Proton SL and TL form-factors: beyond the valence state

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Outline

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2. On the Virtue of the Minkowski Space in Hadron Phenomenology

3. Resolving the Constituent Quark

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5. Conclusions
Motivations for intense theoretical efforts

Forthcoming 12 GeV Experiment at TJLAB

\[ \text{hallaweb.jlab.org/collab/PAC/PAC32/PR12 \,- \, 07 \,- \, 109 \,- \, Ratio.pdf} \]

\begin{equation}
G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4M_N^2} \kappa_N \, F_2^N(Q^2) \quad G_M^N(Q^2) = F_1^N(Q^2) + \kappa_N \, F_2^N(Q^2) \end{equation}

\[ G_M^P(Q^2) > 0 \quad \text{for} \quad 0 > Q^2 > 30 \,(\text{GeV}/c)^2 \]
Relevant questions (in my opinion....)

- The expected, striking zero will be discovered, or not?
- If experimentally confirmed, the unbalancing at \( Q^2 \sim 10 \ (\text{GeV}/c)^2 \) of the Dirac and Pauli proton form factors will open a new window on the dynamical game inside the nucleon, or not?
- In the case of a positive answer to the previous questions, are we getting ready the phenomenological tools (the general framework is obvious: QCD) for interpreting the results and extracting relevant insights?

A methodological remark

In order to extract reliable information, the phenomenological investigation has to cover the widest set of observables and not to face with only one experimental results

- Spacelike form factors of both proton and neutron
- Timelike nucleon form factors, possibly
- Single-spin asymmetries for \( p\bar{p} \rightarrow \ell^+\ell^- \), ....
- ...

Tobias Frederico (ITA)
Proton SL and TL form-factors .....
★★ Our program:
Developing a covariant framework, based on the Bethe-Salpeter Amplitudes of hadrons, in Minkowski space, that allows one to include information on hadron dynamics, through the comparison with the data from processes involving EM probes in both space- and time-like regions.
This could provide a new tool for paving the way from a purely phenomenological microscopic description of the hadronic states to the one with a more consistent dynamical content. In particular in view of the emerging possibility to have solutions of the Bethe-Salpeter Equation in Minkowski space (J. Carbonell and V. Karmanov EPJ A 27, 1 (2006) → PL B 727, 205 (2013); TF, M. Viviani, G Salmè PR D89, 016010 (2014)); W de Paula, TF, R Pimentel, G Salmè, M Viviani, EPJ C 77, 764 (2017)

★★ ★★ Our strategy:
First modeling the quark-photon vertex and the quark-hadron amplitude from an investigation of the pion EM form factor, within a Mandelstam-inspired approach. Then, moving to the nucleon case, producing predictions for SL ratio and FF’s in the timelike region (TL FF’s contain a lot of information on mesonic spectra to be extracted...).
On The Virtue of the Minkowski space in hadron phenomenology

Our physical intuition has been trained through the amusement provided by the Feynman diagrams. Therefore, keeping safe the analytic structure (in Minkowski space) of the dynamical observables is fundamental. Afterall the actual space is Minkowskian...

EM form factor for a $\phi^3$ model. Solid line: calculated by using the Minkowski BS amplitude. Dashed line: Euclidean static approximation (i.e. the Euclidean fourth component remains unchanged). After Karmanov, Mangin-Brinet and Carbonell EPJ A 39, 53 (2009)
LQCD charge and magnetic radii

Alexandrou et al.

Euclidean Lattice access only space-like distances $\rightarrow q^2 < 0$!
Resolving the Constituent Quark: Minkowski space

As well-known, the Light-front framework is very suitable for an investigation of hadron EM form factors in both space- and timelike regions, beyond the valence contribution, since one can exploits the almost "simple" LF vacuum (see Brodsky, Roberts, Shrock and Tandy PRC 82, 022201(R) (2010)4 for the relation with DCSB)

\[
|\text{meson}\rangle = |q\bar{q}\rangle + |q\bar{q}q\bar{q}\rangle + |q\bar{q} g\rangle \ldots
\]

\[
|\text{baryon}\rangle = |qqq\rangle + |qqq q\bar{q}\rangle + |qqq g\rangle \ldots
\]

valence nonvalence

Another benefit

★ The LF boosts do not contain the dynamics, and the initial and final hadronic states, in a given process, can be trivially related to their intrinsic description

Second fundamental ingredient:

★ Playing the game in Minkowski Space allows one to single out contributions to be clearly ascribed to the various components of the hadronic state.
Quick view: Pion SL and TL form factors

\[ q^2 = q^+ q^- - \vec{q}_\perp^2, \quad q^+ = q^0 + q^3 > 0, \quad q^- = q^0 - q^3 \]

Experimental data: R. Baldini et al., EPJC 11, 709 (1999); NPA666, 38 (2000); (private communication); J. Volmer et al., PRL 86, 1713 (2001).
Assuming only the pole contribution for the quark propagator

\[ q^2 = q^+ q^- - q^2_\perp, \quad q^+ = q^0 + q^3 > 0, \quad q^- = q^0 - q^3 \]

**Spacelike Region**

Triangle contr. \quad Pair contr. (Z-diagr.)

\[ \gamma^* \quad \gamma^* \]

\[ k_3 + q \]

\[ P_{N'} \quad P_N' \quad P_N \quad P_{N'} \]

(val.) \quad (a) \quad (b) \quad \uparrow P_N

\[ 0 < k_i^+ < P_N^+ \quad 0 > k_3^+ > -q^+ \]

\[ \times \Rightarrow k \text{ on the mass shell: } k_{on}^- = (m^2 + k^2_\perp)/k^+ \]

**Timelike Region**

\[ P_N \]

\[ P_N' \]

\[ \gamma^* \]

\[ k_3 + q \]

\[ P_{N'} \]

(a) \quad (b)

\[ P_N^+ < k_3^+ + q^+ < q^+ \quad 0 < k_3^+ + q^+ < P_N^+ \]
The Mandelstam Formula for the Nucleon EM FF’s

Spacelike nucleon em form factors are evaluated from the matrix elements of the macroscopic current

\[ \langle \sigma', P_N' | j^\mu | P_N, \sigma \rangle = \bar{U}_N(P_N', \sigma') \left[ -F^N_2(Q^2) \frac{P_N' \mu + P_N\mu}{2M_N} + G^N_M(Q^2) \gamma^\mu \right] U_N(P_N, \sigma) \]

which are approximated microscopically by the Mandelstam formula in Minkowski Space

\[ \langle \sigma', P_N' | j^\mu | P_N, \sigma \rangle = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \sum \left\{ \Phi^\sigma_N(k_1, k_2, k_3', P_N') \times S^{-1}(k_1) S^{-1}(k_2) I^\mu(k_3, q) \Phi^\sigma_N(k_1, k_2, k_3, P_N) \right\} \]

(1)

- \( \Phi^\sigma_N(k_1, k_2, k_3, P_N) \): the Nucleon Bethe-Salpeter amplitude
- \( S(k_i) \): the quark propagator
- \( I^\mu(k_3, q) \): the quark-photon vertex

Chosen Frame: \( q_\perp = 0 \quad q^+ = |q|^2^{1/2} \) and \( q^- = \pm q^+ \) [SL (-) & TL (+)]

N.B. not a Drell-Yan frame! In a DY frame the matrix elements are diagonal in the number of constituents

\[ G^N_E(q^2) = \frac{1}{2} \text{Tr} \left\{ \frac{p_N' + M_N}{2M_N} I^+(q^2) \frac{p_N + M_N}{2M_N} \gamma^+ \right\} , \quad G^N_M(q^2) = \eta \text{Tr} \left\{ \frac{p_N' + M_N}{2M_N} I_x(q^2) \frac{p_N + M_N}{2M_N} \gamma_x \right\} , \quad \eta = -2M_N^2/q^2 \]
Our Model in Minkowski Space

The Dirac structure of the Quark-Nucleon vertex is suggested by an effective Lagrangian (de Araujo et al., PLB B478 (2001) 86)

\[
L_{\text{eff}}(x) = \frac{\epsilon_{abc}}{\sqrt{2}} \int d^4x_1 \, d^4x_2 \, d^4x_3 \mathcal{F}(x_1, x_2, x_3, x) \sum_{\tau_i} \left[ \alpha \, m_N \bar{q}^a(x_1, \tau_1) \gamma^5 q^b_C(x_2, \tau_2) \bar{q}^c(x_3, \tau_3) \
- \frac{(1 - \alpha)}{\sqrt{3}} \bar{q}^a(x_1, \tau_1) \gamma^5 \gamma_\mu q^b_C(x_2, \tau_2) \cdot \bar{q}^c(x_3, \tau_3) \left( -i \, \partial^\mu \right) \right] \psi_N(x, \tau_N) + ... \]

★ In our calculation \( \alpha = 1 \), i.e. no derivative coupling

★★ For the present: \( S(k_i) \rightarrow S_0(k_i) \), but Nakanishi PTIR could open some perspectives of improvements, in Minkowski Space (see in few slides)

★★★ Then, the Nucleon BSA ( \( \rightarrow qqq\)-nucleon vertex) can be approximated as follows

\[
\Phi^\sigma_N(k_1, k_2, k_3, P_N) = \, \iota \left[ S_0(k_1) \, \tau_y \, \gamma^5 \, S_C(k_2) C \otimes S_0(k_3) + \ldots (321) \ldots + \ldots (312) \ldots \right] \times \\
\Lambda(k_1, k_2, k_3) \, \chi_{\tau N} \, U_N(P_N, \sigma)
\]

with \( S_C \) the charge-conjugated of \( S_0 \) and a properly symmetrized Dirac structure obtained

● \( \Lambda(k_1, k_2, k_3) \) describes the vertex-function dependence upon the quark momenta, \( k_i \).
With more Dirac structure more \( \Lambda \) functions.

● \( U_N(P_N, \sigma) \) and \( \chi_{\tau N} \) are the nucleon spinor and isospin eigenstates.

● Quark mass: \( m_u = m_d = 200 \text{ MeV} \) (the same for the SL and TL pion FF)
Spin coupling scheme in the EM current matrix elements

The symmetrisation of the BS amplitude generates four different amplitudes:

The blobs are either the scalar or vector coupling of the quark pair.
Quark-Photon Vertex

\[ I^\mu = I_{IS}^\mu + \tau_z I_{IV}^\mu \]

★ SL: Elastic ch. \( \gamma^* q \rightarrow q \) in the valence region, Pair production ch., \( \gamma^* \rightarrow q\bar{q} \) in the non valence one.

● TL: Pair production contribution only.

N.B. Pair production has two contributions: bare + Vector Meson Dominance terms (\( \rho \)-pole \( \rightarrow \) pion cloud)

\[ I_i^\mu (k, q) = \mathcal{N}_i \left[ \theta(-Q^2) \theta(P_0^+ - k^+) \theta(k^+) \gamma^\mu \right. + \]

\[ + \theta(q^+ + k^+) \theta(-k^+) \left\{ Z_B \mathcal{N}_i \gamma^\mu + Z_{VM}^i \Gamma_{VM}^\mu (k, q, i) \right\} \]

with \( i = IS, IV, \mathcal{N}_{IS} = 1/6 \) and \( \mathcal{N}_{IV} = 1/2 \). The constants \( Z_B \) (bare term) and \( Z_{VM}^i \) (VMD term) are unknown weights to be extracted from the phenomenological analysis of the data.
\[ \Gamma_{VM}^\mu(k, q, IV) \] is the same already used in the pion case. Included up to 20 IV mesons, using the FPZ model (Frederico, Pauli, Zhou model (PRD 66 (2002) 116011), that reproduces the Anisovich-Iachello linear correlation between \( M_{mes}^2 \propto n \)

For the Nucleon, one needs \( \Gamma_{VM}^\mu(k, q, IS) \). The same approach as in the IV case.

We calculate microscopically \( \Gamma_{e^-e^+}^i \) and the amplitudes \( VM + N \rightarrow N \) and \( VM \rightarrow NN\bar{N} \)

A comparison for the IV Mesons (PDG 2012)

<table>
<thead>
<tr>
<th>IV Meson</th>
<th>( \Gamma_{e^+e^-} ) (KeV)</th>
<th>( \Gamma_{e^+e^-}^{\exp} ) (KeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho(770) )</td>
<td>7.01</td>
<td>( 7.04 \pm 0.06 )</td>
</tr>
<tr>
<td>( \rho(1450) )</td>
<td>0.97</td>
<td>( 0.4 - 1.8 )</td>
</tr>
<tr>
<td>( \rho(1700) )</td>
<td>0.99</td>
<td>( &gt; 0.30 - 0.4 )</td>
</tr>
</tbody>
</table>

A comparison for the IS Mesons (PDG 2012)

<table>
<thead>
<tr>
<th>IS Meson</th>
<th>( \Gamma_{e^+e^-} ) (KeV)</th>
<th>( \Gamma_{e^+e^-}^{\exp} ) (KeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega(782) )</td>
<td>0.603</td>
<td>( 0.60 \pm 0.02 )</td>
</tr>
<tr>
<td>( \omega'(1420) )</td>
<td>0.095</td>
<td>( 0.12 - 0.16 )</td>
</tr>
<tr>
<td>( \omega''(1650) )</td>
<td>0.090</td>
<td>-</td>
</tr>
</tbody>
</table>

N.B. The total decay for mesons heavier than the first three ones, has been fixed to \( \Gamma_{tot} = 150 MeV \).
Momentum Dependence of the Bethe-Salpeter Amplitudes

The onshell vertex function, that fully determines the triangle term, → valence component, with two of quark on the \( k^- \)-shell

\[
\int \frac{dk_1^-}{2\pi} \int \frac{dk_2^-}{2\pi} \Phi^{BSA}(k_1, k_2; P_N) = \psi^{val}_N(\xi_1, \xi_2, k_1\perp, k_2\perp) \sim P^+_N \frac{\Lambda_V(\xi_1, \xi_2, k_1\perp, k_2\perp)}{[M_N^2 - M_0^2(1, 2, 3)]}
\]

\( M_0(1, 2, 3) \equiv \) free mass of the three-quark system

The Nucleon valence component is approximated \textit{a la} Brodsky-Lepage (i.e. PQCD inspired, but not restricted to a PQCD regime !!)

\[
\psi^{val}_N(\xi_1, \xi_2, k_1\perp, k_2\perp) \sim P^+_N \mathcal{N} \frac{(9 m^2)^{7/2}}{(\xi_1 \xi_2 \xi_3)^p [\beta^2 + M_0^2(1, 2, 3)]^{7/2}}
\]

\( \mathcal{N} \) a normalization constant.

The power \( 7/2 \) and the parameter \( p = 0.13 \) are chosen such that the asymptotic decrease of the triangle contribution to \( G^N_E \) is faster than the dipole

N.B. Only the triangle diagram determines the nucleon magnetic moments, and furthermore they weakly depend on \( p \). Then, \( \beta = 0.65 \) can be fixed through \( \mu_p(n) \)

Proton: 2.87 (Exp. 2.793) Neutron: -1.85 (Exp. -1.913)
The non-valence vertex, that contributes to the pair-product term, depends on the available invariants.

SL: (i) free mass of quarks 1 and 2, $M_0(1, 2)$, and (ii) the free mass of the $N-\bar{q}$ system $M_0(N, \bar{3})$

\[
\Lambda_{NV}^{SL}(k_1, k_2, k_3) = [g_{12}]^2 \cdot [g_{N\bar{3}}]^{7/2 - 2} \cdot \left( \frac{k_{12}^+}{P_N^+} \right) \left( \frac{P_N^+}{k_3^+} \right) \left( \frac{P_N^+}{k_3^+} \right)
\]

\[k_{12}^+ = k_1^+ + k_2^+\]

\[g_{AB} = \frac{(m_A m_B)}{[\beta^2 + M_0^2(A, B)]}\]

TL: (i) free mass of antiquarks 1 and 2, $M_0(\bar{1}, \bar{2})$, and (ii) the free mass of the $N-\bar{q}\bar{q}$ system $M_0(N, \bar{1}\bar{2})$ (Nucleon - anti diquark system)

\[
\Lambda_{NV}^{TL}(k_1, k_2, k_3) = 2 \cdot [g_{\bar{1}\bar{2}}]^2 \cdot [g_{N\bar{1}\bar{2}}]^{3/2} \cdot \left( \frac{-k_{12}^+}{P_N^+} \right) \left( \frac{P_N^+}{k_3'^+} \right) \left( \frac{P_N^+}{k_3'^+} \right)
\]
Adjusted parameters fixed in the SL region only! \((m_q = 200 \text{ MeV ab initio})\)

★ the weights for the Pair production terms:

- \(Z_B = Z_{VM}^{IV} = 2.283\)
- \(Z_{VM}^{IS}/Z_{VM}^{IV} = 1.12\)

★★ the two parameters in the vertex function projected in the SL valence and non valence regions

- \(p = 0.13\) in the valence amplitude
- \(r = 0.17\) in the non valence amplitude

By using in the fitting procedure the experimental data (updated to 2009) for \(\mu_p G_E^P/G_M^P, G_E^n, G_M^P\) and \(G_M^n\) (only 3 sets in the most recent calculations)

\[
\Rightarrow \chi^2 = 1.7
\]

Results from PLB 671, 153 (2009)

\[
r_p = (0.903 \pm 0.004) \text{ fm} \quad r_p^{exp} = (0.895 \pm 0.018) \text{ fm}
\]

\[
- \left[ dG_E^P(Q^2)/dQ^2 \right]_{Q^2=0}^{th} = (0.501 \pm 0.002) \left( \frac{c}{\text{GeV}} \right)^2 \quad \left[ \text{exp.} = (0.512 \pm 0.013) \left( \frac{c}{\text{GeV}} \right)^2 \right]
\]
Solid line: full calculation $\equiv F_\triangle + Z_B F_{bare} + Z_{VM} F_{VMD}$ (de Melo et al PLB 671, 153 2009)
Dotted line: $F_\triangle$ (triangle contribution only)
Data: JLAB - Hall A Collab. before 2009
Interference between triangle and $Z$-diagram contributions, i.e. higher Fock components produces our zero.

Red line: only $G^n_E$, $G^n_M$ and $G^n_M$ in the fit for fixing the 4 parms
Low-$Q^2$ data: Paolone et al, PRL 105, 072001 (2010) and Ron et al, PRC 84 055204 (2011)
The zero is predicted by $G^n_E$, $G^n_M$ and $G^n_M$, within our model!
New Cloët et al calculation in few slides

Tobias Frederico (ITA)

Proton SL and TL form-factors ......

* New Cloët et al calculation in few slides

SL Nucleon form factors: $G_E^n$, $G_M^p$, $G_M^n$

Solid line: full calculation $\equiv \mathcal{F}_\Delta + Z_B \mathcal{F}_{bare} + Z_{VM} \mathcal{F}_{VMD}$

Dotted line: $\mathcal{F}_\Delta$ (triangle contribution only)

$G_D = 1/\left[1 - q^2/(0.71 \ (\text{GeV}/c)^2)\right]^2$

The Pair-production contribution is essential for the result !!
Proton and Neutron effective form factor in the TL region

★★ Parameter free result ★★

Parameter free like the new evaluation of the SL $\mu_p G_E^p / G_M^p$

Solid line: full calculation - Dotted line: bare production (no VM).
Proton: Missing strength at $q^2 = 4.5 \text{ (GeV/c)}^2$ and $q^2 = 8 \text{ (GeV/c)}^2$
Neutron: Dashed line: solid line arbitrarily $\times 2$. 

$$G_{\text{eff}}(q^2) = \sqrt{\frac{2\tau |G_M(q^2)|^2 + |G_E(q^2)|^2}{2\tau + 1}}$$

(2)
TL proton and neutron polarization orthogonal to the scattering plane: no polarized electron beam!

\[ P_y(\theta_{CM}) = -\sin(2\theta_{CM}) \frac{\Im m\{G_E(q^2)G_M^*(q^2)\}}{D \sqrt{\tau}} \]

\[ \tau = \frac{q^2}{4M_N^2} \text{ and } D = [1 + \cos^2(\theta_{CM})] |G_M(q^2)|^2 + \sin^2(\theta_{CM}) \frac{|G_E(q^2)|^2}{\tau} \]

LF Constituent Quark Model

TL proton and neutron polarization orthogonal to incident beams in the scattering plane: polarized electron beam!

\[ P_x(\theta_{CM}) = P_e \ 2\sin(\theta_{CM}) \ \Re \{ G_E(q^2) G^*_M(q^2) \} \ \frac{1}{D \sqrt{T}} \]

**LF Constituent Quark Model**

TL proton and neutron polarization along the incident beams: polarized electron beam!

\[ P_z(\theta_{CM}) = P_e\ 2\cos(\theta_{CM})\ \frac{|G_M(q^2)|^2}{D} \]

LF Constituent Quark Model

Cloët, Roberts and Thomas (PRL 111, 101803 (2013)) have recently emphasized the role of the Nucleon self-energy in determining the position of the zero. The Nucleon Faddeev amplitude is a Euclidean momentum space solution with a proper kernel, with dressed quarks and diquark dof.

\[ \alpha = 2.0 \]  
\[ \alpha = 1.8 \]  
\[ \alpha = 1.4 \]  
\[ \alpha = 1.0 \]  

\[ \mu_p G_{Ep}/G_{Mp} \]  
\[ Q^2 (\text{GeV}^2) \]
Conclusions & Perspectives

- Minkowski Space allows one to perform reliable calculations based on the physical intuition.

- A relativistic Constituent Quark Model, based on a phenomenological Ansatz for the Nucleon Bethe-Salpeter amplitude, has been applied for evaluating the nucleon EM form factors, in SL and TL regions.

- A microscopical Vector Meson Dominance model has been implemented through a realistic approach quite successful in reproducing the vector meson masses and EM widths.

- Only 4 adjusted parameters are necessary to get a very description of $G^p_M$, $G^n_E$ and $G^n_M$ in the SL region and predicts a zero for the SL ratio $\mu_p G^p_E/G^p_M$ around $Q^2 \sim 9 \ (GeV/c)^2$. The interference between the valence and non valence component (Pair production) of the proton state is the cause.

- TL Nucleon ff’s are also predictions! The comparison with experimental data for the proton points to missing strength around 4.5 and 8 (GeV/c)$^2$ Calculations of the TL polarizations show interesting structures, related both to the realistic description of the SL nucleon ff’s and to the VMD.

- Extension of the model to obtain off-shell SL and TL form factors?