Glueball masses on the lattice with exponentially improved statistical precision

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FAIR Lattice QCD Days, 24/11/2009, GSI-Darmstadt

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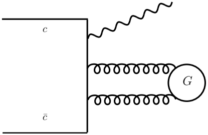
Based on Computer Physics Comm. 180:813-818 and 180:819-826, 2009

- Motivations for glueball studies
- Exponential growth of the signal to noise ratio
- The Yang-Mills theory
- Decomposition of the path integral and boundary conditions
- Symmetry-Constrained Monte Carlo
- Results
- Conclusions and outlook

Motivations for glueball studies

- Self-interaction of gluon $\rightarrow$ Do bound states of gluon exist ?
- From the theoretical point of view the question is cleanly posen in the pure YM theory
- Experimentally: search of extra states (resonances) not in the quark model (cold be glueballs, tetraquarks ... exotic)
  - no electric charge (no direct coupling to γ), no flavor (flavor blind decay mode)
- Most exploited channel, J/Ψ radiative decays

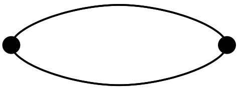
  BES, SLAC (MARK), FAIR ?



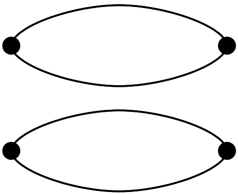
- Several resonances, f_0's for the 0^{++} and η's for the 0^{-+} between 1.5 and 2 GeV.
- Precise theoretical predictions are needed to safely identify these states as glueballs.
- In the full theory glueballs mix with $q\overline{q}$ states !! [TχL, UKQCD]

Exponential growth of the signal to noise ratio (Parisi '84, Lepage '89)

Consider a point to point correlation function interpolating (eg) a meson. The signal is given by the expectation value of



while the variance is given by the expectation value of

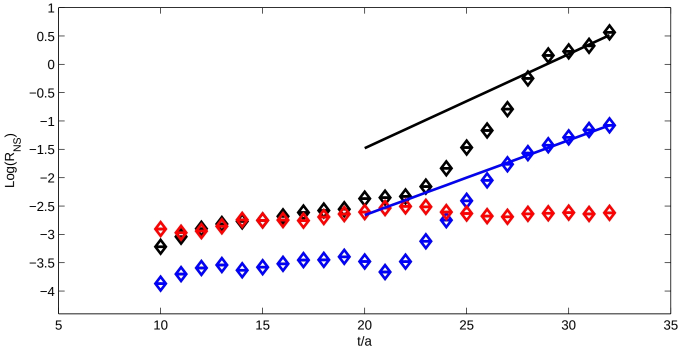


Luckily Wick-contractions are done *before squaring*, for the variance. Then a multi-pion state dominates, otherwise it would be the vacuum (as for YM).

pion $R_{NS} \propto const$

ρ $R_{NS} \propto \exp((m_\rho - m_\pi)t)$

N $R_{NS} \propto \exp((m_N - \frac{3}{2}m_\pi)t)$



O(2000) quenched confs ($\beta = 6.2$, $\kappa = 0.1526$) in APE, hep-lat/9611021

- For an operator interpolating a parity odd glueball

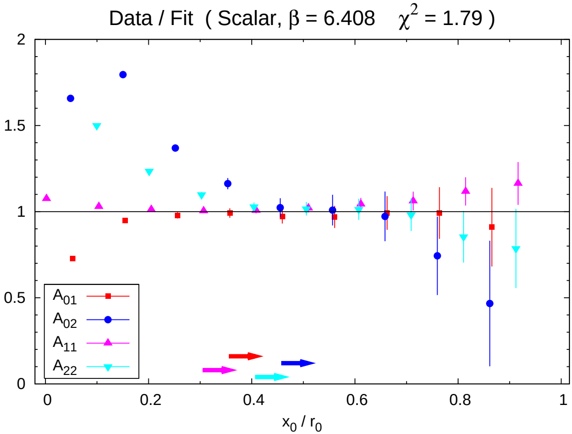
$$C_{O_G}(t) = \langle O_G(t) O_G(0) \rangle \rightarrow |\langle 0|O_G(0)|G^-\rangle|^2 \, e^{-M_{G^-}\, t} + \ldots$$

the variance can be estimated as

$$\sigma^2 = \langle O_G^2(t) O_G^2(0) \rangle - \langle O_G(t) O_G(0) \rangle^2 \rightarrow \langle 0|O_G^2(0)|0\rangle^2 + \ldots$$

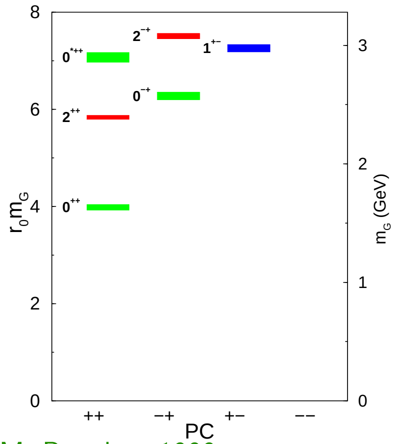
- The noise to signal ratio at large time separations is given by

$$R_{NS}(t) \rightarrow \frac{\langle 0|O_G^2(0)|0\rangle}{|\langle 0|O_G(0)|G^-\rangle|^2} \, e^{M_{G^-}\, t} + \ldots$$



H. Meyer, JHEP 0901:071,2009.

Signal up to 0.5 fm at most.



C. Morningstar and M. Peardon, 1999 ....

Nice results, which however we believe need to be checked concerning systematic effects. In particular a single state dominance in the correlation function for large x_0 (in fm.) is not always observed. Rather, compromises between excited states contributions at short time- and large errors at large time-separations.

- On a given gauge configuration symmetries as parity are not preserved. All states are allowed to propagate despite the quantum numbers of the source.
- In the gauge average each configuration gives a contribution $O(e^{-M_{G-}t})$ to the signal and $O(1)$ to the variance.

This suggests one should introduce some sort of projectors on the relevant states, but it is not clear what that means in the path integral approach of Monte Carlo simulations.

Decomposition of the path integral and boundary conditions

with periodic boundary conditions $Z = \int D_3[V]\langle V|e^{-T\hat{H}}\hat{P}_G|V\rangle$

$$Z = Z^+ + Z^- \ , \quad Z^{\pm} = e^{-E_0 T}\left[\frac{1 \pm 1}{2} + \sum_{n=1} w_n^{\pm}\, e^{-E_n^{\pm} T}\right]$$

We introduce a parity transformation

$$\hat{\wp}\,|V\rangle = |V^{\wp}\rangle \ , \qquad V_k^{\wp}(\mathbf{x}) = V_k^{\dagger}(-\mathbf{x} - \hat{k}) \ ,$$

with $\hat{\mathrm{V}}_k(\mathbf{x})|V\rangle = V_k(\mathbf{x})|V\rangle$ and

$$Z^{tw} = \int D_3[V]\langle V|e^{-T\hat{H}}\hat{P}_G|V^{\wp}\rangle =$$

$$\sum_G \int D_3[V]\langle V|G\rangle\langle G|e^{-T\hat{H}}\hat{\wp}|G\rangle\langle G|V\rangle = Z^+ - Z^-$$

so $\frac{Z-Z^{tw}}{2} = Z^{-}$ and $\frac{Z^{-}}{Z}$ for large T should be dominated by the lightest parity odd glueball.

Our strategy consists in computing the ratio $\frac{Z^{tw}}{Z}$ where the boundary conditions in Z^{tw} are parity twisted

So far anyway the exponential problem remains unsolved ...

The Transfer matrix elements $\langle V_{x_0+1}|\,\hat{T}\,\hat{P}_G|V_{x_0}\rangle$ give the probability for the state V_{x_0} to evolve into the state V_{x_0+1} within a timeslice.

We introduce Parity eigenstates

$$|V,\pm\rangle = \frac{1}{\sqrt{2}}\Big[|V\rangle \pm |V^{\wp}\rangle\Big]$$

with Transfer matrix elements

$$\langle s', V_{x_0+1}|\hat{\mathrm{T}}|V_{x_0}, s\rangle = 2\,\delta_{s's}\ \mathrm{T}^s\Big[V_{x_0+1}, V_{x_0}\Big]$$

$$\mathrm{T}^s\Big[V_{x_0+1}, V_{x_0}\Big] = \frac{1}{2}\Big\{\mathrm{T}\Big[V_{x_0+1}, V_{x_0}\Big] + s\,\mathrm{T}\Big[V_{x_0+1}, V_{x_0}^{\wp}\Big]\Big\}\ .$$

The definition can be easily generalized to thick time-slices of size d and the ratio of such Transfer matrix elements can be numerically computed by O(V) MC simulations for each choice of the boundaries V_{x_0} and V_{x_0+d}.

By dividing the time extent T into several thick time-slices of size d, Z^s can be obtained as the product of the ratios $\frac{\mathrm{T}^s}{\mathrm{T}}$ integrated over the boundary configurations

$$\frac{Z^s}{Z} = \frac{1}{Z} \int \mathrm{D}_4[U]\, e^{-S[U]}\, \mathrm{P}^s_{m,d}\Big[T, 0\Big] \text{, with } T = m * d$$

$$\mathrm{P}^s_{m,d}\Big[T, 0\Big] = \prod_{i=0}^{m-1} \frac{\mathrm{T^s}[V_{x_0+(i+1)\cdot d}, V_{x_0+i\cdot d}]}{\mathrm{T}[V_{x_0+(i+1)\cdot d}, V_{x_0+i\cdot d}]}$$

At this point a multilevel algorithm can be used to achieve an exponential error reduction, similarly to what was done by Lüscher and Weisz '01 for the Polyakov Loops

The insertion of $\mathrm{T}^s[V_{y_0}, V_{x_0}]$ in the path integral plays the role of a projector and allows the propagation of states with a given parity only.

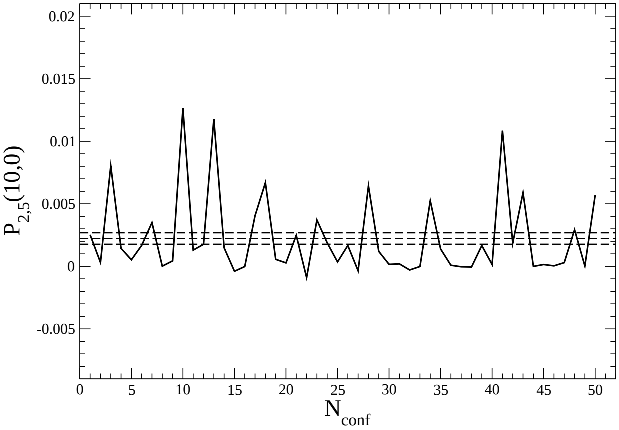
As we need to compute $P^{-}_{m,d}[T,0]$ for each boundary configuration generated with the weight $e^{-S[U]}$ we are performing MC simulations within a MC simulation $\rightarrow$ We have a V^2 algorithm.

We extract the effective mass from

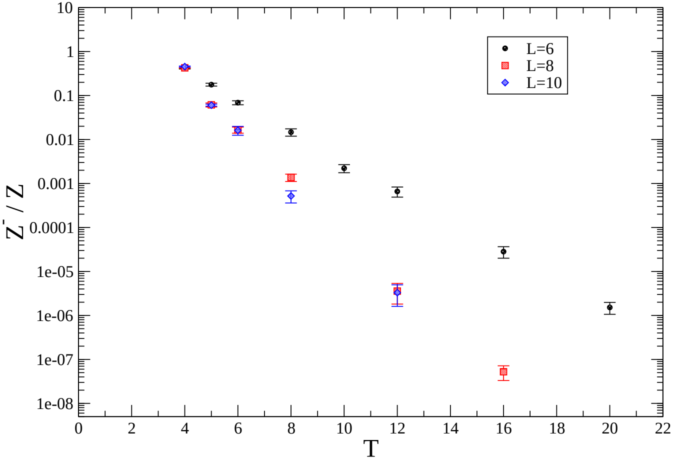
$$m^{eff}_{G^-}(T) = -\frac{1}{T}\ln\left(\frac{Z^-}{Z}(T)\right)$$

For a given precision on that, the algorithm scaling with T is proportional to $\simeq e^{2m_{G^-}d}\cdot(\frac{T}{d})^2$ to be compared with the $\simeq e^{2m_{G^-}T}$ scaling of the standard technique

Results (Wilson action, $\beta = 5.7$)

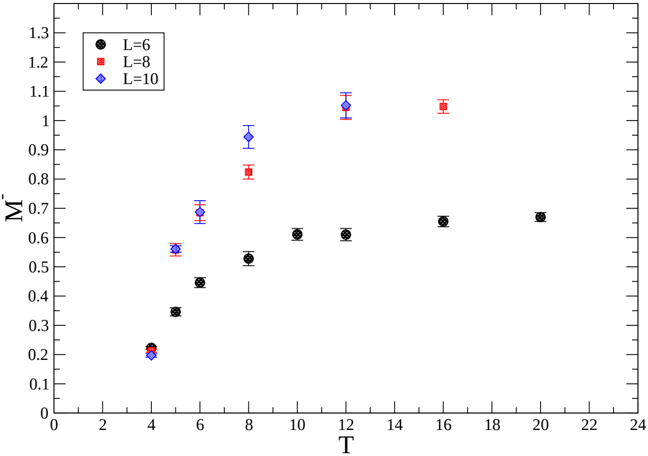


MC history of $P^-_{2,5}$ from the $L = 6,\ T = 10$ and $\beta = 5.7$ run



We could follow the exponential decay over 7 orders of magnitude.

By increasing the precision on the points at large T ($T/a > 12$) we could also extract the multiplicity of the state, which is not accessible in other approaches. As now we have asummed it to be 1.



Signal up to more than 3 fm separation !

Finite size effects clearly visible for $L < 1.7$ fm .

For $J^{PC} = 0^{-+}$ we estimate $r_0 m_{G^-} = 3.07(7)$ $\beta = 5.7$, Wilson. [Preliminary]

Conclusions and outlook

- The noise to signal exponential problem can be solved by enforcing the propagation in time of states with the desired quantum numbers only
- We have tested the strategy in the pure-YM case for the mass of the first parity-odd state
- In the future we plan to extend the computation to larger volumes and finer lattice spacings. The existence of a bound glueball state could then be proven in a theoretically sound way (a single state dominating a correlation function over large time separations).
- but also to consider other symmetries (C-parity rather similar, O(3) a bit more complicated)
- Fermions ?