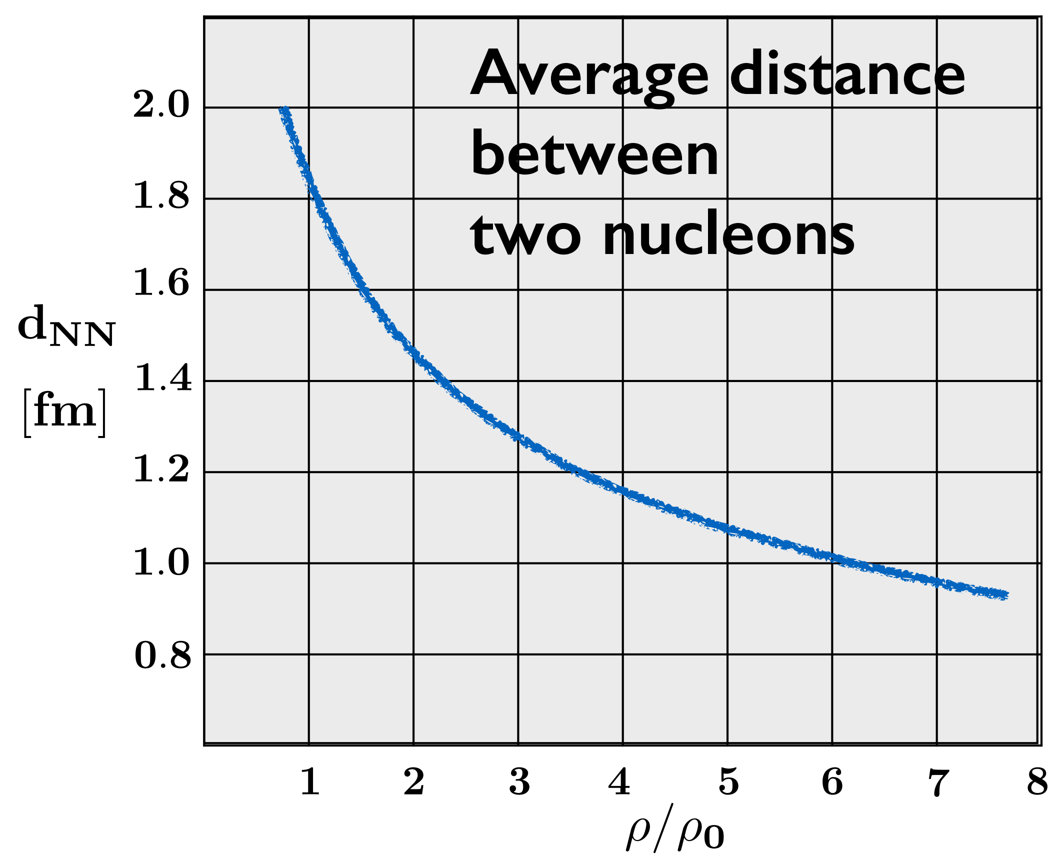
$$\rho_{core} \sim 7\,\rho_0$$

$$\mathrm{P}_{core} \simeq \frac{1}{2}\,\mathcal{E}_{core} \simeq 0.7\ \mathrm{GeV/fm^3}$$

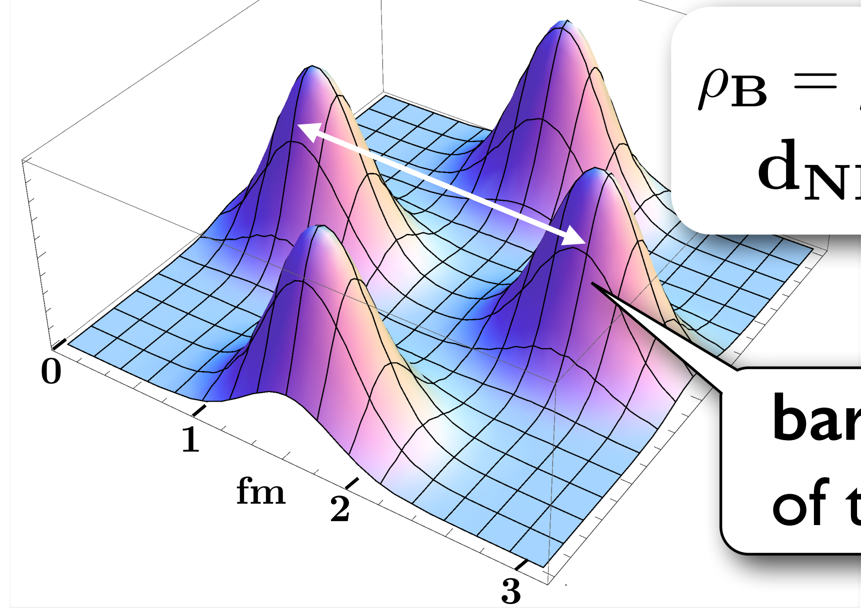
Chiral Quark-Soliton
model
Note again:
compact core
radius R ~ 1/2 fm

M.V. Polyakov, P. Schweitzer
Int. J. Mod. Phys. A33 (2018) 1830025

Densities and Distance Scales in Baryonic Matter



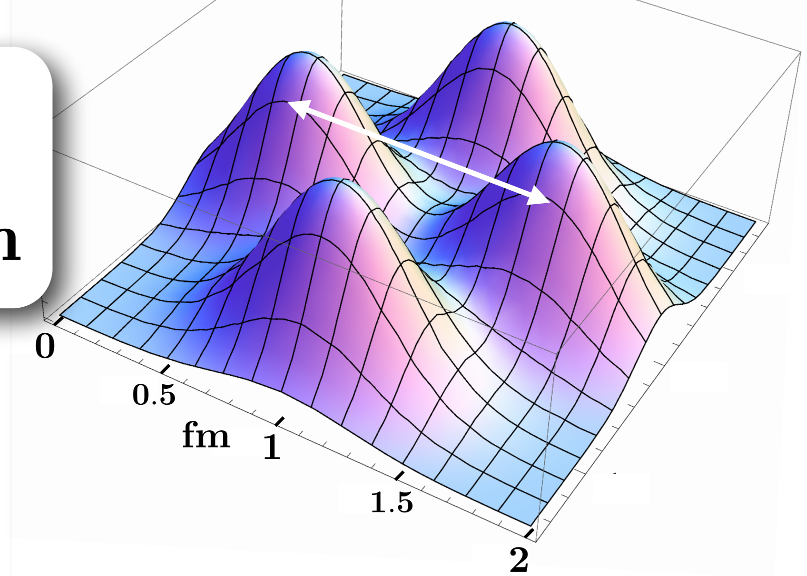
normal nuclear matter



$$\rho_B = \rho_0 = 0.16\ \mathrm{fm}^{-3}$$
$$d_{NN} \simeq 1.8\ \mathrm{fm}$$

baryonic core of the nucleon

neutron star core matter



$$\rho_B \sim 5\,\rho_0$$
$$d_{NN} \sim 1\ \mathrm{fm}$$

- (Multi-)pion fields in space between baryonic sources (ChEFT)
- Quark cores of nucleons overlap (percolate) at baryon densities $\rho_B > 5\,\rho_0$

NUCLEAR FORCES from LATTICE QCD

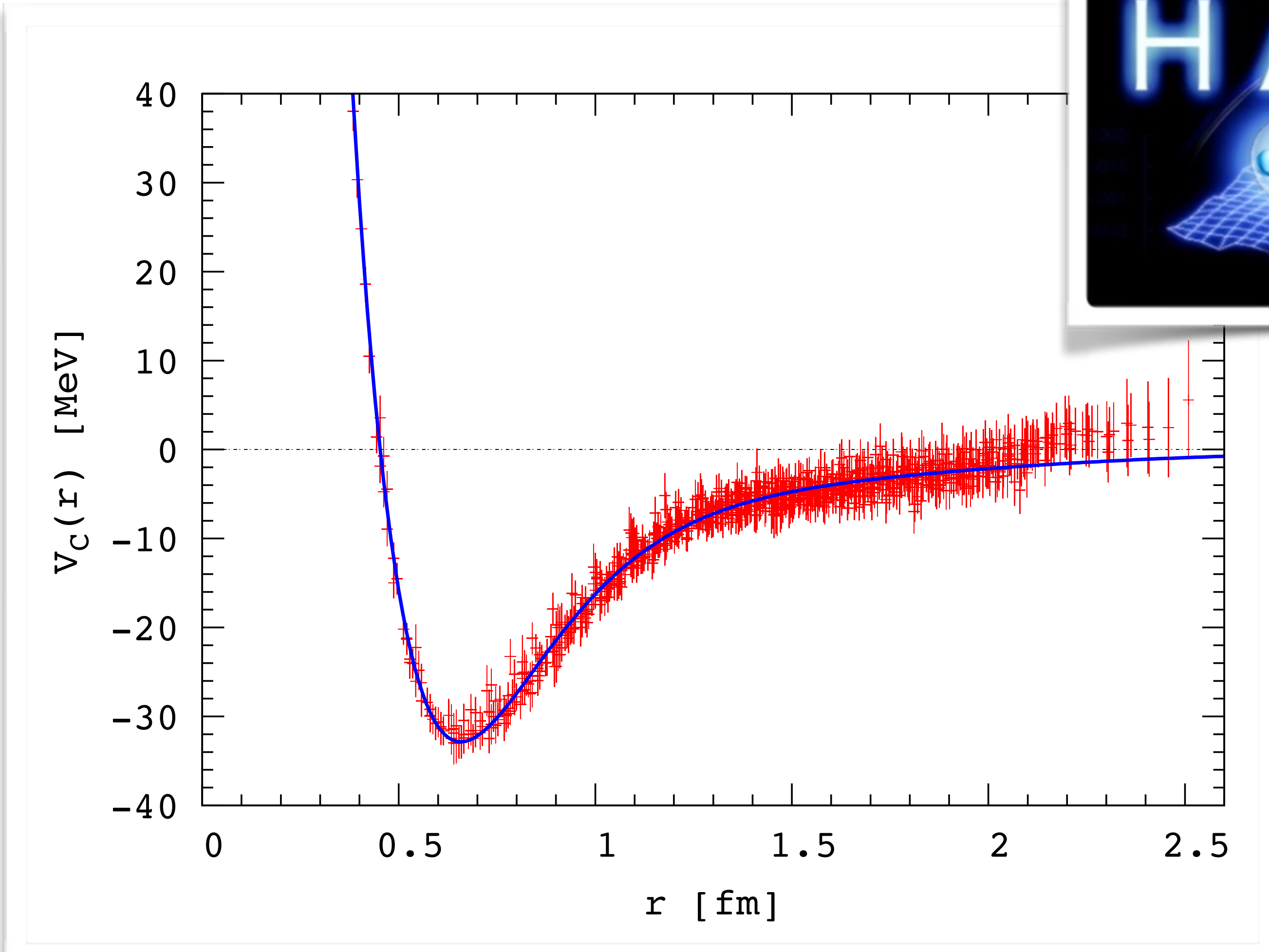
NN Central Potential (S = 0, I = 1)
deduced from LQCD two-nucleon (6-quark) correlation function

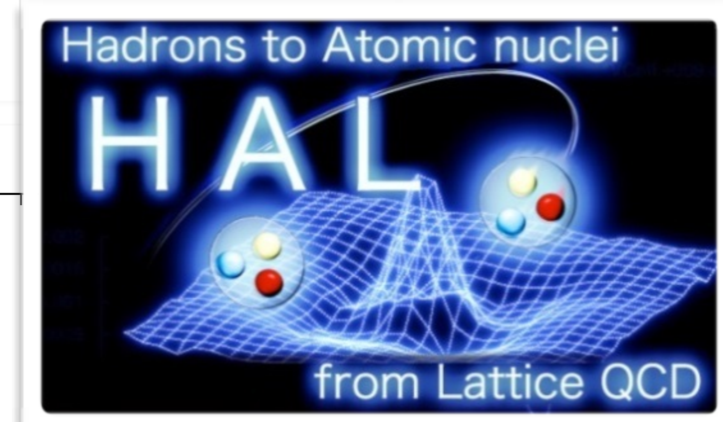
still:
unphysically large
u- and d-quark
masses

…but :
stable 0.5 Fermi
repulsive core

S. Aoki, T. Hatsuda, N. Ishii
Prog. Theor. Phys. 123 (2010) 89

S. Aoki
Eur. Phys. J. A49 (2013) 81





now:

towards
physical
quark
masses

- **Compression of baryonic matter is energetically expensive**

2.

Chiral Effective Field Theory and related approaches to the Nuclear Many-Body Problem and Neutron Star Matter

- **Pions** and **Nucleons** as "active" degrees as freedom in a Nuclear Fermi Sea

- **Perturbative methods:**
  Chiral Effective Field Theory
  and
  Nuclear Many-Body Problem
  (baryon densities $\rho \lesssim 2\,\rho_0$ **)**

- **Non-perturbative methods:**
  Chiral Nucleon-Meson Field Theory
  and
  Functional Renormalisation Group
  (towards higher densities : $\rho \sim 5\,\rho_0$ **)**

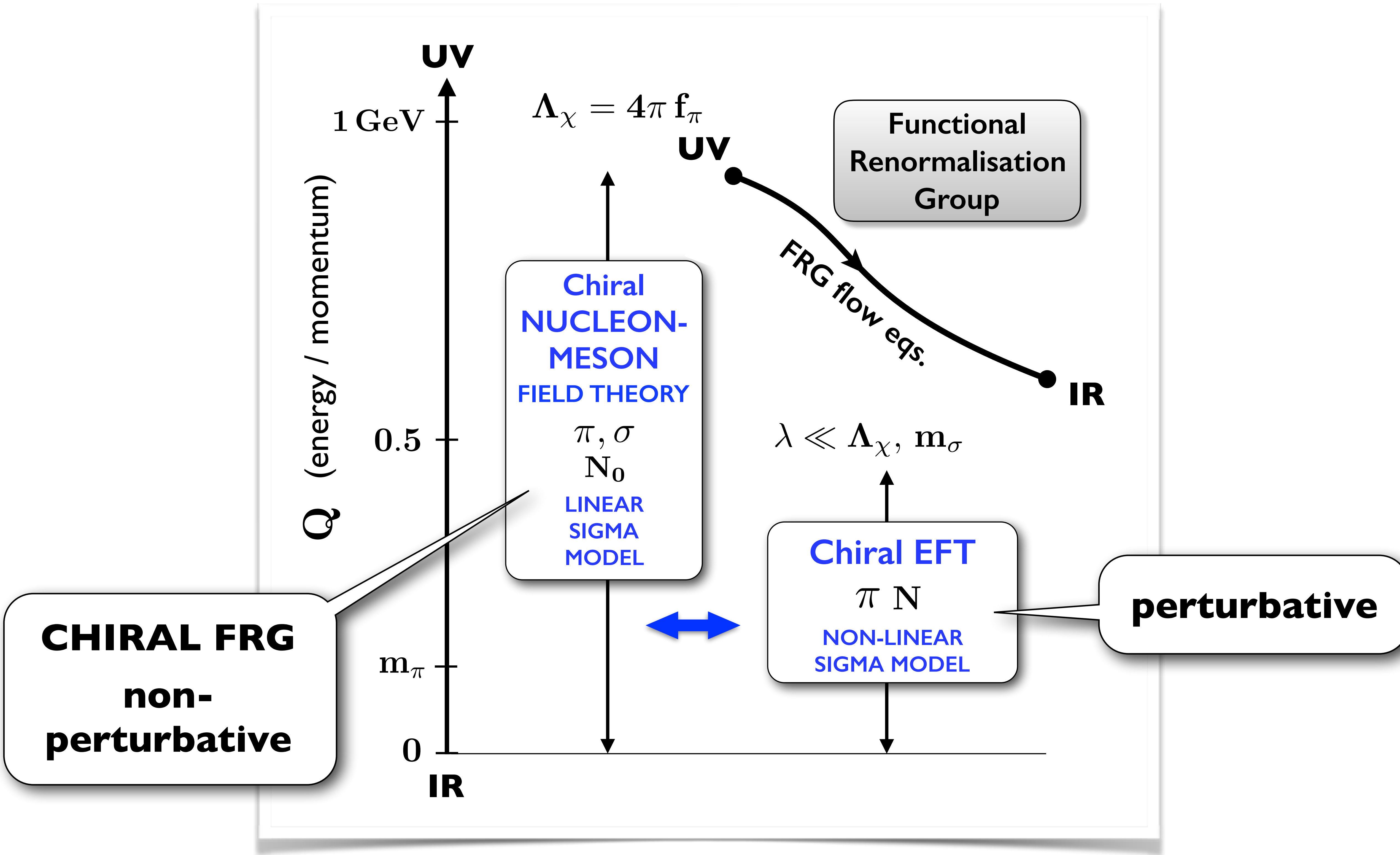
Minimal Conditions to be satisfied by **Chiral Symmetry** - based theories of **Dense Baryonic Matter**

- **Vacuum :**
  - **Low-energy theorems of spontaneously broken CHIRAL SYMMETRY**
    (Gell-Mann, Oakes, Renner ; Goldberger-Treiman ; ...)
  - **Low-energy pion-pion and pion-nucleon scattering**
  - **Realistic NN interaction**
- **Low density** $(\rho \lesssim 2\,\rho_0)$ **:**
  - **Realistic EoS of symmetric nuclear matter and neutron matter**
  - **Asymmetric nuclear matter and symmetry energy**
  - **Nuclear thermodynamics : liquid-gas phase transition**
- **High density :**
  - **Neutron star maximum mass** $\mathrm{M}_{max} \gtrsim 2\ \mathrm{M}_{\odot}$
  - **Neutron star radii** $11\,\mathrm{km} \lesssim \mathrm{R} \lesssim 13\,\mathrm{km}$
  - **Tidal deformability constraints from neutron star mergers (GW signals)**

T. Zhao, J.M. Lattimer : Phys. Rev. D98 (2018) 063020

E.R. Most et al. : Phys. Rev. Lett. 120 (2018) 261103

Theoretical FRAMEWORKS and METHODS



PIONS and NUCLEI
in the context of LOW-ENERGY QCD

- **CONFINEMENT** of quarks and gluons in hadrons
- Spontaneously broken **CHIRAL SYMMETRY**

LOW-ENERGY QCD

At (energy and momentum) scales $\mathrm{Q} < 4\pi\,\mathrm{f}_\pi \sim 1\,\mathrm{GeV}$

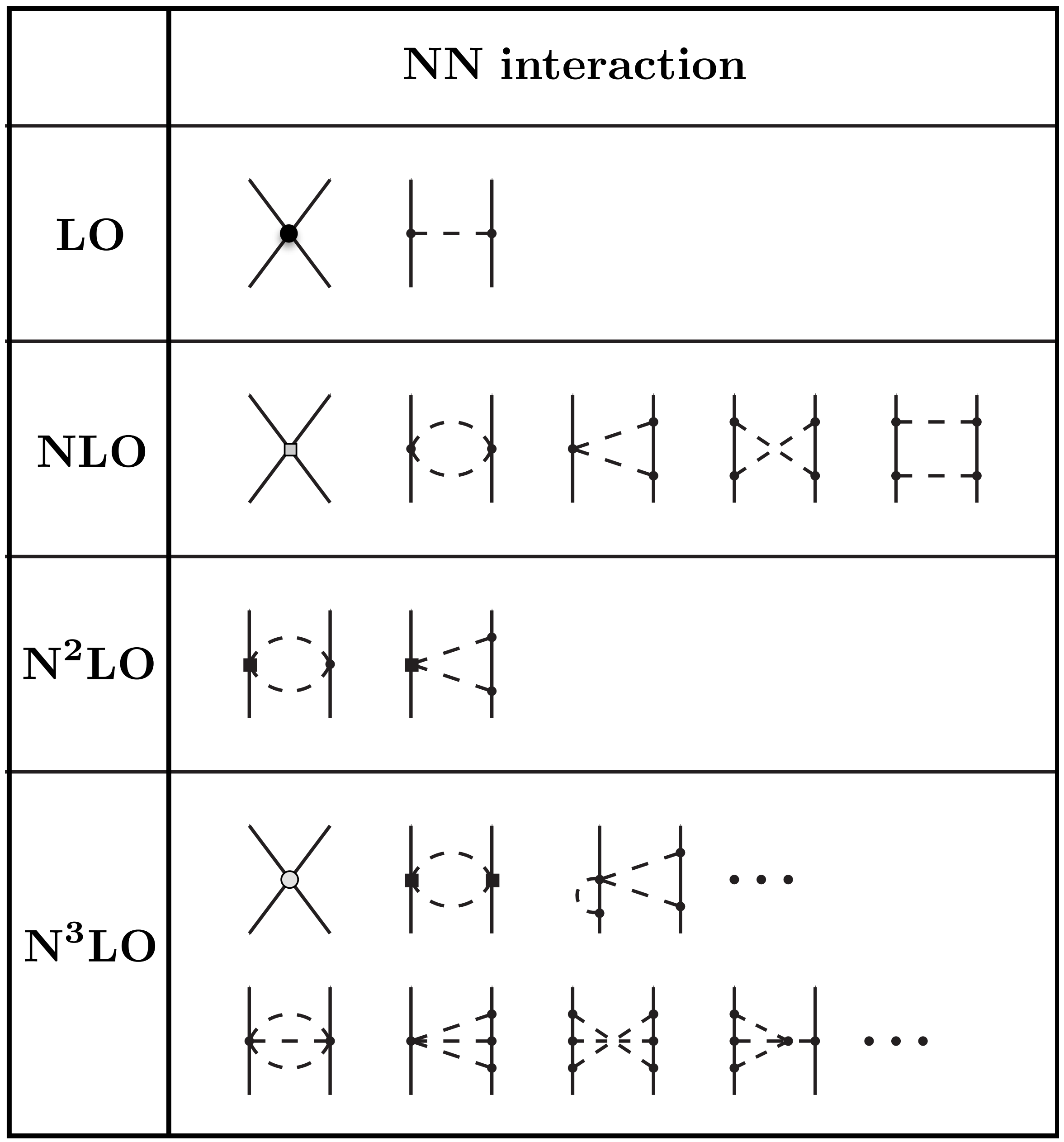
is realised as an

Effective Field Theory

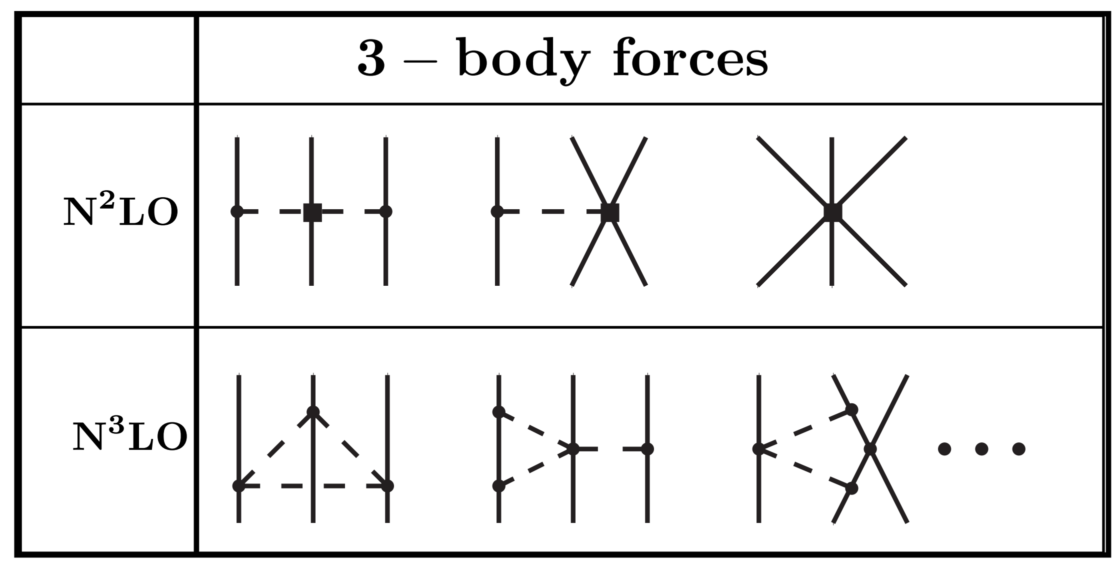
of Nambu-Goldstone Bosons (PIONS) coupled to NUCLEONS as (heavy) Fermion sources

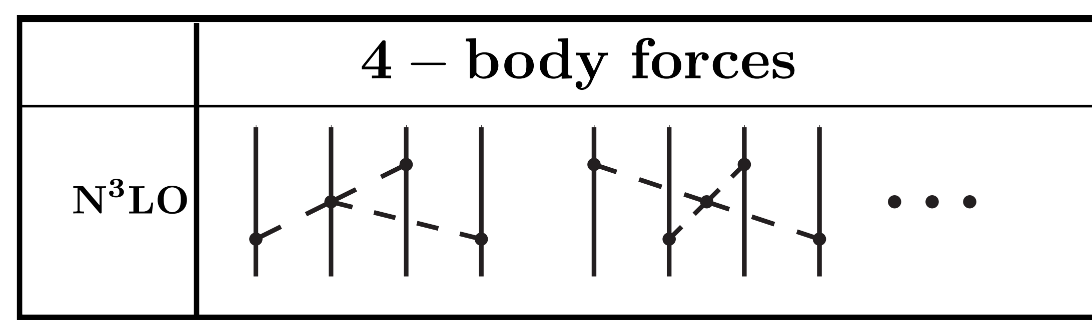
NUCLEON-NUCLEON INTERACTION from CHIRAL EFFECTIVE FIELD THEORY

Weinberg Bedaque & van Kolck Bernard, Epelbaum, Kaiser, Meißner ; . . .



- **Systematically organized hierarchy in powers of $\frac{Q}{\Lambda}$**
  (Q: momentum, energy, pion mass)

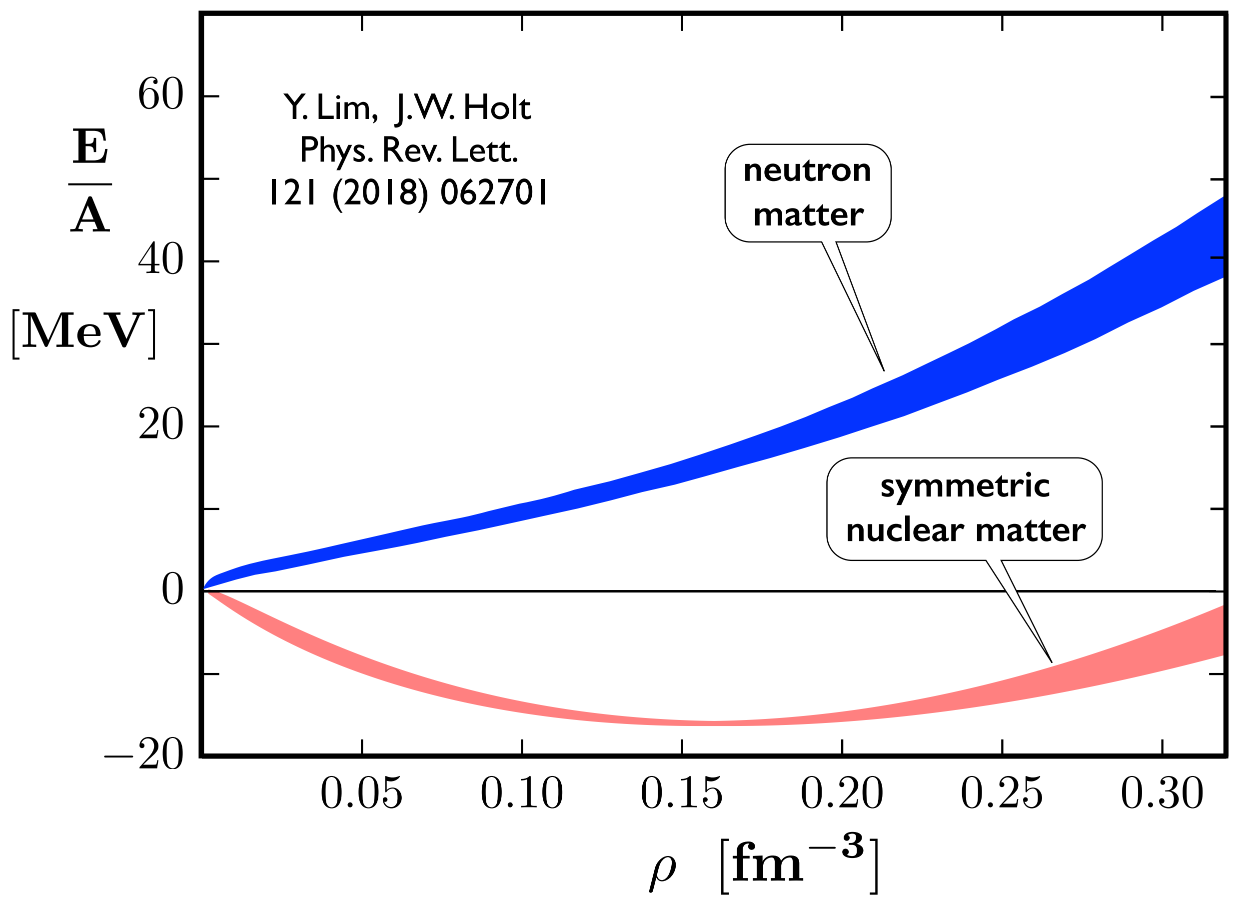




- **NN interaction state-of-the-art: N^4LO plus convergence tests at N^5LO**

NEUTRON and NUCLEAR MATTER from CHIRAL EFT

- N3LO chiral NN interactions + N2LO 3-body forces
- Many-body perturbation theory (3rd order) J.W. Holt, N. Kaiser Phys. Rev. C95 (2017) 034326



Perturbative Chiral EFT :

applicable up to baryon densities

$\rho \sim 2\,\rho_0$

- **Agreement with advanced many-body calculations**
  (e.g. Quantum Monte Carlo computations - S. Gandolfi et al.: EPJ A50 (2014) 10)

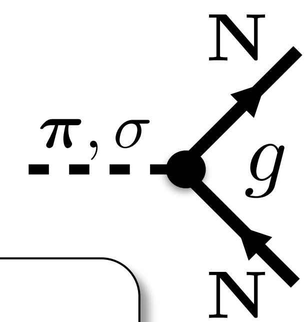
C. Wellenhofer, J.W. Holt, N. Kaiser, W.W.: Phys. Rev. C89 (2014) 064009, C92 (2015) 015801

- **Further recent developments: N4LO** F. Sammarruca et al.: arXiv:1807.06640

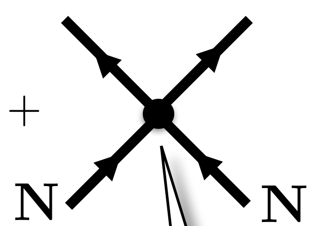
Mesons, Nucleons, Dense Baryonic Matter
and
Functional Renormalisation Group

- **Chiral nucleon - meson Lagrangian**

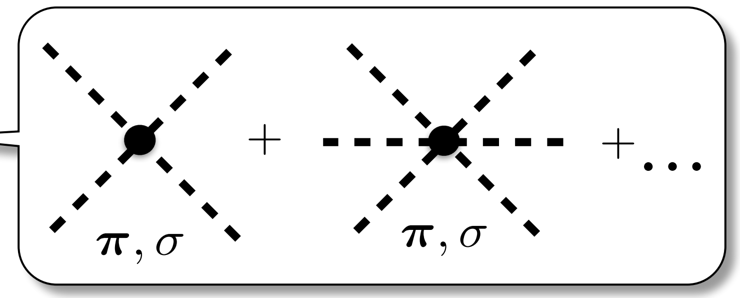
$$\mathcal{L} = \bar{\mathrm{N}}\, i\gamma_\mu \partial^\mu \mathrm{N} + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}\right) +$$



$+$



$- \, \mathcal{U}(\boldsymbol{\pi}, \sigma)$



isoscalar & isovector current-current interactions mediated by heavy vector fields

- **Nambu-Goldstone boson** $\boldsymbol{\pi}$ and "**heavy**" σ

- **Potential** $\mathcal{U}(\sigma, \boldsymbol{\pi})$ **: polynomial** in $\chi = \boldsymbol{\pi}^2 + \sigma^2$ constructed to reproduce **vacuum physics** and **equilibrium nuclear matter**

- **Symmetry-breaking mass term + $\chi^2 \log \chi$ term**

Review: M. Drews, W.W. : Prog. Part. Nucl. Phys. 91 (2017) 347

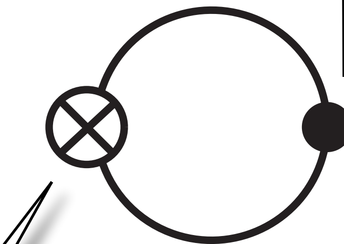
Renormalisation Group strategies

k-dependent action

full propagator

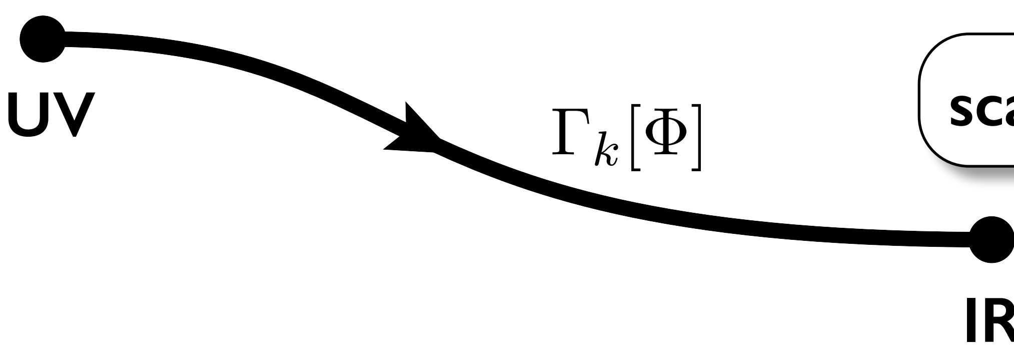
Wetterich's FRG flow equations

$$k\,\frac{\partial \Gamma_k[\Phi]}{\partial k} = \frac{1}{2}\mathrm{Tr}\left[k\frac{\partial R_k}{\partial k}\cdot\left(\Gamma_k^{(2)}[\Phi]+R_k\right)^{-1}\right] =$$



scale regulator R_k

$$\Gamma_{k=\Lambda}[\Phi] = S$$

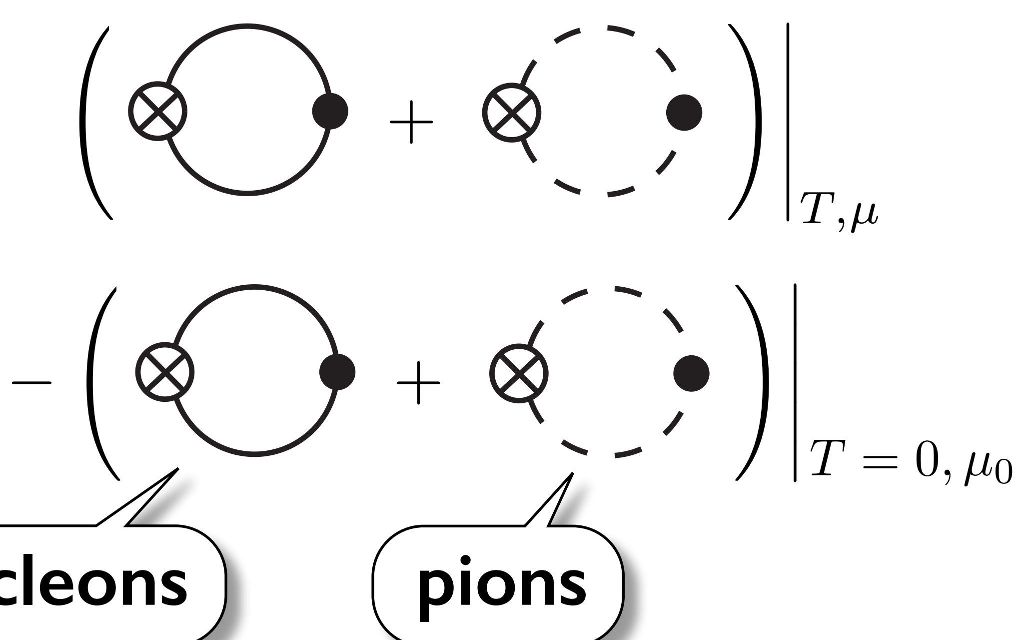


$$\Gamma_{k=0}[\Phi] = \Gamma[\Phi]$$

C. Wetterich:
Phys. Lett. B 301 (1993) 90

- **Thermodynamics:**

$$k\,\partial_k\bar{\Gamma}_k(T,\mu) =$$

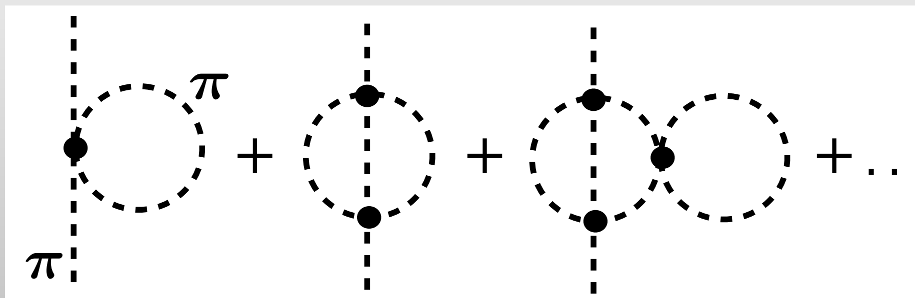


Non-perturbative treatment of :

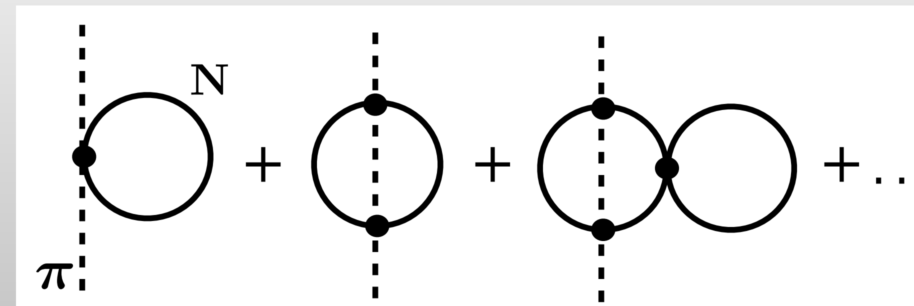
- **multi-pion exchange processes**
- **nucleon-hole excitations**
- **multi-nucleon correlations**

Classes of **mesonic** and **baryonic** fluctuations treated non-perturbatively in the **FRG** approach

- **Pion propagator** $$\mathbf{D}_\pi(\mathbf{x_1},\mathbf{x_2}) = \left[\frac{\delta^2\Gamma}{\delta\boldsymbol{\pi}(\mathbf{x_1})\,\delta\boldsymbol{\pi}(\mathbf{x_2})}\right]^{-1}$$

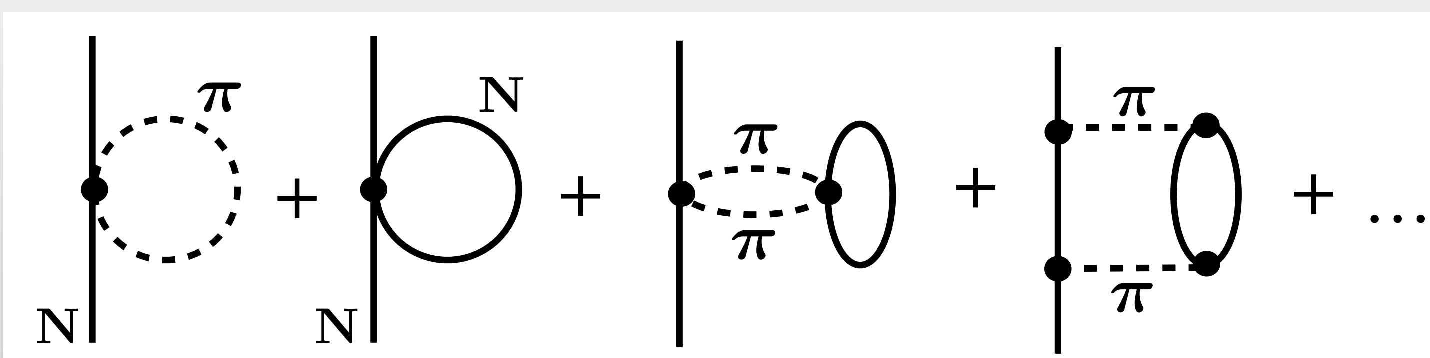


Pion propagation in **pionic heat bath**



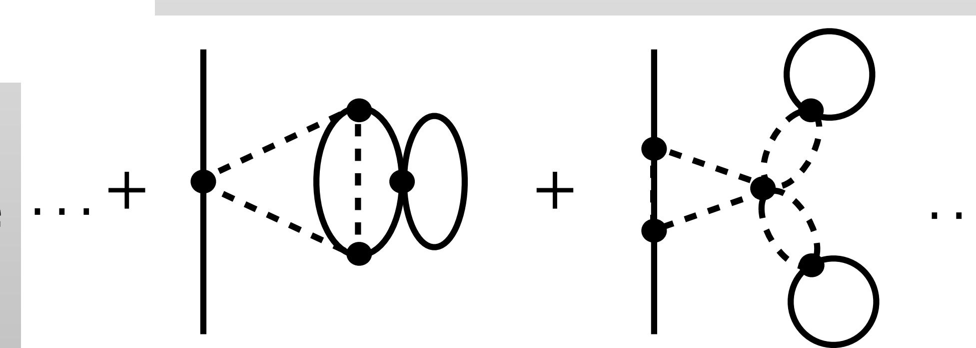
Pion propagation in **nuclear Fermi sea**

- **Nucleon propagator** $$\mathbf{D}_\mathbf{N}(\mathbf{x_1},\mathbf{x_2}) = \left[\frac{\delta^2\Gamma}{\delta\mathbf{N}(\mathbf{x_1})\,\delta\mathbf{N}^\dagger(\mathbf{x_2})}\right]^{-1}$$



thermal pion cloud | Hartree (MF) | Two-pion exchange | 2nd order tensor force

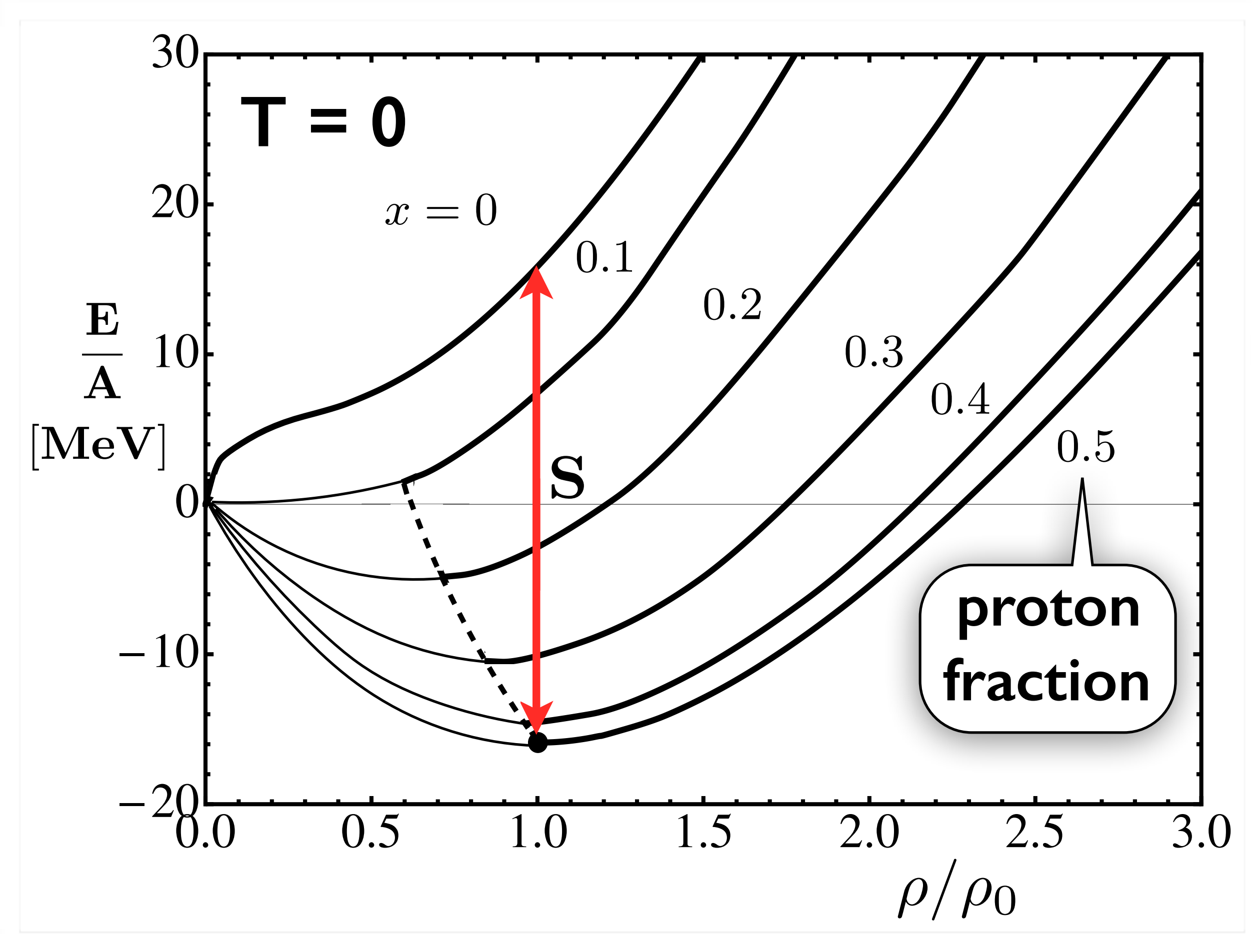
loops & ladders involving **nucleonic particle-hole excitations**



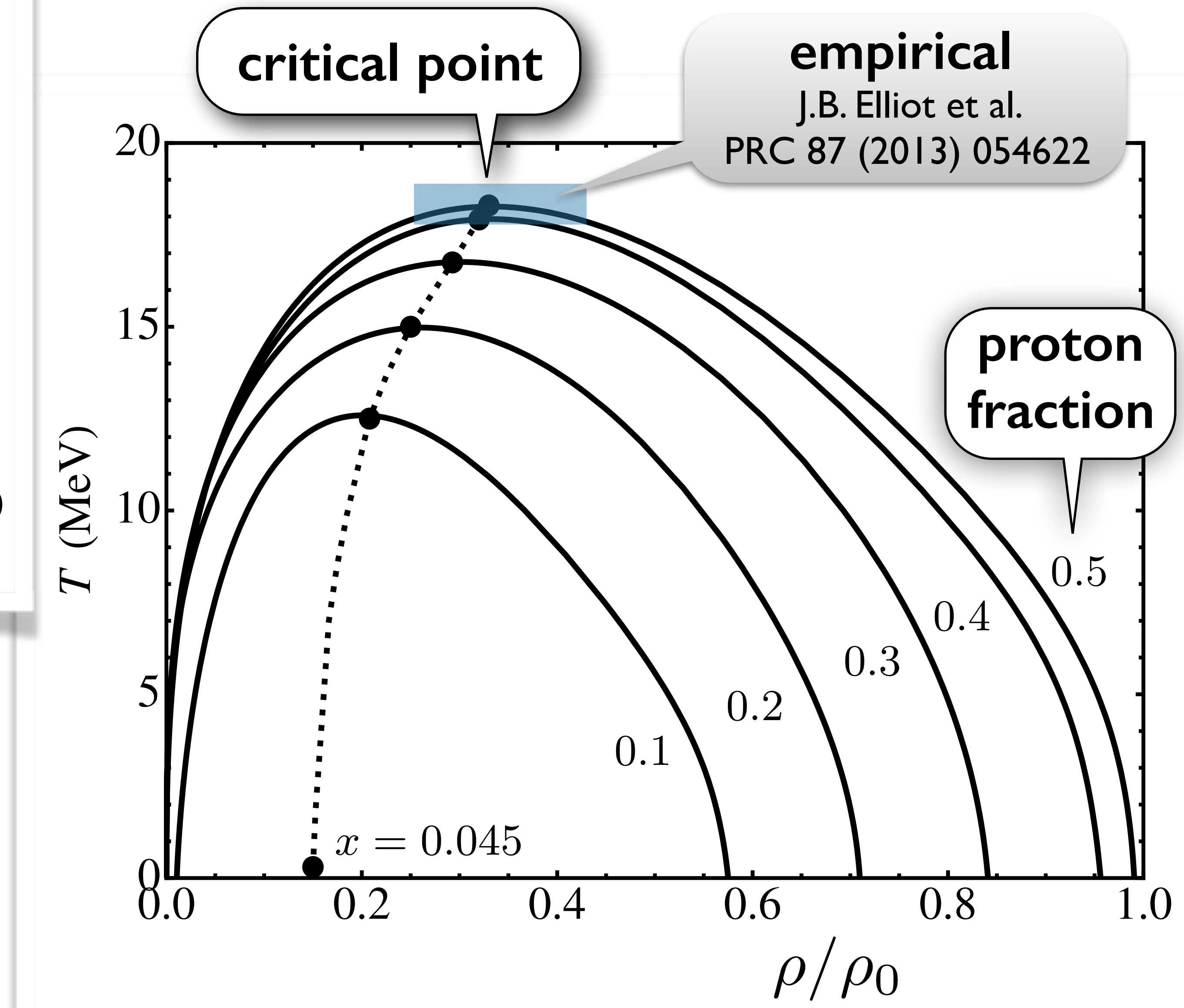
multi-nucleon correlations

From symmetric to asymmetric nuclear matter
in the Chiral FRG approach

M. Drews, W.W.   Phys. Lett. B738 (2014) 187   Phys. Rev. C91 (2015) 035802



- **Liquid-gas phase transition:**
  Evolution of coexistence regions from **symmetric** to **asymmetric** nuclear matter



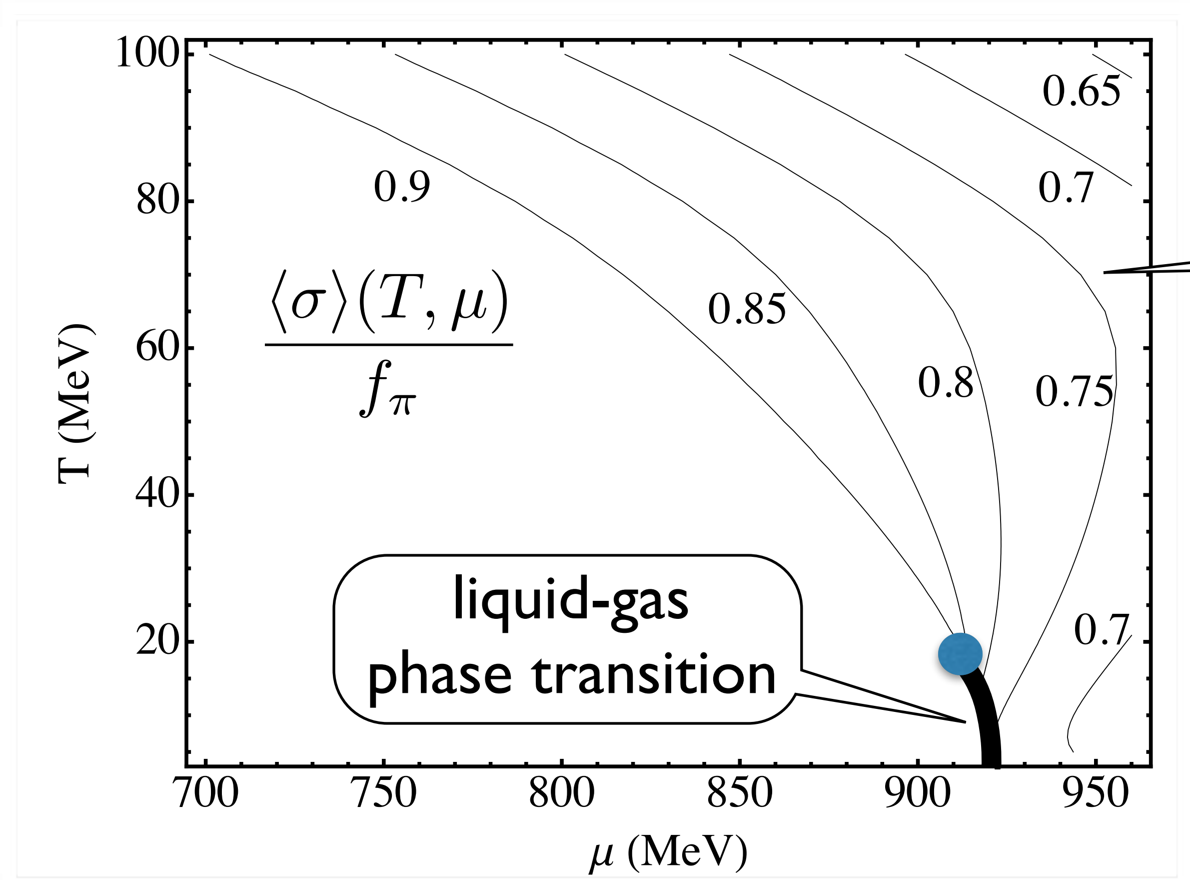
- **Symmetry energy** $\mathbf{S = 32\,MeV}$
- **FRG results (non-perturbative)** consistent with (perturbative) **Chiral EFT calculations**

NUCLEAR THERMODYNAMICS : CHIRAL ORDER PARAMETER in NUCLEAR MATTER

- Chiral Nucleon-Meson field theory and **F**unctional **R**enormalisation **G**roup
- Chiral order parameter : $\langle\sigma\rangle_{T,\mu} = f_\pi^*(T,\mu)$

Chiral FRG

symmetric nuclear matter



lines of const. $\langle\sigma\rangle$

M. Drews, W.W.

Phys. Rev. C91(2015)035802

Prog. Part. Nucl. Phys. 93 (2017) 69

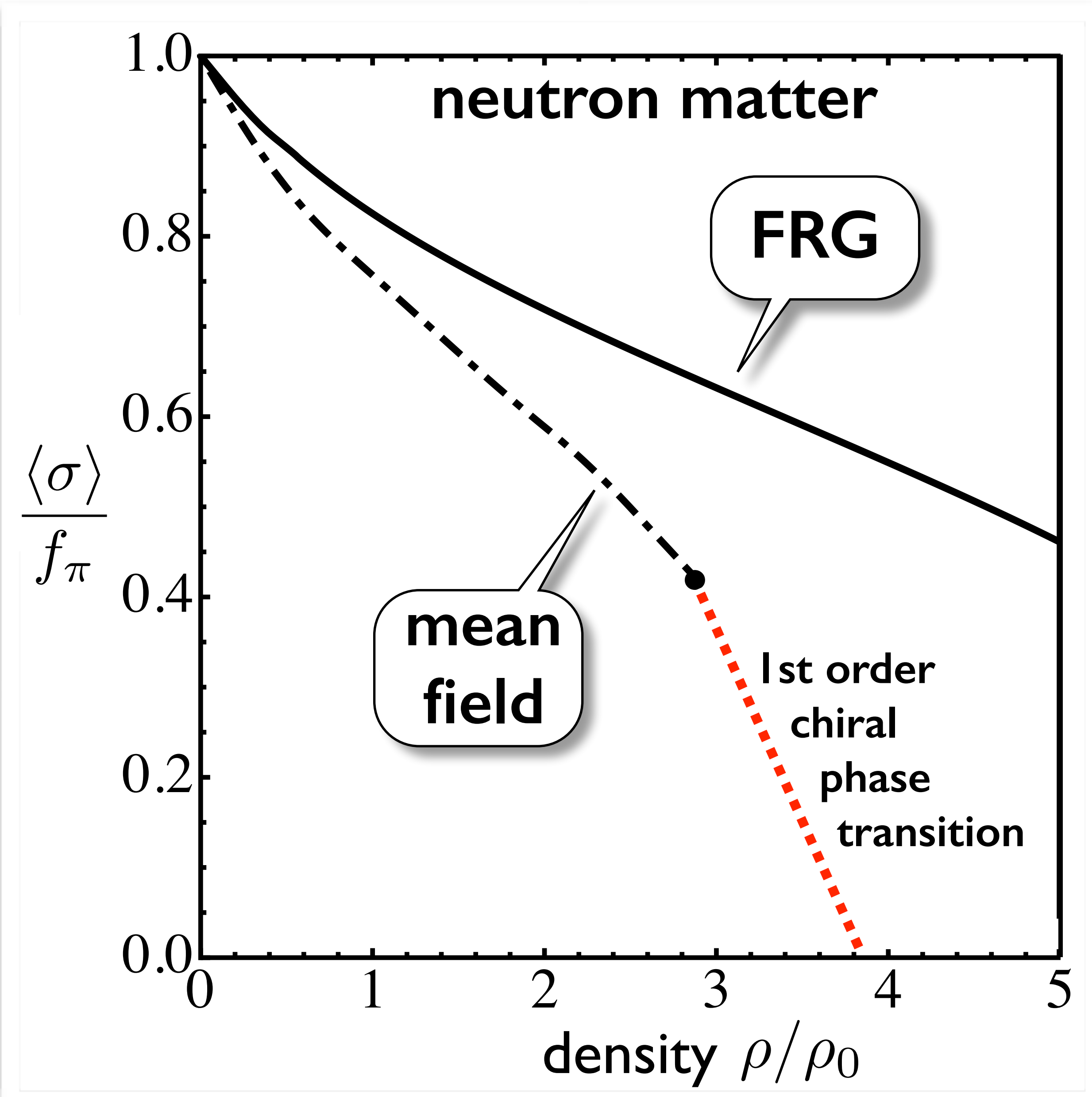
- **No first-order chiral phase transition in the range**

$$\mathrm{T} \lesssim 100\ \mathrm{MeV} \qquad \mu_B \lesssim 1\ \mathrm{GeV}$$

CHIRAL ORDER PARAMETER in NEUTRON MATTER

- **Chiral Nucleon-Meson field theory** and **F**unctional **R**enormalization **G**roup

M. Drews, W.W. Phys. Rev. C91 (2015) 035802 Prog. Part. Nucl. Phys. 93 (2017) 69



- **Chiral order parameter : sigma field**

↕

in-medium pion decay constant

$$\langle\sigma\rangle_\rho = f_\pi^*(\rho)$$

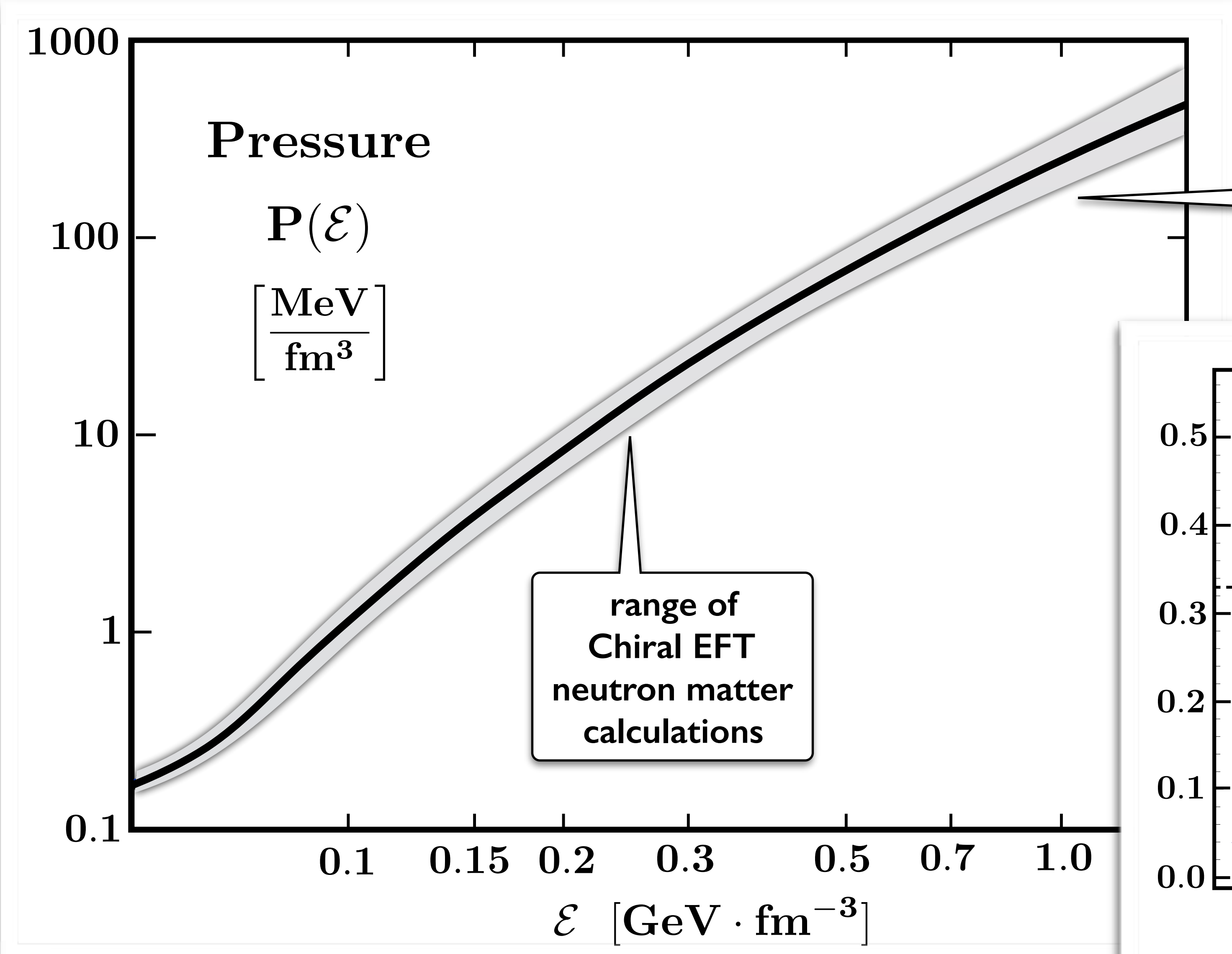
Important role of **fluctuations**
(pionic, nucleon-hole, many-body correlations)
beyond mean-field approximation:

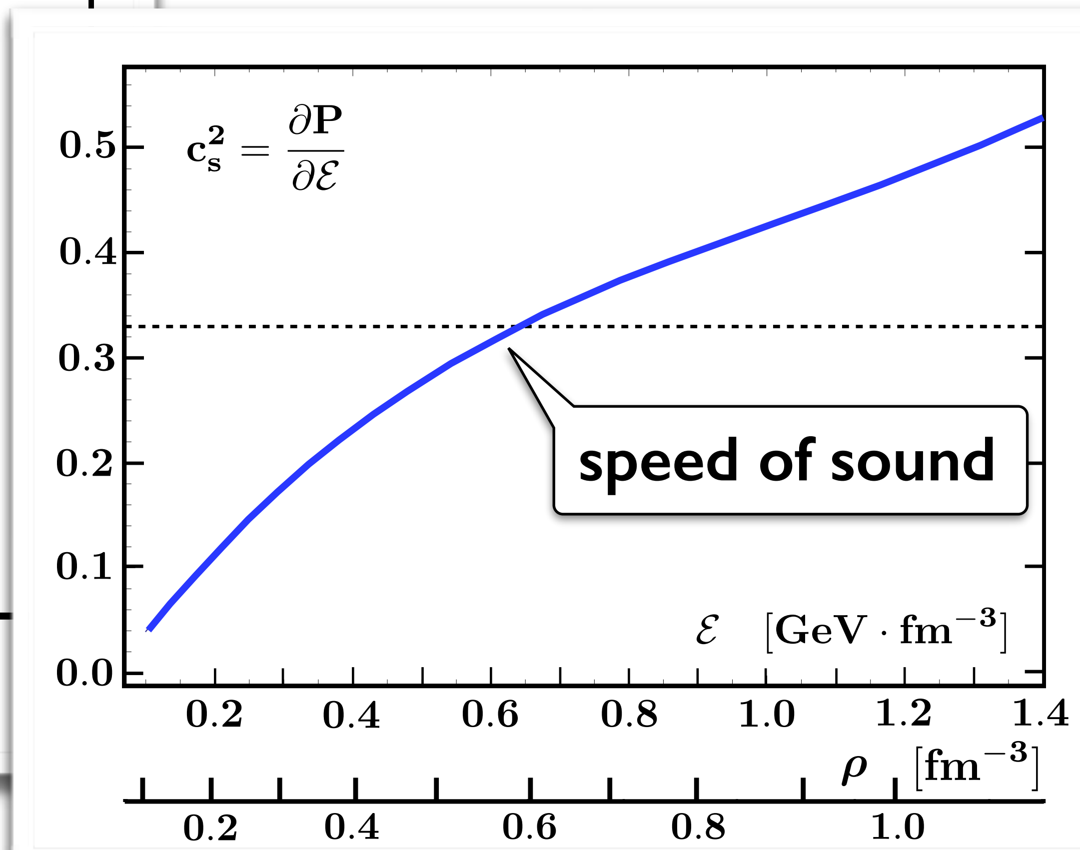
NO first-order chiral phase transition
in the **n-star density range**

NEUTRON STAR MATTER
Equation of State

- Chiral Nucleon-Meson Field Theory
- **FRG calculations** with inclusion of beta equilibrium

M. Drews, W.W. Phys. Rev. C91(2015) 035802 Prog. Part. Nucl. Phys. 93 (2017) 69

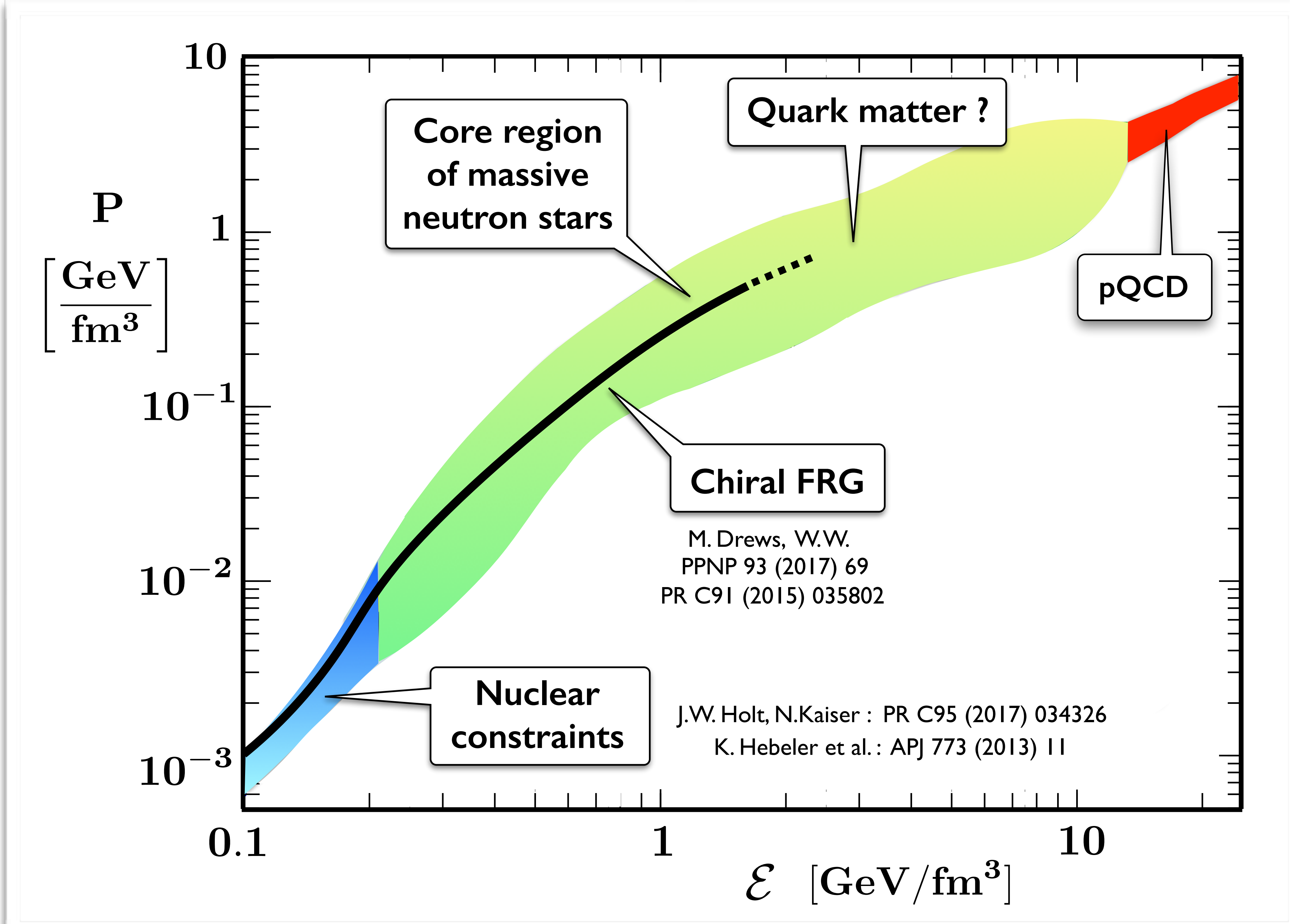




NEUTRON STAR MATTER   Equation of State

- incl. $M_{max} \gtrsim 2\, M_\odot$ + **GW constraints** and extrapolation to **pQCD limit**

A. Kurkela et al. : Astroph. J. 789 (2014) 127   A. Annala et al. : PRL 120 (2018) 172703   A. Vuorinen : arXiv:1807.04480



3.

Neutron Star Matter as a Relativistic Fermi Liquid

- **Derive Landau Fermi-liquid parameters from EoS**
- **Discuss sound velocity and strength of correlations**
- **Compare with other well-known Fermi liquid:** $^3\mathrm{He}$

(Bengt Friman & W.W. - work in progress)

Basics of (Relativistic) Fermi-Liquid Theory

G. Baym, S.A. Chin : Nucl. Phys. A262 (1976) 527

T. Matsui : Nucl. Phys. A370 (1981) 365

- **Variation of the energy (T = 0)** $n_p = \Theta(\mu - \varepsilon_p)$

$$\delta E = V \delta \mathcal{E} = \sum_p \varepsilon_p \, \delta n_p + \frac{1}{2V} \sum_{pp'} \mathcal{F}_{pp'} \delta n_p \delta n_{p'} + \dots$$

quasiparticle energy $\varepsilon_p = \dfrac{\delta E}{\delta n_p}$

quasiparticle interaction

$$\mathcal{F}_{pp'} = V \frac{\delta^2 E}{\delta n_p \delta n_{p'}} = f_{pp'} + g_{pp'} \, \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}'$$

- **Landau effective mass** $m^* = \sqrt{p_F^2 + M^2(\rho)} \qquad (M = g\langle\sigma\rangle)$

- **Density of states at Fermi surface** $N(0) = \dfrac{m^* \, p_F}{\pi^2}$

Landau parameters

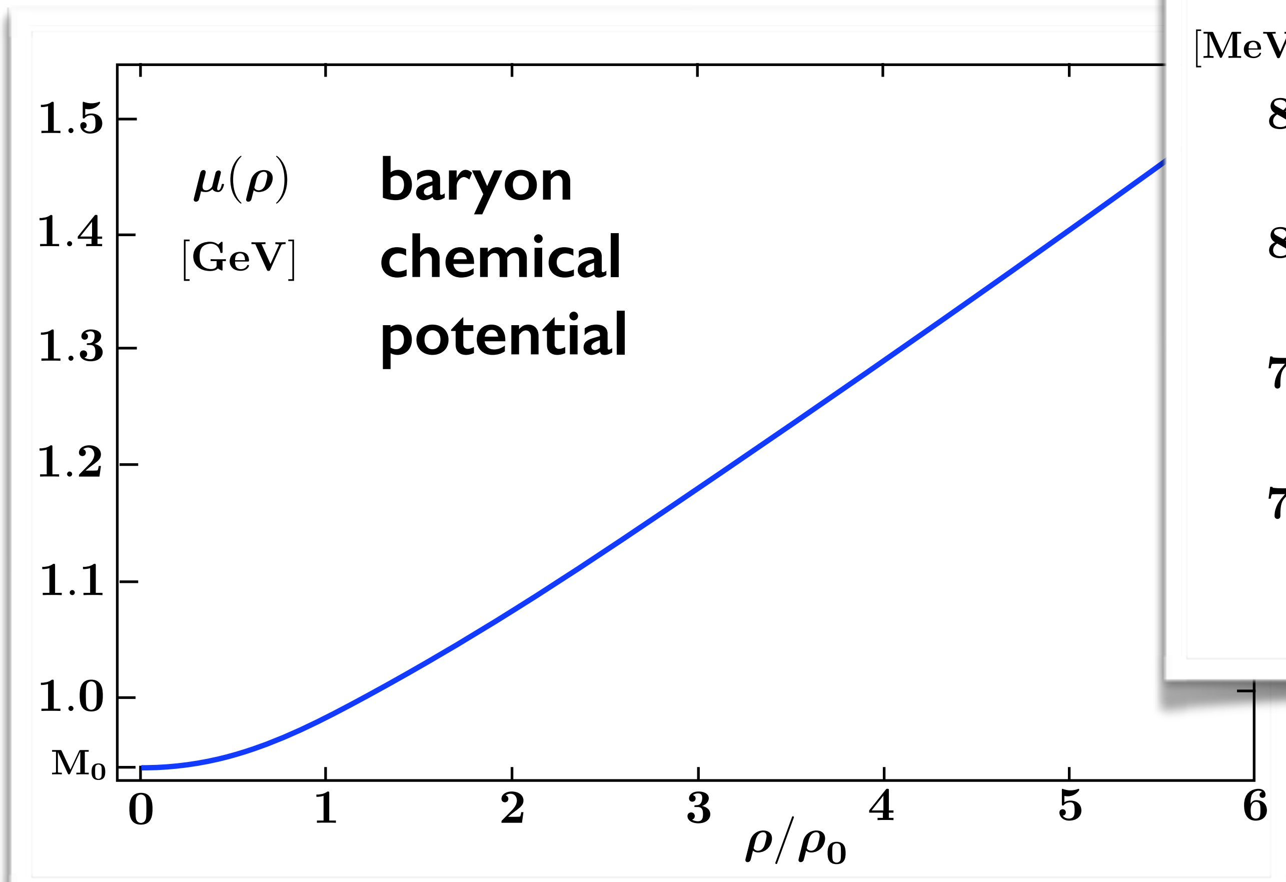
$$f_{pp'} = \sum_{\ell=0}^{\infty} f_\ell \, P_\ell(\cos\theta_{pp'}) \qquad F_\ell = N(0) \, f_\ell$$

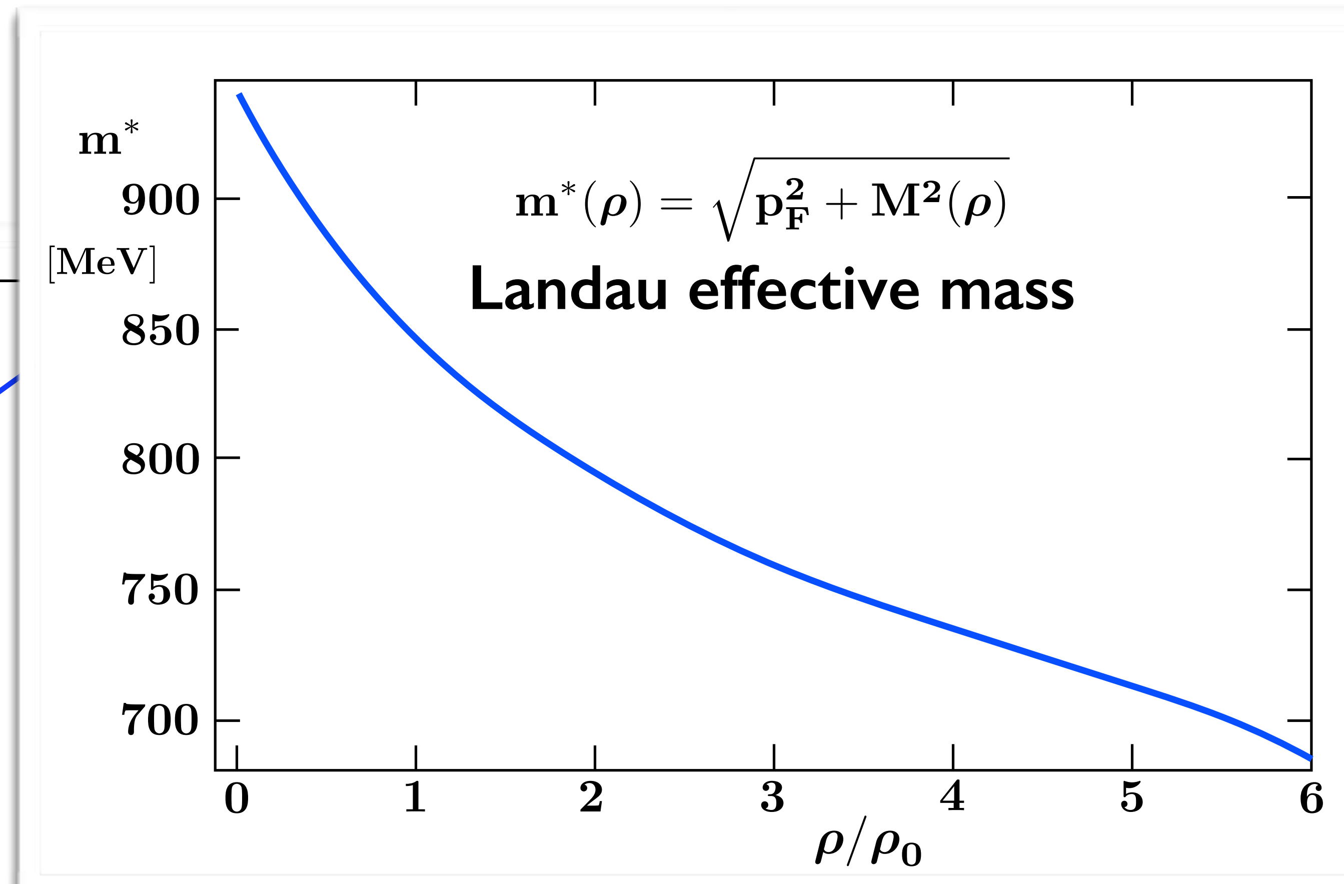
Relativistic Fermi-Liquid Theory (contd.)

- **Leading Landau parameters (T=0)**

$$1 + F_0(\rho) = \frac{p_F\, m^*(\rho)}{\pi^2}\,\frac{\partial^2 \mathcal{E}(\rho)}{\partial \rho^2} = \frac{m^*(\rho)}{3p_F^2}\,\mathcal{K}(\rho)$$ **(incompressibility)**

$$1 + \frac{F_1(\rho)}{3} = \frac{m^*(\rho)}{\mu}$$





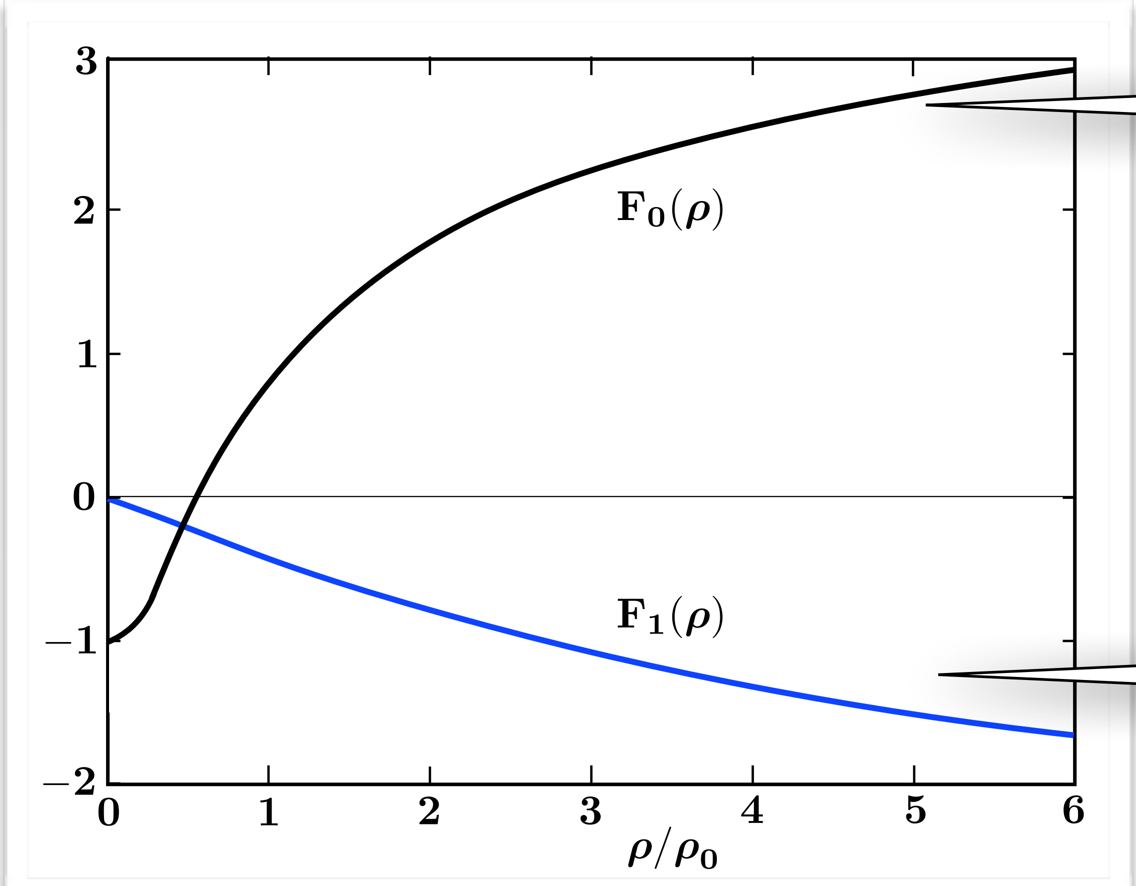
Chiral FRG calculation

LANDAU FERMI-LIQUID PARAMETERS

- Deduced from **neutron star matter EoS** calculated using Chiral Nucleon-Meson Field Theory & **F**unctional **R**enormalisation **G**roup

neutron star matter

strongly correlated Fermi liquid
...
but not "extreme" !



strongly repulsive correlations

decreasing effective Fermion mass

- Low densities $\rho \lesssim \rho_0$: F_0 in good agreement with ChEFT results

J.W. Holt, N.Kaiser, W.W. : Phys. Rev. C87 (2013) 014338

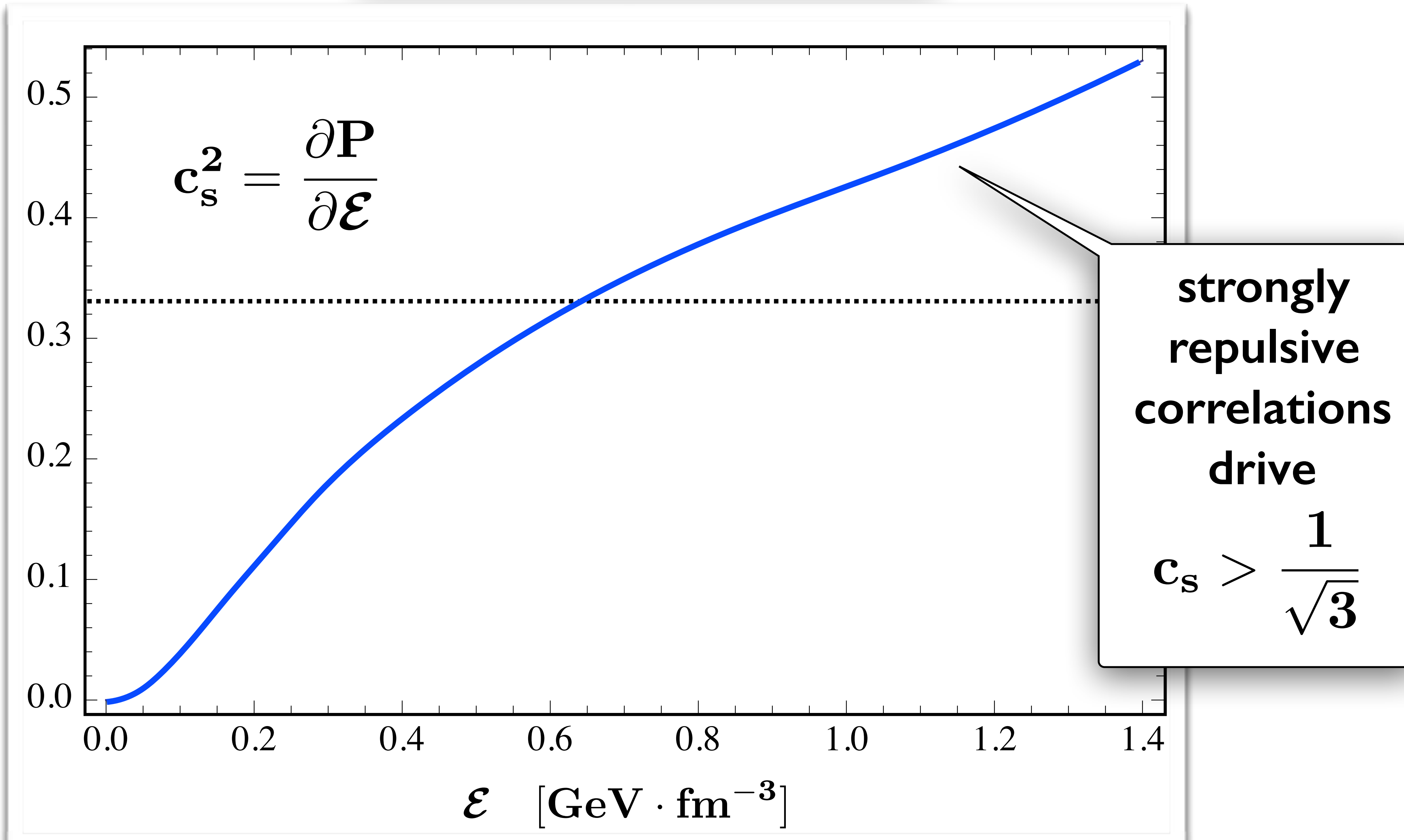
- Comparison with **liquid helium3** at pressures P = (0 - 30) bar:

$$F_0(^3\mathrm{He}) \sim 10 - 70 \qquad F_1(^3\mathrm{He}) \sim 5 - 13$$

Relativistic **Fermi-Liquid Theory** (contd.)

- **Speed of sound**

$$c_s^2 = \frac{1}{3}\left(\frac{p_F}{\mu}\right)^2 \frac{1 + F_0}{1 + \frac{1}{3}F_1}$$

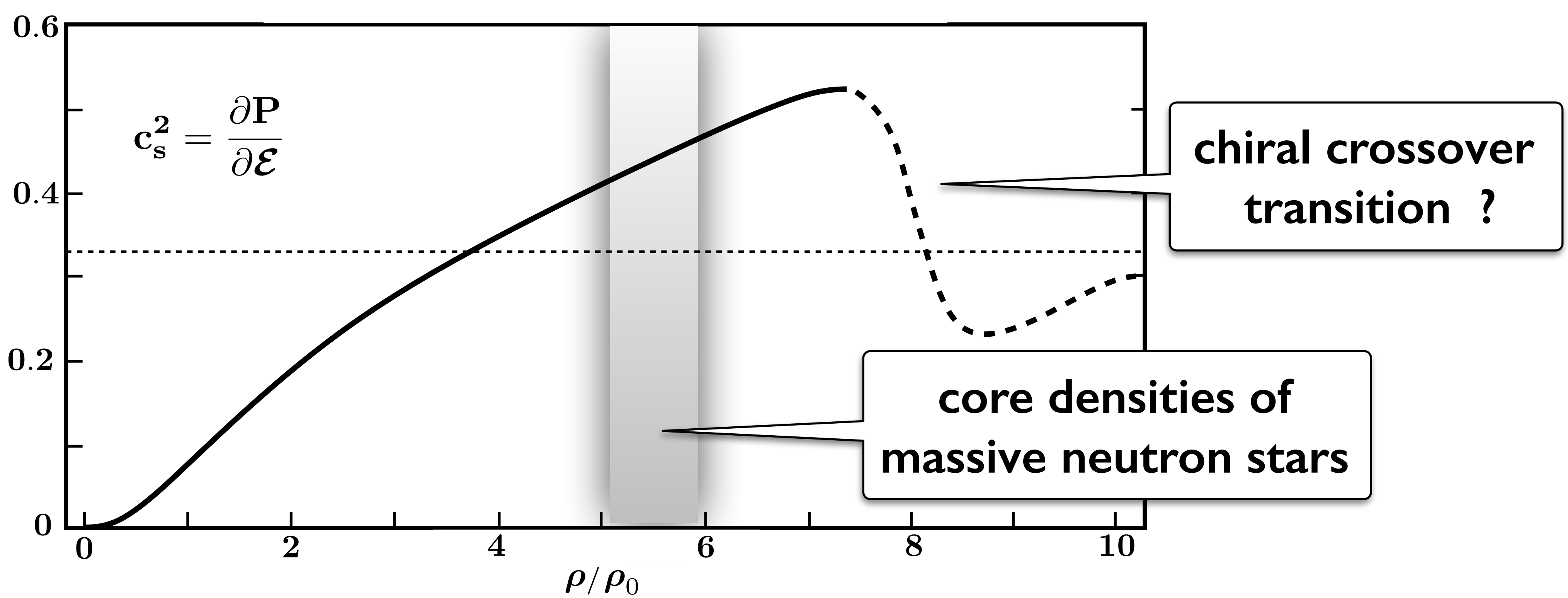


SUMMARY / CONCLUDING REMARKS

- ChEFT + many-body perturbation theory works for $\rho \lesssim 2\,\rho_0$
- Chiral nucleon-meson field theory combined with (non-perturbative) FRG works also at higher densities $(\rho \to 5\,\rho_0)$
- **No chiral phase transition** in neutron star matter

  Hadronic EoS (Nambu-Goldstone phase) consistent with constraints from neutron stars and mergers ($\mathrm{M}_{max} \simeq 2\,\mathrm{M}_\odot$, tidal deformability)
- **Neutron star matter** : strongly correlated **Fermi liquid**

  … but not as strongly correlated as liquid helium !
- Speed of sound



Appendix :

Supplementary Materials

Chiral FRG details

Mesons, Nucleons, Nuclear Matter
and
Functional Renormalization Group

- **Chiral nucleon - meson model** $\qquad \boldsymbol{\Psi} = (\psi_\mathbf{p}, \psi_\mathbf{n})^\top$

$$\begin{aligned}\mathcal{L} = \; & \bar{\boldsymbol{\Psi}}\, i\gamma_\mu \partial^\mu \boldsymbol{\Psi} + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}\right) \\ & - \bar{\boldsymbol{\Psi}}\left[g(\sigma + i\gamma_5\, \boldsymbol{\tau} \cdot \boldsymbol{\pi}) + \gamma_\mu \left(g_v\, v^\mu + g_\tau\, \boldsymbol{\rho}^\mu\right)\right]\boldsymbol{\Psi} \\ & - \boldsymbol{\mathcal{U}}(\sigma, \pi) + \frac{1}{2}\, m_V^2 \left(v_\mu v^\mu + \boldsymbol{\rho}_\mu \boldsymbol{\rho}^\mu\right) \\ & \cancel{- \frac{1}{4}\left[F^{(v)}_{\mu\nu} F^{(v)\mu\nu} + \boldsymbol{F}^{(\rho)}_{\mu\nu} \cdot \boldsymbol{F}^{(\rho)\mu\nu}\right]}\end{aligned}$$

- **Potential** $\boldsymbol{\mathcal{U}}(\sigma, \boldsymbol{\pi})$ constructed to reproduce standard nuclear thermodynamics around equilibrium

- **Pionic fluctuations** and **nucleonic particle-hole** excitations treated **non-perturbatively** using **FRG**

M. Drews, T. Hell, B. Klein, W.W. Phys. Rev. D88 (2013) 096011

M. Drews, W.W. Phys. Lett. B738 (2014) 187 Phys. Rev. C91 (2015) 035802 PPNP 93 (2017) 69

Fixing the input

explicit chiral symmetry breaking

- **Potential** $\mathcal{U}(\sigma,\boldsymbol{\pi}) = \mathcal{U}_0(\chi) - m_\pi^2 f_\pi(\sigma - f_\pi)$

Chiral invariant part :

$$\mathcal{U}_0(\chi) = \sum_n a_n(\chi - \chi_0)^n + \chi^2 \ln\chi \text{ term}$$

expanded in powers of

$$\chi = \frac{1}{2}(\sigma^2 + \boldsymbol{\pi}^2) \qquad \chi_0 = f_\pi^2/2 \text{ (vacuum)}$$

$n_{max} = 4$

$a_1,\ a_2$
determined by vacuum constants $(f_\pi,\ m_\pi,\ m_\sigma)$

$a_3,\ a_4$
adjustable parameters

- **Scalar** (“sigma”) **field:**
mean field (chiral **order parameter**) plus **fluctuating** pieces.
σ mass: **NOT** to be identified with “$\sigma(500)$”: $\mathbf{m_\sigma \simeq 0.9\ GeV}$

- **Nucleon mass:** $m_N = g\sqrt{2\chi}$

$\ldots$ in vacuum: $m_N = g\, f_\pi$ (Goldberger - Treiman)

Fixing the input
(contd.)

- **Vector fields** encode **short-distance NN dynamics**:

  self-consistently determined background **mean fields** (non-fluctuating)

  (**NOT** to be identified with physical ω and ρ mesons)

  Effective chemical potentials $\mu^{\text{eff}}_{\mathbf{n,p}} = \mu_{\mathbf{n,p}} - g_v\, v_0 \pm g_\tau\, \rho^3_0$

  Relevant quantities:

  $$G_v = \frac{g_v^2}{m_V^2}\ ,\ G_\tau = \frac{g_\tau^2}{m_V^2} \longleftrightarrow$$ **contact terms in ChEFT**

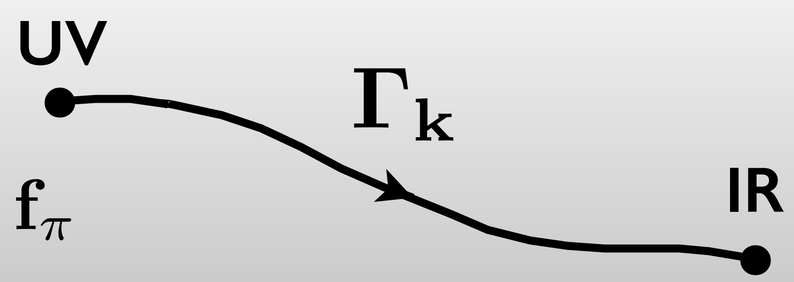
  $$G_\tau \sim G_v/4 \sim 1\,\mathrm{fm}^2$$

- **Parameters**

  determined by nuclear matter properties and symmetry energy

Flow equations in practice

UV scale:
action $\mathbf{\Gamma_{k=\Lambda}}$
$\mathbf{\Lambda \simeq \Lambda_\chi = 4\pi\, f_\pi}$



"full"
effective action
$\mathbf{\Gamma_{k=0}}$

- Simplifying assumptions and approximations :
  derivative expansion & no "k-running" of Yukawa coupling

- **k-dependent effective action :**

$$\Gamma_k = \int d^4x \Big[\bar{\boldsymbol{\Psi}} i\gamma_\mu \partial^\mu \boldsymbol{\Psi} + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2}\partial_\mu \boldsymbol{\pi} \partial^\mu \boldsymbol{\pi}$$

$$- \bar{\boldsymbol{\Psi}} \left[g\,(\sigma + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}) + \gamma_0 \left(g_v\, v_0 + g_\tau\, \rho_0^3\, \tau_3\right) \right] \boldsymbol{\Psi} - \boldsymbol{\mathcal{U}_k} \Big]$$

k-dependent potential

Flow equations in practice
(contd.)

k-dependent potential:

$$k\frac{\partial \mathcal{U}_k}{\partial k}(T,\mu_{\rm p},\mu_{\rm n},\chi,v_0,\rho_0^3) =$$

$$= \frac{k^5}{12\pi^2}\left\{\frac{1+2n_B(E_\sigma)}{E_\sigma} + \frac{3+2n_B(E_\pi)}{E_\pi} - 4\sum_{i=n,p}\frac{1-n_F(E_N-\mu_i^{\rm eff})}{E_N}\right\}$$

$$\chi = \frac{1}{2}(\sigma^2 + \boldsymbol{\pi}^2)$$

$$E_\pi^2 = k^2 + \mathcal{U}_k'(\chi)\,, \qquad E_\sigma^2 = k^2 + \mathcal{U}_k'(\chi) + 2\chi\,\mathcal{U}_k''(\chi)\,,$$

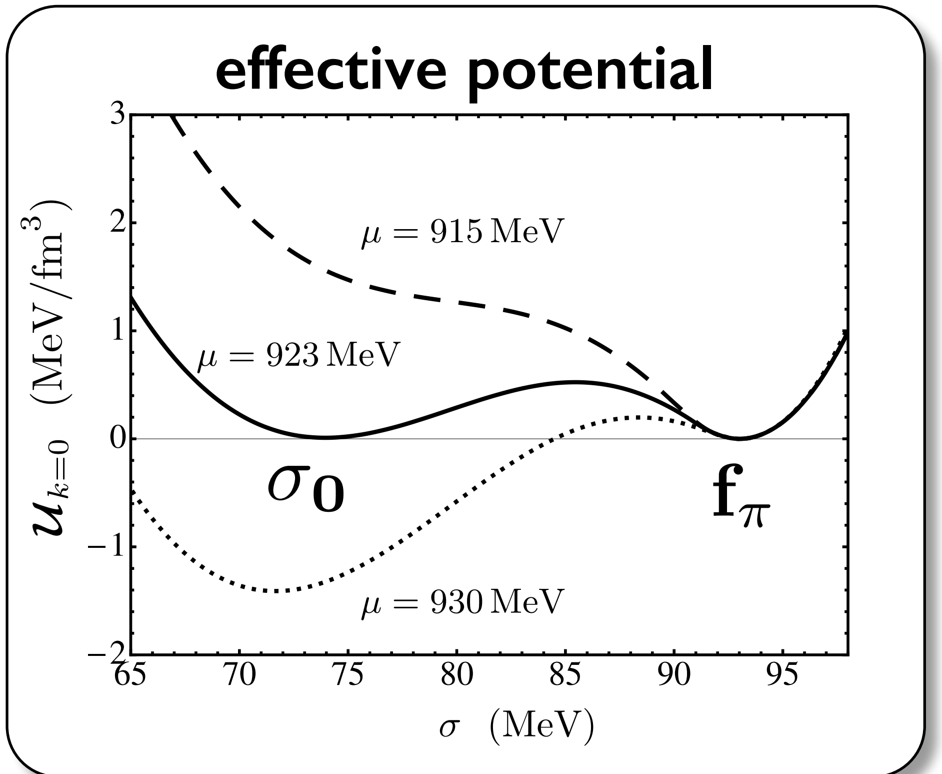
$$\mathcal{U}_k'(\chi) = \frac{\partial \mathcal{U}_k(\chi)}{\partial \chi}\,, \qquad E_N^2 = k^2 + 2g^2\chi\,,$$

$$\mu_{\mathbf{n,p}}^{\rm eff} = \mu_{\mathbf{n,p}} - g_v\,v_0 \pm g_\tau\,\rho_0^3 \qquad n_{\rm B}(E) = \frac{1}{{\rm e}^{E/T}-1}\,,\; n_{\rm F}(E) = \frac{1}{{\rm e}^{E/T}+1}\,.$$

… plus vector field equations,
then full system of coupled differential equations solved on a grid.

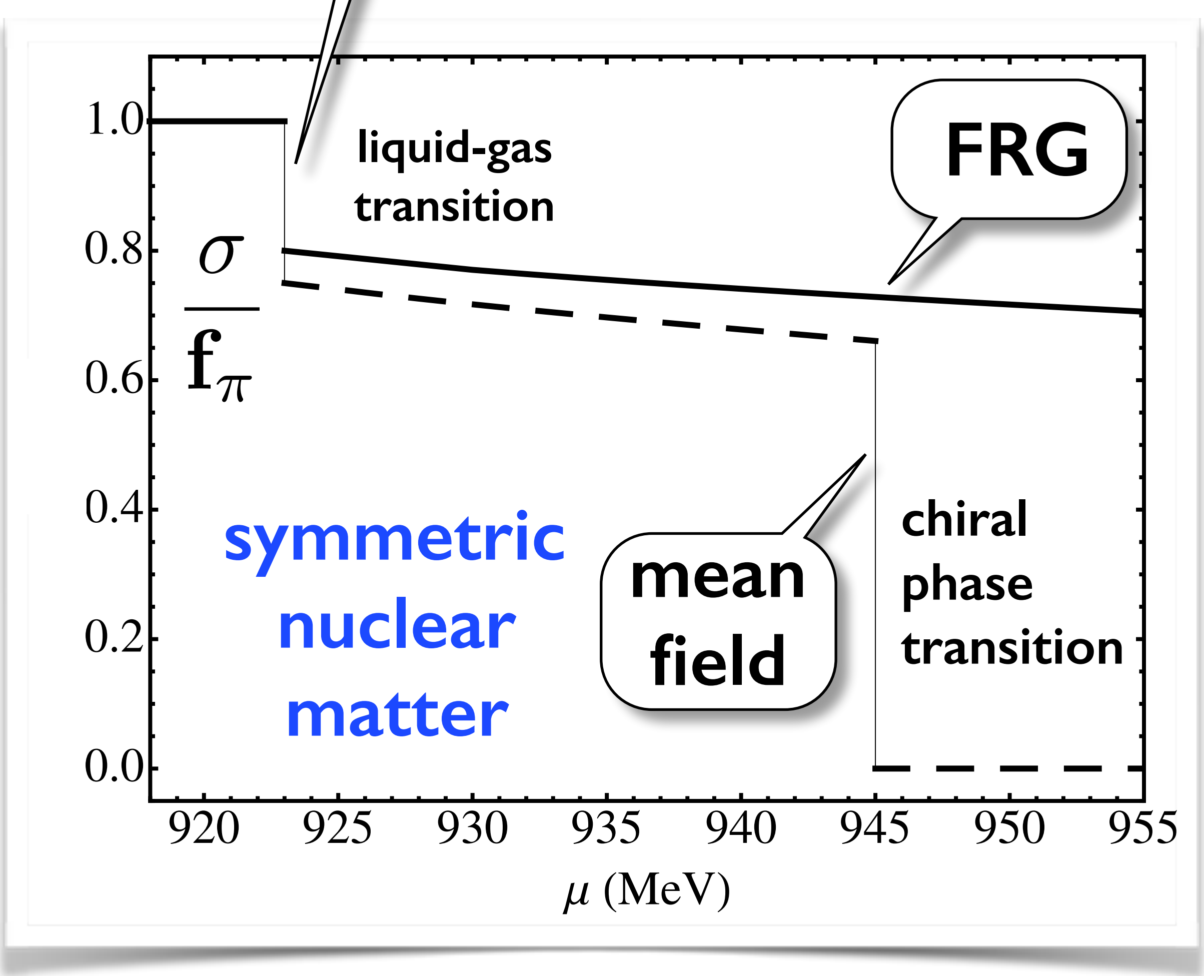
Chiral Order Parameter

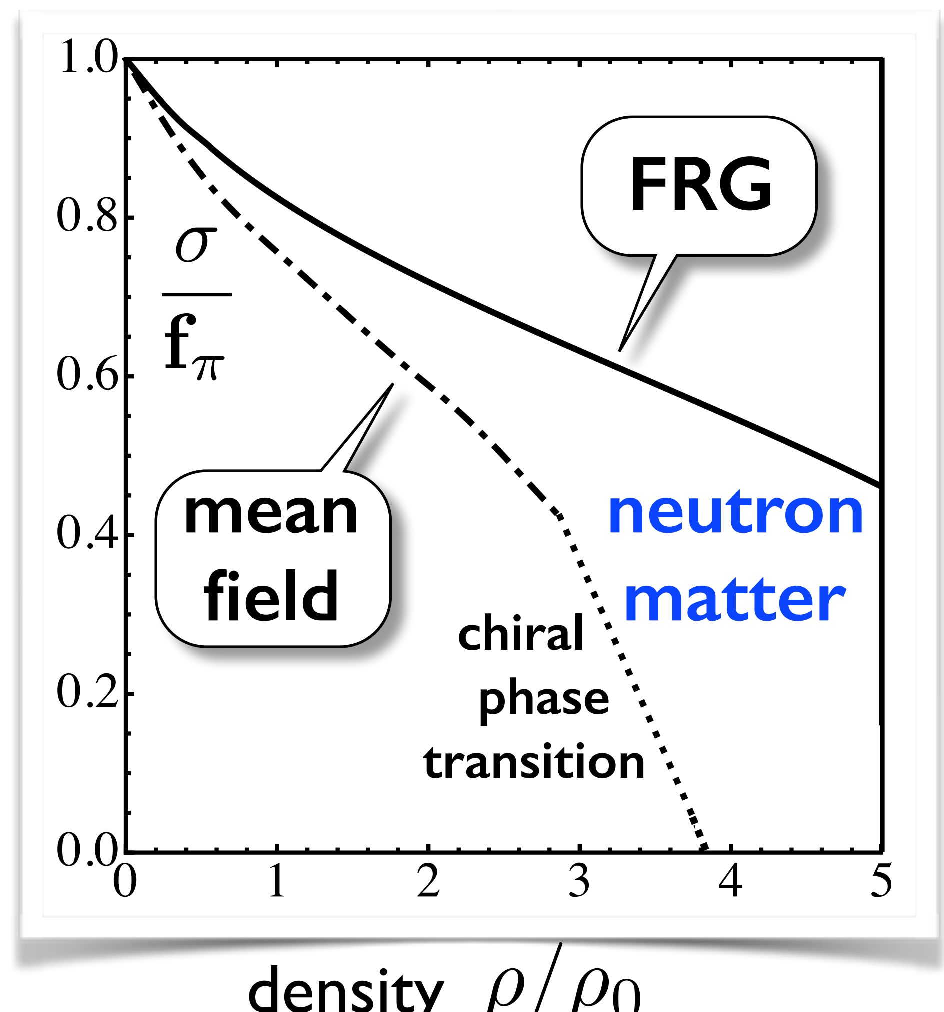
M. Drews, W.W.
Phys. Rev. C91 (2015) 035802



important role of **fluctuations**
beyond mean-field approximation:

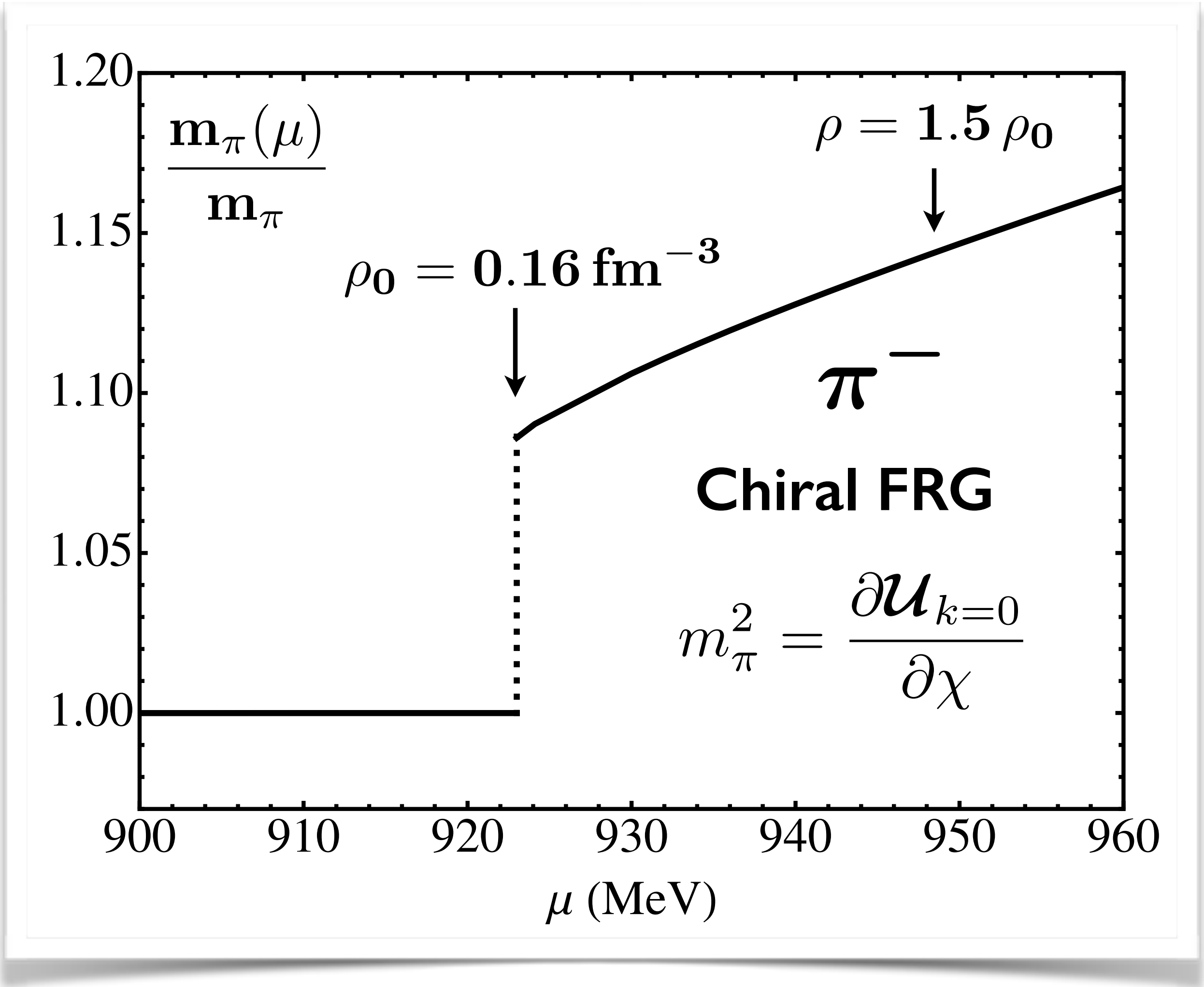
DISAPPEARANCE of
first-order chiral phase transition



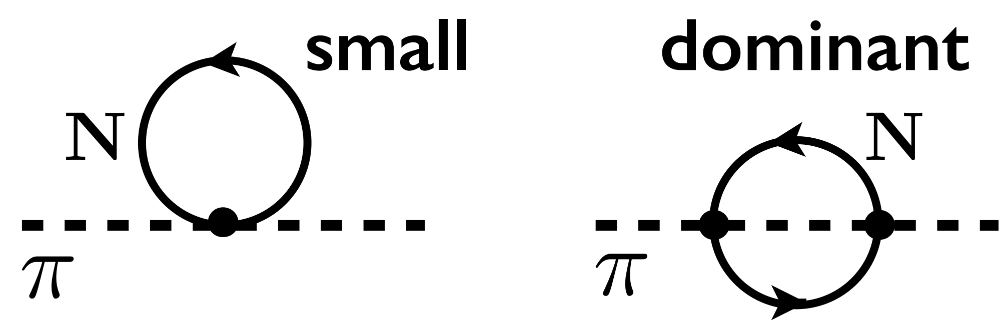


Test case: **in-medium pion mass**

- Contact with phenomenology :
  compare with **s-wave pion-nuclear optical potential** from **pionic atoms**



S-wave π^- optical potential



$$U(\rho) = -\frac{2\pi}{m_\pi}\left[b_0 - (b_0^2 + 2b_1^2)\left\langle\frac{1}{r}\right\rangle\right]\rho$$

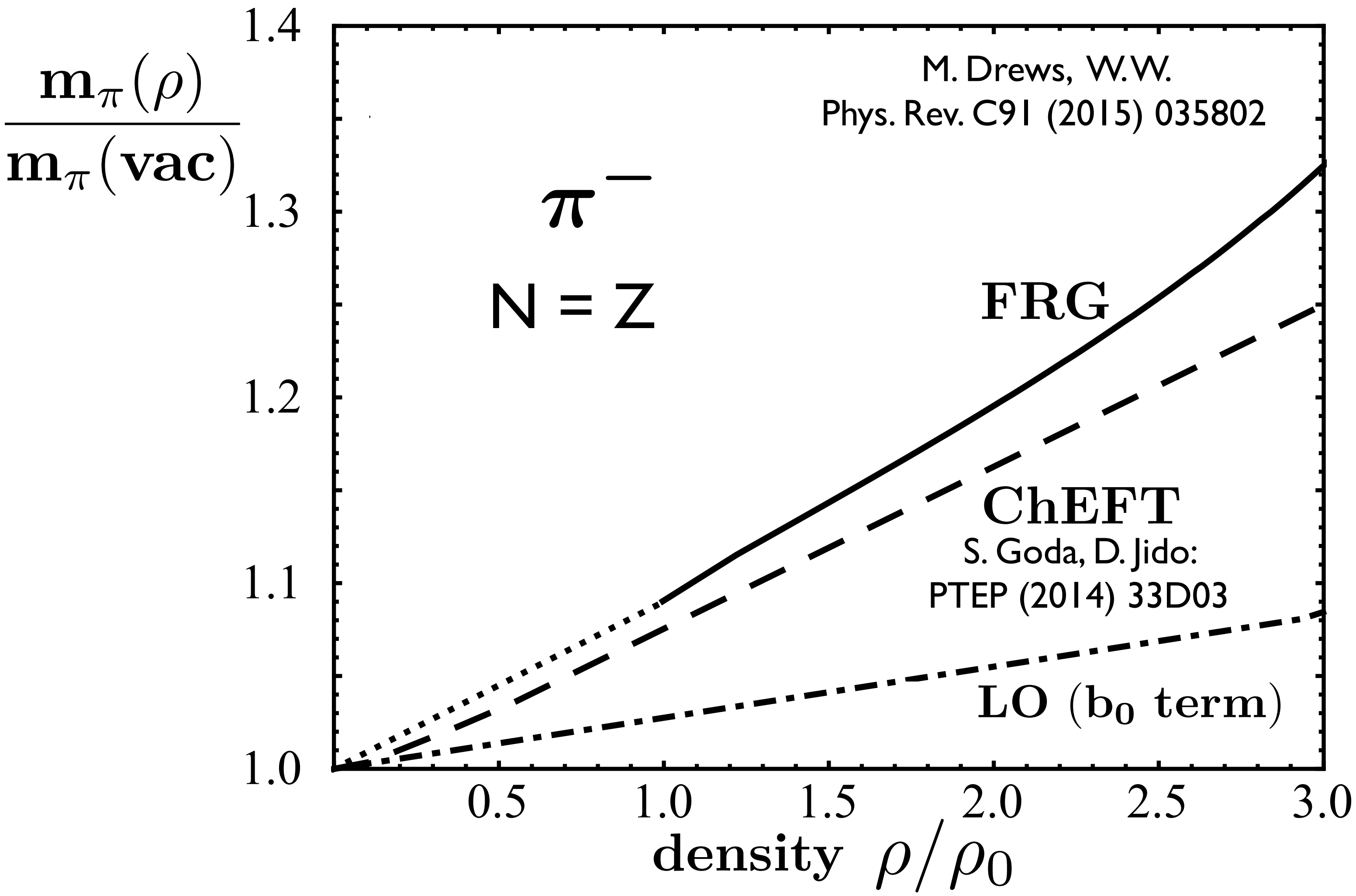
$$\frac{m_\pi(\rho)}{m_\pi} \simeq 1 + \frac{U(\rho)}{m_\pi} \simeq 1.1\,\frac{\rho}{\rho_0}$$

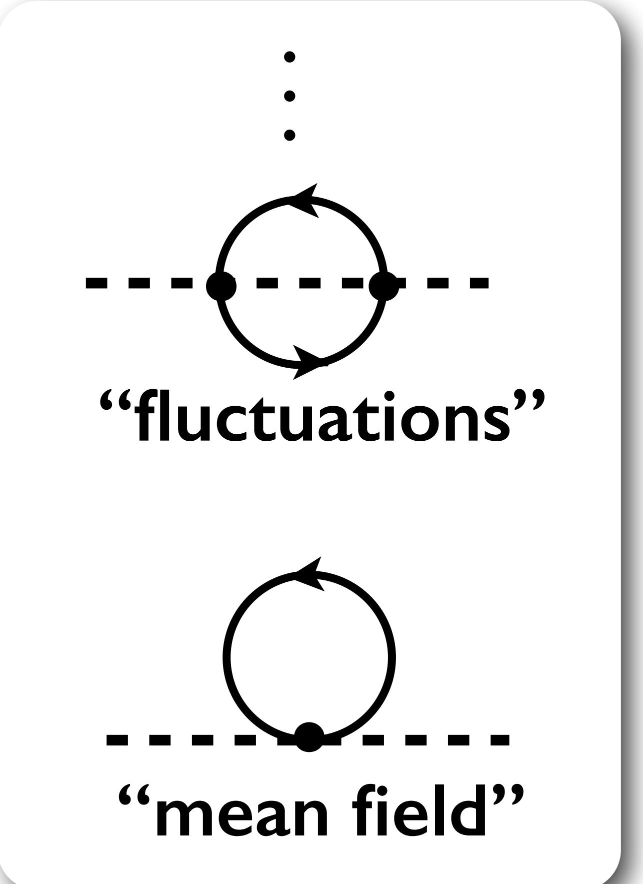
- Good agreement of **FRG** calculation with empirical **in-medium pion mass shift,** both in sign and magnitude

In-medium pion mass (contd.)

- **Non-perturbative FRG** result in comparison with in-medium **Chiral Perturbation Theory**

$$m_\pi^2 = \frac{\partial \mathcal{U}_{k=0}}{\partial \chi}$$





- In-medium **ChPT (NLO)**:

$$m_\pi^* = m_\pi \left\{ 1 + \rho \frac{b^+}{2m_\pi^2} - \frac{g_A^2 k_F^4}{24\pi^4 f_\pi^4} F\left(\frac{m_\pi}{2k_F}\right) + \left[\frac{1}{8} + m_\pi^2 \left(\frac{b^+}{2m_\pi^2} - \frac{g_A^2}{8m_N} \right)^2 \right] \frac{2k_F^4}{\pi^4 f_\pi^4} \right\}$$

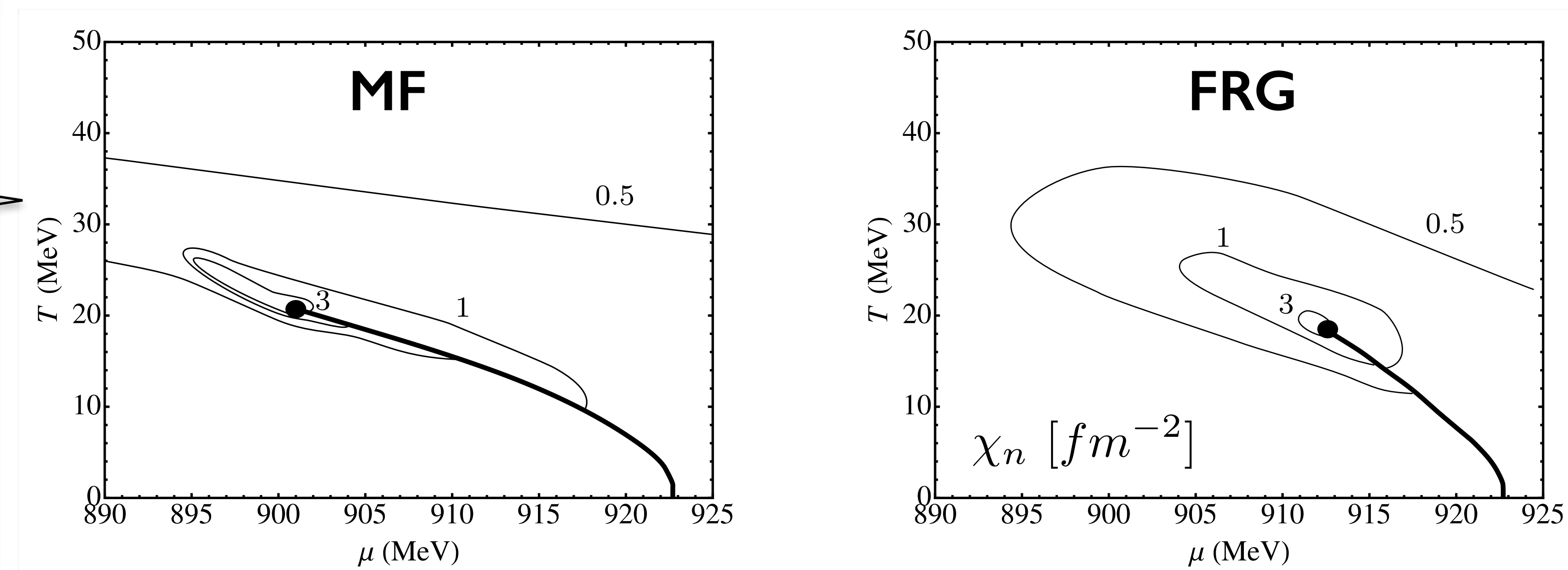
Susceptibilities

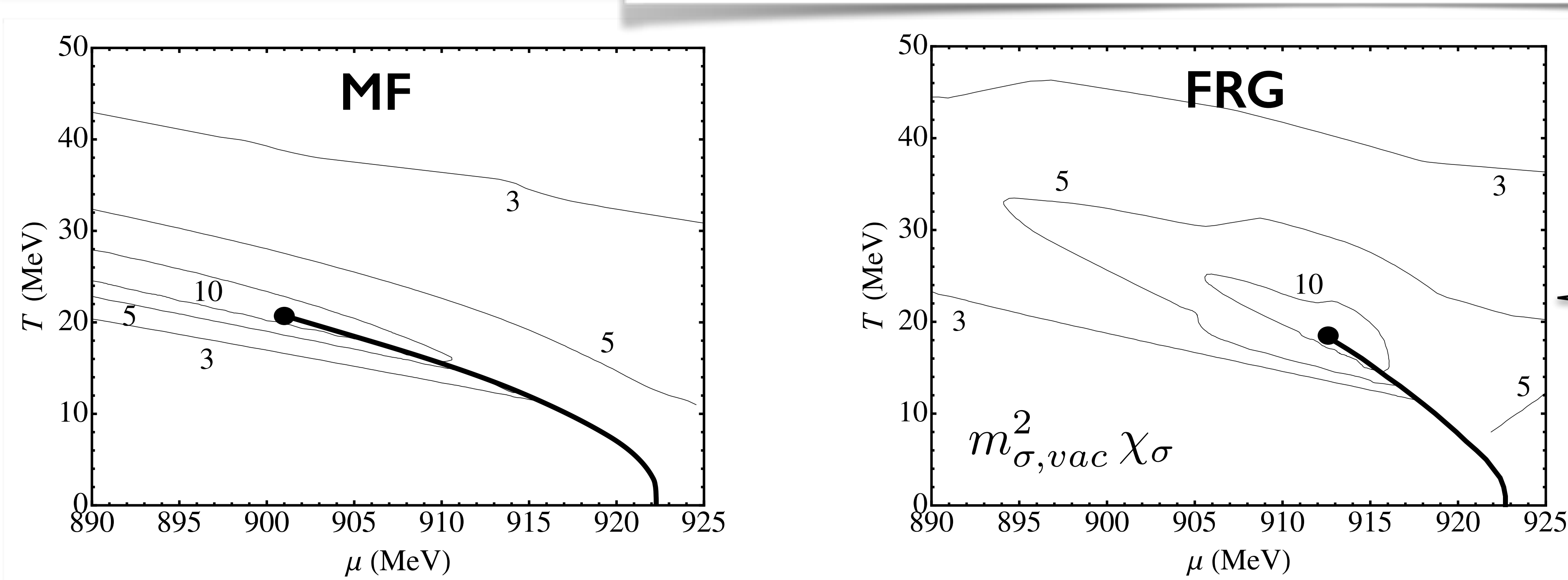
- Mean-field (**MF**) versus full **FRG** calculations

Test case: liquid-gas first-order phase transition

- Baryon no. susceptibility

$$\chi_n(T,\mu) = -\frac{\partial^2 U}{\partial \mu^2} = \frac{\partial n}{\partial \mu}$$





- Chiral susceptibility

$$\chi_\sigma(T,\mu) = m_\sigma^{-1}(T,\mu)$$