Lattice QCD studies of hadron structure

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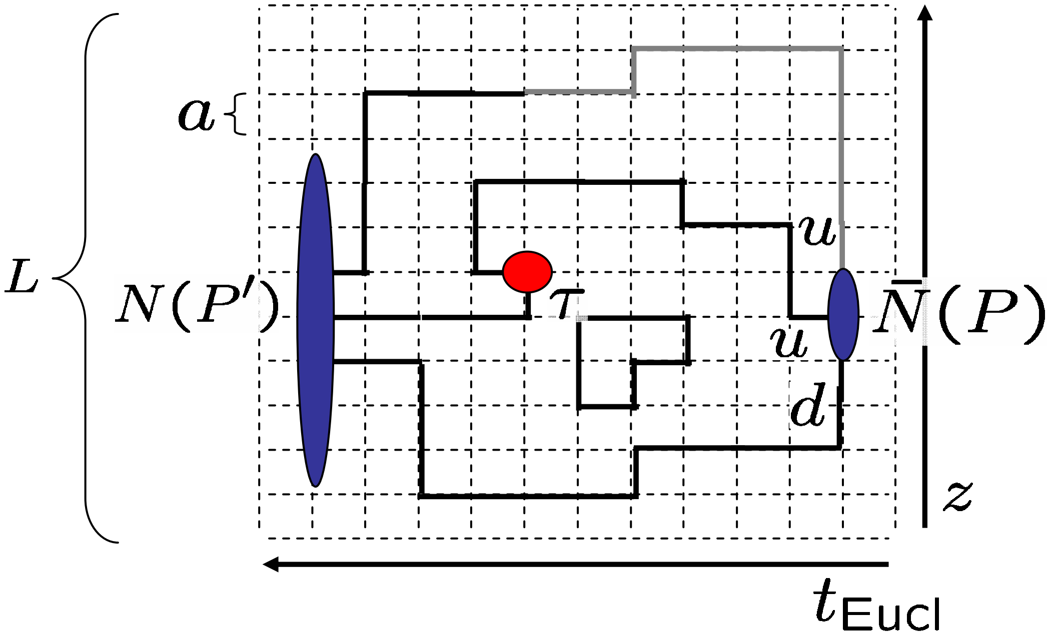
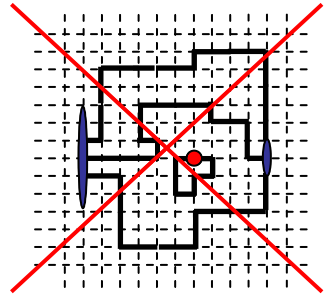
excellence cluster universe

in collaboration with
LHPC and QCDSF/UKQCD

Lattice QCD calculations of hadron structure





not relevant for u-d

= vector-, axialvector-, *quark spin flip-,* (spin-2) *graviton-, „spin-n“* coupling

$$C_{3pt}(P', P, \tau) \leftrightarrow e^{-E'(T-\tau)-E\tau} \langle P', \Lambda' | \mathcal{O} | P, \Lambda \rangle \propto g_A, \Delta\Sigma, F_1(t), F_2(t), \langle x \rangle, A_{20}(t), \ldots$$

$t = \Delta^2 (\hat{=} q^2)$

$$\mathcal{O} \hat{=} \overline{q}(x) \Gamma D^\mu D^\nu \ldots q(x)$$

Overview of numerical results

form factors – nucleon rms radii and (anomalous) magnetic moment — $F_1, F_2(Q^2) \rightarrow \langle r^2 \rangle_{1,2},\ \mu$

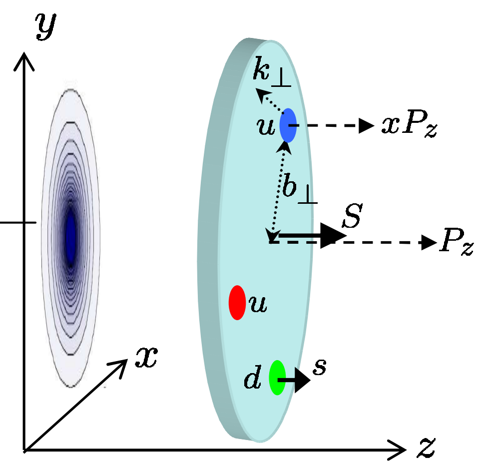
axial vector coupling constant & tensor charge — $g_A,\ g_T$

quark momentum fraction — $\langle x \rangle_q$

GPDs – generalized form factors incl.
form factors of the energy momentum tensor — $H, E(x,\xi,t) \rightarrow A(t),\ B(t),\ C(t)$

decomposition of the nucleon spin — $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$

correlations between momenta, positions and spins



QCDSF improved Wilson action parameters

- $N_f = 2$ dynamical Wilson - fermions with (NP) clover - improvement
- only connected contributions

- lattice spacing fixed using $m_N \leftrightarrow r_0 = 0.467\text{fm}$
- three projectors

$$\tilde{\Gamma}_{\text{unpol}} = \frac{1}{2}(1+\gamma_0),$$

$$\tilde{\Gamma}_{1,2} = \frac{1}{2}(1+\gamma_0)\gamma_5\gamma_{1,2}$$

- three sink - momenta $p' = (0,0,0),(1,0,0),(0,1,0)$
- non – perturbative operator renormalization

#	β	κ	L	a[fm]	L[fm]	m_π [GeV]	$m_\pi L$
1	5.20	0.13420	16	0.0856	1.37	1.348	9.4
2	5.20	0.13500	16	0.0856	1.37	0.956	6.6
3	5.20	0.13550	16	0.0856	1.37	0.67	4.7
6	5.25	0.13460	16	0.0794	1.27	1.225	7.9
7	5.25	0.13520	16	0.0794	1.27	0.949	6.1
8	5.25	0.13575	24	0.0794	1.91	0.635	6.1
9	5.25	0.13600	24	0.0794	1.91	0.457	4.4
11	5.29	0.13400	16	0.0753	1.2	1.511	9.2
12	5.29	0.13500	16	0.0753	1.2	1.102	6.7
13	5.29	0.13550	12	0.0753	0.9	0.945	4.3
14	5.29	0.13550	16	0.0753	1.2	0.874	5.3
15	5.29	0.13550	24	0.0753	1.81	0.857	7.8
16	5.29	0.13590	12	0.0753	0.9	0.883	4.
17	5.29	0.13590	16	0.0753	1.2	0.66	4.
18	5.29	0.13590	24	0.0753	1.81	0.629	5.8
19	5.29	0.13620	24	0.0753	1.81	0.414	3.8
21	5.29	0.13632	32	0.0753	2.41	0.282	3.4
22	5.29	0.13632	40	0.0753	3.01	0.276	4.2
23	5.29	0.13640	40	0.0753	3.01	0.168	2.6
24	5.40	0.13500	24	0.0672	1.61	1.183	9.7
25	5.40	0.13560	24	0.0672	1.61	0.917	7.5
26	5.40	0.13610	24	0.0672	1.61	0.648	5.3
27	5.40	0.13625	24	0.0672	1.61	0.558	4.6
28	5.40	0.13640	24	0.0672	1.61	0.451	3.7
29	5.40	0.13660	32	0.0672	2.15	0.255	2.8

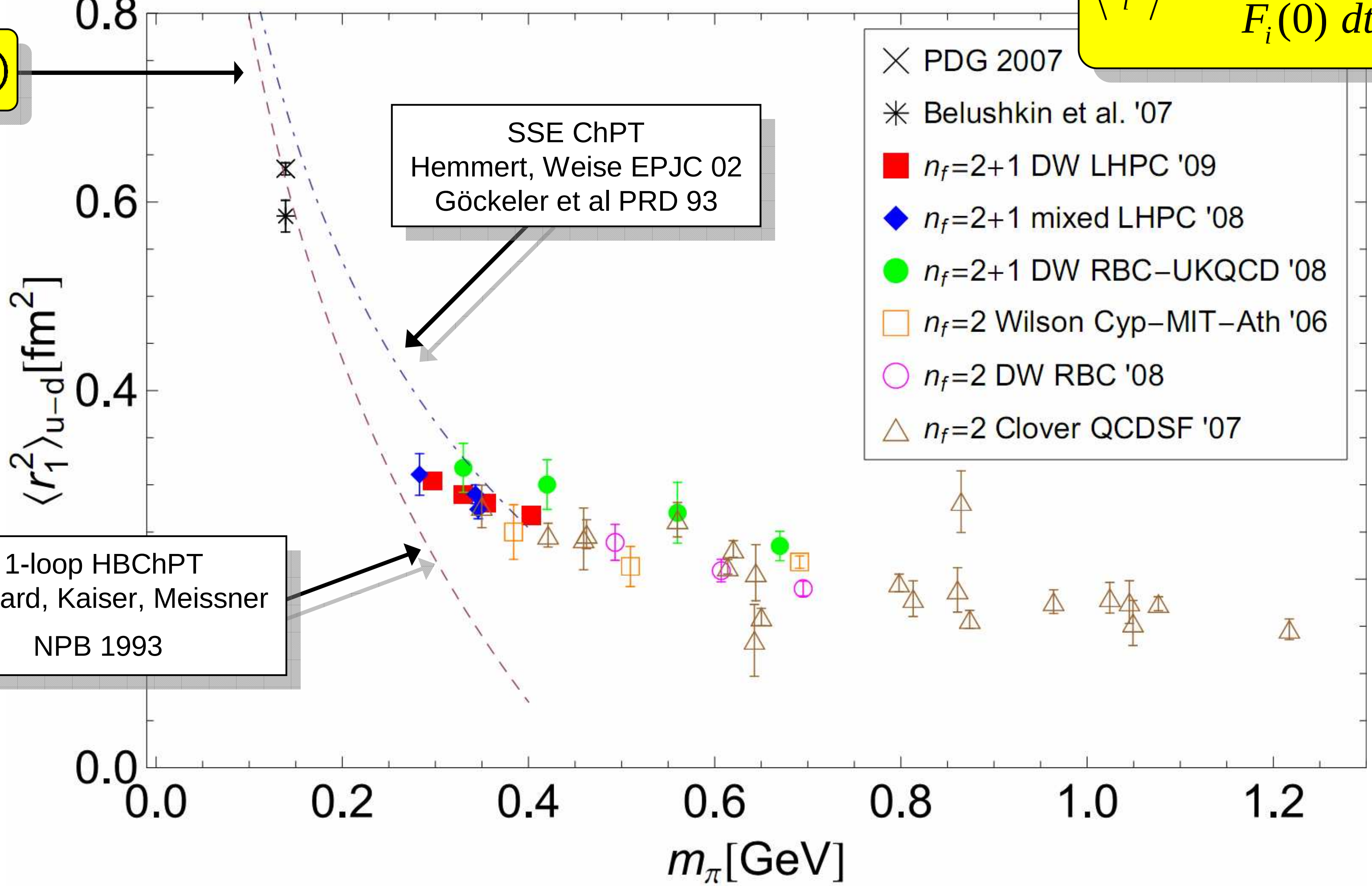
Proton mean square radii

Dirac and Pauli FFs

$$\langle P'|\overline{q}\gamma_\mu q|P\rangle = \overline{U}(P')\Big\{\gamma_\mu F_1(t) + i\frac{\sigma_{\mu\nu}\Delta^\nu}{2m_N}F_2(t)\Big\}U(P)$$

$$\langle r_i^2\rangle = -\frac{6}{F_i(0)}\frac{d}{dt}F_i(t)\Big|_{t=0}$$

$\ln(m_\pi/\lambda)$



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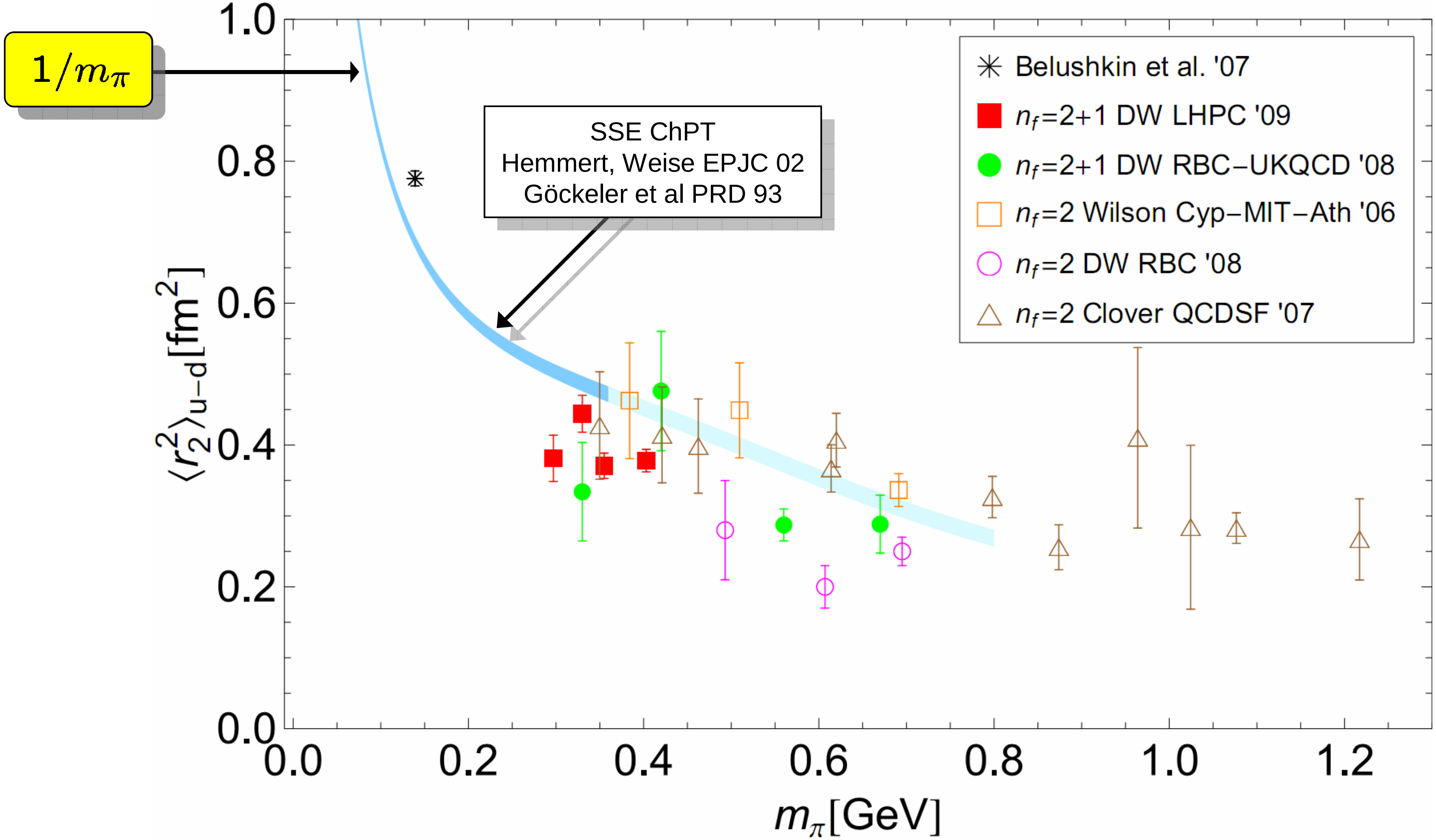

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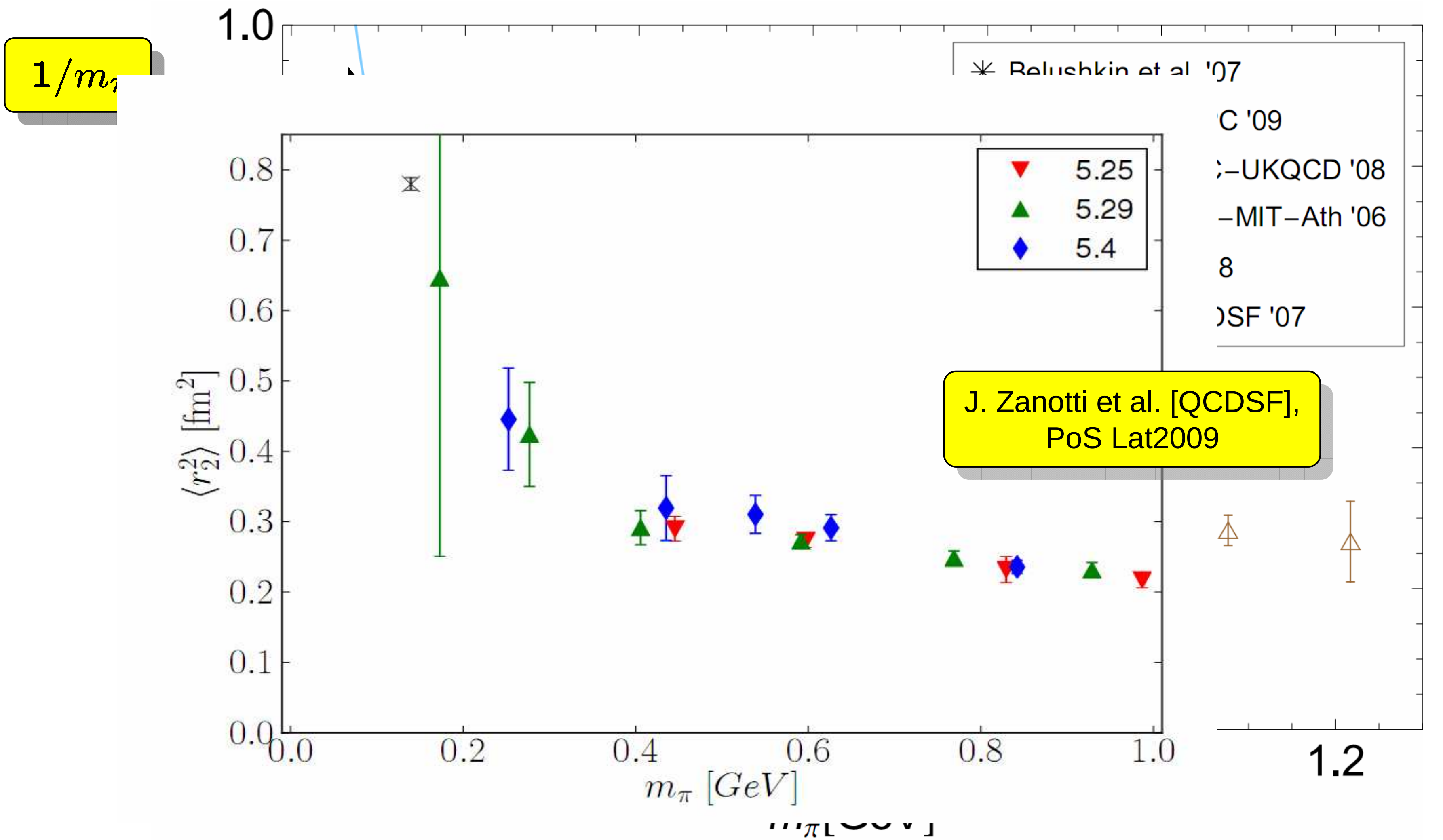
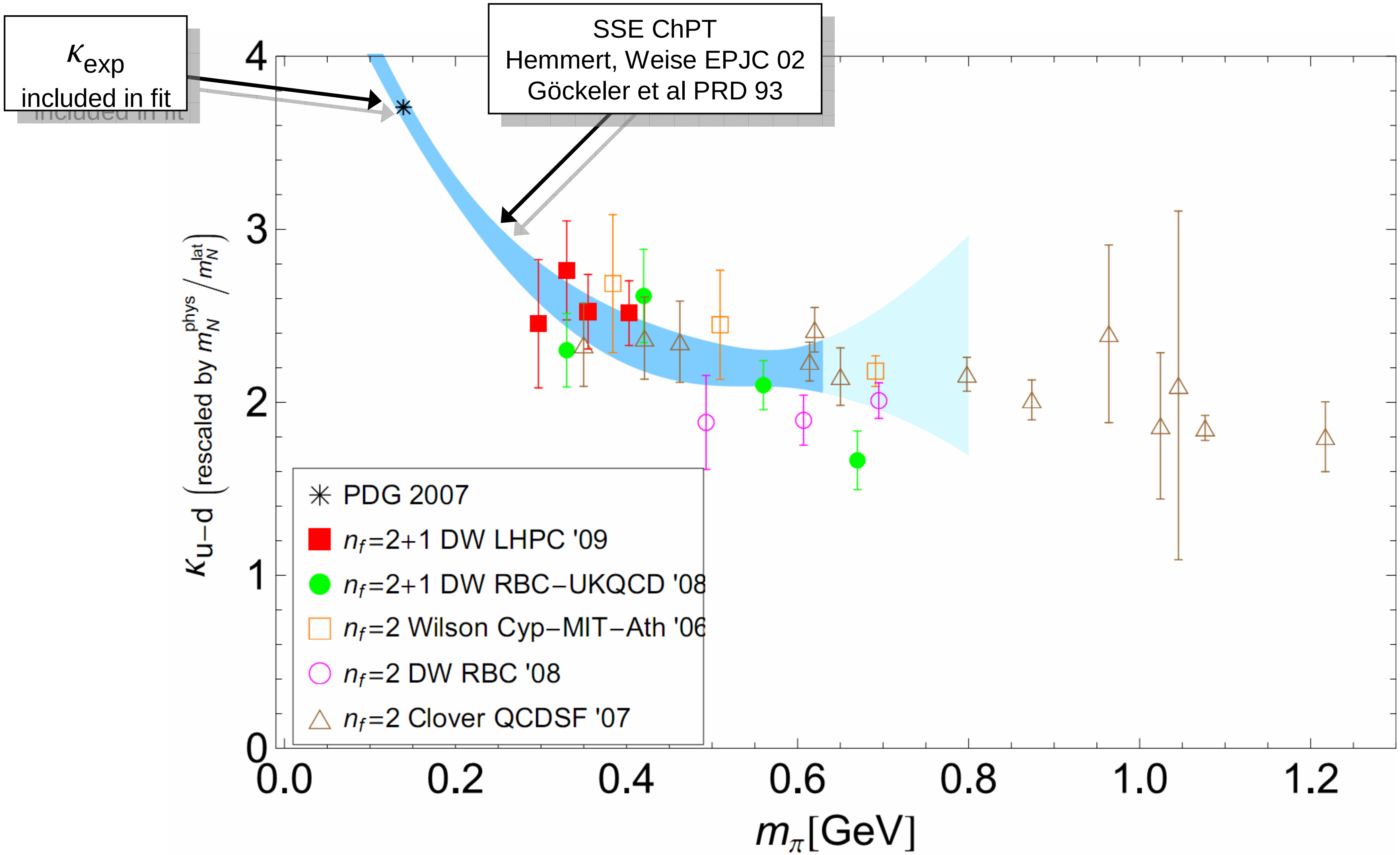


J. Zanotti et al. [QCDSF], PoS Lat2009

Nucleon isovector anomalous magnetic moment

(anomalous) magnetic moment → $\kappa = F_2(Q^2 \to 0) = \mu - F_1(0)$



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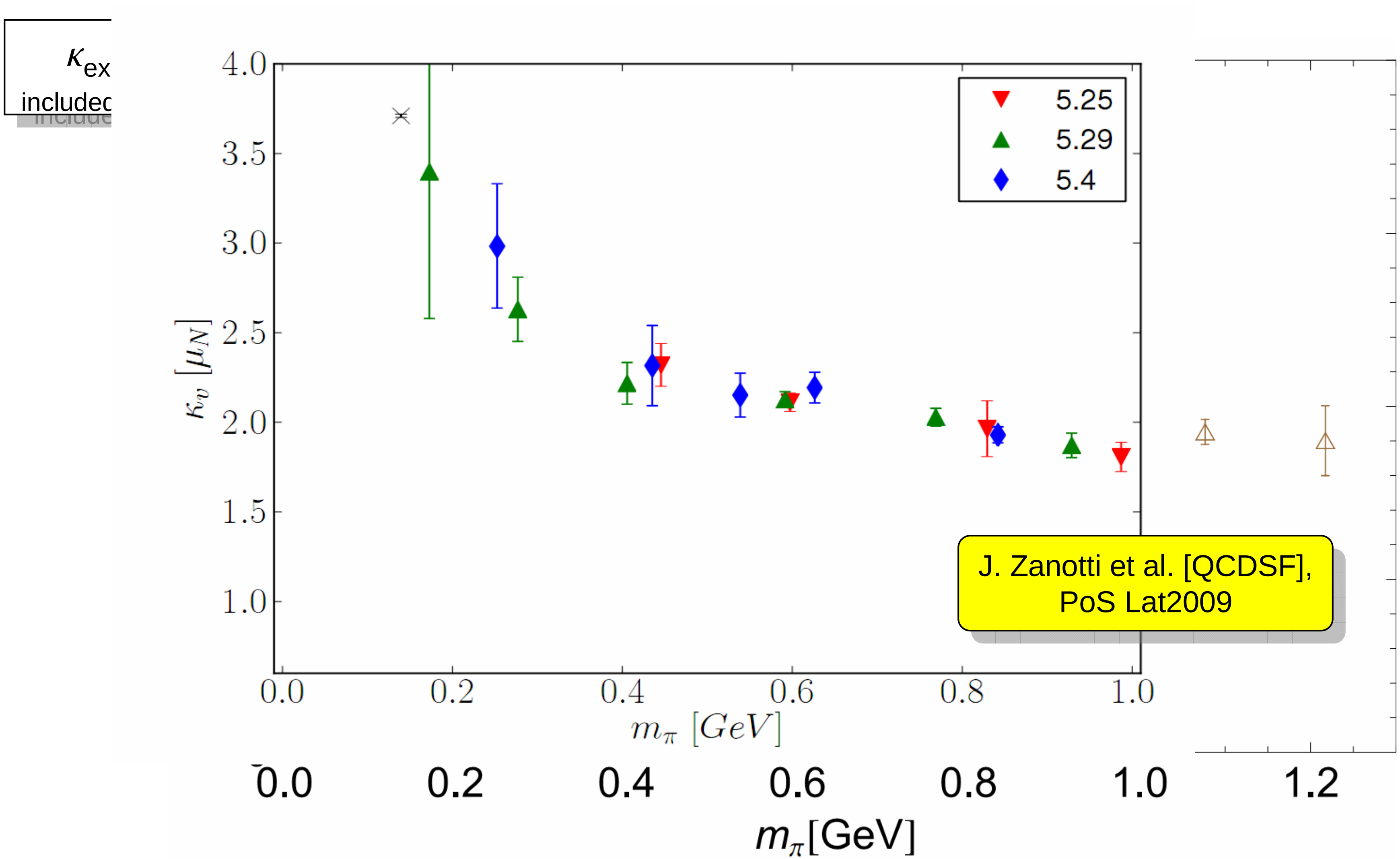
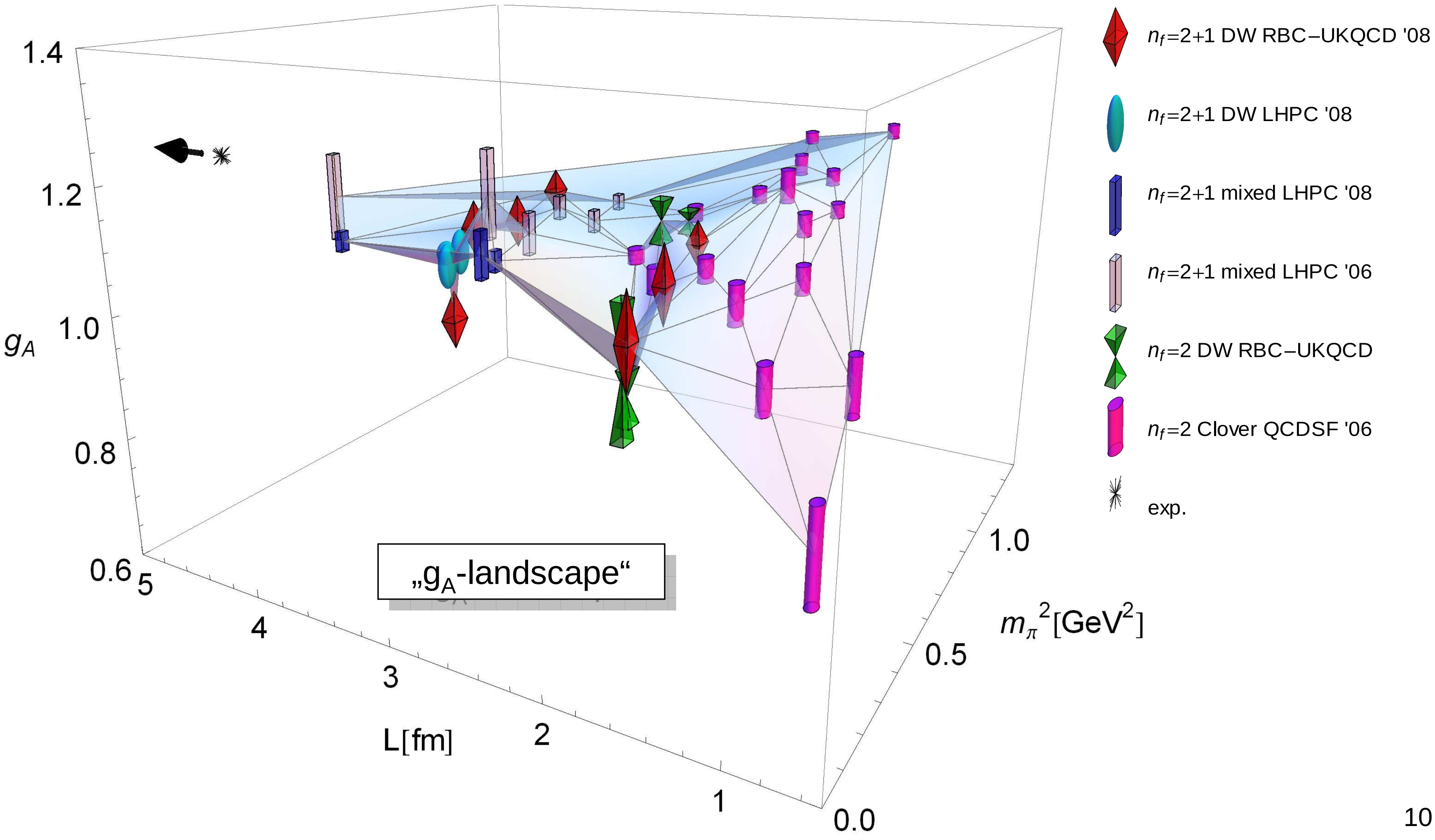


J. Zanotti et al. [QCDSF], PoS Lat2009

Nucleon axial vector coupling constant published data

$$\langle P|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|P\rangle = g_A\bar{U}(P)\gamma_\mu\gamma_5 U(P)$$



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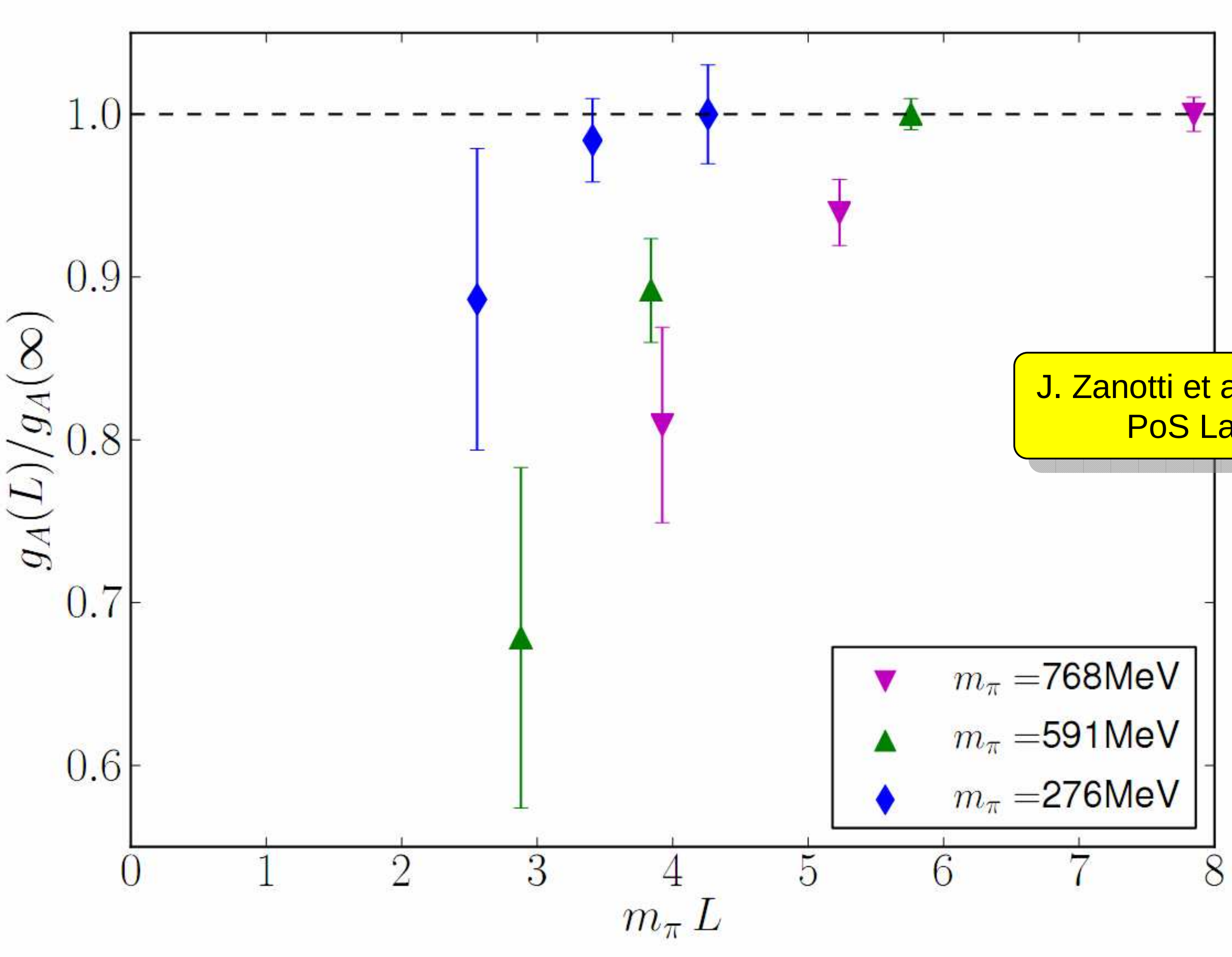
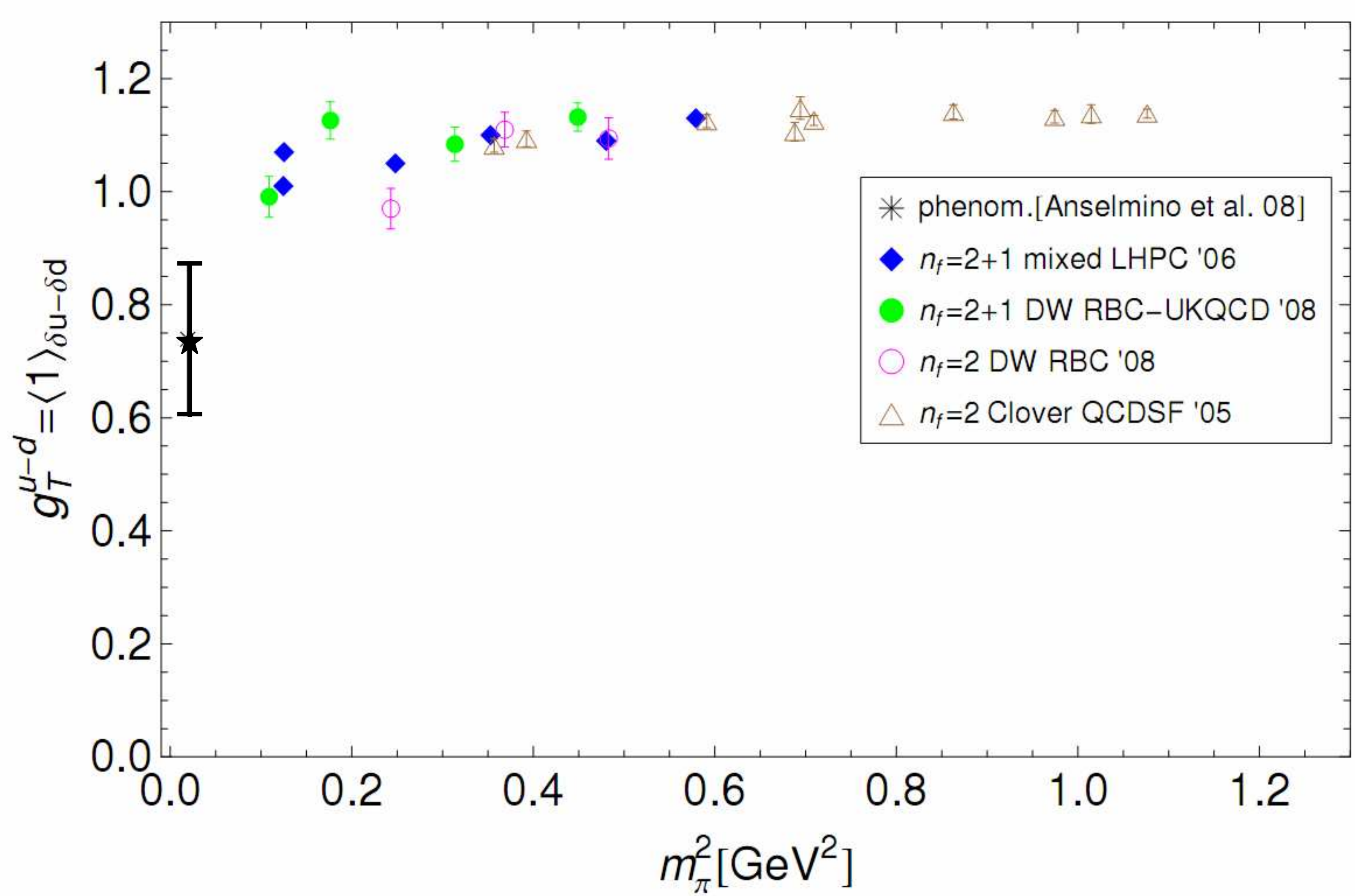


J. Zanotti et al. [QCDSF], PoS Lat2009

Tensor charge

$$\langle P|\bar{q}i\sigma_{\mu\nu}q|P\rangle = g_T\bar{U}(P)i\sigma_{\mu\nu}U(P) \longleftrightarrow g_T = A_{T10}(0) = \int_{-1}^{+1} dx\,\delta q(x) = \langle 1\rangle_{\delta q} - \langle 1\rangle_{\delta\bar{q}}$$



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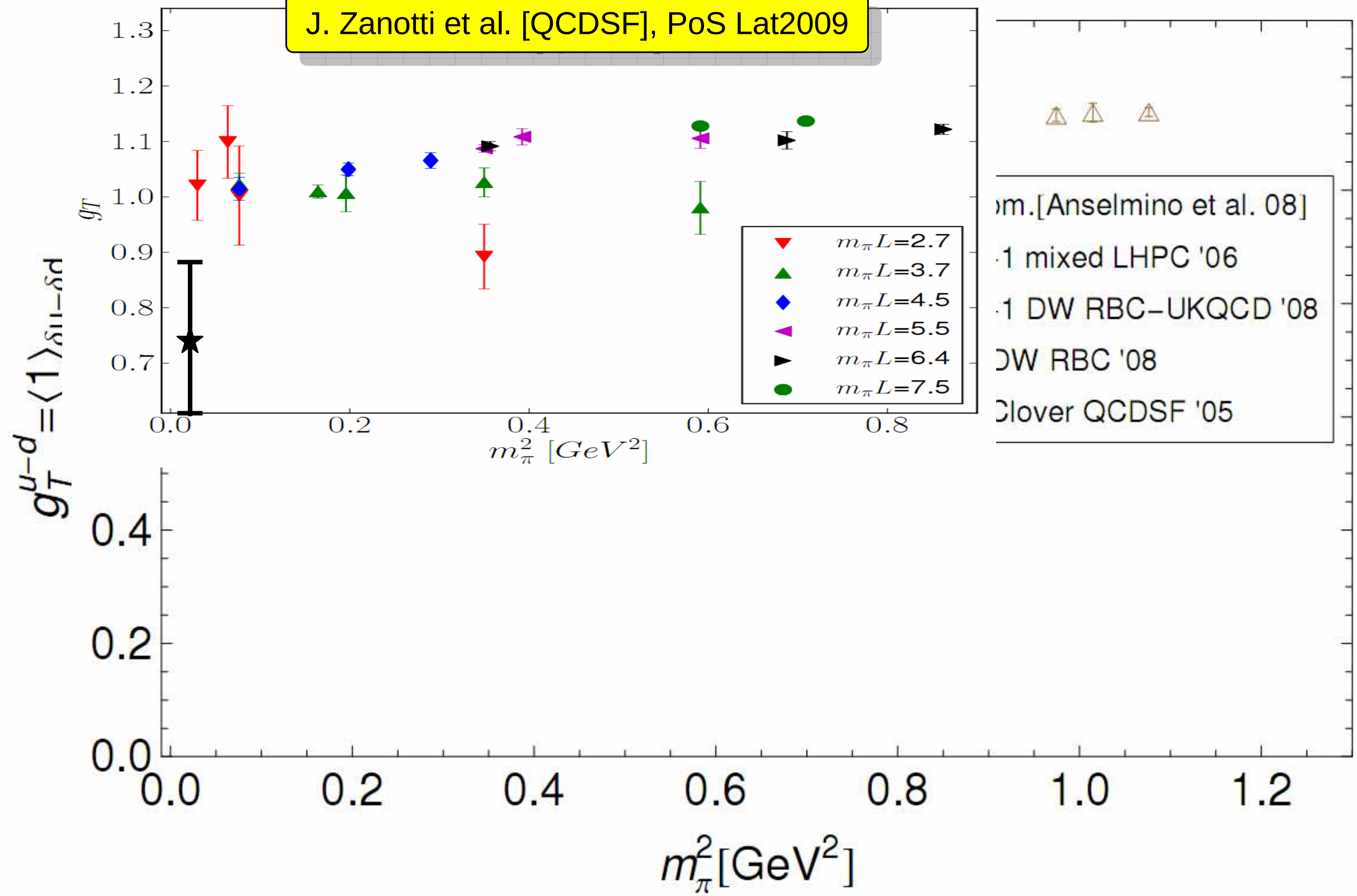
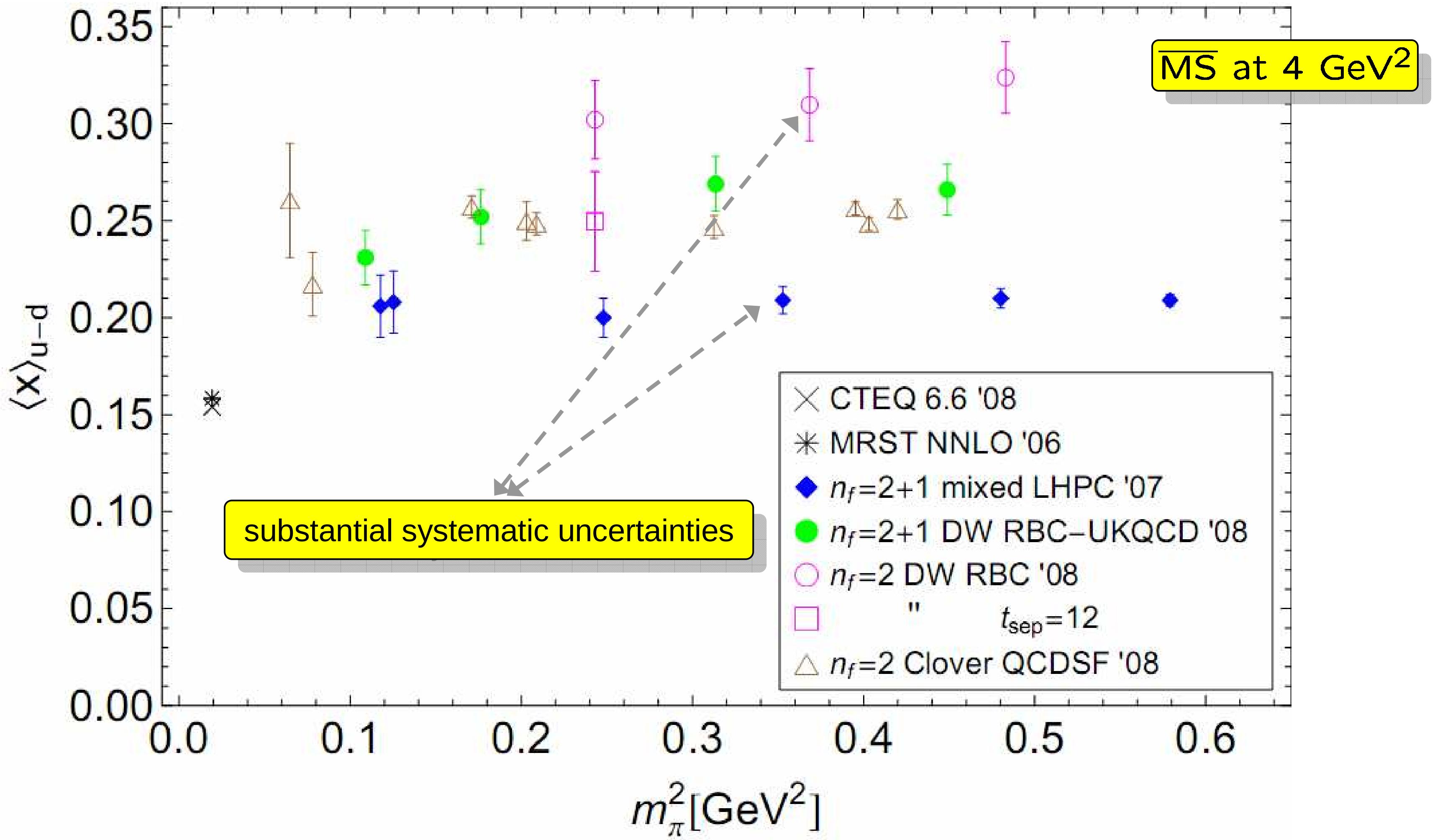


Momentum fraction of quarks in the nucleon

$$\langle P|\overline{q}\gamma^{\{\mu}D^{\nu\}}q|P\rangle = \overline{U}(P)\gamma^{\{\mu}P^{\nu\}}U(P)\langle x\rangle$$

$$\langle x\rangle = A_{20}(0) = \int_{-1}^{+1} dx\, x\, q(x) = \langle x\rangle_q + \langle x\rangle_{\overline{q}}$$



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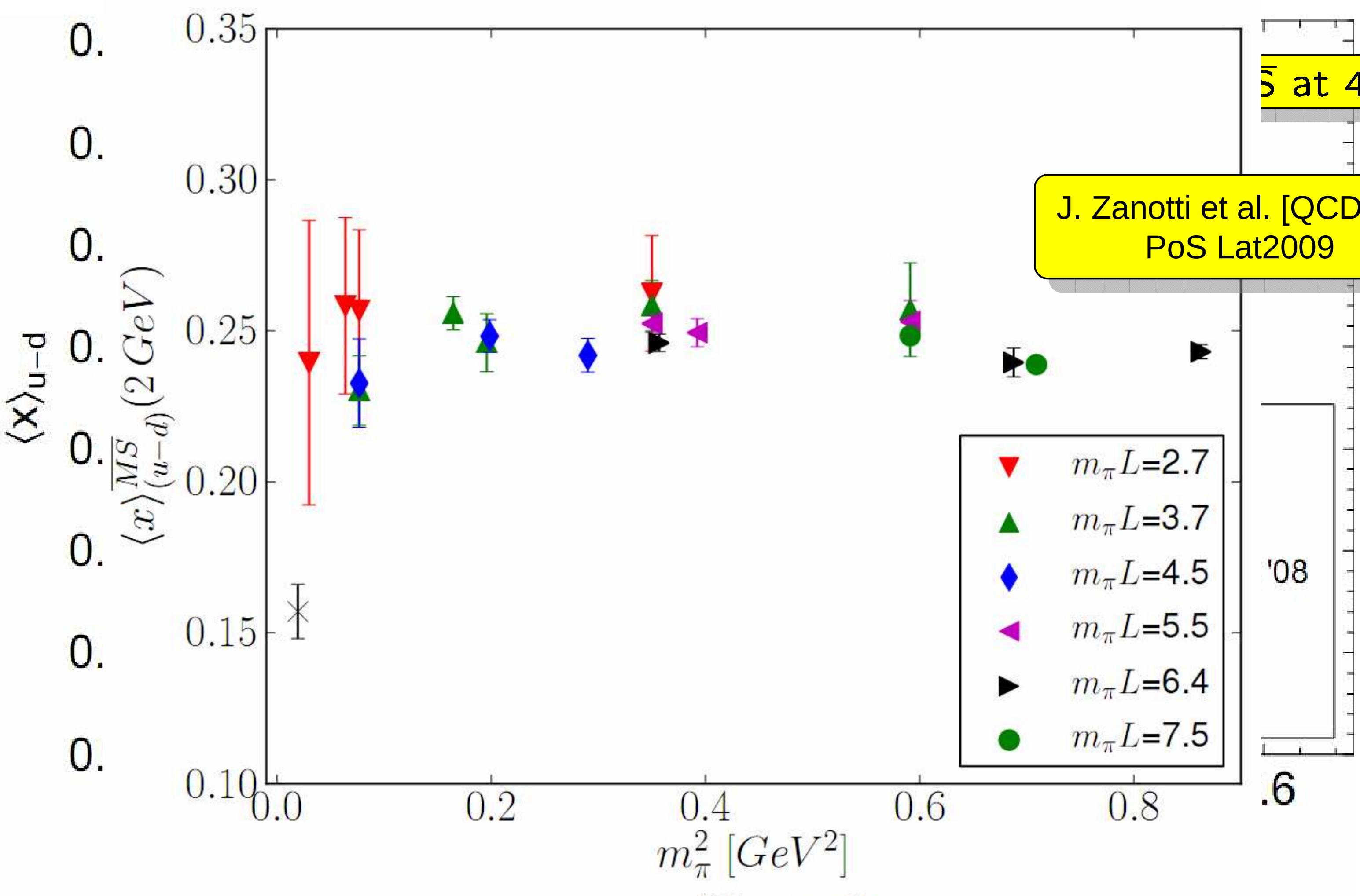
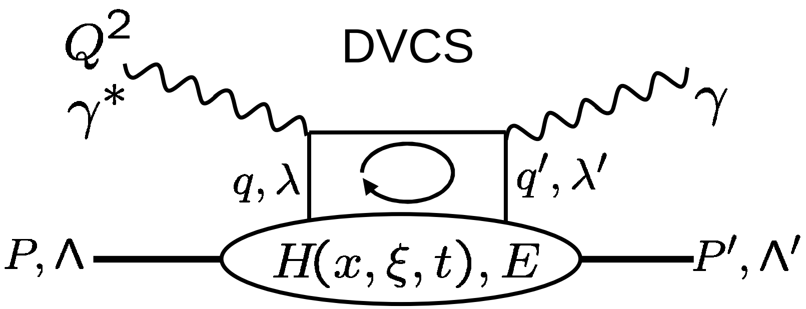


J. Zanotti et al. [QCDSF], PoS Lat2009

x^{n-1}- (Mellin-) moments of GPDs



$$\int_{-1}^{1} dx\, x^{n-1} H(x,\xi,t) = A_{n0}(t) + (2\xi)^2 A_{n2}(t) + \ldots$$

$$\int_{-1}^{1} dx\, x^{n-1} E(x,\xi,t) = B_{n0}(t) + (2\xi)^2 B_{n2}(t) + \ldots$$

$$\left\langle x^{n-1} \right\rangle_q = A_{n0}(t=0)$$

$$\left\langle x^{n-1} \right\rangle_{\Delta q} = \widetilde{A}_{n0}(t=0)$$

$$\ldots$$

$$F_1(Q^2) = A_{10}(t=-Q^2)$$

$$F_2(Q^2) = B_{10}(t=-Q^2)$$

graviton-coupling
spin-2 coupling

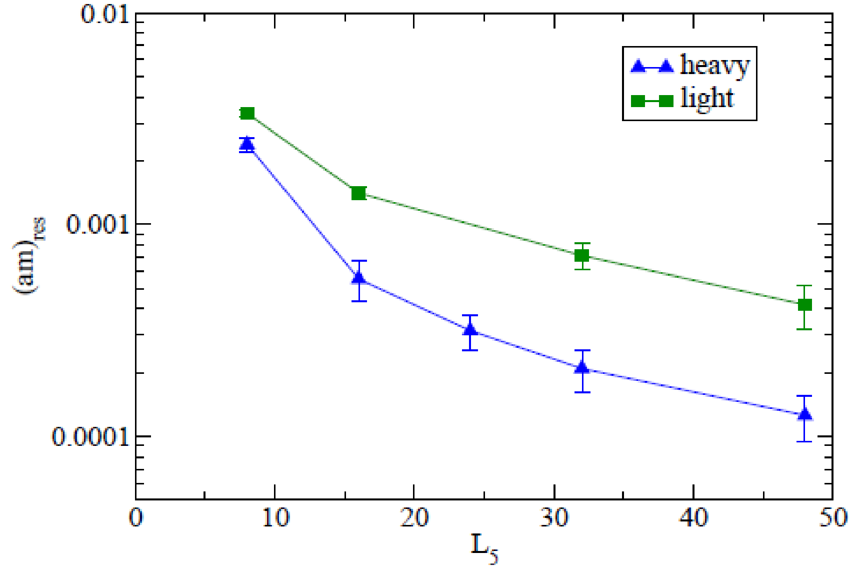
$$\langle P'|T^{\mu\nu}|P\rangle = \overline{U}(P')\Big\{\gamma^\mu \overline{P}^\mu A_{20}(\Delta^2) + \frac{i\sigma^{\mu\rho}\Delta_\rho \overline{P}^\nu}{2m_N} B_{20}(\Delta^2) + \frac{\Delta^\mu\Delta^\nu}{m_N} C_{20}(\Delta^2)\Big\} U(P)$$

$$\frac{1}{2} = S_z = \frac{1}{2}\Big(\sum_q A^q_{20}(0) + A^g_{20}(0) + \sum_q B^q_{20}(0) + B^g_{20}(0)\Big)$$

$$= \sum_q J_q + J_g = \sum_q \frac{1}{2}\Delta\Sigma_q + \sum_q L_q + J_g$$

LHPC mixed action lattice parameters

- domain - wall - fermions on a staggered "Asqtad" staggered sea ("hybrid" formalism) with HYP - smearing
- use of staggered quarks is "a matter of taste"
- $N_f = 2+1$, but only connected contributions
- $Ls = 16$, $m_{res} \leq 0.1 m_q$
- inverse lattice - spacing is $a^{-1} \approx 1.6$ GeV
- pion masses as low as 300MeV in volumes $\leq (3.5\text{fm})^3$
- one projector $\tilde{\Gamma}_{pol} = \frac{1}{4}(1+\gamma_0)(1-\gamma_5\gamma_3)$
- two sink - momenta $p' = (0,0,0), (-1,0,0)$



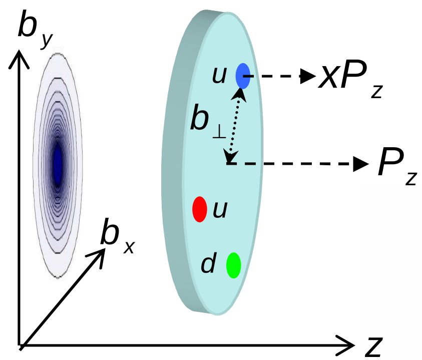
operator renormalization: $Z_{\mathcal{O}} = \frac{Z_{\mathcal{O}}^{pert}}{Z_A^{pert}} Z_A^{nonpert}$ → $\overline{MS}$ at 4 GeV^2

of „measurements“ increased by factor 8 compared to PRD 77 094502 (2008)

dataset	Ω	#	$(am)_q^{Asqtad}$	$(am)_q^{DWF}$	$(am)_\pi^{Asqtad}$	$(am)_\pi^{DWF}$	$(am)_N^{Asqtad}$	$(am)_N^{DWF}$	m_π^{DWF} [MeV]
1	$20^3 \times 32$	425	0.050/0.050	0.0810	0.4836(2)	0.4773(9)	1.057(5)	0.986(5)	758.9(1.4)
2		350	0.040/0.050	0.0478	0.4340(3)	0.4293(10)	1.003(3)	0.938(8)	682.6(1.6)
3		564	0.030/0.050	0.0644	0.3774(2)	0.3747(10)	0.930(3)	0.869(6)	595.8(1.6)
4		486	0.020/0.050	0.0313	0.3109(2)	0.3121(11)	0.854(3)	0.814(7)	496.2(1.7)
5		655	0.010/0.050	0.0138	0.2242(2)	0.2243(10)	0.779(6)	0.730(12)	356.6(1.6)
6	$28^3 \times 32$	270	0.010/0.050	0.0138		0.2220(9)		0.766(15)	352.3(1.4)
7	20^3x32	460	0.007 / 0.05			0.1842(7)			292

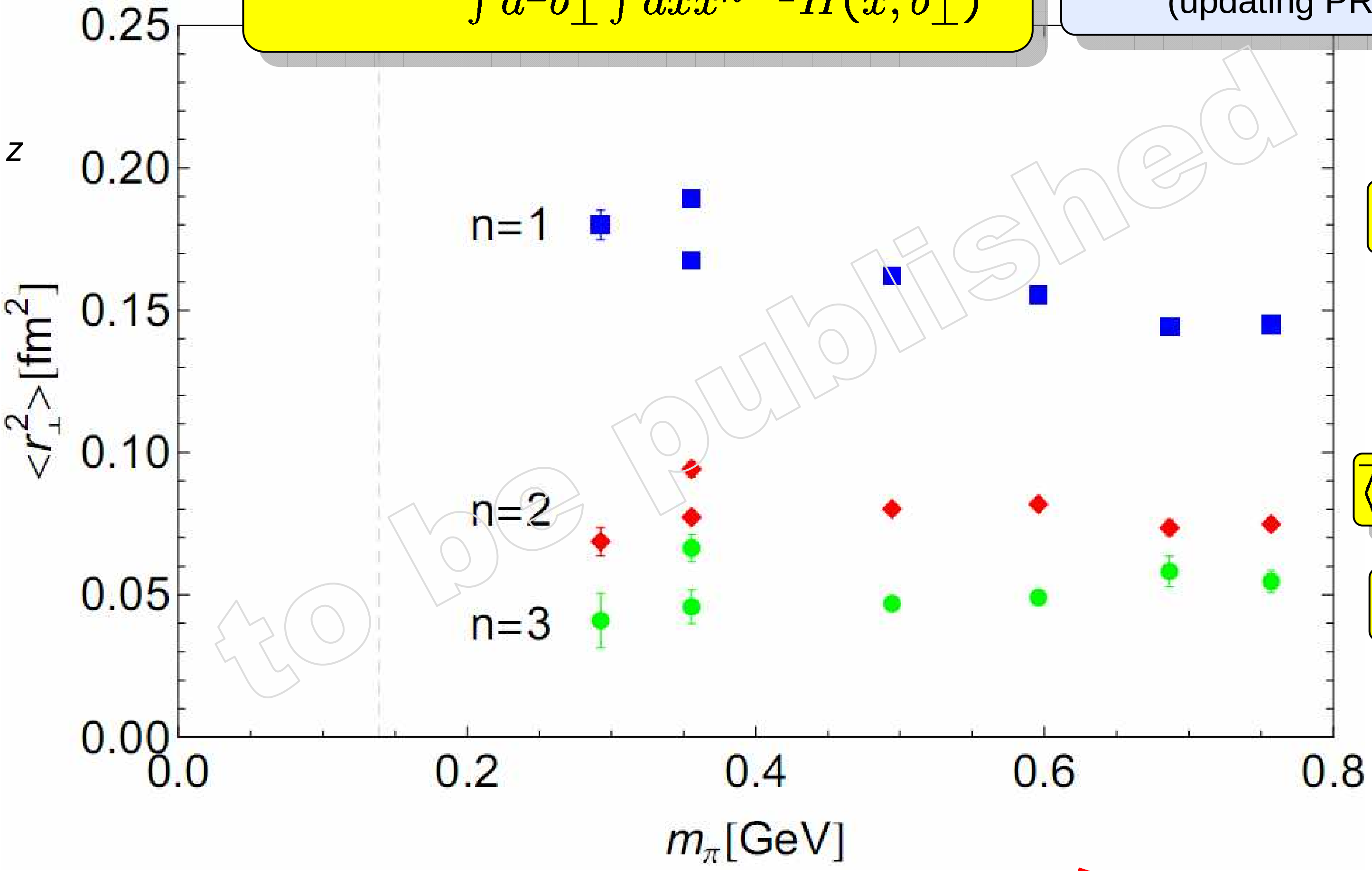
Generalized mean square radii of the nucleon

correlations in x and $b_\perp$



$$\langle r_\perp^2 \rangle_n = \frac{\int d^2b_\perp b_\perp^2 \int dx x^{n-1} H(x,b_\perp)}{\int d^2b_\perp \int dx x^{n-1} H(x,b_\perp)}$$

LHPC n_f=2+1 mixed preliminary (updating PRD 2008)



$\overline{\langle x \rangle} \approx 0.2$

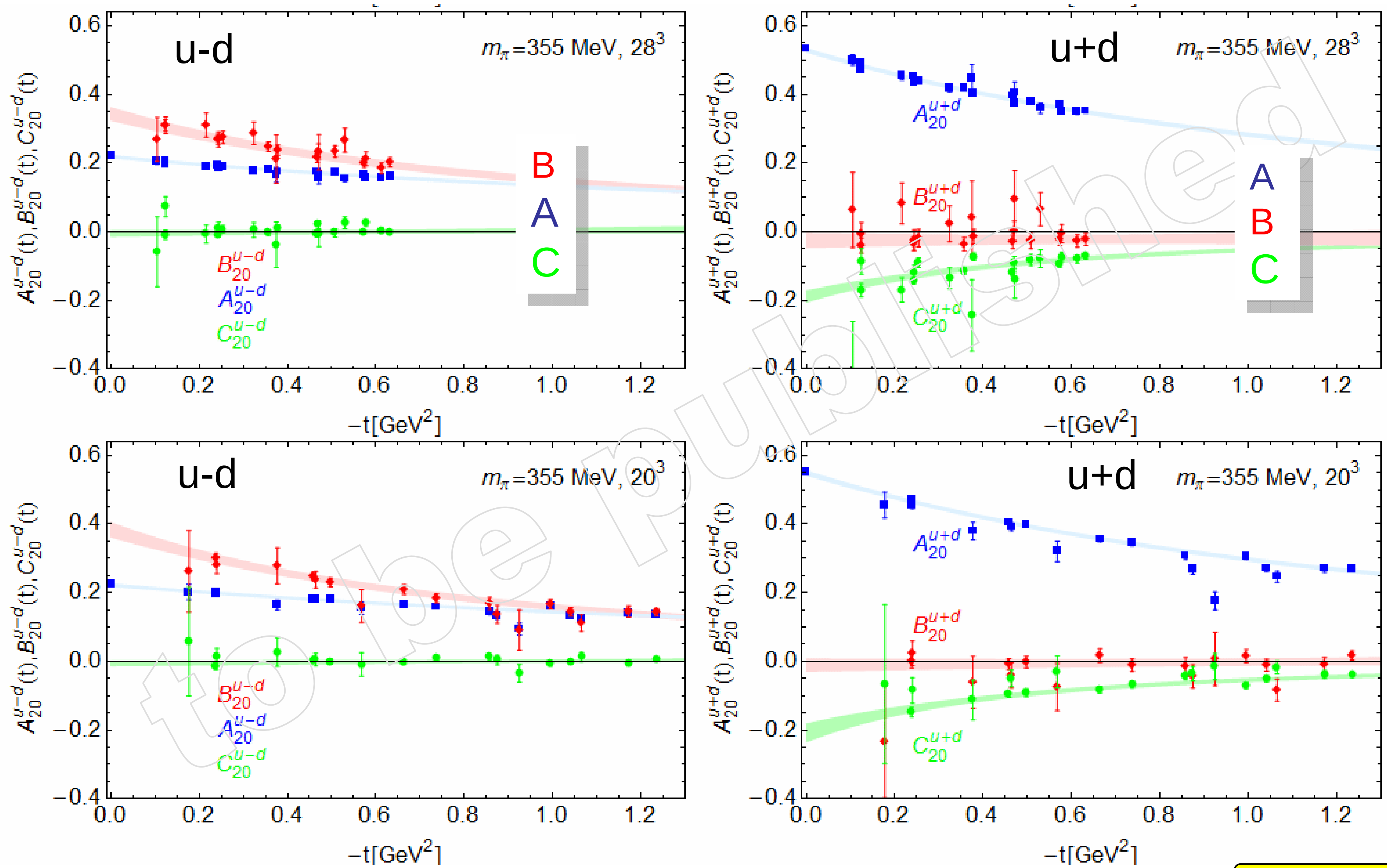
$\overline{\langle x \rangle} \approx 0.25$

$\overline{\langle x \rangle} \approx 0.4$

strong correlations in x and $b_\perp$ ↔ no factorization of GPDs in x and t ↔ ~~$H(x,0,t) = f(x)h(t)$~~

A, B, C

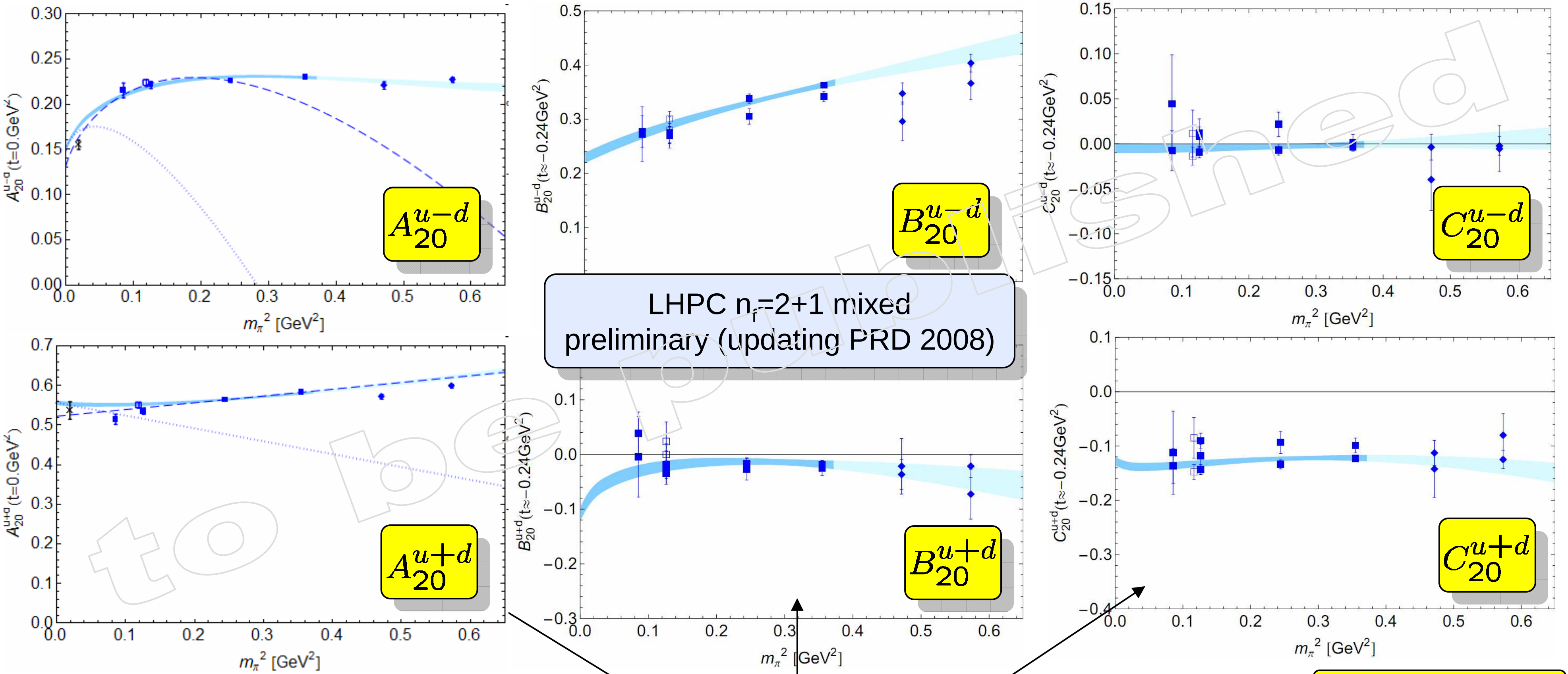
LHPC n_f=2+1 mixed; tbp
(updating PRD 2008, 0810.1933)



$\overline{MS}$ at 4 GeV^2

disconnected contributions are not included↔
only u-d is „exact"

Form factors of the energy momentum tensor



$\overline{\mathrm{MS}}$ at 4 GeV^2

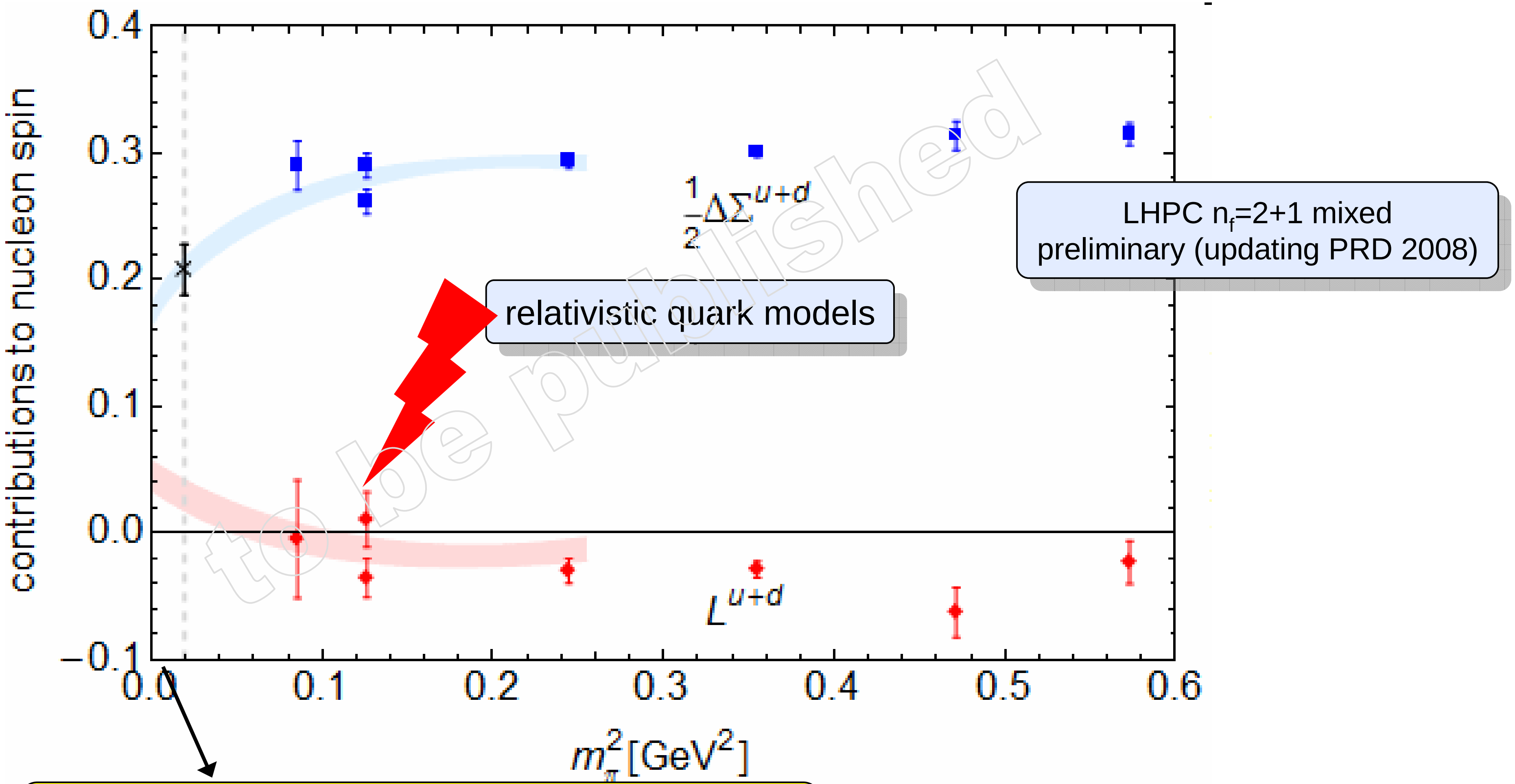
chiral extrapolation based on covariant BChPT by Dorati, Gail, Hemmert NPA 2008

$A_{20}(t,m_\pi), B_{20}(t,m_\pi), C_{20}(t,m_\pi)$ → with common parameter → $A_{20}^0 = A_{20}(t=0,m_\pi=0) = \langle x \rangle^0$

Nucleon spin structure and spin sum rule

$J_q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right)$ — $L_q \equiv J_q - \Delta\Sigma_q/2$

$\overline{\text{MS}}$ at 4 GeV^2



$\Delta\Sigma^{u+d}/2 \approx 0.21 \pm 0.006 \approx 42\%$ of $1/2$
$L^{u+d} \approx 0.030 \pm 0.012 \approx 6\%$ of $1/2$

$J^{u+d} \approx 0.238 \pm 0.008 \approx 48\%$ of $1/2$

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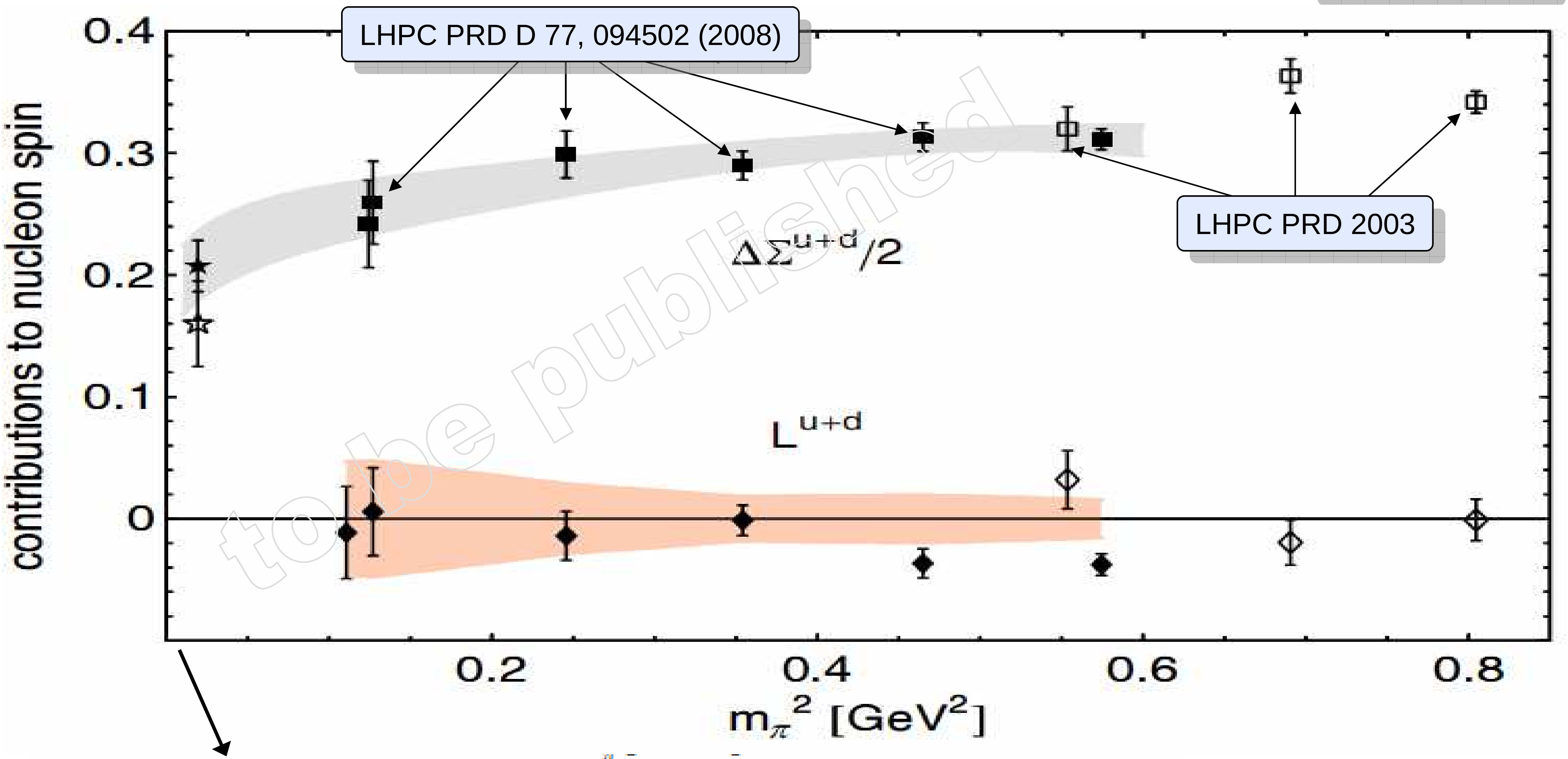

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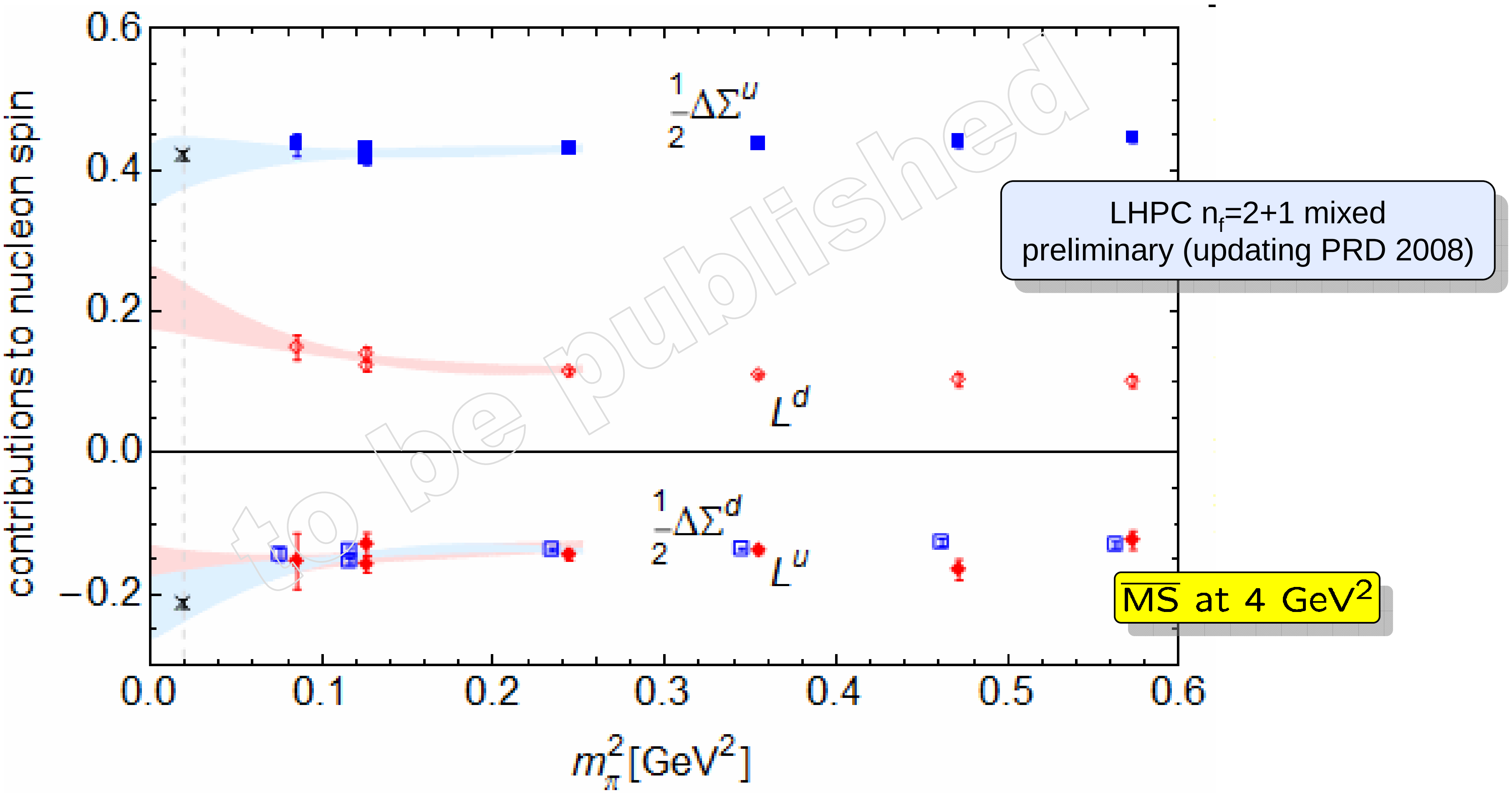


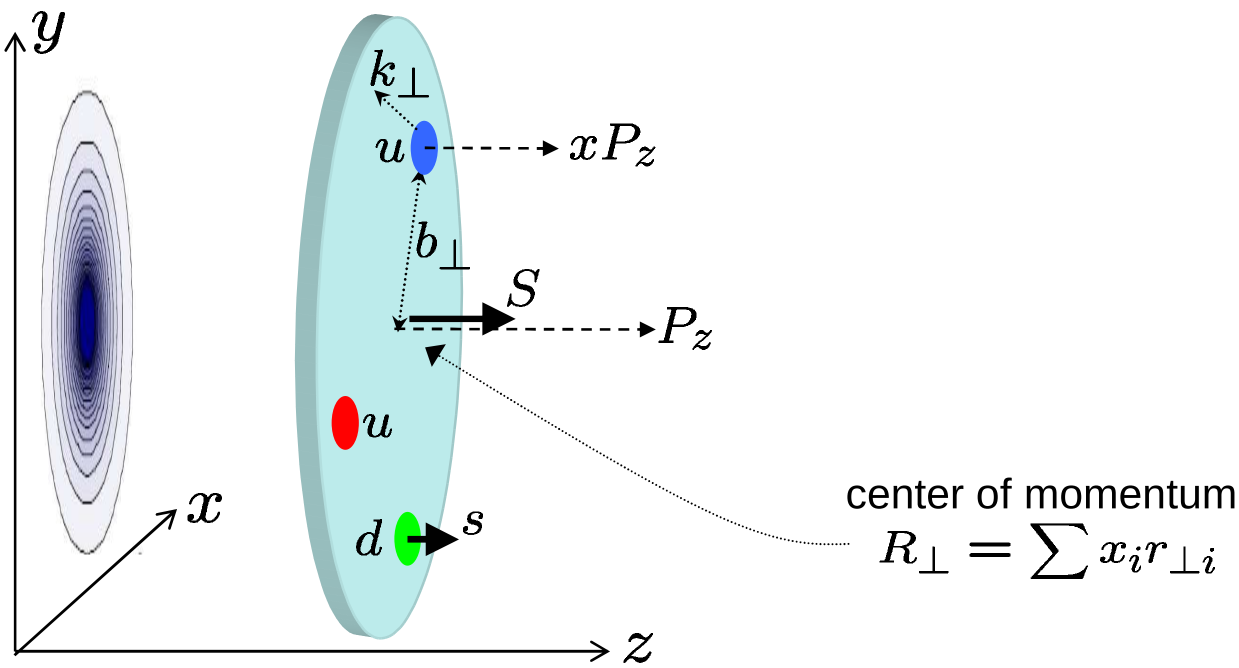
$J^u \approx 0.236 \pm 0.006 \,\hat{=}\, 48\%$ of $1/2$
$J^d \approx 0.002 \pm 0.004$

$L^d \approx -L^u \approx 0.185 \pm 0.06 \approx 36\%$ of $1/2$
$L^{u+d} \approx 0.030 \pm 0.012 \approx 6\%$ of $1/2$

pioneering lattice calculations by Gadiyak, Ji and Jung in 2001

$\kappa^{u+d} = 3\kappa^{p+n} = -0.36$

Correlations between momenta, positions, spins



generalized parton distributions (GPDs)

$H(x, b_\perp) \overset{\text{FT}}{\leftrightarrow} H(x, \xi = 0, t)$

$b_\perp \leftrightarrow -\Delta_\perp \hat{=} |t|^{\frac{1}{2}}$

Burkardt PRD 2000

Fourier-transformation

transverse momentum dependent PDFs

$f(x, k_\perp) \overset{\text{FT}}{\leftrightarrow} A(l \cdot P, l^2)$

$k_\perp \leftrightarrow l_\perp$

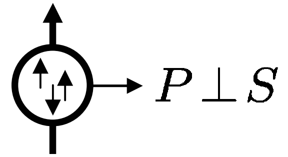
$x \leftrightarrow l \cdot P$

$\langle P | \bar{q}(l) \Gamma U q(0) | P \rangle$

Transversely polarized quarks in transversely polarized nucleons

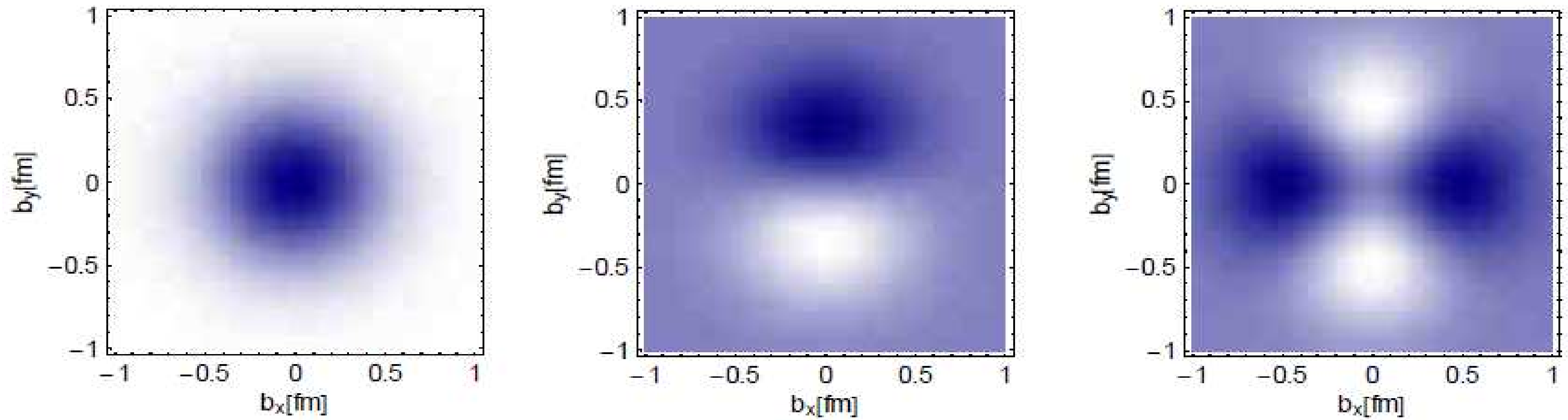
probability density
for (transversely polarized) quarks in
a (transversely polarized) proton

multipole-expansion



Diehl / PhH EPJC 2005

$$\langle P^+, 0_\perp, S_\perp | \hat{\rho}_T(x, b_\perp; s_\perp) | P^+, 0_\perp, S_\perp \rangle$$
$$= \frac{1}{2}\Big\{ H + s^i_\perp S^i_\perp \Big(H_T - \frac{1}{4m^2} \Delta_b \widetilde{H}_T \Big) - \epsilon_{ij} S^i_\perp b^i_\perp \frac{1}{m} E' - \epsilon_{ij} s^i_\perp b^i_\perp \frac{1}{m} \overline{E}'_T + s^i_\perp (2 b^i_\perp b^j_\perp - b^2_\perp \delta^{ij}) S^j_\perp \frac{1}{m^2} \widetilde{H}''_T \Big\}$$

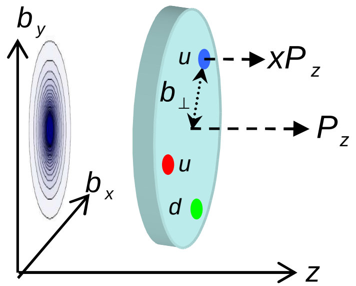


monopole

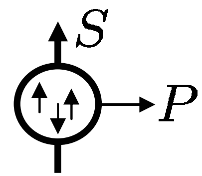
dipole

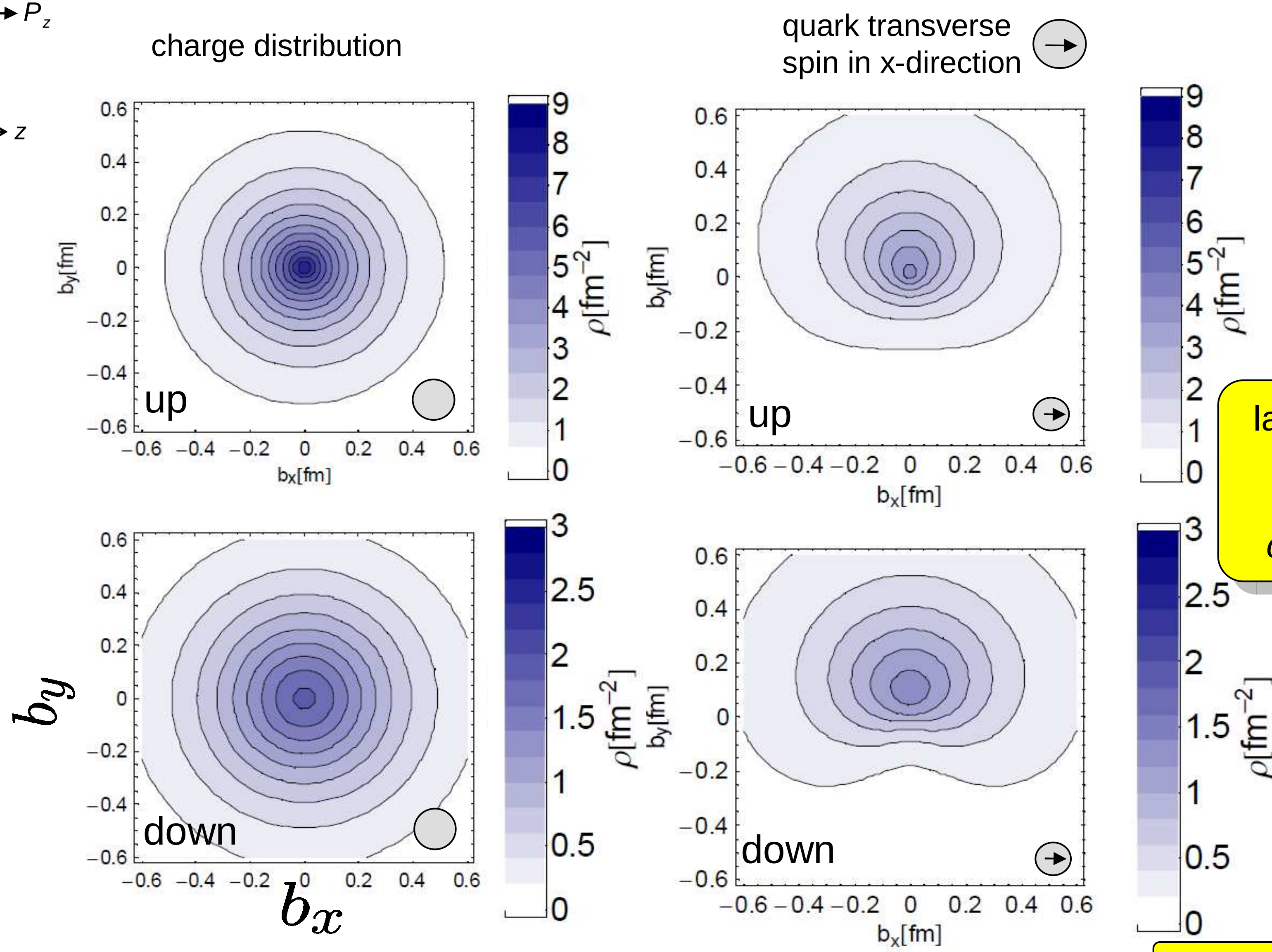
quadrupole

Transverse spin densities in the proton



QCDSF n_f=2 Clover PRL 2007





lattice calc. of quark *spin-flip couplings*

$\overline{\text{MS}}$ at 4 GeV^2

Intrinsic transverse momentum densities of the nucleon

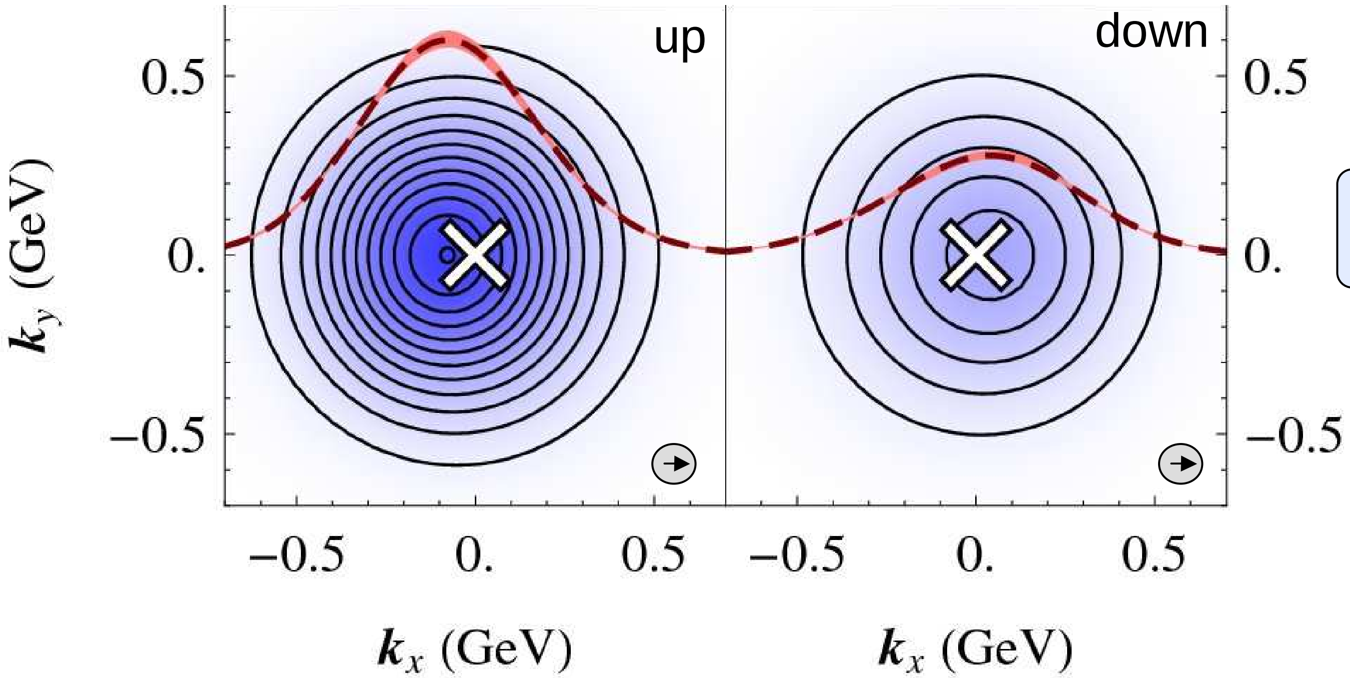
first exploratory lattice study of transverse momentum distributions of quarks in the nucleon using non-local operators

not identical to TMDs in exp.

$$\rho(x, k_\perp; \Lambda, s_\perp) = \frac{1}{2}(f_1 + \Lambda \frac{k_\perp \cdot s_\perp}{m_N} h_{1L}^\perp)$$

Diehl, PhH EPJC 44 (2005)

Boglione, Mulders PRD 60 (1999)



PhH, B. Musch et al. arXiv:0908.1283

~~$h_{1L}^\perp(x, k_\perp) \leftrightarrow$ GPDs~~

genuine effect of intrinsic transverse momentum of quarks

Challenges and Prospects

getting close to the physical pion mass – but still far from the physical point ?

continuum limit – appears smooth so far, but could turn into a rough ride

systematic studies of volume dependence promising

routinely include disconnected diagrams

full singlet renormalization/evolution ↔ mixing with gluon operators (LATTICE `09 plenary talk by Renner)

great potential in (non-trivial) pion mass dependence in combination with ChPT

requires strongly improved statistics (feasible at not too small pion masses)

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References: QCDSF PoS(LAT2006)120, 0710.1534, PRL 98 222001 (2007), PRL 2008 (0708.2249),
Zanotti et al. Pos(LAT2009), Brömmel et al EPJC 2007; LHPC PRD 77, 094502 (2008), 0810.1933;
Diehl&Hägler EPJC hep-ph/0504175;
Musch et al. 0811.1536; Musch arXiv:0907.2381; PhH, Musch et al. arXiv:0908.1283