QCD thermodynamics on the lattice: Approaching the continuum limit with physical quark masses

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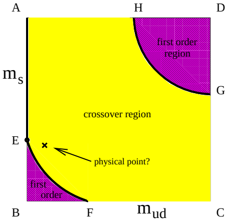
Outline

1. Introduction
2. The T>0 QCD transition
3. Discrepancy: 2006 litarature
4. New results: Wuppertal-Budapest & 'hotQCD'
5. Summary

Phase diagram and its uncertainties



physical quark masses: important for the nature of the transition

n_f=2+1 theory with m_q=0 or ∞ gives a first order transition

intermediate quark masses: we have an analytic cross over (no χPT)

F.Karsch et al., Nucl.Phys.Proc. 129 ('04) 614; G.Endrodi et al. PoS Lat'07 182('07);

de Forcrand, S. Kim, O. Philipsen, Lat'07 178('07)

continuum limit is important for the order of the transition:

n_f=3 case (standard action, N_t=4): critical $m_{ps}\approx$300 MeV

different discretization error (p4 action, N_t=4): critical $m_{ps}\approx$70 MeV

the physical pseudoscalar mass is just between these two values

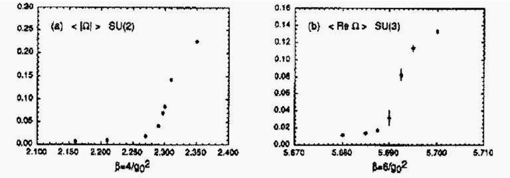
Finite-size scaling theory

problem with phase transitions in Monte-Carlo studies

Monte-Carlo applications for pure gauge theories ($V = 24^3 \cdot 4$)

existence of a transition between confining and deconfining phases:

Polyakov loop exhibits rapid variation in a narrow range of β



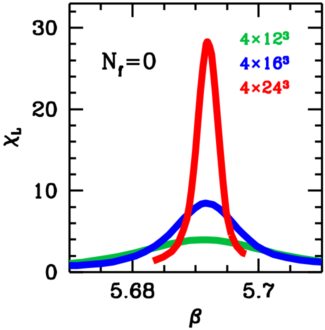
• theoretical prediction: SU(2) second order, SU(3) first order

$\Longrightarrow$ Polyakov loop behavior: SU(2) singular power, SU(3) jump

data do not show such characteristics!

Finite size scaling in the quenched theory

look at the susceptibility of the Polyakov-line
first order transition (Binder) $\Longrightarrow$ peak width $\propto$ 1/V, peak height $\propto$ V



finite size scaling shows: the transition is of first order

The nature of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

finite size scaling study of the chiral condensate (susceptibility)

$$\chi=(T/V)\partial^2\log Z/\partial m^2$$

phase transition: finite V analyticity $V\to\infty$ increasingly singular
(e.g. first order phase transition: height $\propto$ V, width $\propto$ 1/V)
for an analytic cross-over χ does not grow with V

two steps (three volumes, four lattice spacings):
a. fix V and determine χ in the continuum limit: a=0.3,0.2,0.15,0.1fm
b. using the continuum extrapolated χ_{max}: finite size scaling

How to get rid of the discretization errors?

renormalize the susceptibility the same way as the pressure

$$p(T) \propto \log Z(T{\neq}0)/V_4 - \log Z(T{=}0)/\bar{V}_4$$

$p(T)$ has a continuum limit and we can use $m_r{=}Z_m{\cdot}m$

$$\chi_r(T) = \partial^2/(\partial m_r^2)\ [\log Z(T{\neq}0)/V_4 - \log Z(T{=}0)/\bar{V}_4]$$

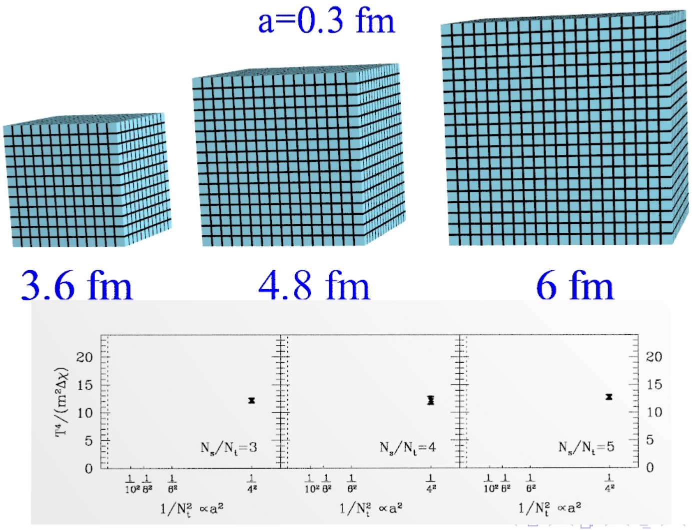
construct a quantity in continuum: Z_m drops out from $m^2\partial^2/\partial m^2$

$$\Longrightarrow m_r^2{\cdot}\chi_r(T) = m^2{\cdot}[\chi(T{\neq}0) - \chi(T{=}0)]$$

we will study a dimensionless combination of it:

$$T^4/m^2 \cdot [\chi(T \neq 0) - \chi(T = 0)]$$

Approaching the continuum limuit



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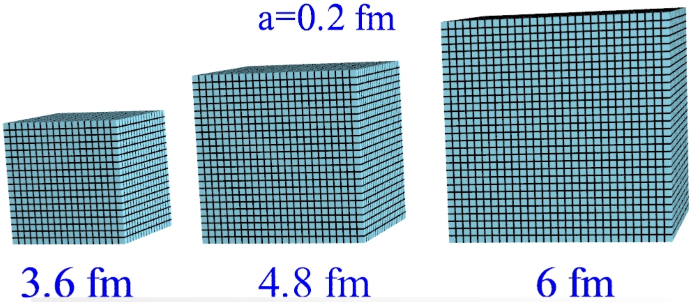

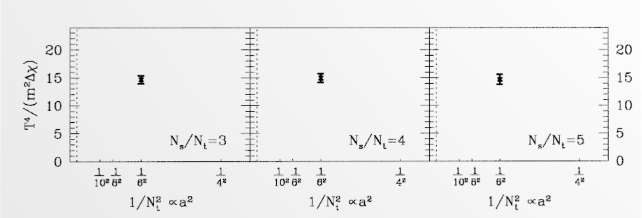

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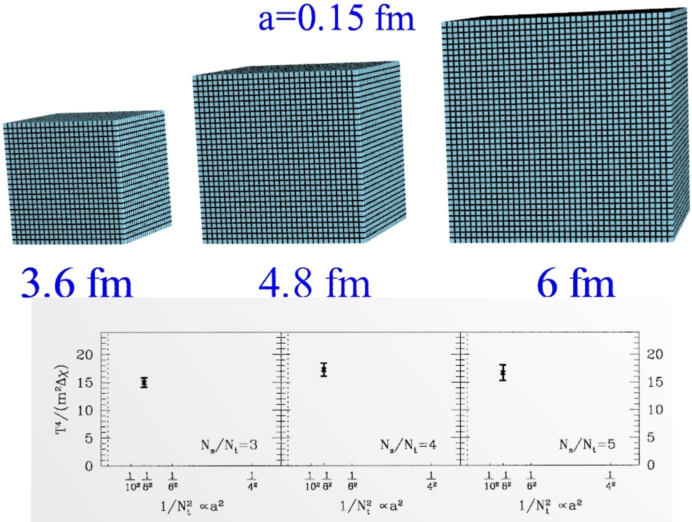
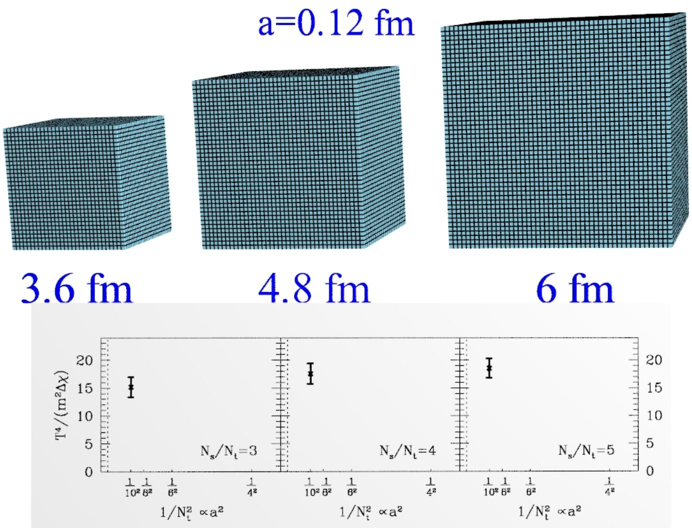
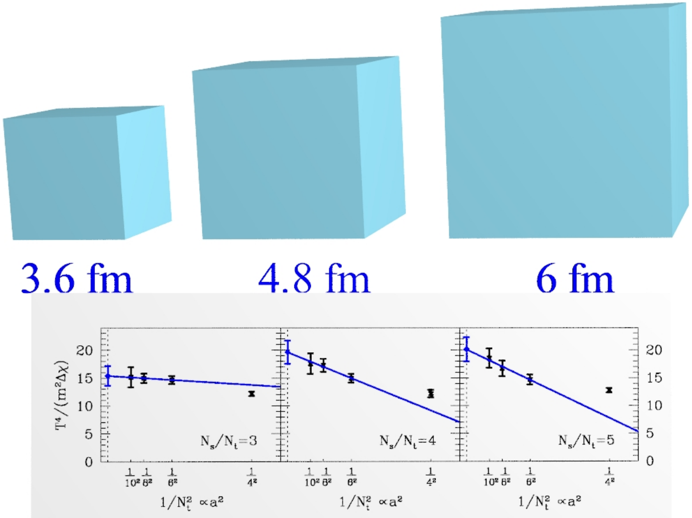


Approaching the continuum limuit

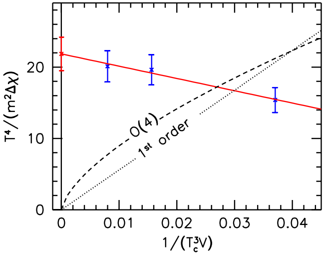


Approaching the continuum limuit



The nature of the QCD transition: result

- finite size scaling analysis with continuum extrapolated $T^4/m^2\Delta\chi$

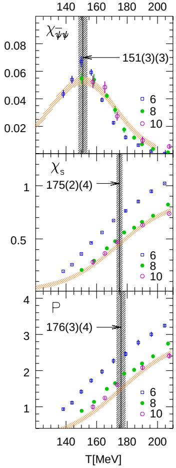


the result is consistent with an approximately constant behavior for a factor of 5 difference within the volume range
chance probability for 1/V is 10^{-19} for O(4) is $7 \cdot 10^{-13}$
continuum result with physical quark masses in staggered QCD:

the QCD transition is a cross-over

The transition temperature: results and scaling

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46



Chiral susceptibility

T_c=151(3)(3) MeV

ΔT_c=28(5)(1) MeV

Quark number susceptibility

T_c=175(2)(4) MeV

ΔT_c=42(4)(1) MeV

Polyakov loop

T_c=176(2)(4) MeV

ΔT_c=38(5)(1) MeV

Literature: discrepancies between T_c

Bielefeld-Brookhaven-Riken-Columbia Coll. (+MILC='hotQCD'):

M. Cheng et.al, Phys. Rev. D74 (2006) 054507

T_c from $\chi_{\bar{\psi}\psi}$ and Polyakov loop, from both quantities:

T_c=192(7)(4) MeV

Wuppertal-Budapest group (WB):

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46

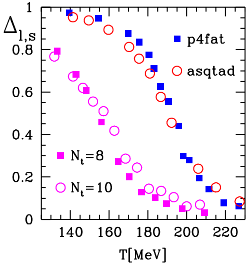
chiral susceptibility: T_c=151(3)(3) MeV
Polyakov and strange susceptibility: T_c=175(2)(4) MeV

'chiral T_c': ≈40 MeV; 'confinement T_c': ≈15 MeV difference

both groups give continuum extrapolated results with physical m_π

Literature: discrepancies between T dependences

Reason: shoulders, inflection points are difficult to define?
Answer: no, the whole temperature dependence is shifted



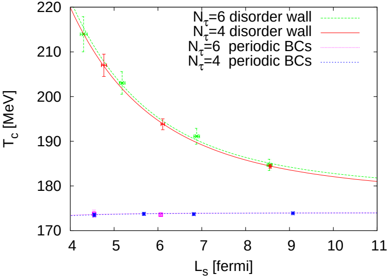
for chiral quantities ≈35 MeV; for confinement ≈15 MeV
this discrepancy would appear in all quantities (eos, fluctuations)

150 MeV transition temperature: isn't it a bit too small?
lattice works in V$\rightarrow\infty$, which gives much smaller T_c

T_c strongly depends on the geometry

nanotube-water doesn't freeze, even at hundred degrees below 0^oC

exploratory study: A. Bazavov and B. Berg, Phys.Rev. D76 (2007) 014502
use 'confined' spatial boundary conditions: more like experiments



large deviation (upto 30 MeV) from the infinite volume limit
if V$\rightarrow \infty$ is 150 MeV a 100 fm^3 system might have 170 MeV

Possible reasons for the discrepancy

"Non-lattice artefact/formulation" related reasons

a. bug in the codes

b. systematic errors are largely underestimated

"Lattice artefact/formulation" related reasons

a. the pion mass is not small enough:
'hotQCD' 230MeV $\Rightarrow$ shift of 5 MeV, WB: 135 MeV pseudogoldstone

b. not small enough lattice spacings: new 'hotQCD'/WB upto N_t=8/12

c. actually it is not QCD, what we are studying
(most large scale thermodynamics studies use staggered fermions)

Discretization errors in the transition region

we always have discretization errors: nothing wrong with it as long as

a. result: close enough to the continuum value (error subdominant)
b. we are in the scaling regime (a^2 in staggered)

various types of discretization errors $\Rightarrow$ we improve on them (costs)

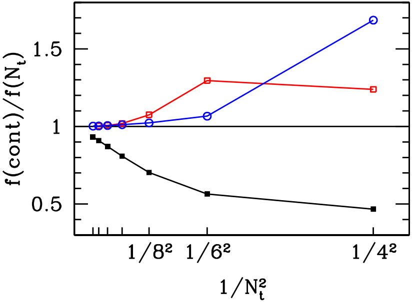
we are speaking about the transition temperature region
interplay between hadronic and quark-gluon plasma physics
smooth cross-over: one of them takes over the other around T_c

both regimes (low T and high T) are equally important
improving for one: T$\gg T_c$, doesn't mean improving for the other: T$<T_c$

example: 'expansion' around a Stefan-Boltzman gas (van der Waals)
for water: it is a fairly good description for T$\gtrsim$300^o
claculate the boiling point: more accuracy needed for the liquid phase

Examples for improvements, consequences

how fast can we reach the continuum pressure at T=∞?



p4 action is essentially designed for this quantity T$\gg$$T_c$

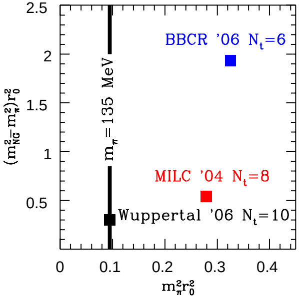
asqtad designed mostly for T=0 physics (but good at high T, too)

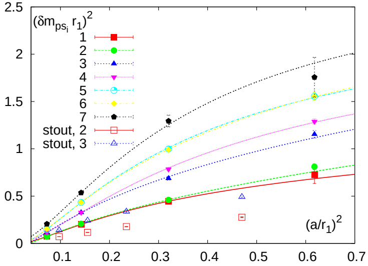
stout-smeared one-link converges slower but in the a^2 scaling regime (e.g. extrapolation from N_t=8,10 provides a result within about 1%)

Chiral symmetry breaking and pions

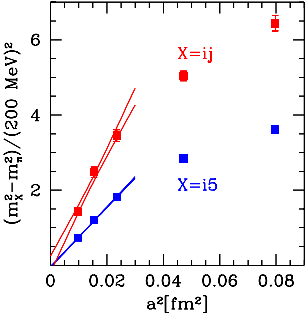
transition temperature for remnant of the chiral transition:
balance between the chirally broken and chirally symmetric sectors
chiral symmetry breaking: 3 pions are the pseudo-Goldstone bosons

staggered QCD: 1 pseudo-Goldstone instead of 3 (taste violation)
staggered lattice artefact ⇒ disappears in the continuum limit
WB: stout-smeared improvement is designed to reduce this artefact





Scaling for the pion splitting



scaling regime is reached if a^2 scaling is observed

asymptotic scaling starts only for N_t>8 ($a \lesssim 0.15$ fm): two messages

a. N_t=8,10 extrapolation gives 'p' on the ≈1% level: good balance

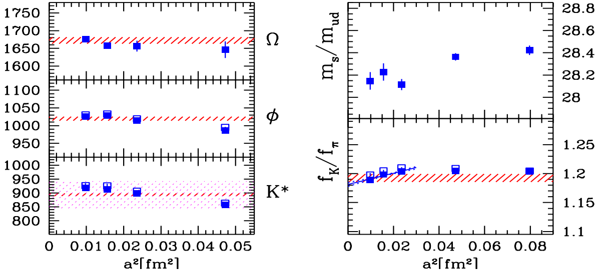
b. stout-smeared improvement is designed to reduce this artefact

most other actions need even smaller 'a' to reach scaling

Setting the scale in lattice QCD

in lattice QCD we use g,m_{ud} and m_s in the Lagrangian ('a' not)
measure e.g. the vacuum mass of a hadron in lattice units: $M_\Omega a$
since we know that M_Ω=1672 MeV we obtain 'a' and T=1/N_ta

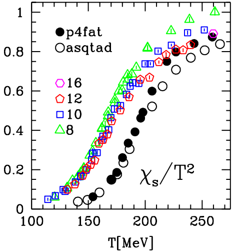
Y.Aoki et al. [Wuppertal-Budapest Collaboration] arXiv:0903.4155



independently which quantity is taken (we used physical masses)
⇒ one obtains the same '*a*' and T, result is safe

T>0 results: strange susceptibility

Compare the Wuppertal-Budapest results [Y.Aoki et al. JHEP 0906:088,2009] with 'hotQCD'

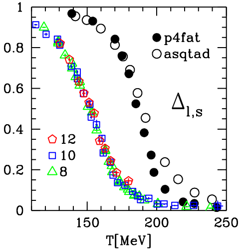


'hotQCD' results are on N_t=8, WB results are on N_t=8,10,12,(16)
'hotQCD': results with two different actions are almost the same
WB: for large T one extrapolates according to the known a^2 behaviour
WB: no change in the lattice results compared to our 2006 paper
note, that the experimental value of f_K decreased by 3% since 2006

about 20 MeV difference between the results

T>0 results: chiral condensate

Compare the Wuppertal-Budapest results [Y.Aoki et al. JHEP 0906:088,2009] with 'hotQCD'



'hotQCD' results are on N_t=8, WB results are on N_t=8,10,12
'hotQCD': results with two different actions are almost the same
WB: no lattice spacing dependence observed for N_t=8,10,12
WB: no change in the lattice results compared to our 2006 paper

about 35 MeV difference between the results

transition temperatures for various observables

	$\chi_{\bar{\psi}\psi}/T^4$	$\chi_{\bar{\psi}\psi}/T^2$	$\chi_{\bar{\psi}\psi}$	$\Delta_{l,s}$	L	χ_s
WB'09	146(2)(3)	152(3)(3)	157(3)(3)	155(2)(3)	170(4)(3)	169(3)(3)
WB'06	151(3)(3)	-	-	-	176(3)(4)	175(2)(4)
BBCR	-	192(4)(7)	-	-	192(4)(7)	-

renormalized chiral susceptibility, renormalized chiral condensate
Polyakov loop and strange quark number susceptibility

no change compare to our 2006 data (errors are reduced)
note, that the experimental value of f_K decreased by 3% since 2006
Particle Data Group now gives f_K=155.5(2)(8)(2) MeV (error 0.5%)

r_0 is not directly measurable:

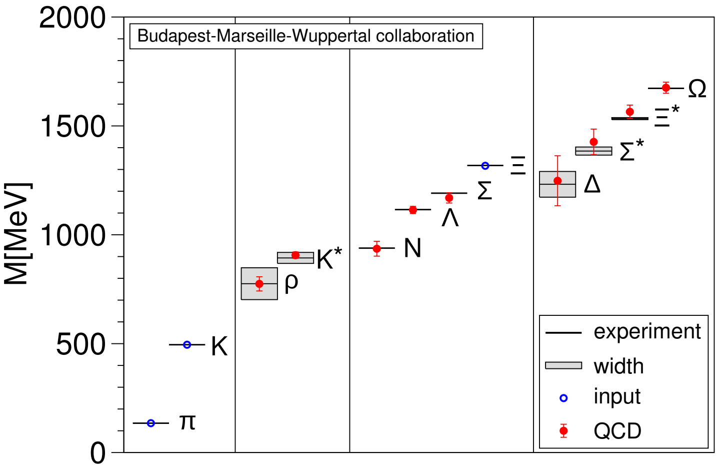
ETM:0.444(4) fm, QCDSF:0.467(6) fm,

HPQCD&UKQCD:0.469(7) fm, PACS-CS:0.492(6)(+7) fm

Summary

- the T>0 QCD transition is an analytic cross-over
- new (2009) results for the transition temperature
- three major improvements since 2006
  a. at T=0 all simulations are done with physical quark masses
  b. to verify that the results are independent of the scale setting we use 5 experimentally well-known quantites: f_K,f_π,m_{K^*},m_Ω,m_Φ
  c. even smaller lattice spacings: N_t=12 (in one case N_t=16)
- all findings are in complete agreement with our 2006 results
- Particle Data Group reduced the experimental value of f_K: 3%
- discrepancy between Wuppertal-Budapest & 'hotQCD' results
  a. for the remnant of the deconfinement transition: about 20 MeV
  b. for the remnant of the chiral transition: about 35 MeV
  $\Rightarrow$ finding the reason: task for the future
- Wilson fermions: theoretically cleaner option

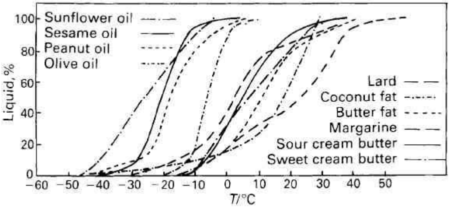
Final result for the hadron spectrum

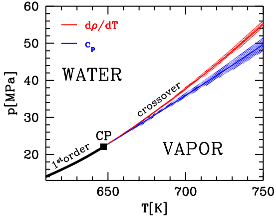


The nature of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

analytic transition (cross-over) $\Rightarrow$ it has no unique T_c:
examples: melting of butter (not ice) & water-steam transition





above the critical point c_p and $d\rho/dT$ give different T_cs.
QCD: chiral & quark number susceptibilities or Polyakov loop
they result in different T_c values $\Rightarrow$ physical difference