The QCD phase transition probed by fermionic boundary conditions

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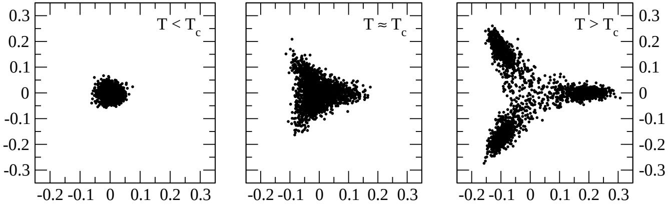
SFB/
TR 55

QCD phase transition

finite temperature, $\mu = 0$, quenched

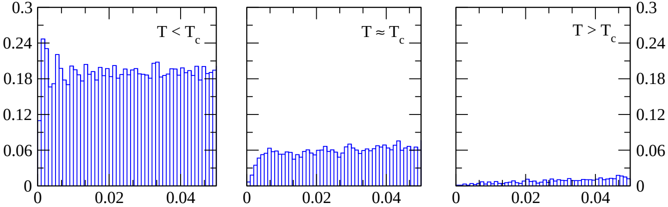
Polyakov loop trace tr P:



'deconfinement'

center symmetry broken

chiral condensate $\equiv$ spectral density of the Dirac operator D at $\lambda = 0$:



'spectral gap'

chiral symmetry restored

same T_c in quenched case: related!?

Fermionic boundary conditions

phase 'twisted' temporal boundary conditions:

$$\psi(x_0+\beta,\vec{x}) = z\,\psi(x_0,\vec{x}), \qquad z = e^{i\phi} \quad \text{imag. chemical potential}$$

as a probe, physical: $z=-1, \phi=\pi$ Gattringer '06

realised on the lattice:

$$U_0 \to z U_0 \quad \text{at some time slice}$$

formally: link U_0 in $U(N_c)$

- a numerical fact:

for $T > T_c$ a chiral condensate remains for $\phi = -\text{angle(P)}$

Chandrasekharan, Christ '95; Meisinger, Ogilvie '95; Stephanov '96

Gattringer, Rakow, A. Schäfer, Söldner '02; Gattringer, S. Schäfer '03

Bornyakov, Luschevskaya, Morozov, Polikarpov, Ilgenfritz, Müller-Preussker '08; Kovacs '08

where constant links ['vacua'] with same P allow for zero modes

The idea

- different closed loops W get different factors:

  $P \to e^{i\phi} P$ as well as all loops winding once

  $P^\dagger \to e^{-i\phi} P^\dagger$ as well as all loops winding minus once

  plaquettes stay as well as all 'trivial' loops

  (reconstruction of P from $D_\phi^{N_0}$ with three ϕ's FB et al. '06)

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- all gauge invariant observables can be expanded in closed loops

propagator:

$$\mathrm{Tr}\,\frac{1}{m+D_\phi} = \frac{1}{m}\sum_{k=0}^{\infty}\frac{(-1)^k}{m^k}\,\mathrm{Tr}(D_\phi)^k$$
$$= \frac{1}{m}\sum_{\text{loops } W} c(m;W)\ \mathrm{tr}\prod_W U_\mu(x)\ e^{i\phi q(W)}$$

$q(W)$: how many times the loop winds around $[0,\beta]$
staggered, $D \sim U$: $c = \frac{\pm 1}{(2am)^{\text{length(W)}}}$

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- different factors of $e^{i\phi}$ can be extracted via a Fourier transform

A new observable

FB et al. '08

$$\tilde{\Sigma}_1 \equiv \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi} \frac{1}{V} \left\langle \mathrm{Tr} \frac{1}{m + D_\phi} \right\rangle = \frac{1}{mV} \sum_{\substack{\text{loops } W \\ \text{winding once}}} c(m; W) \left\langle \mathrm{tr} \prod_W U_\mu(x) \right\rangle$$

dual condensate — dressed Polyakov loops

massless limit:

$$\lim_{m \to 0} \lim_{V \to \infty} \tilde{\Sigma}_1 = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi} \langle \bar{\psi}\psi \rangle_\phi$$

chiral condensate with boundary angle ϕ integrated over

massive limit:

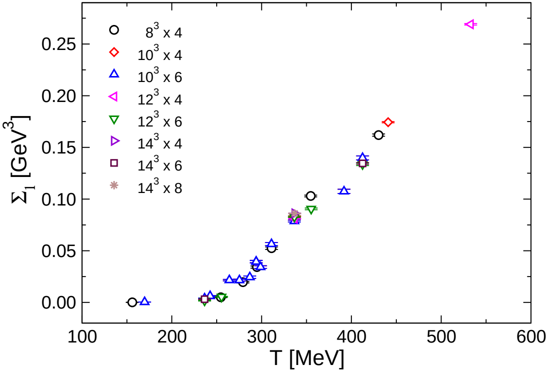
$$\lim_{m \to \infty} \tilde{\Sigma}_1 \sim \langle \mathrm{tr}\, P \rangle$$

thin Polyakov loop (shortest) detours suppressed by m

Order parameter

$m = 100 MeV$, quenched case:

FB et al. '08



different lattice spacings and volumes
(staggered valence fermions)

same pattern as for the conventional Polyakov loop

same transformation under center symmetry

needs less renormalisation!?

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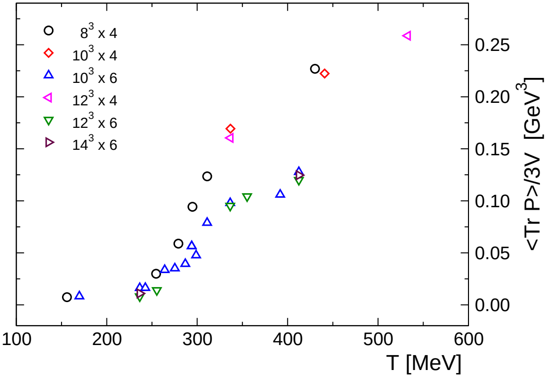

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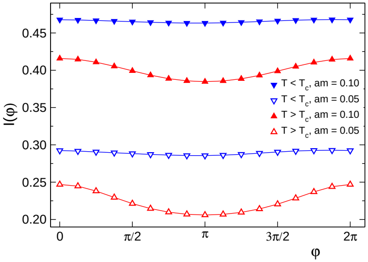
Mechanism

fermions respond differently to bc.s in confined and deconfined phase

propagator = integrand of $\tilde{\Sigma}_1$ without Fourier factor: FB et al. '08



real tr P (non-real tr P: plot shifted by $\pm 2\pi/3 \Rightarrow 2\pi/3$-periodicity)

question: $\tilde{\Sigma}_1$ behaves like P, but $\langle\bar{\psi}\psi\rangle$ opposite to P?!

conf. phase: chiral condensate ✓ but ϕ-independent $\stackrel{\textit{Fourier}}{\longrightarrow} \tilde{\Sigma}_1 = 0$

deconf. phase: chiral condensate for periodic bc.s $\stackrel{\textit{Fourier}}{\longrightarrow} \tilde{\Sigma}_1 \neq 0$

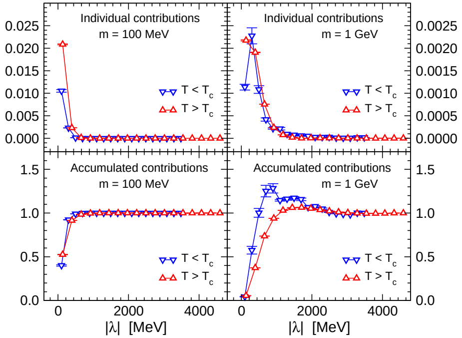
Spectral representation

Tr means sum over all eigenmodes:

$$\tilde{\Sigma}_1 \equiv \int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{-i\phi} \frac{1}{V} \left\langle \mathrm{Tr} \frac{1}{m + D_\phi} \right\rangle = \int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{-i\phi} \frac{1}{V} \left\langle \sum_i \frac{1}{m + \lambda_\phi^{(i)}} \right\rangle$$

truncate the ev sum: IR dominance

FB et al. '08



expected: lowest modes most sensitive to bc.s, λ in denominator convergence in the continuum Synatschke, Wipf, Langfeld '08

all powers of D_ϕ Fourier transformed:

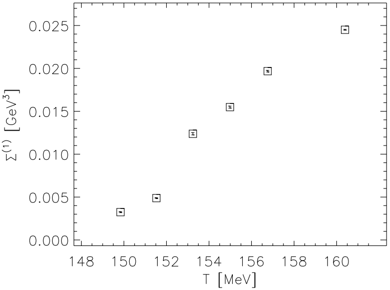
$$\int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{-i\phi} \mathrm{Tr}\,(D_\phi)^k$$

- transform like the Polyakov loop under center symmetry
- converge
  higher eigenvalues λ_ϕ less and less sensitive to bc.s ϕ
  heat kernel methods

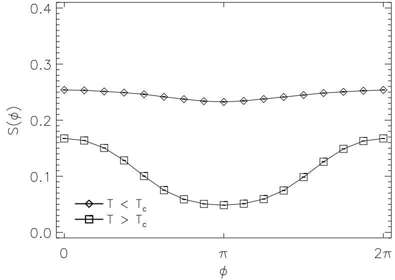
fermion determinant prefers real tr *P*: no strict center symmetry

`preliminary` (on MILC configurations): FB et al. '09

order parameter:



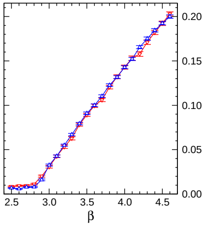
integrand:



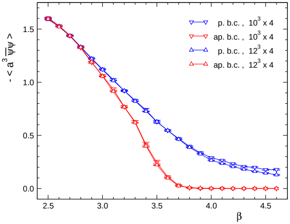
as expected: behaviour in confined phase like in deconfined one
with small real Polyakov loop cf. Söldner '07

testbed since: $T_c^{\text{chiral}} \sim 4\,T_c^{\text{deconf}}$ $(SU(2))$ Bilgici, Gattringer, Ilgenfritz, Maas '09

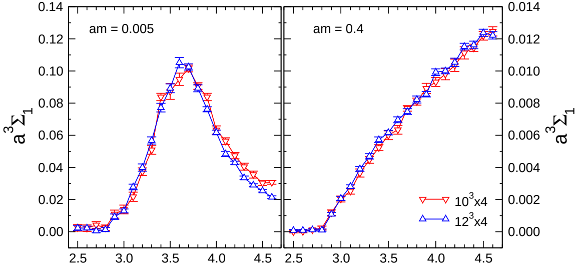
adjoint Polyakov loop:



per. and antiper. condensates:

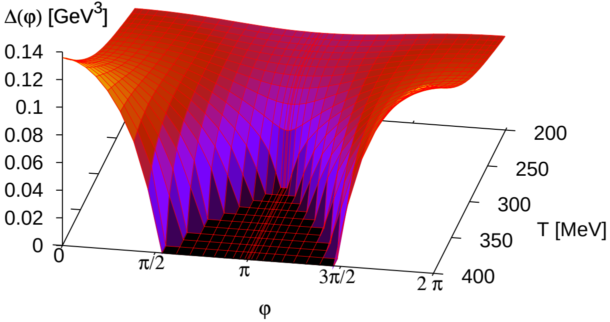


$\tilde{\Sigma}_1$ for small and large mass:



loop integrals with ϕ-shifted Matsubara frequencies   Fischer, Müller '09

integrand of $\tilde{\Sigma}_1$ without Fourier factor in the chiral limit:



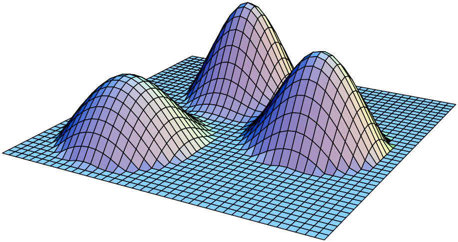
inf. volume limit, larger T's . . .

Relevant excitations!?

calorons $\equiv$ instantons at finite temperature

Harrington, Shepard '78

Kraan, van Baal; Lee, Lu '98



topological (action) density for total charge $Q = 1$ in $SU(3)$

substructure: N_c constituents = magn. monopoles/dyons

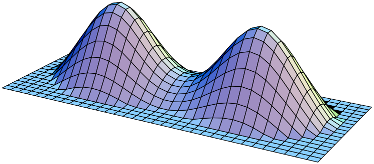
masses governed by asymptotic Polyakov loop

$$P_\infty = \lim_{|\vec{x}|\to\infty} P(\vec{x}) \ldots \text{holonomy}$$

conjecture: holonomy $\operatorname{tr} P_\infty \leftrightharpoons$ order parameter $\langle \operatorname{tr} P \rangle$

$\Rightarrow$ dyon massses sensitive to the phase of QCD

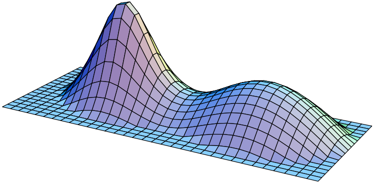
confined phase



[SU(2): 2 dyons]

equal masses

deconfined phase



heavy vs. light

fermionic zero modes: $\psi^0_{\phi\simeq 0}$ at light dyon, $\psi^0_{\phi\simeq\pi}$ at heavy dyon

mechanism above T_c: heavy dyons suppressed FB '09

$\Rightarrow \langle\bar\psi\psi\rangle_{\phi\simeq\pi}$ suppressed, $\langle\bar\psi\psi\rangle_{\phi\simeq 0}$ stays ✓ Bornyakov et al. '09

$\Rightarrow$ top. susceptibility suppr. ✓