Towards charmonium spectroscopy at threshold from lattice QCD

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FAIR Lattice QCD Days

Introduction

- New experimental results for Charmonium are motivating lattice studies.
- In principle lattice QCD can help to shed light on the nature of the excited spectrum of $c\bar{c}$ states.

- Control the statistical and systematic errors in determining energy levels to resolve a tower of radial and orbital states?
- Discrete group theory to understand particle identification in the observed spectrum.
- Understand states above threshold.

Determining the $c\bar{c}$ spectrum: a strategy

- Spin assignment: identification of the observed excitations in lattice data
- Use dynamical anisotropic lattices
  - $N_f = 2$,
  - $3 + 1$ anisotropy, $\xi = a_s/a_t = 3.5$
  - relativistic charm quarks, $a_t m_c \ll 1$ and $a_s m_c < 1$. Good since m_c light for HQET.
- Use "distillation"
- Use extended operators, smeared operators and variational analysis
  - allows us to extract multiple excitations

Spin assignment from lattice states (1)

- The lattice explicitly breaks $O(3)$ rotational symmetry to a finite 48-element sub-group O_h, the cubic point group.
- Eigenstates of the lattice hamiltonian will simultaneously be eigenstates of O_h only, so are classified according to their irreducible representation under action of O_h.
- Parity is still a good quantum number (use sub-script g and u for even/odd)
- O_h has 10 irreps:

Irrep	A_{1g}, A_{1u}	A_{2g}, A_{2u}	E_g, E_u	T_{1g}, T_{1u}	T_{2g}, T_{2u}
Dim	1	1	2	3	3

Spin assignment from lattice states (2)

- Continuum spin assignment is made by forming the representations of O_h subduced from $O(3)$. Multiplicities:

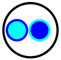
J	0	1	2	3	4
A_1	1	0	0	0	1
A_2	0	0	0	1	0
E	0	0	1	0	1
T_1	0	1	0	1	1
T_2	0	0	1	1	1

- Degeneracies across lattice irreps as continuum is approached

- Difficulty for charmonium: small hyperfine splittings, near-degeneracy of many states with different spin assignments.
- A pathological example: it would be hard to distinguish near-degenerate triplet of spin 0,1,2 from spin 4. Both these systems have the same lattice irrep content: $A_1 \oplus T_1 \oplus E \oplus T_2$. Can a radial excitation of P-wave $\bar{c}c$ be distinguished from 4^{++} F-wave?

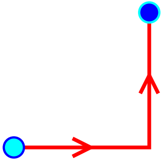
Operator construction

- Lattice operators are bilinears with path-ordered products between the quark and anti-quark field; different offsets, connecting paths and spin contractions give different projections into lattice irreps.
- This analysis uses a sub-set of the operators given below.



$\mathcal{O}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)\psi_\beta(x)$

$\mathcal{O}^{i}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)U_i(x)\psi_\beta(x+\hat{\imath})$



$\mathcal{O}^{ij}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)U_i(x)U_j(x+\hat{\imath})\psi_\beta(x+\hat{\imath}+\hat{\jmath})$

- This gives all S and P wave irreps, some D and beyond.
- In progress: more complete basis construction.

Simulation Details

Use an anisotropic action, $\xi = a_s/a_t = 3.5$. full details in [Phys.Rev.D79:034502,2009]

- Fermion action: anisotropic clover stout-link.
- Gauge action: Symanzik-improved, tree-level tadpole coefficients. ($\mathcal{O}(\alpha_s^4, a_t^2, g^2 a_s^2)$).

$\xi = a_s/a_t$ is non-perturbatively tuned [PRD 74 014505 (2006)].

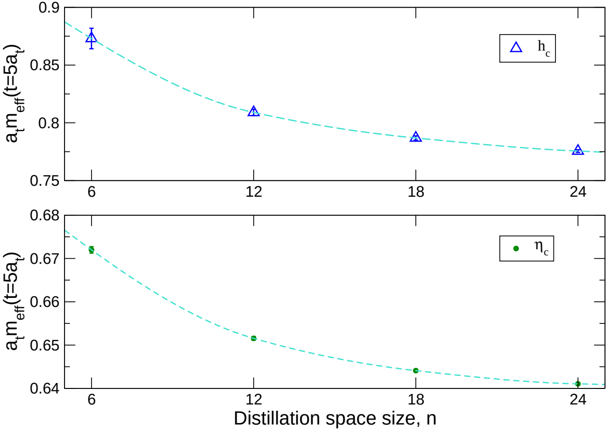
The HSC has a range of volumes and quark masses to $32^3 \times 256$ with $m_\pi \sim$ 220MeV.

Simulation Parameters

Flavours	$N_f = 3$.
Light quarks	m_π/m_ρ= 0.66
Lattice volume	$12^3 \times 96$
$a^{-1}_{(m_\Omega)}$	4.76GeV^{-1}

from diluting to distilling...

Distillation (talk yesterday).
Might ask how big should the distillation space be?



- Effective mass for both S-wave (η_c) and P-wave (h_c) still falling as N is increased.

Charmonium meets the GPU

- For this test, using CUDA implementation of SW inverter implemented by Clark and Babich and linked to `chroma` system by Joó
- Sustains 100 GFlops (single precision) for inversion on Tesla C1060 (=1.5c/MFlop)
- Double precision solves almost as fast (via algorithmic tricks)

- Current software limitation - only $N = 24$ possible on $12^3 \times 96$ lattice on desktop
- Inversions from all time-slices for $N = 24$ requires about 3 hours per configuration

Determination of energies using the variational method

Use the variational analysis of Michael, NPB 259, 58 (1985) and Lüscher and Wolf, NPB339, 222, (1990).

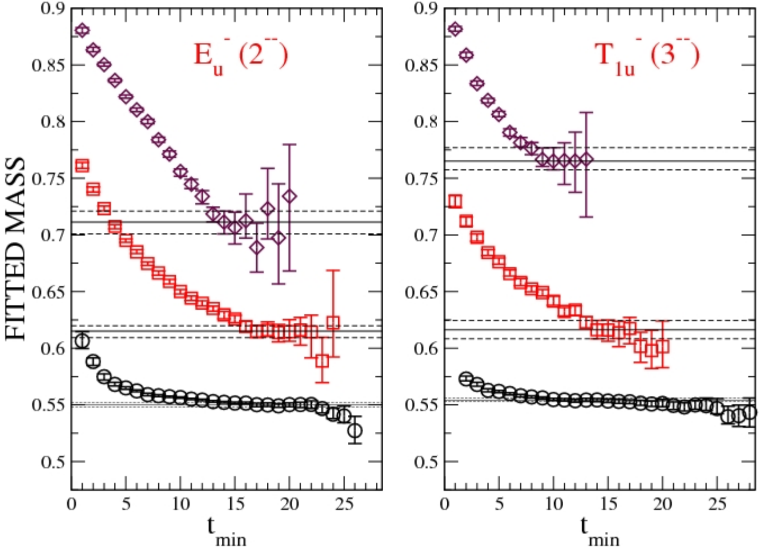
- Create a basis of interpolating operators for the state of interest, $\mathcal{O}_\alpha$, $\alpha = 1, 2, \ldots, n$ using point and extended operators as well as different smearings.
- construct the matrix

$$C_{\alpha\beta} = \langle 0|\mathcal{O}_\alpha(t)\mathcal{O}^\dagger_\beta(0)|0\rangle$$

- solve for the eigenvalues λ_α of the matrix $C(t_0)^{-1/2}C(t)C(t_0)^{-1/2}$ where t_0 is some small reference time, $(\lambda_1 \geq \lambda_2 \geq \lambda_3 \ldots)$
- Then

$$\lim_{t\to\infty} \lambda_\alpha(t, t_0) = e^{-(t-t_0)E_\alpha}[1 + O(e^{(-t\Delta E_\alpha)}]$$

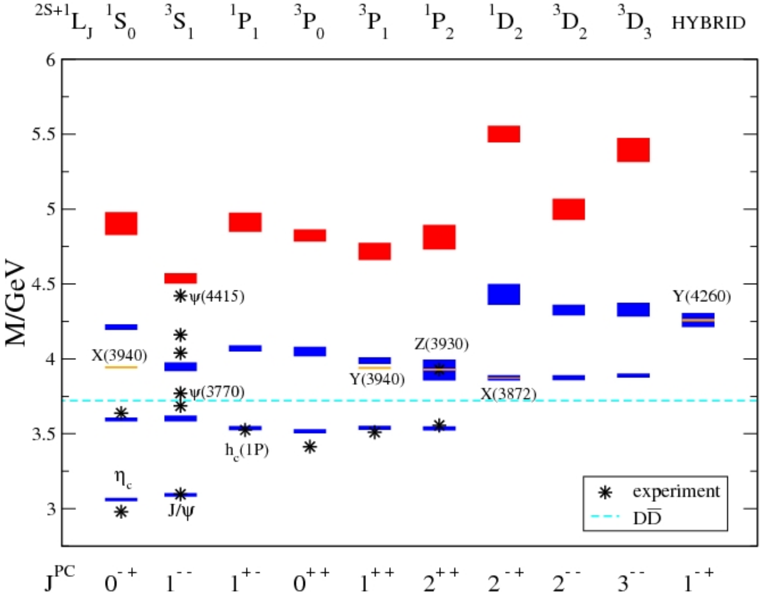
We expect to see plots like ...



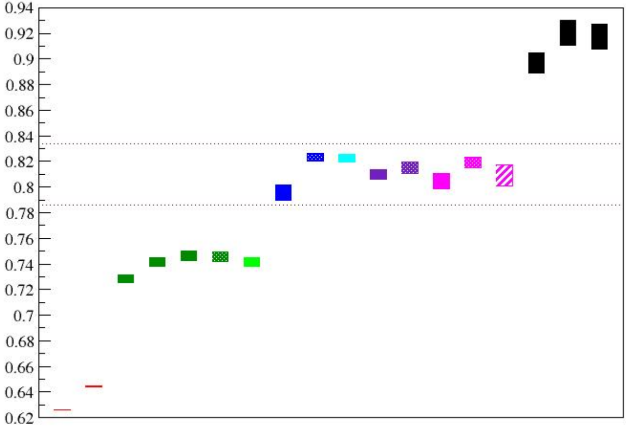
... but better.

From previous work

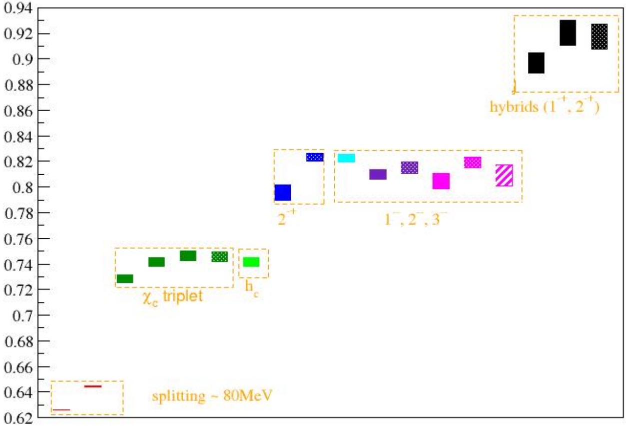
The $c\bar{c}$ spectrum from “old” all-2-all data.



The spectrum: preliminary

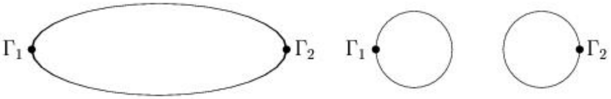


The spectrum: preliminary



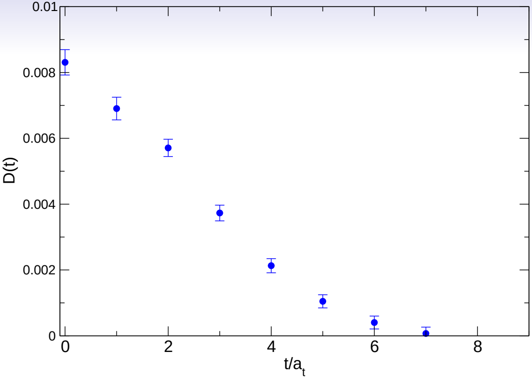
Including disconnected diagrams (1)

$c\bar{c}$ interpolating operators are singlet ($\bar{c}\Gamma c$) $\rightarrow$ bubble diagrams in Wick contractions.



OZI suppressed $\Rightarrow$ small. Unless other nonperturbative effects play a role.

Usually non-singlet (connected) correlators are calculated. The disconnected diagrams require all-to-all propagators.

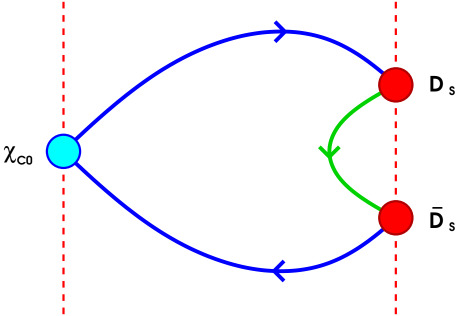


The disconnected contribution to η_c.

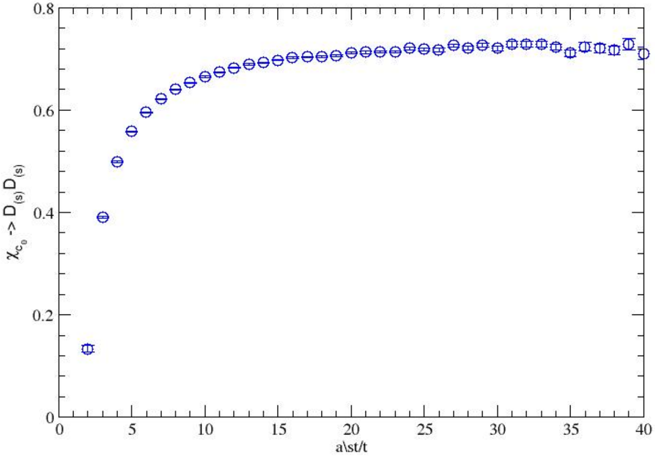
- Fit $(1 - D/C)$ to resolve $m_{\eta_c(C-D)} \approx m_{\eta_c(C)}$
- Mixing of the non-singlet with a nearby glueball (as in Morningstar & Peardon PRD60, 034509) $\longrightarrow$ states repel? Predicts the singlet contribution raises m_{η_c} and lowers $m_{J/\psi}$?
- All very preliminary yet - an analysis is underway.

Including threshold effects:

Calulating threshold effects is (relatively) straightforward with distillation. Eg. one needs to determine the S-wave mixing



The effective mass for $\chi_{c_0} \rightarrow D_s\bar{D}_s$



Conclusions

- This is a first application of distillation to charmonium. Small lattices, heavy (light) quarks, better tuning of m_c, better determination of a^{-1}, bigger distillation space.
- The combination of anisotropic lattices, distillation and variational fitting techniques mean the orbital, radial and gluonic excitations of Charmonia can be resolved.
- Additional operators will give a definitive spin identification of calculated excited states. In progress.
- A similar analysis is in progress for the heavy-light ($D_{(s)}$) system
- Simulations at finer lattice spacings and at lighter sea quark masses are underway.
- Distillation allows us to investigate threshold effects.
- GPU implementation is proving very efficient!