recent lattice QCD results @ $\mu_B > 0$

Swagato Mukherjee

BROOKHAVEN NATIONAL LABORATORY

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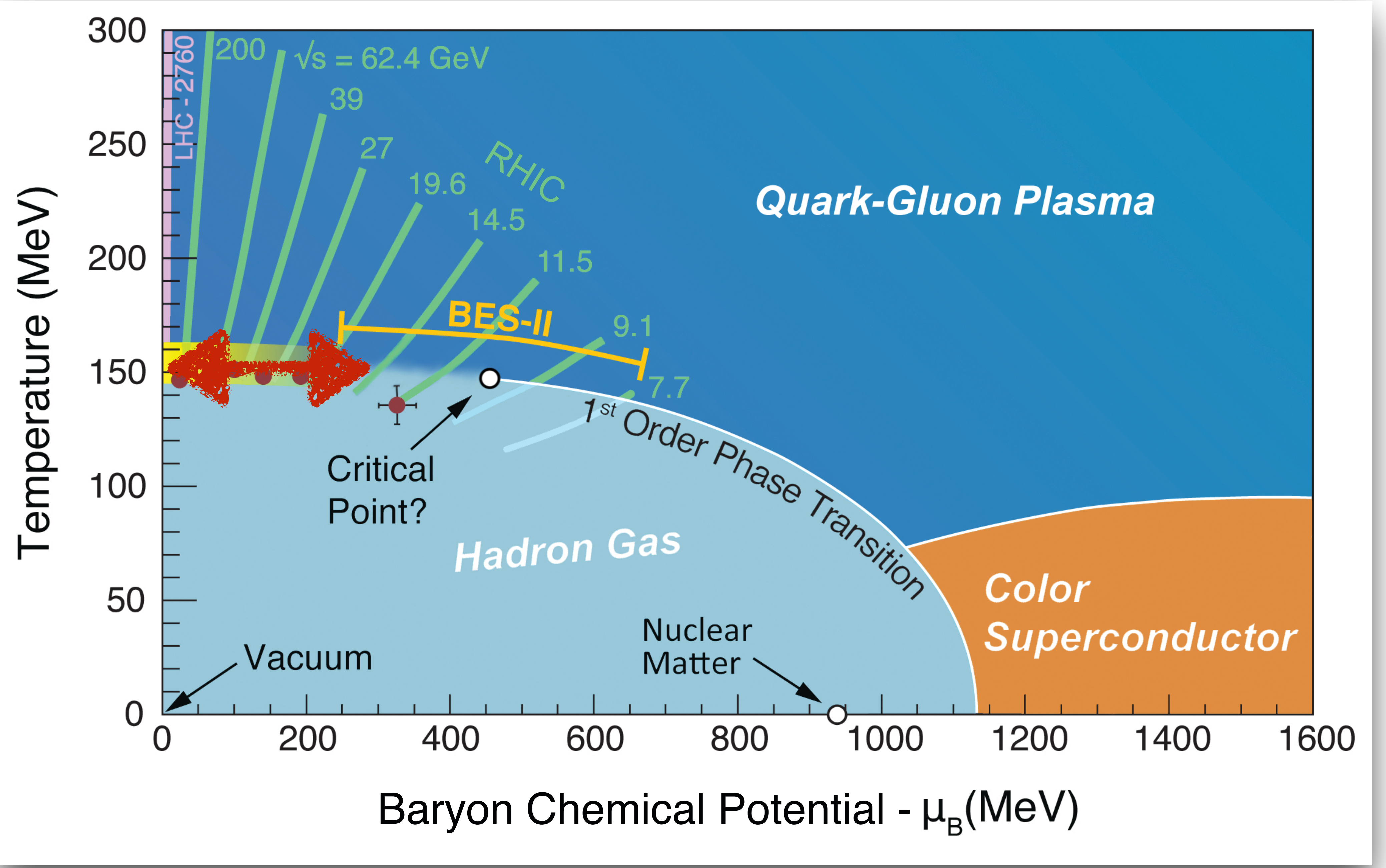
QCD chiral crossover temperature: $T_c(\mu_B)$

HotQCD: arXiv:1812.08235



$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2^B \left(\frac{\mu_B}{T_c(0)}\right)^2 - \kappa_4^B \left(\frac{\mu_B}{T_c(0)}\right)^4 + \mathcal{O}\left(\mu_B^6\right)$$

chiral order parameter:
$$\Sigma = \frac{1}{f_K^4}\left[m_s\langle\bar{u}u + \bar{d}d\rangle - (m_u + m_d)\langle\bar{s}s\rangle\right]$$

order parameter susceptibility:
fluctuation of order parameter
$$\chi^\Sigma = m_s\left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right)\Sigma$$

disconnected chiral susceptibility:
fluctuation of light quark condensate
$$\chi = \frac{m_s^2}{f_K^4}\left[\langle(\bar{u}u + \bar{d}d)^2\rangle - (\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)^2\right]$$

Taylor expansion in chemical potential:
$$\Sigma(T,\mu_X) = \sum_{n=0}^{\infty}\frac{C_{2n}^\Sigma(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n} \qquad C_0^\Sigma(T) = \Sigma(T,0)$$

… and for $\chi(T,\mu_X)$:
$$C_{2n}^\chi(T) \qquad C_0^\chi(T) = \chi(T,0)$$

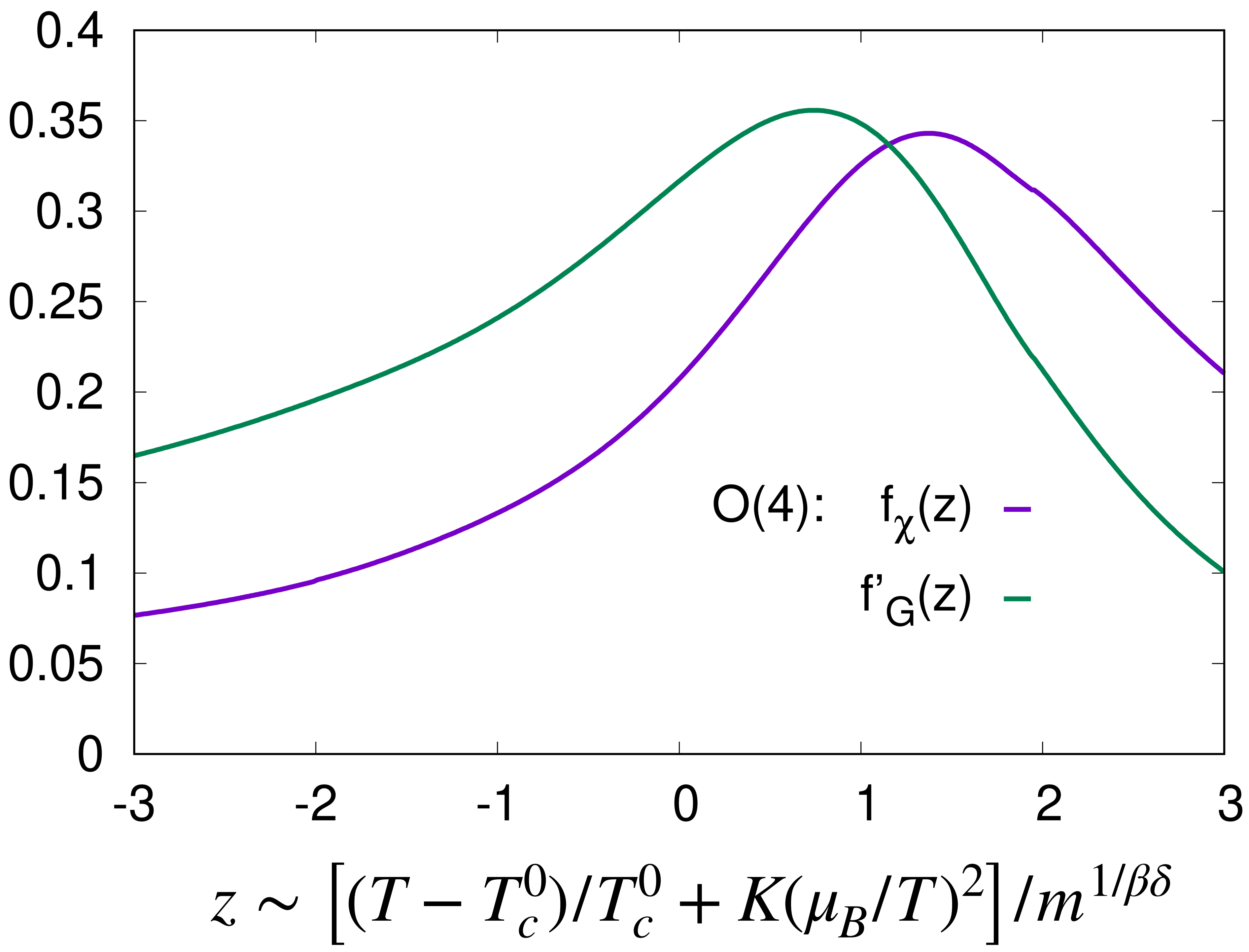
$$\mu = 0: \quad C_0^\Sigma(T),\ C_2^\Sigma(T),\ C_0^\chi(T),\ C_2^\chi(T),\ \chi^\Sigma(T)$$

$$\Sigma(T,\mu_B) \sim m^{1/\delta} f_G$$

$$\chi(T,\mu_B), \chi^{\Sigma}(T) \sim m^{(1-\delta)/\delta} f_\chi$$

$$\partial_T C_0^{\Sigma}(T),\, C_2^{\Sigma}(T) \sim m^{(\beta-1)/\beta\delta} f'_G$$

$$\partial_T \chi^{\Sigma}(T),\; \partial_T C_0^{\chi}(T),\, C_2^{\chi}(T) \sim m^{(\beta-\beta\delta-1)/\beta\delta} f'_\chi$$



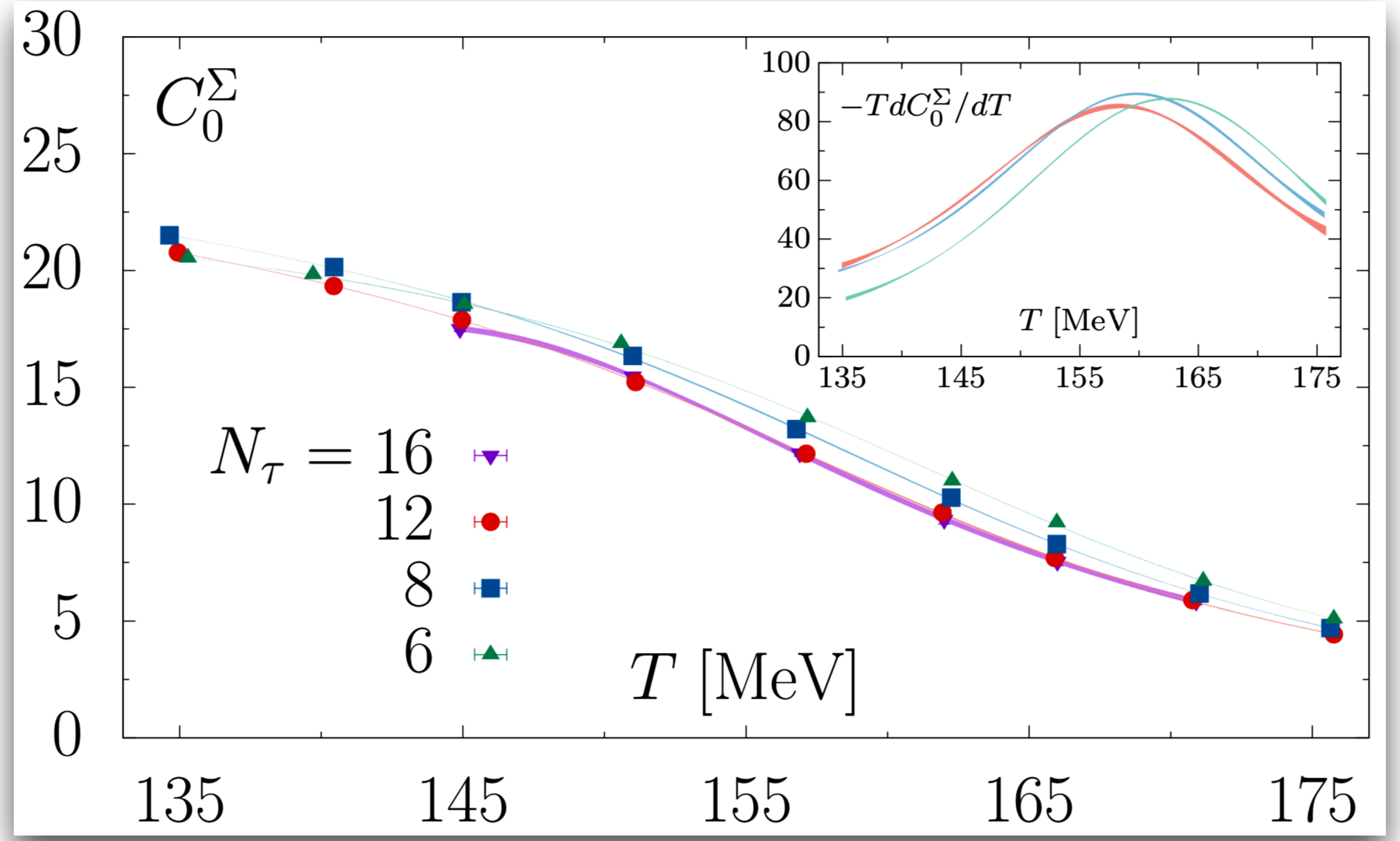
$$\mu = 0$$

$$\partial_T^2 C_0^{\Sigma}(T) = 0 \qquad \partial_T C_2^{\Sigma}(T) = 0$$

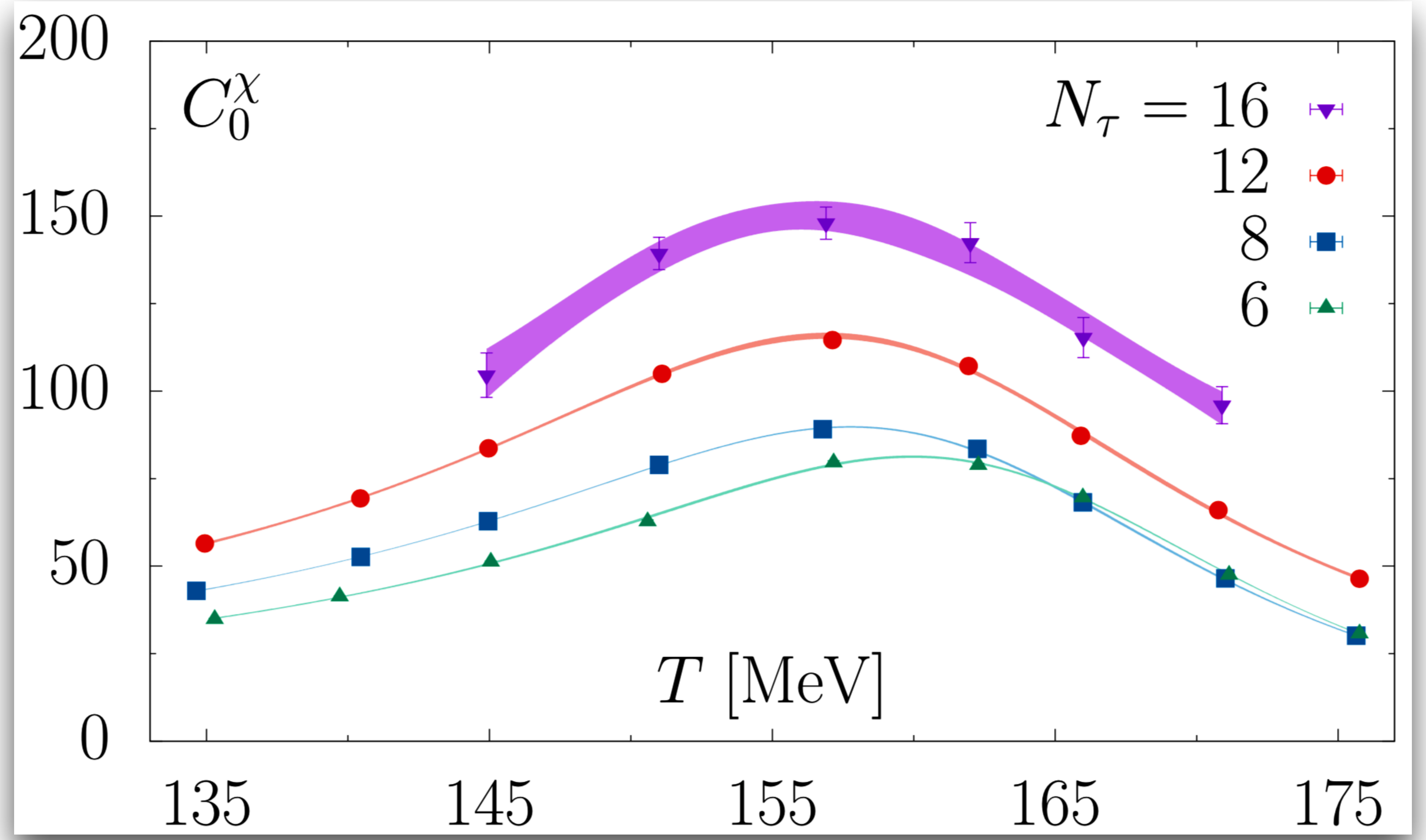
$$\partial_T C_0^{\chi}(T) = 0 \qquad C_2^{\chi}(T) = 0$$

$$\partial_T \chi^{\Sigma}(T) = 0$$

- m = 0: all these susceptibilities will diverge at a unique transition temperature
- m > 0: crossover, different susceptibilities can lead to different crossover temperatures

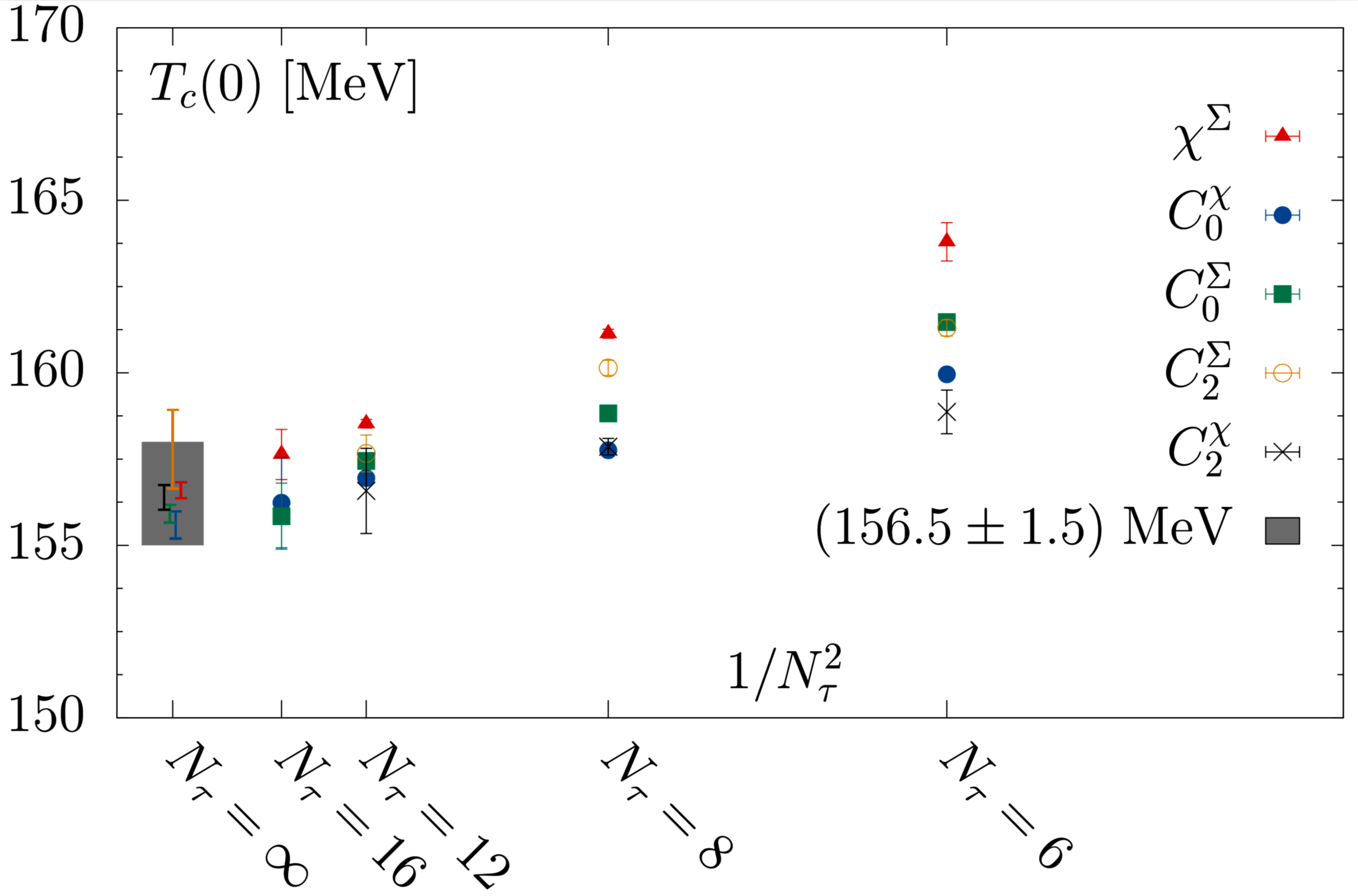


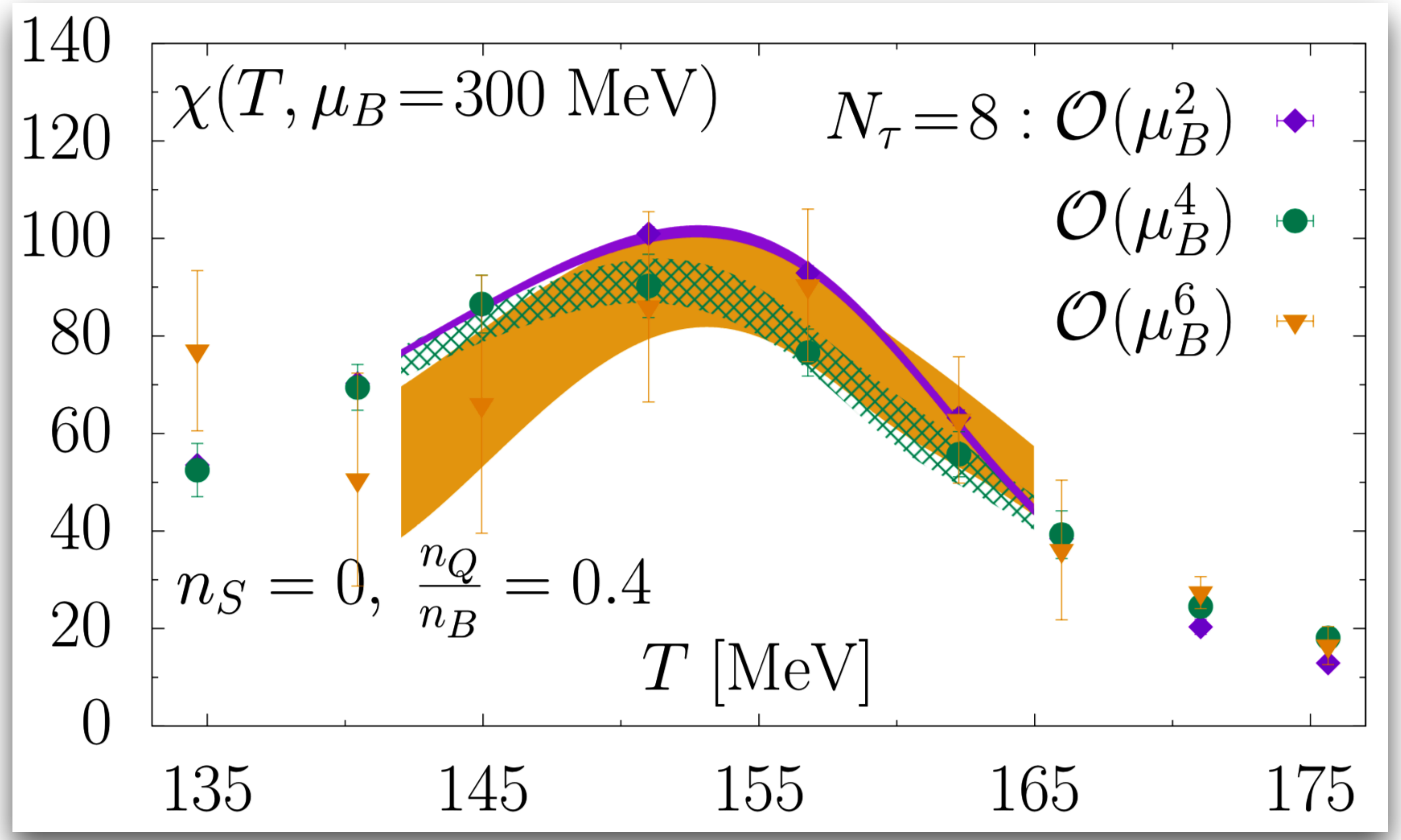
chiral condensate
order parameter



chiral susceptibility
order parameter fluctuation

$T_c(\mu_B = 0) = 156.5 \pm 1.5\ MeV$



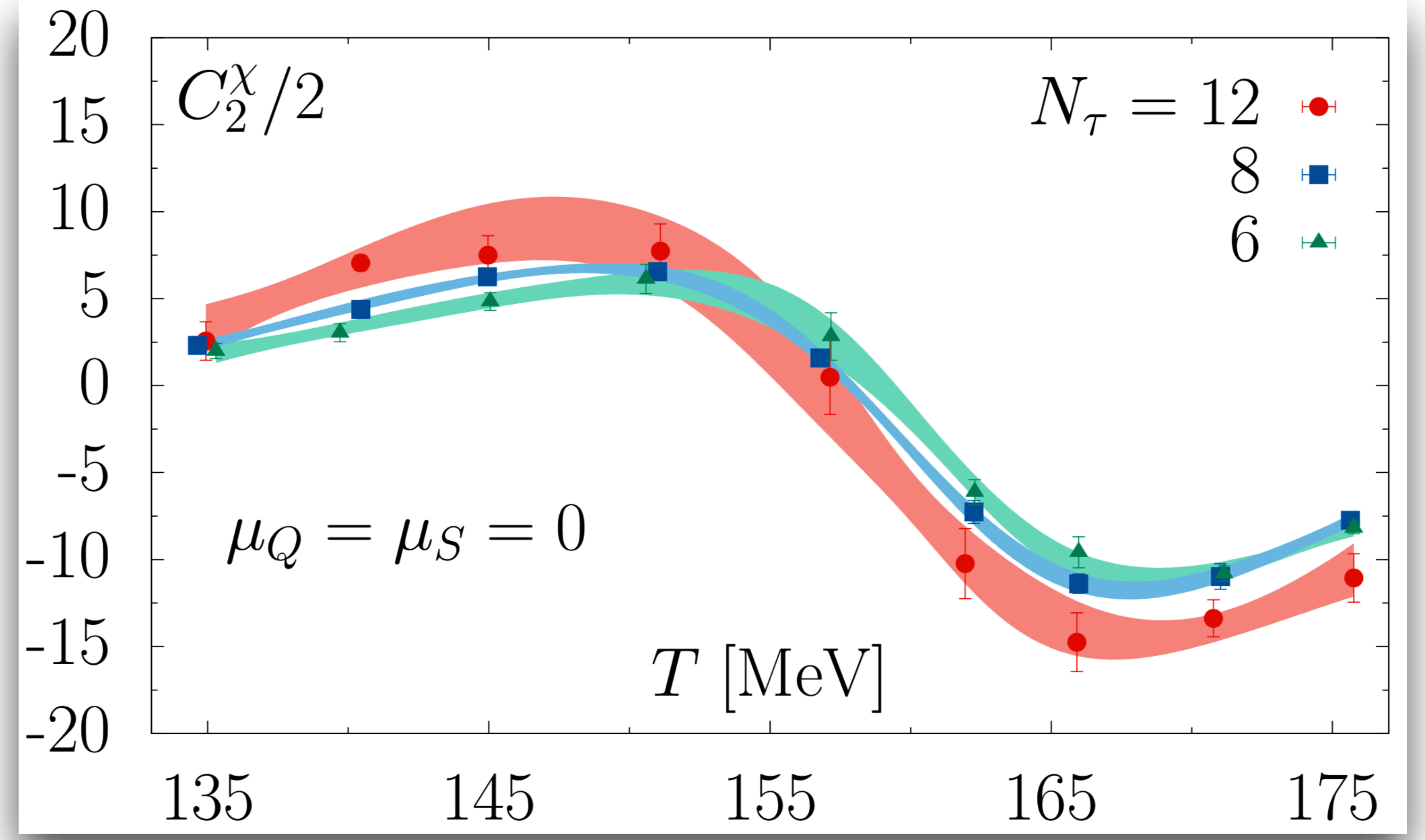


chiral susceptibility

order parameter fluctuation

$n_S = 0,\ n_Q/n_B = 0.4$: heavy-ion collision like strangeness neutrality & charge-to-baryon ratio

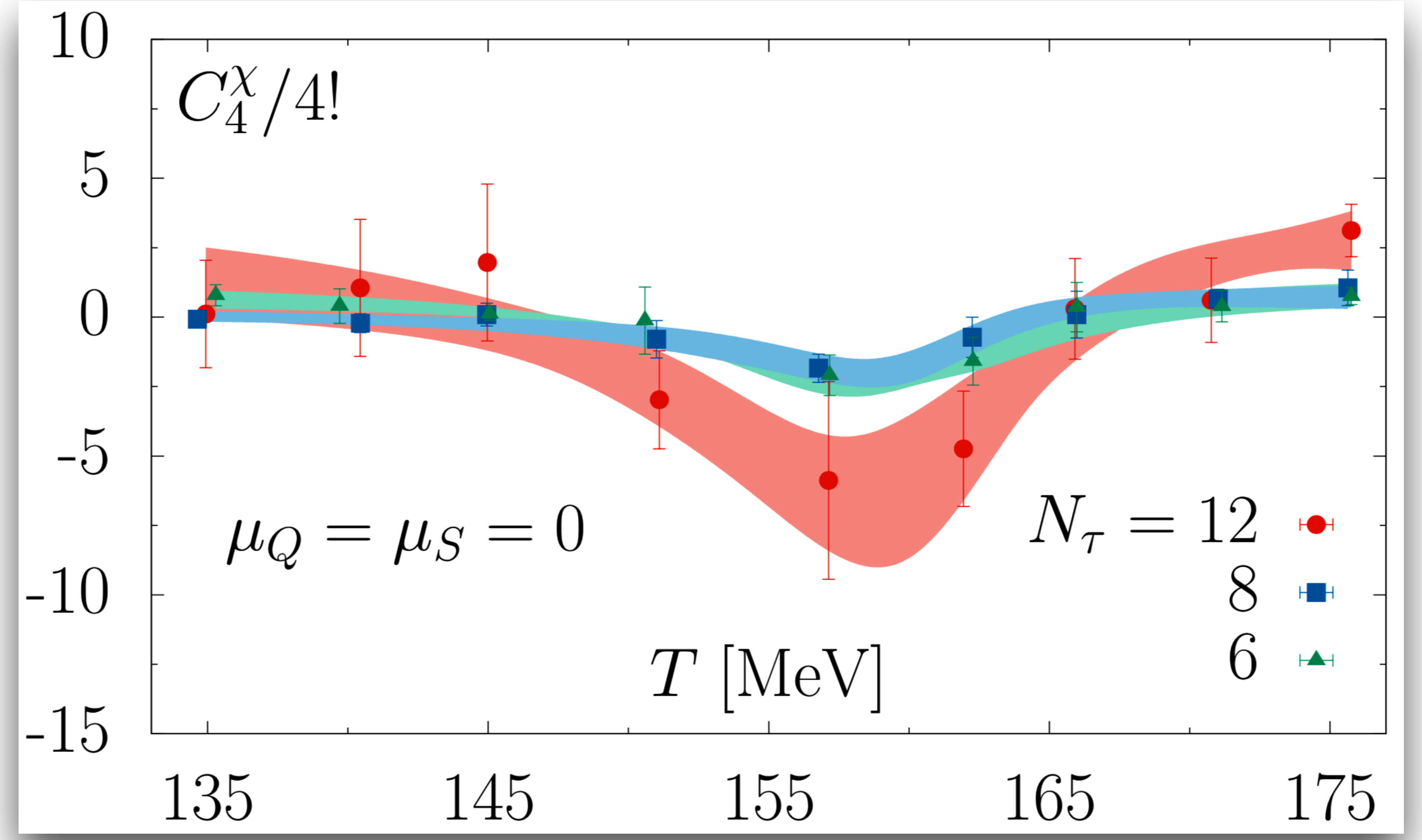
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2^B \left(\frac{\mu_B}{T_c(0)}\right)^2 - \kappa_4^B \left(\frac{\mu_B}{T_c(0)}\right)^4 + \mathcal{O}\left(\mu_B^6\right)$$



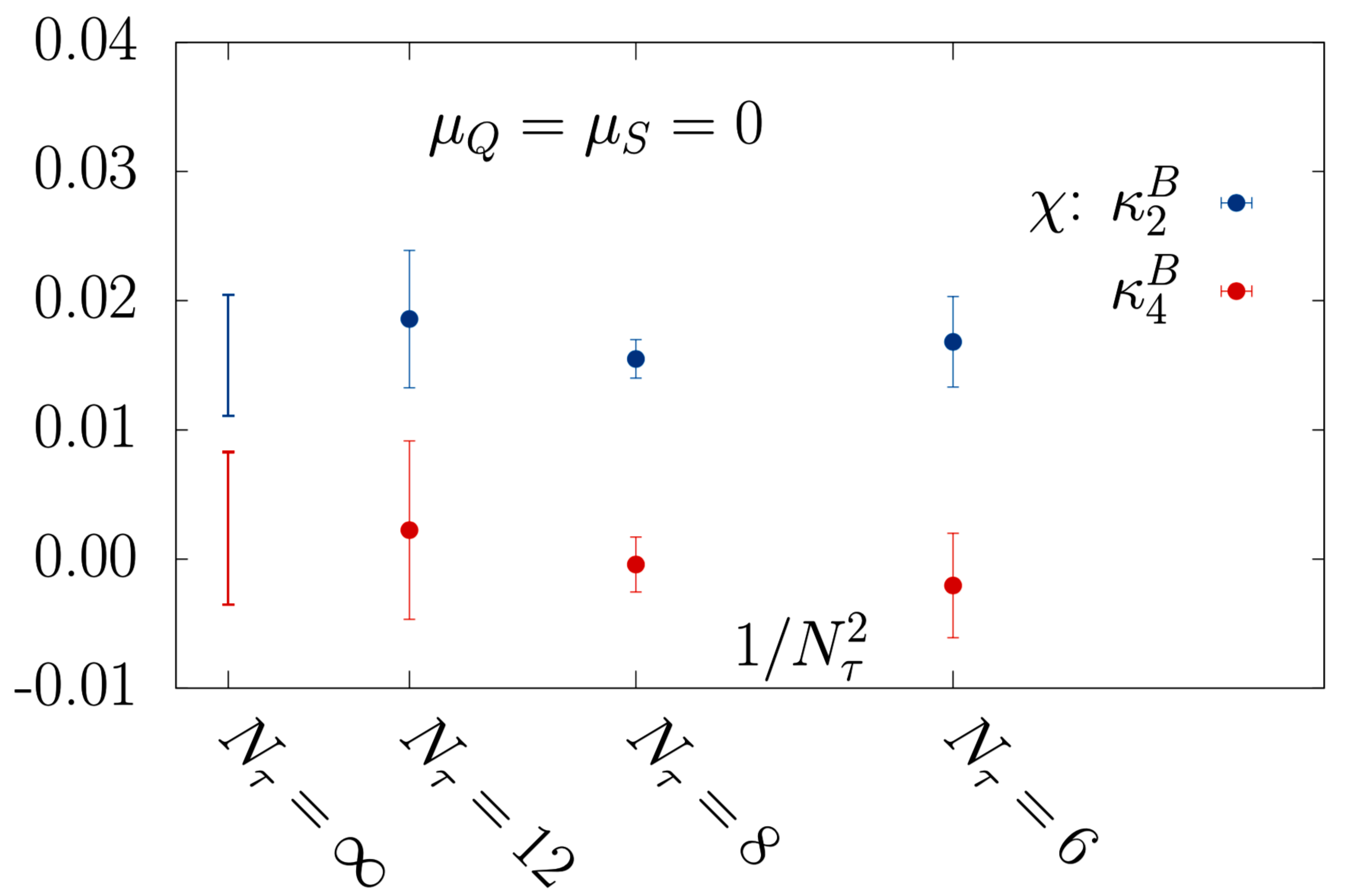
$$\partial_T \chi(T,\mu_B) = \partial_T C_0^\chi(T) + \partial_T C_2^\chi(T) + \partial_T C_4^\chi(T) + \cdots = 0$$

$$\kappa_2^X = \frac{1}{2T^2 \partial_T^2 C_0^X} \left[T\partial_T C_2^X - 2C_2^X\right]$$

$$\kappa_4^X = \frac{1}{24T^2 \partial_T^2 C_0^X} \left[-72\kappa_2^X C_2^X - 4C_4^X + T\partial_T C_4^X + 12\kappa_2^X \left(4T\partial_T C_2^X - T^2 \partial_T^2 C_2^X + \kappa_2^X T^3 \partial_T^3 C_0^X\right)\right]$$



$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2^B \left(\frac{\mu_B}{T_c(0)}\right)^2 - \kappa_4^B \left(\frac{\mu_B}{T_c(0)}\right)^4 + \mathcal{O}\left(\mu_B^6\right)$$

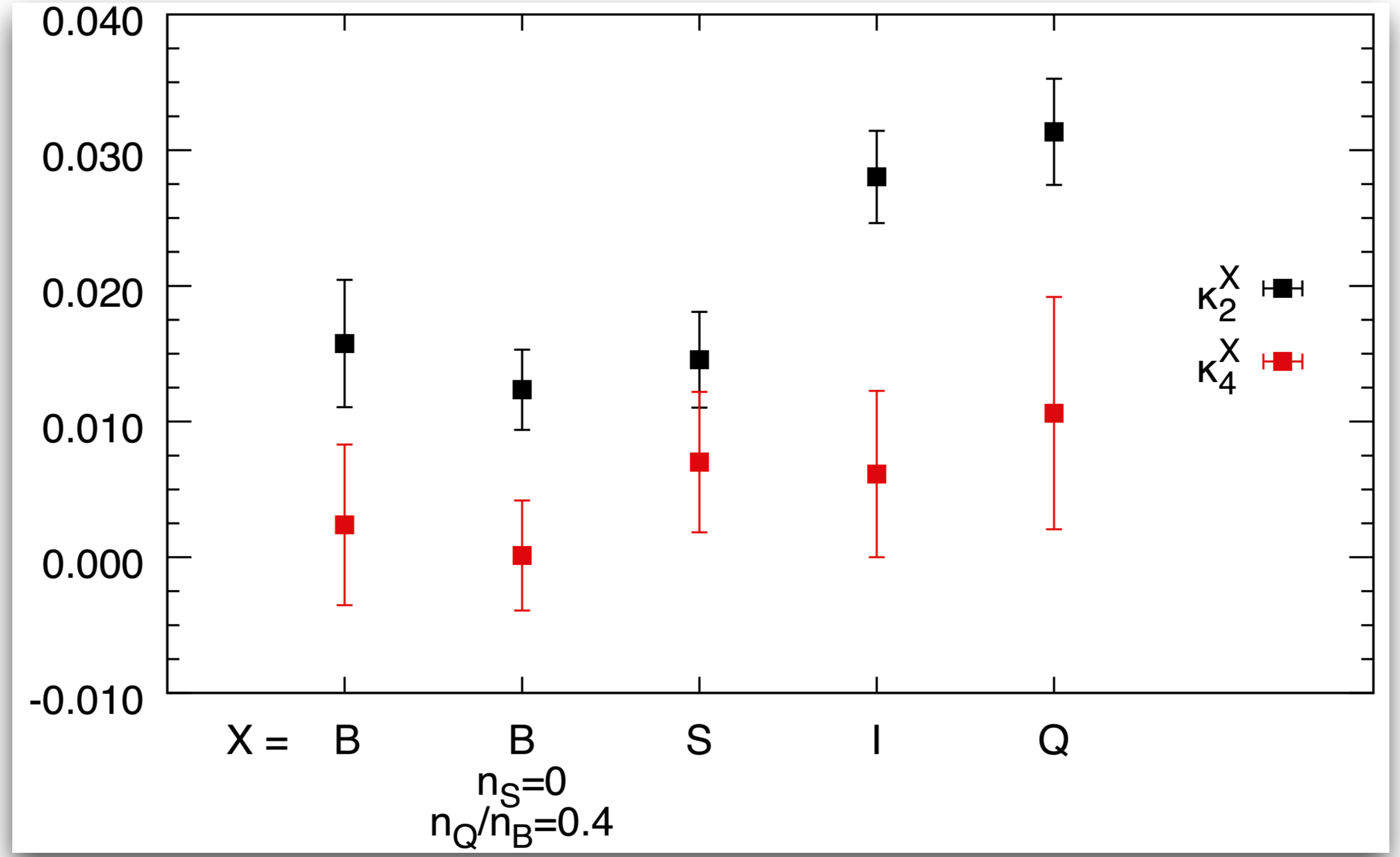


- 4^{th} order corrections order of magnitude smaller than 2^{nd}

chiral crossover surface: $T_c(\mu_X)$

$$\frac{T_c(\mu_X)}{T_c(0)} = 1 - \kappa_2^B \left(\frac{\mu_X}{T_c(0)}\right)^2 - \kappa_4^B \left(\frac{\mu_X}{T_c(0)}\right)^4 + \mathcal{O}\left(\mu_X^6\right)$$

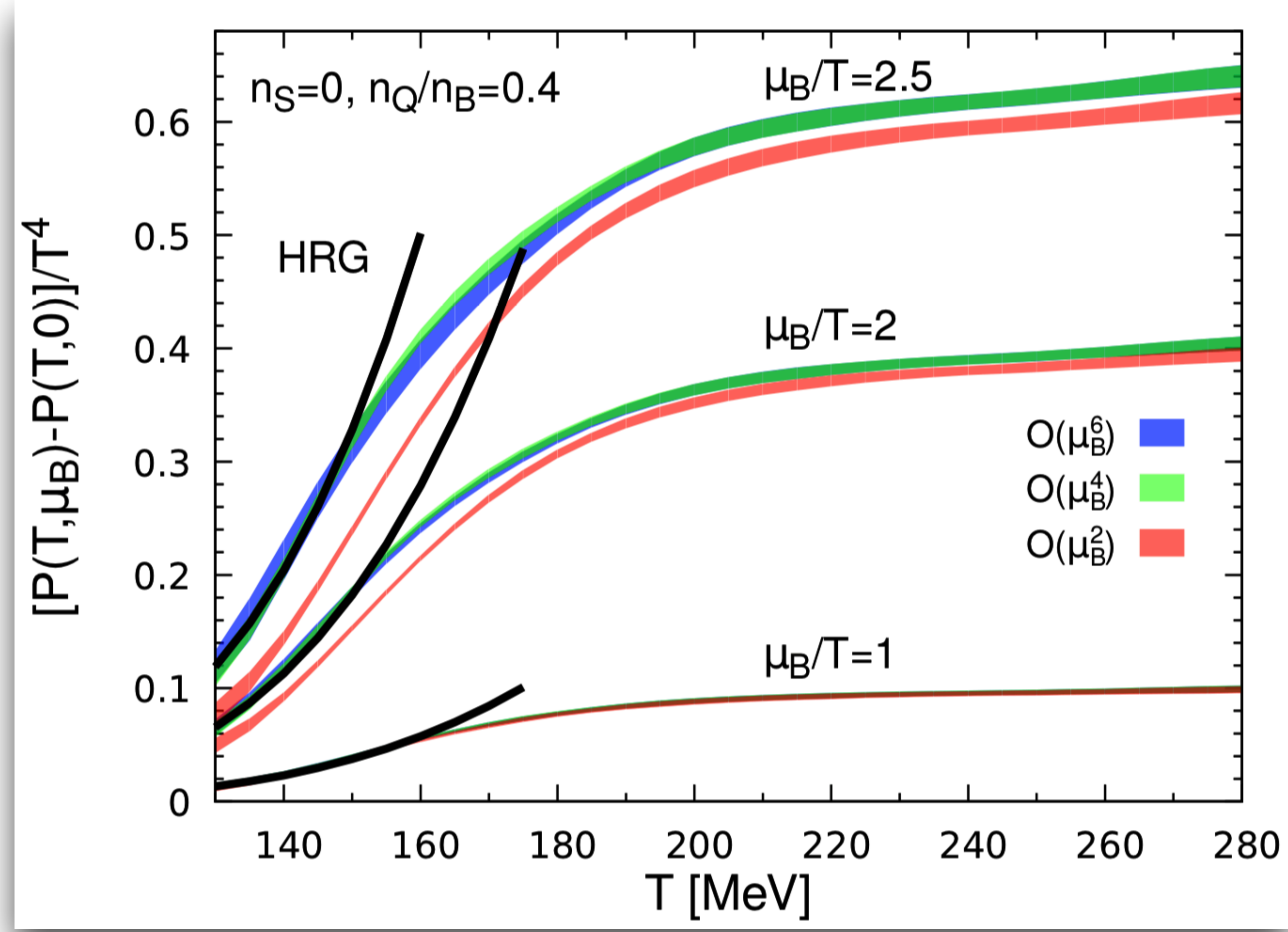
- X=B: baryon
- X=Q: electric charge
- X=S: strangeness
- X=I: isospin



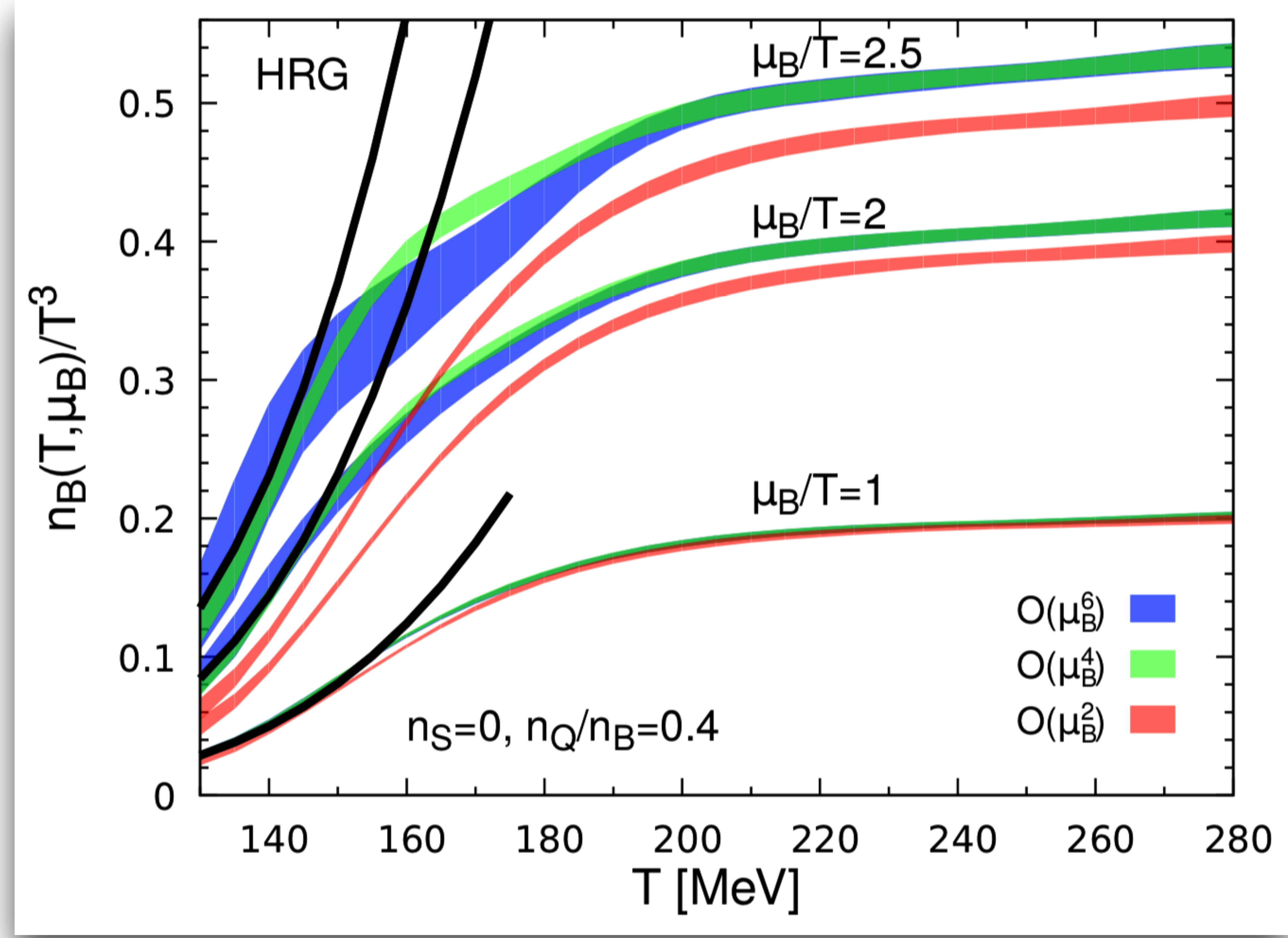
QCD equation of state @ $\mu_B > 0$

HotQCD: Phys. Rev. D95, no.5, 054504 (2017)

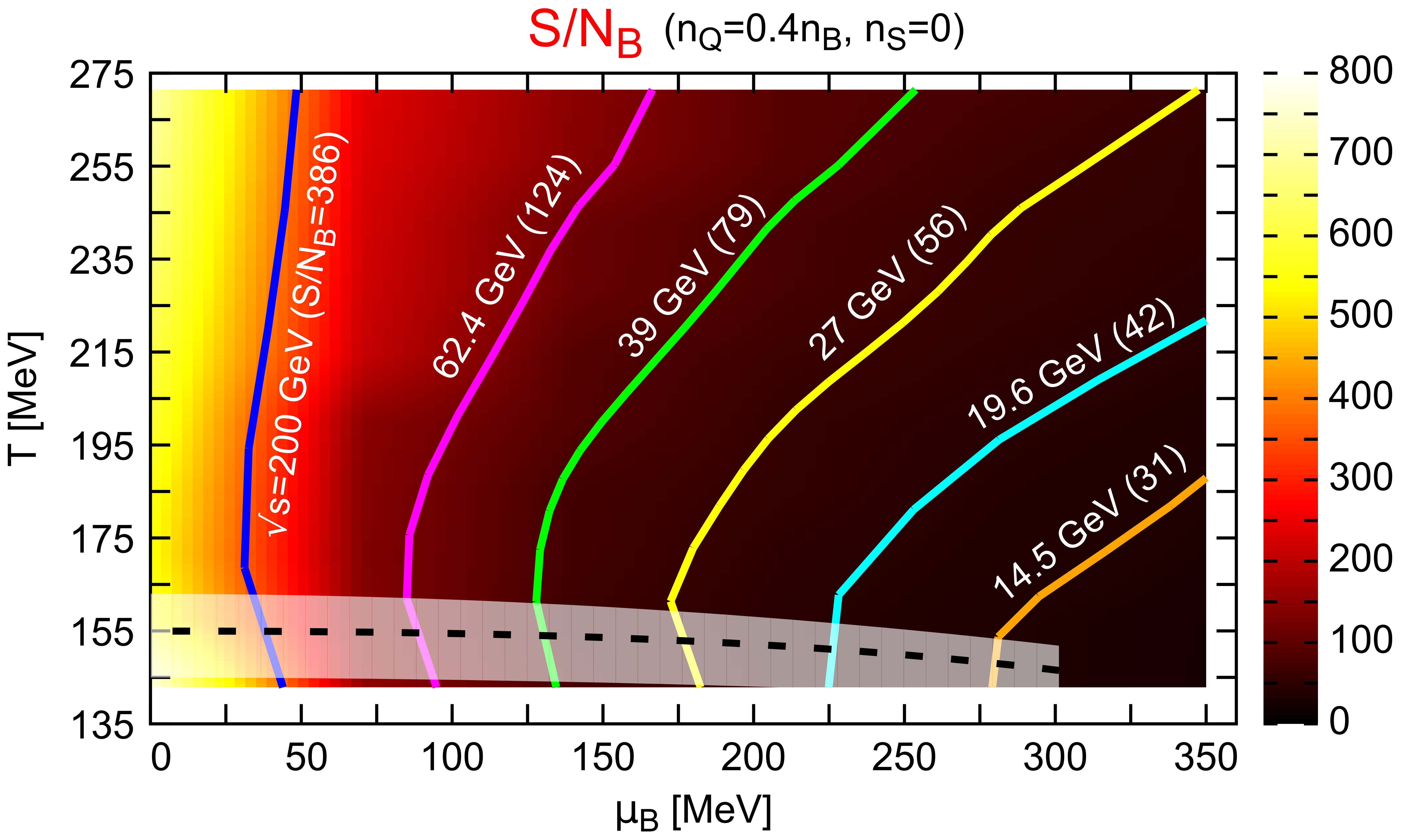
pressure difference



net baryon number density



- Taylor expansion up to: $\mathcal{O}(\mu_B^6)$
- controlled up to: $\mu_B/T \lesssim 2$



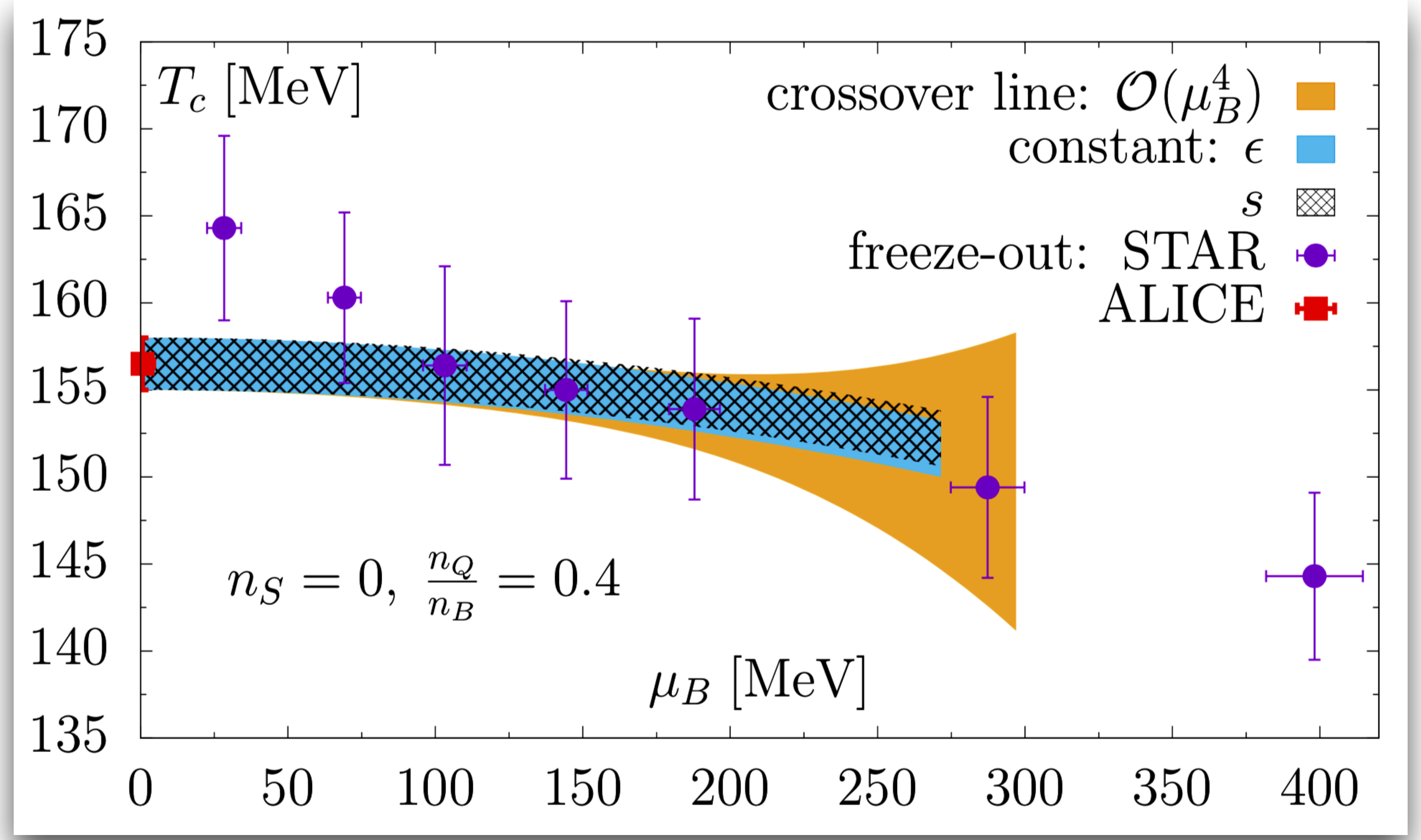
approximate trajectories in heavy-ion collisions

QCD phase boundary

HotQCD: arXiv:1812.08235

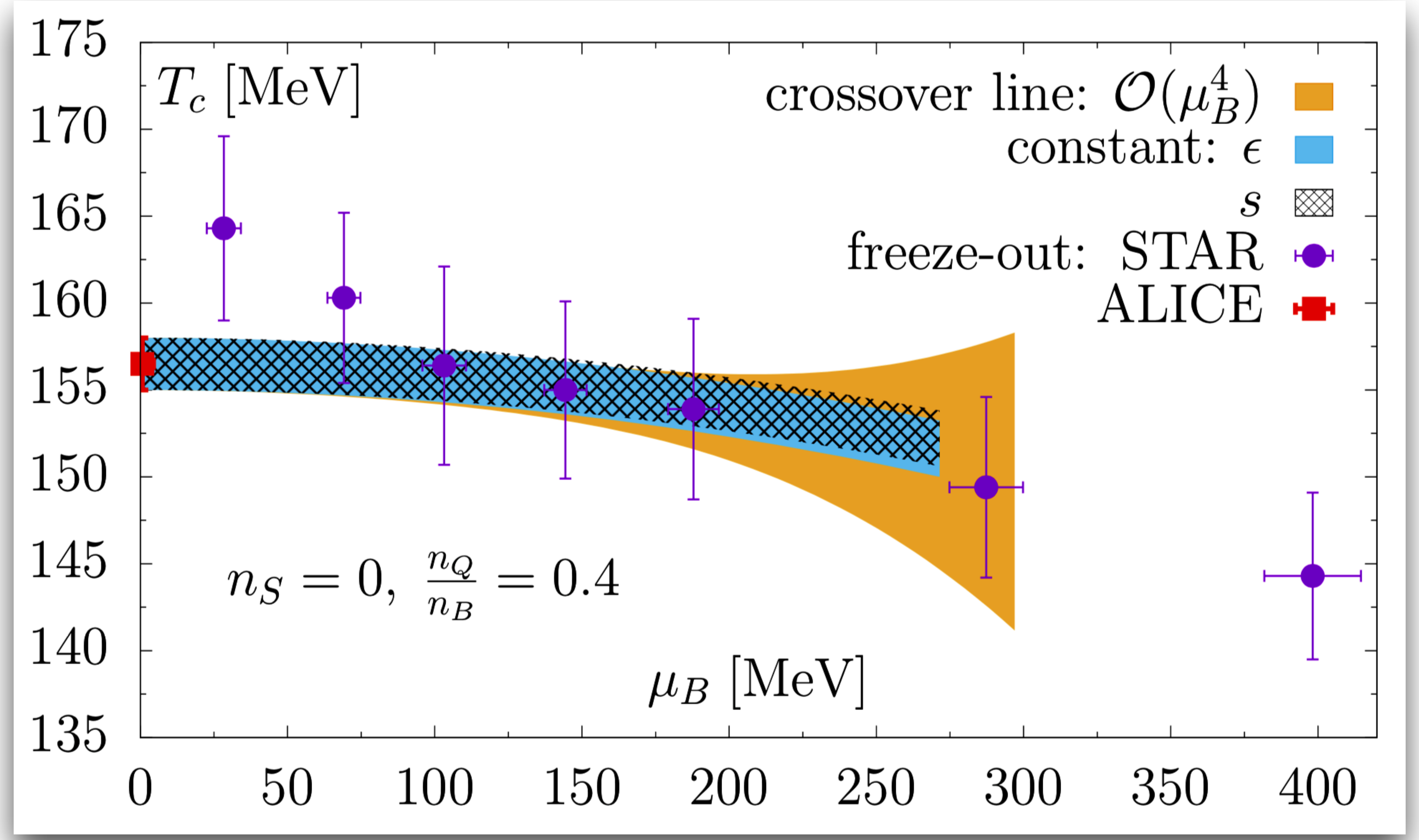
HotQCD: Phys. Rev. D95, no.5, 054504 (2017)

- freeze-out line coincides with the chiral crossover
- along the chiral crossover energy density & entropy density remains constant



2^{nd} order fluctuations around QCD phase boundary

HotQCD preliminary

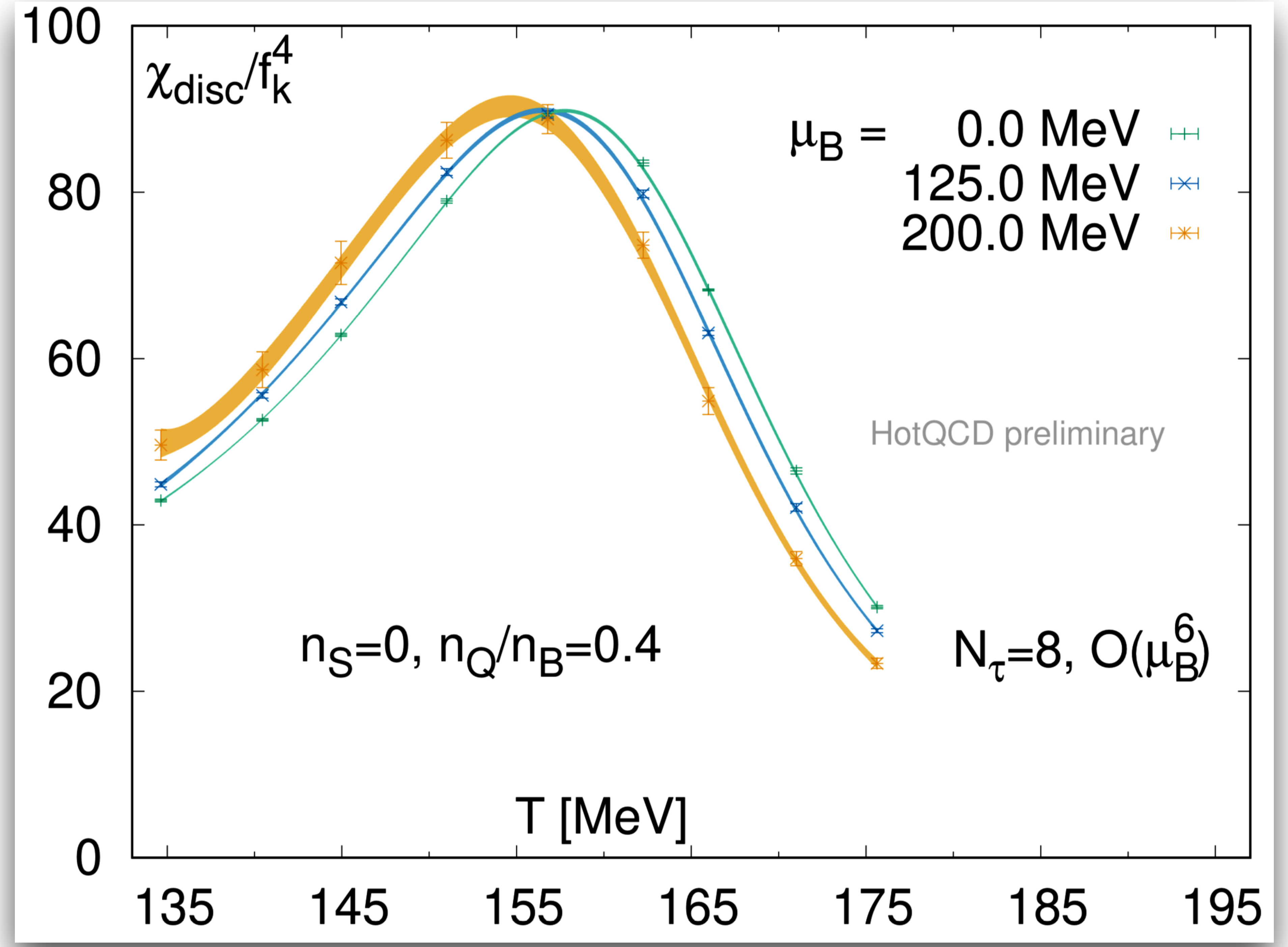


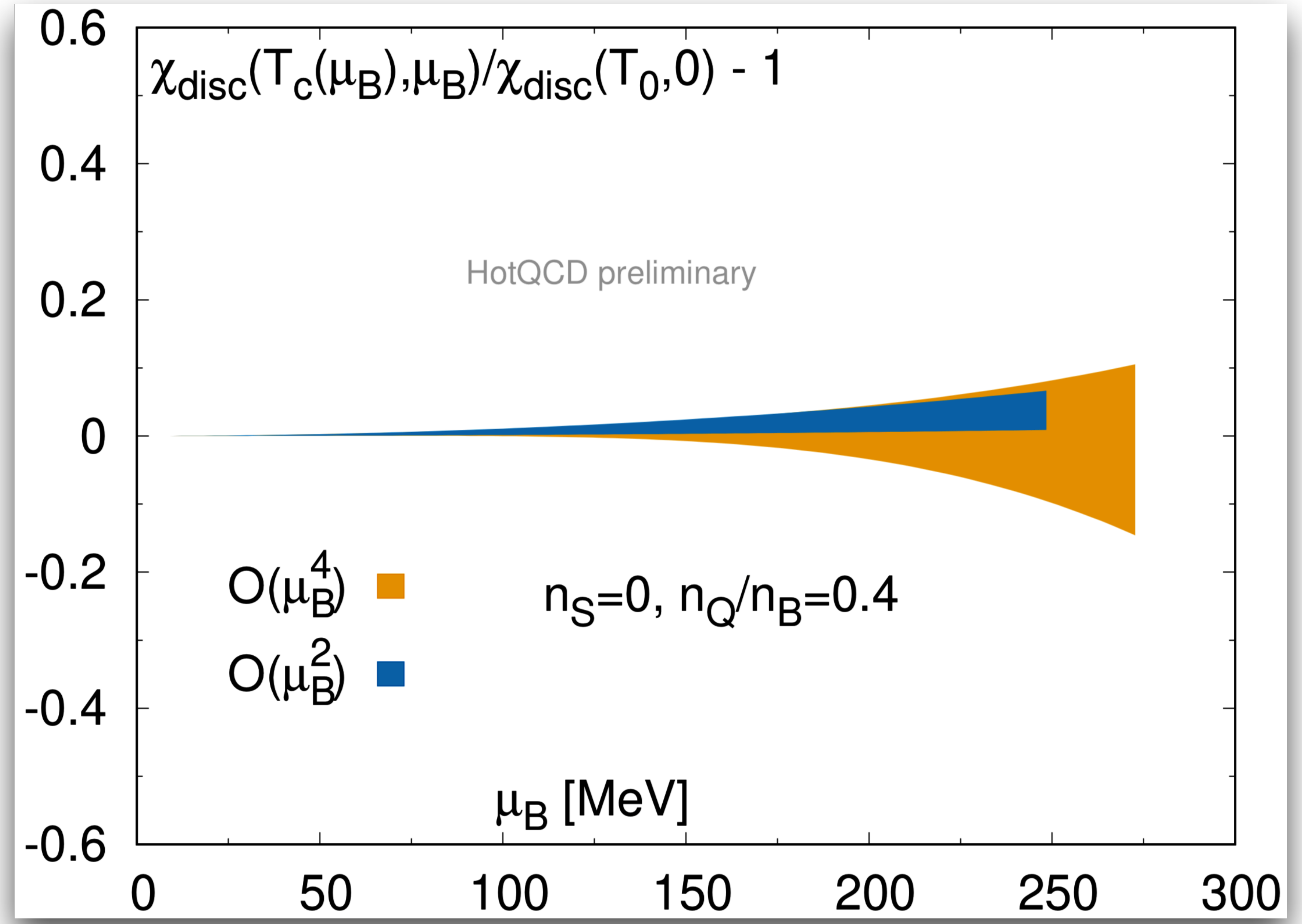
signs of enhanced fluctuations around the phase-boundary ?

chiral susceptibility

$$\chi_{disc} \sim \partial^2 \ln \mathscr{Z} / \partial m_q^2$$

along $T_c(\mu_B)$



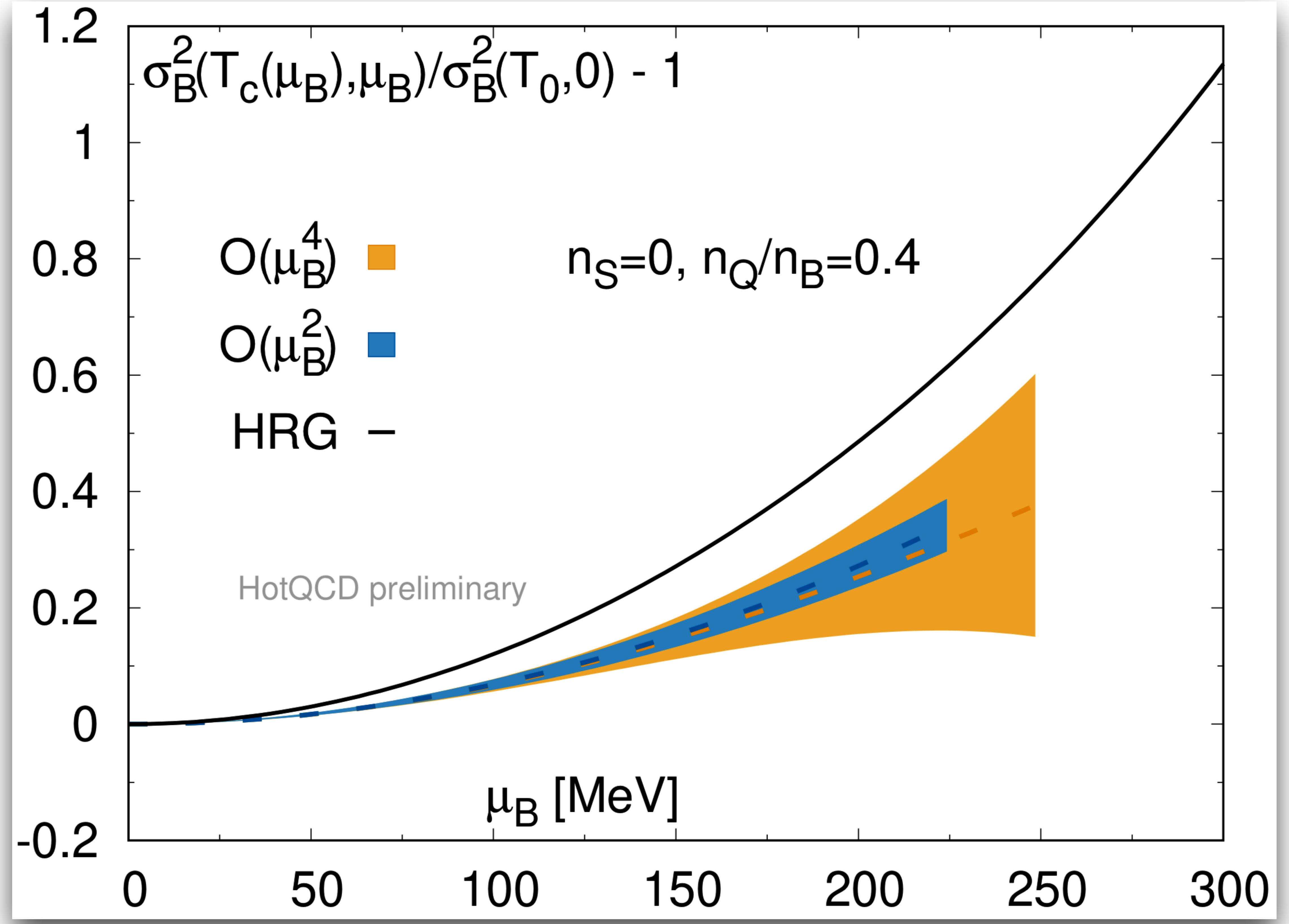


- little change in peak-height & width with increasing $\mu_B > 0$: no indication of transition becoming stronger

net baryon number fluctuations

along $T_c(\mu_B)$

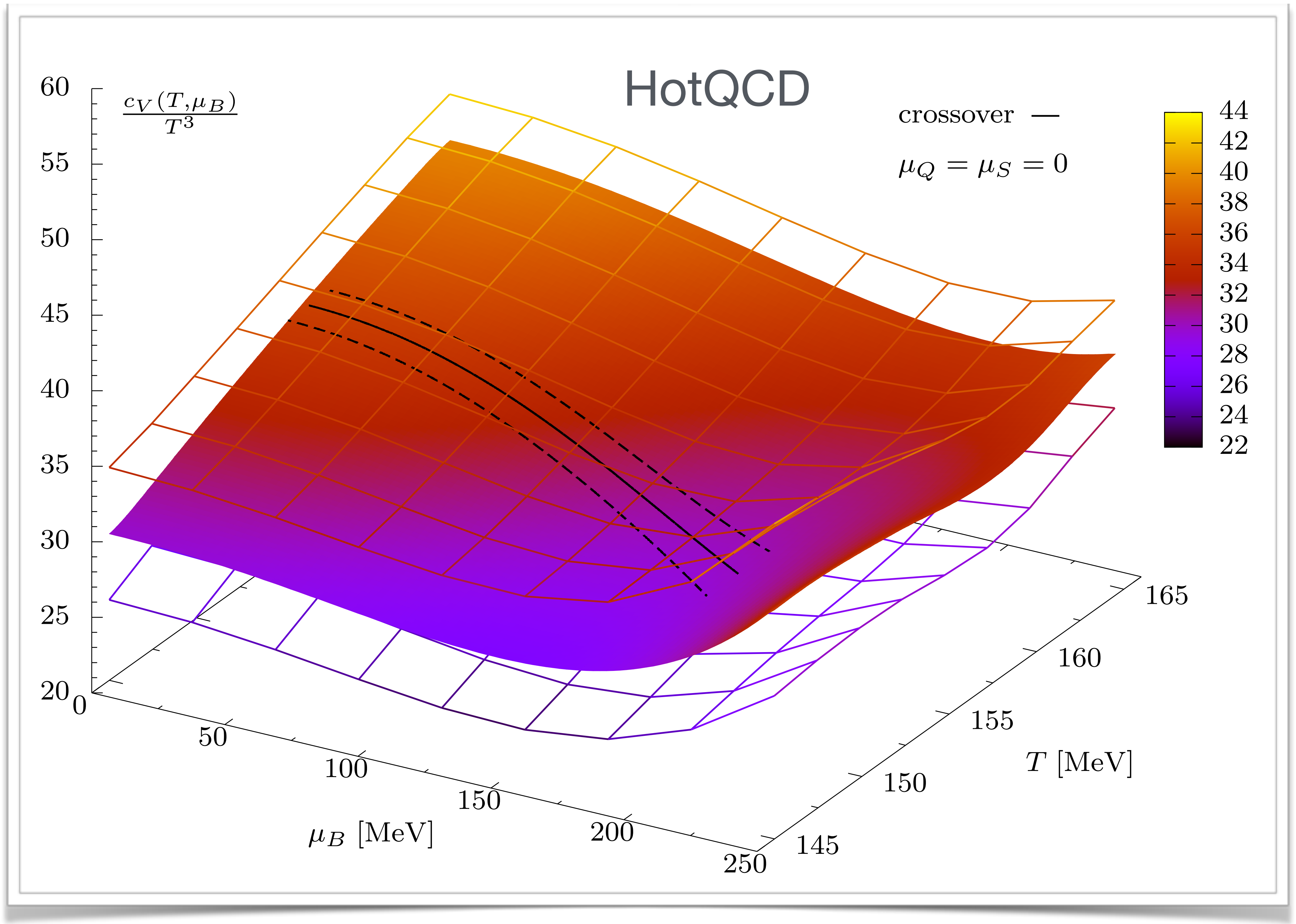
$$\frac{\sigma_B^2}{Vf_K^3} = \frac{1}{Vf_K^3}\frac{\partial \ln Z}{\partial \hat{\mu}_B^2} = \sum_{n=0}^{\infty} \frac{c_n^B}{n!}\hat{\mu}_B^n$$



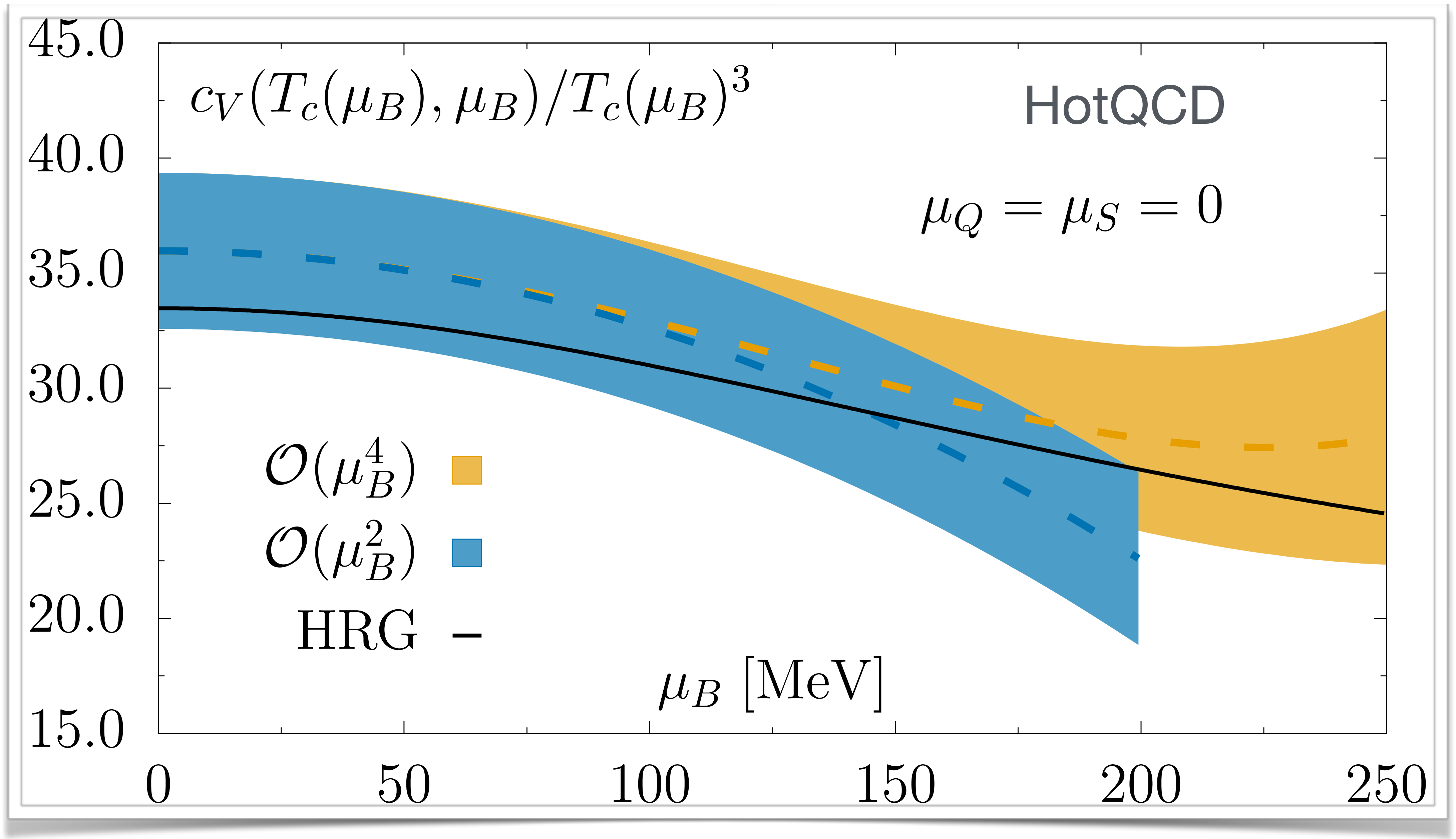
- increase remains less than (ideal) hadron gas resonance gas model (HRG)

specific heat @ constant volume

$$c_V = T\left(\frac{\partial s}{\partial T}\right)_{n_B}$$



along $T_c(\mu_B)$



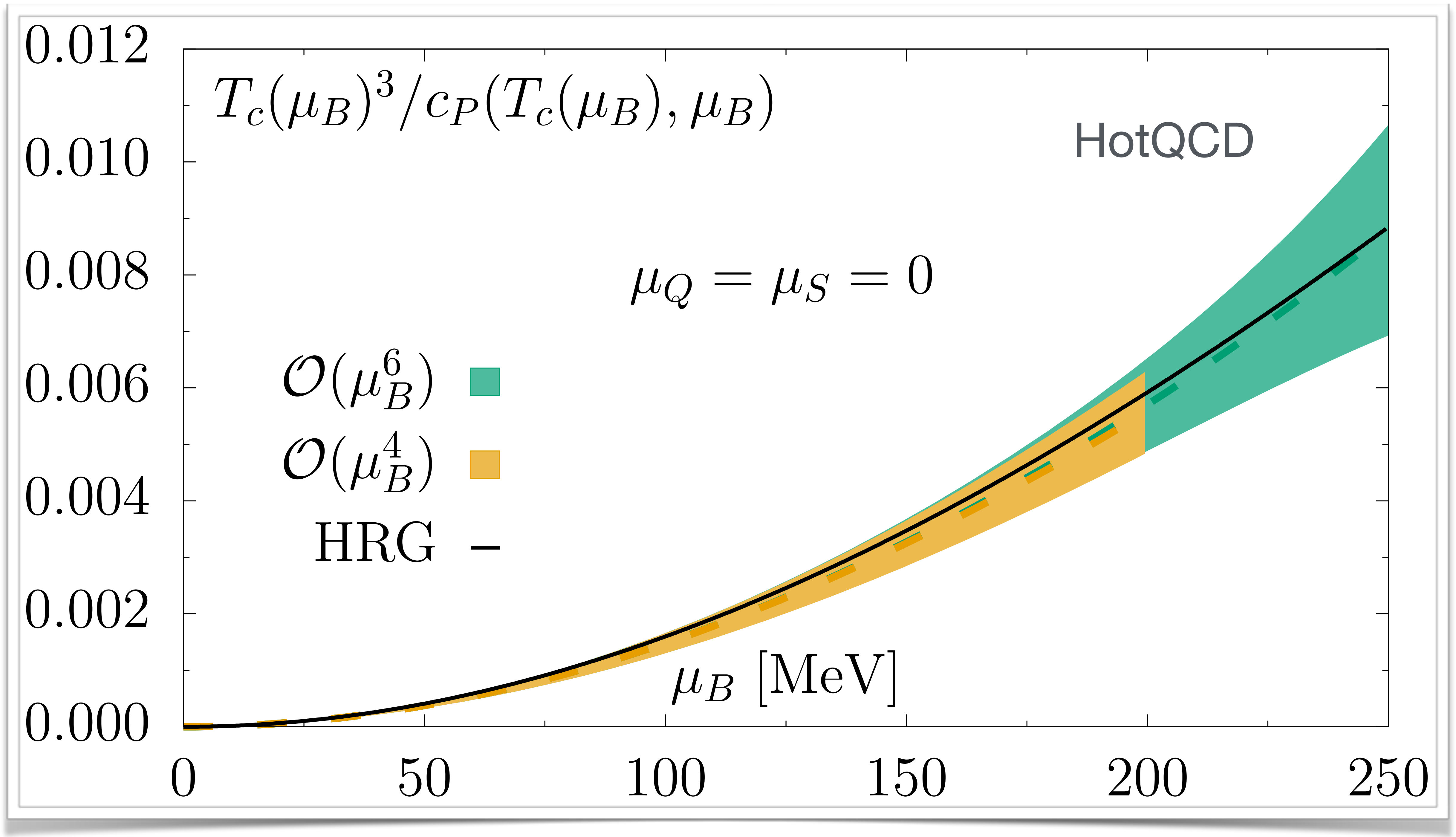
- no increase above HRG

(inverse) specific heat @ constant pressure

$$c_p = \frac{T}{(s/n_B)} \left[\frac{\partial (s/n_B)}{\partial T} \right]_p$$

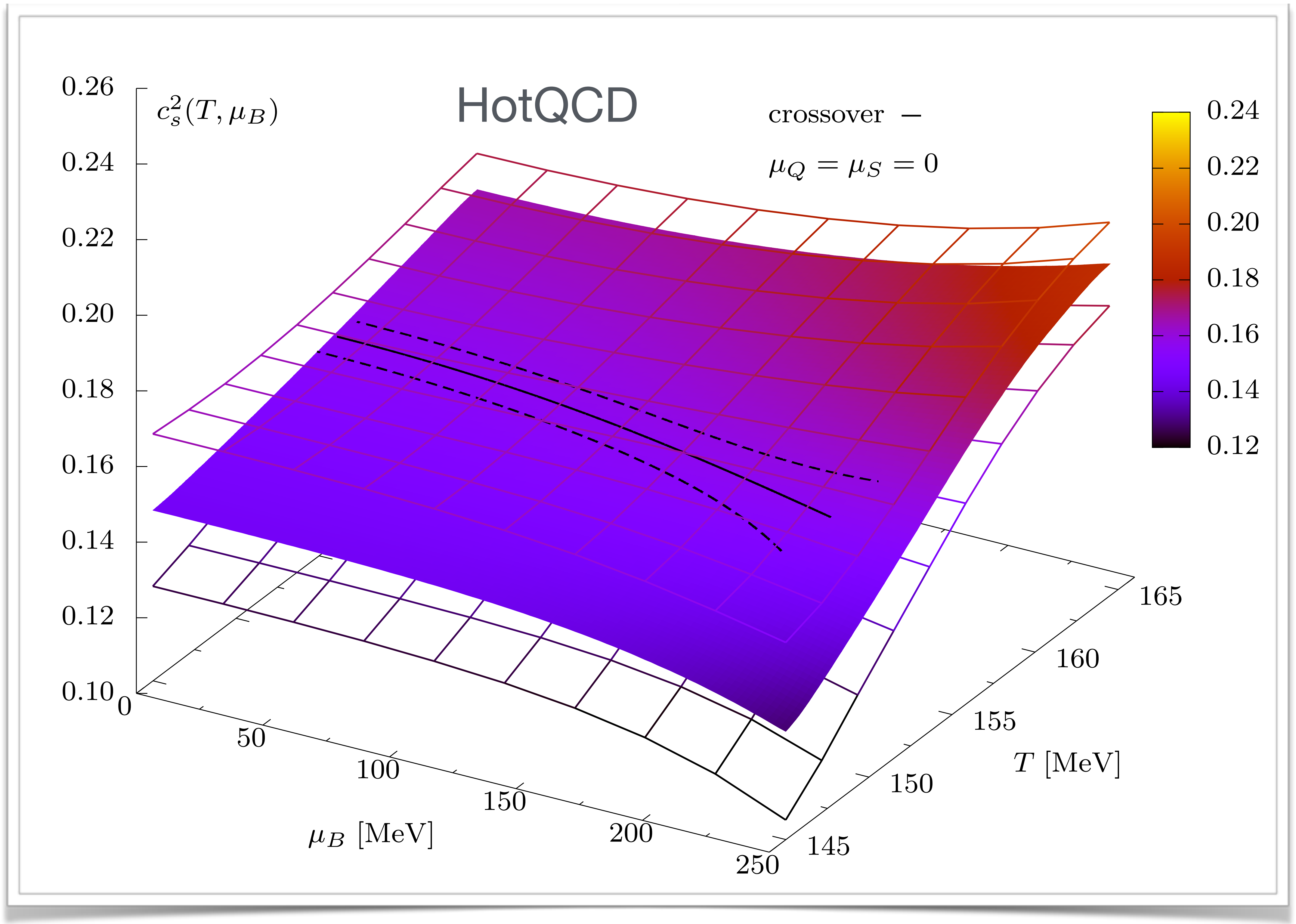
- no increase above HRG

along $T_c(\mu_B)$

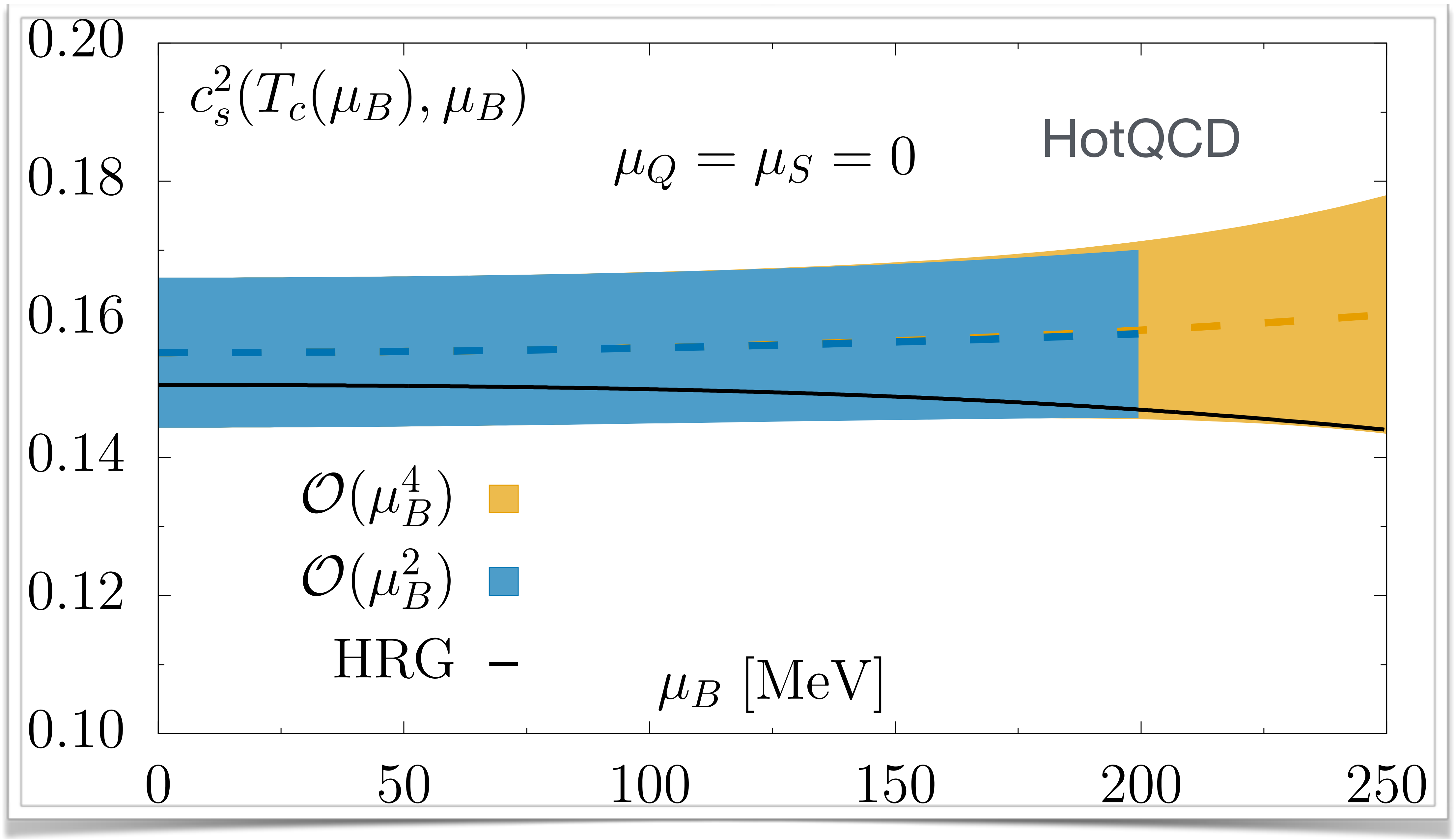


isentropic speed of sound

$$c_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_{s/n_B}$$



along $T_c(\mu_B)$



- no decrease below HRG

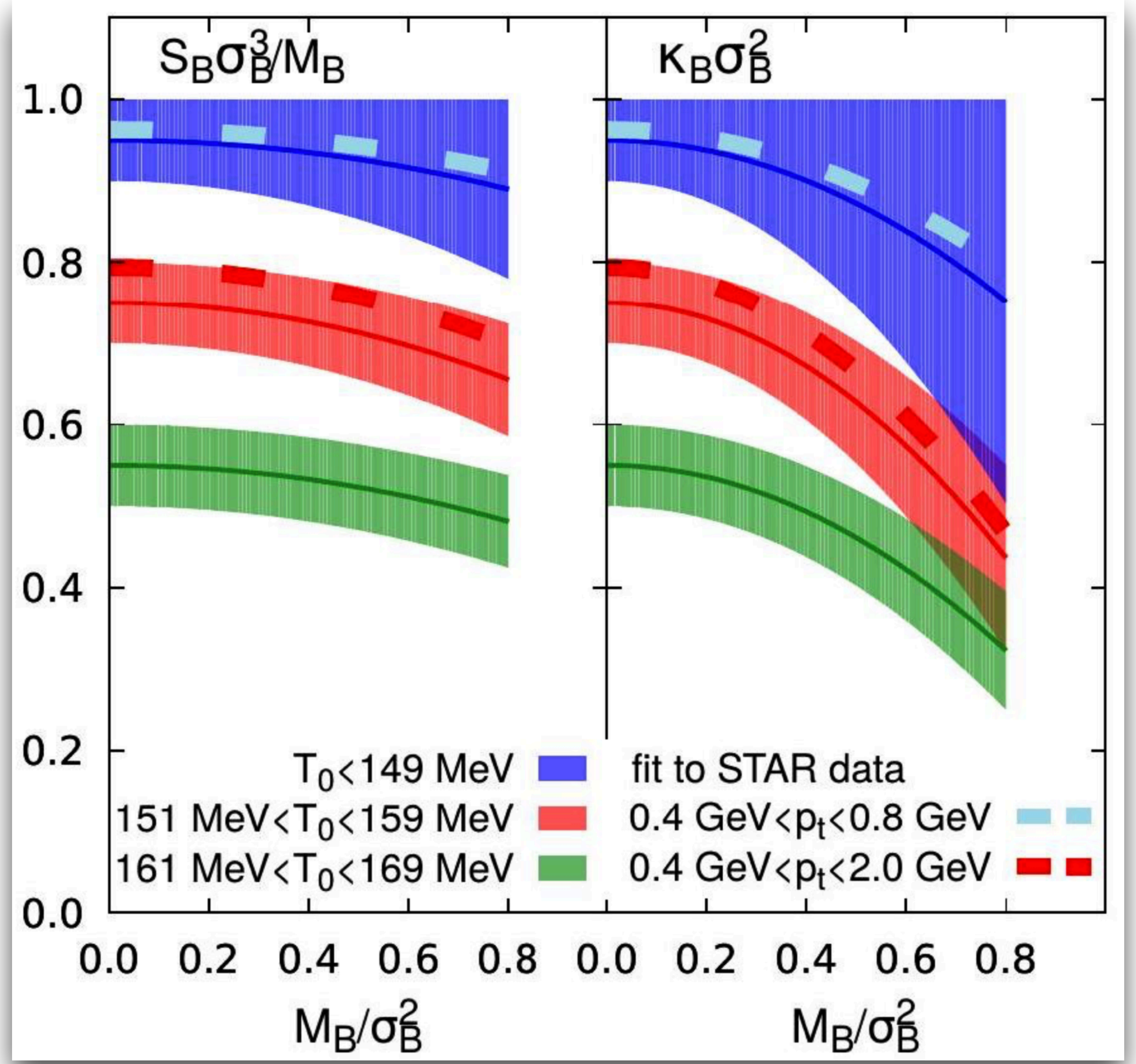
higher order fluctuations around QCD phase boundary

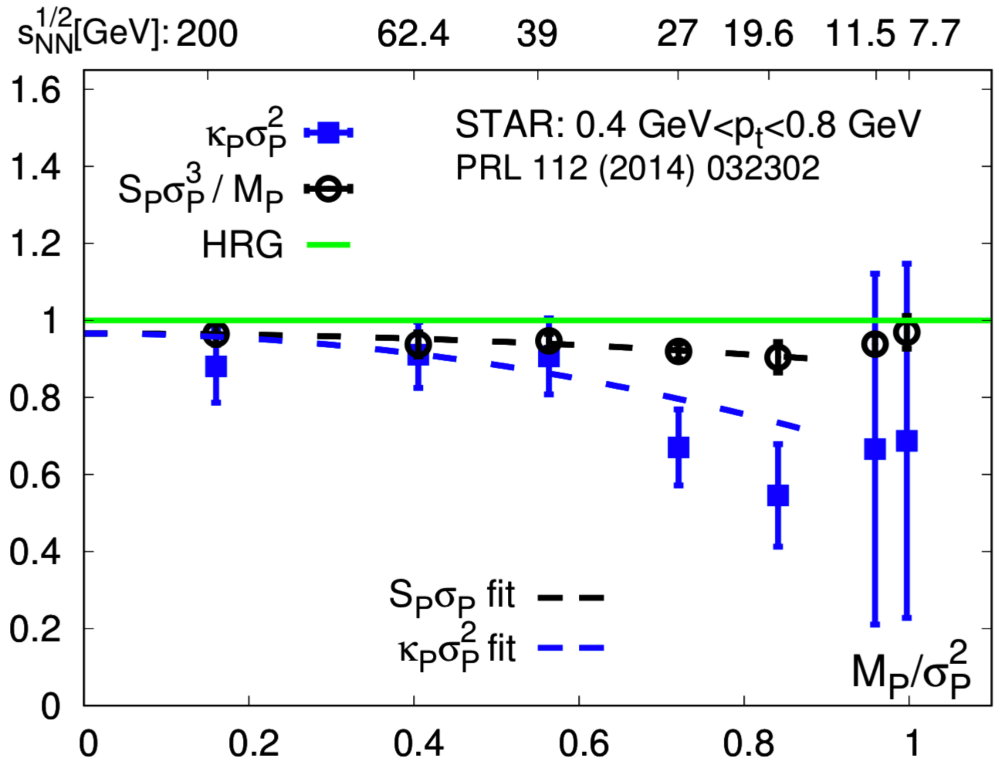
HotQCD: Phys.Rev. D96, no.7, 074510 (2017)

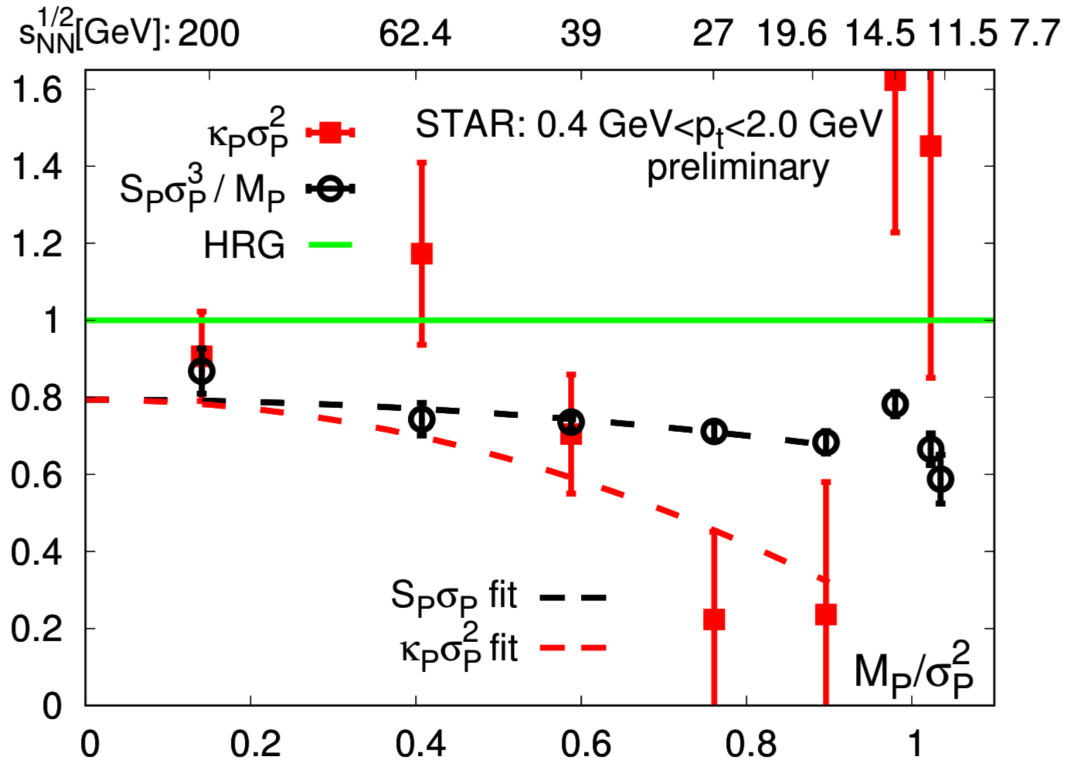
$$M_B/\sigma_B^2 = \chi_1^B/\chi_2^B \leftrightarrow \mu_B$$

$$S_B\sigma_B^3/M_B = \chi_3^B/\chi_2^B$$

$$\kappa_B\sigma_B^2 = \chi_4^B/\chi_2^B$$



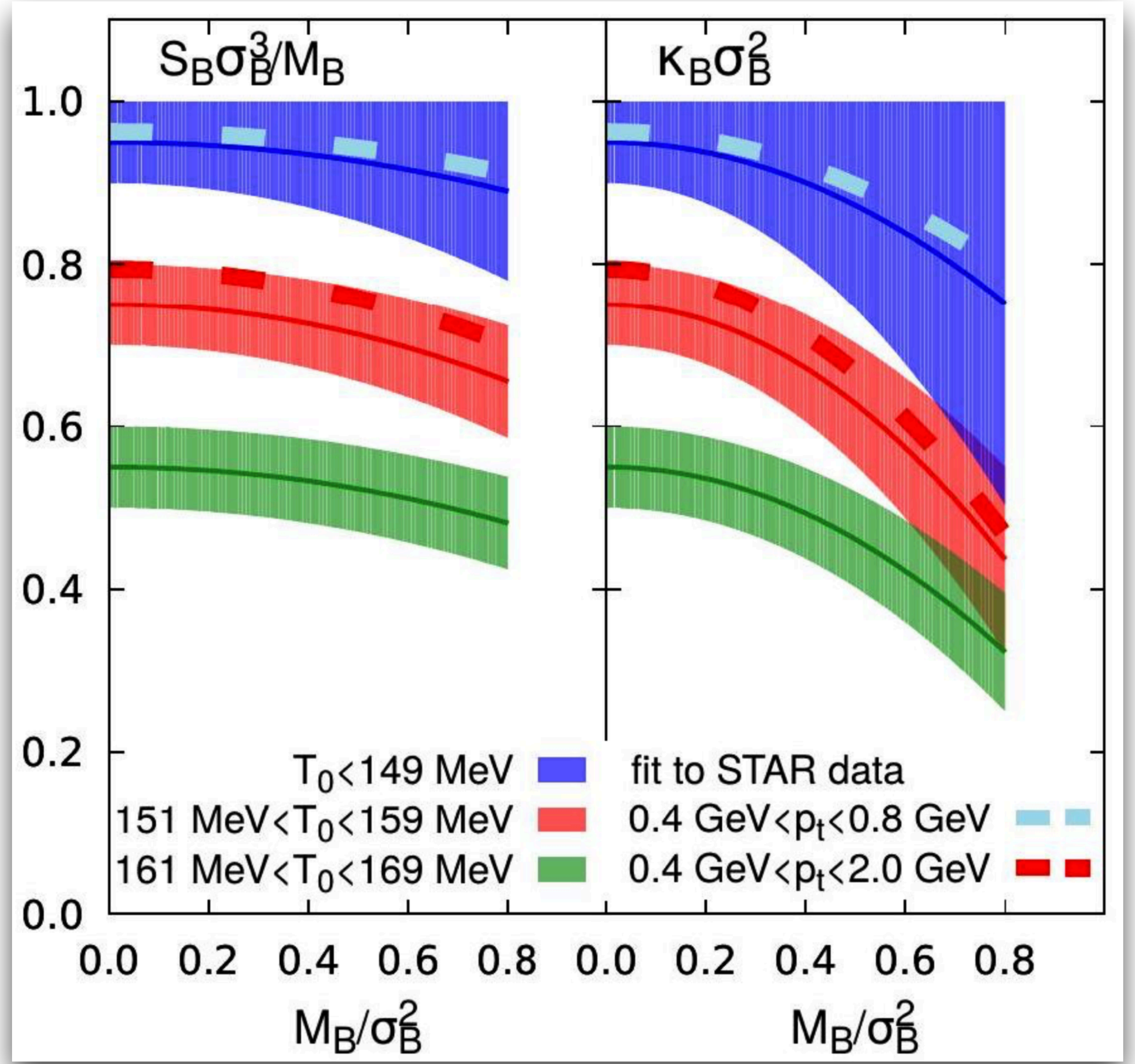




$$\sqrt{s} \gtrsim 19.6\ GeV:$$

cumulants of net-p fluctuations are consistent with equilibrium QCD

… but not an apples-to-apples comparison



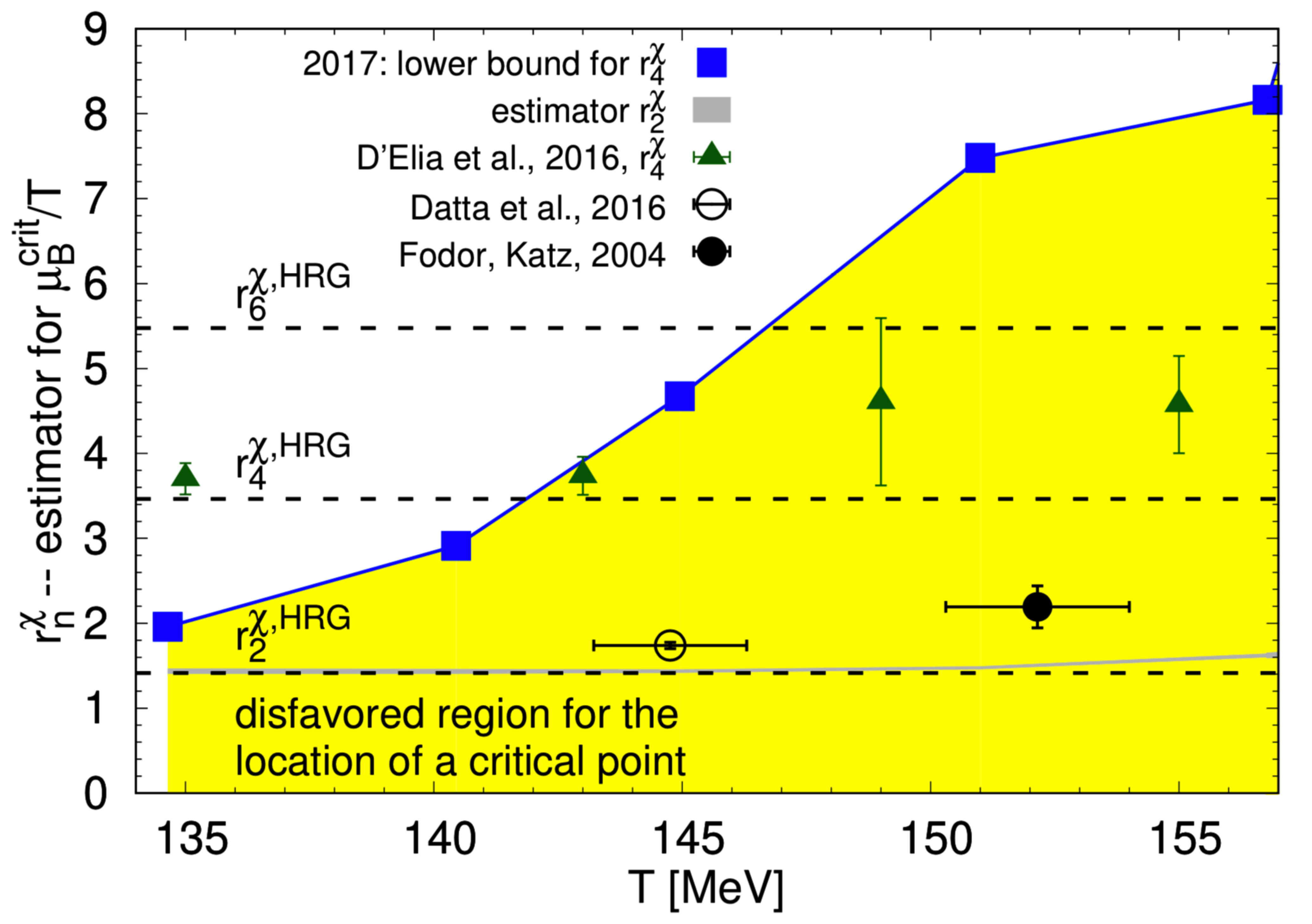
constraining the location of QCD critical point ?

HotQCD: Phys. Rev. D95, no.5, 054504 (2017)

- radius of convergence for baryon number susceptibility:

$$r_{2n}^{\chi} = \sqrt{2n(2n-1)\left|\frac{\chi_{2n}^B}{\chi_{2n+2}^B}\right|}$$

$$T \geq 155\ MeV:\ \ \chi_6^B < 0$$



$$135\ MeV \lesssim T \lesssim 155\ MeV:\ \mu_B/T \gtrsim 2$$

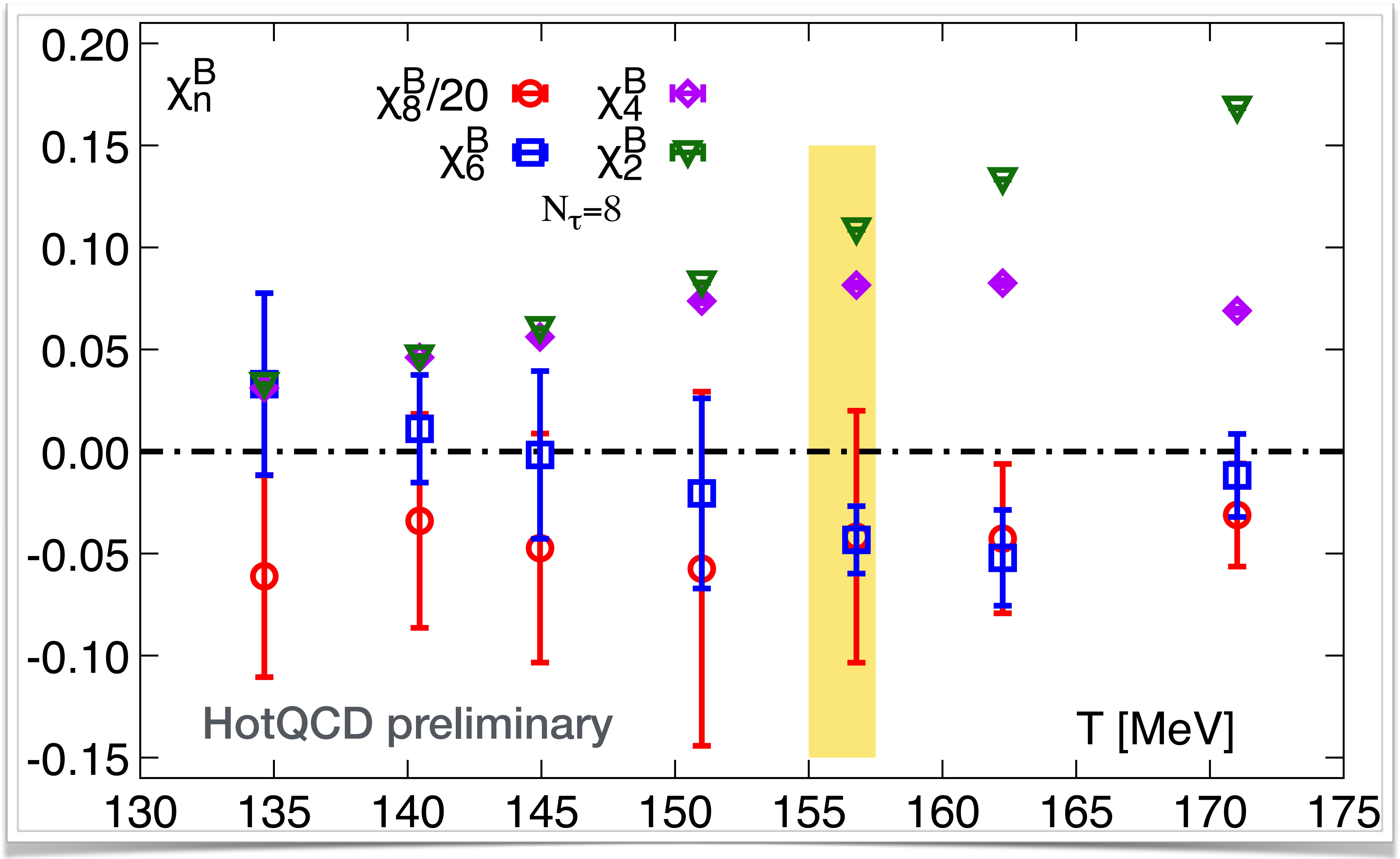
constraining the location of QCD critical point ?

HotQCD very preliminary

- radius of convergence for baryon number susceptibility:

$$r_{2n}^{\chi} = \sqrt{2n(2n-1)\left|\frac{\chi_{2n}^B}{\chi_{2n+2}^B}\right|}$$

$$T \geq 135\ MeV:\ \chi_8^B < 0\ ?$$



$T \geq 135$ MeV : nearest singularity in complex μ_B ?

consistent with $T_c^0 = 132^{+3}_{-6}\ MeV$

summary

- precise chiral crossover temperature $T_c(\mu_B = 0) = 156.5 \pm 1.5\ MeV$
- $T - \mu_B$ phase-boundary up to $\mu_B \lesssim 300\ MeV$
- QCD equation of state for $\mu_B/T \lesssim 2$
- 2^{nd} order fluctuations not enhanced above (ideal) HRG
- QCD critical point: $T \lesssim 135\ MeV$?