2+1 flavor QCD at high temperatures

I Equation of State

II Transition

III Critical behavior ?!

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for the
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I Equation of State

the equation of state is $\epsilon(p)$

equivalent to knowing pressure $p(T)$ and energy density $\epsilon(T)$

$\Rightarrow$ entropy density $s = (\epsilon + p)/T$ and other quantities through thermodynamic relations

start from energy-momentum tensor $\dfrac{\Theta^{\mu}_{\mu}(T)}{T^4} = \dfrac{\epsilon - 3p}{T^4} = T\dfrac{d}{dT}(p/T^4)$

where $\dfrac{p}{T^4} = \dfrac{1}{VT^3}\ln Z(T,V) - \lim_{T\to 0}\dfrac{1}{VT^3}\ln Z(T,V)$ $\quad T = 0$ subtraction takes care of UV divergencies

now $\quad Z(T,V;g,m_l,m_s) = Z(N_\tau,N_\sigma,a;\beta = 6/g^2,\hat{m}_l,\hat{m}_s) \to Z(N_\tau,N_\sigma,a;\beta,m_\pi,m_K)$

and tune bare lattice parameters $\hat{m}_l, \hat{m}_s$ with β such that $m_{\pi,K} = \text{const} \Rightarrow \hat{m}_{l,s}(\beta), a(\beta)$

$\Rightarrow$ central quantity

$$\frac{\Theta^{\mu}_{\mu}(T)}{T^4} = -R_\beta(\beta) N_\tau^4 \left(\left\langle \frac{d\bar{S}}{d\beta} \right\rangle_T - \left\langle \frac{d\bar{S}}{d\beta} \right\rangle_{T=0} \right)$$

with $\quad R_\beta(\beta). = T\dfrac{d\beta}{dT} = -a\dfrac{d\beta}{da}$ $\quad$ (non-perturbative) β function

then $$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{1}{T'^5}\Theta^{\mu}_{\mu}(T')$$

action $S = \beta\, S_G \;+\; 2\,\hat{m}_l(\beta)\;\bar{\psi}_l\psi_l \;+\; \hat{m}_s(\beta)\;\bar{\psi}_s\psi_s \;+\; \beta\ \text{independent}$

such that Θ^{μ}_{μ} consists of three pieces (`asqtad` slightly more complicated: needs $du_0/d\beta$)

$$\frac{\Theta^{\mu\mu}_G(T)}{T^4} = R_\beta\; N_\tau^4\; \Delta\langle \bar{S}_G\rangle$$

where $\Delta\langle \mathcal{O}\rangle = \langle \mathcal{O}\rangle_{T=0} - \langle \mathcal{O}\rangle_T$

$$\frac{\Theta^{\mu\mu}_F(T)}{T^4} = -R_\beta\, R_m\; N_\tau^4\; \left\{\, 2\,\hat{m}_l\, \Delta\langle\bar{\psi}\psi\rangle_l \;+\; \hat{m}_s\, \Delta\langle\bar{\psi}\psi\rangle_s \,\right\}$$

$$\frac{\Theta^{\mu\mu}_h(T)}{T^4} = -R_\beta\, R_h\; N_\tau^4\; \hat{m}_s\, \Delta\langle\bar{\psi}\psi\rangle_s$$

need: non-perturbative β functions $R_\beta(\beta),\;\; R_m(\beta) = \dfrac{1}{\hat{m}_l(\beta)}\dfrac{d\hat{m}_l(\beta)}{d\beta}$

$$R_h(\beta) = \frac{1}{h(\beta)}\frac{dh(\beta)}{dh} = 0$$

at $h(\beta) = \dfrac{\hat{m}_l}{\hat{m}_s} = \text{const}$

"action differences" $\Delta\bar{S}_G,\, \Delta\langle\bar{\psi}\psi\rangle_{l,s}$

Simulation parameters

- $N_F = 2+1$: two degenerate u/d quarks + strange quark
- RHMC algorithm
- lattice sizes $T>0$: $N_\sigma/N_\tau = 4$ $N_\tau = 4,6,8$ for $m_l/m_s = 1/10$ ($m_\pi \simeq 220$ MeV)

  $N_\tau = 8,12$ for $m_l/m_s = 1/20$ ($m_\pi \simeq 150$ MeV)

  each $T>0$ run accompanied by $T=0$ run, for subtractions, scales

  $T=0$: $N_\sigma/N_\tau \leq 1$ same N_σ as for $T>0$

- statistics $\mathcal{O}(10k-60k)$ for $T>0$, each $(\beta, \hat{m}_l, \hat{m}_s)$

  $\mathcal{O}(5k)$ for $T=0$, each $(\beta, \hat{m}_l, \hat{m}_s)$

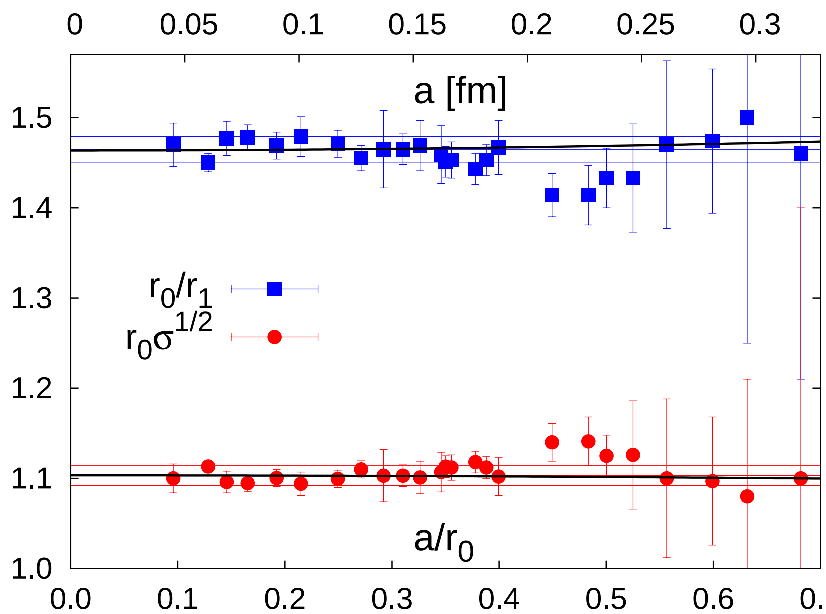
- actions `p4fat3` on $N_\tau = 4,6,8$

  `asqtad` on $N_\tau = 6,8,12$

⋆ T = 0 scale taken from $\Upsilon\, 2S - 1S$ splitting [A. Gray et al.] via the heavy quark potential $V(r)$

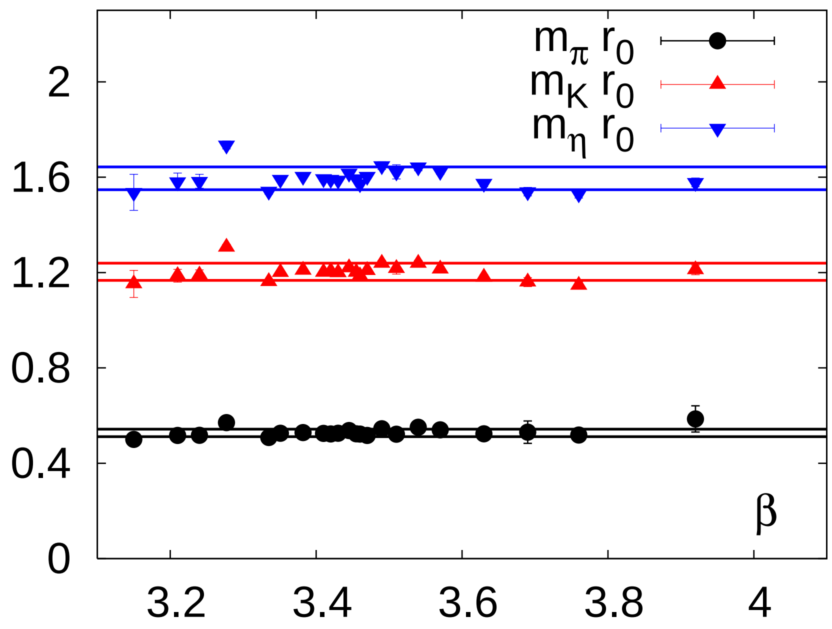
$$r_{0,1}/a \quad \text{from} \quad r^2 \frac{dV(r)}{dr}\Big|_{r=r_{0,1}} = 1.65(1.0)$$

for absolute values (in MeV) we use $r_0 = 0.469(7)$ fm [A. Gray et al.]



2 %

⋆ fine tune $\hat{m}_l(\beta)$

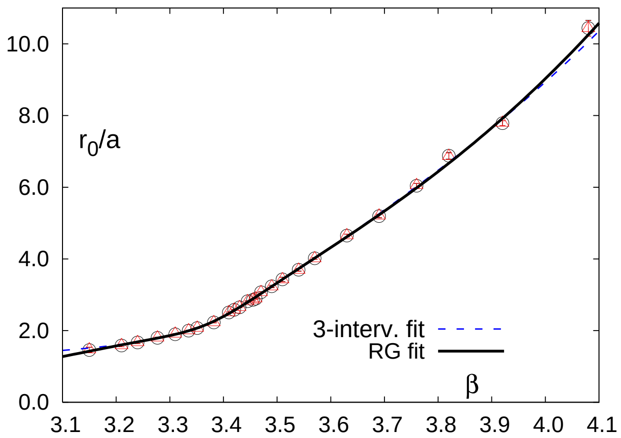


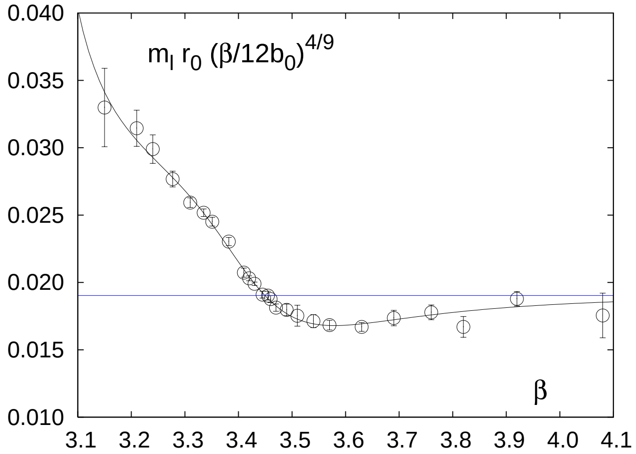
3 %

$m_K \simeq m_K^{\mathrm{phys}}$

$m_\pi \simeq 220\,\mathrm{MeV}$

p4 action, $m_l = 0.1\, m_s$





p4 action

$m_l = 0.1\, m_s$

Allton inspired parametrization with rational fct. in $\hat{a}(\beta) = R_\beta^{(2-loop)}(\beta)/R_\beta^{(2-loop)}(\beta = 3.4)$

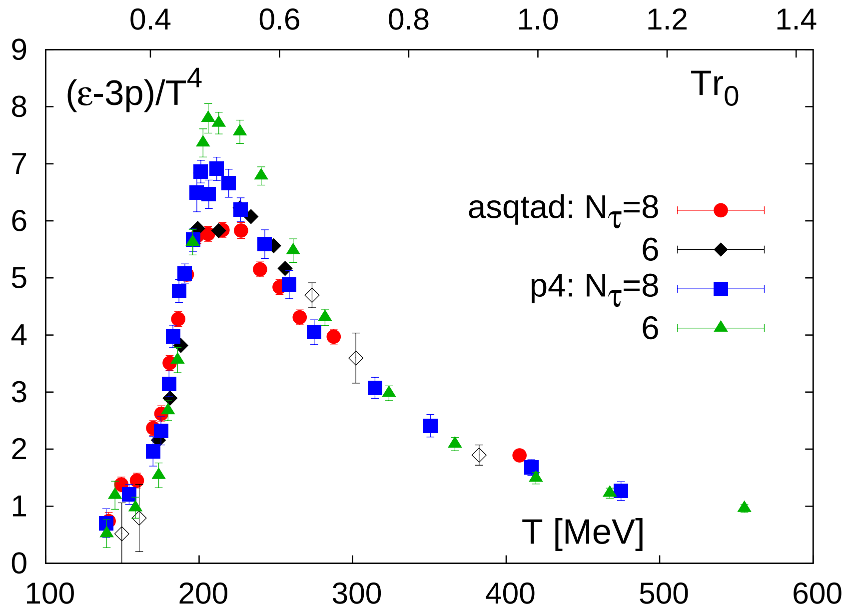
$$\frac{a}{r_0} = a_r R_\beta^{(2-loop)} \frac{1 + b_r \hat{a}^2 + c_r \hat{a}^4 + d_r \hat{a}^6}{1 + e_r \hat{a}^2 + f_r \hat{a}^4} \qquad \Rightarrow \qquad R_\beta = \frac{r_0}{a} \left(\frac{d r_0/a}{d\beta} \right)^{-1}$$

$$\hat{m}_l = a_m R_\beta^{(2-loop)} \left(\frac{12 b_0}{\beta} \right)^{4/9} \frac{1 + b_m \hat{a}^2 + c_m \hat{a}^4 + d_m \hat{a}^6}{1 + e_m \hat{a}^2 + f_m \hat{a}^4 + g_m \hat{a}^6} \qquad \Rightarrow \qquad R_m$$

for asqtad there is a general parametrization for a large set of $T = 0$ data

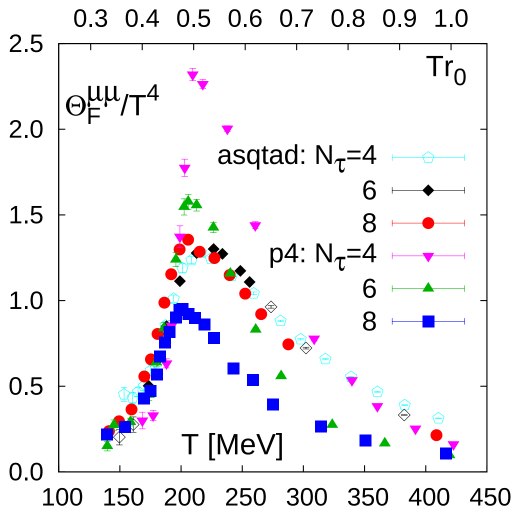
results: $m_l = 01\, m_s$

$\Theta^\mu_\mu(T)/T^4$ (the central quantity)



⋆ very good agreement between p4fat3 and asqtad actions

⋆ except in the peak region, here differences dominantly due to fermionic part Θ_F



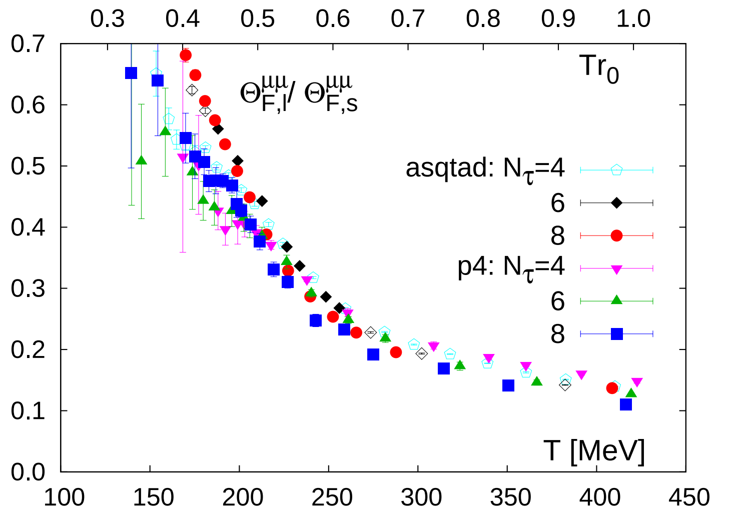
there: 15 % Θ_F contribution

$\Rightarrow$ 5 % overall

affected by R_m at low β

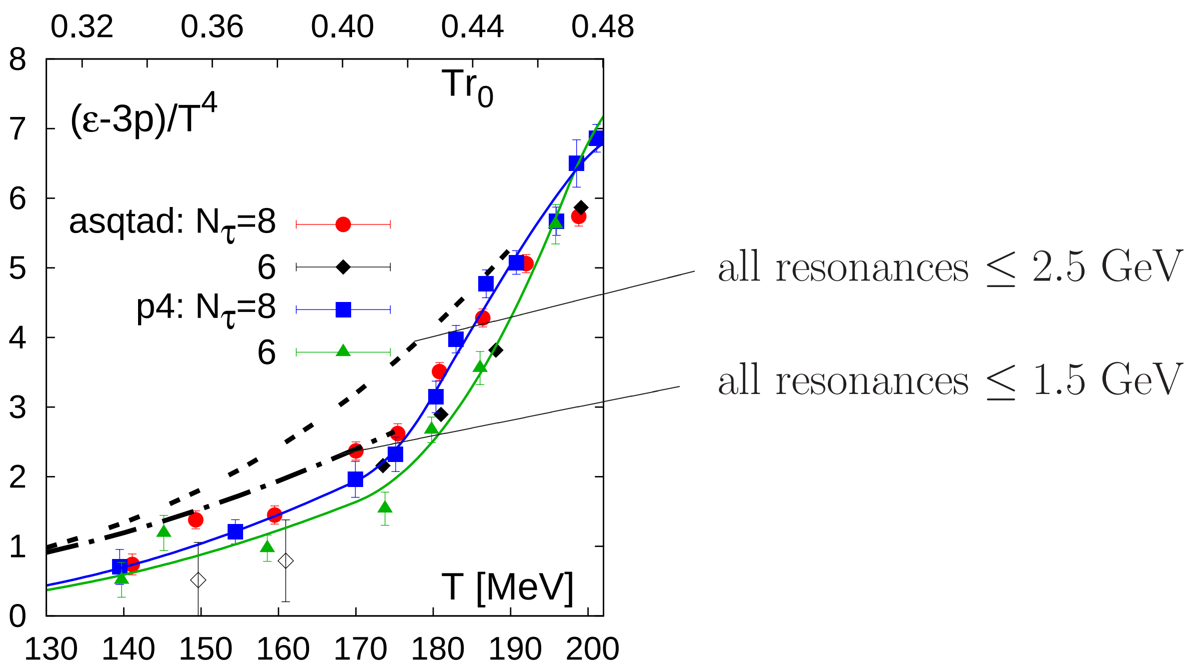
$\rightsquigarrow$ drops out in $\Theta_{F,l}/\Theta_{F,s}$

remaining differences due to slight mismatch in m_s^{phys}



results $m_l = 0.1\, m_s$

low T region:



model to compare with:

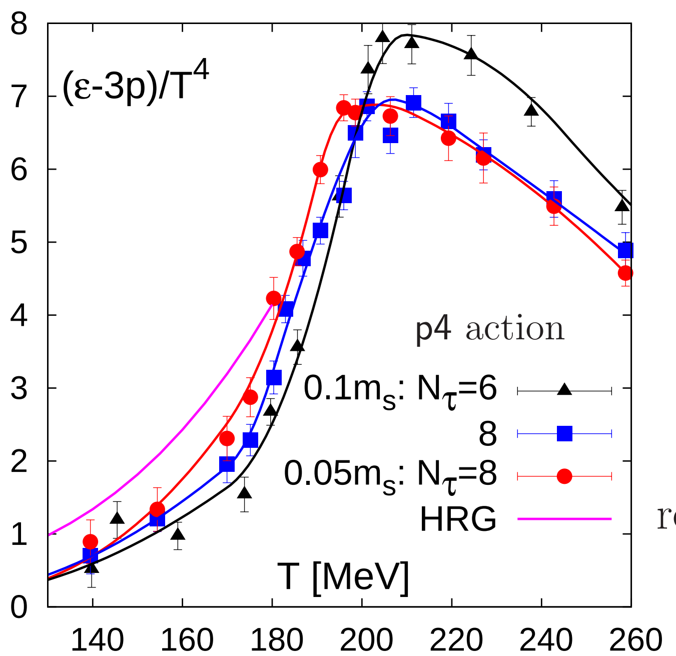
hadron resonance gas (HRG)

$$\left(\frac{\epsilon - 3p}{T^4}\right)_{low\,T} = \sum_{m_i \le m_{max}} \frac{d_i}{2\pi^2} \sum_{k=1}^{\infty} (\pm)^{k+1} \frac{1}{k} \left(\frac{m_i}{T}\right)^3 K_1(km_i/T)$$

- ⋆ non-negligible contribution of heavy resonances in HRG
- ⋆ reducing discretization effects lowers the crossover temperature

results $m_l = 0.05\, m_s$

low T region:



resonances ≤ 2.5 GeV

model to compare with:

hadron resonance gas (HRG)

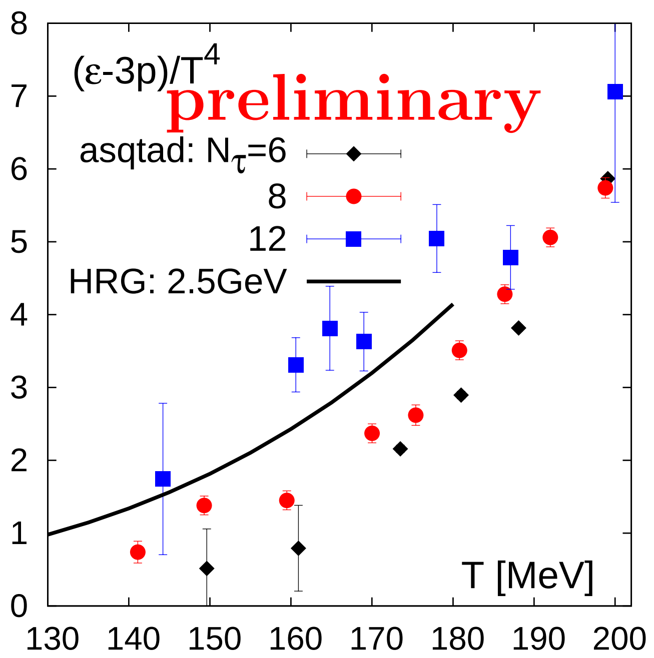
$$\left(\frac{\epsilon - 3p}{T^4}\right)_{low\,T} = \sum_{m_i \leq m_{max}} \frac{d_i}{2\pi^2} \sum_{k=1}^{\infty} (\pm)^{k+1} \frac{1}{k} \left(\frac{m_i}{T}\right)^3 K_1(k m_i/T)$$

- ⋆ reducing discretization effects lowers the crossover temperature
- ⋆ diminishing quark mass lowers the crossover temperature
- ⋆ smaller quark masses are expected to raise $\epsilon - 3p$ somewhat at low T

results

low T region:

$N_\tau = 12,\ m_l = 0.05\, m_s$



} $m_l = 0.1 m_s$

$m_l = 0.05 m_s$

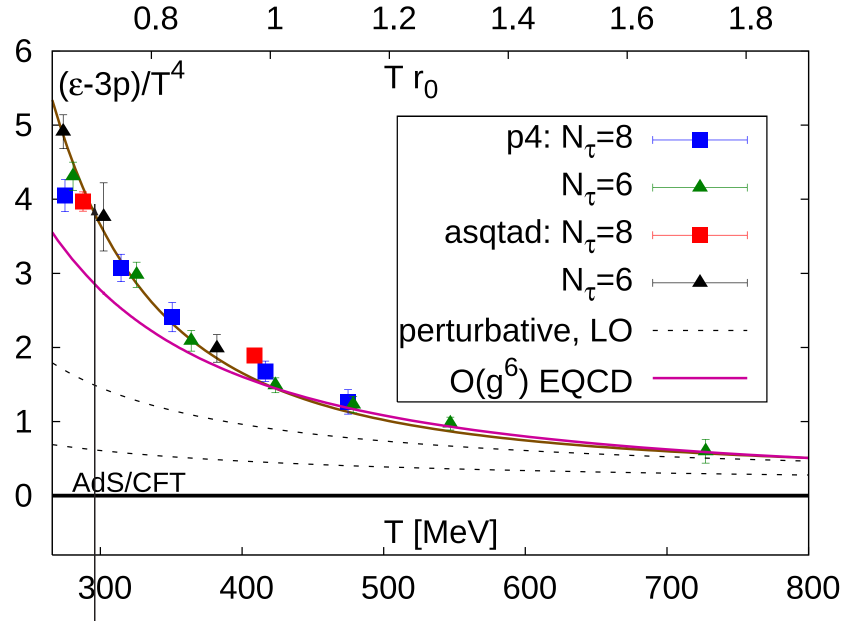
model to compare with:

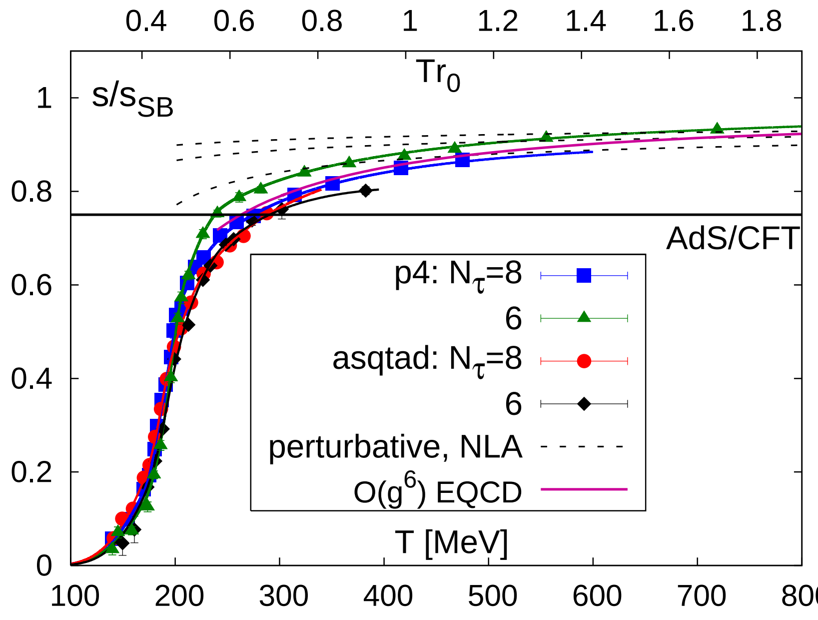
hadron resonance gas (HRG)

$$\left(\frac{\epsilon - 3p}{T^4}\right)_{low\,T} = \sum_{m_i \le m_{max}} \frac{d_i}{2\pi^2} \sum_{k=1}^{\infty} (\pm)^{k+1} \frac{1}{k} \left(\frac{m_i}{T}\right)^3 K_1(k m_i/T)$$

- ⋆ reducing discretization effects lowers the crossover temperature
- ⋆ diminishing quark mass lowers the crossover temperature
- ⋆ smaller quark masses are expected to raise $\epsilon - 3p$ somewhat at low T, but effect is small
- ⋆ reducing discretization effects from $N_\tau = 8 \to 12$ seems to raise $\epsilon - 3p$, preliminary

results: $m_l = 0.1\, m_s$ high T





courtesy P.Petreczky

$$\left(\frac{\epsilon - 3p}{T^4}\right)_{high\,T} = \frac{3}{4} b_0 g^4(T) + \frac{d_2}{T^2} + \frac{d_4}{T^4}$$

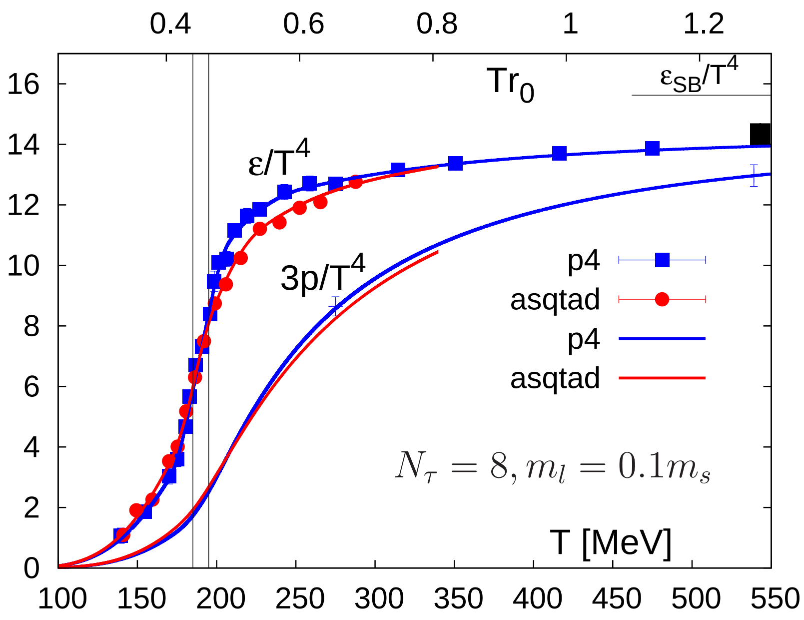
$$s/T^3 = (\epsilon + p)/T^4$$

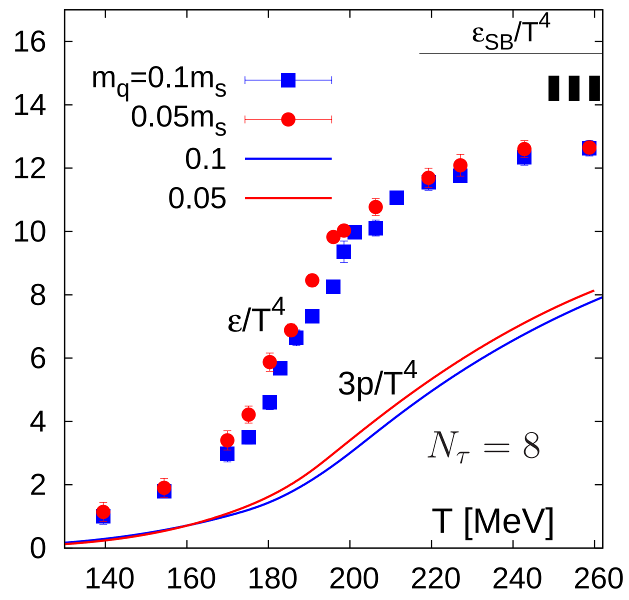
EQCD: 3d effective theory,
with perturbative matching to 4d
Kajantie et al.; Laine, Schröder

also agreement with resummed pert.theory
Blaizot et al.

results

pressure and energy density



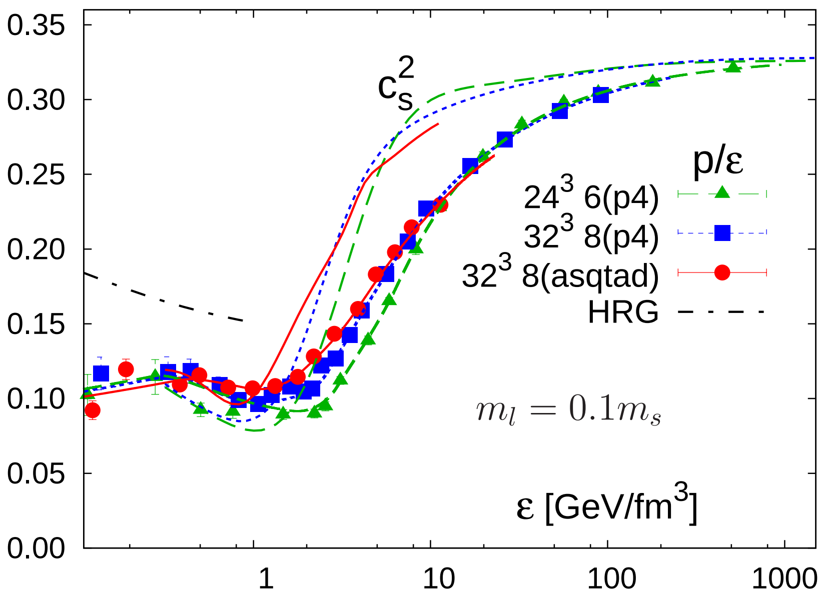


- ⋆ good agreement between **`asqtad`** and **`p4fat3`** actions
- ⋆ integration error: indicated by little bars to the right
- ⋆ in comparison with **Stefan-Boltzmann**: 10 % below at 2 - 3 T_c
- ⋆ quark mass effects negligible at high T

vertical lines denote $N_\tau = 8$ transition region 185 MeV $< T <$ 195 MeV for $m_l = 0.1m_s$

results

speed of sound $$c_s^2 = \frac{p}{\epsilon} + \epsilon \frac{d(p/\epsilon)}{d\epsilon}$$



⋆ good agreement between asqtad and p4fat3 actions

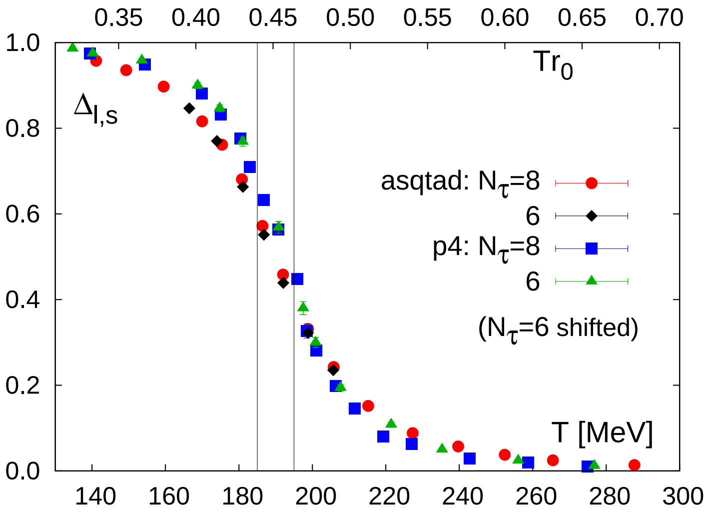
⋆ softest point corresponds to $\epsilon \simeq 1$ GeV/fm^3

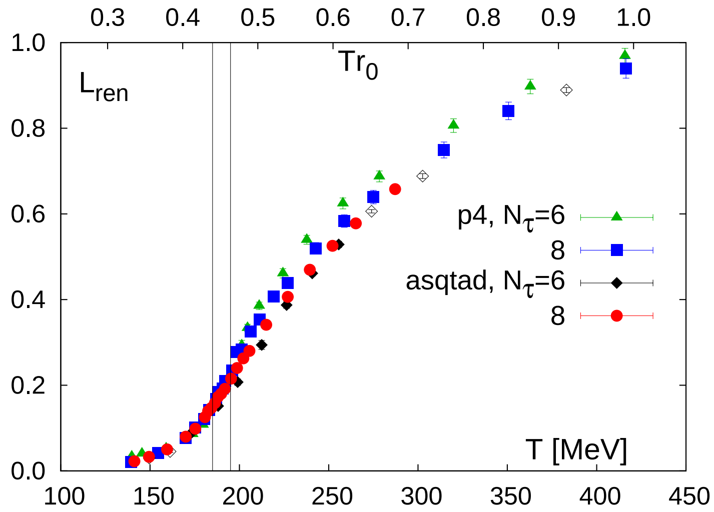
II Transition

comparison with chiral condensate and Polyakov loop

– chiral condensate $\langle\bar{\psi}\psi\rangle$ true order parameter in the chiral limit $m_q \to 0$

– Polyakov limit $L \sim \exp(-F_q/T)$ true order parameter in the static quark limit $m_q \to \infty$





– both need renormalization

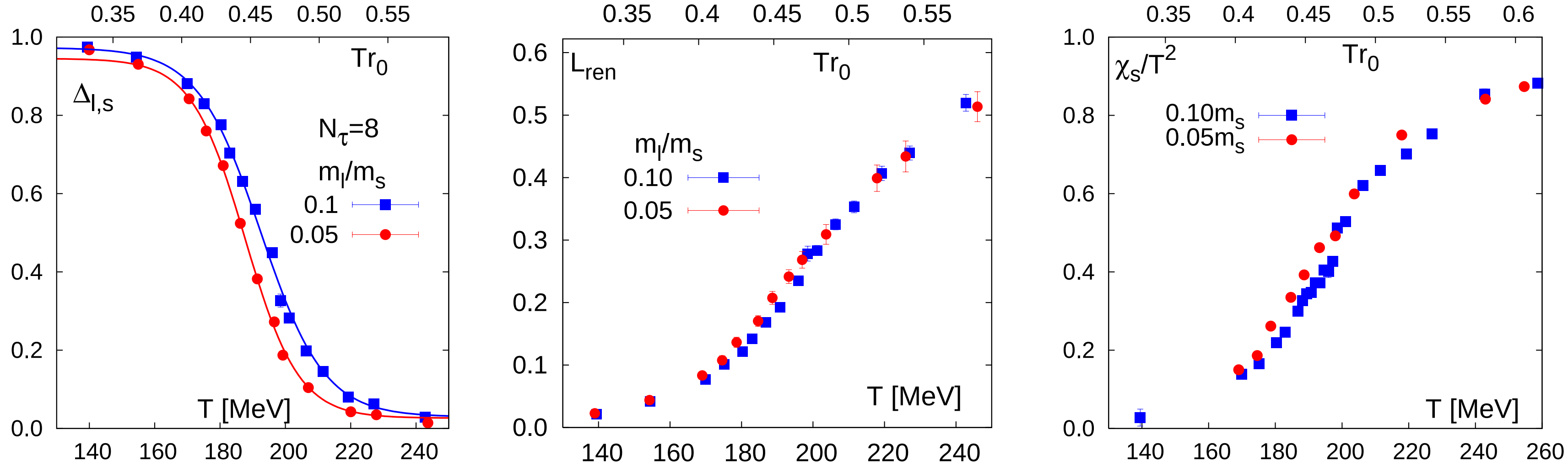
$$\Delta_{l,s}(T) = \frac{\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}$$

subtracting power-law additive and cancelling multiplicative UV divergencies

UV renormalization via normalization of static quark potential

significant changes in the same temperature range as EoS going to $m_l = 0.05\, m_s \;\leftrightarrow\; m_\pi \simeq 150$ MeV ($N_\tau = 8$, `p4` action, `asqtad` $T=0$ running)

strange quark number susceptibility



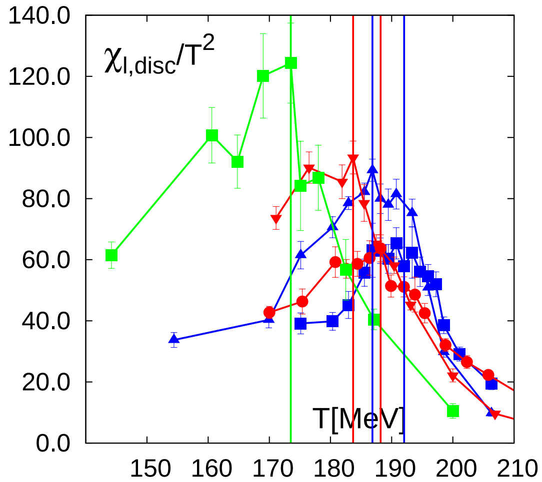
$$\chi_s(T) \sim \frac{1}{T^3 V} \left. \frac{\partial^2 \ln Z(T,\mu_s)}{\partial \mu_s^2} \right|_{\mu_s=0}$$

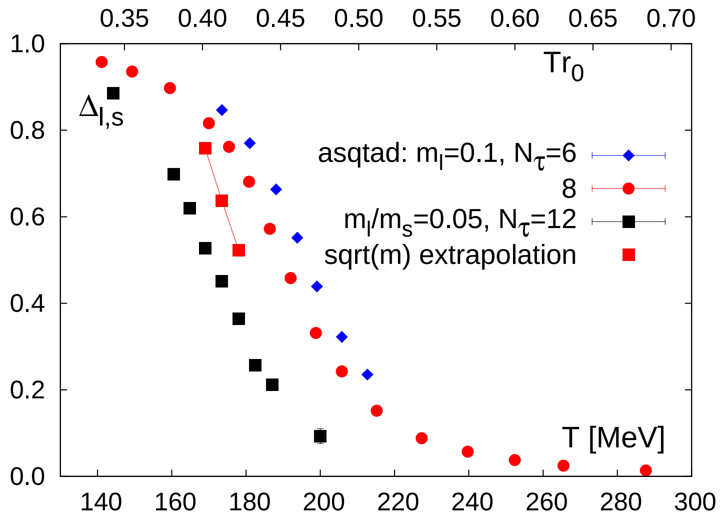
at low T: $\sim \exp(-m_K/T)$
not too sensitive to criticality

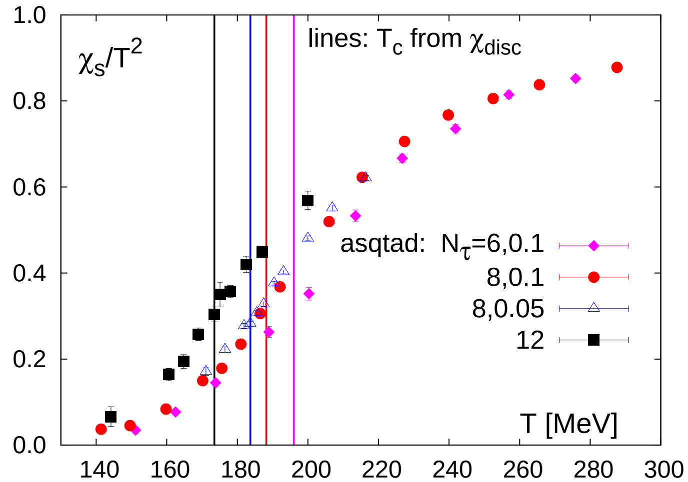
– as in EoS: overall shift by about -5 MeV due to smaller quark mass going to $N_\tau = 12$ ($m_l = 0.05\, m_s$, asqtad action)

disc. chiral susceptibility

$\chi_{l,disc} \sim \langle (\bar\psi\psi)^2 \rangle_l - \langle \bar\psi\psi \rangle_l^2$







asqtad: $N_\tau = 8$, $N_\tau = 12$
p4: $N_\tau = 8$
vertical lines mark $\chi_{l,max}$

red boxes:
$N_\tau = 12$ at $m_l = 0.1 m_s$
through $\sqrt{m_l}$ extrapolation
(see next)

note the asymmetry around $\chi_{l,max}$

more work under way,

so far, a transition temperature $\lesssim$170 MeV at physical π mass suggested

III Critical behavior ?!

phase diagram of QCD at $\mu_q = 0$

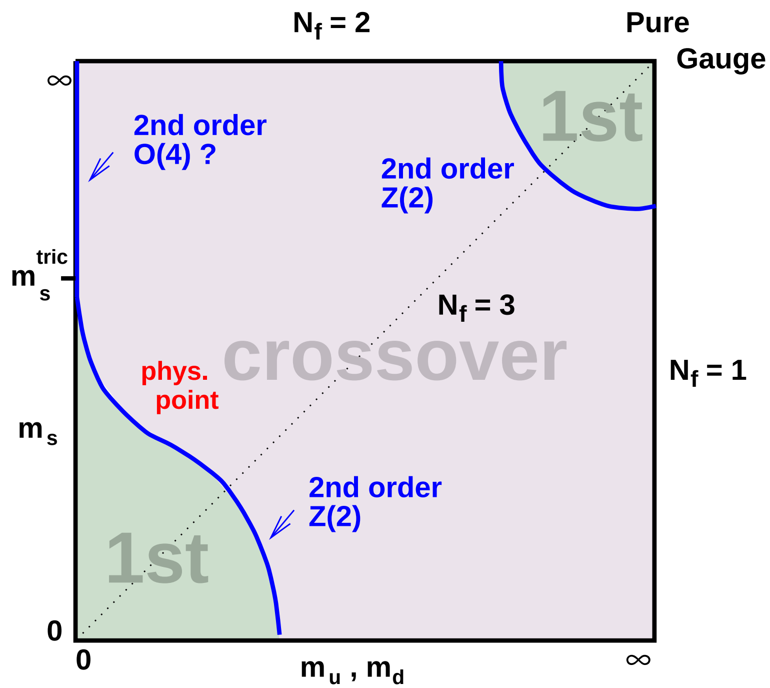
- chiral symmetry of 2 flavor QCD is:

$$SU_L(2)\times SU_R(2) \simeq O(4)$$

- hence, if m_s is large enough in 2+1 QCD

  expect universal behavior as of 3d $O(4)$ spins
  in the vicinity of finite T chiral limit, $m_l \rightarrow 0$
  [Pisarski, Wilczek, ...]

- so far no clear evidence from numerical simulations and there are caveats due to the fate of the $U_A(1)$
  [long list of refs.]



- staggered fermions preserve a flavor non-diagonal $U(1)$ part of chiral symmetry even at $a > 0$

  $\Rightarrow$ search for $O(2)$ critical behavior in the vicinity of chiral limit also at $a > 0$

magnetic equation of state in O(N) spin models

magnetization $M = h^{1/\delta} f_G(z)$

where $z = t/h^{1/\beta\delta}$

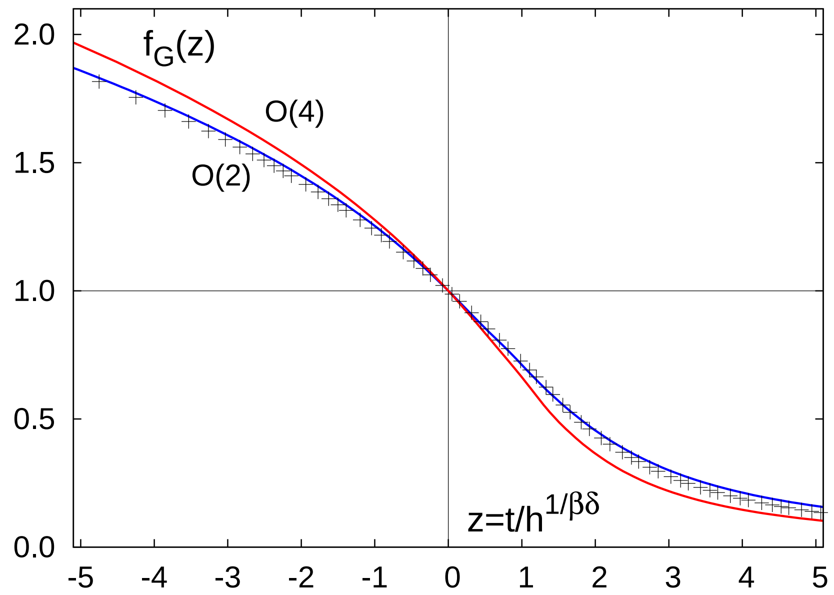
$f_G(z)$ universal scaling function

$t = \frac{1}{t_0}\frac{T - T_c}{T_c}$ reduced temperature

$h = \frac{H}{h_0}$ external field

β, δ universal critical exponents

t_0, h_0 non-universal normalizations



note: difference between O(2) and O(4) tiny

$f_G(z), \beta, \delta$ known to high precision Engels et al., condensed matter literature

in the limit $z \to -\infty$, corresponding to $h \to 0$ and $t < 0$

$f_G(z) \to (-z)^\beta \{1 + \tilde{c}_2(-z)^{-\beta\delta/2}\}$ corresponding to $M(t,h) \to M(t,0) + c_2(t)\sqrt{h}$

Goldstone effect

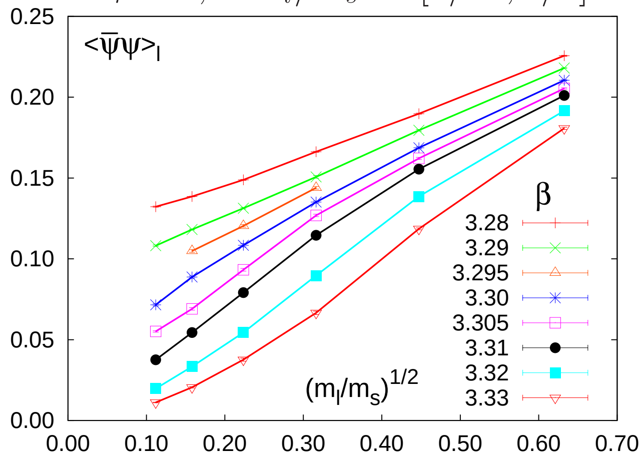
magnetic equation of state in QCD

here $M \sim \langle\bar{\psi}\psi\rangle_l \qquad H \sim m_l$

we use $M_b = m_s\langle\bar{\psi}\psi\rangle_l$

$$M = m_s\left[\langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s\right]$$

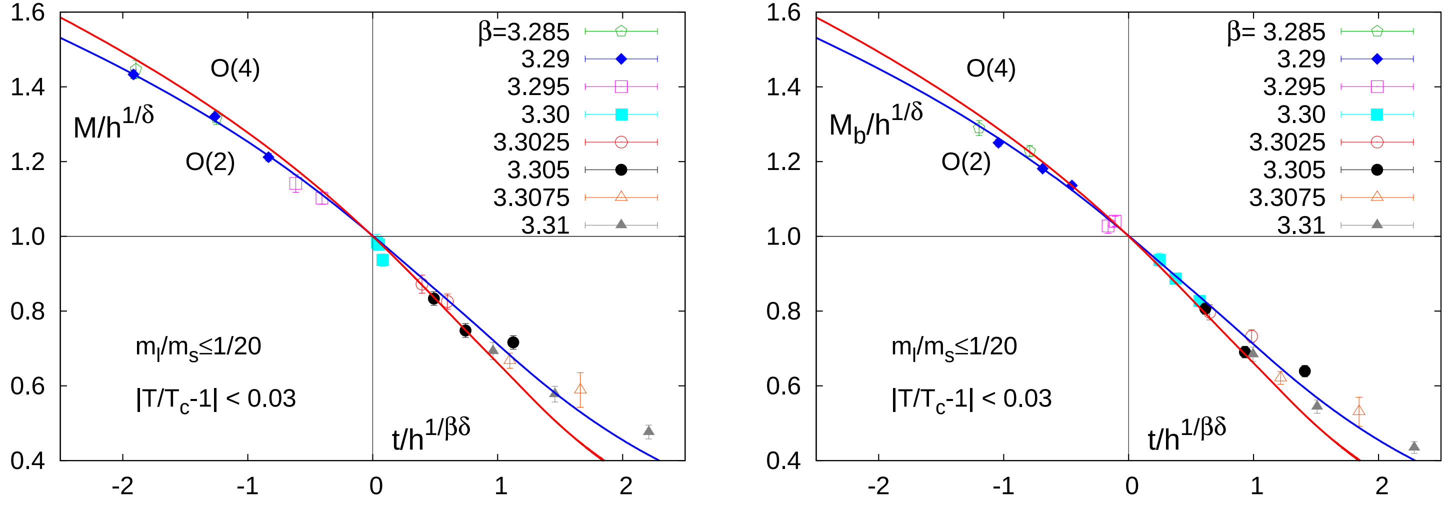
$N_\tau = 4, \quad m_l/m_s \in [1/80, 2/5] \quad m_s$ fixed



⋆ clear evidence for $\sqrt{m_l}$ dependence at low β
i.e. for Goldstone effect

– fit to the $O(2)$ scaling function $f_G(z)$, 3 fit parameters: $t_0, h_0, T_c(m_l = 0)$

m_l/m_s	m_π
1/80	75 MeV
1/40	105 MeV
1/20	150 MeV



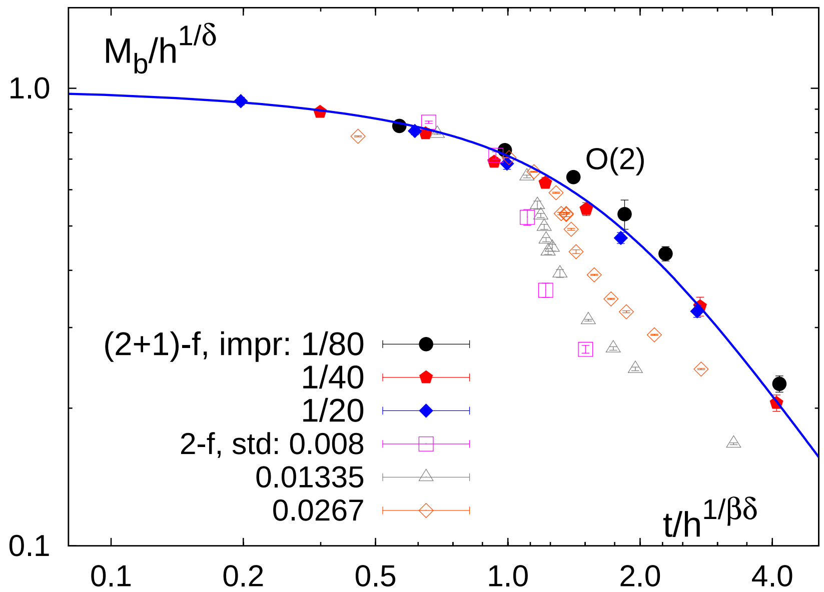
consistency with $O(N)$ scaling behavior at small quark masses

comparison to staggered data in the standard discretization

until now

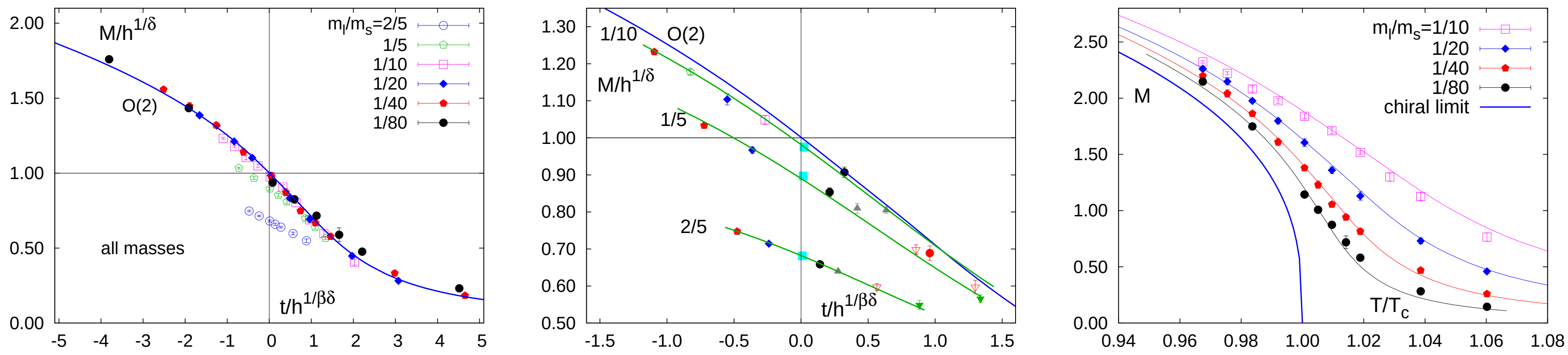
comparison with magn. EoS
for $z > 0$ only

similar difficulties with fits for critical exponents



MILC, Pisa data

scaling violations



– are clearly visible in the region of small $|z|$ i.e. large m_l and small t, and can be parametrized as

$$M(t,h) = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + a_t th + b_1 h + b_3 h^3 + b_5 h^5$$

since the scale parameters t_0, h_0 have been determined above

– for the linear term: $b_1 = 0$ for M (subtracted condensate)
$b_1 \neq 0$ for M_b

–> curves and chiral extrapolation for $M(T)$

⋆ scaling violations are small for physical values of the quark mass

Summary

- Equation of State
  - extending analysis to $m_\pi \simeq 150$ MeV on $N_\tau = 12$ lattices
  - at low T: reducing lattice spacing and quark mass leads to approaching HRG
  - at high T: making contact with (resummed) perturbation theory and EQCD

- Transition
  - reducing a and m_l affects the studied observables in the same way
  - suggest a continuum extrapolated value of $T_c \lesssim 170$ MeV preliminary

- critical behavior ?
  - finally, strong indications for $O(N)$ scaling of the magnetic equation of state !
  - although at large lattice spacing, hints at physical point possibly being in the attraction basin of a second order chiral phase transition
  - optimistic signal for further studies of the QCD phase diagram