First results for the hadron spectrum using a new quark smearing method

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The Hadron Spectrum Collaboration:

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Overview

- Physics projects of HSC
- Vacuum ensembles
- Measurements
    - Isovector and isoscalar mesons
    - Excited states
    - Spin
    - New method: distillation
- Results
    - Isovector meson spectrum
    - First look at isoscalar mesons
- Conclusions

Hadron Spectrum Collaboration - physics projects

- Map the light hadron spectrum
  - Isovector mesons including hybrids and exotics.
  - Isoscalar mesons and the glueballs.
  - Nucleon spectrum and excitations.
  - Mesons and baryons with strange quarks.
  - Charmonium.
- Make progress towards computing widths of resonances
  - First step - multi-meson states
- Compute radiative transitions for GlueX photoproduction

HadSpec lattice ensembles

- Quark field dynamics included in the importance sampling
- 2+1 dynamical flavours
- Anisotropic lattice to enhance temporal resolution.
  Non-perturbatively tuned to $a_s/a_t = 3.5$.
- Tree-level Symanzik-improved gauge action
- Sheikholeslami-Wohlert quark action
- Spatial stout-link background for quark propagation

Volume	$a_t m_s^0$	$a_t m_l^0$	m_π/m_ρ
$16^3 \times 128$	-0.0742	-0.0742	0.6880(18)
$16^3 \times 128$	-0.0742	-0.0808	0.571(5)
$16^3 \times 128$	-0.0742	-0.0830	0.490(6)
$24^3 \times 128$	-0.0742	-0.0840	0.447(4)

Hadron spectroscopy (1)

- **Masses** of (colourless) QCD bound-states can be computed by measuring **two-point functions**. The Euclidean two-point function is

$$C(t) = \langle 0|\Phi(t)\Phi^\dagger(0)|0\rangle$$

- The time-dependence of the operator, Φ is given by $\Phi(t) = e^{Ht}\Phi e^{-Ht}$, so

$$C(t) = \langle \Phi|e^{-Ht}|\Phi\rangle$$

inserting a complete set of energy eigenstates gives

$$C(t) = \sum_{k=0}^{\infty}\langle \Phi|e^{-Ht}|k\rangle\langle k|\Phi\rangle = \sum_{k=0}^{\infty}|\langle \Phi|k\rangle|^2 e^{-E_k t}$$

- Then $\lim_{t\to\infty} C(t) = Ze^{-E_0 t}$
- If the large-time exponential fall-off of the correlation function can be observed, the energy of the state can be measured.

Hadron spectroscopy (2)

- The energies of **excited states** can be computed reliably too.
- Tracking sub-leading exponential fall-off works sometimes but a more efficient method is to use a matrix of correlators. With a set of N operators $\{\Phi_1, \Phi_2, \ldots\}$ (with the same quantum numbers), compute all elements of

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

- Now solve the generalised eigenvalue problem

$$C(t_1)v = \lambda C(t_0)v$$

  for different t_0 and t_1. [M. Lüscher + U. Wolff, C. Michael]
- The method constructs an optimal linear combination to form a ground-state, and then constructs a set of operators that are orthogonal to it.
- The second eigenvector can not have overlap with the ground-state at large t, and will fall to the first excited energy.

Spin on the lattice

- Eigenstates of the hamiltonian simultaneously form irreducible representations of $SO(3)$, the rotation group. Spin is a good quantum number.
- The lattice hamiltonian does not have $SO(3)$ symmetry. It is symmetric under the discrete sub-group of **rotations of the cube**, O_h. This group has 48 elements (once parity is included) and ten irreducible representations.
- The eigenstates of the lattice hamiltonian therefore have a good "quantum letter"; $A_1^{u,g}, A_2^{u,g}, E^{u,g}, T_1^{u,g}, T_2^{u,g}$
- Can we deduce the continuum spin of a state? With some caveats, yes.
- A pattern of degeneracies must be found and matched against the representations of O_h subduced from $SO(3)$.

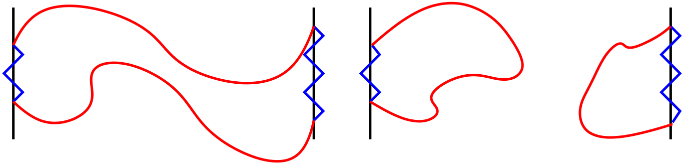
Isoscalar meson correlation functions

- If we are interested in measuring isoscalar meson masses, extra diagrams must be evaluated, since four-quark diagrams become relevant. The Wick contraction yields extra terms, since

$$\langle \psi_i \bar{\psi}_j \psi_k \bar{\psi}_l \rangle = M_{ij}^{-1} M_{kl}^{-1} - M_{il}^{-1} M_{jk}^{-1}$$

- Now

$$\langle 0|\Phi_{I=0}(t)\Phi^{\dagger}_{I=0}(0)|0\rangle =$$
$$\langle 0|\Phi_{I=1}(t)\Phi^{\dagger}_{I=1}(0)|0\rangle - \langle 0|\mathrm{Tr}\ M^{-1}\Gamma U_{\mathcal{C}}(t)\mathrm{Tr}\ M^{-1}\Gamma U_{\mathcal{C}}(0)|0\rangle$$



The need for better (statistical) precision

- Excited states fall faster than ground-states.
- Gluonic excitations (in hybrids, isoscalar states, glueballs) are intrinsically noisy.
- Maiani-Testa - no direct access to decay matrix elements from Euclidean field theory but decay widths can be inferred indirectly from the dependence of the energy spectrum on the lattice volume (Lüscher).

Want to get as much information about quark propagation as possible

Smearing - an essential ingredient for precision

- To build an operator that projects effectively onto a low-lying hadronic state need to use smearing
- Instead of the creation operator being a direct function applied to the fields in the lagrangian first smooth out the UV modes which contribute little to the IR dynamics directly.
- A popular gauge-covariant smearing algorithm; Jacobi/Wuppertal smearing: Apply the linear operator

$$\Box_J = \exp(\sigma \Delta^2)$$

- Δ^2 is a lattice representation of the 3-dimensional gauge-covariant laplace operator on the source time-slice

$$\Delta^2_{x,y} = 6\delta_{x,y} - \sum_{i=1}^{3} U_i(x)\delta_{x+\hat{\imath},y} + U_i^\dagger(x-\hat{\imath})\delta_{x-\hat{\imath},y}$$

- Correlation functions look like $\mathrm{Tr}\ \Box_J M^{-1} \Box_J M^{-1} \Box_J \ldots$

Redefine smearing

- After tuning the free parameter σ it turns out $\Box_J$ is a very low rank operator.
- The choice of smearing operator is arbitrary, provided
    1. It is a scalar operator
    2. It is gauge covariant
    3. It is a function of only field on time-slice t (or perhaps a few nearest neighbours?)
- Redefine smearing to be a projection operator onto a low-dimensional space of fields:

$$\Box = \sum_{k=1}^{M} v^{(k)} \otimes v^{(k)*}$$

- This is distillation.
- How to choose v? One simple choice is to use the lowest M eigenvectors of Δ^2

Distilled correlation functions

- Why is this helpful? Look at correlation functions such as an isovector meson two-point function

$$C_{AB}(t_1, t_0) = \mathrm{Tr}\ \square(t_1)\Gamma_1\square(t_1)M_u^{-1}(t_1, t_0)\square(t_0)\Gamma_0\square(t_0)M_d^{-1}(t_0, t_1)$$

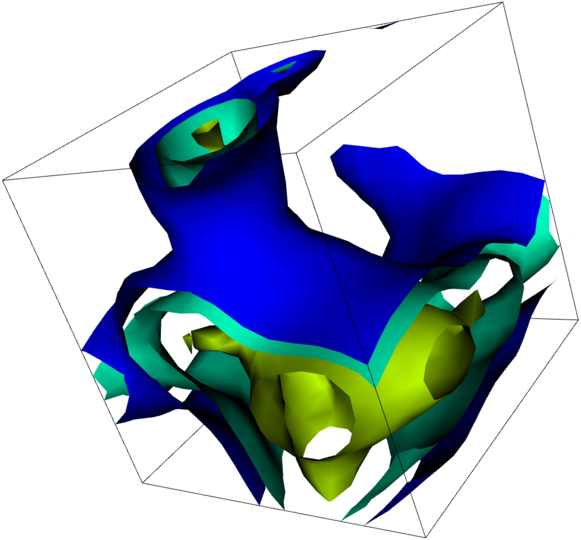
- $\Gamma_{1,2}$ are creation operators that make mesons with appropriate quantum numbers
- Inserting the definition of the distillation operator, the correlation function becomes a trace over a product of rank-M matrices.

$$C_{AB}(t_1, t_0) = \mathrm{tr}\ \Phi_1(t_1)\tau(t_1, t_0)\Phi_0(t_0)\tau(t_0, t_1)$$

with

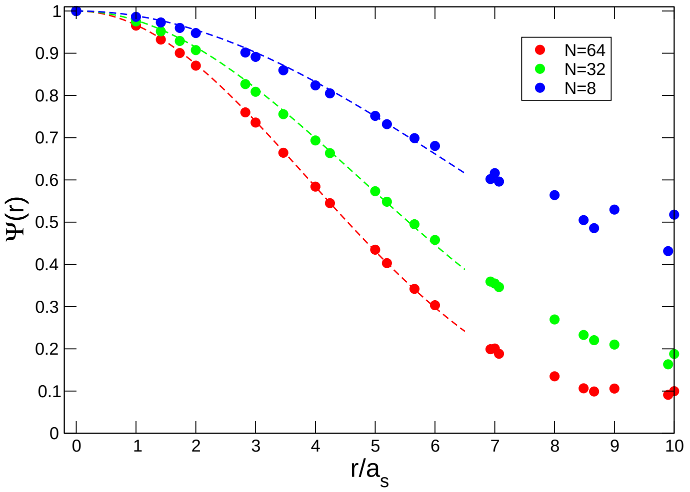
$$\Phi_a^{(i,j)} = v^{(i)*}\Gamma_a v^{(j)} \text{ and } \tau_a^{(i,j)} = v^{(i)*}(t_1)M^{-1}(t_1, t_0)v^{(j)}(t_0)$$

The lowest eigenvector of the laplace operator



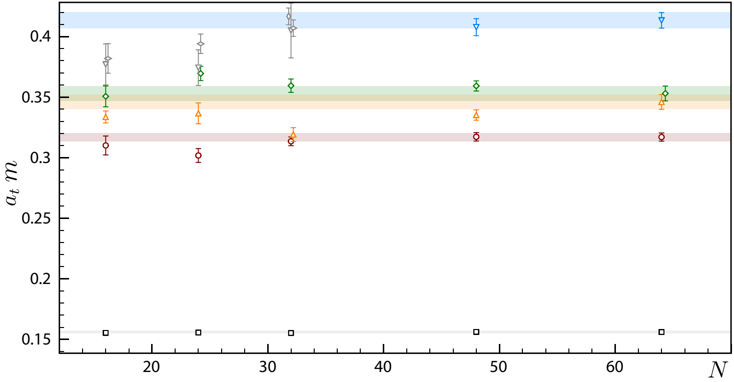
- Localised mode - size is confinement scale

Distribution of distillation operator



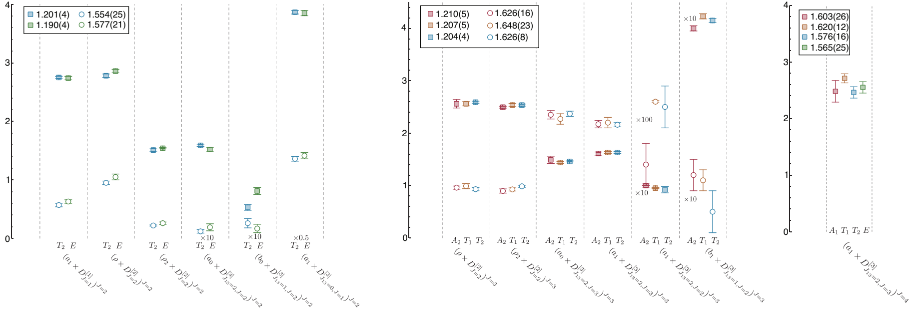
- Distillation operator is rotationally symmetric and gaussian

How big should the distillation space be?



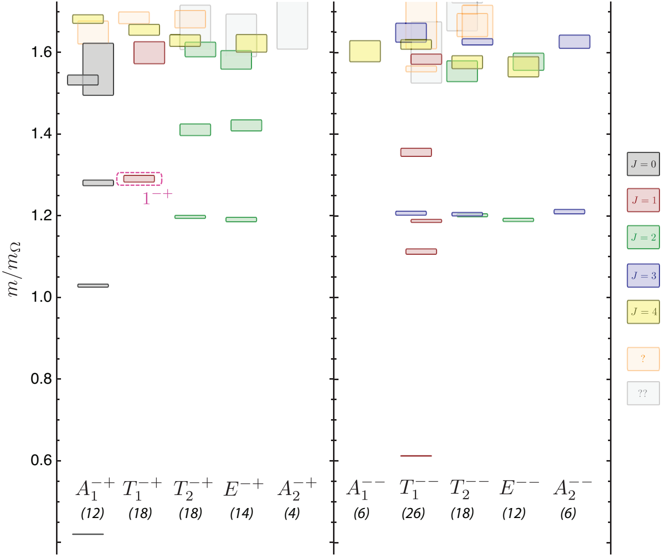
- 16^3 spatial volume. Volume approx 2fm.
- Still big, but feasible here - now in production
- Volume dependence a problem: Cost $\propto V^2$

Continuum spin identification

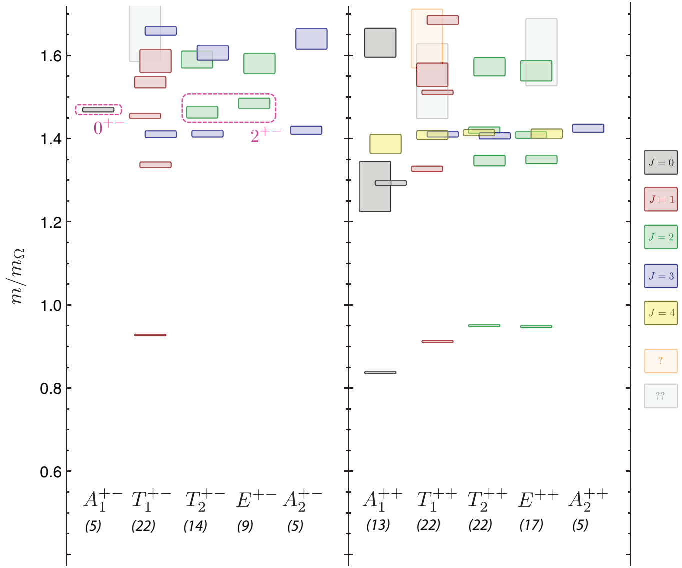


- Take continuum operator basis and then subduce, using a lattice representation of the derivative.
- Overlap of operator onto state - should be the same for all polarisations
- Renormalisation - mild because fields are smeared.
- It seems to work! Test with well-established cases
- First identification of spin-4 state

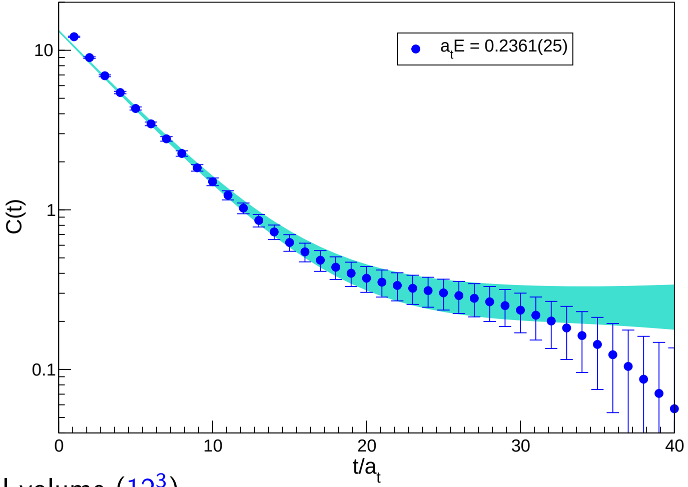
Isovector meson spectrum: PC=−+ and −−



Isovector meson spectrum: PC=$+-$ and $++$



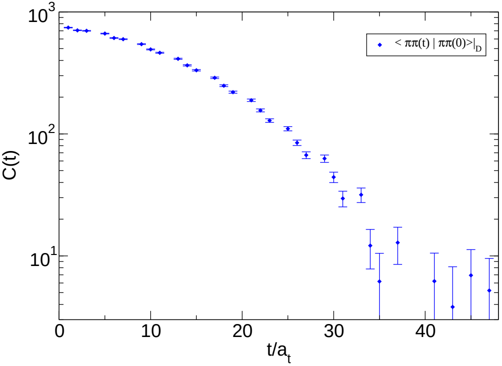
Isoscalar A_1^{-+} correlation function

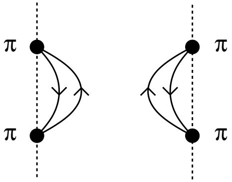


- Small volume (12^3)
- Fit requires a constant: $C(t) = A_0 + A_1 e^{-A_2 t}$ - volume artefact?
- Precise result - $a_t E$ determined to 1% instead of 10%

$\pi\pi_{I=0}$ - $\pi\pi_{I=0}$ correlation function (disconnected part)

Initial tests on small lattices





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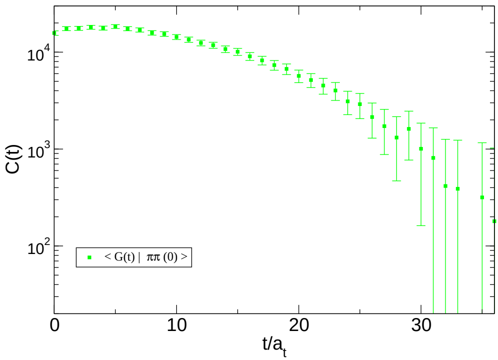
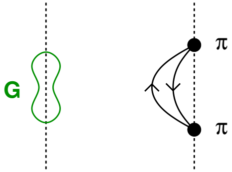
Glueball - $\pi\pi$ correlation function

Initial test on small lattices





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Conclusions

- Hadron Spectrum Collaboration making progress determining the spectrum of excited isovector mesons using anisotropic lattice combined with new measurement techniques.
- Recent progress - developed a new algorithm (distillation) for quark smearing that facilitates precise spectroscopy measurements.
- Spin identification seems to work. Identified three exotic hybrids and three spin-4 states.
- Distillation enables isoscalar correlation function measurements
- Problem: V^2 scaling - solutions under investigation
- Next step - multi-hadron operators to investigate resonances then analyse data from multiple volumes