FAIR LATTICE QCD DAYS

GSI DARMSTADT

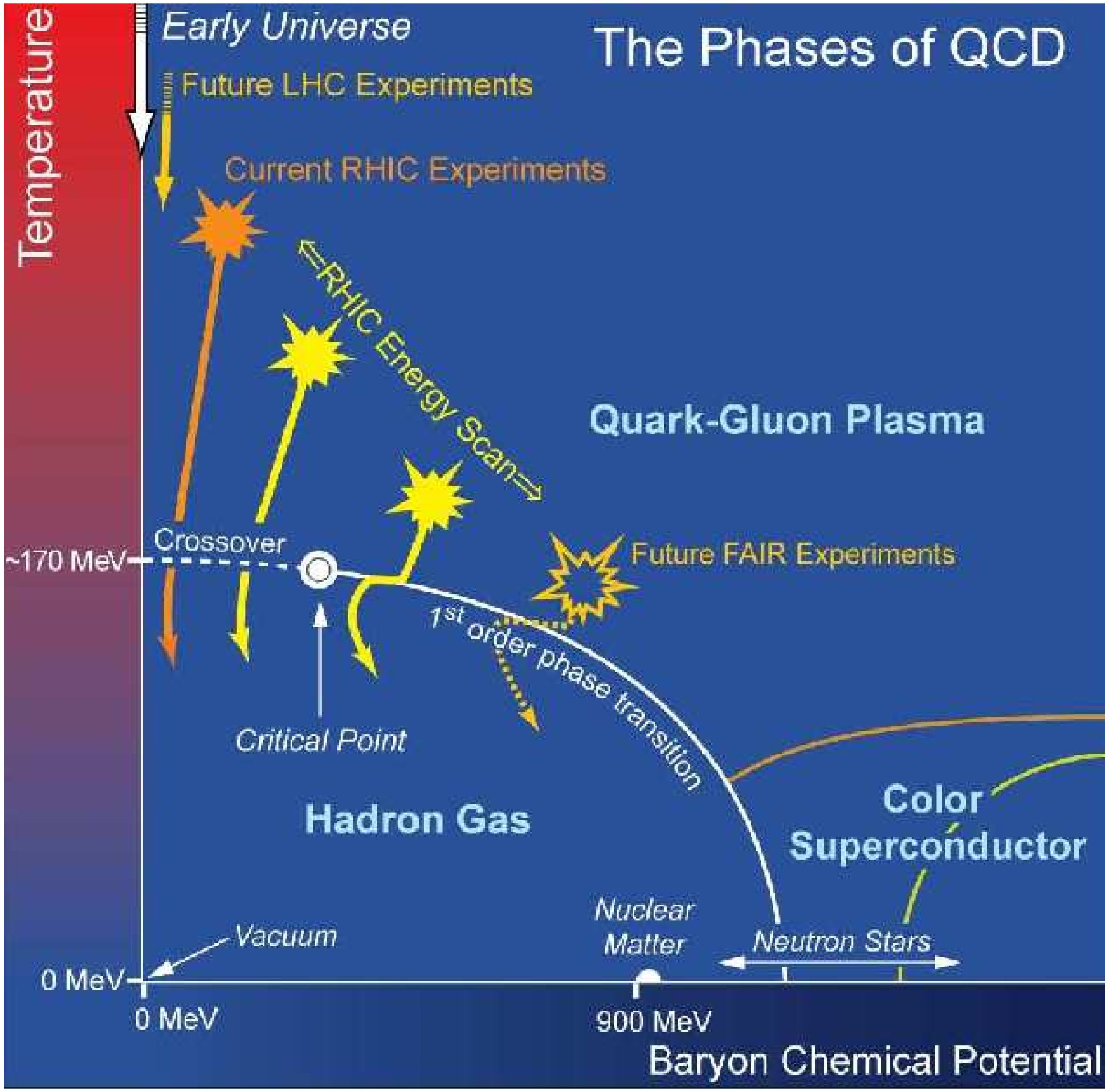
November 23–24 2009

Lattice QCD Thermodynamics with twisted mass Wilson fermions

Maria Paola Lombardo

The tmfT Collaboration

- E.M. Ilgenfritz, K. Jansen, M. P. Lombardo, M. Müller-Preussker, M. Petschlies, O. Philipsen and L. Zeidlewicz, **Phys.Rev.D80**:094502,2009
- M. Müller-Preussker, E.M. Ilgenfritz, K. Jansen,,M. P. Lombardo, O. Philipsen, L. Zeidlewicz, M. Kirchner, M. Petschlies, D. Schulze and C.Urbach PoS **LAT2009**
- E.M. Ilgenfritz, K. Jansen, M. P. Lombardo, M. Müller-Preussker, M. Petschlies, O. Philipsen and L. Zeidlewicz, PoS **LAT2008** (2008) 206
- E. M. Ilgenfritz, M. Müller-Preussker,M. Petschlies, A. Sternbeck, K. Jansen, M. P. Lombardo, O. Philipsen, and Lars Zeidlewicz PoS **LAT2007** (2007) 238
- E. M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, K. Jansen, I. Wetzorke, M. P. Lombardo and O. Philipsen, PoS **LAT2006** (2006) 140



From NSAC Long Range Plan

PLAN

- INTRODUCTION
- TWISTED MASS : MODEL ANALYSIS
- RESULTS for $N_t = 8$: Fixed Twisted Mass
- RESULTS for $N_t = 8$: Twisted Mass at Work
- TOWARDS THE CHIRAL/CONTINUUM LIMIT :
  Nt=10, Nt=12 Maximal Twist:
  Preliminary
- COMPARISON WITH OTHER RESULTS, WILSON and STAGGERED
- PERSPECTIVES

TWO FLAVORS: REASONS OF INTEREST

Useful to complete the QCD Phase diagram

Universality Issue

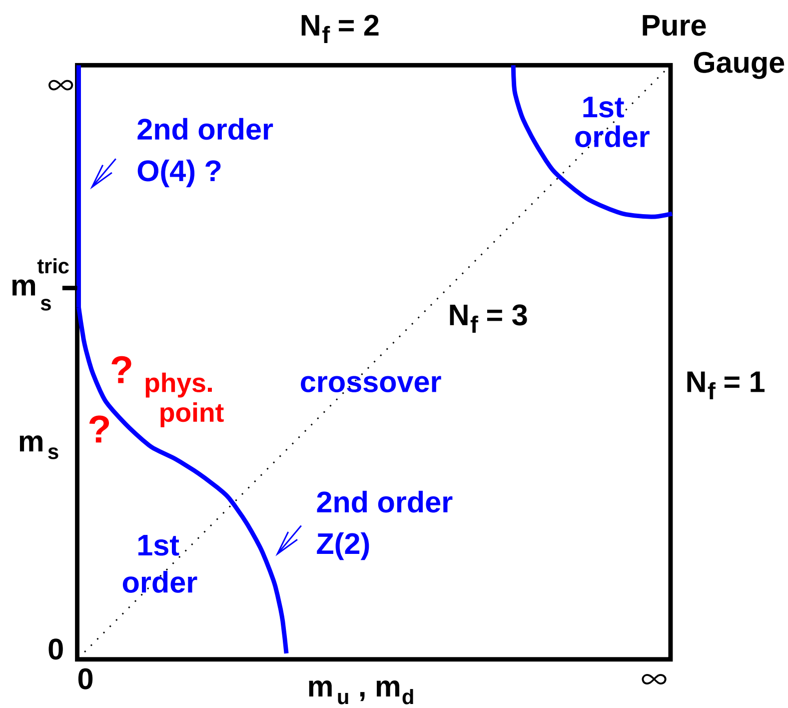
Physical Crossover Temperature :

quantitative estimate of N_f dependence

$N_f = 2$ relevant for $N_f = 2 + 1$

Phase transitions of QCD in the up=down, strange plane

This talk : $N_f = 2$



	$U(1)_A$ anomaly	suppressed anomaly at T_c
QCD	$SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$	$U(N_f)_L \otimes U(N_f)_R \to U(N_f)_V$
$N_f = 1$	crossover or first order	O(2) or first order
$N_f = 2$	O(4) or first order	$U(2)_L \otimes U(2)_R / U(2)_V$ or first order
$N_f \geq 3$	first order	first order
aQCD	$SU(2N_f) \to SO(2N_f)$	$U(2N_f) \to O(2N_f)$
$N_f = 1$	O(3) or first order	U(2)/O(2) or first order
$N_f = 2$	SU(4)/SO(4) or first order	first order

Universality class of QCD transition with N_f light quarks

Pisarski, Wilczek; original discussion
Basile, Pelissetto, Vicari 2005; RG analysis

STATUS

The Universality Class of Nf=2 QCD from the Lattice:

Staggered Fermions:

- *O(2) or O(4) with Nt=8 but scaling window very narrow - other behaviours cannot be ruled out* Kogut Sinclair 2004
- *O(2) at strong coupling very high precision low masses* Chandrasekharan Strouthos 2005
- *First order (O(2) / O(4) ruled out)* Pisa Group, 2000-2008
- *O(4) scaling visible with largish masses,* T. Mendez 2005–2008

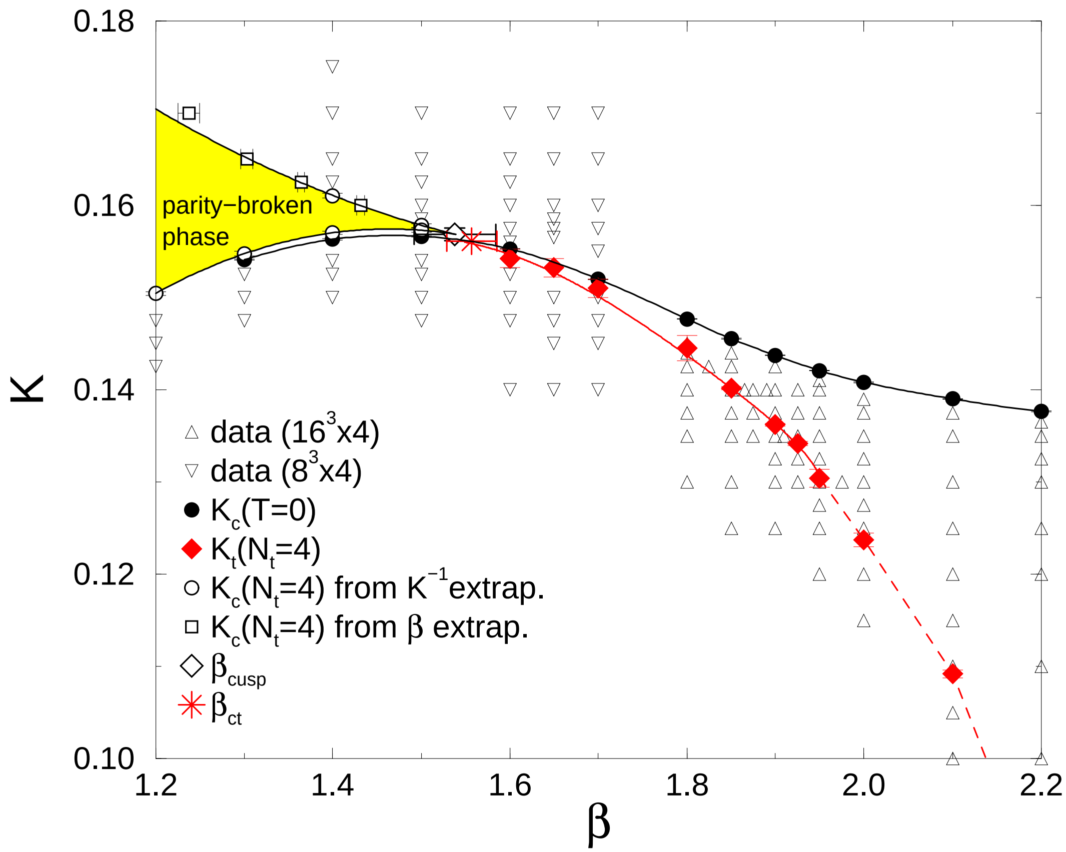
Wilson fermions:

- *Apparently compatible with O(4) scaling Nt=4-* CP-Pacs 2001

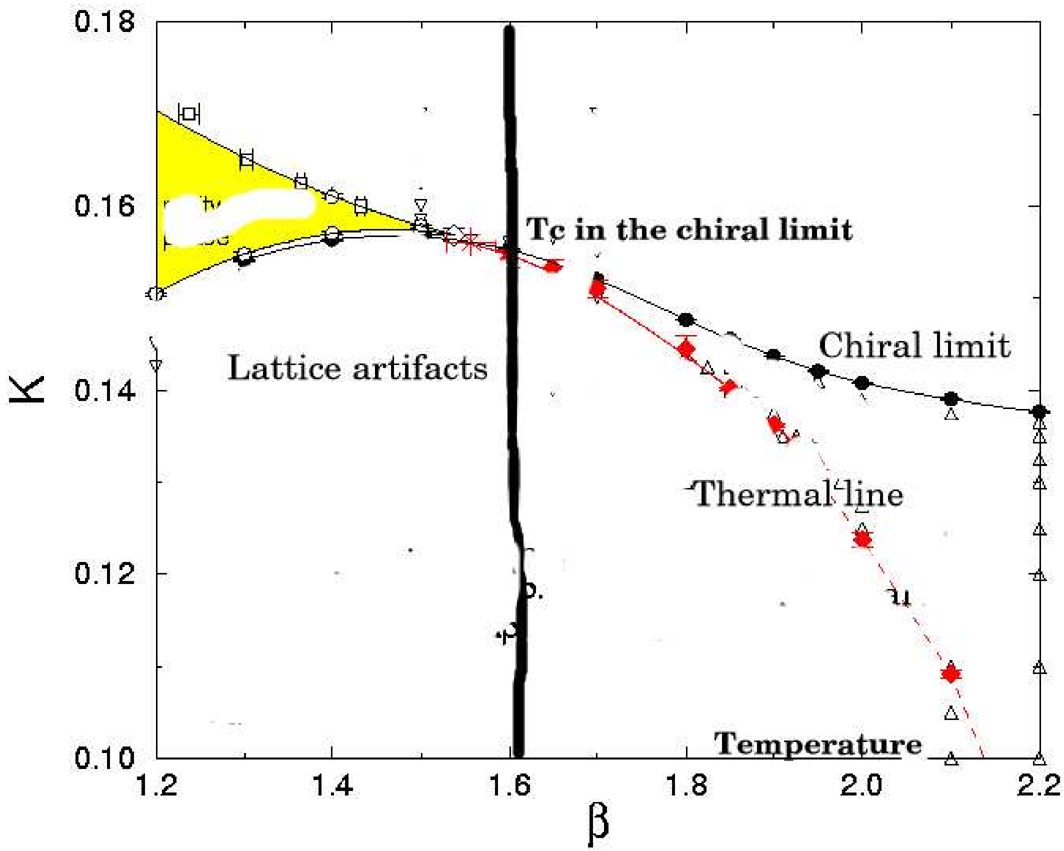
The Pseudocritical Temperature of QCD : E. Laermann and Z. Fodor talks

Phases of Wilson Fermions :Early Lattice results

A. Ali Khan *et al.* [CP-PACS Collaboration], Phys. Rev. D 63, 034502 (2001); Phys. Rev. D 64, 074510 (2001)



Phases of Wilson Fermions : Sketchy view



Twisted Mass QCD [**ETMC Collaboration**]

- Fermion sector

$$S_q = \sum_x \left\{ (\overline{\chi}_x [\mu_\kappa + i\gamma_5 \tau_3 a\mu] \chi_x) - \frac{1}{2} \sum_{\mu=\pm1}^{\pm4} \left(\overline{\chi}_{x+\hat{\mu}} U_{x\mu} [r + \gamma_\mu] \chi_x \right) \right\} \ ,$$

$\mu_\kappa \equiv am_0 + 4 = 1/2\kappa$, am_0 the bare "untwisted" quark mass in lattice units and μ the twisted quark mass;

- Gauge sector: Symanzik improved

$$S_g = \beta \sum_x \left(c_0 \sum_{\mu<\nu;\, \mu,\nu=1}^{4} \left\{ 1 - \frac{1}{3} \operatorname{Re} U_{x\mu\nu}^{1\times1} \right\} + c_1 \sum_{\mu\neq\nu;\, \mu,\nu=1}^{4} \left\{ 1 - \frac{1}{3} \operatorname{Re} U_{x\mu\nu}^{1\times2} \right\} \right) \ ,$$

$U_{x\mu\nu}^{1\times2}$ is the planar rectangular (1×2), and we use tree-level Symanzik improved gauge action (tlSym) $c_1 = -1/12$.

TWISTED MASS : MODEL ANALYSIS

Twisted Mass : A model study Mike Creutz,T=0 1996, Finite T, 2007

Effective potential for Wilson fermions

$$V(\vec{\pi},\sigma,L) = \lambda(\sigma^2+\vec{\pi}^2-v^2)^2+\alpha_0(K^2-K_c(\beta)^2)\sigma+\alpha_1\sigma^2+m_t\pi_3$$

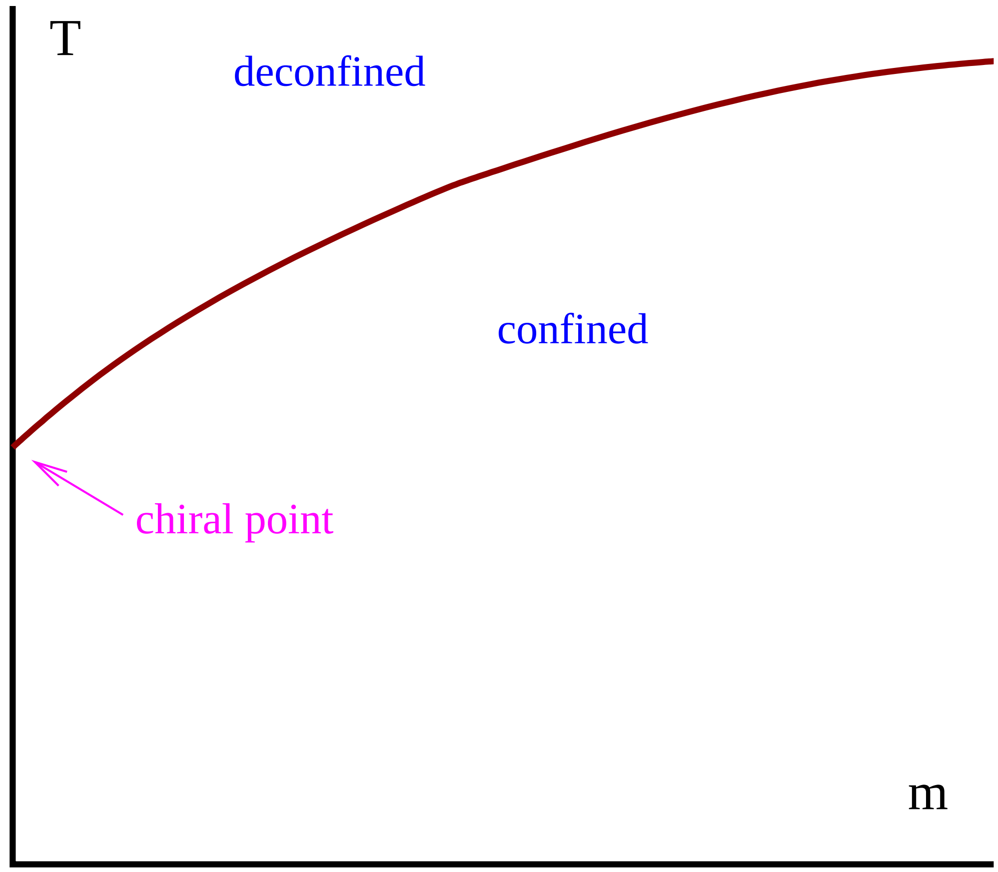
- The first line is the "linear" sigma model with $O(4)$ symmetry.
- The second line is the mass term.
- The third term is a chiral symmetry breaking **lattice artifact**
- The m_t term is the twisted mass. Without the α_1 term this can be rotated into the α_0 mass term along the curve of constant $m_t^2+\alpha_0^2(K^2-K_c^2)^2$.
- **If $K=K_c$ lattice artifacts are reduced**

$$m_q^2 = m_t^2+\alpha_0^2(K^2-K_c^2)^2+O(a)$$

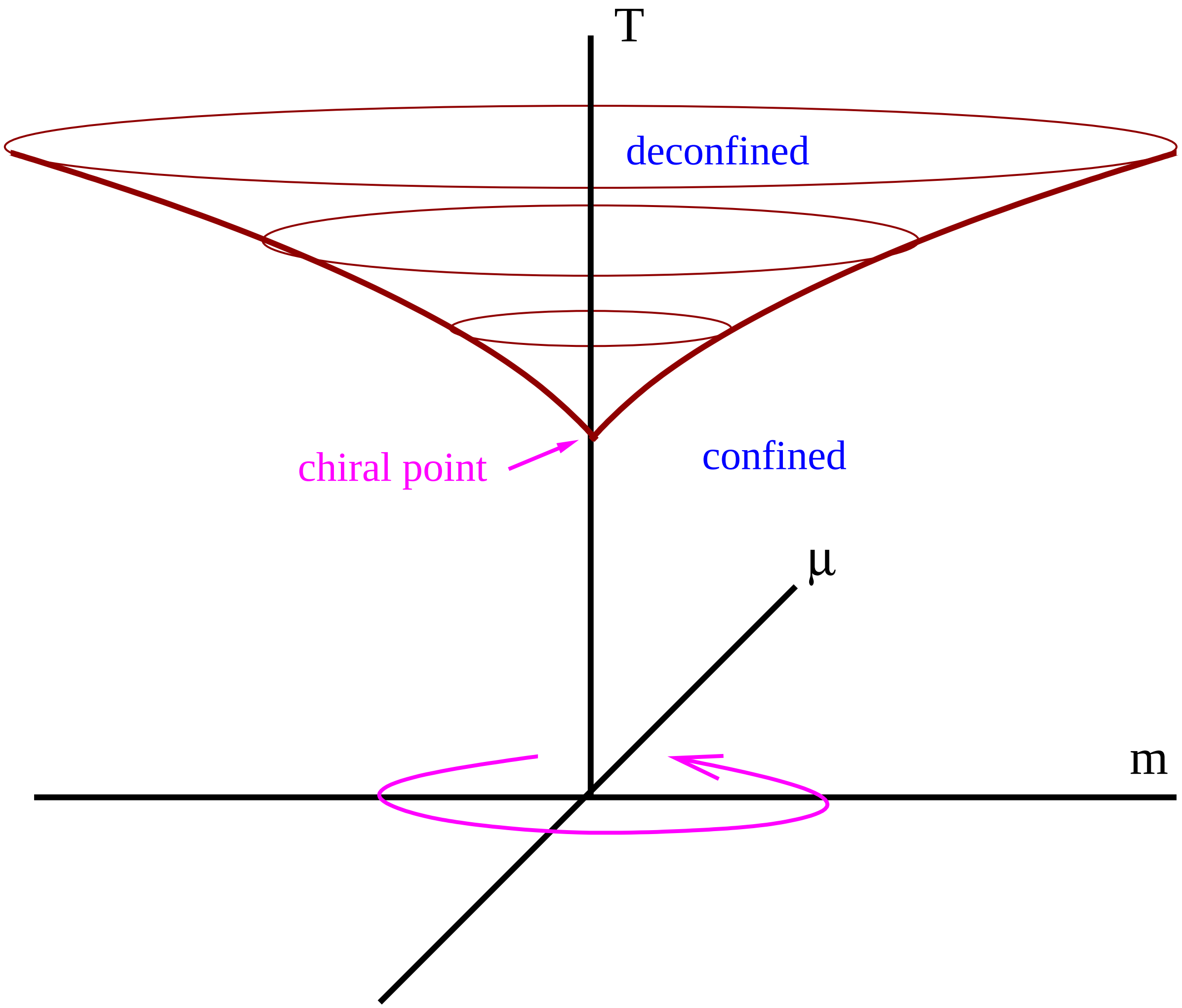
.

Twisted Mass and Finite Temperature - M. Creutz 2007

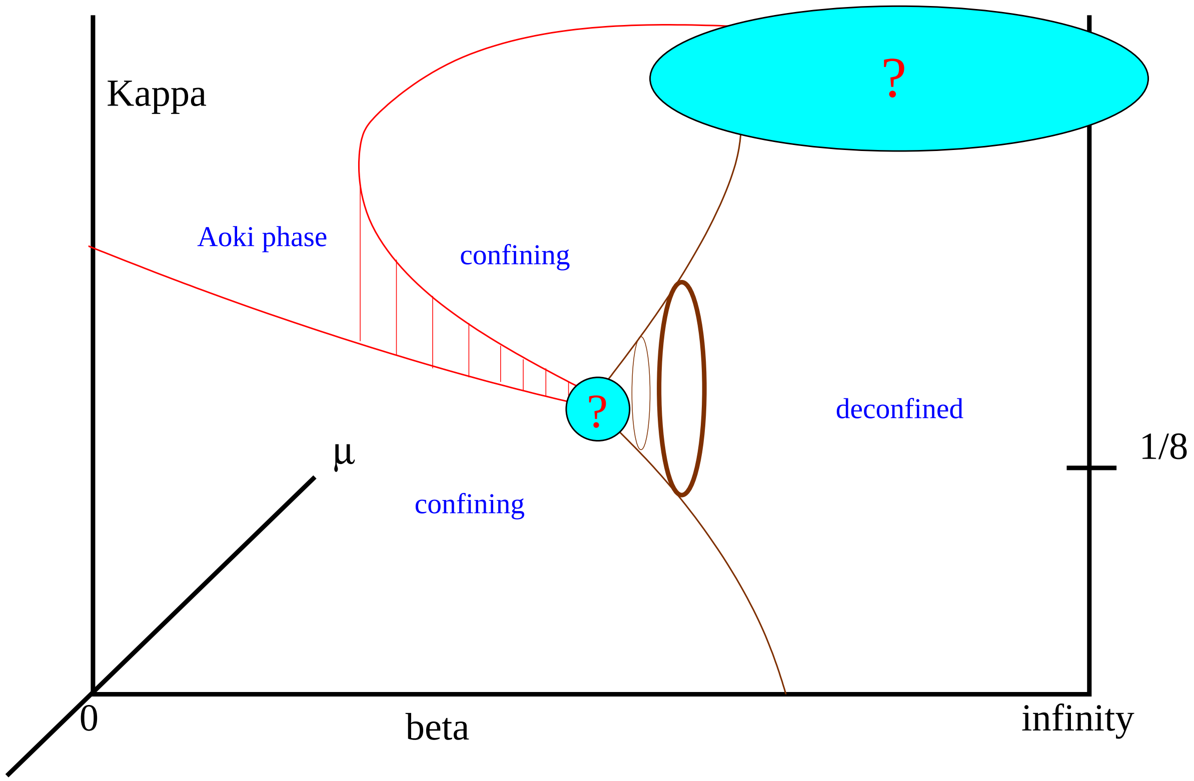
Without twisted mass:



With twisted mass: the 'cone'



A fixed N_t section



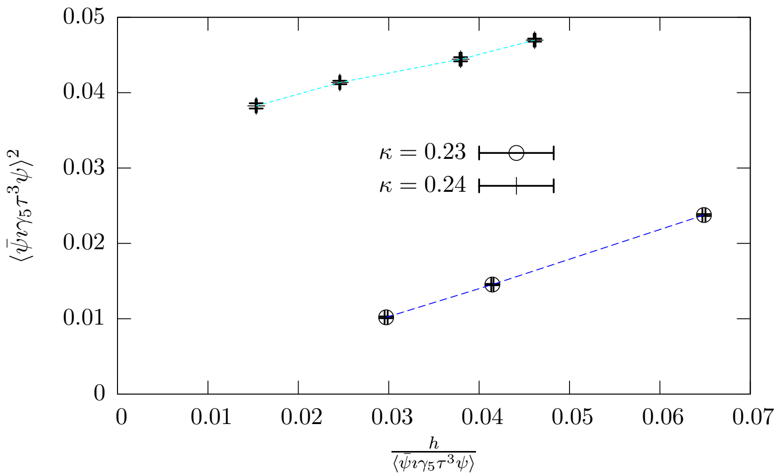
RESULTS FOR $N_t = 8$

Fixed Twisted Mass

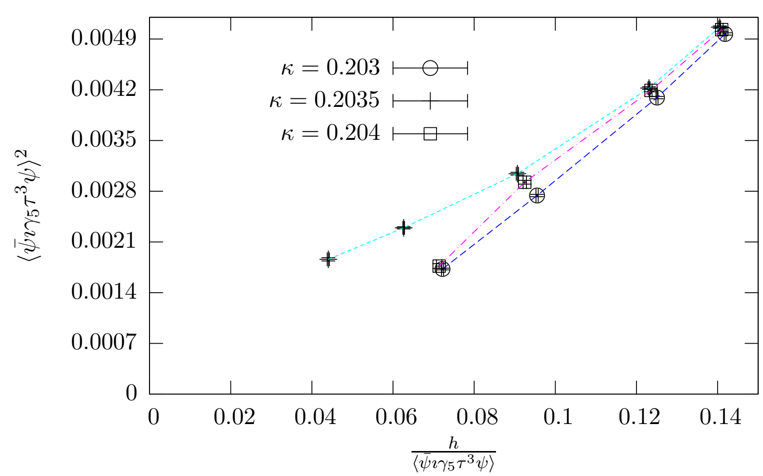
Strong Coupling: The Aoki Phase

- $\beta \in \{1.8, 3.0, 3.4\}$.
- Order parameter

$$\lim_{h \to 0} \lim_{N_\sigma \to \infty} \langle \bar{\psi} i \gamma_5 \tau^3 \psi \rangle_{N_\sigma, h} \neq 0 \,.$$

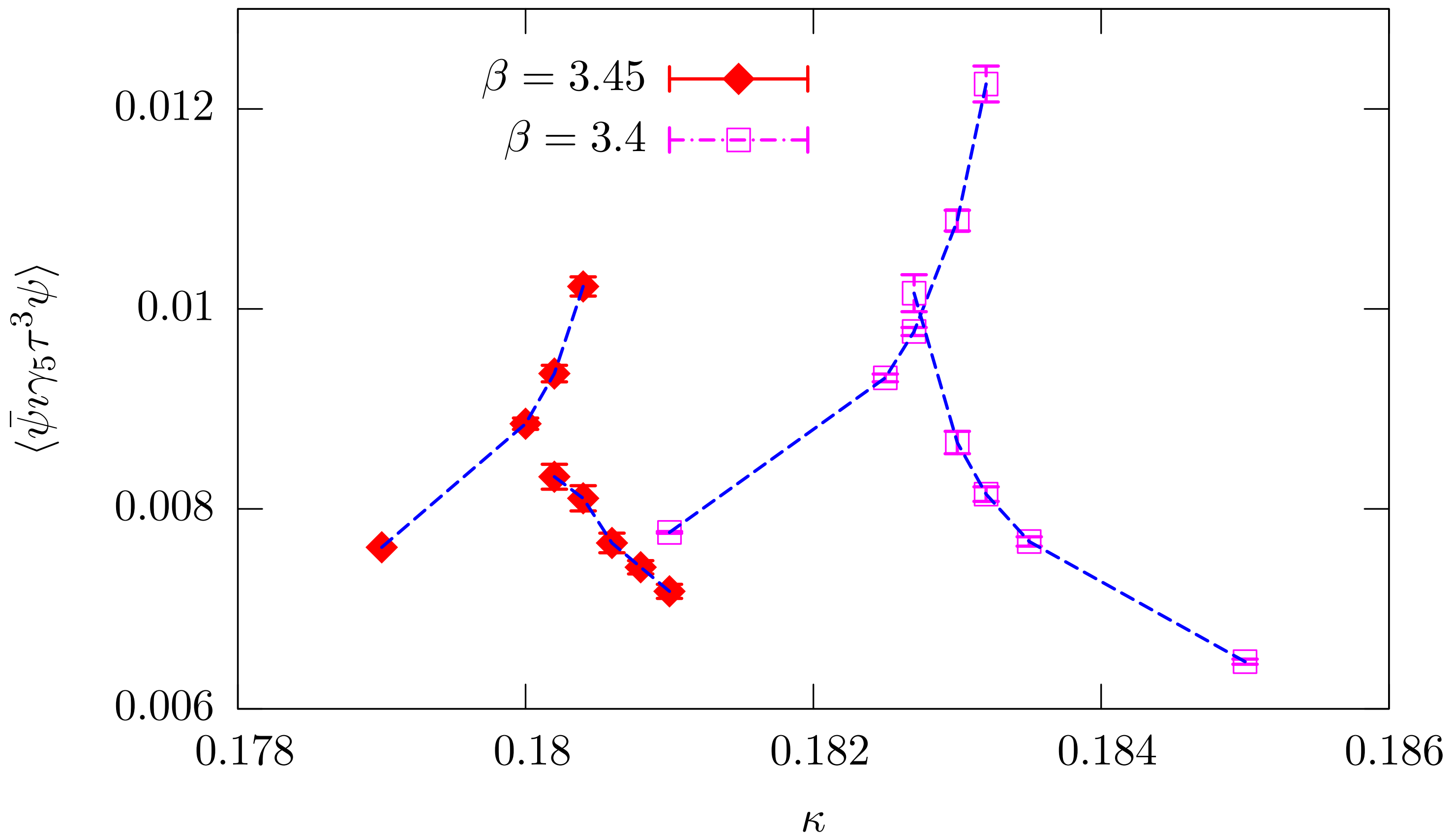


$\beta = 1.8$

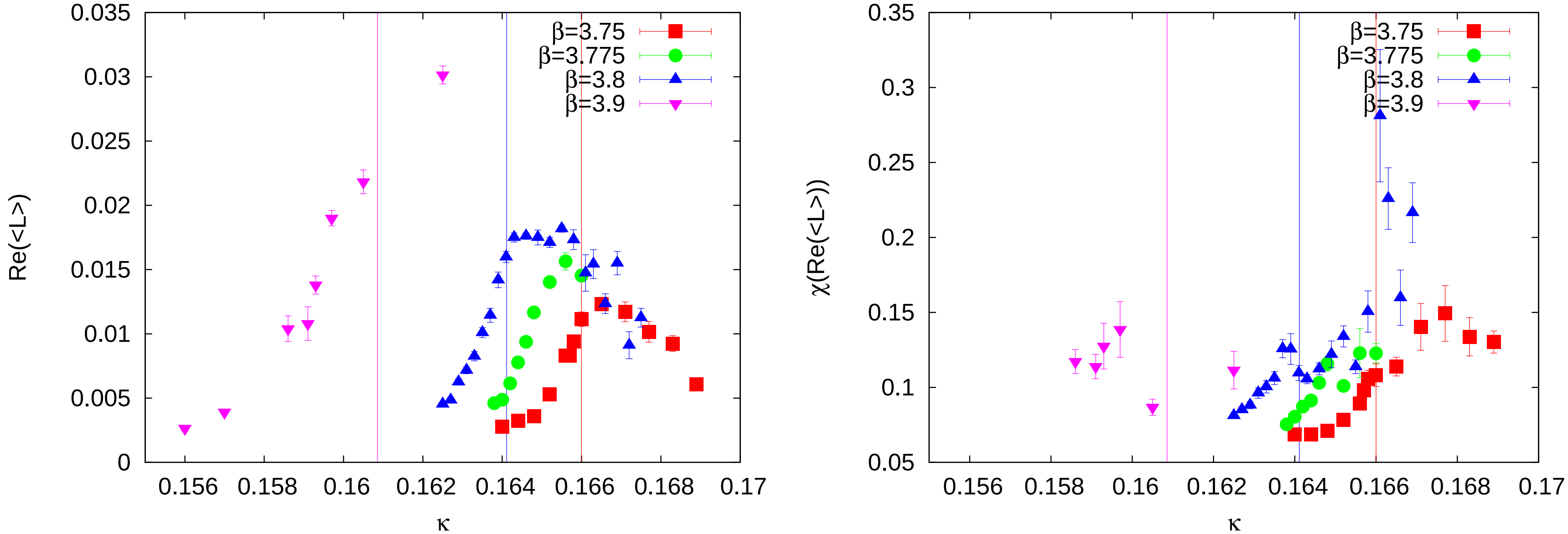


$\beta = 3.0$

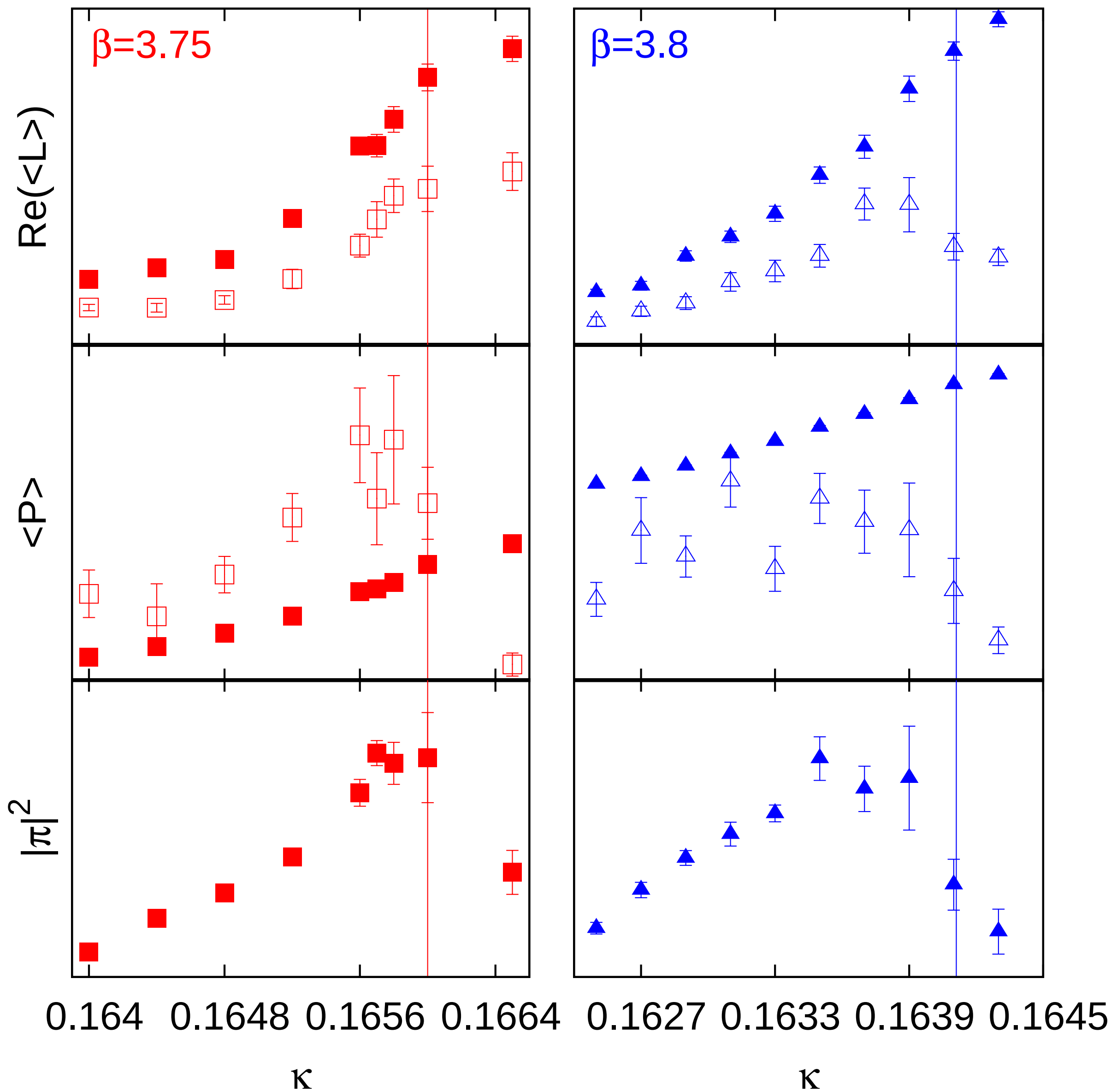
The intermediate region $\beta = 3.4, 3.45, 3.65$: a first order transition



The thermal line at $N_t = 8$



The vertical lines mark $\kappa_c(T = 0, \beta)$ for $\beta = 3.9$, 3.8, 3.75 from left to right.

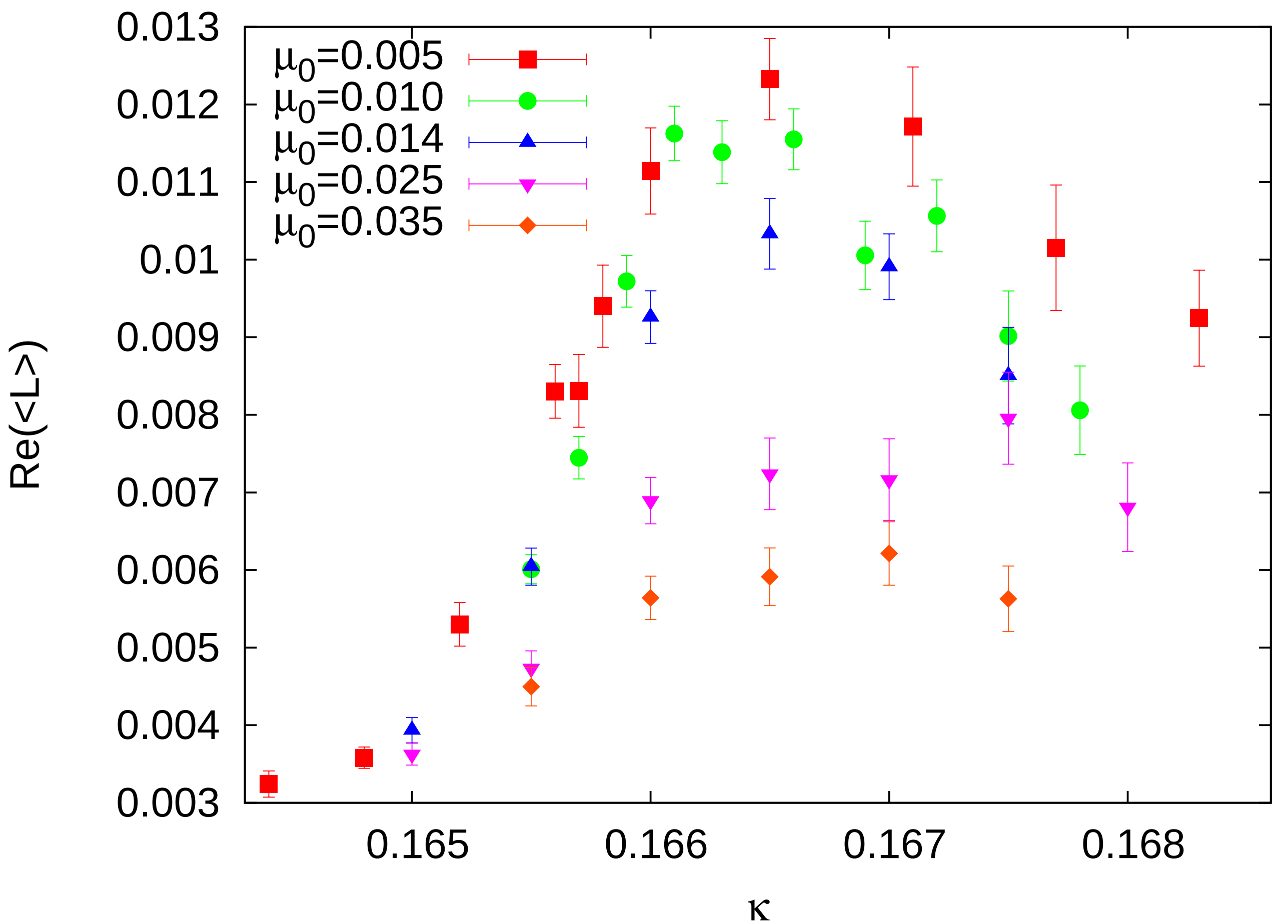


β=3.75
β=3.8
Re(<L>)
<P>
|π|^2
0.164
0.1648
0.1656
0.1664
0.1627
0.1633
0.1639
0.1645
κ
κ

RESULTS FOR $N_t = 8$:

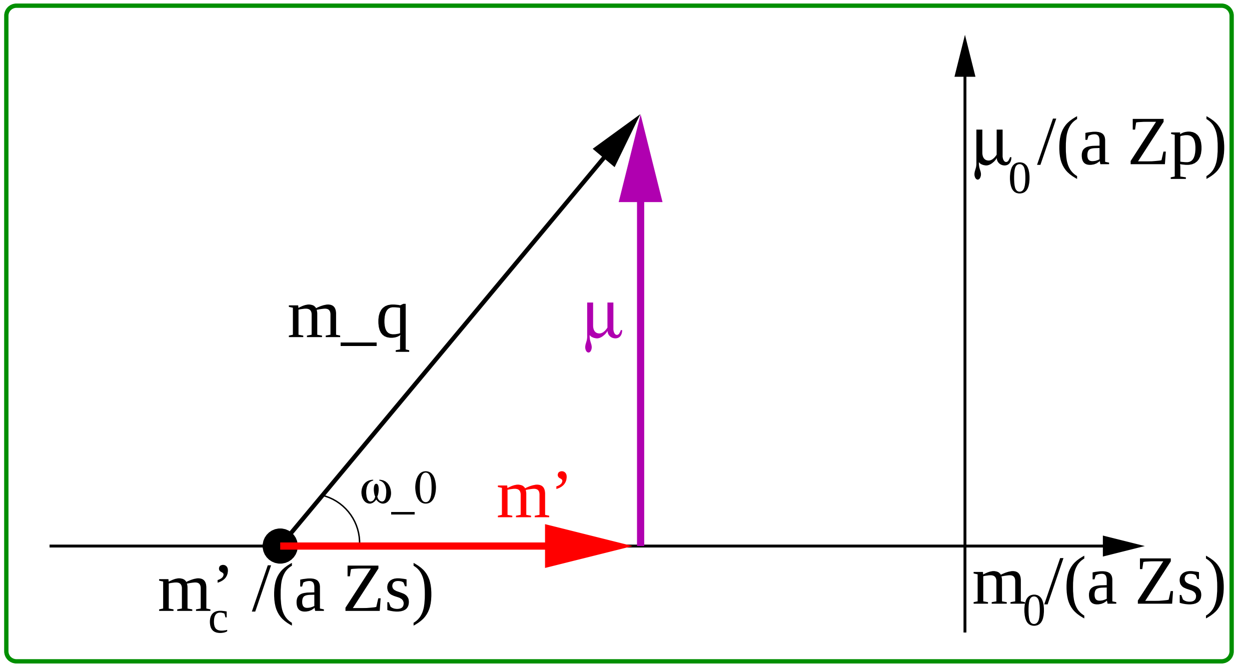
Twisted Mass at Work

$\beta = 3.75$: THE LOWEST MASS POINT : VARYING μ



BACK TO THE ACTION:

LO (Lattice χPT) : m_π^2 = K: ELLYPSES

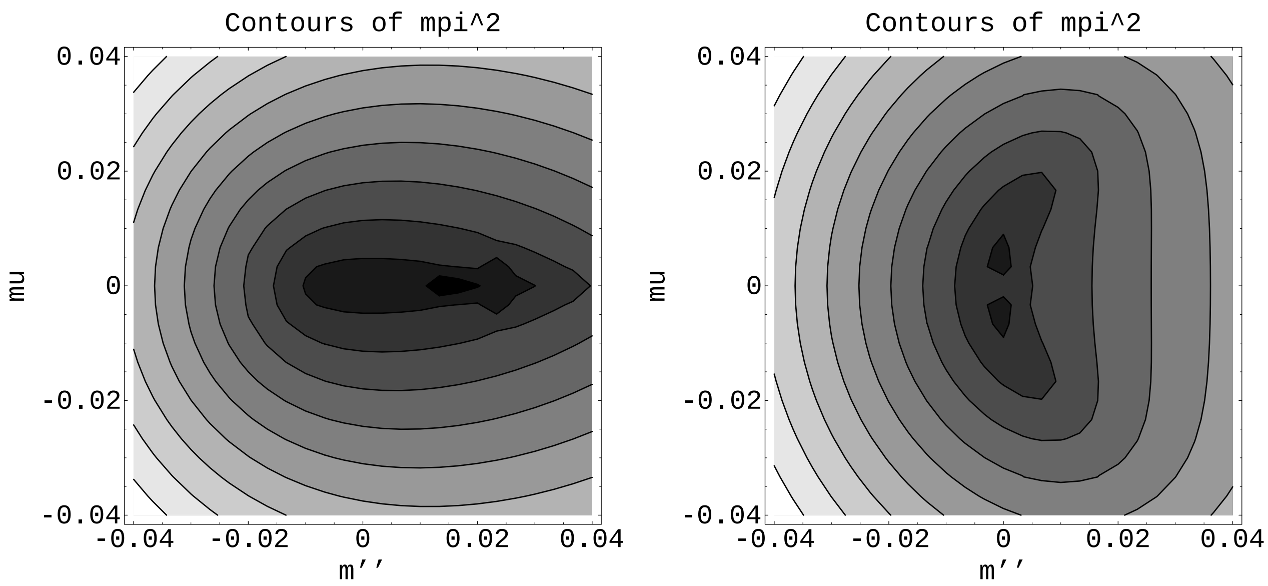


The contour plots of $m^2_{\pi\pm}$ in twisted mass plane are elypses with axes renormalized by the Z factors. (figure after Sharpe 2006)

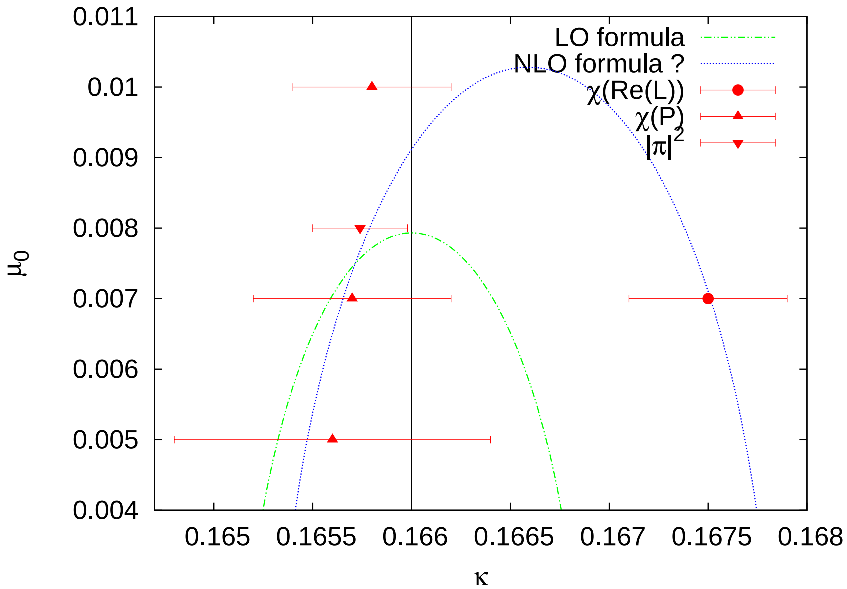
NLO Lattice χPT : $m_\pi^2 = K$: DISTORTED ELLYPSES

The general form of NLO results for observables is

$$m^2_{\pi\pm} = \widehat{m_q}\left[1 + \frac{1}{2}L_\pi + \frac{16}{f^2}\widehat{m_q}(2L_{68} - L_{45})\right]$$
$$+ \widehat{m_q}\cos\omega_0(2\delta_W - \delta_{\tilde{W}}) + 2(\cos\omega_0)^2 w' \,. \qquad (1)$$



Contour plots of $m^2_{\pi\pm}$ in twisted mass plane, from NLO (LχPT) for Aoki-phase (left) and first-order (right) scenarios [**Sharpe 2006**]

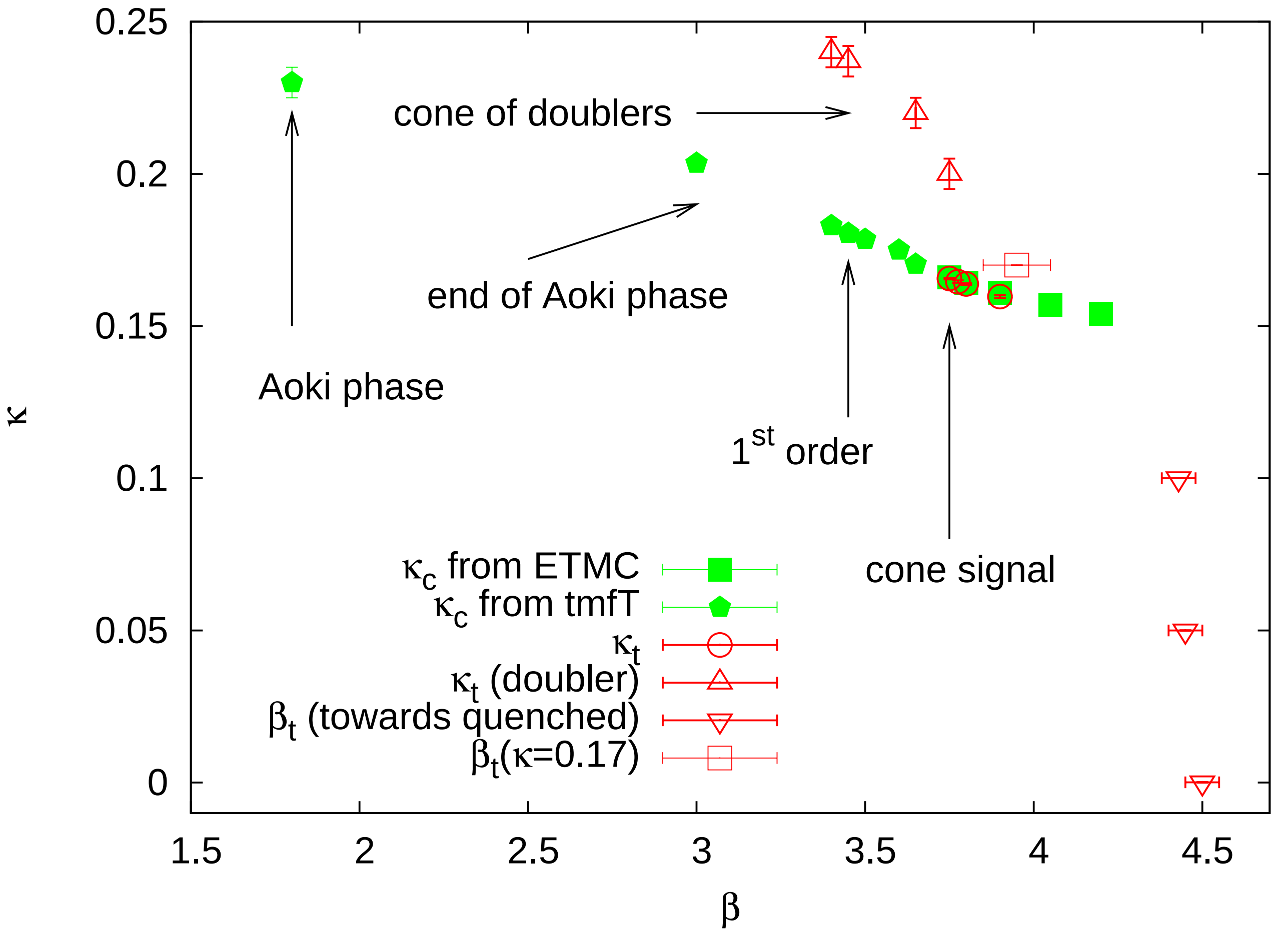


Simplified NLO formula

$$m_q^2 = \left(\frac{1}{Z_P^2} \mu^2 + \frac{1}{Z_S^2} \frac{1}{4} \left(\frac{1}{\kappa} - \frac{1}{\kappa_c} \right)^2 \right) (1 + K \cos\omega)^2 \ .$$

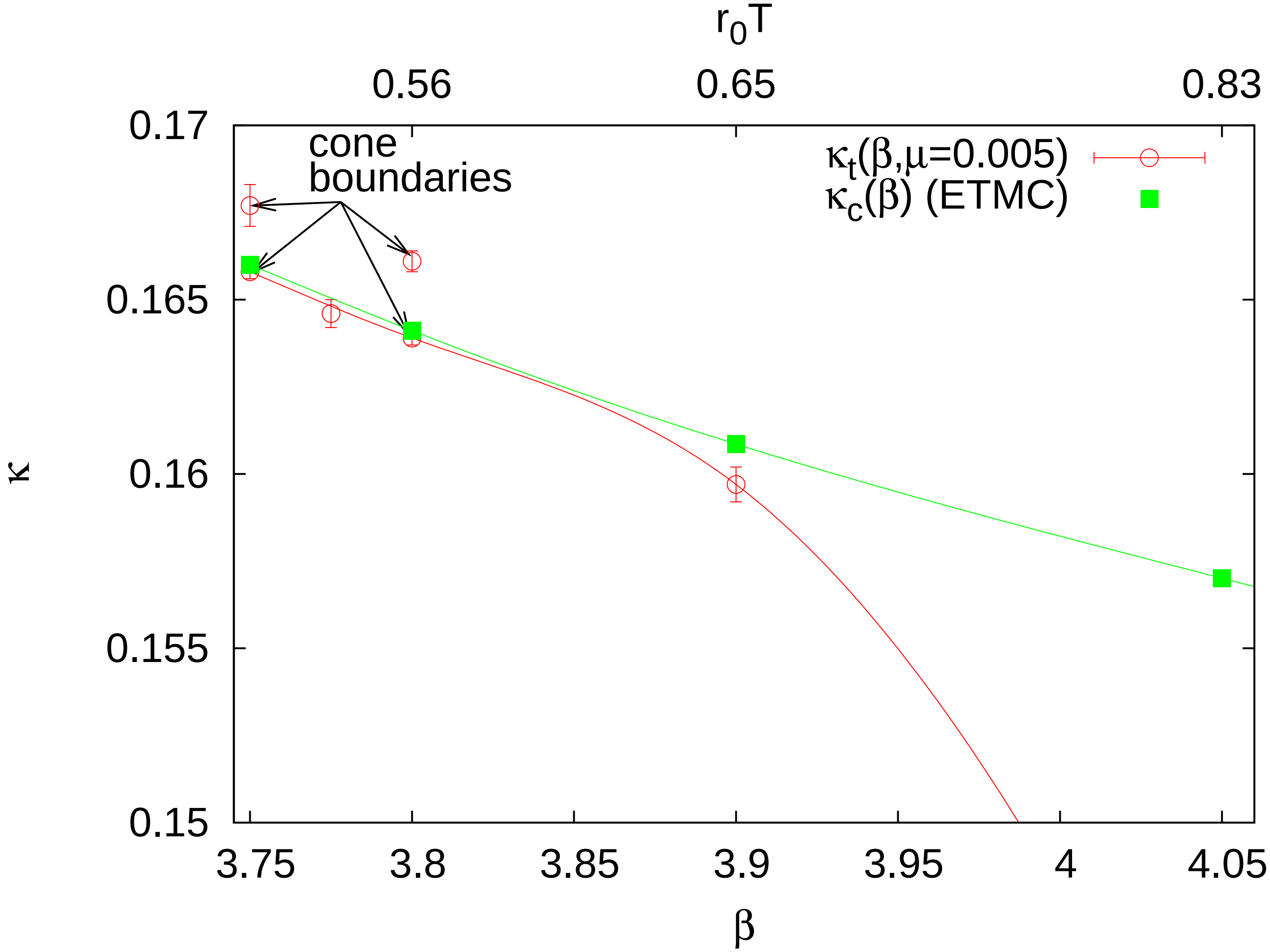
with $m_q = 0.028$, $K = 0.5$, $Z_S = 0.6$, $Z_P = 0.3$ and $\kappa_c = 0.1660$ is consistent with the data points.

SUMMARY $N_t = 8$



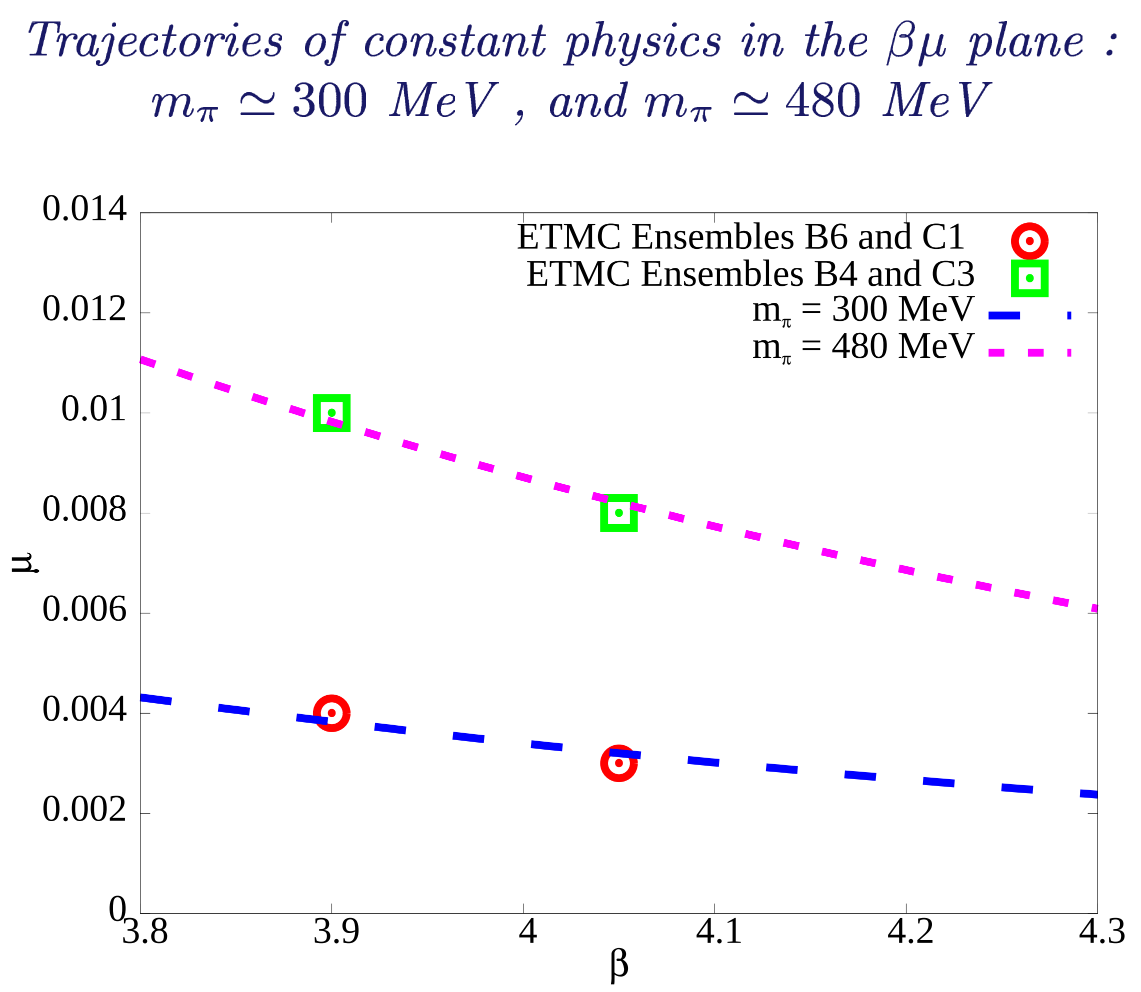
0.25
0.2
0.15
0.1
0.05
0
1.5
2
2.5
3
3.5
4
4.5
κ
β
cone of doublers
end of Aoki phase
Aoki phase
1st order
cone signal
κc from ETMC
κc from tmfT
κt
κt (doubler)
βt (towards quenched)
βt(κ=0.17)

THE THERMAL LINE for $N_t = 8$

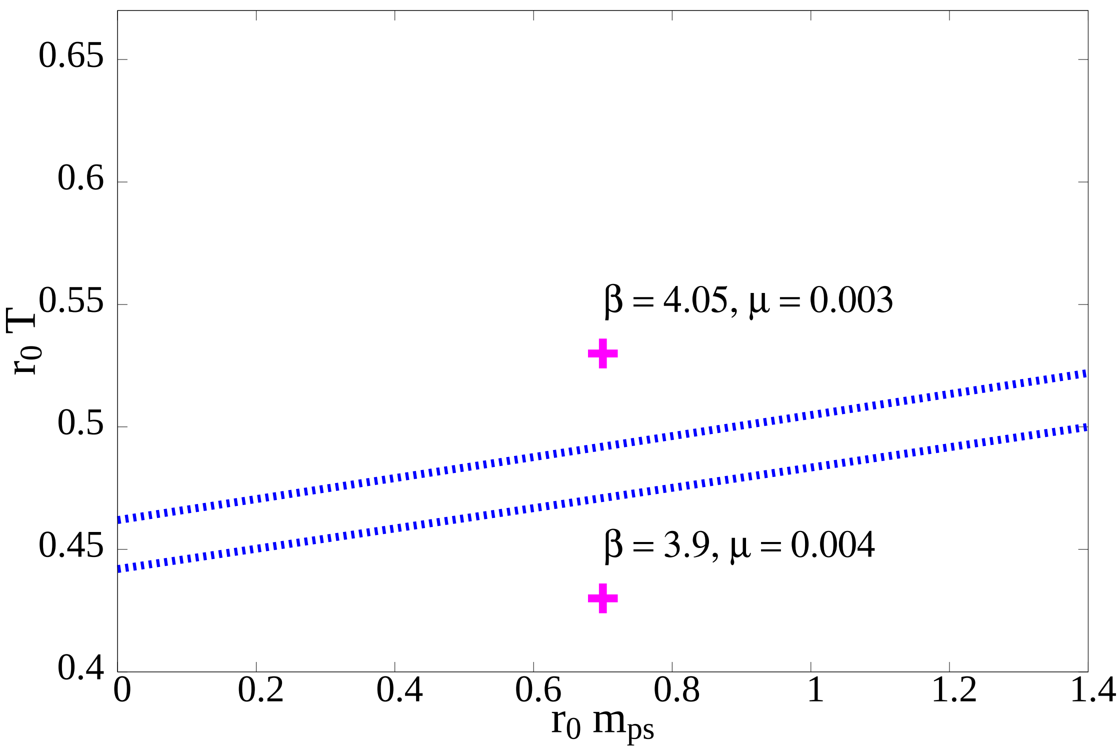


TOWARDS THE CHIRAL/CONTINUUM LIMIT

Nt=10, Nt=12, Maximal Twist: Preliminary


Trajectories of constant physics in the βμ plane :
m_π ≃ 300 MeV , and m_π ≃ 480 MeV
ETMC Ensembles B6 and C1
ETMC Ensembles B4 and C3
m_π = 300 MeV
m_π = 480 MeV
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
3.8
3.9
4
4.1
4.2
4.3
β
μ

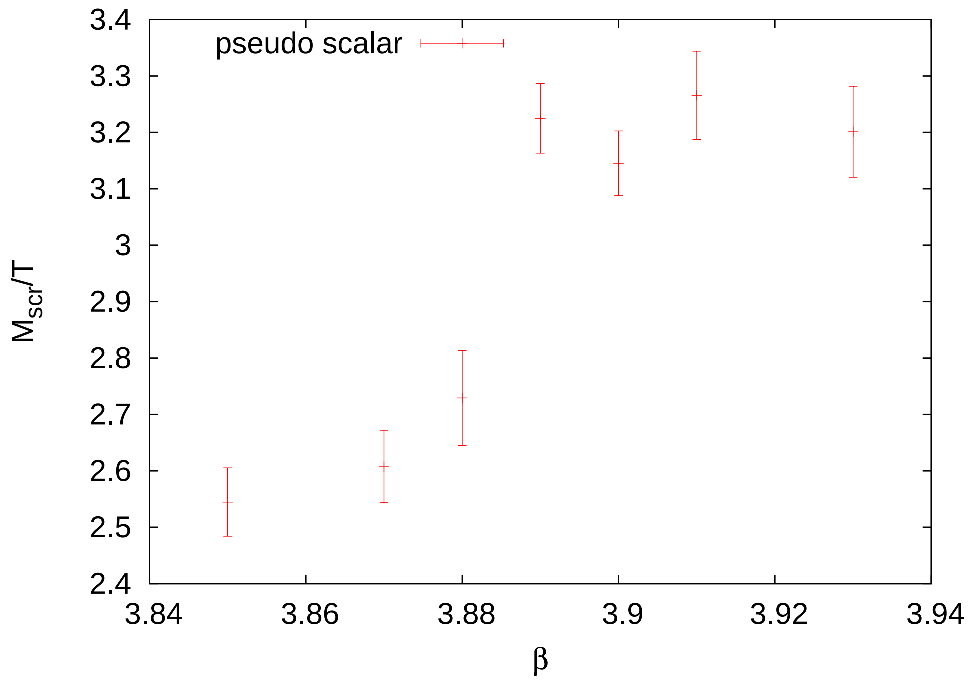
Critical band $r_0(T - T_c) = a(r_0 m_{ps})^d$ – *Example of parameter choices for twisted mass*



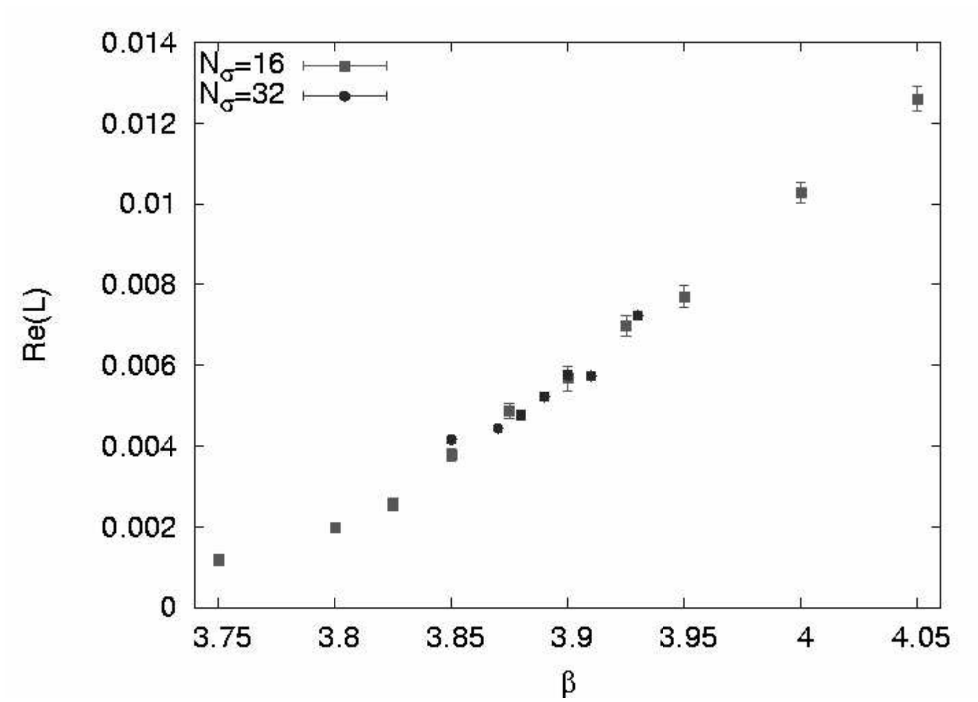
The two points corresponds to $m_\pi \simeq 300\ MeV,\ N_t = 12$

ONGOING SIMULATIONS AT MAXIMAL TWIST:

- Simulations on a $N_t = 10$, $N_s = 16, 32$
- $\mu = 0.006$, $k = k_c$, $3.85 < \beta < 4.05$
- About four months on a 1 Teraflop apeNEXT
- Located critical β
- Very modest size dependence - suggesting crossover
- Simulations on a $N_t = 12$, $N_s = 32$ in progress



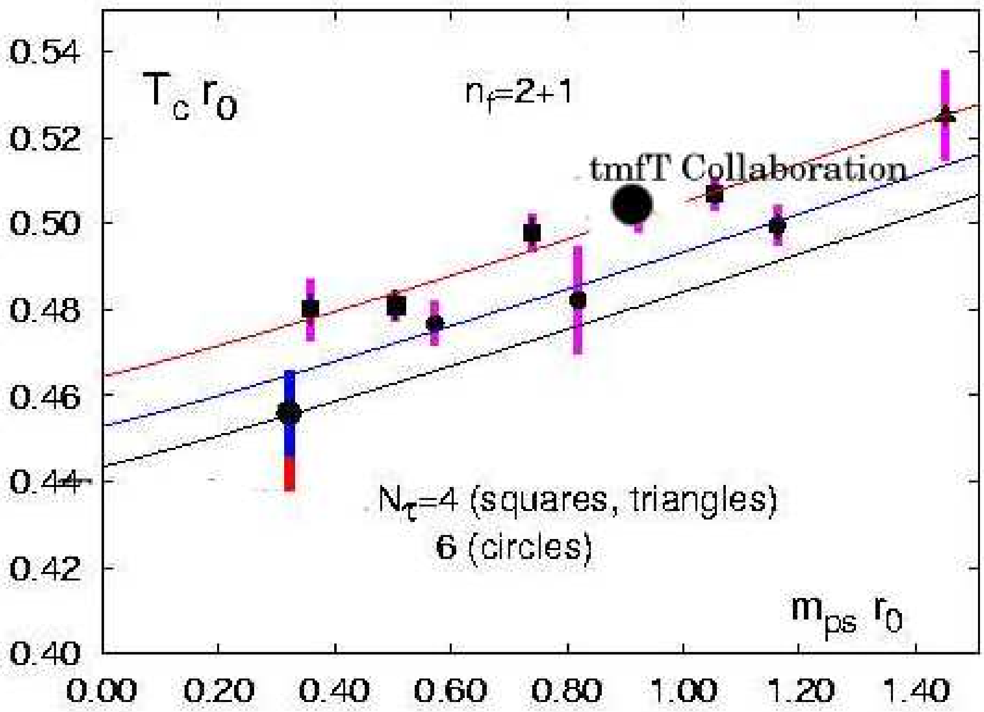
pseudo scalar
M_{scr}/T
β
3.4
3.3
3.2
3.1
3
2.9
2.8
2.7
2.6
2.5
2.4
3.84
3.86
3.88
3.9
3.92
3.94



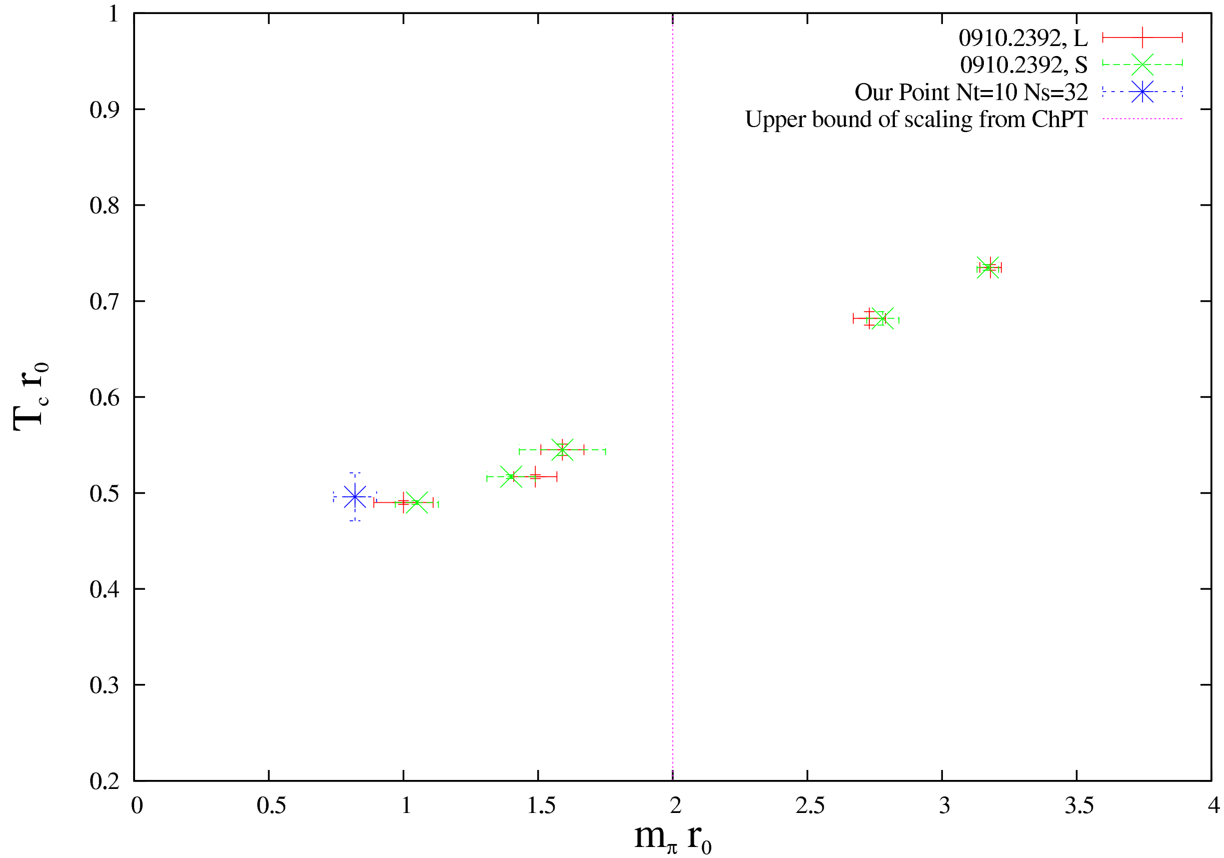
N_σ=16
N_σ=32
Re(L)
β
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
3.75
3.8
3.85
3.9
3.95
4
4.05

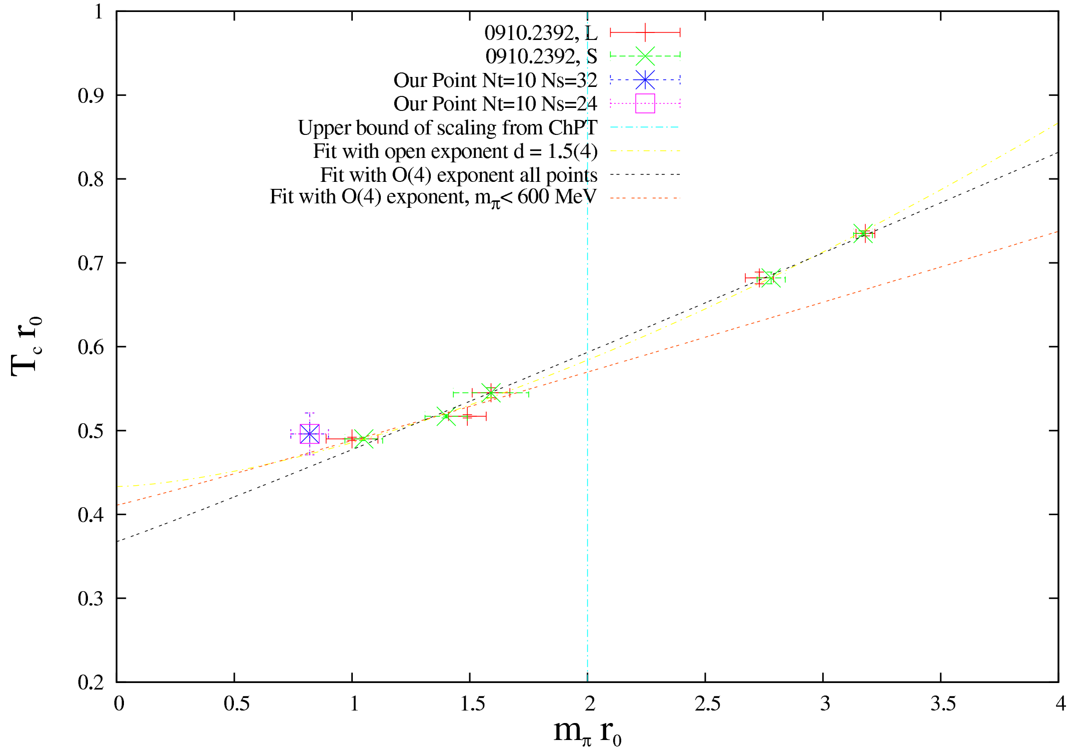
COMPARISON WITH STAGGERED, WILSON RESULTS

COMPARE STAGGERED [**F. Karsch's 2008 Review**]



COMPARE WILSON [Bornyakov et al 2009]

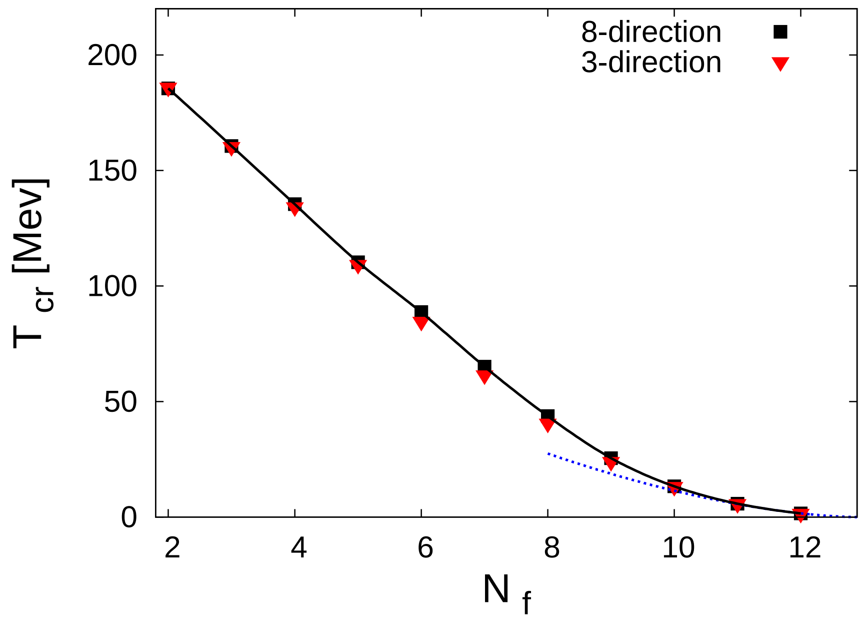


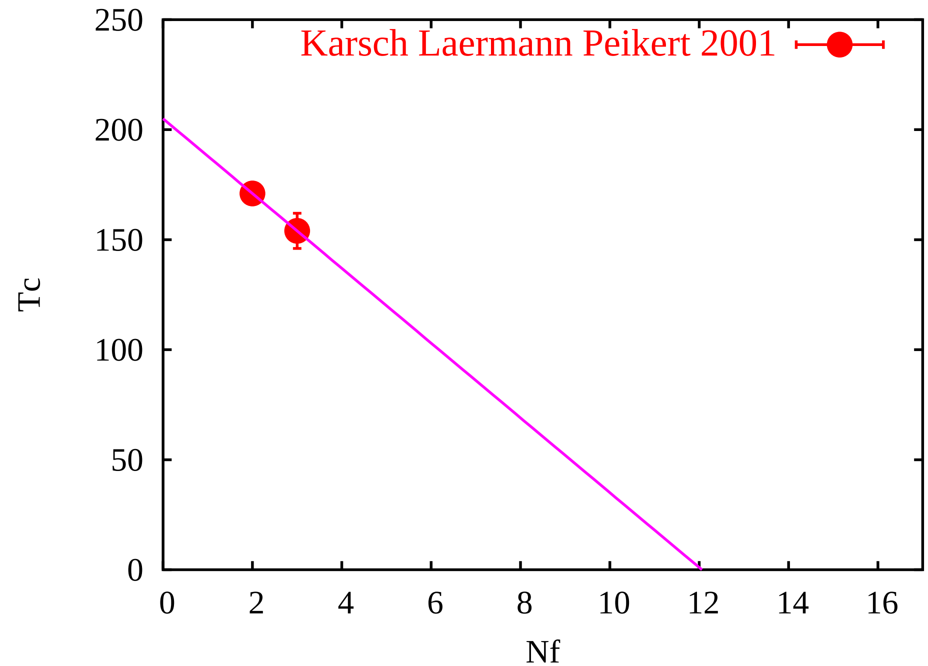


- Bornyakov et al.: constrained O(4) exponent, full mass range
- However : only justifiable within ChPT
- Restricted range $m_\pi < 600$ MeV i.e. $m_\pi r_0 < 2$ gives higher T_c
- In agreement with fits with open exponent in the entire mass range
- $m_\pi r_0 \simeq 0.8$ seems to have enough discriminating power

ASIDE : Comment on N_f dependence of T_c

Known to be linear extrapolating to zero at about Nf=12.

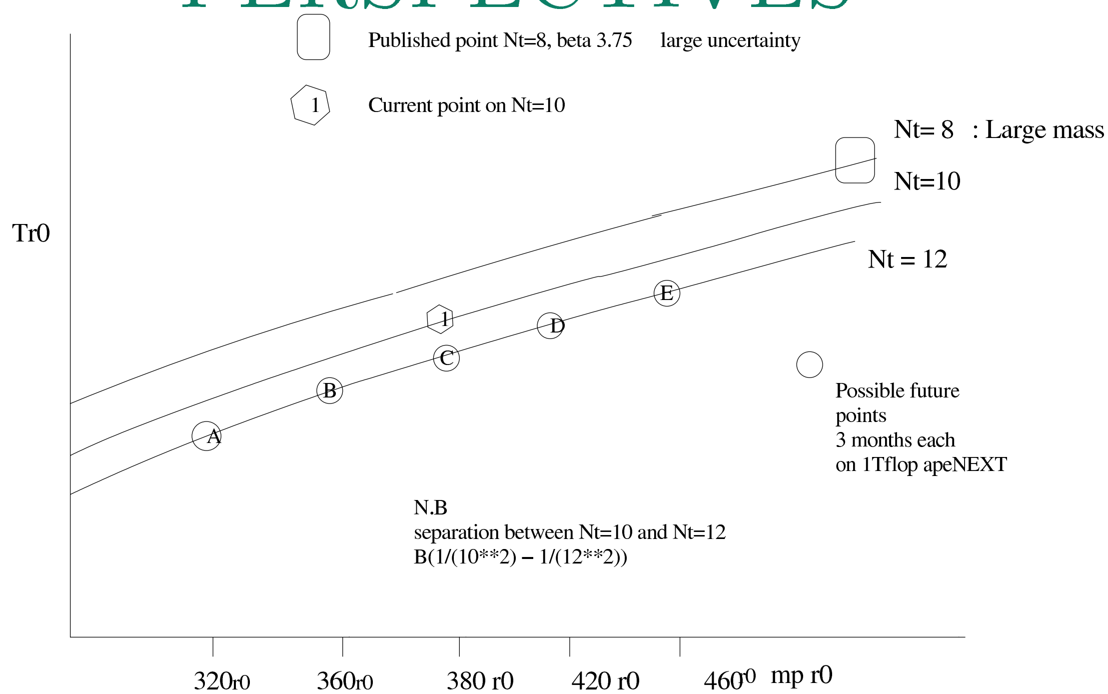




Braun, Gies 2007 **Karsch, Laermann, Peikert 2001**

- Strange quark effect on T_c order a few percent, below current statistics and systematic uncertainty.
- Semiquantitative estimate of the correction should be possible.

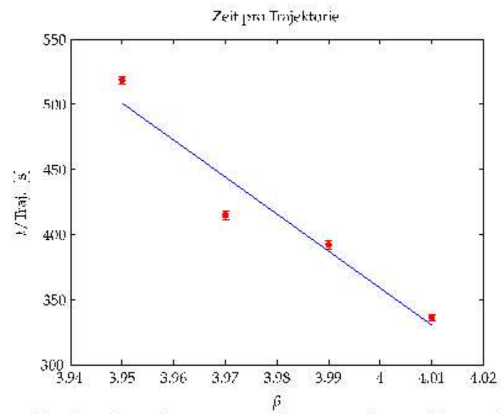
PERSPECTIVES



COMPUTATIONAL REQUIREMENTS

- Each point within a beta scan requires approx. 5000 Trajectories
- On a 500 Gflop substained Computer 400 trajectories per day
- Making about 8 Days per point : i.e. each point costs 4 Teraflop/day
- A scan of 15 Points costs 60 Teraflop Day $\simeq$ 2 Teraflop Month
- Temporary Conclusion : EACH POINT ON THE PSEUDOCRITICAL LINE COSTS ABOUT 2 TERAFLOP MONTH.

Time estimate 12×32^3, 256 nodes



The plot shows the average runtime per trajectory. The number of CG iterations is quite dependent on β. The first approx. 300 trajectories were used (while thermalization) to tune the HMC by adjusting the number of integrator steps and the preconditioning mass. The linear regression could be used to estimate the runtime for some more β-values.

$$t(\beta) = 11725 - 2842\beta$$

We can run a 12h/256nodes-job every day on each account. Using 4 accounts that makes approx. 400 trajectories per day.

IN BRIEF

- Structure of the phase space understood.
  - Agreement with model calculations
  - Agreement with lattice NLO ChPT
- Measured thermal line at large mass 500 MeV $< m_\pi$, at the onset of continuum physics.
- New data at $N_t = 10$, in progress $N_t = 12$ at $m_\pi = 380$ MeV at the lower end of current Wilson simulations with comparable lattice spacing.
- O(10) new points along the (pseudo)critical line could allow estimate of the (pseudo)critical temperature in the continuum, chiral limit: **Estimate cost: 2 Teraflop Year**