(Old and) New developments for QCD at imaginary chemical potential

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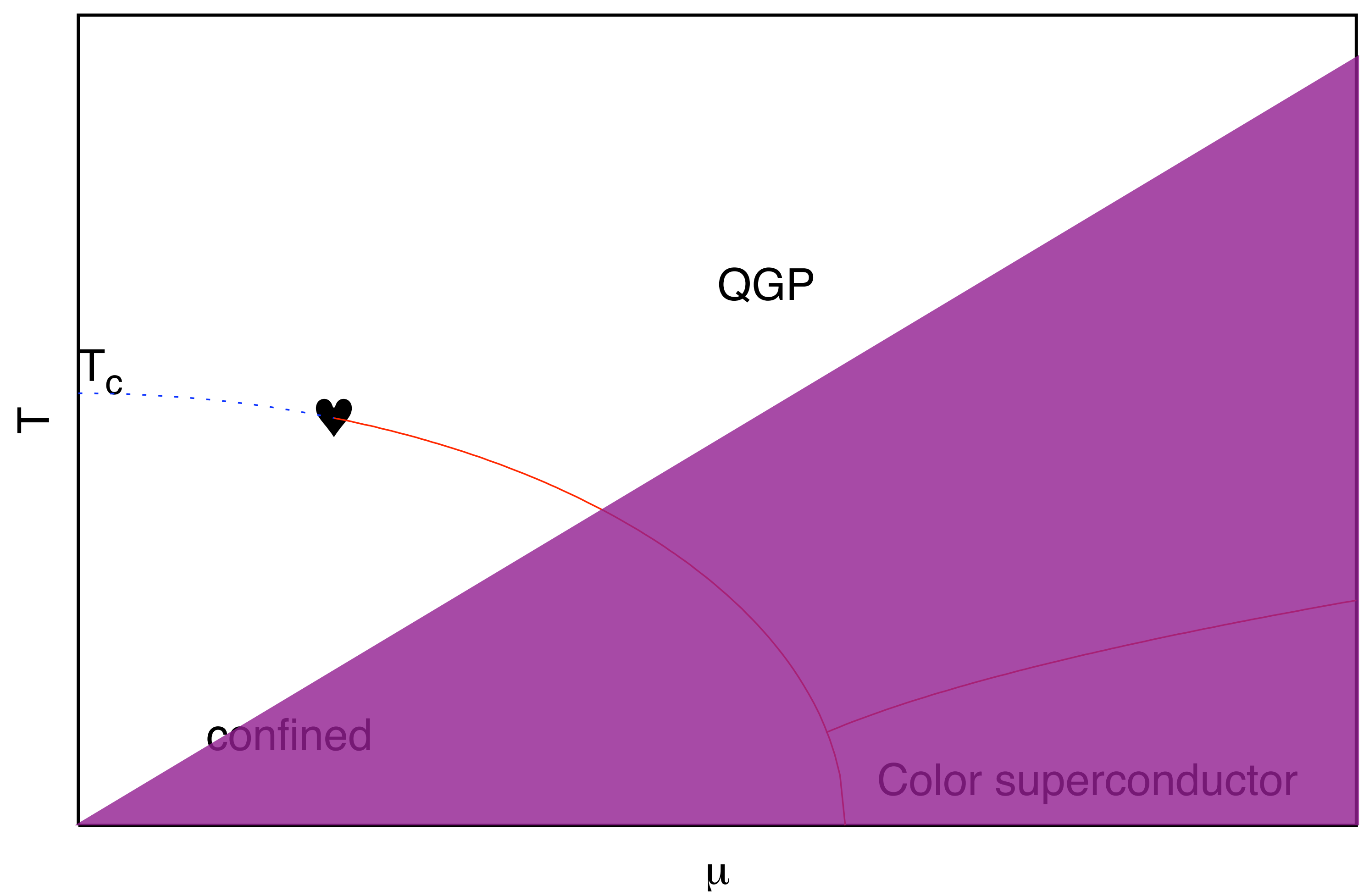
- Introduction: summary on QCD phase diagram
- taking imaginary μ more seriously
- Triple, critical and tri-critical structures at $\mu = i\frac{\pi T}{3}$
- Implications for the QCD phase diagram

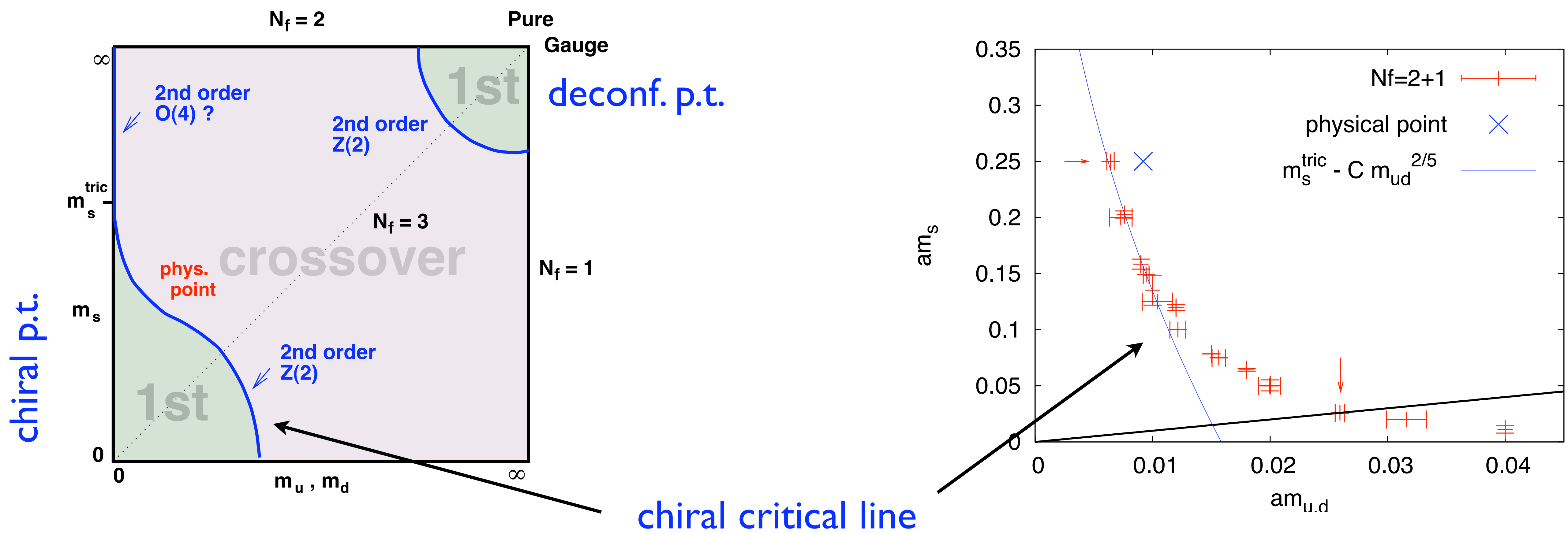
in collaboration with Ph. de Forcrand (ETH/CERN)

The calculable region of the phase diagram



- 2001-present: sign problem not solved, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \;\; (\mu = \mu_B/3)$
- Upper region: equation of state, screening masses, quark number susceptibilities etc. under control
- Here: phase diagram itself, most difficult!

Hard part: order of p.t., arbitrary quark masses $\mu = 0$



- physical point: crossover in the continuum — Aoki et al 06
- chiral critical line on $N_t = 4, a \sim 0.3$ fm — de Forcrand, O.P. 07
- consistent with tri-critical point at $m_{u,d} = 0, m_s^{\rm tric} \sim 2.8T$
- But: $N_f = 2$ chiral O(4) vs. 1st still open — Di Giacomo et al 05, Kogut, Sinclair 07
  $U_A(1)$ anomaly! — Chandrasekharan, Mehta 07, RBC-BI 09

The 'sign problem' is a phase problem

$$Z = \int DU \, [\det M(\mu)]^f \, e^{-S_g[U]}$$

importance sampling requires
positive weights

Dirac operator: $\not{D}(\mu)^\dagger = \gamma_5 \not{D}(-\mu^*)\gamma_5$

$\Rightarrow \det(M)$ complex for SU(3), $\mu \neq 0$

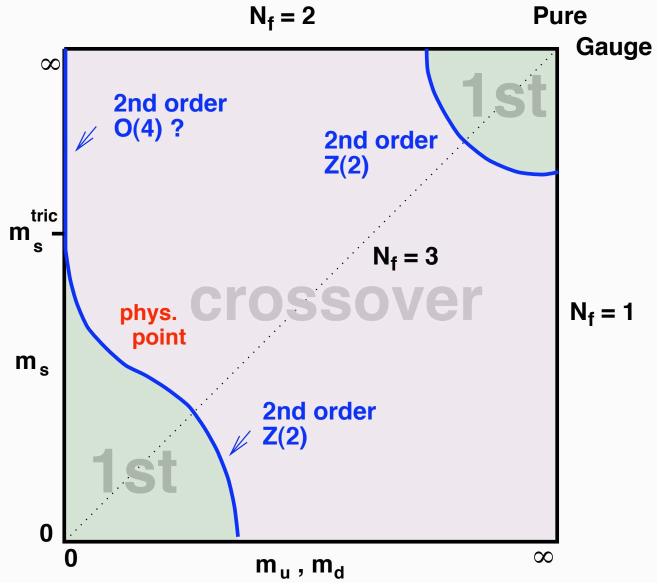
$\Rightarrow$ real positive for SU(2), $\mu = i\mu_i$

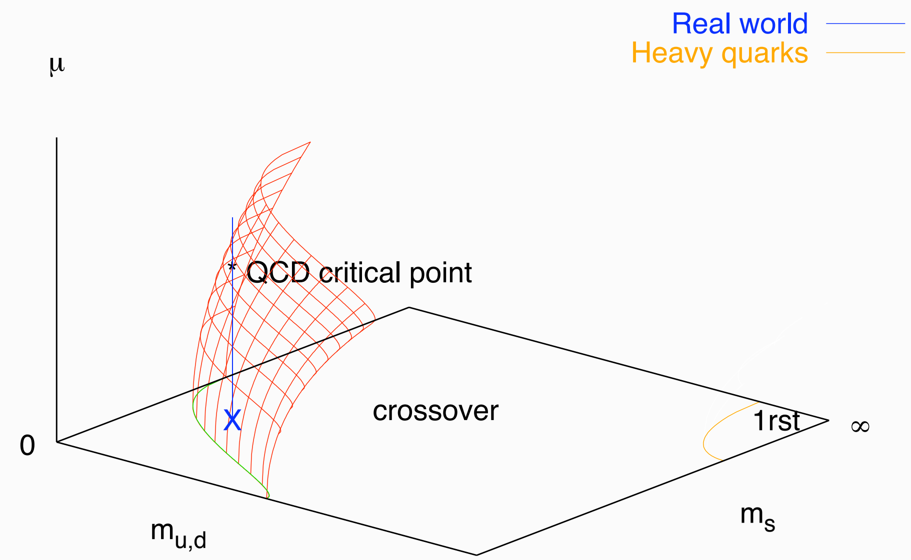
$\Rightarrow$ real positive for $\mu_u = -\mu_d$

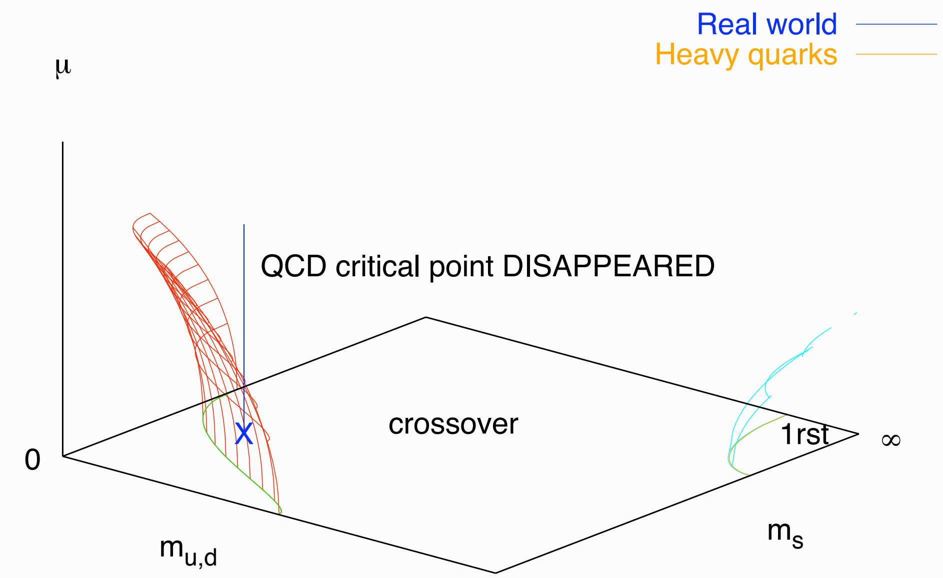
Cut-off effects: $T = \frac{1}{aN_t}$

Continuum limit: $N_t \to \infty, a \to 0$

Finite density: chiral critical line ⟶ critical surface

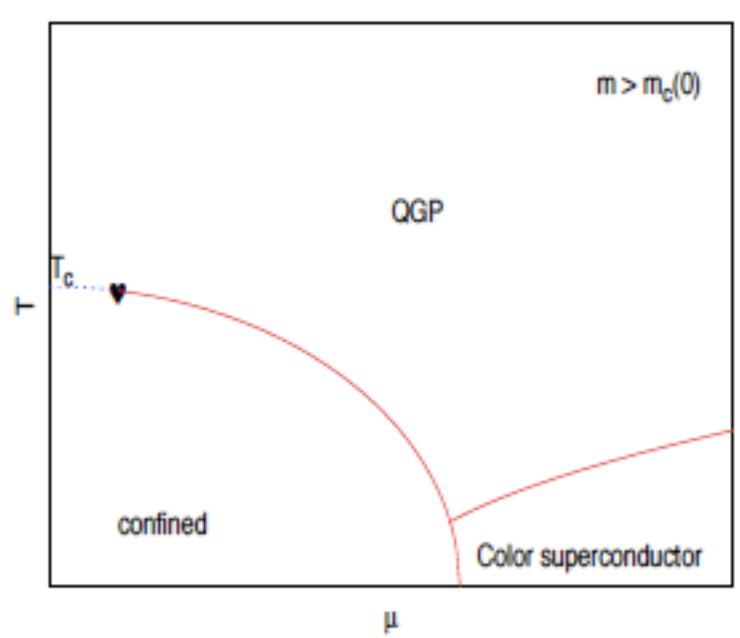


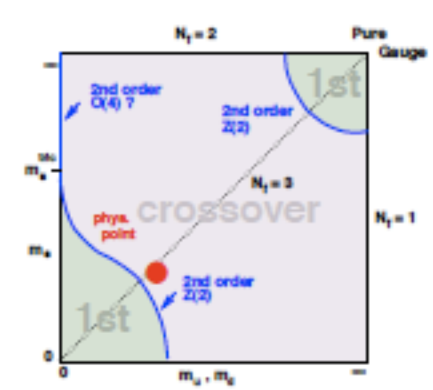


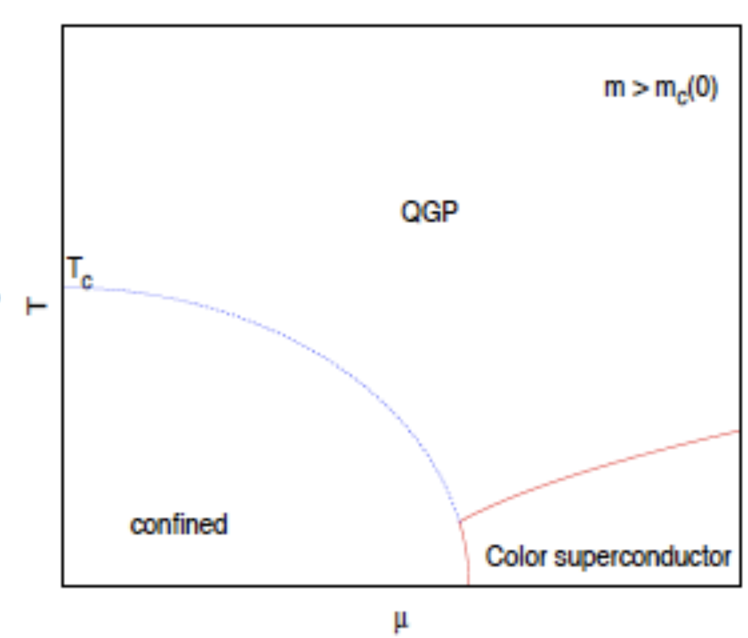


$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} \mathbf{c_k} \left(\frac{\mu}{\pi T}\right)^{2k}$$

$c_1 > 0$ $c_1 < 0$





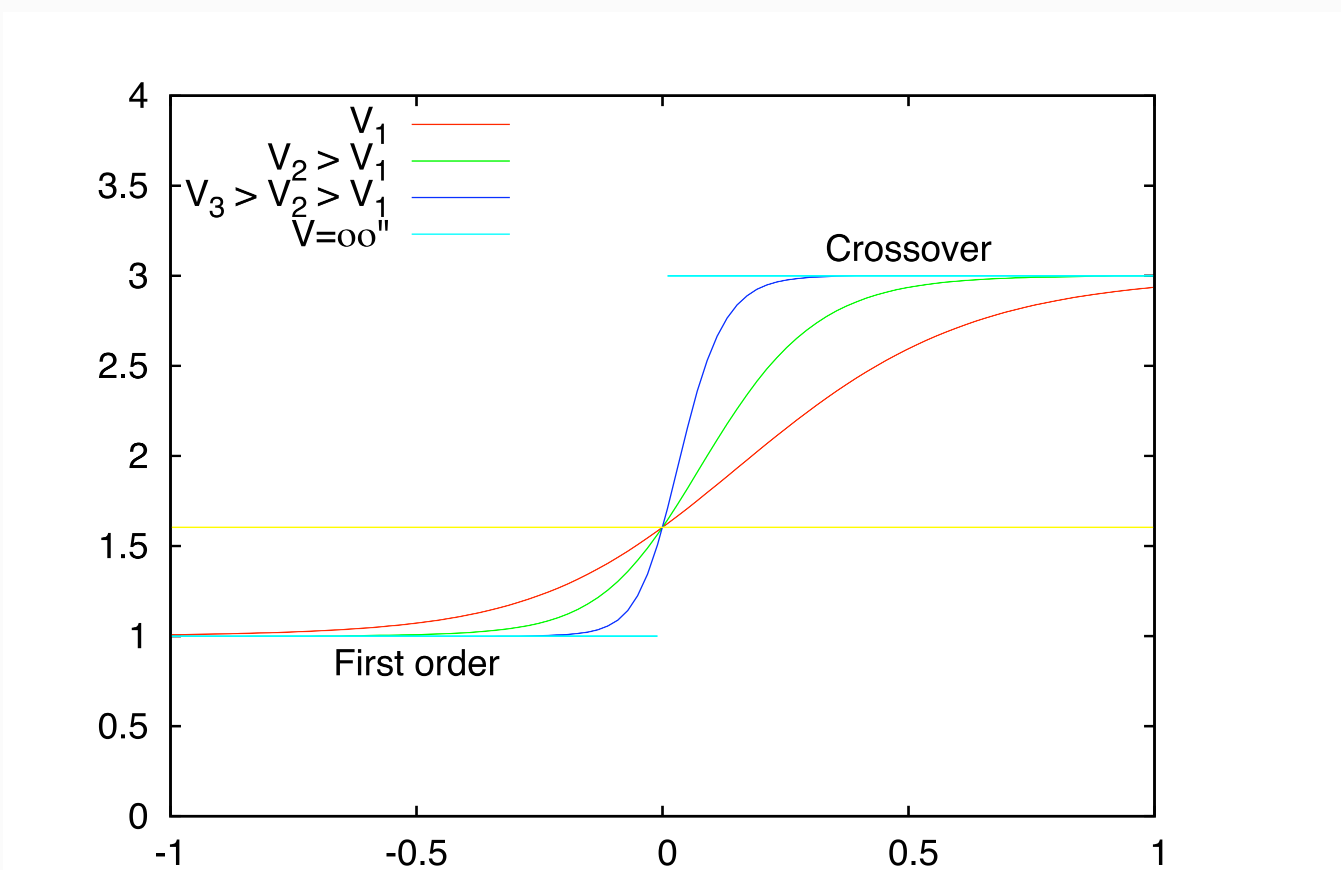


How to identify the order of the phase transition

$$B_4(\bar{\psi}\psi) \equiv \frac{\langle(\delta\bar{\psi}\psi)^4\rangle}{\langle(\delta\bar{\psi}\psi)^2\rangle^2} \xrightarrow{V\to\infty} \begin{cases} 1.604 & \text{3d Ising} \\ 1 & \text{first-order} \\ 3 & \text{crossover} \end{cases}$$

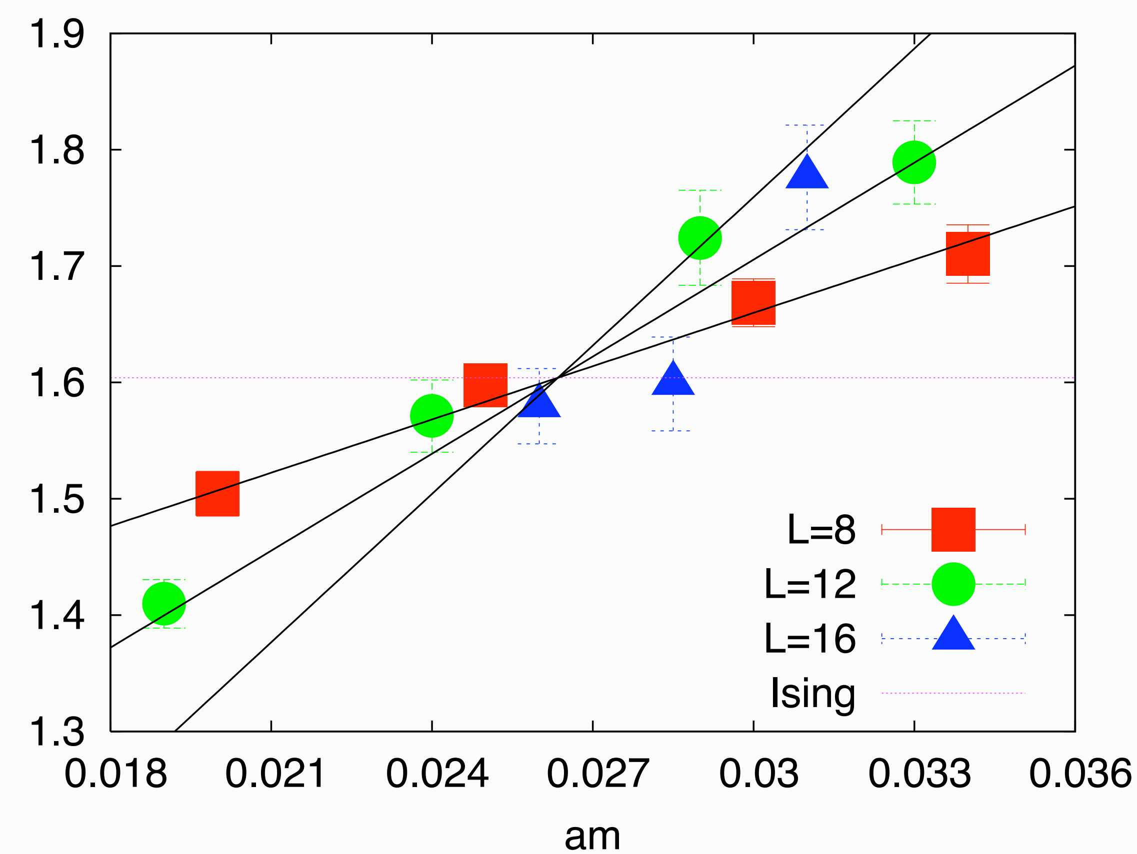
$\mu = 0:$ $\quad B_4(m,L) = 1.604 + bL^{1/\nu}(m - m_0^c), \quad \nu = 0.63$

B_4

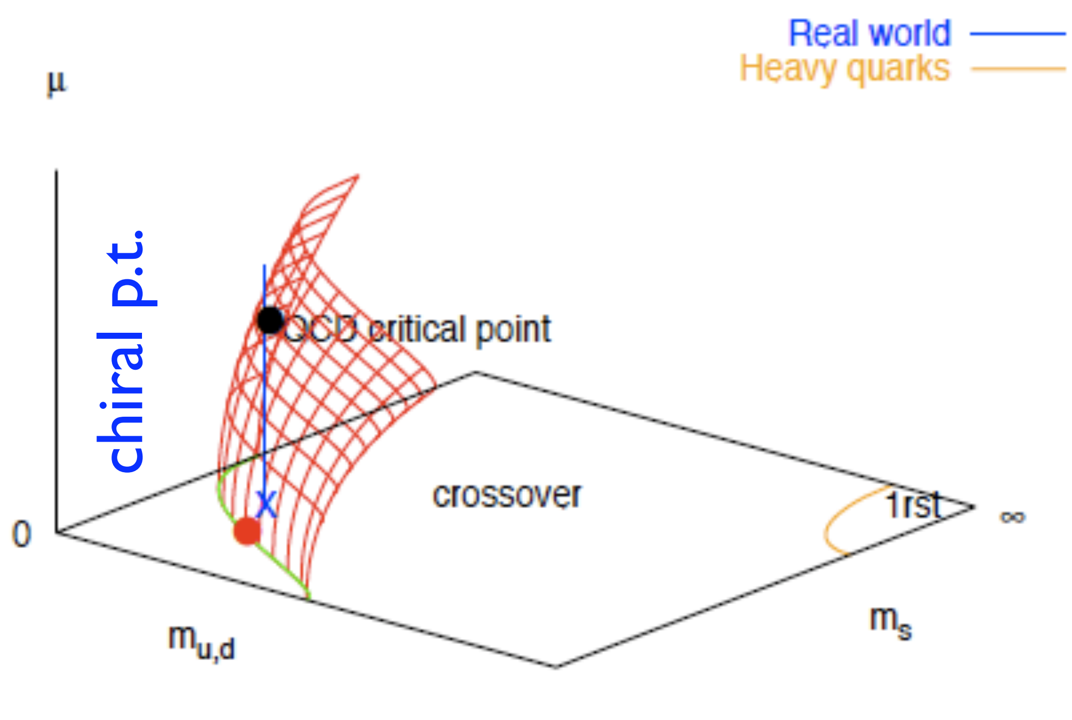


$x - x_c$

parameter along phase boundary, $T = T_c(x)$



$\mu \neq 0$, conservative: follow chiral critical line ➡ surface



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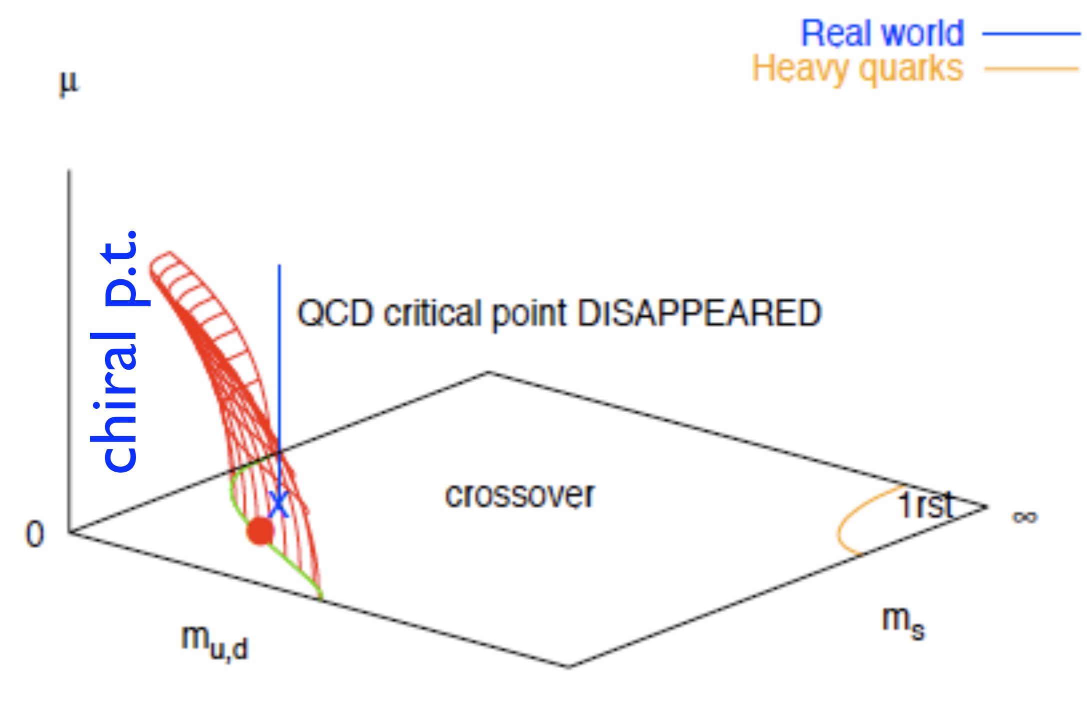


$c_1 < 0$

$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

$$\frac{d\,am^c}{d(a\mu)^2} = -\frac{\partial B_4}{\partial (a\mu)^2} \Big/ \frac{\partial B_4}{\partial am}$$

hard/easy

Numerical results for $N_f = 3, N_t = 4$

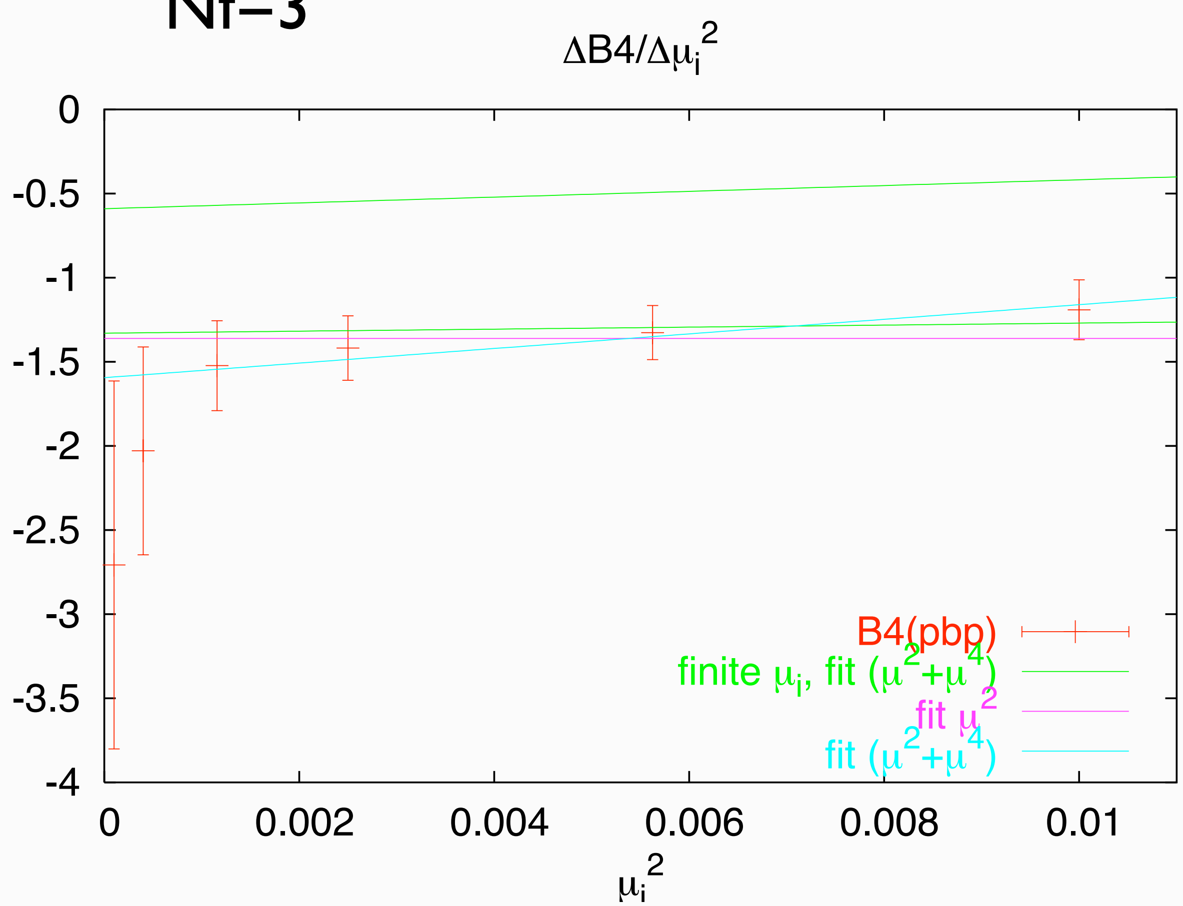
de Forcrand, O.P. 08

unimproved staggered fermions, RHMC algorithm

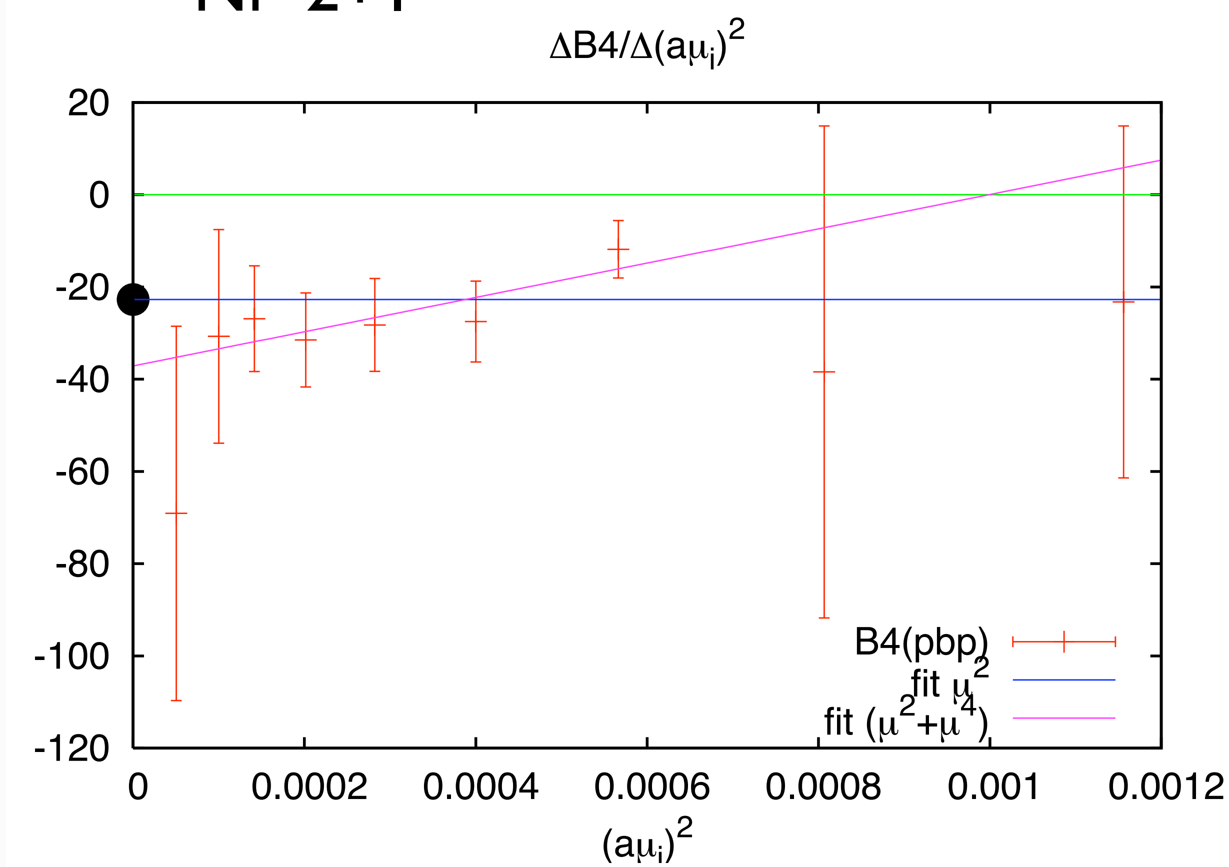
I. imag. μ : $8^3 \times 4, 42$ pairs $(am, a\mu_i)$ > 20 million traj., 18 unconstrained dof's in fits

II: deriv. at $\mu = 0$: $8^3, 12^3 \times 4 \quad m_\pi L \gtrsim 3, 4.5$ > 5 million, 0.5 million traj.

Nf=3



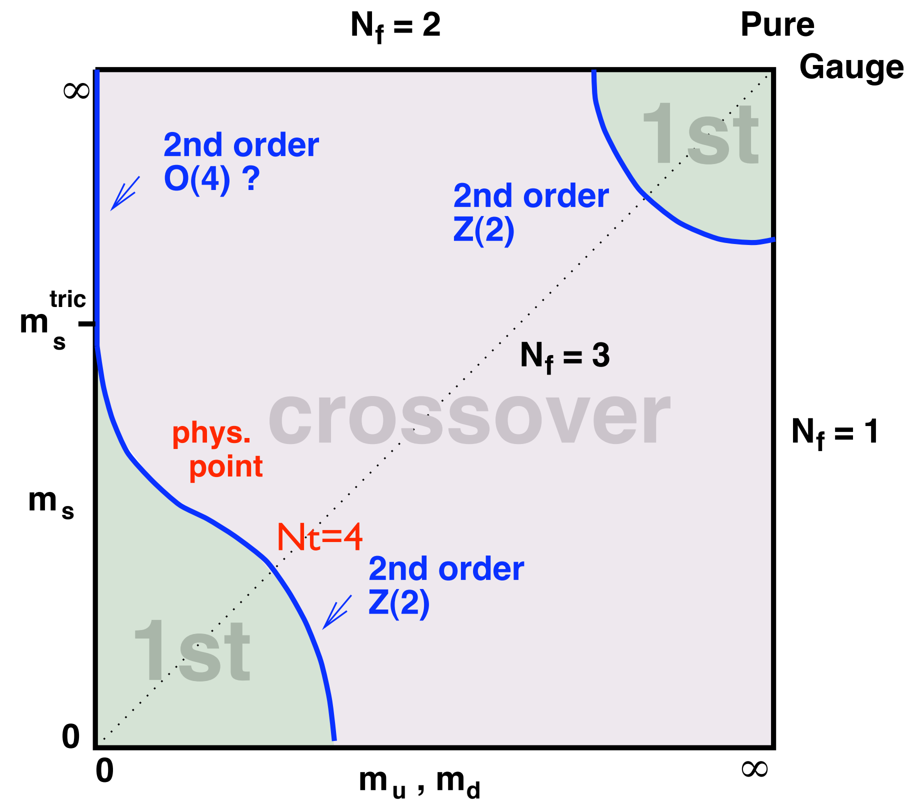
Nf=2+1



$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3) \left(\frac{\mu}{\pi T}\right)^2 - 47(20) \left(\frac{\mu}{\pi T}\right)^4 - \ldots$$

Exotic scenario!

Towards the continuum: $N_t = 6, a \sim 0.2$ fm



$$\frac{m_\pi^c(N_t = 4)}{m_\pi^c(N_t = 6)} \approx 1.77 \qquad N_f = 3$$

de Forcrand, Kim, O.P. 07
Endrodi et al 07

- Physical point deeper in crossover region as $a \to 0$
- Cut-off effects stronger than finite density effects!
- Curvature of critical surface consistent with zero
- No chiral critical point at small density

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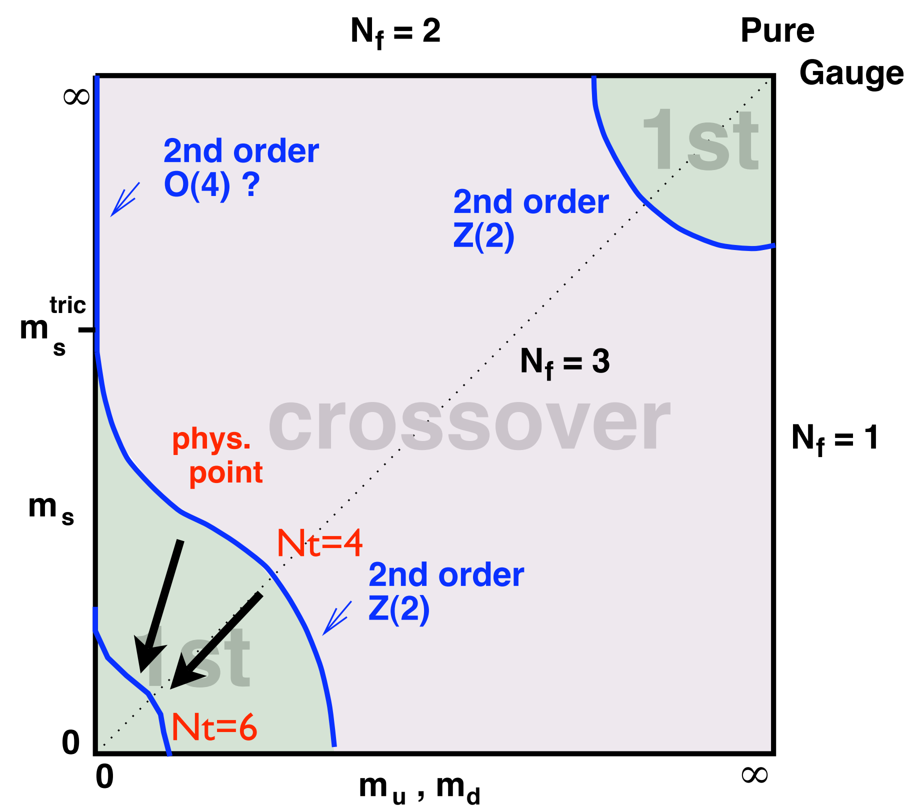

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QCD at complex μ: general properties

$$Z(V,\mu,T) = \mathrm{Tr}\left(\mathrm{e}^{-(\hat{H}-\mu\hat{Q})/T}\right); \quad \mu = \mu_r + \mathrm{i}\mu_i; \quad \bar{\mu} = \mu/T$$

exact symmetries: μ-reflection and μ_i-periodicity

Roberge, Weiss

$$Z(\bar{\mu}) = Z(-\bar{\mu}), \qquad Z(\bar{\mu}_r, \bar{\mu}_i) = Z(\bar{\mu}_r, \bar{\mu}_i + 2\pi/N_c)$$

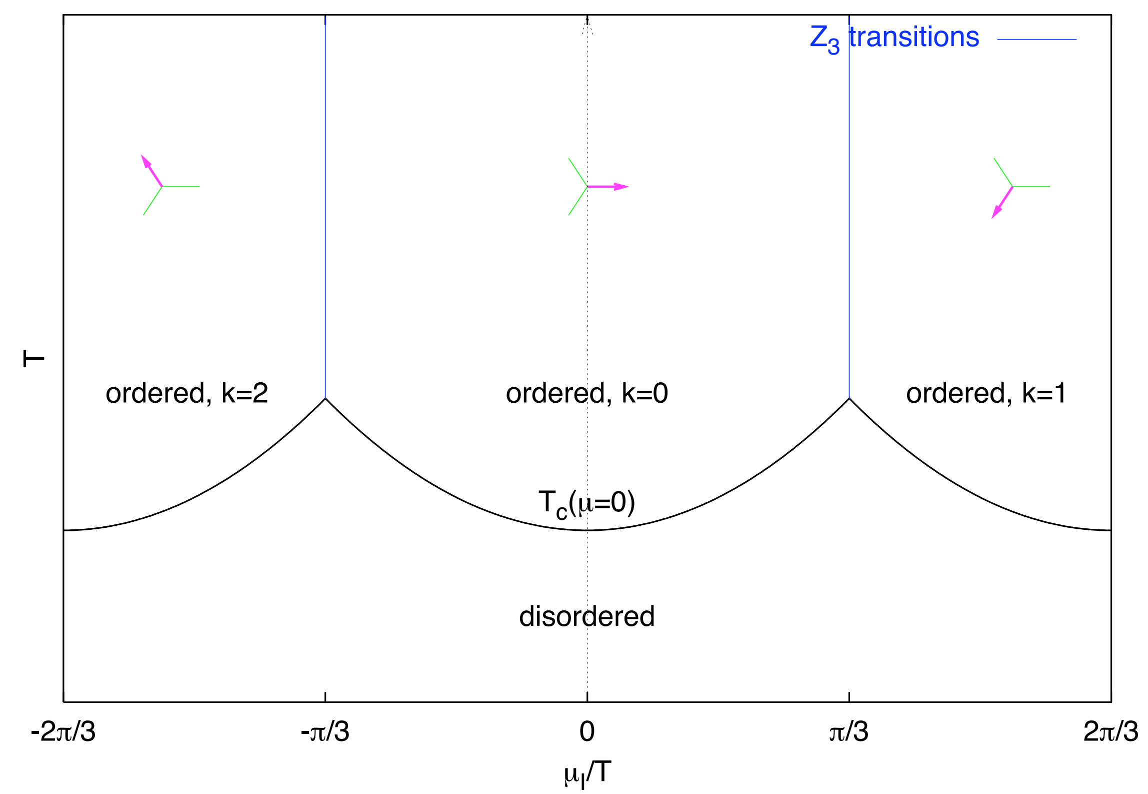
Imaginary μ phase diagram:

Z(3)-transitions: $\bar{\mu}_i^c = \frac{2\pi}{3}\left(n + \frac{1}{2}\right)$

1rst order for high T, crossover for low T

analytic continuation within:

$|\mu|/T \leq \pi/3 \Rightarrow \mu_B \lesssim 550\mathrm{MeV}$



So far: $$\langle O \rangle = \sum_n^N c_n \bar{\mu}_i^{2n} \Rightarrow \mu_i \longrightarrow -\mathrm{i}\mu_i$$

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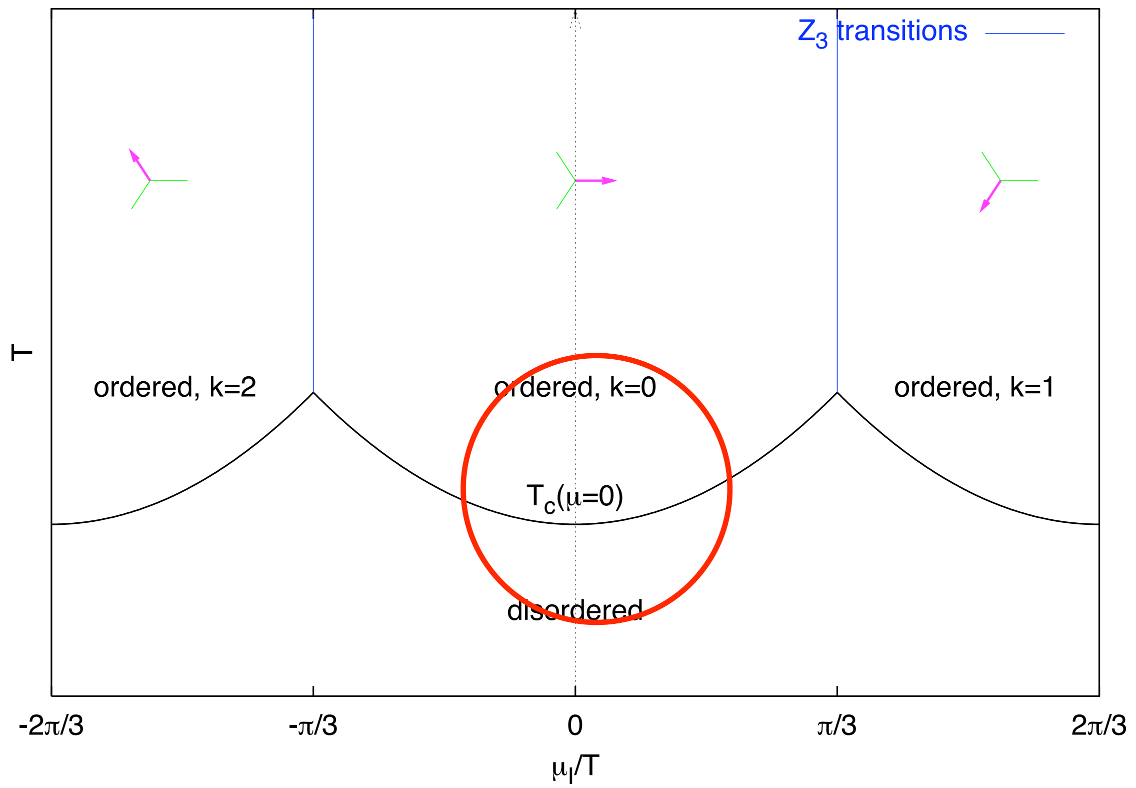

So far: $$\langle O \rangle = \sum_n^N c_n \bar{\mu}_i^{2n} \Rightarrow \mu_i \longrightarrow -\mathrm{i}\mu_i$$ chiral/deconf. transition

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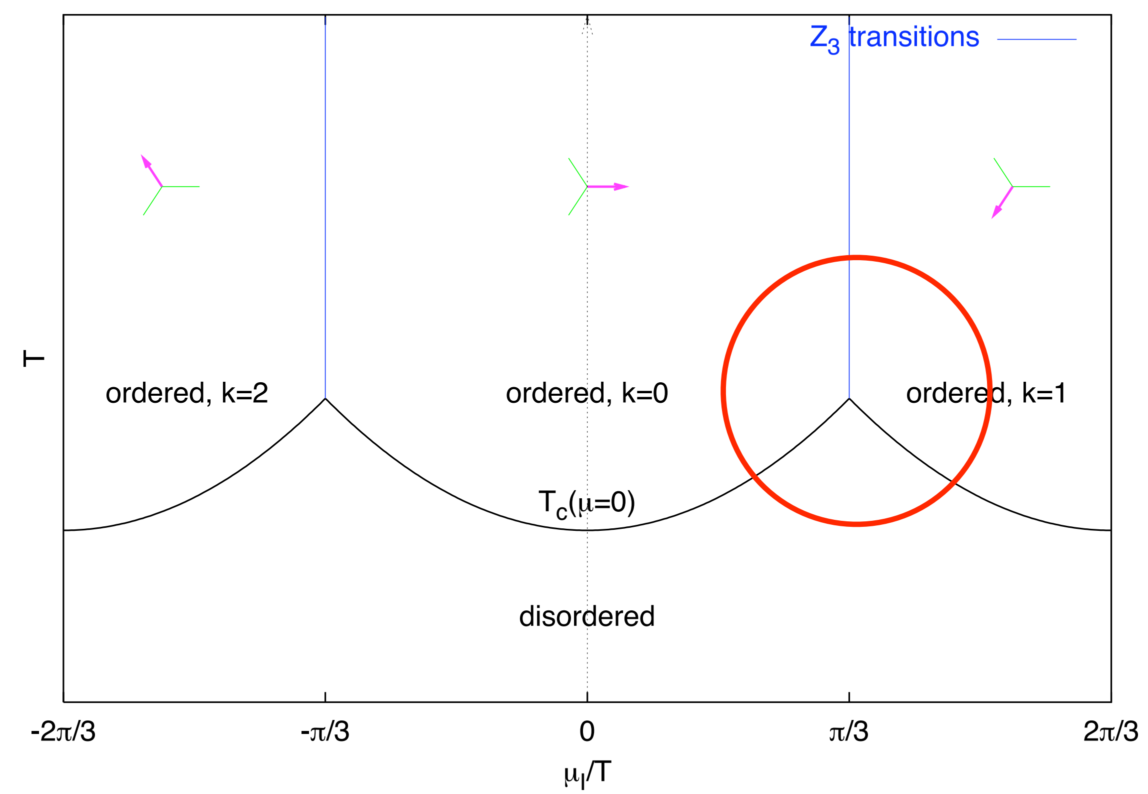

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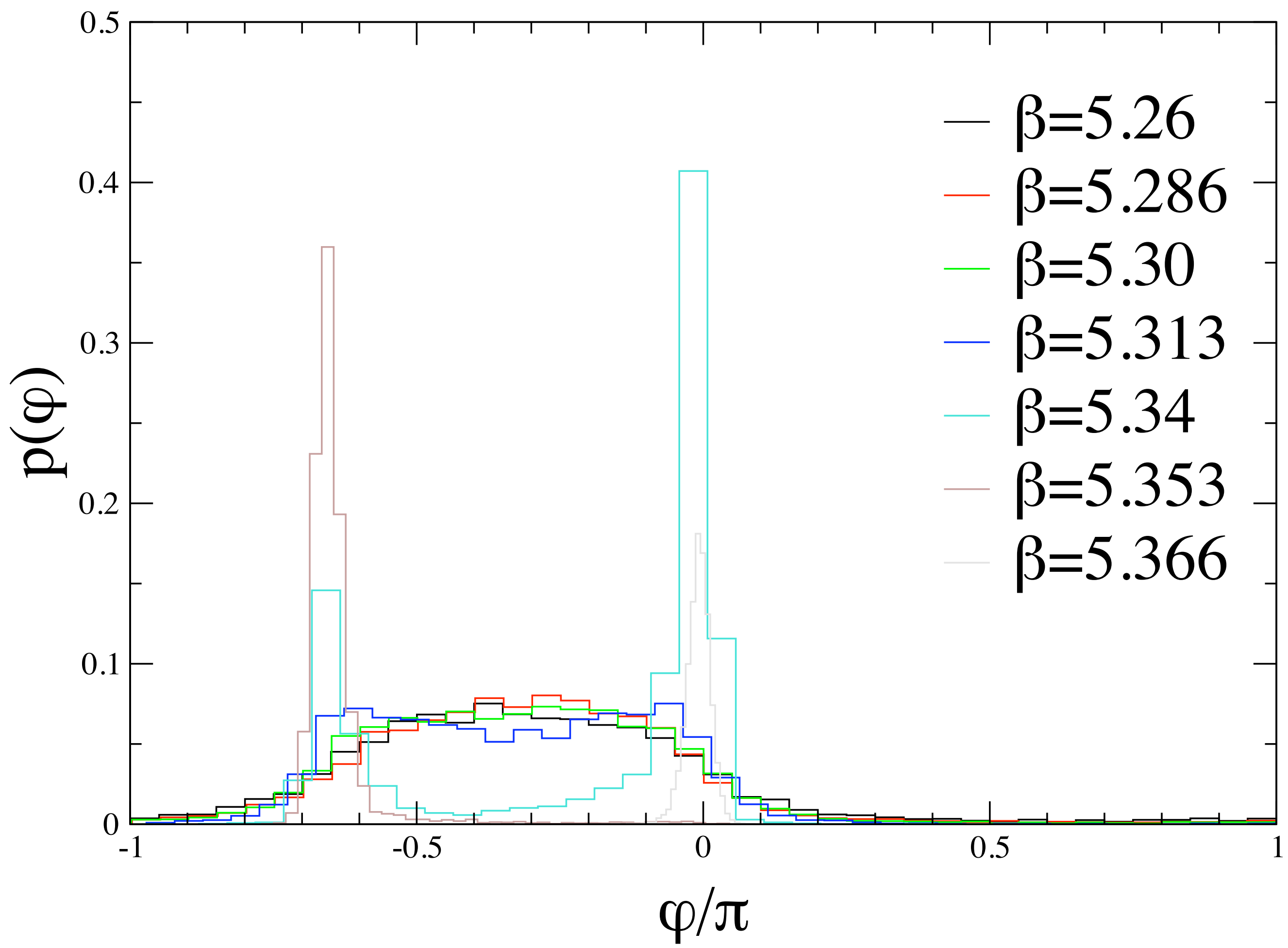
Now: endpoint of Z(N) transition

The Z(N) transition numerically

Nf=2: de Forcrand, O.P. 02

Nf=4: D'Elia, Lombardo 03

Sectors characterised by phase of Polyakov loop: $\langle L(x) \rangle = |\langle L(x) \rangle| \, \mathrm{e}^{i\varphi}$



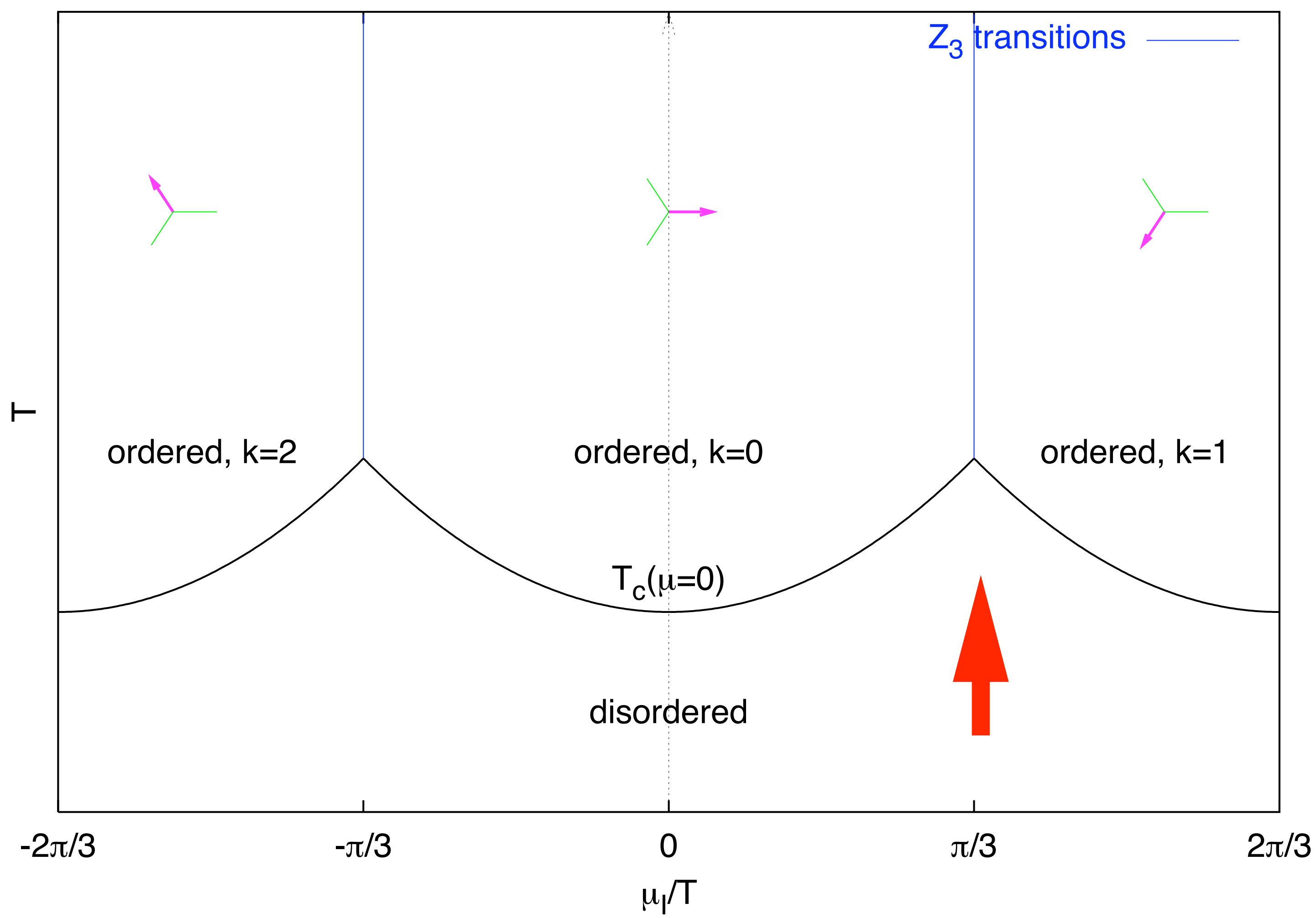
Low T: crossover High T: first order p.t.

The nature of the Z(3) end point at $\mu = i\frac{\pi T}{3}$

Nf=2: D'Elia, Sanfilippo 09 Here: Nf=3

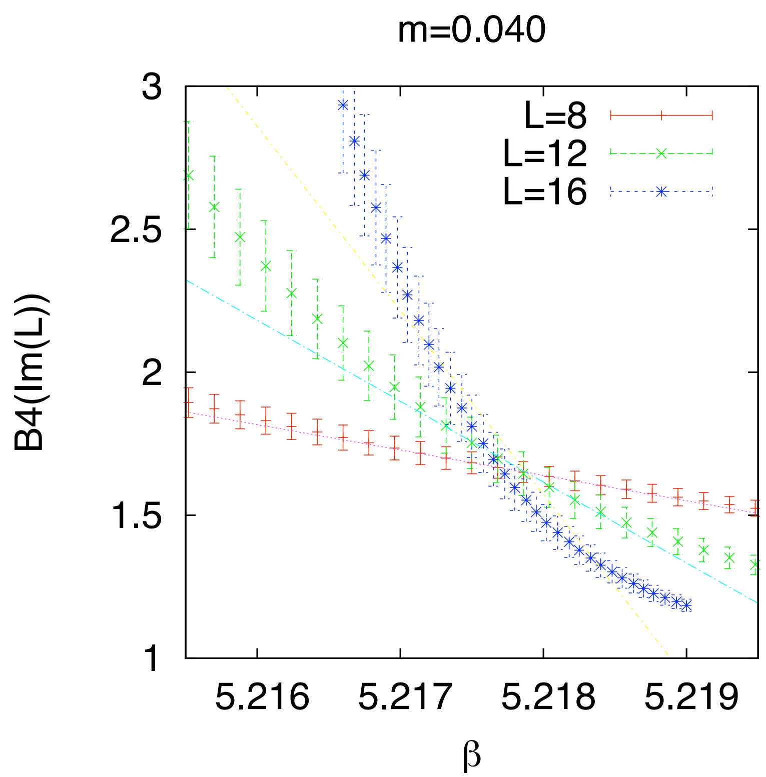
Strategy: fix $\frac{\mu_i}{T} = \frac{\pi}{3}, \pi$, measure Im(L), |Im(L)| is order parameter at $\frac{\mu_i}{T} = \pi$

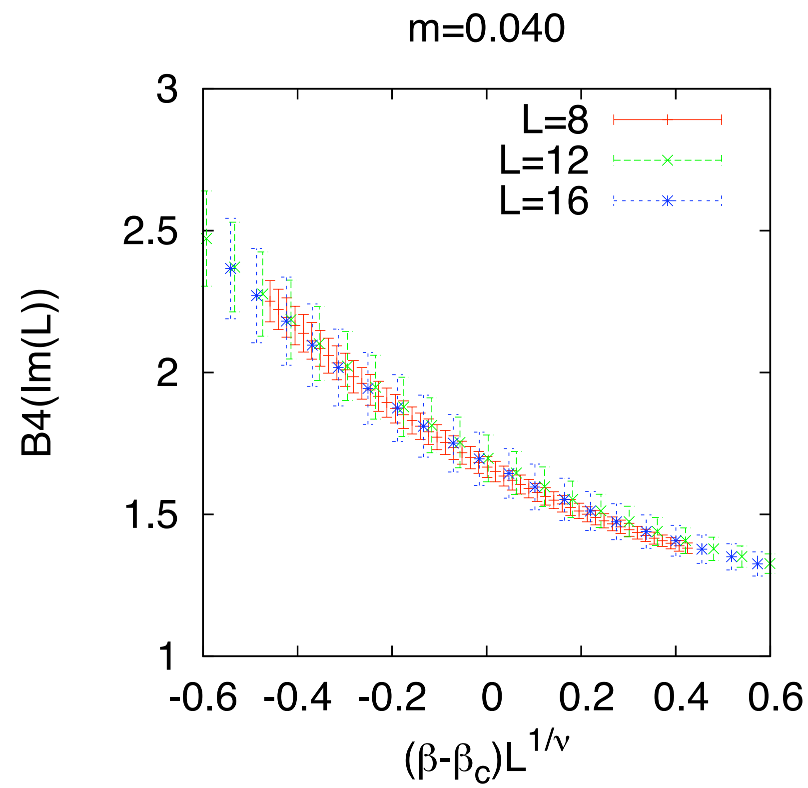
determine order of Z(N) branch/end point as function of m



$$B_4 = \frac{\langle \delta \mathrm{Im}(L)^4 \rangle}{\langle \delta \mathrm{Im}(L)^2 \rangle^2}$$

Results:





$\nu = 0.33$

$$B_4(\beta, L) = B_4(\beta_c, \infty) + C_1(\beta - \beta_c)L^{1/\nu} + \dots$$

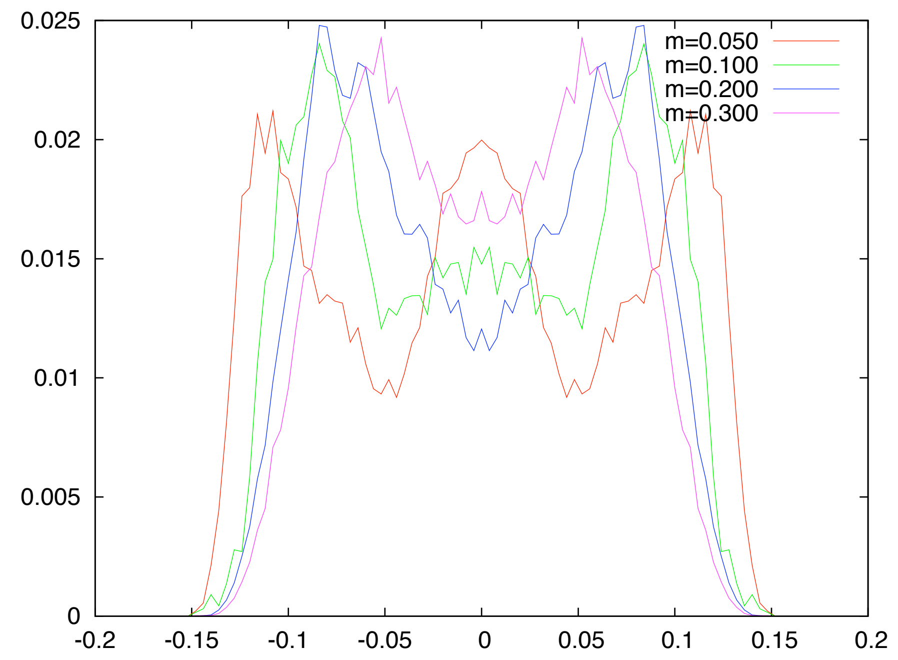
B4 at intersection has large finite size corrections, ν more stable:

$\nu = 0.33, 0.5, 0.63$

for 1st order, tri-critical, 3d Ising

am	β_c	ν	
0.04	5.21778(3)	0.37(3)	first order
0.05	5.23352(3)	0.35(2)	
0.20	5.3961(2)	0.51(5)	
0.30	5.45861(5)	0.65(2)	3d Ising
0.50	5.5427(3)	0.59(4)	
2.00	5.67744(8)	0.347(9)	first order

Details of RW-point: distribution of Im(L)

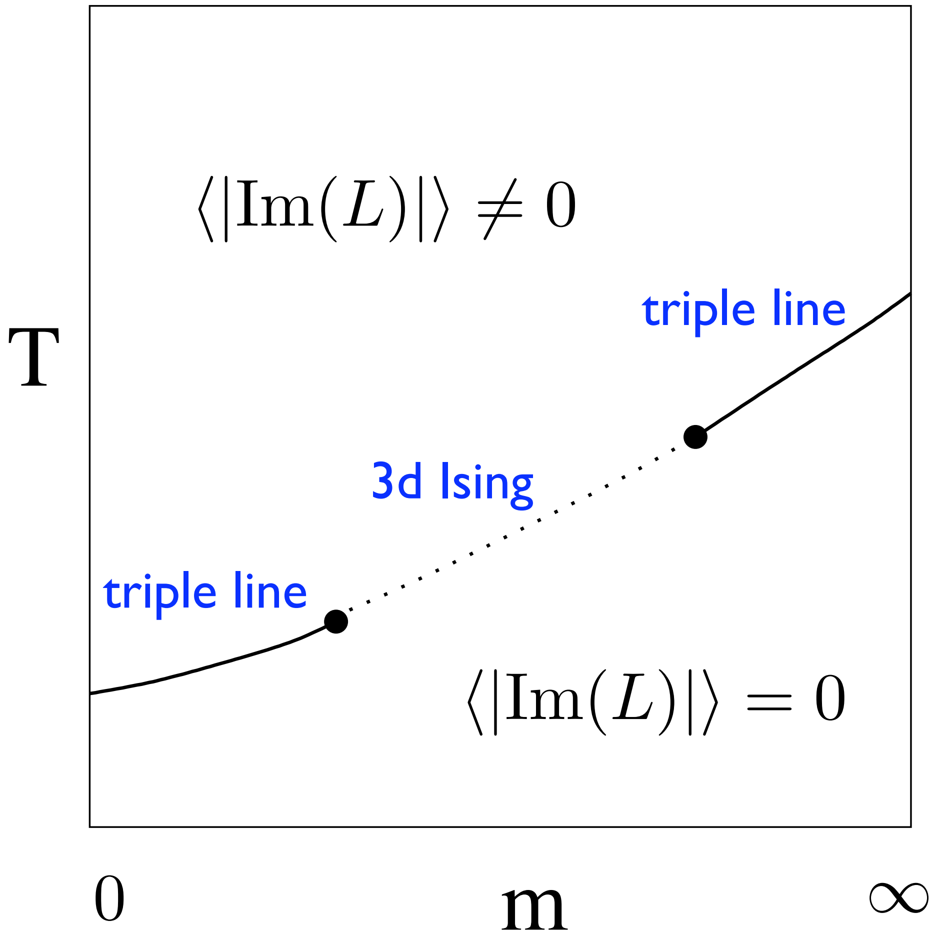


Small+large masses: three-state coexistence — triple point

Intermediate masses: middle peak disappears — Ising distribtion in magn. direction

→ tri-critical point in between!

Phase diagram at $\mu = i\frac{\pi T}{3}$



Nf=2: light and intermediate masses, 1st and 3d Ising behaviour D'Elia, Sanfilippo 09

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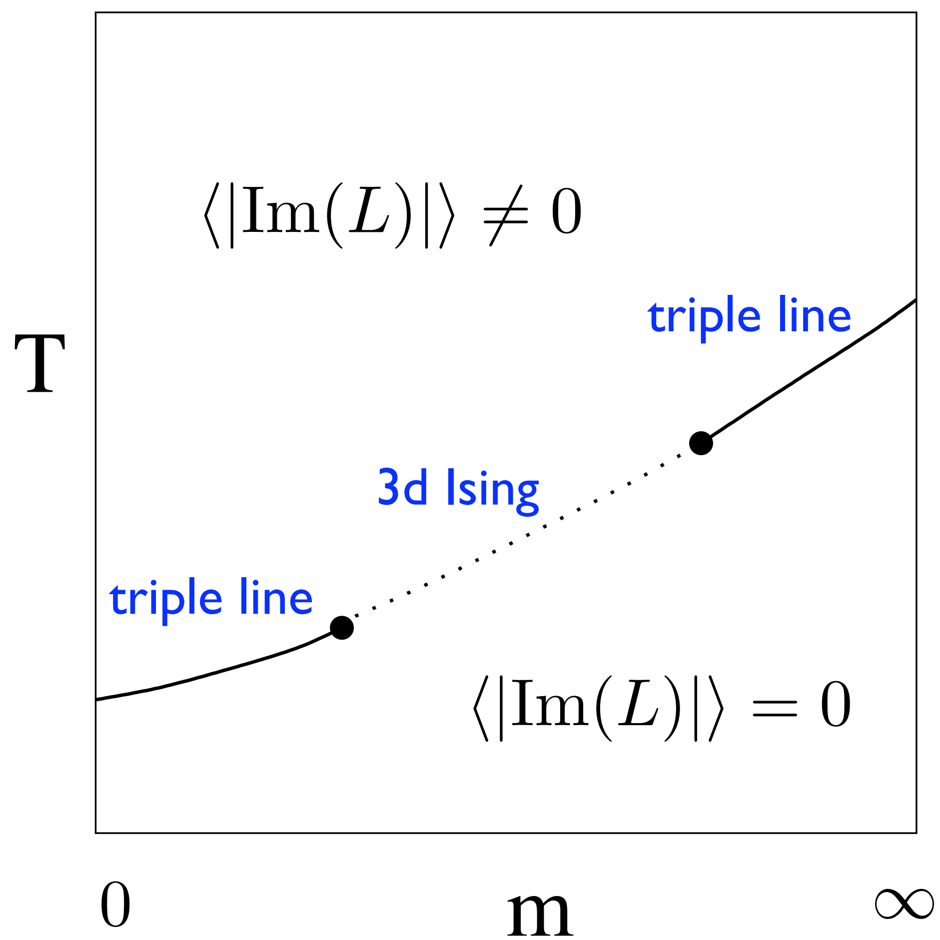
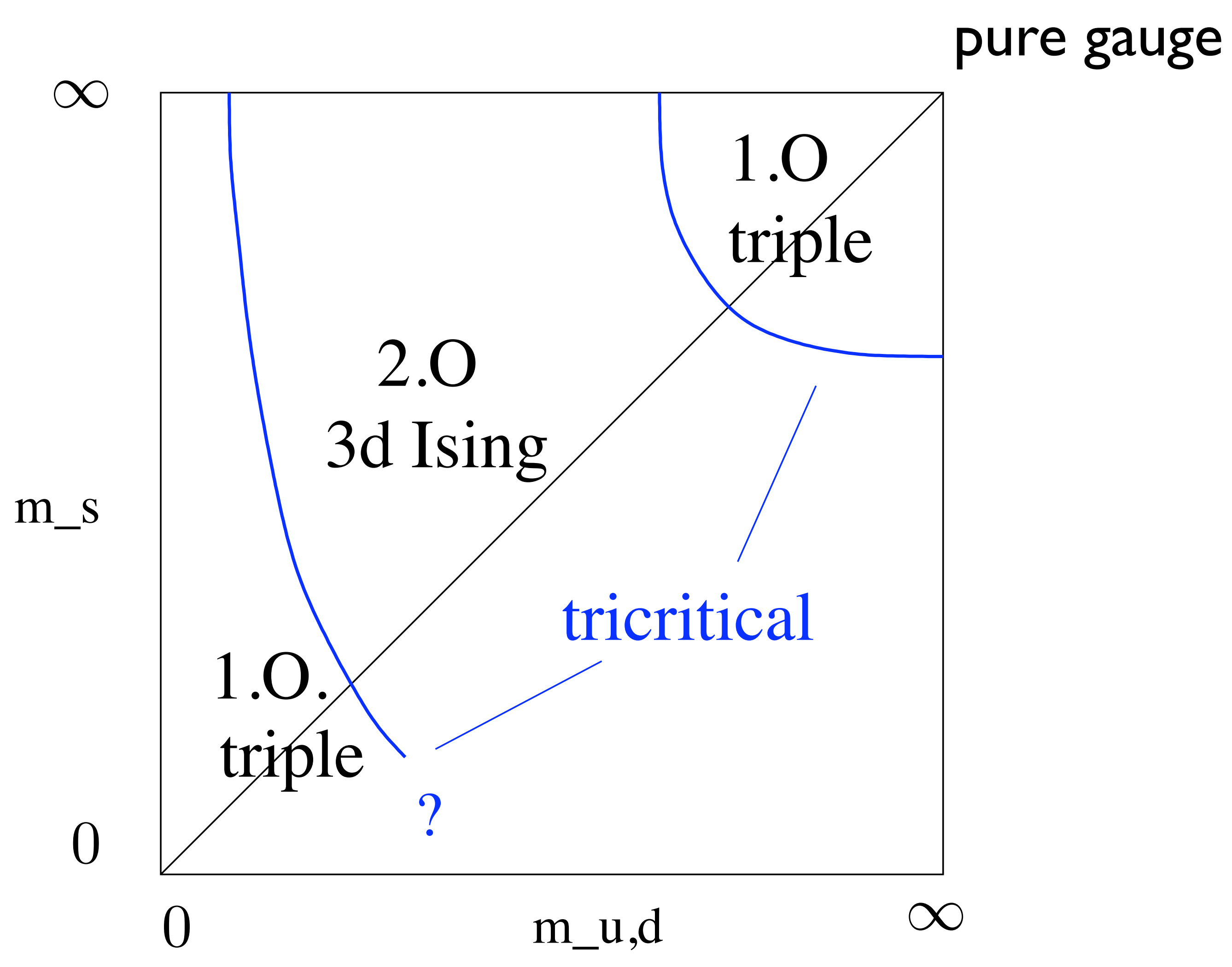


Cut-off effects?

-location of lines, tric. points strongly affected
-qualitative structure stable, universality!
 (up to tric. points merging or on boundary?)

Nf=2: light and intermediate masses, 1st and 3d Ising behaviour   D'Elia, Sanfilippo 09

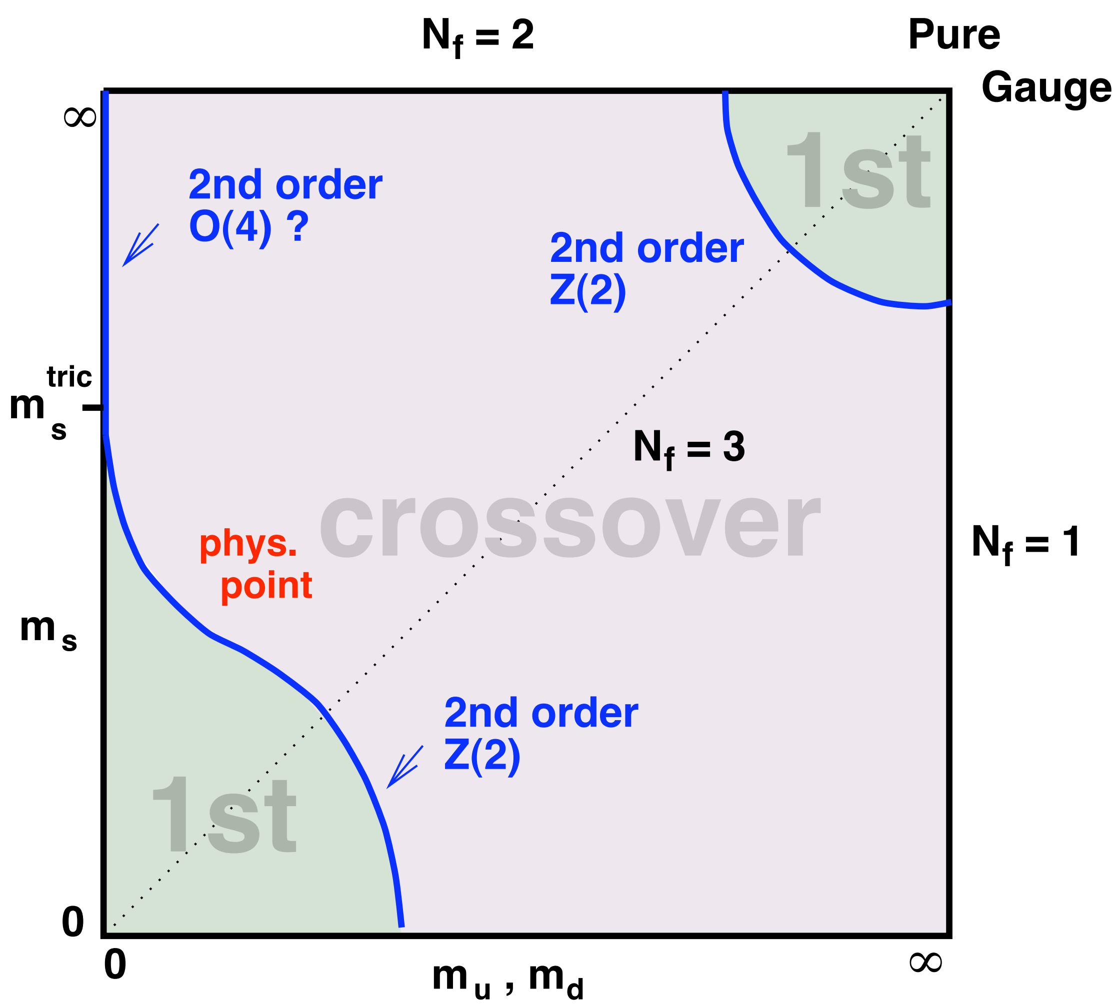
Generalisation: nature of the Z(3) endpoint for Nf=2+1

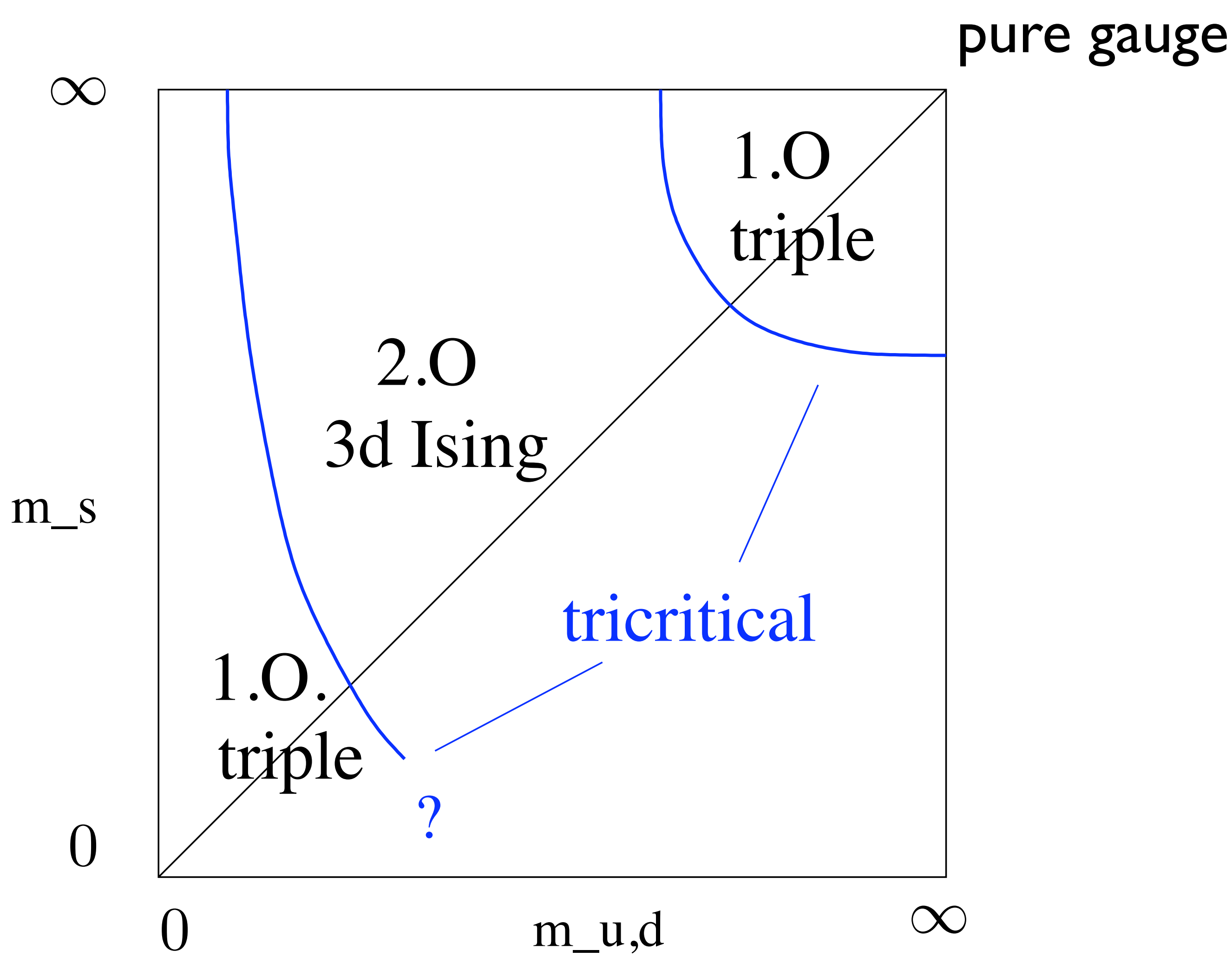


-Diagram computable with standard Monte Carlo, continuum limit feasible!

-Benchmarks for PNJL, chiral models etc.

Connection with zero and real μ



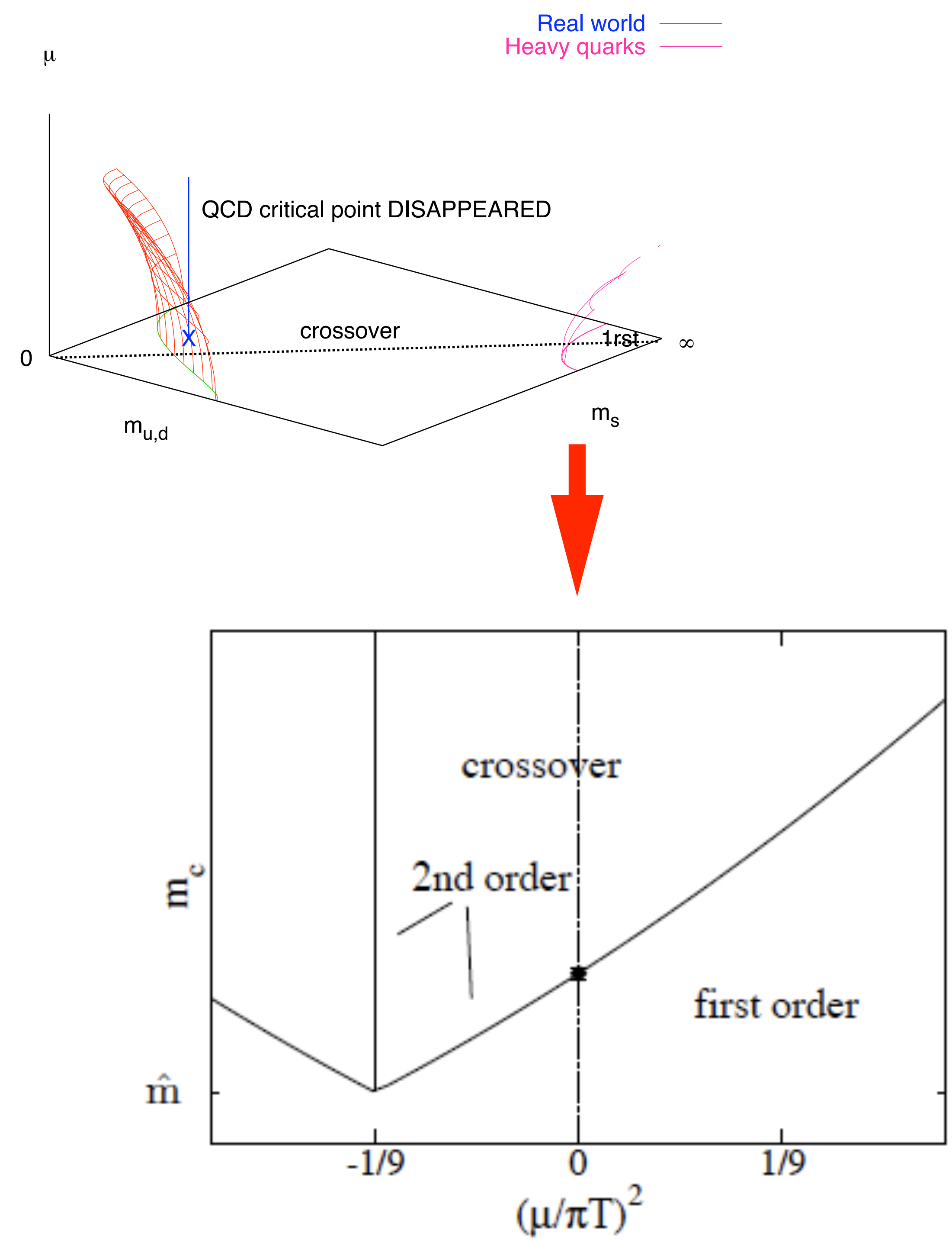


$\mu = 0$

$\mu = i\frac{\pi T}{3}$

-Connection computable with standard Monte Carlo!

Example: critical surface for heavy quarks



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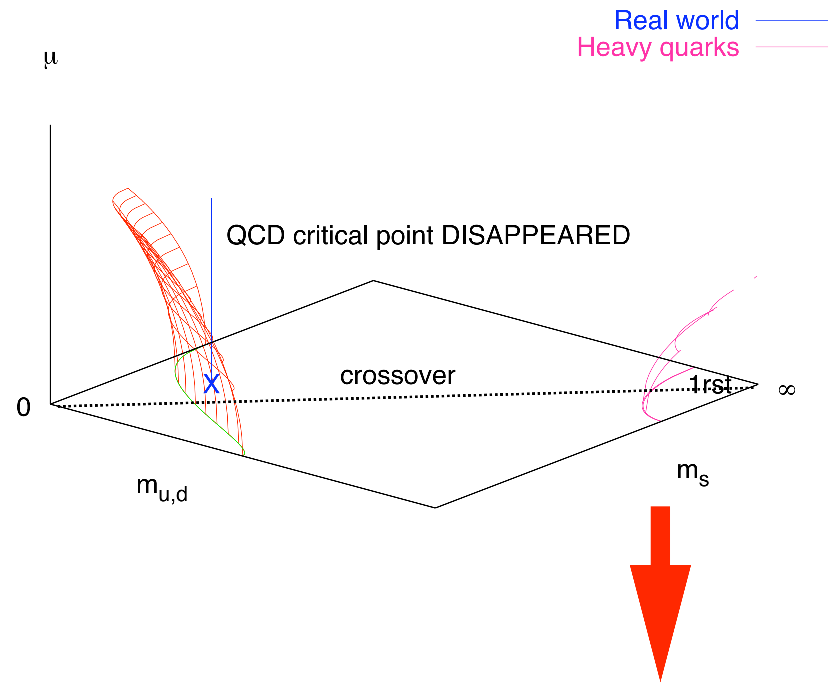
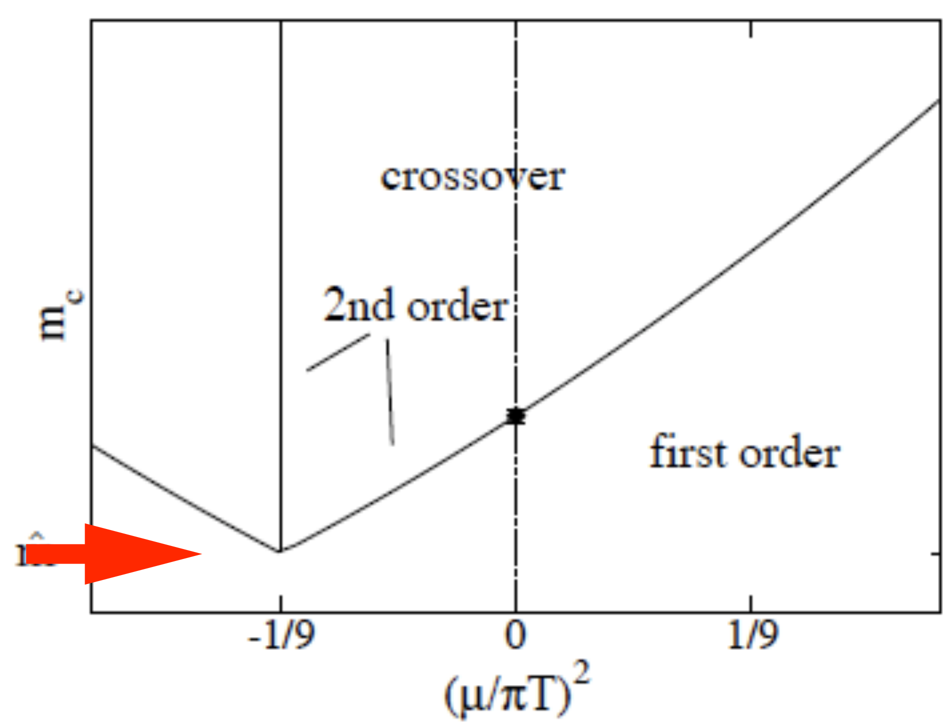




tricritical point!

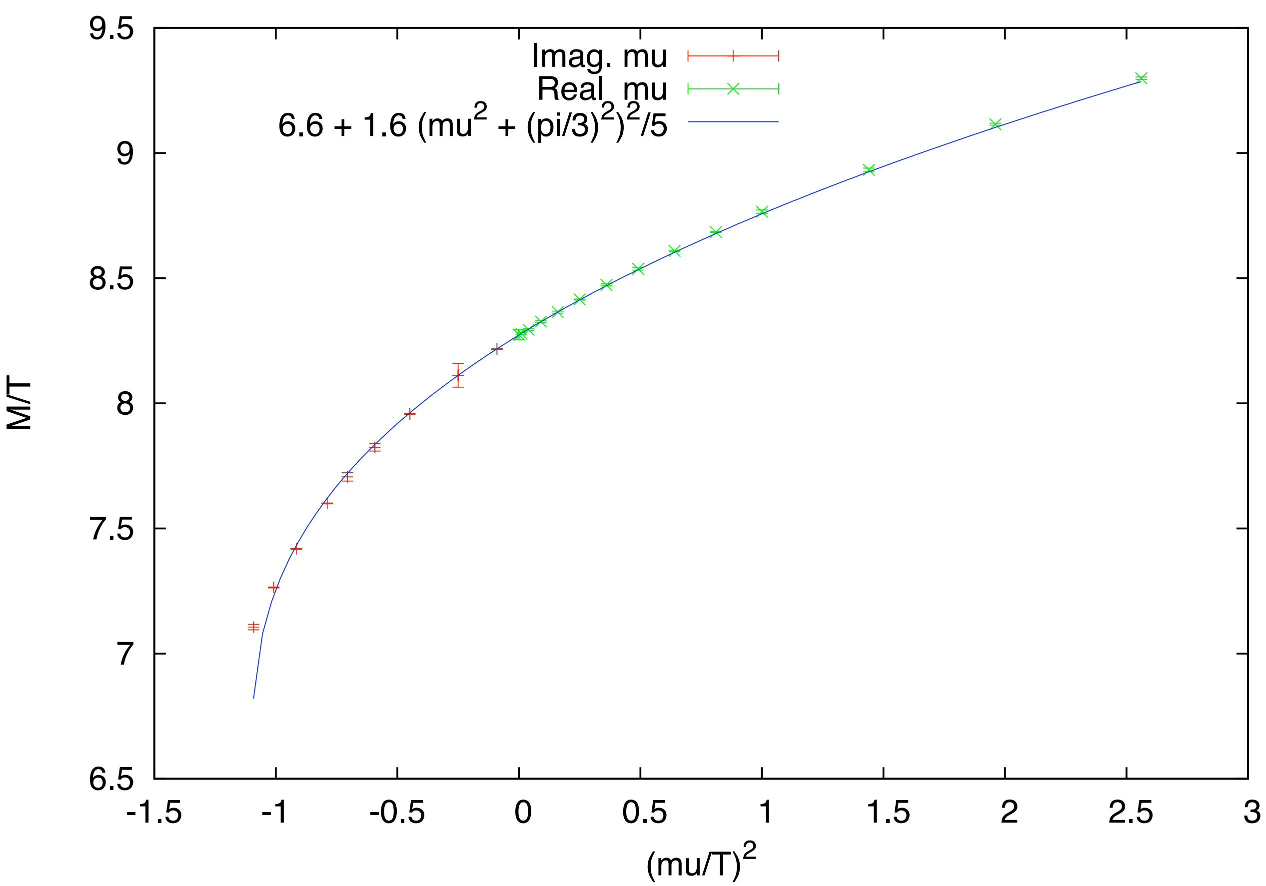
$m \to \infty$: QCD$\to$ theory of Polyakov lines $\to$ universality class of 3d 3-state Potts model

(3d Ising, Z(2))

small μ/T: sign problem mild, doable for real μ!

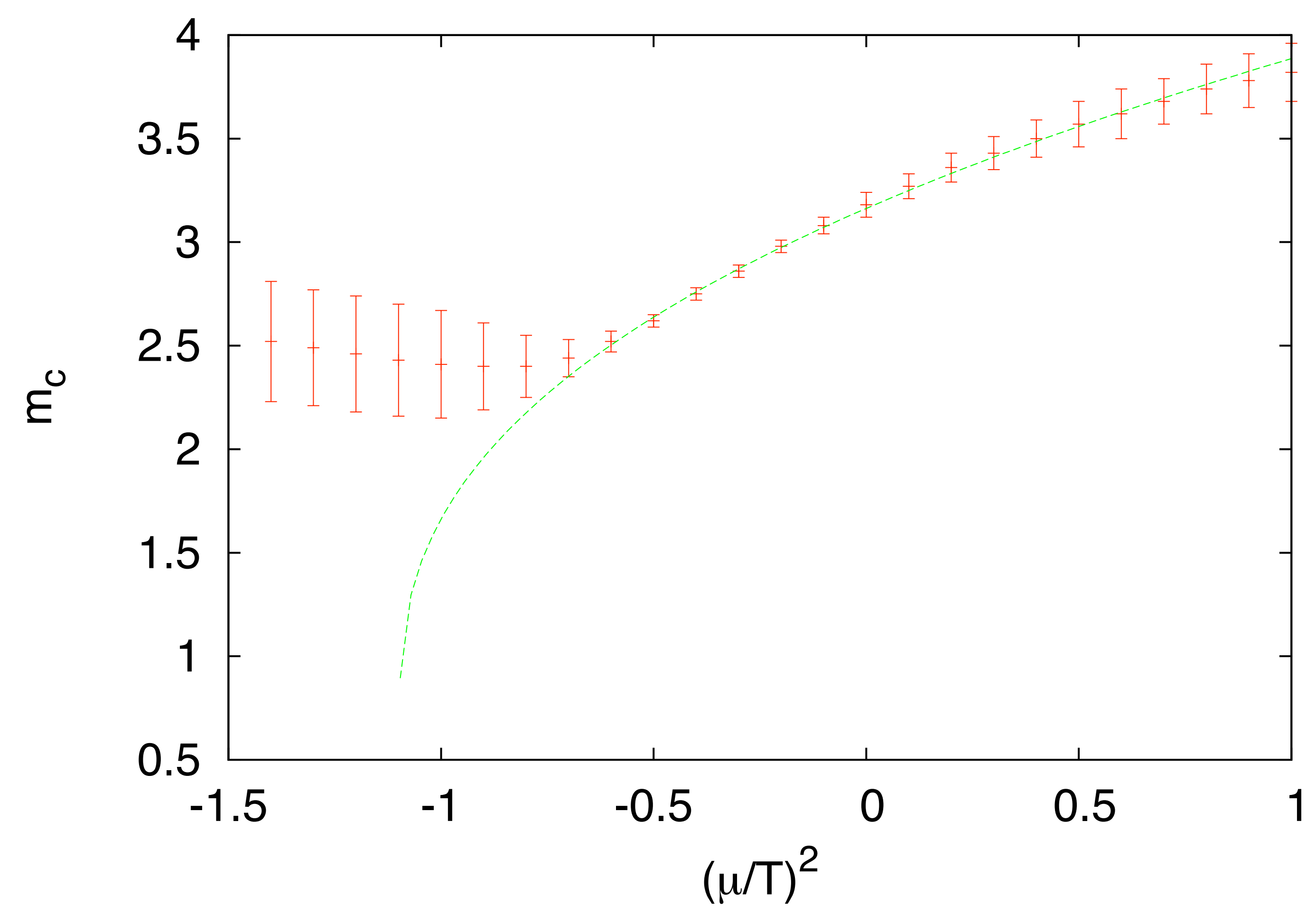
de Forcrand, Kim, Kratochvila, Takaishi

Potts:



QCD, Nt=1, strong coupling series:

Langelage, O.P. 09



tri-critical scaling:

$$\frac{m_c}{T}(\mu^2) = \frac{m_{tric}}{T} + K\left[\left(\frac{\pi}{3}\right)^2 + \left(\frac{\mu}{T}\right)^2\right]^{2/5}$$

exponent universal

Conclusions

- For lattices with a~0.3 fm no chiral critical point for $\mu/T \lesssim 1$
- CEP scenario not yet clear: exploring uncharted territory!
- Z(N) transition at imaginary chem. pot. connects with chiral/deconf. transition
- Curvature of deconfinement critical surface determined by tri-critical scaling
- Check if same holds for chiral critical surface!