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Fluctuation analysis for the STAR experiment --- multi-dimensional unfolding of net-number and temperature including volume fluctuation ---

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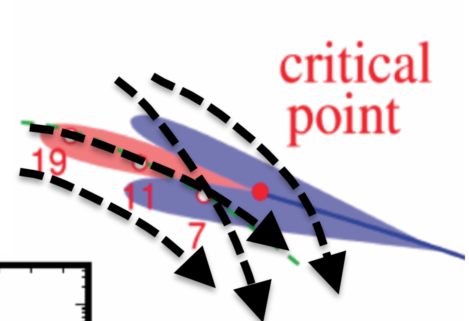
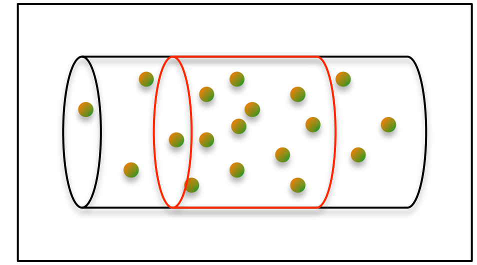
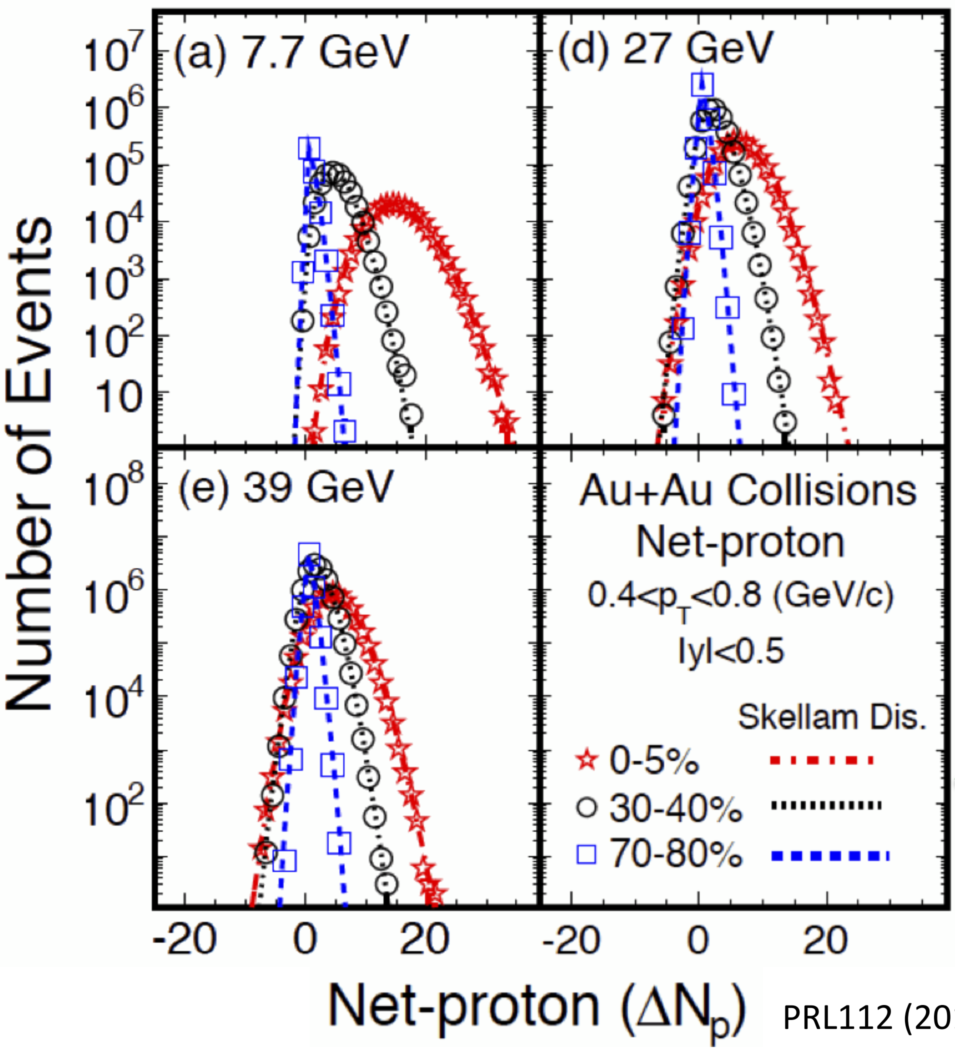
- Looking for a critical fluctuation in conserved number and possibly also in temperature distribution simultaneously with unfolding method including volume fluctuation
- Volume fluctuation study with UrQMD by T. Sugiura, T.N., S.E.
- Another solution for non-binomial effect by T. Nonaka, M.K., S.E.

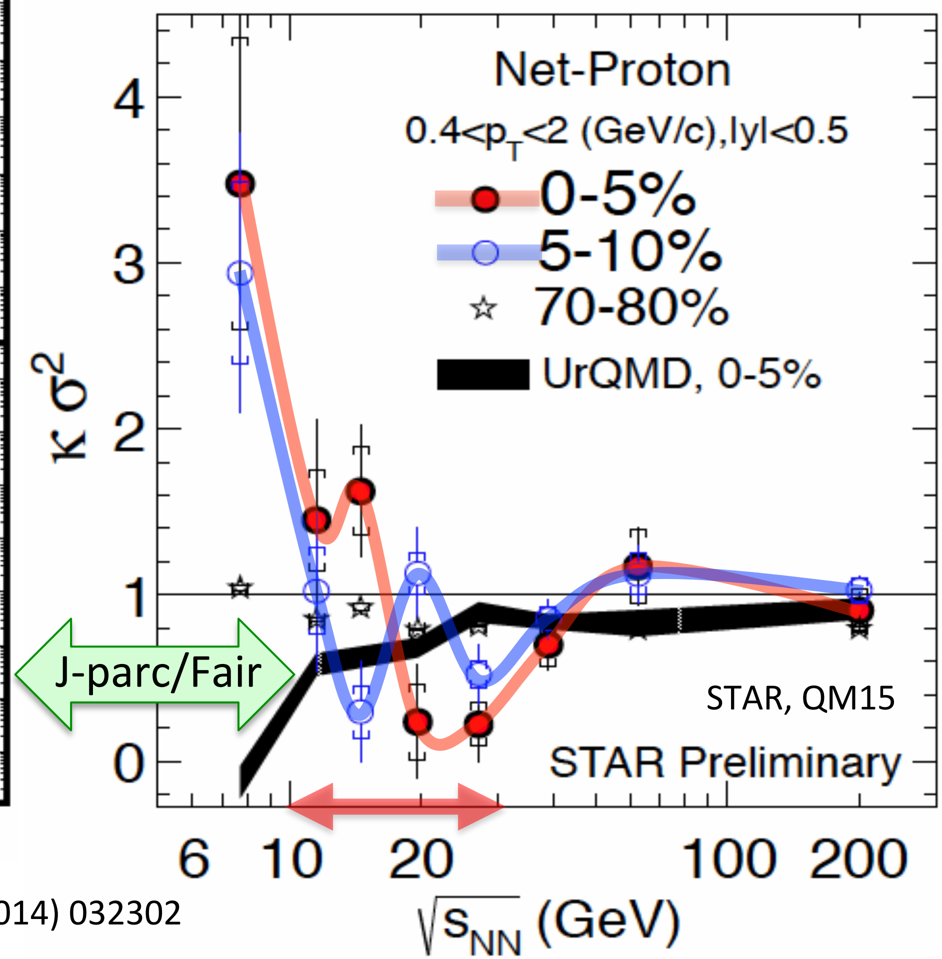
Possible signal from Critical Point?







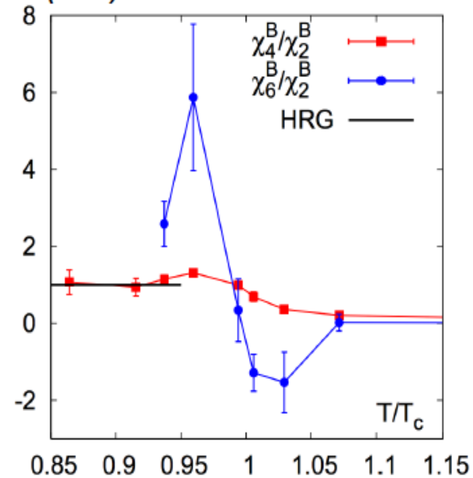
PRL112 (2014) 032302



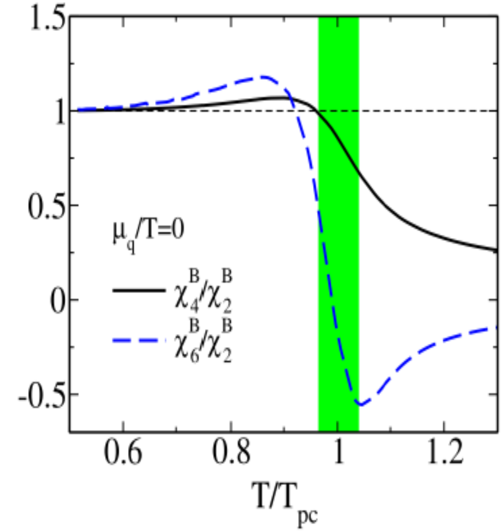
6^{th}-order cumulants of net-proton and net-charge

Higher-order cumulants are expected to be more sensitive to the critical fluctuation than lower orders.
Even more statistics needed though ...

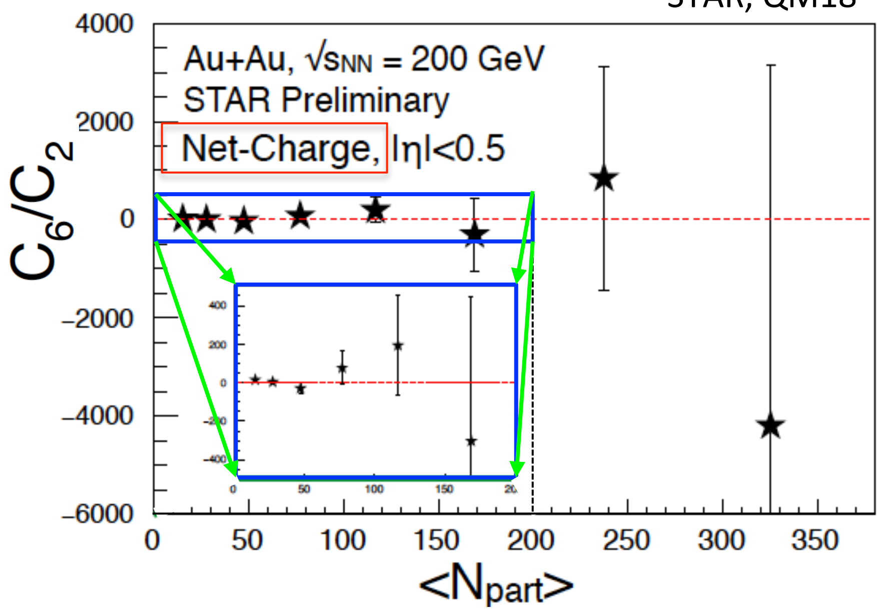
Cheng et al, Phys. Rev. D 79, 074505 (2009) : Lattice QCD



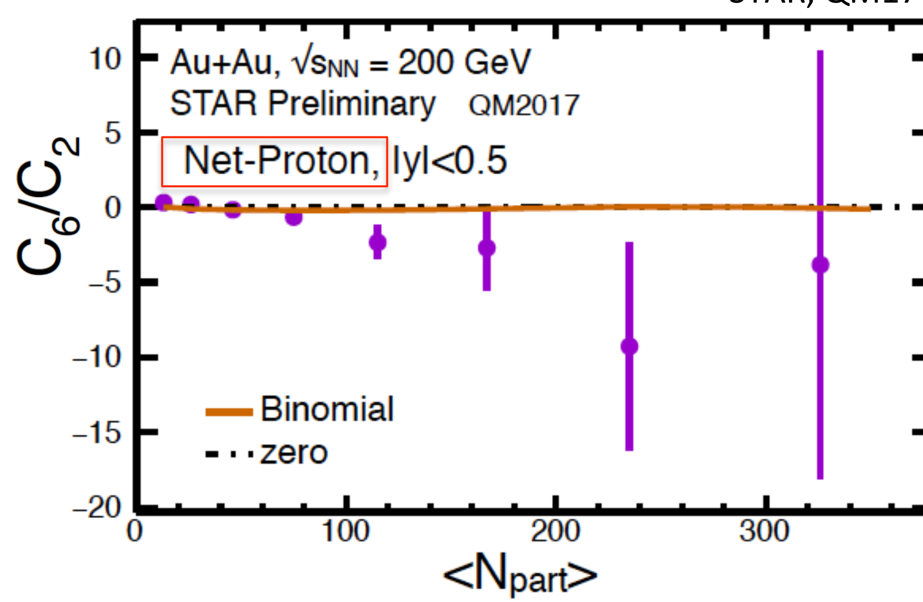
Friman et al, Eur. Phys. J. C (2011) 71:1694 : O(4) scaling functions



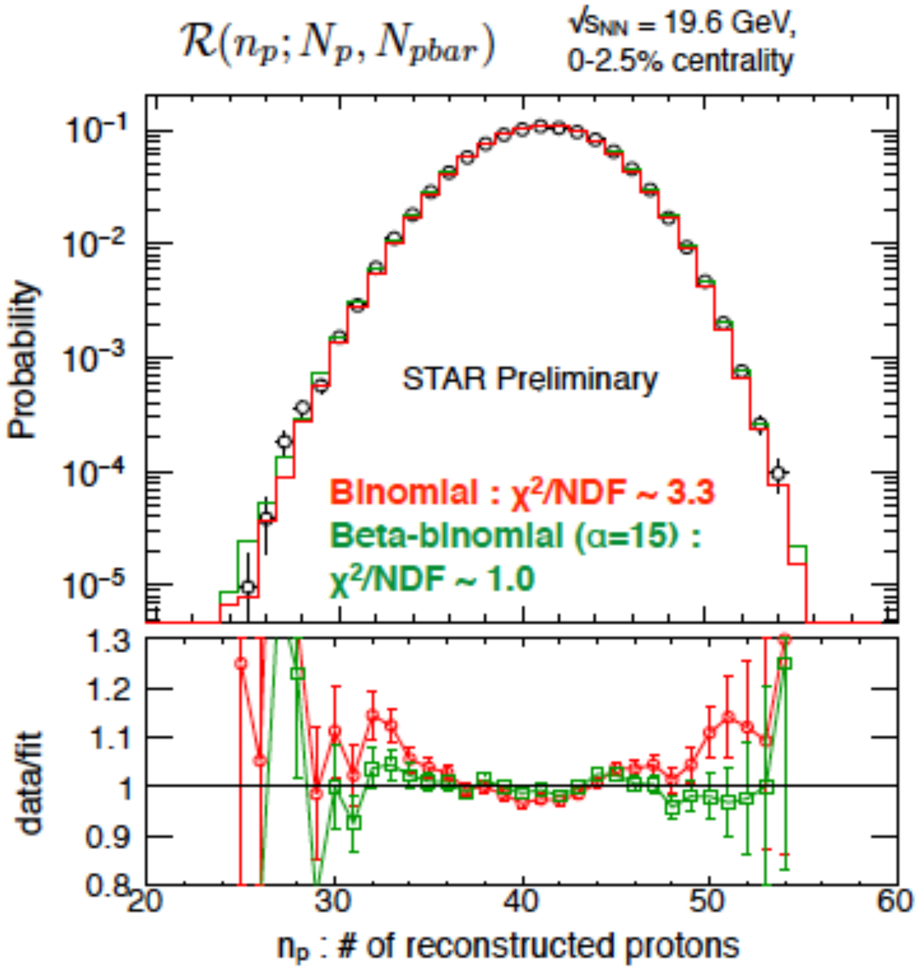
STAR, QM18

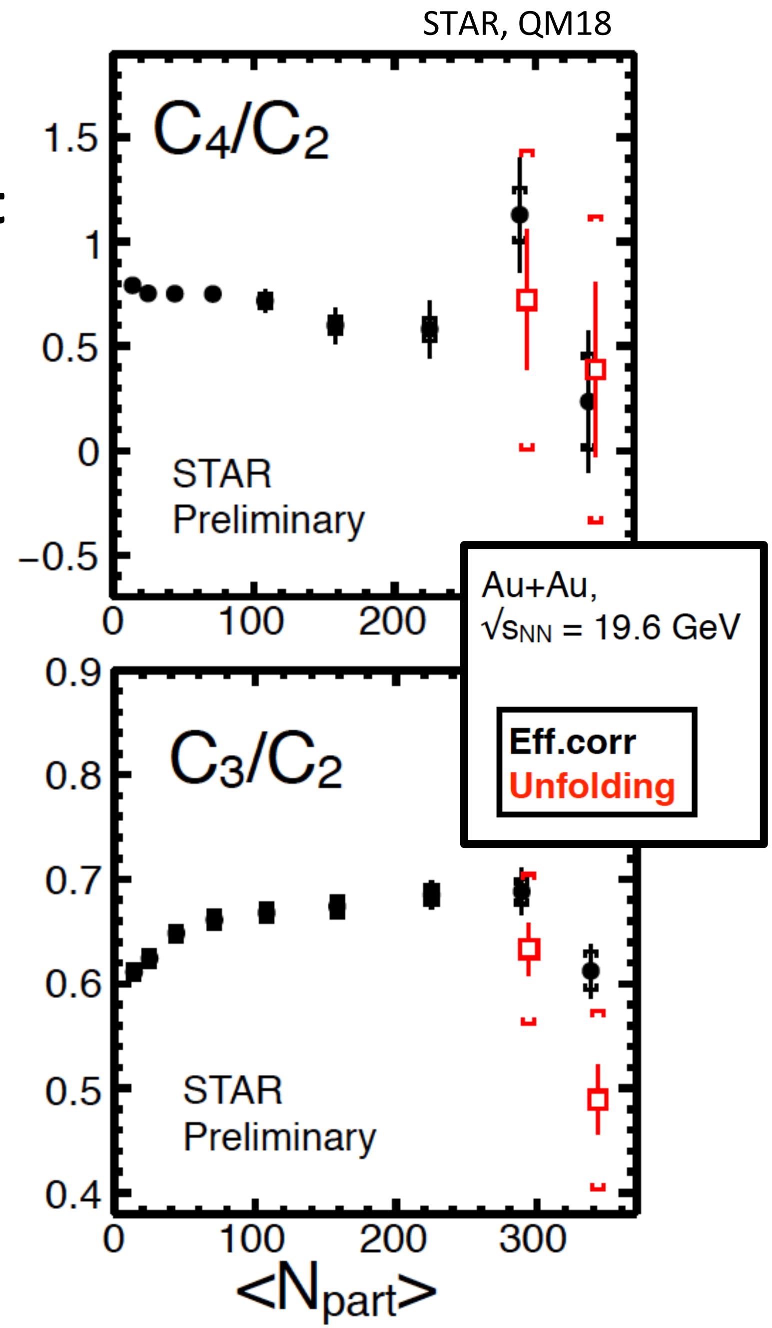


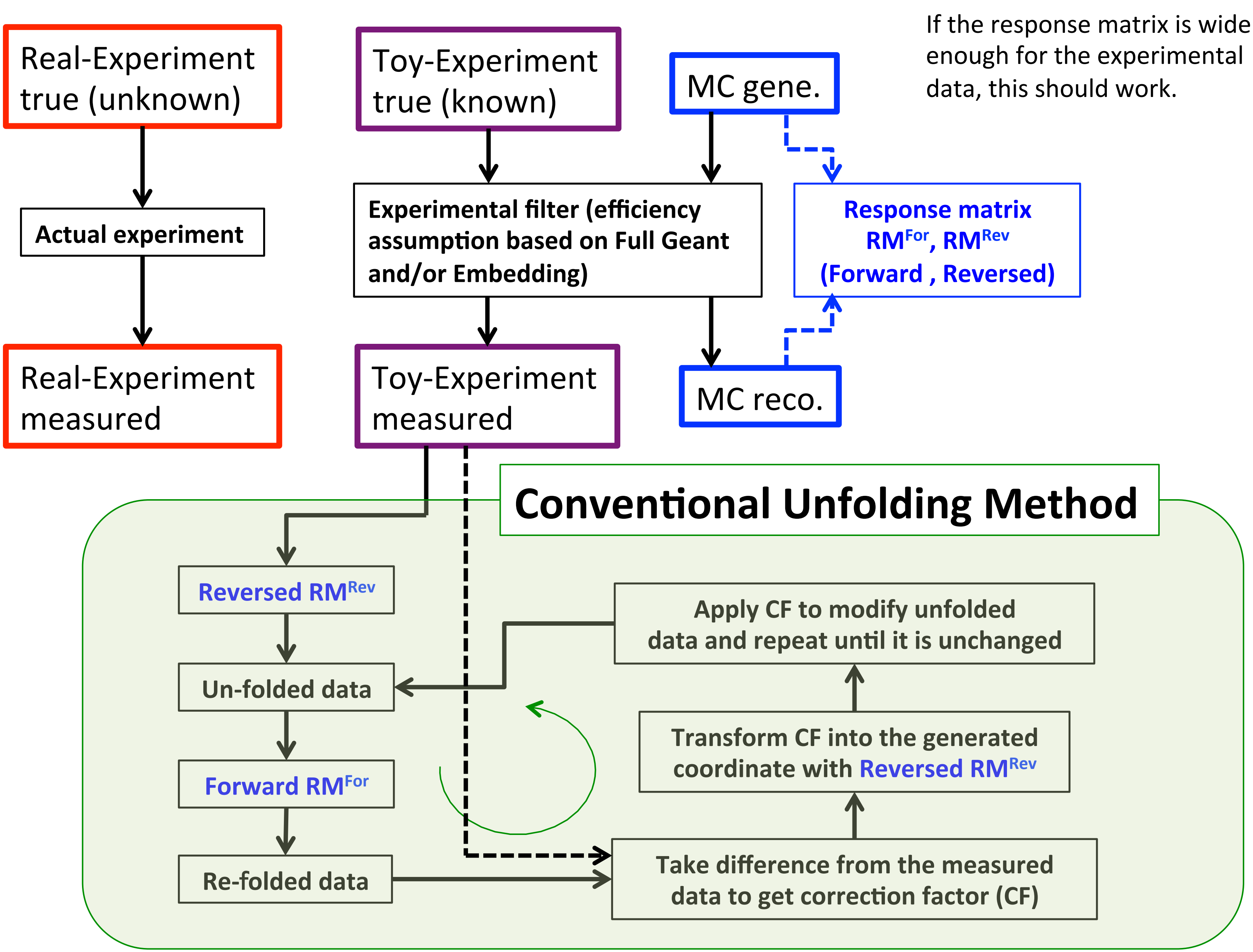
STAR, QM17



Unfolding result of net-proton distribution with non-binomial effect





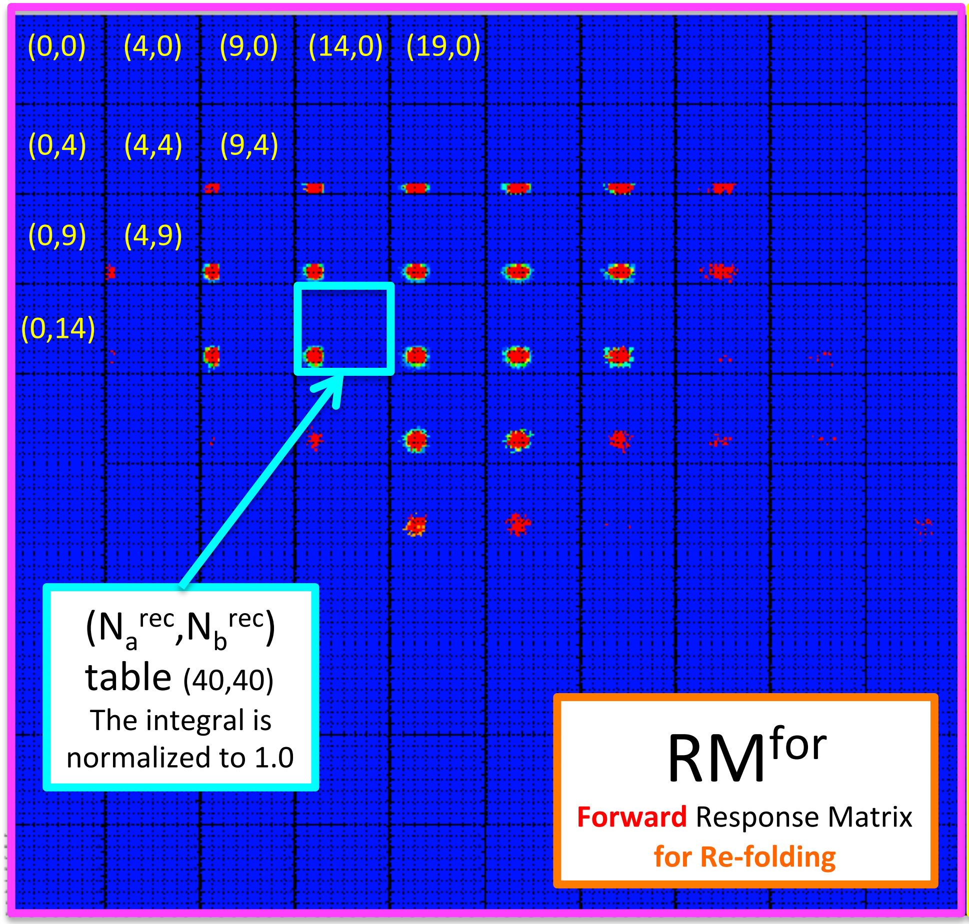

Real-Experiment true (unknown)
Actual experiment
Real-Experiment measured
Toy-Experiment true (known)
MC gene.
Experimental filter (efficiency assumption based on Full Geant and/or Embedding)
Response matrix
RM^{For}, RM^{Rev}
(Forward , Reversed)
If the response matrix is wide enough for the experimental data, this should work.
Toy-Experiment measured
MC reco.
Conventional Unfolding Method
Reversed RM^{Rev}
Un-folded data
Forward RM^{For}
Re-folded data
Apply CF to modify unfolded data and repeat until it is unchanged
Transform CF into the generated coordinate with Reversed RM^{Rev}
Take difference from the measured data to get correction factor (CF)

Response Matrix (including MC input weighting)

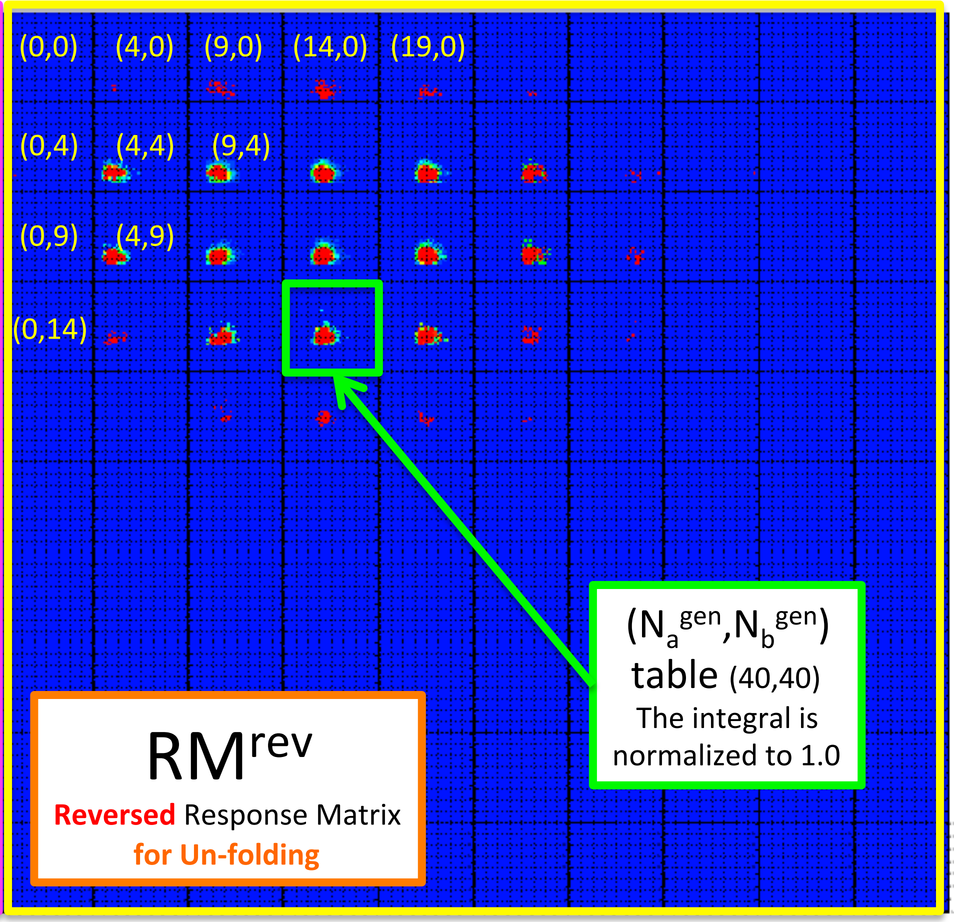
Forward Matrix $(MC^{gen} \text{--->} MC^{rec})$

Reversed Matrix $(MC^{rec} \text{--->} MC^{gen})$

(N_a^{gen}, N_b^{gen}) (0-39, 0-39)

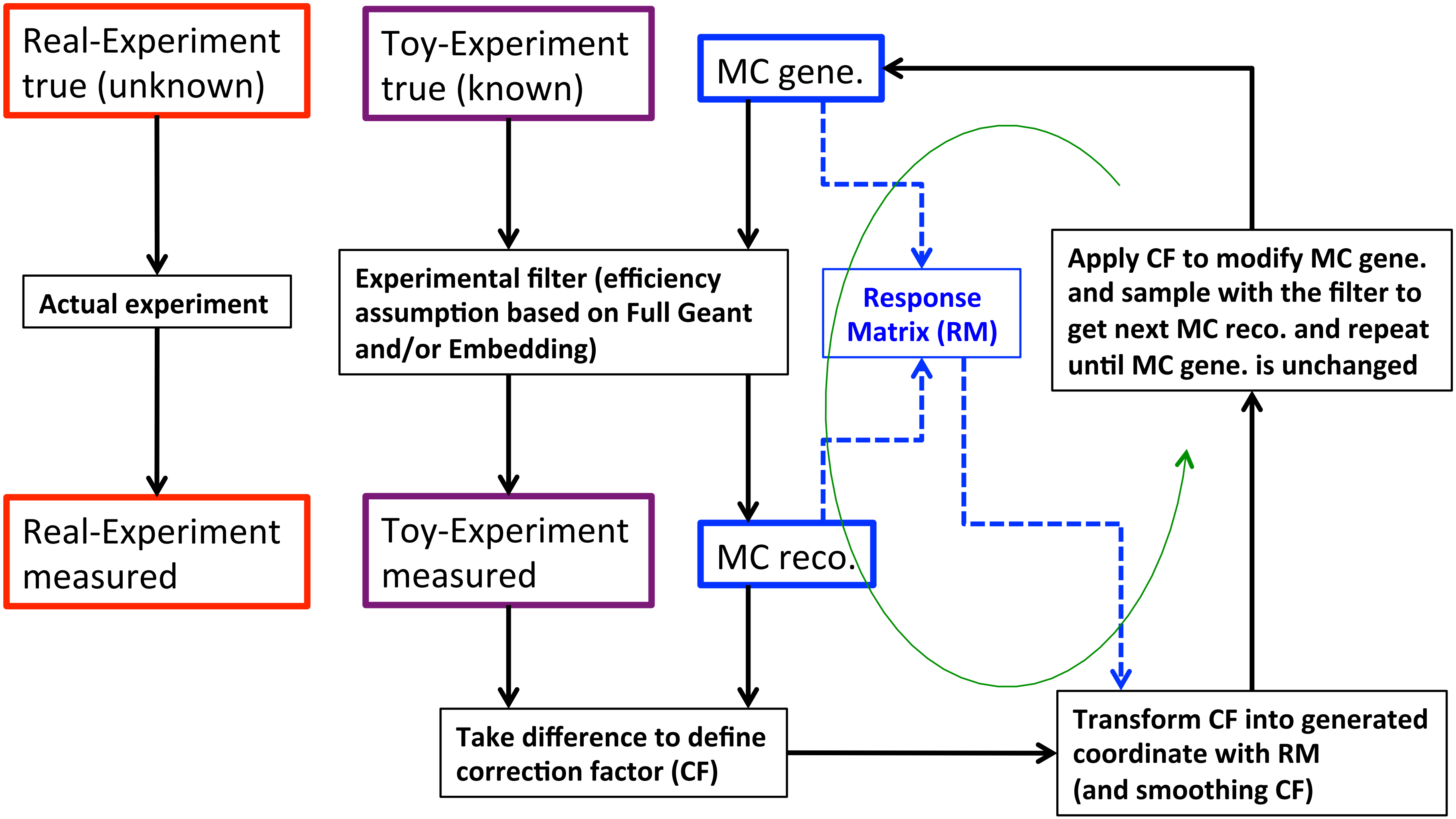


(N_a^{rec}, N_b^{rec}) (0-39, 0-39)



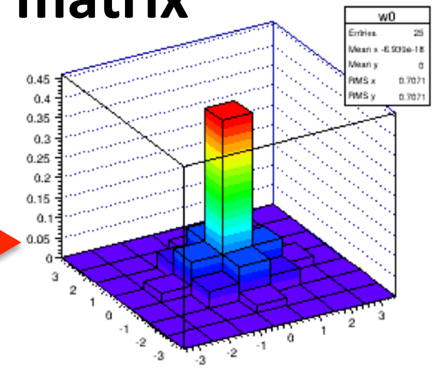
“Incremental unfolding” procedure

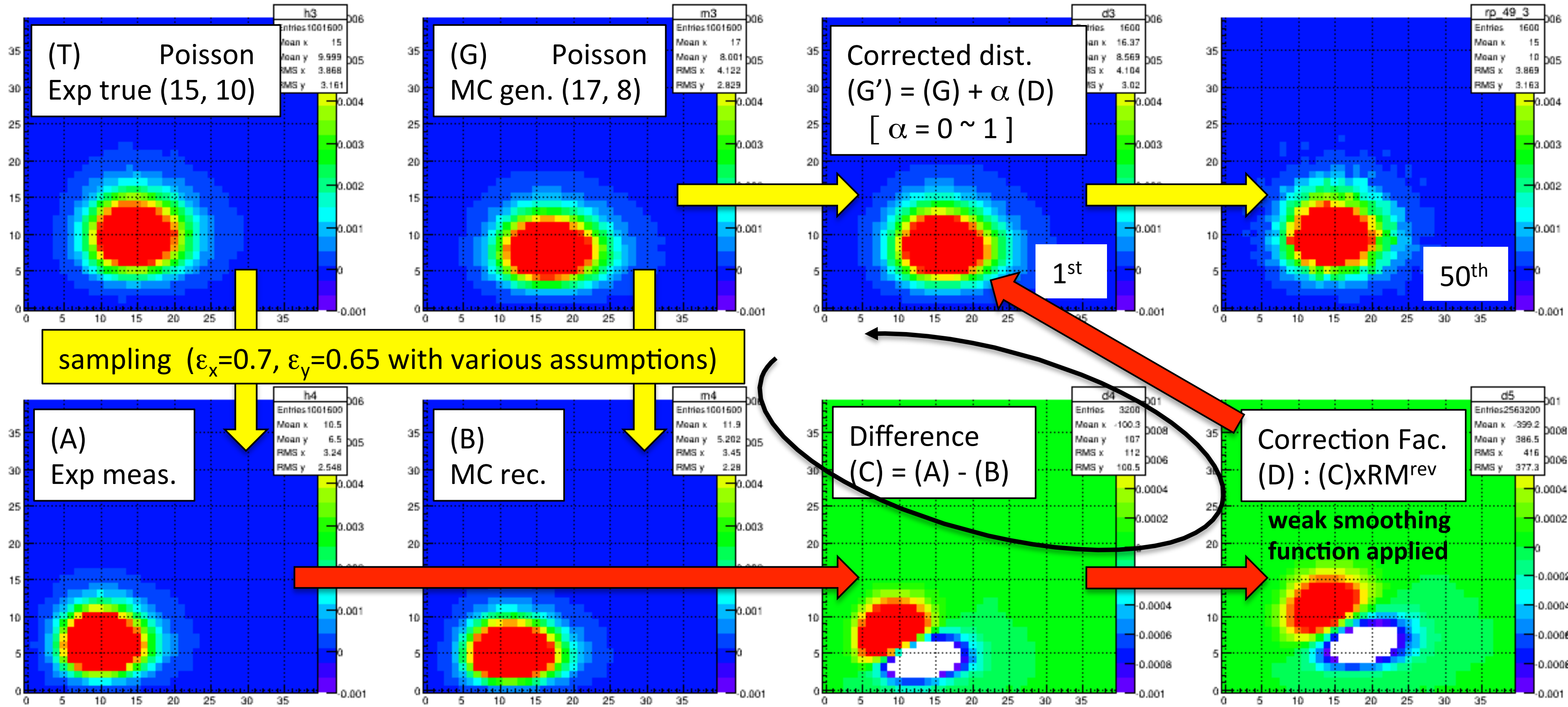
This will become feasible, once the response matrix can be parameterized based on the simulation.



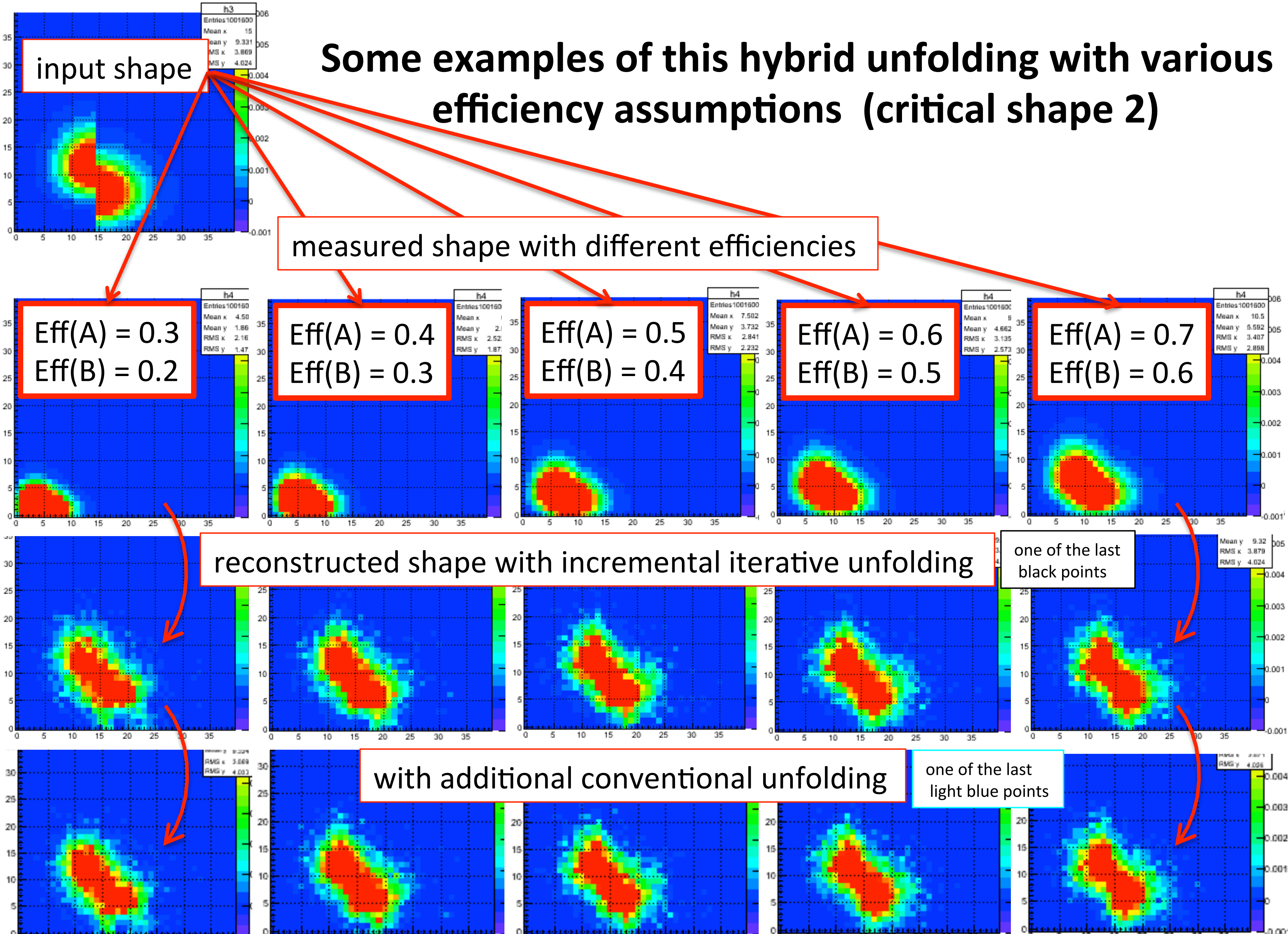
Incremental unfolding by updating the response matrix

(1) True is unkown, there is only exp meas. (A)
(2) generate mc_gen (G) and sample mc_rec (B) distributions
(3) take difference between exp_meas. (A) and mc_rec (B) to define (C)
(4) transform (C) to (D) by RM^{rev} and smoothing function
(5) apply this correction factor (D) to mc_gen (G) for next iteration
(6) repeat above steps (2)-(5) iteratively



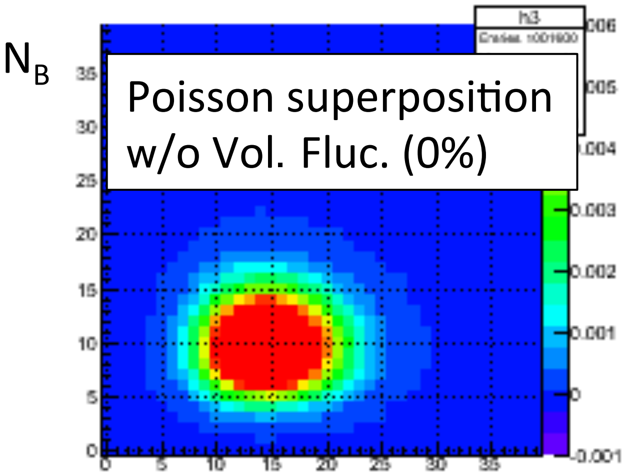


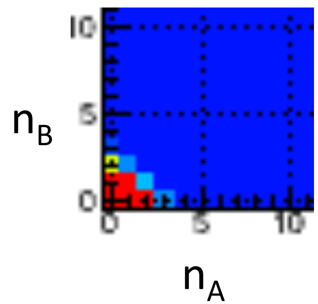
Some examples of this hybrid unfolding with various efficiency assumptions (critical shape 2)



Unfolding with Volume Fluctuation (V.F.)

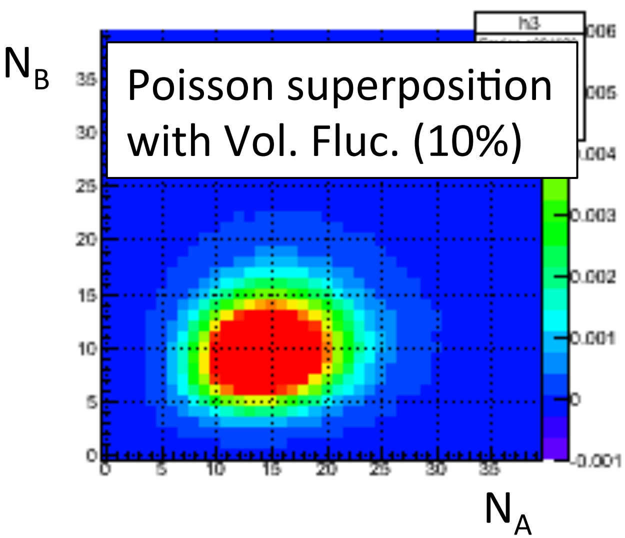
Volume fluctuation is assumed to be known precisely according to Glauber (or any other initial) model. V.F. naturally induces a positive correlation between N_A vs N_B. Gaussian fluctuation is used in this toy model simulation.

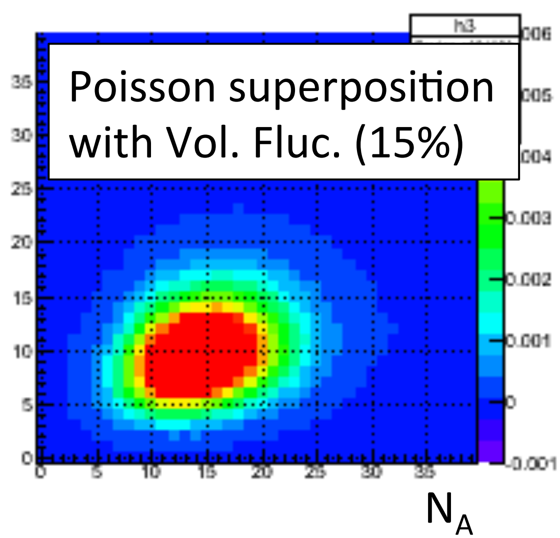


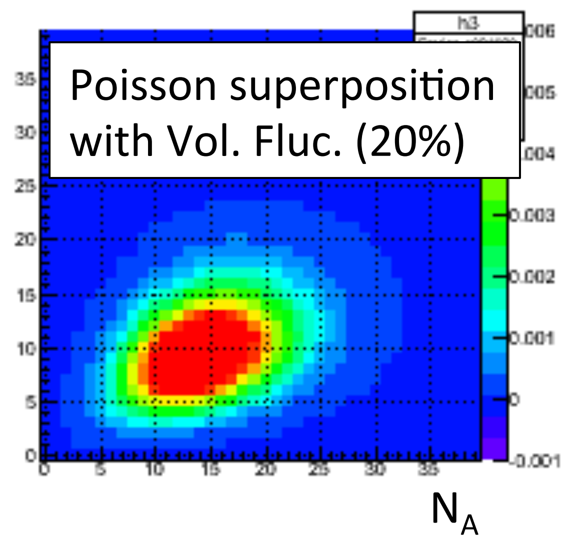


“n_A vs n_B” distribution per source will be extracted, where the number of sources “n_S” has well known fluctuation from VF.

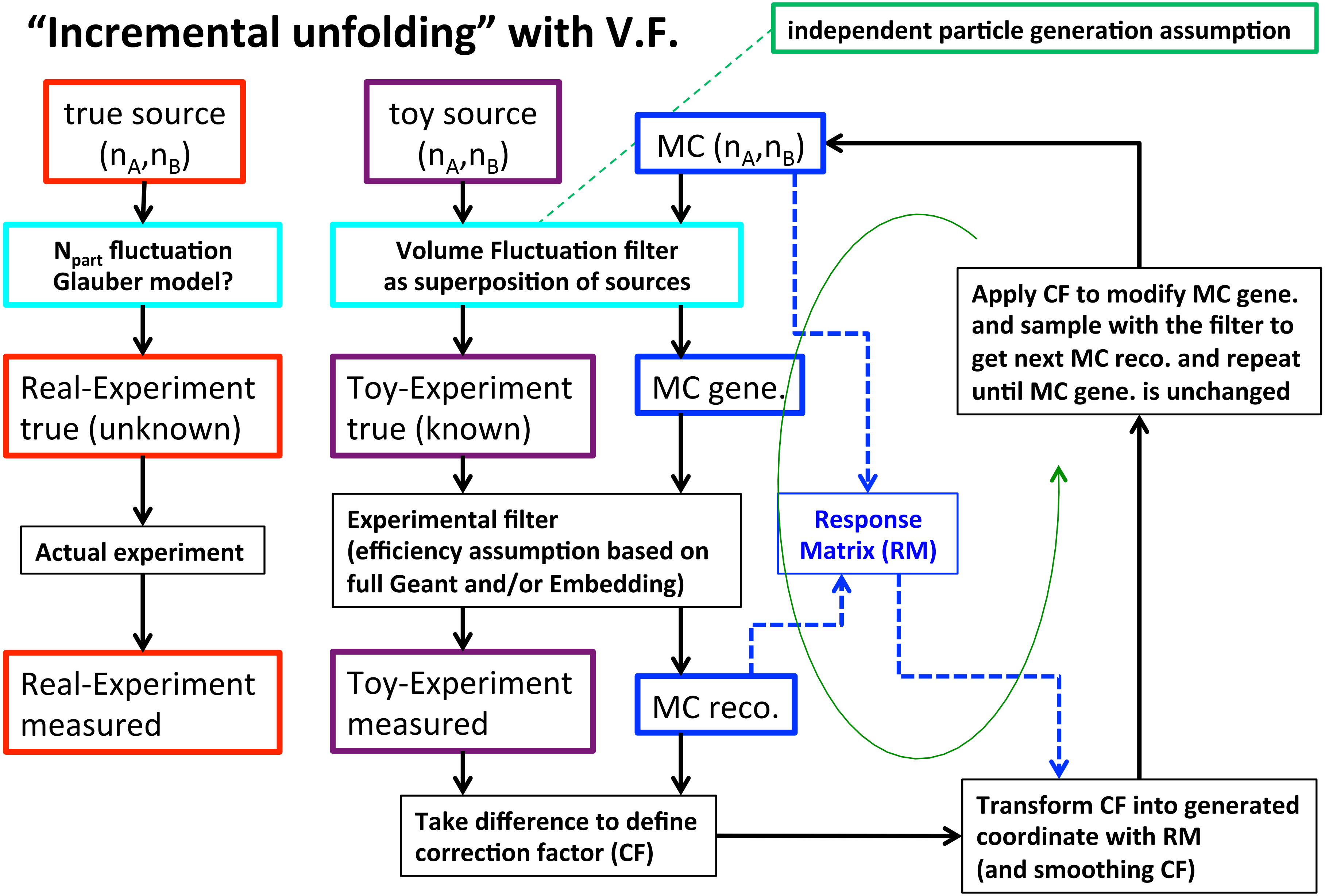
Poisson source : $\langle n_A \rangle = 0.15$, $\langle n_B \rangle = 0.1$
Gaussian fluc. : $\langle n_S \rangle = 100$ +/- V.F.(%)
$N_A = n_A \times n_S$, $N_B = n_B \times n_S$







"Incremental unfolding" with V.F.



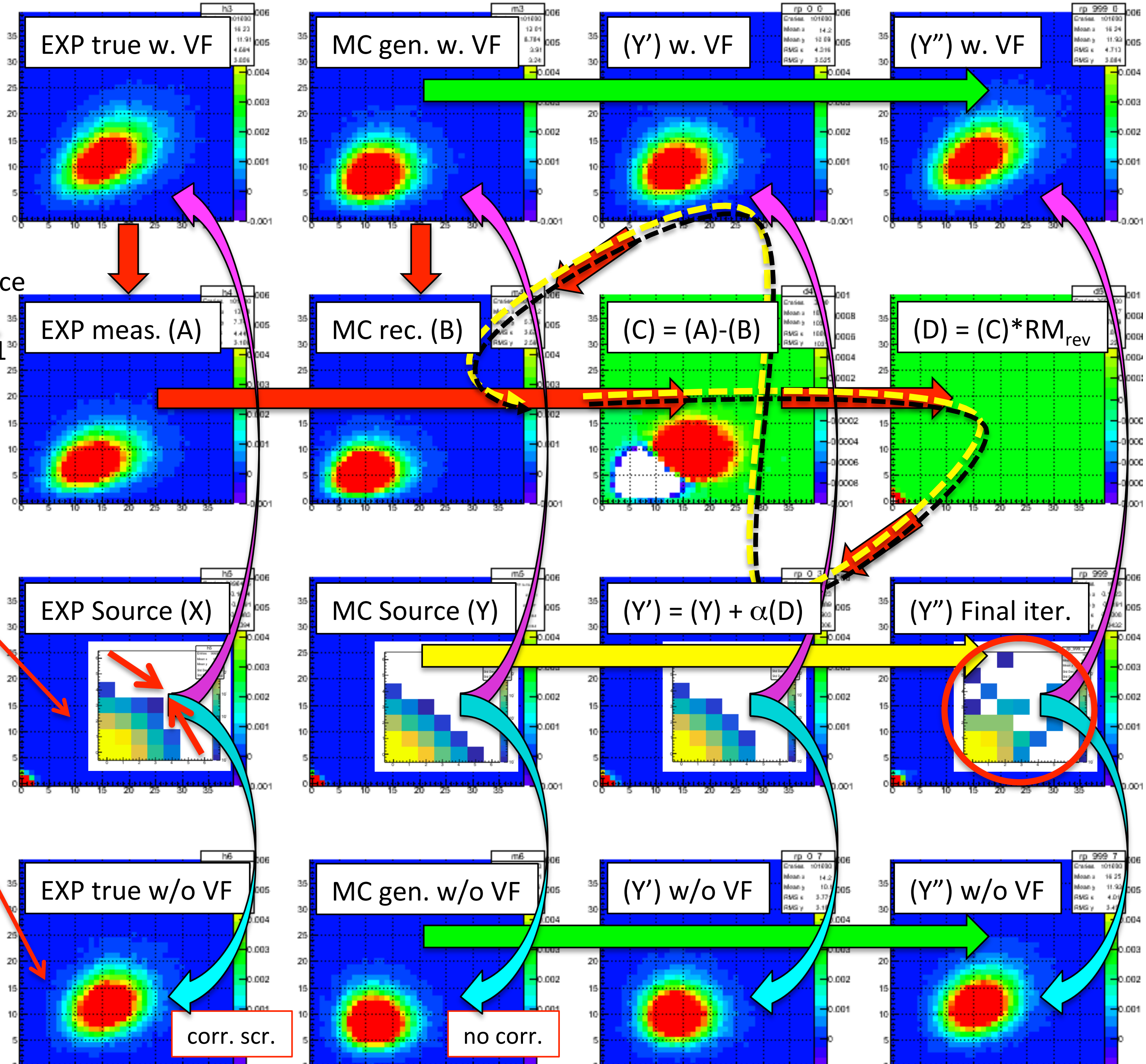
Positively Correlated Source with V.F.

- Modified Poisson Source (positive correlation) : $\langle n_A \rangle \sim 0.15$, $\langle n_B \rangle \sim 0.1$
- Gaussian Vol. fluc. : $\langle n_S \rangle = 100 \pm 15(\%)$
- $\varepsilon_x=0.85$, $\varepsilon_y=0.65$

Superposition of sources with volume fluctuation

Recovery of Source distribution

Superposition of fixed # of sources



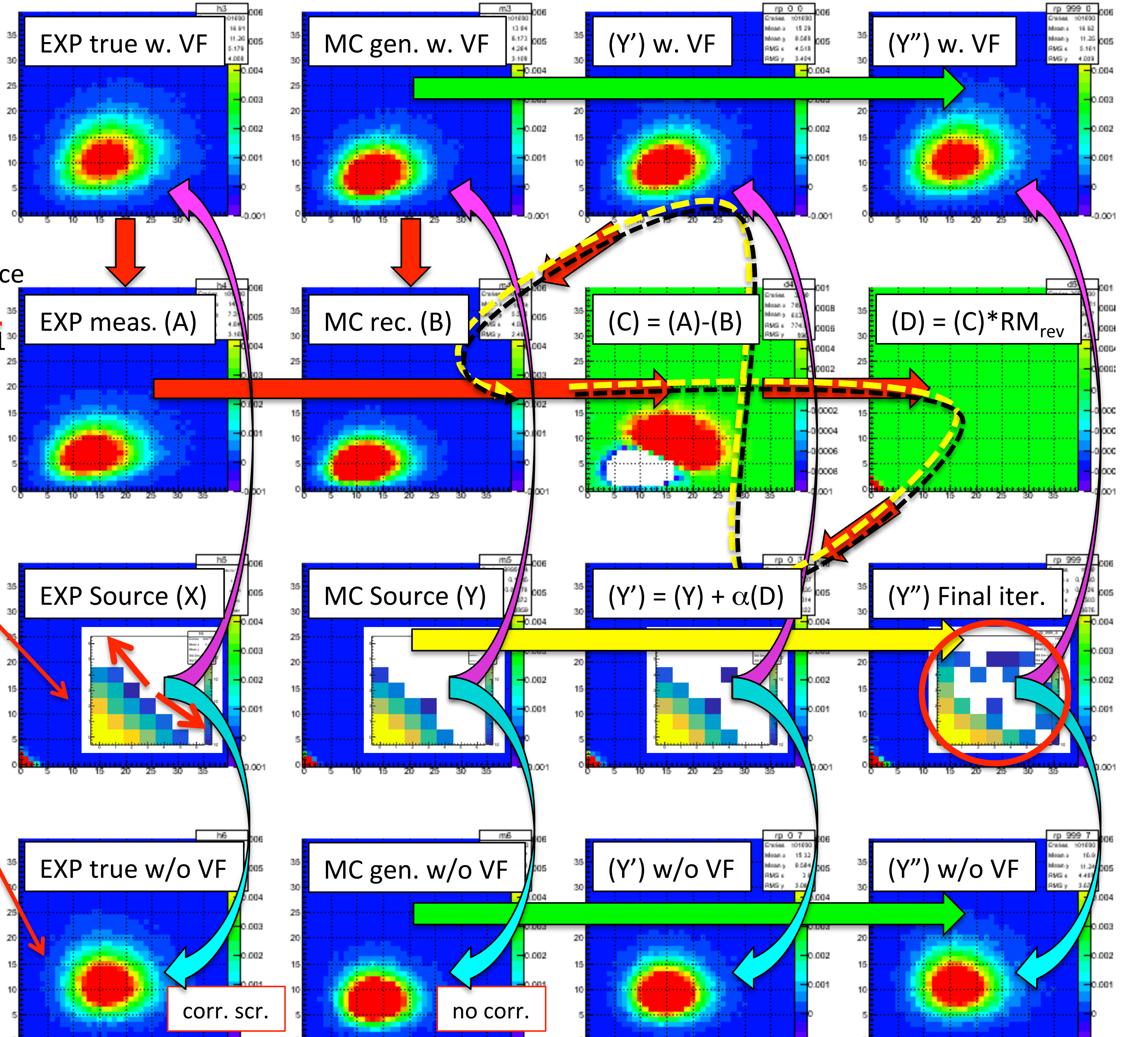
Negatively Correlated Source with V.F.

- Modified Poisson Source (negative correlation) : $\langle n_A \rangle \sim 0.15$, $\langle n_B \rangle \sim 0.1$
- Gaussian Vol. fluc. : $\langle n_S \rangle = 100 \pm 15(\%)$
- ε_x=0.85 , ε_y=0.65

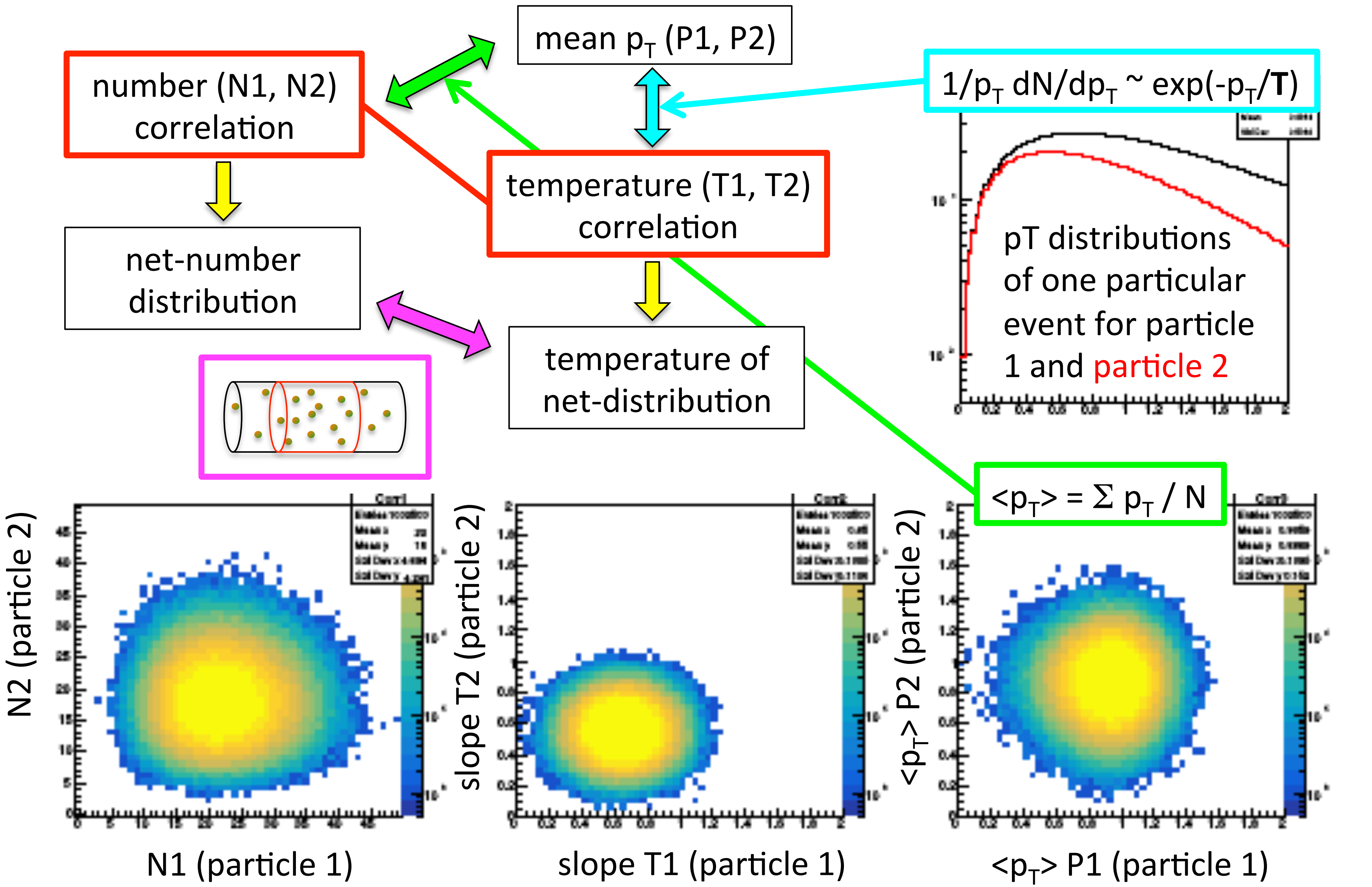
Superposition of sources with volume fluctuation

Recovery of Source distribution

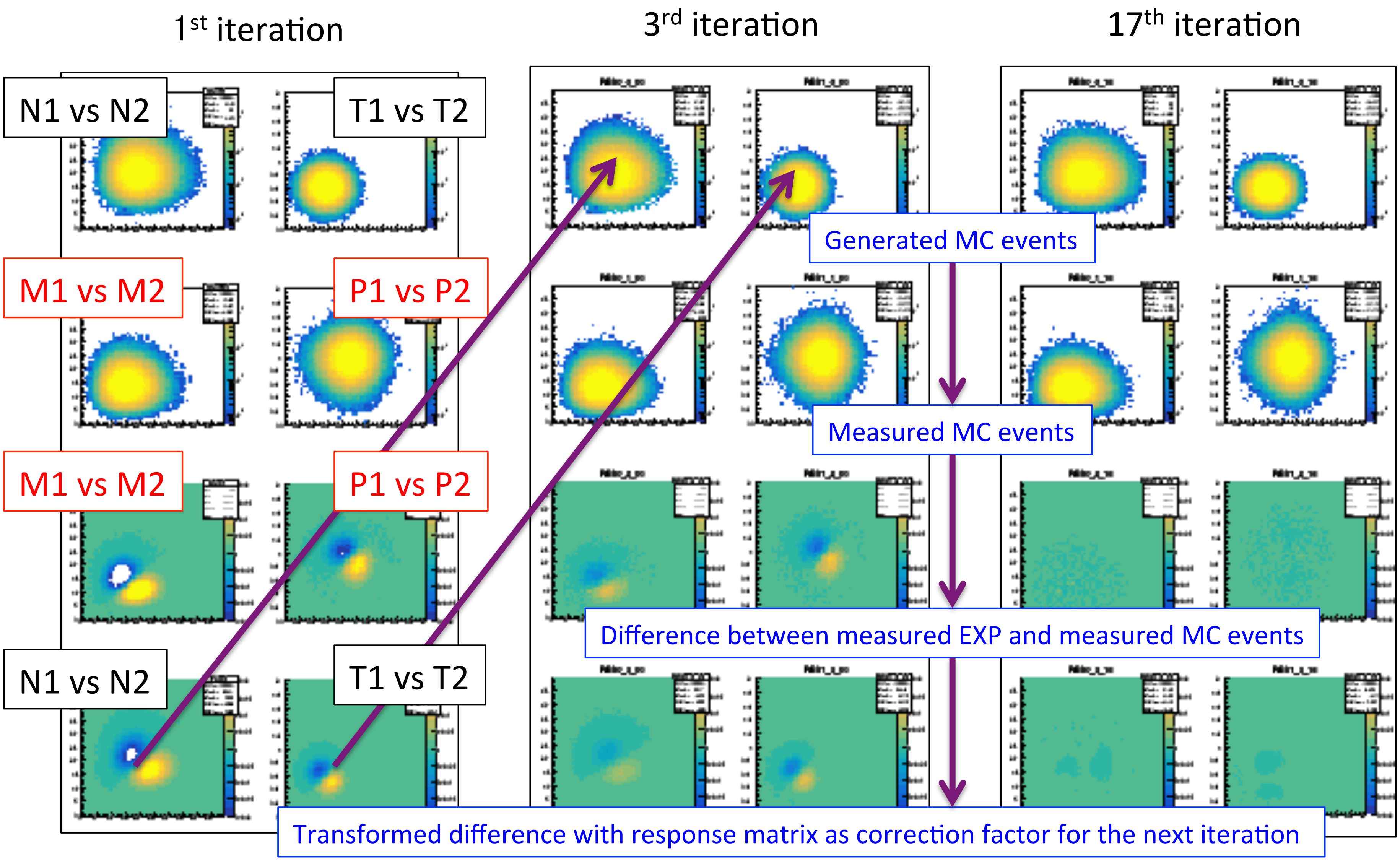
Superposition of fixed # of sources



Extraction of net-number and temperatures simultaneously



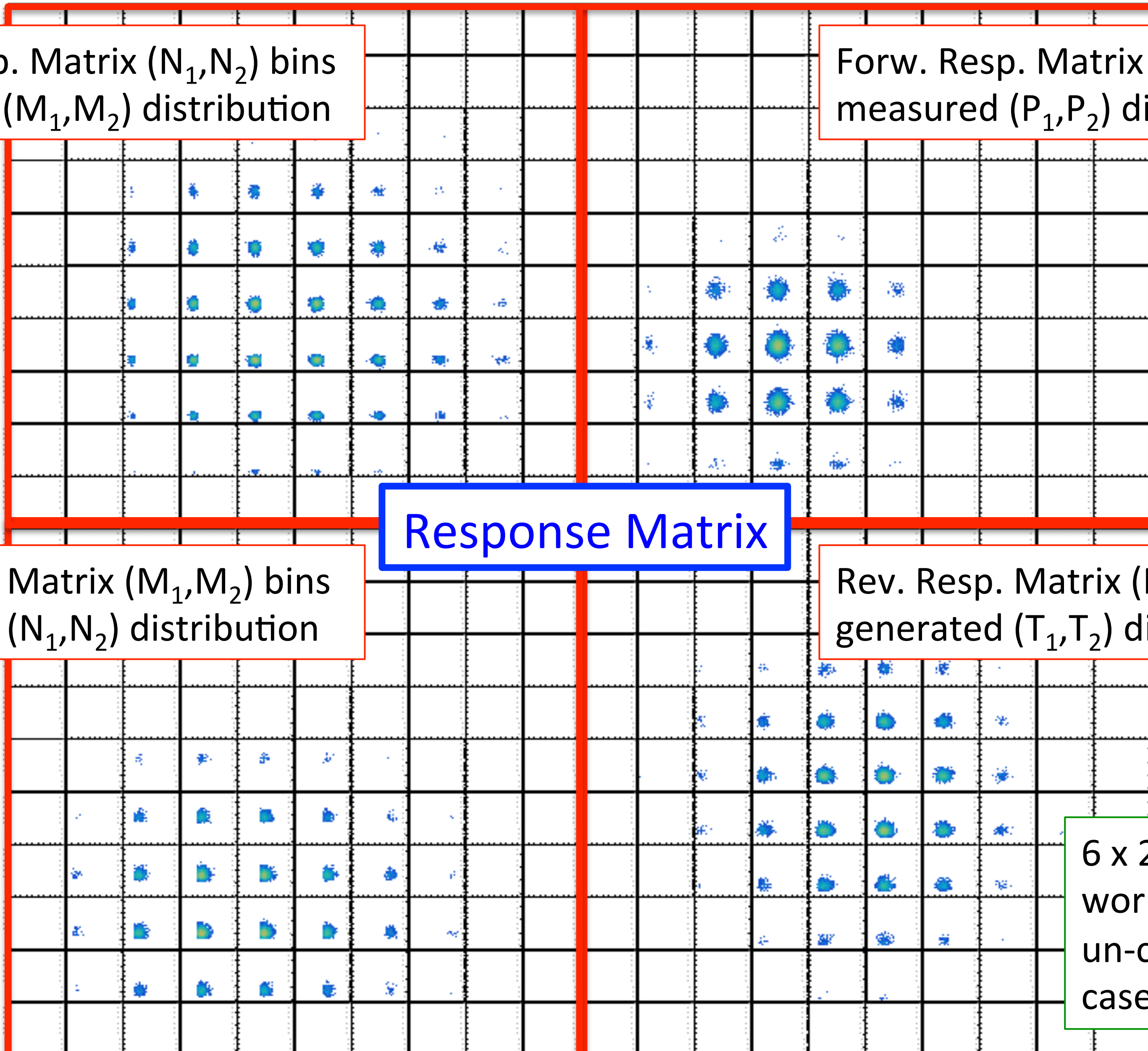
Test iteration without correlations



2 out of total 6 2D-projections are
shown for visualization purpose.

Forw. Resp. Matrix (N_1,N_2) bins
measured (M_1,M_2) distribution

Forw. Resp. Matrix (T_1,T_2) bins
measured (P_1,P_2) distribution



Response Matrix

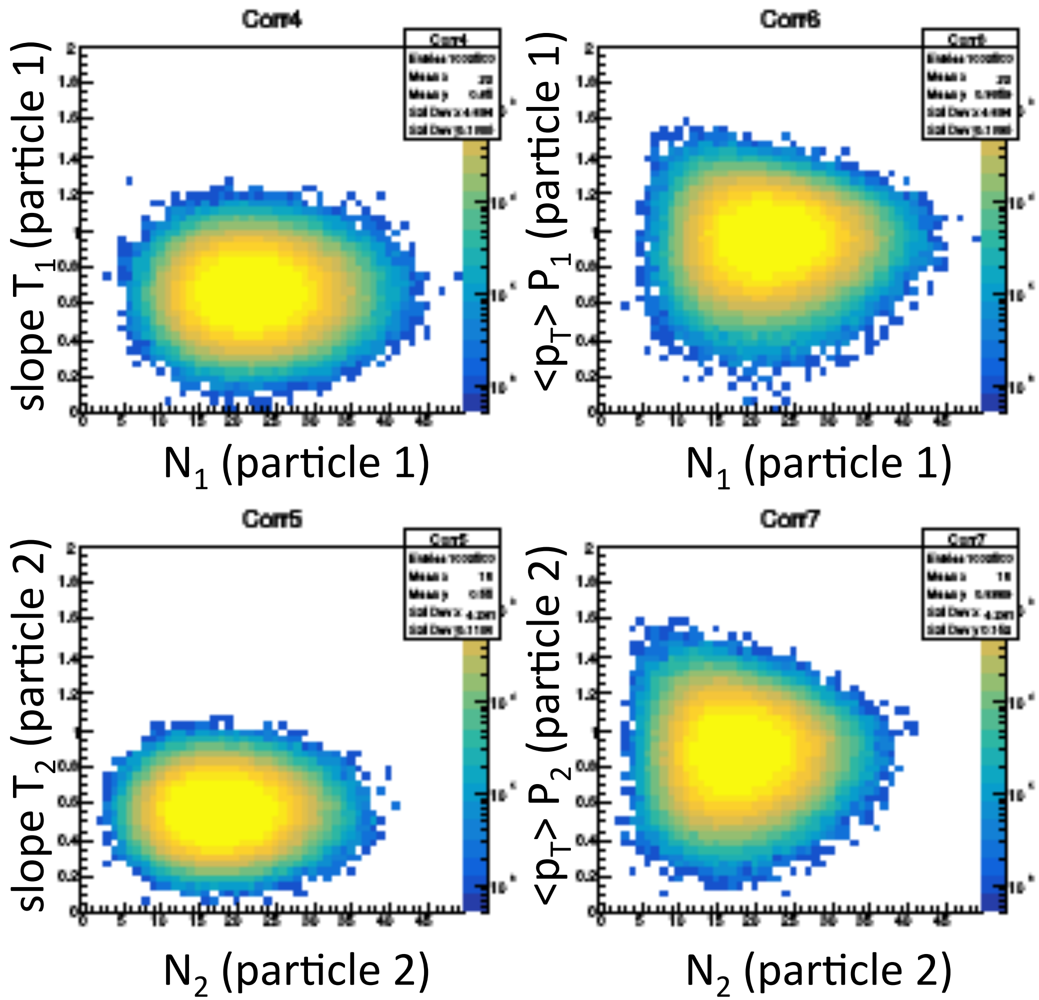
Rev. Resp. Matrix (M_1,M_2) bins
generated (N_1,N_2) distribution

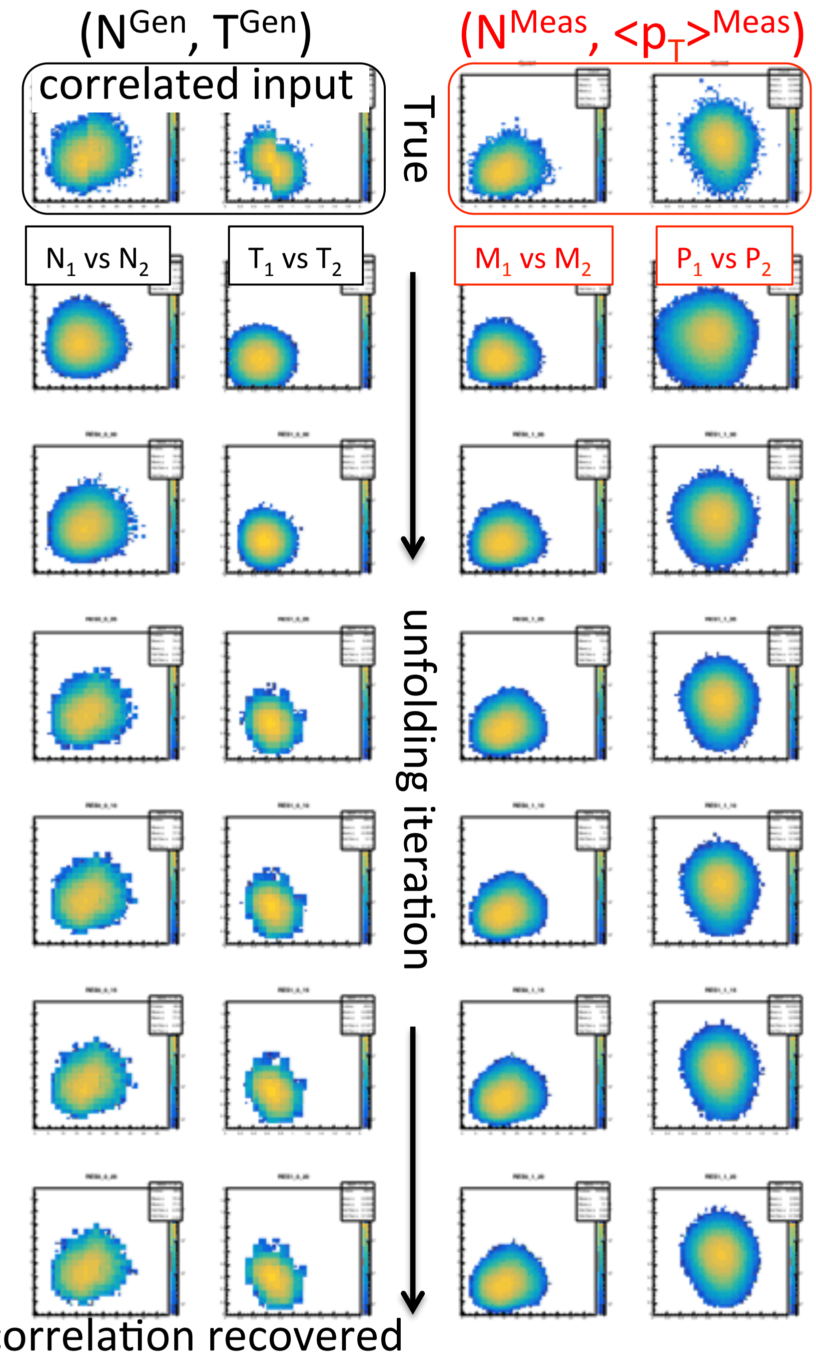
Rev. Resp. Matrix (P_1,P_2) bins
generated (T_1,T_2) distribution

6 x 2D-RM's
work only in
un-correlated
cases

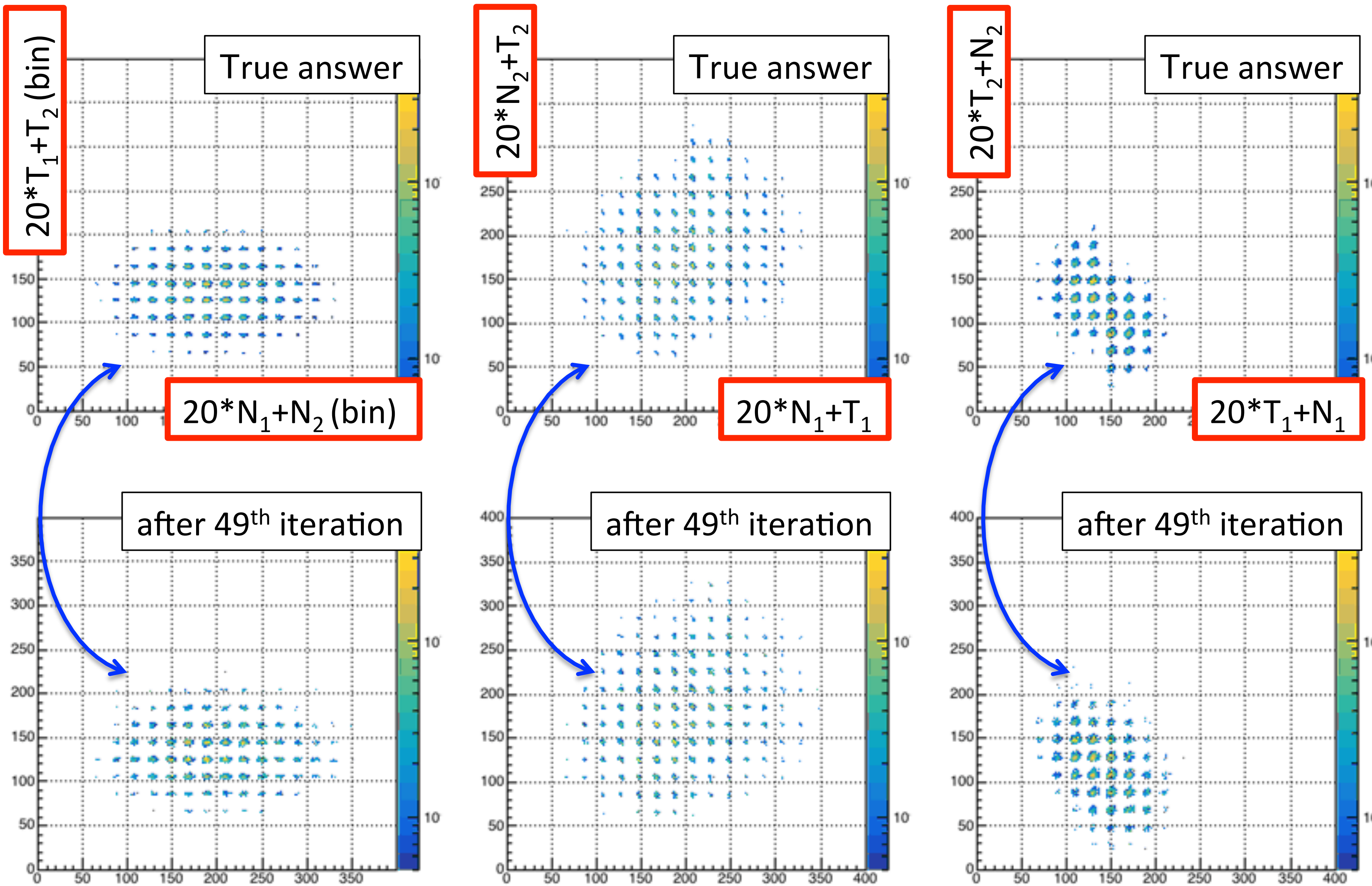
4D-Unfolding of temperature and net-number distributions for higher order fluctuation

$dN/dp_T \sim p_T \exp(-p_T/T)$ $\langle p_T \rangle = \Sigma\, p_T / N$





2D view of 4-dimentional unfolding



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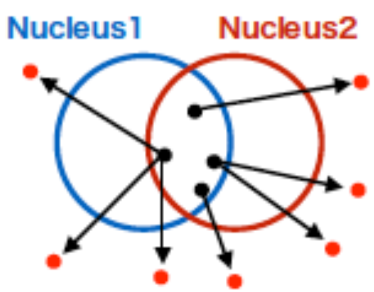
Volume fluctuation correction (VFC)

1

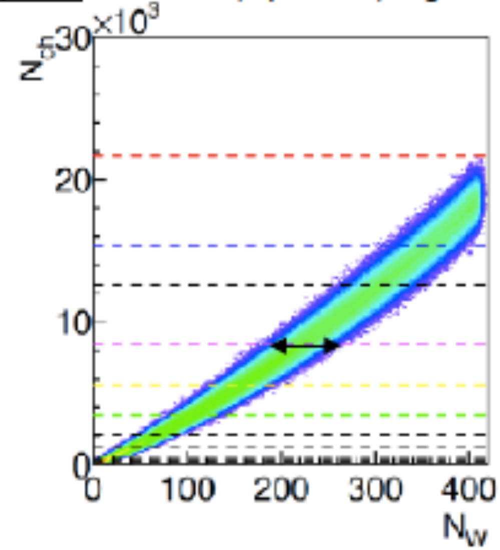
Nuclear Physics A Volume 960, April 2017, Pages 114-130

- Nw is fluctuating even if final state multiplicity is fixed.
  →**CBWC may not be enough to eliminate VF**



Independent particle production (IPP) model

- ● : initial source
- ● : final state multiplicity



- Under the IPP model, measured cumulants include additional term from VF which can be written by Nw cumulants.
- We can know **true cumulants by subtracting additional terms (VFC)**

Measured cumulant | True cumulant

Δn : number of net-particle

Additional term caused from N_W fluctuation

$$\kappa_1(\Delta N) = \langle N_W \rangle \kappa_1(\Delta n)$$

$$\kappa_2(\Delta N) = \langle N_W \rangle \kappa_2(\Delta n) + \langle \Delta n \rangle^2 \kappa_2(N_W)$$

$$\kappa_3(\Delta N) = \langle N_W \rangle \kappa_3(\Delta n) + 3\langle \Delta n \rangle \kappa_2(\Delta n) \kappa_2(N_W) + \langle \Delta n \rangle^3 \kappa_3(N_W)$$

$$\kappa_4(\Delta N) = \langle N_W \rangle \kappa_4(\Delta n) + 4\langle \Delta n \rangle \kappa_3(\Delta n) \kappa_2(N_W) + 3\kappa_2^2(\Delta n) \kappa_2(N_W) + 6\langle \Delta n \rangle^2 \kappa_2(\Delta n) \kappa_3(N_W) + \langle \Delta n \rangle^4 \kappa_4(N_W)$$

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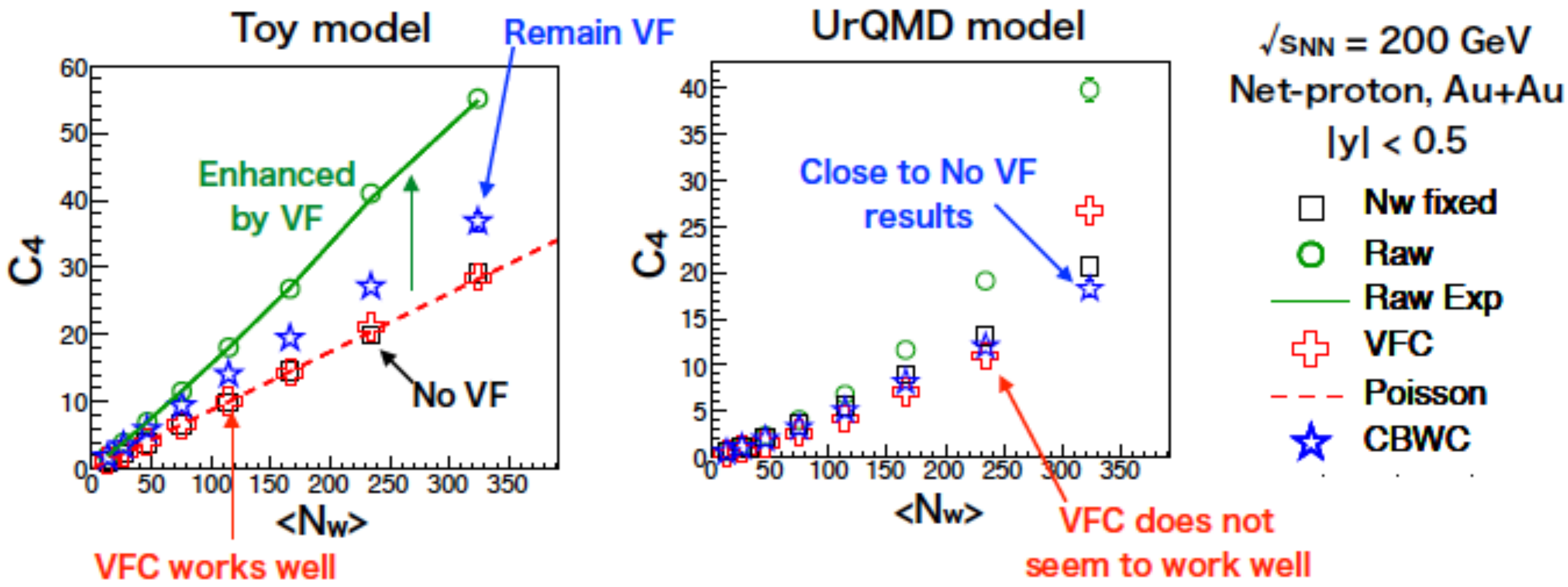
Comparison between toy model and UrQMD

2

arXiv:1903.02314, T.Sugiura, T.Nonaka and S.Esumi



- **VFC works well** by definition **in toy model.**
- However, **does not** seem to **work well in UrQMD**

- ✓ Independent Particle Production model is expected to be broken in UrQMD
- ✓ Multiplicity correlation between centrality and net-proton in UrQMD

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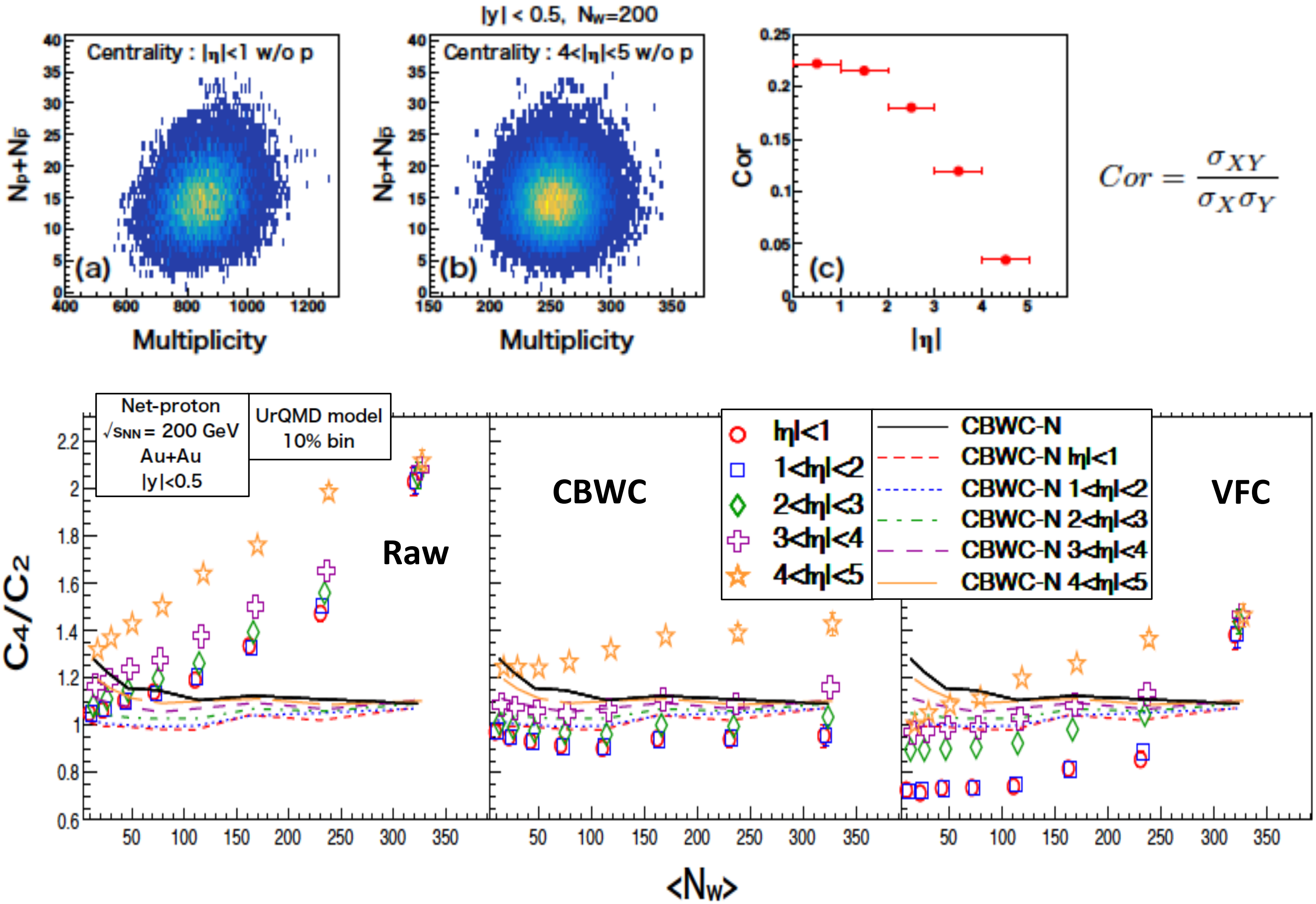
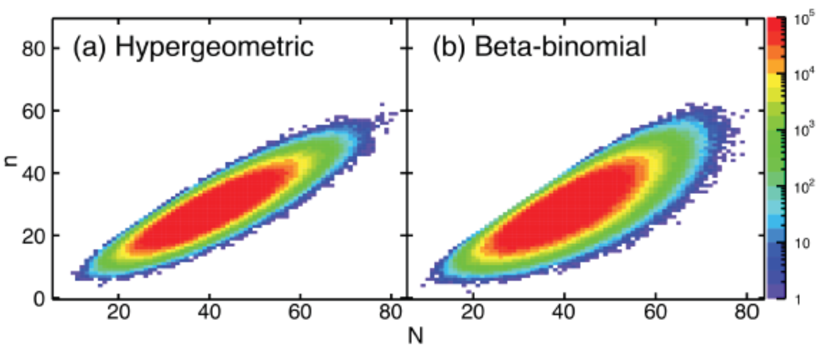
Multiplicity correlation



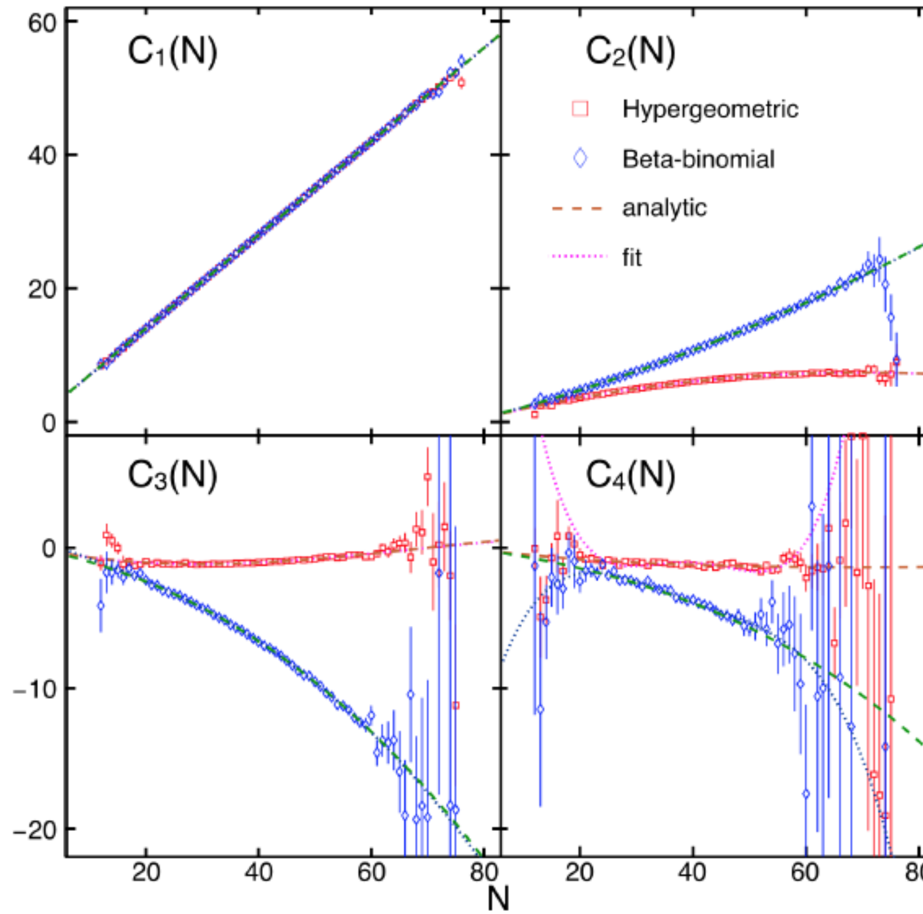
$$Cor = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

Analytical approach of efficiency correction with non-binomial effects

NIM A906 (2018) 10-17, arXiv:1805.00279
T. Nonaka, M. Kitazawa, S.E.



$$\begin{bmatrix} \langle\langle n \rangle\rangle \\ \langle\langle n^2 \rangle\rangle \\ \vdots \\ \langle\langle n^L \rangle\rangle \end{bmatrix} = \begin{bmatrix} r_{10} \\ r_{20} \\ \vdots \\ r_{L0} \end{bmatrix} + \boldsymbol{R} \begin{bmatrix} \langle N \rangle \\ \langle N^2 \rangle \\ \vdots \\ \langle N^L \rangle \end{bmatrix}$$



If the fitting works with the L-th order polynomial function of N, the "R" can be determined in order to solve the equations.

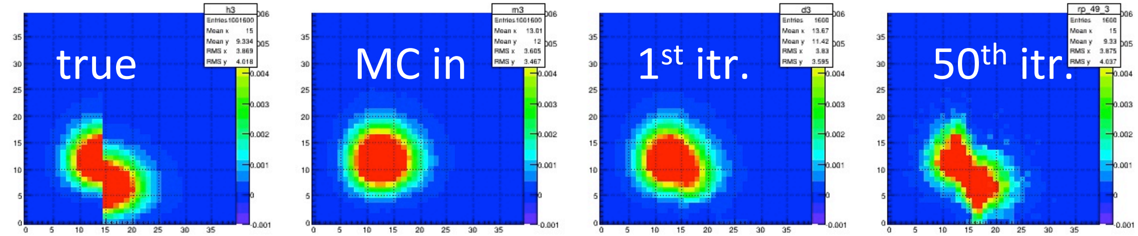
Summary

- unfolding of net-distribution including volume fluctuation and simultaneously extracting the temperature distribution
- Volume fluctuation vs correlated particle production
- Analytical approach to attack non-binomial efficiency

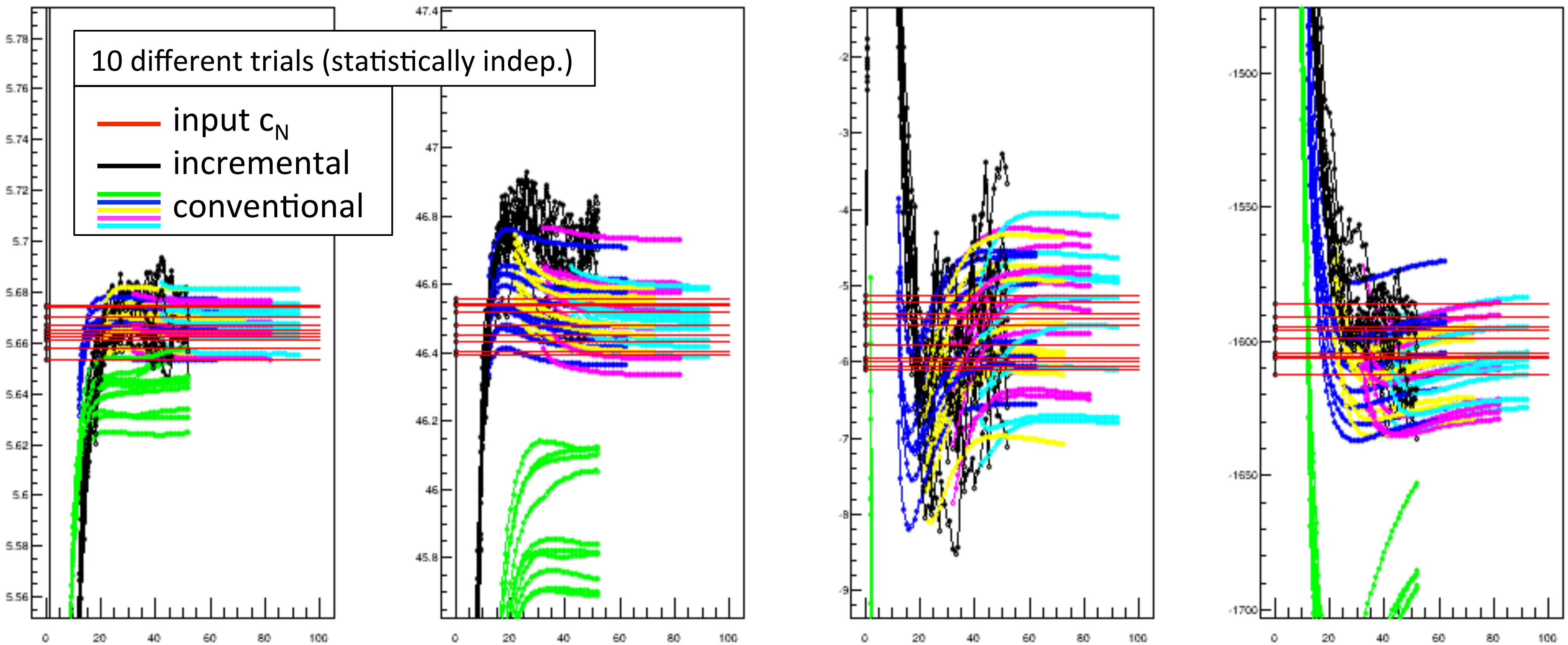
Hybrid iteration

--- with critical signal ---

(ε_x=0.7, ε_y=0.65)



C_1 = 5.66+/- 0.01 C_2 = 46.5 +/- 0.1 C_3 = -6.5 +/- 0.5 C_4 = -1600 +/- 15

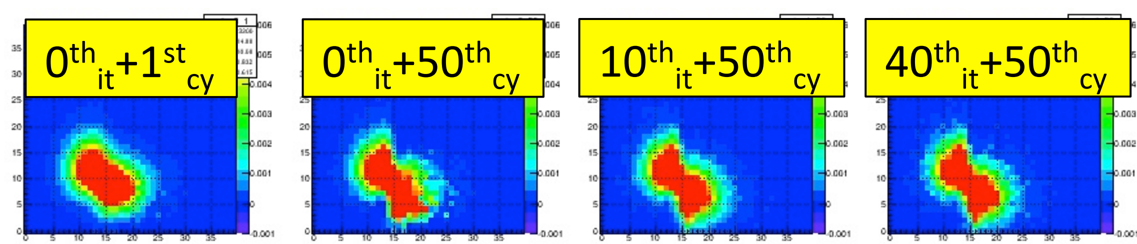


number of iteration

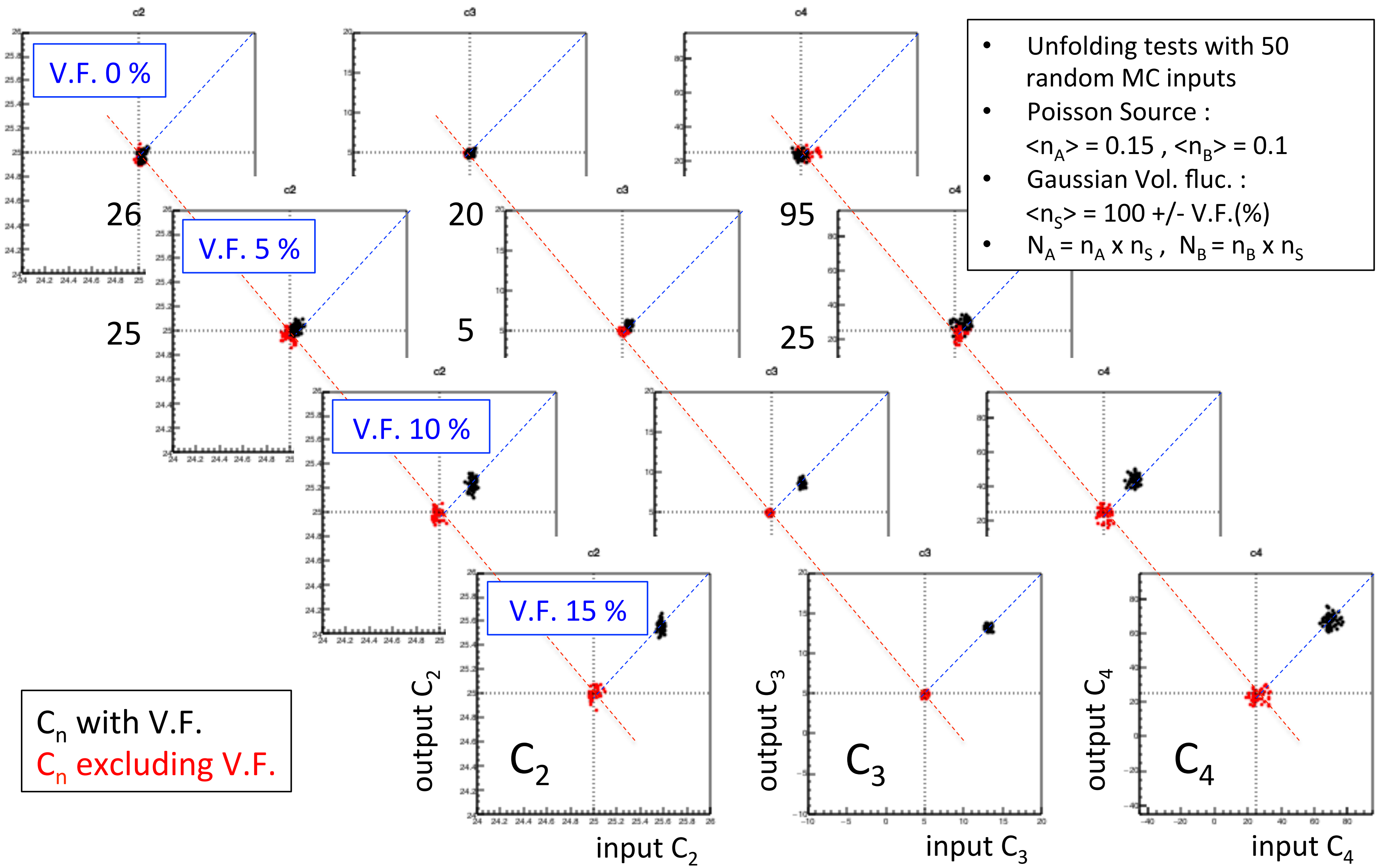
response matrix from

original, 10^{th}, 20^{th}, 30^{th}, 40^{th}

incremental unfolding

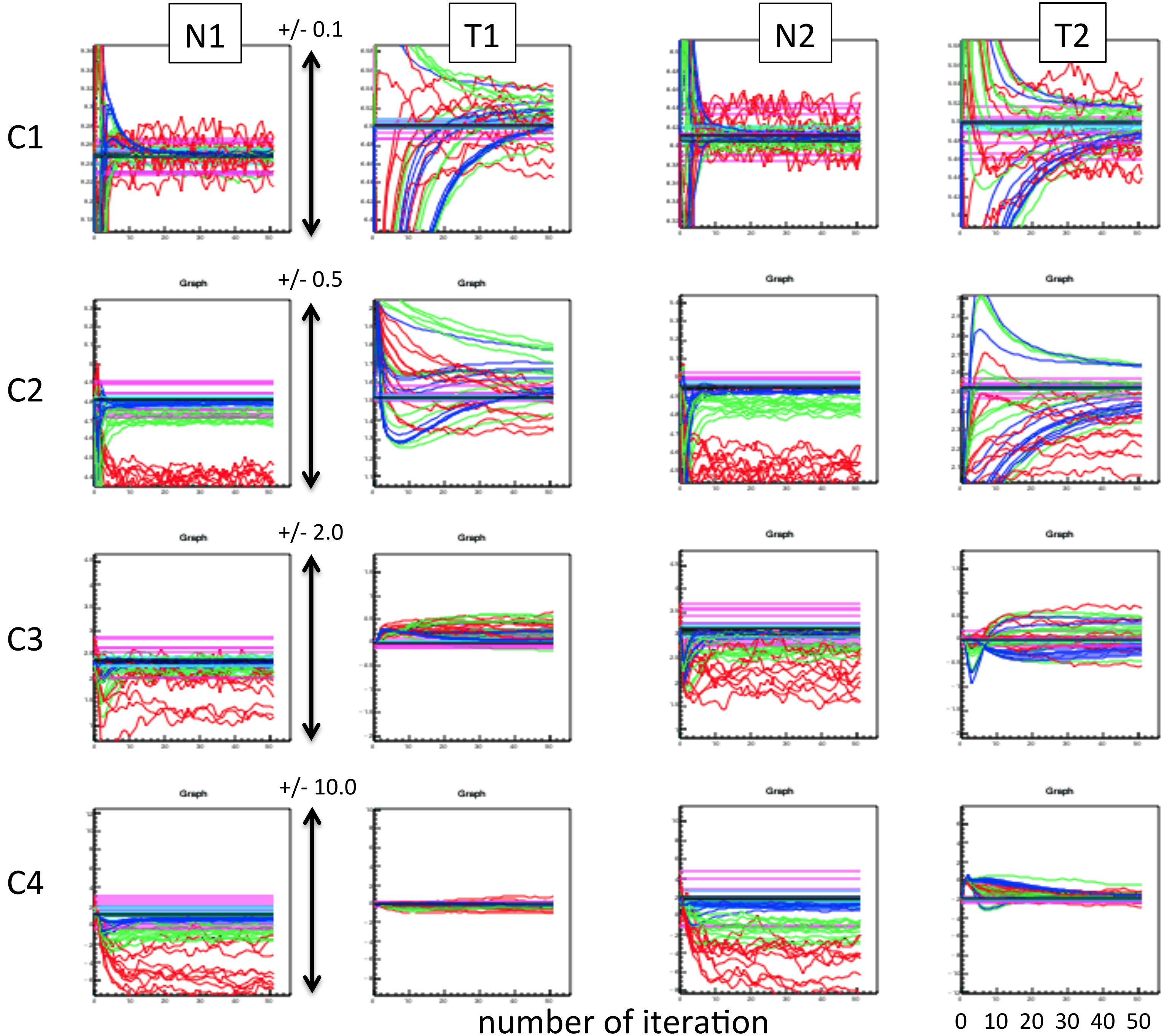


Extracted Cumulant (C_n) after unfolding iterations with V.F.



- Unfolding tests with 50 random MC inputs
- Poisson Source : $\langle n_A \rangle = 0.15$, $\langle n_B \rangle = 0.1$
- Gaussian Vol. fluc. : $\langle n_S \rangle = 100$ +/- V.F.(%)
- $N_A = n_A \times n_S$, $N_B = n_B \times n_S$

C_n with V.F.
C_n excluding V.F.



test sample with correlations

10k True
100k MC

100k True
1M MC

1M True
10M MC

2 particle/bin
0.1 GeV/bin
are not scaled
correctly yet...