Hadron structure with Wilson fermions on the Wilson cluster

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Introduction

Ongoing long-term project to compute hadronic correlation functions – **at fine lattice spacings, and with full control over the systematic errors**

FORM FACTORS, STRUCTURE FUNCTIONS, GENERALIZED PARTON DISTRIBUTIONS, . . .

$N_f = 2$ flavours of (non-perturbatively) $O(a)$ improved Wilson quarks

Preliminary results: obtained at three quark masses ($\kappa = 0.13640, 0.13650, 0.13660$) on a $96 \cdot 48^3$ lattice at $\beta = 5.5$

$\rightarrow$ lattice spacing $a = 0.06\ fm$, lattice size $L = 2.9\ fm$

In the current runs: smallest pion mass around $360\ MeV$, which corresponds to $m_\pi L = 5.3$

Maintaining $\underline{m_\pi L > 3}$ is a necessary condition to control finite-volume effects and obtain significant results

Approaching the chiral limit ($m_\pi \rightarrow 135\ MeV$) will require <u>very large lattices</u> and substantial computational efforts

The work is part of the ***CLS*** *project ("Coordinated Lattice Simulations")*

Introduction

CLS: generate a set of ensembles for QCD with two dynamical flavours for a variety of lattice spacings ($a \approx 0.04, 0.06, 0.08\ fm$) and volumes, such that the continuum limit can be taken in a controlled manner

CLS: Berlin - CERN - DESY - Madrid - Mainz - Milan - Rome - Valencia
$\longrightarrow$ share configurations and technology

WE NEED TO HAVE FULL CONTROL OVER ALL SYSTEMATICS

Continuum limit of lattice QCD with dynamical quarks still poorly understood
$\longrightarrow$ no continuum limit for many phenomenologically interesting observables

There are not many systematic scaling studies of hadronic quantities

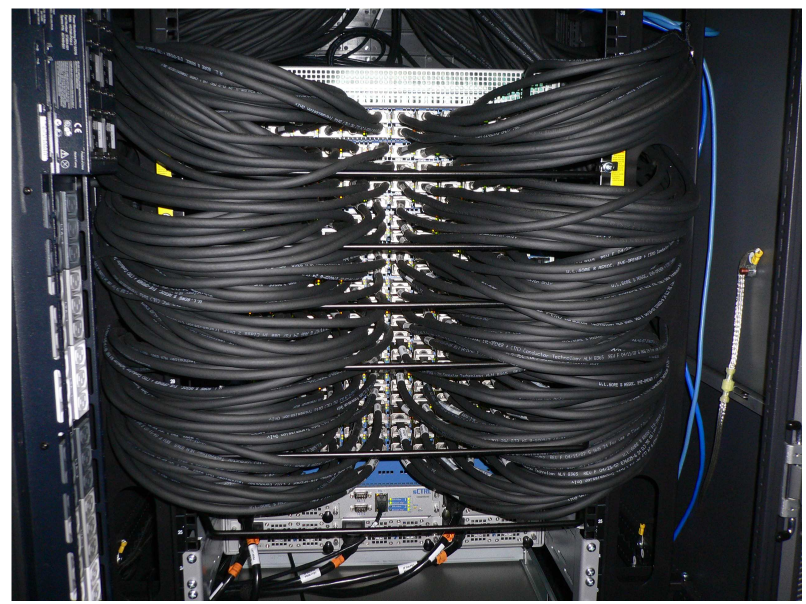
Many results obtained at one or two values of the lattice spacing only

$m_\pi L$ is often dangerously small (≤ 3)

To tune the masses of the light quarks towards their physical values and at the same time keep the numerical effort in the simulations at a manageable level:
deflation accelerated DD-HMC algorithm

DD-HMC algorithm on commodity cluster hardware

The Wilson Cluster



The Wilson Cluster

Cluster platform Wilson, at the Inst. for Nuclear Physics in the Univ. of Mainz

Fully commissioned in NOVEMBER 2008

`Exclusively used for lattice QCD`

280 nodes, each equipped with two AMD 2356 QuadCore processors

$\Rightarrow$ 2240 cores, clocked at $2.3\,GHz$

Sustained performance: up to $3.6\,TFlops$ (depending on local system size)

Cost: $1.1\,Million\ euros$

$\Rightarrow$ cost-effectiveness of about $0.30\,euros/MFlops$ (sustained)

Each core: $1\,GByte$ of memory $\rightarrow$ cluster's total memory" $2.24\,TBytes$

Communication between nodes: realised via an Infiniband network and switch

The compute nodes are placed in water-cooled server racks

The required cooling capacity per compute speed is $20\,kW/TFlops$

Runs

By now: about <u>one year of production runs</u>

We have generated configurations at $\beta = 5.5$ on lattices of size $96 \cdot 48^3$
$\rightarrow \quad a = 0.06\ fm$, $L = 2.9\ fm$

The length of one Hybrid Monte Carlo trajectory was set to $\tau = 0.5$

Symanzik improvement, with $c_{sw} = 1.75150$

Calculation of quark propagators, extended propagators, 2-point and 3-point correlation functions, . . .

Analysis

In the future: $0.04\ fm \leq a \leq 0.08\ fm$

Meson physics and baryon physics

The baryon project

Extensive project for the computation of matrix elements of baryons

Code for baryonic correlators (2pt, 3pt) : we developed several routines which extend the freely available code by Martin Lüscher (based on DD-HMC)

Observables:

- FORM FACTORS
- STRUCTURE FUNCTIONS
- GENERALIZED PARTON DISTRIBUTIONS
- ....

The generic structures of the operators which measure the moments of structure functions are

$$\overline{\psi}\gamma_\mu D_{\mu_1} \ldots D_{\mu_n}\psi, \quad \overline{\psi}\gamma_\mu\gamma_5 D_{\mu_1} \ldots D_{\mu_n}\psi$$

for unpolarized and polarized structure functions respectively, and

$$\overline{\psi}\sigma_{\mu\nu}\gamma_5 D_{\mu_1} \ldots D_{\mu_n}\psi$$

for the transversity structure function

Some technical points

At this stage of the project:

- Calculation of matrix elements (3-point correlators): need to generate the quark propagator $S(y, x)$ from every source x to every other sink y

  $\Rightarrow$ would require $L^3 \cdot T$ inversions of the Dirac operator

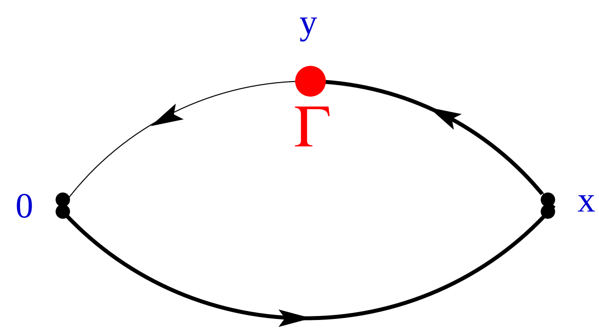
  Solution: extended propagators
- Low transferred momenta $\longrightarrow$ twisted boundary conditions
- Better interpolating operators: Jacobi smearing, stochastic sources . . .
- *SSE3* rewriting of the most frequently used functions

Not yet completely “settled”:

- disconnected diagrams
- twisted boundary conditions for some baryonic correlators

Extended propagators

Extended source method: we need to compute



$$\mathrm{Tr}\left(\sum_{\vec{y};y_0=\tau} e^{i\,\vec{q}\cdot\vec{y}}\,S(0,y)\,O(y)\sum_{\vec{x};x_0=t} e^{-i\,\vec{p}\cdot\vec{x}}\,S(y,x)\,\gamma_5\,S(x,0)\,\gamma_5\right)$$

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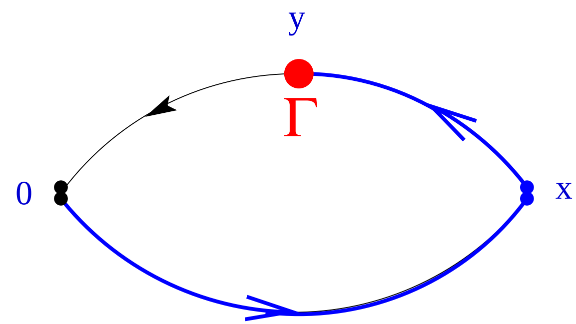

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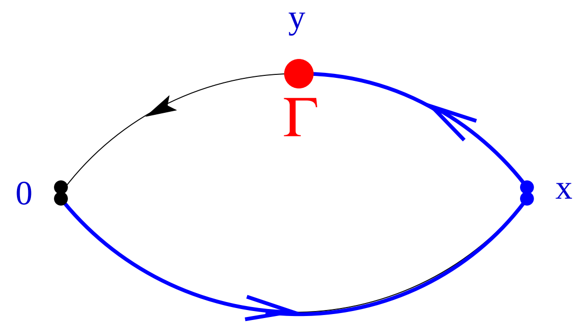

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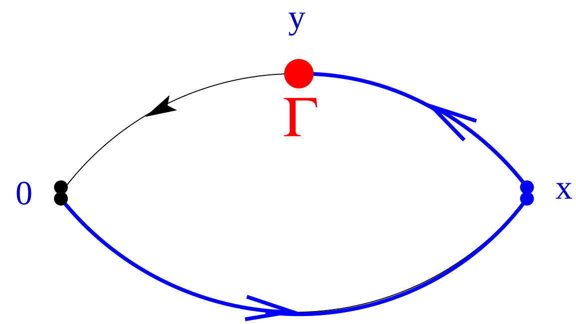

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The extended propagator can then be obtained by a simple additional inversion (for each choice of the final momentum $\vec{p}$):

$$\sum_{y} M(z,y)\,\Sigma(y,0) = e^{-i\,\vec{p}\cdot\vec{z}}\,\gamma_5\, S(z,0)\Big|_{z_0=t}$$

Extended propagators

Changing the properties of the sink, i.e., simulating:

- several final momenta , or
- a different field interpolator , or
- a different smearing for the sink

requires the computation of new extended propagators and becomes rapidly rather expensive

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We have however chosen to define the extended propagators through a fixed sink rather than through a fixed current

A fixed current would be indeed even more expensive, because it requires a new inversion for

- each different value of the momentum transfer , or
- every new type of operator (scalar, vector, . . .)

Baryons

Standard interpolating operators for the nucleon, the Δ and the Ω

The current that we use for the nucleon, and the Δ^+ and Δ^0 particles of the spin-$3/2$ decuplet, is given by

$$J_\gamma(x) = \epsilon^{abc} \left(u^a(x)\,\Gamma\,d^b(x)\right)\, u^c_\gamma(x)$$

For the nucleon $\Gamma = C\gamma_5$, while for the Δ^+ and Δ^0 one must use $\Gamma = C\gamma_\mu$

The current

$$J_\gamma(x) = \epsilon^{abc} \left(u^a(x)\,\Gamma\,u^b(x)\right)\, u^c_\gamma(x)$$

is used for the Δ^{++} particle, with $\Gamma = C\gamma_\mu$

If the u quarks are replaced by the d or s flavor we obtain the Δ^- or Ω^- baryons, respectively

2-point correlator for the nucleon:

$$-\epsilon^{abc}\,\epsilon^{a'b'c'}\,\Gamma_{\alpha\beta}\,(\overline{\Gamma}^T)_{\alpha'\beta'}\,S^{bb'}_{\beta\beta'}\left(S^{aa'}_{\alpha\alpha'}\,S^{cc'}_{\gamma\gamma'} - S^{ac'}_{\alpha\gamma'}\,S^{ca'}_{\gamma\alpha'}\right)$$

where $S = S(x,0)$ and $\overline{\Gamma} = \gamma_0\Gamma^\dagger\gamma_0$

Baryons

2-point correlator for the Ω^- (and Δ^{++} and Δ^- as well):

$$
\begin{aligned}
-\epsilon^{abc}\,\epsilon^{a'b'c'}\,\Gamma_{\alpha\beta}\,(\overline{\Gamma}^T)_{\alpha'\beta'} \quad \cdot \quad & \Big\{ S^{bb'}_{\beta\beta'} \left(S^{aa'}_{\alpha\alpha'}\, S^{cc'}_{\gamma\gamma'} - S^{ac'}_{\alpha\gamma'}\, S^{ca'}_{\gamma\alpha'} \right) \\
& + S^{ba'}_{\beta\alpha'} \left(-S^{ab'}_{\alpha\beta'}\, S^{cc'}_{\gamma\gamma'} + S^{ac'}_{\alpha\gamma'}\, S^{cb'}_{\gamma\beta'} \right) \\
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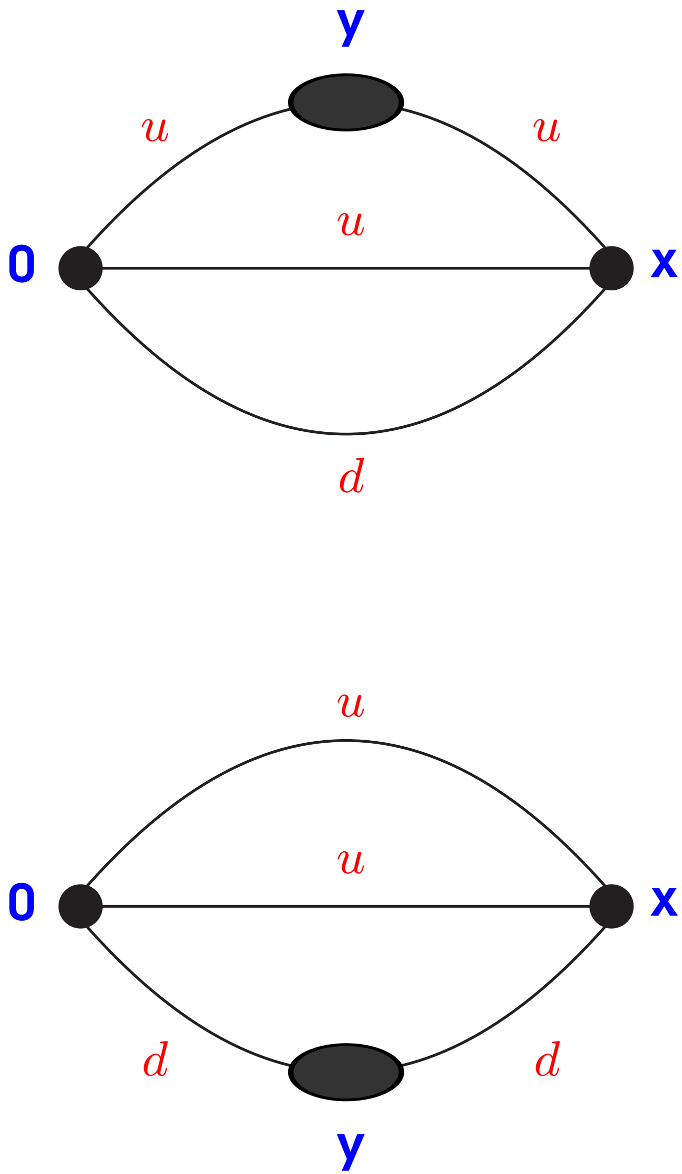
\end{aligned}
$$

The extended propagator for a proton when an u quark is attached to the operator at y is

$$
\begin{aligned}
\Sigma_u(0,y)^{xy}_{\rho\sigma} \quad = \quad & -\epsilon^{abc}\,\epsilon^{xb'c'}\,\Gamma_{\alpha\beta}\,(\overline{\Gamma}^T)_{\rho\beta'}\, \left(\overline{S}^{ay}_{\alpha\sigma}\, S^{bb'}_{\beta\beta'}\, S^{cc'}_{\gamma\gamma'} - \overline{S}^{cy}_{\gamma\sigma}\, S^{bb'}_{\beta\beta'}\, S^{ac'}_{\alpha\gamma'} \right) \\
& -\epsilon^{abc}\,\epsilon^{a'b'x}\,\Gamma_{\alpha\beta}(\overline{\Gamma}^T)_{\alpha'\beta'}\, \left(\overline{S}^{cy}_{\gamma\sigma}\, S^{bb'}_{\beta\beta'}\, S^{aa'}_{\alpha\alpha'} - \overline{S}^{ay}_{\alpha\sigma}\, S^{bb'}_{\beta\beta'}\, S^{ca'}_{\gamma\alpha'} \right) \cdot \delta_{\gamma'\rho}
\end{aligned}
$$

where $S = S(x,0)$ and $\overline{S} = S(x,y)$

Baryons



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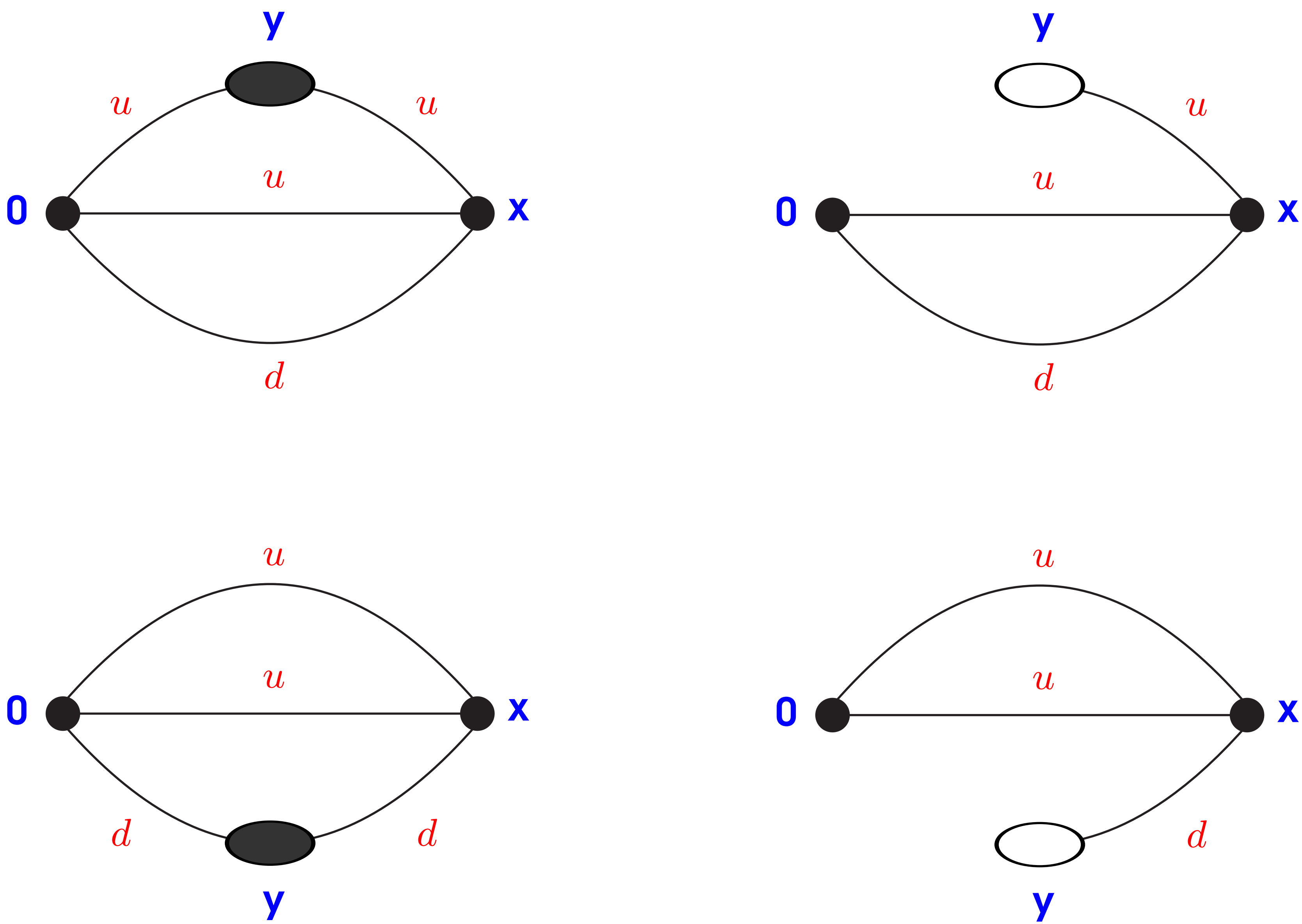

Baryons

The corresponding source satisfies

$$\sum_y \Sigma_u(0,y)^{xy}_{\rho\sigma}\, M^{yz}_{\sigma\tau}(y,z) = (\eta_u)^{xz}_{\rho\tau}(0,z)$$

and is consequently given by

$$\begin{aligned}
(\eta_u)^{xz}_{\rho\tau} \quad = \quad & -\epsilon^{abc}\,\epsilon^{xb'c'}\,\Gamma_{\alpha\beta}\,(\overline{\Gamma}^T)_{\rho\beta'}\,\left(\delta^{az}\,\delta_{\alpha\tau}\,S^{bb'}_{\beta\beta'}\,S^{cc'}_{\gamma\gamma'} - \delta^{cz}\,\delta_{\gamma\tau}\,S^{bb'}_{\beta\beta'}\,S^{ac'}_{\alpha\gamma'}\right) \\
& -\epsilon^{abc}\,\epsilon^{a'b'x}\,\Gamma_{\alpha\beta}\,(\overline{\Gamma}^T)_{\alpha'\beta'}\,\left(\delta^{cz}\,\delta_{\gamma\tau}\,S^{bb'}_{\beta\beta'}\,S^{aa'}_{\alpha\alpha'} - \delta^{az}\,\delta_{\alpha\tau}\,S^{bb'}_{\beta\beta'}\,S^{ca'}_{\gamma\alpha'}\right)\cdot\delta_{\gamma'\rho} \\
& = -\epsilon^{zbc}\,\epsilon^{xb'c'}\,\Gamma_{\tau\beta}\,(\overline{\Gamma}^T)_{\rho\beta'}\,S^{bb'}_{\beta\beta'}\,S^{cc'}_{\gamma\gamma'} \\
& \quad +\epsilon^{abz}\,\epsilon^{xb'c'}\,\Gamma_{\alpha\beta}\,(\overline{\Gamma}^T)_{\rho\beta'}\,S^{bb'}_{\beta\beta'}\,S^{ac'}_{\alpha\gamma'}\cdot\delta_{\gamma\tau} \\
& \quad -\epsilon^{abz}\,\epsilon^{a'b'x}\,\Gamma_{\alpha\beta}\,(\overline{\Gamma}^T)_{\alpha'\beta'}\,S^{bb'}_{\beta\beta'}\,S^{aa'}_{\alpha\alpha'}\cdot\delta_{\gamma'\rho}\cdot\delta_{\gamma\tau} \\
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Similar formulae hold when it is the d quark to be attached to the operator at y

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`All this has been already implemented`

Twisted boundary conditions

On a finite lattice with periodic boundary conditions, a limited number of discrete momenta:

$$\vec{p} = \frac{2\pi}{aL}\,\vec{n}$$

The lowest available non-zero momentum is in general already too large: for a lattice with $a = 0.05\ fm$ and $L = 64$ one gets

$$\sqrt{p^2_{\min}} \simeq 400\ MeV$$

A severe limitation when one is interested in the physics <u>at low momenta</u> . . .

Need the near-forward region to extract radii (derivative at zero momentum)

One also does not wish to use models for the momentum dependence

Furthermore: there are big gaps between neighboring momenta

Need then a better resolution for form factors, . . . (fit to a function)

We can overcome this limitation by using non-periodic boundary conditions for the fields

$\Rightarrow$ <u>twisted</u> boundary conditions (Bedaque; Sachrajda & Villadoro, . . .)

Twisted boundary conditions

It is not necessary to use periodic boundary conditions – is just simpler

One can choose different boundary conditions, provided that the action

$$\mathcal{L}_\psi = \overline{\psi}(x)\,(\not{D} + M)\,\psi(x)$$

still maintains its single-valuedness

$M =$ diagonal mass matrix $(u, d, s, \ldots)$

Flavor twisted boundary conditions (in the spatial directions):

$$\psi(x + L\hat{k}) = U_k\,\psi(x) \qquad (k = 1, 2, 3)$$

where the unitary matrices U_k must satisfy

$$[U_k, \not{D}] = [U_k, M] = 0$$

U_k must then be diagonal :

$$U_k = e^{i\theta_k} = e^{i\theta_k^a t^a}$$

t^a: generators in the Cartan subalgebra of flavor $U(N)_V$ which commutes with M

Twisted boundary conditions

Redefine

$$\psi(x) = e^{i\frac{\theta_k}{L}x_k}\,\chi(x) = V(x)\,\chi(x)$$

where $\chi(x)$ now obeys periodic boundary conditions: $\chi(x+L\hat{k}) = \chi(x)$

In this new variable:

$$\mathcal{L}_\chi = \overline{\chi}(x)\,(\not{D} + V^\dagger(x)\,\not{\partial}\,V(x) + M)\,\chi(x) = \overline{\chi}(x)\,(\not{D}^{b.c.} + M)\,\chi(x)$$

where

$$D_\mu^{b.c.} = D_\mu + iB_\mu$$

B is a constant background field: $B_\mu = \dfrac{\theta_\mu}{L} \quad (B_0 = 0)$

We now work with periodic quark fields coupled to a constant vector field B_μ, with charges given by the phases in the twisted boundary conditions

Free twisted quark propagator:

$$S^{bc}(x,\theta) = \langle\chi(x)\overline{\chi}(0)\rangle = \frac{1}{L^3}\sum_{\vec{p}=\frac{2\pi}{L}\vec{n}}\int\frac{dp_0}{2\pi}\,\frac{e^{ipx}}{i(\not{p}+\not{B})+M}$$

$\rightarrow$ the quark momentum is now given by

$$\vec{p} = \frac{2\pi}{aL}\vec{n} + \frac{\vec{\theta}}{aL}$$

Twisted boundary conditions

Now, also lower momenta can propagate on the lattice

We can produce a continuous hadron momentum: the momentum transfer can thus be continuously varied

$\longrightarrow$ arbitrarily low

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There is a breaking of isospin symmetry induced by the boundary conditions

The field B_μ breaks cubic symmetry, and also all symmetries which do not commute with it, like flavor $SU(3)$, I^2, ...

However, it does not break I_z, strangeness and the electric charge

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Flavor twisting produces long-range interactions that modify the physics

This generates finite volume corrections , which can be estimated using chiral effective theories

Sachrajda & Villadoro (2004), for mesons: these corrections remain exponentially small with the volume, also in the case of partially twisted boundary conditions (for quantities without final-state interactions)

Twisted boundary conditions

Partially twisted boundary conditions: implemented only in the valence sector, while the sea quarks remain periodic at the boundary

Enormous gain: no need in unquenched simulations to generate new gauge configurations for each value of $\vec{\theta}$

Furthermore: if the sea quarks u and d are twisted differently, then with fully twisted boundary conditions one must use lattice fermions for which the determinant is positive definite for each single flavor

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$\vec{\theta}$ is diagonal $\Rightarrow$ π^0 commutes with B_μ $\Rightarrow$ no twist!

For $\pi^\pm$: shift in momentum given by

$$\frac{\vec{\theta}_u - \vec{\theta}_d}{L}$$

Momentum transfer:

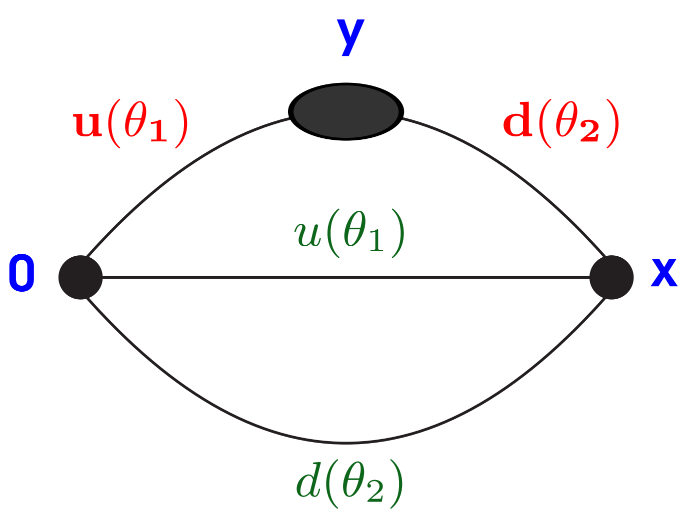
$$\vec{q} = \frac{2\pi}{aL}\vec{n} + \frac{\delta\vec{\theta}}{aL}$$

where now $\delta\vec{\theta}$ is the difference in the twist angles of the flavors changed

Twisted boundary conditions

Twisting is easy for matrix elements like the transition form factors, where the initial and final particles are different

$p \longrightarrow n$ transition:



$$\longrightarrow \quad \frac{\vec{\theta}_1 - \vec{\theta}_2}{L}$$

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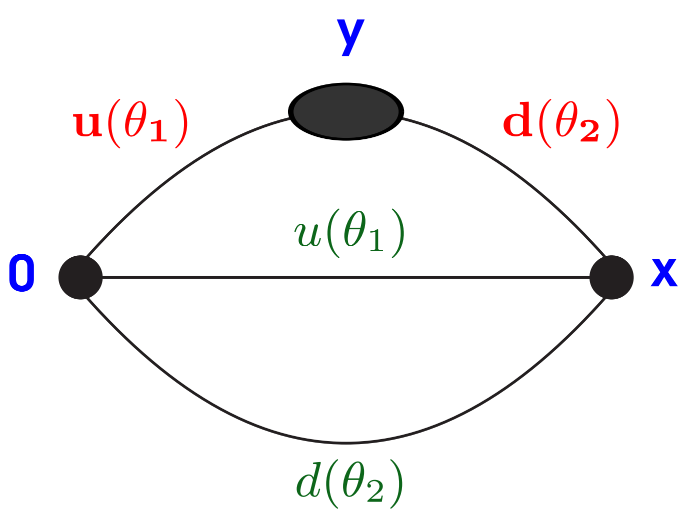


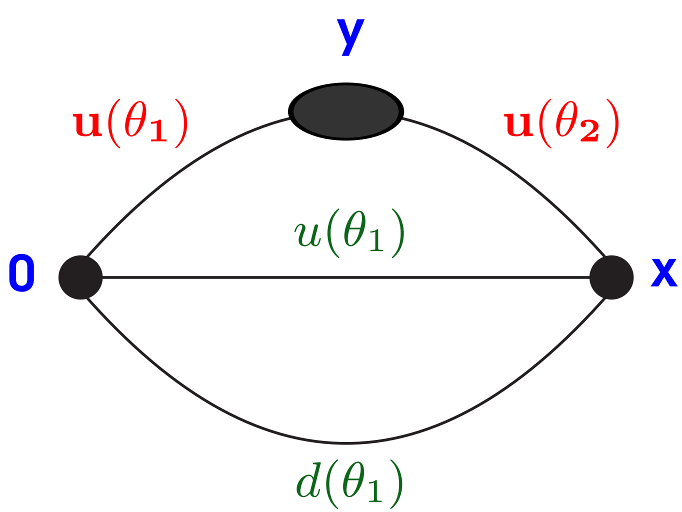
$$\longrightarrow \quad \frac{\vec{\theta}_1 - \vec{\theta}_2}{L}$$

More complicated, instead, when the final particle is the same as the initial one

For scattering form factors: the active quarks, that couple to the current insertion, are of the same flavor

Twisted boundary conditions

Proton form factor:



$$\longrightarrow \qquad \frac{\vec{\theta}_1 - \vec{\theta}_2}{L}$$

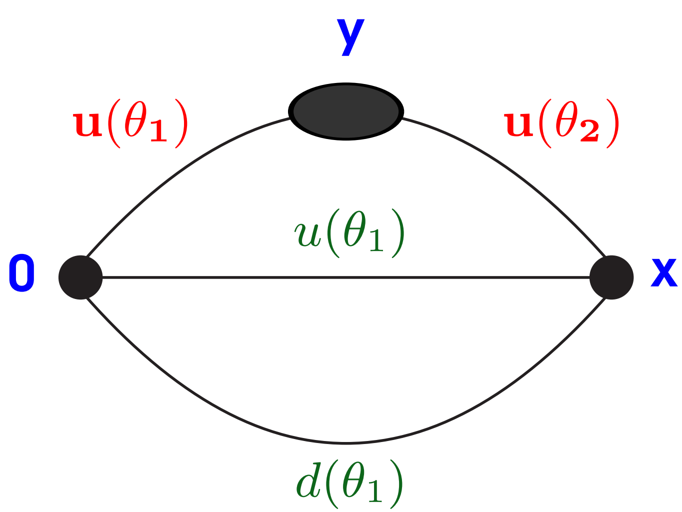
Need then introduce extra fictitious flavors, differing only in their boundary conditions

However: now the finite volume corrections depend on an unphysical and unknown parameter, g_1 (an artefact of the enlarged valence flavor group)

A lattice calculation of g_1 will be necessary to control the systematic uncertainty from volume effects in this approach

Twisted boundary conditions

Proton form factor:



$$\longrightarrow \quad \frac{\vec{\theta}_1 - \vec{\theta}_2}{L}$$

Need then introduce extra fictitious flavors, differing only in their boundary conditions

However: now the finite volume corrections depend on an unphysical and unknown parameter, g_1 (an artefact of the enlarged valence flavor group)

A lattice calculation of g_1 will be necessary to control the systematic uncertainty from volume effects in this approach

Other ideas can be useful…

Twisted boundary conditions

For the pion: one can also use isospin symmetry (Sachrajda et al., 2007)

The valence strange quark plays no role for the pion form factor $\rightarrow$ implement partial quenching with

$$m_u^V = m_u^S = m_d^V = m_d^S = m_s^V \neq m_s^S = m_s^{phys}$$

Then, from flavor $SU(3)$ symmetry of the valence quarks one gets

$$\langle \pi^+ | \overline{u}\gamma_\mu u \, |\pi^+\rangle = -\langle \pi^+ | \overline{d}\gamma_\mu d \, |\pi^+\rangle = \langle \pi^+ | \overline{u}\gamma_\mu s \, |\bar{K}^0\rangle$$

$\Rightarrow$ simulate (and twist) the $K \rightarrow \pi$ matrix element

This is then equal to the sought-for pion form factor

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For the nucleon: Tiburzi (2006) pointed out that, using vector flavor $SU(3)$,

$$\langle p|\,\overline{u}\gamma_\mu d\,|n\rangle = \langle p|\,\frac{2}{3}\,\overline{u}\gamma_\mu u - \frac{1}{3}\,\overline{d}\gamma_\mu d\,|p\rangle - \langle n|\,\frac{2}{3}\,\overline{u}\gamma_\mu u - \frac{1}{3}\,\overline{d}\gamma_\mu d\,|n\rangle$$

Also showed that the magnetic form factor can be computed as

$$\langle p(\vec{q})\downarrow|J_3^1 + iJ_3^2|n(\vec{0})\uparrow\rangle = \frac{-iq}{2M}\,F_2(q^2) \qquad \left(J_\mu^a = \overline{\psi}T^a\gamma_\mu\psi\right)$$

Twisted boundary conditions

Finite volume corrections: in the case of baryons can become pronounced for small twist angles (Tiburzi)

They then decrease like powers of the volume, instead of exponentially small corrections

The same seems to happen for the magnetic contributions

Also: one must use heavy baryon chiral perturbation theory for the study of these volume corrections

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Other drawback (also for mesons) : technique limited only to connected contributions

Twisted boundary conditions cannot be applied to disconnected diagrams (i.e., self-contractions)

Then, for these diagrams $2\pi/L$ remains the only option as to the minimum momentum – no continuous momentum

Topology

Issue: critical slowing down – especially for the topological charge

Simulations performed as part of the CLS project have revealed a severe case of critical slowing down in the topological charge

Steep rise of the autocorrelation time as a function of the lattice spacing

It was observed that at $\beta = 5.7$ (where $a \approx 0.04\ fm$) tunnelling between topological sectors is strongly suppressed

In our simulations at $\beta = 5.5$ (run N3) the topological charge is not stuck at zero, and produces a distribution which is reasonably symmetric

Similar observations were made at the other values of the quark masses used in our simulations

Thus, unlike the situation encountered at the larger $\beta = 5.7$, our topological charge does not appear to be stuck in a particular sector

The distribution of topological charge is not pathological

While this may be accidental, we can take confidence that the composition of our ensembles is apparently not strongly biased

Better interpolating operators

We also investigate the effectiveness of stochastic noise sources and Jacobi smearing

Point source: the hadron correlators can be quite noisy

an unambiguous identification of the asymptotic behaviour is then quite difficult

Aims :

- reduce the level of statistical noise
- enhance the spectral weight of the desired state in the spectral decomposition of the correlator

To enhance and tune the projection onto the ground state of interpolating operators in a given channel: we have implemented Jacobi smearing, supplemented by fat link variables (APE or HYP procedure)

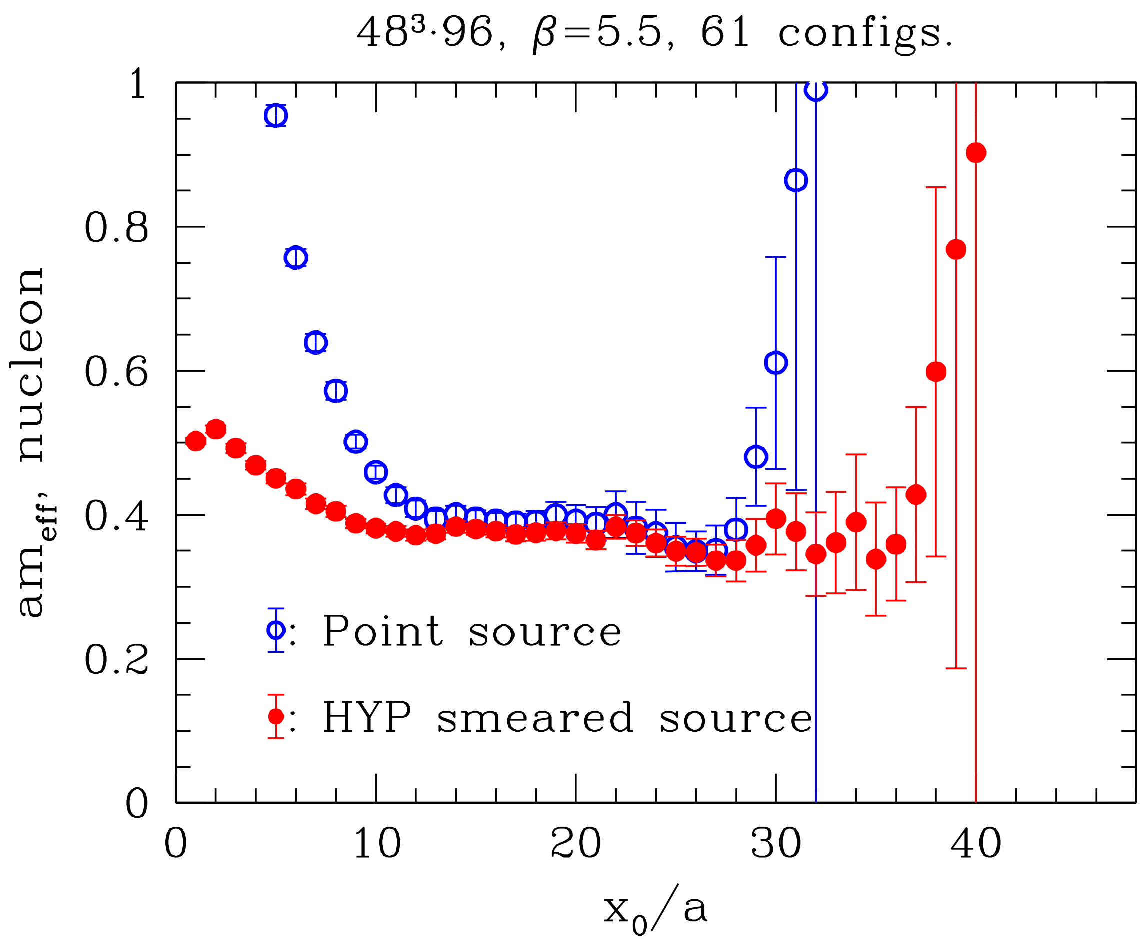
Particularly important for baryons

Jacobi smearing: now implemented also at the sink

While we found much better plateaus when using smeared links of either type, HYP smearing appears to have a slight advantage

Better interpolating operators

Effective mass plots for the nucleon, with point and HYP-Jacobi sources



Not only the contribution of excited states is reduced – also the plateau extends to larger timeslices if HYP-Jacobi smearing is applied

`Room for further improvement via better tuning of the smearing parameters`

Better interpolating operators

We have also implemented stochastic noise sources (“all-to-all”), with the generalised “one-end-trick”

Generalized one-end-trick: choose a spin-diagonal random source vector

The noise source has support only on a particular spin component and timeslice

For every hit (every choice of random source) one must perform four inversions (one for each spin component)

Compared with point sources, numerical costs are reduced by a factor three per hit

In the pion channel we see that random noise sources lead to a significant enhancement of the statistical signal

A similar improvement is, unfortunately, not observed in the vector channel

For baryons: in order to reach a given statistical accuracy, the numerical effort is at least as large as for point sources (even with various dilution schemes)

The method does not seem to be useful for the determination of baryonic ground state masses

Setting the scale

Setting of the overall scale: using the Ω^- baryon

The mass of the Ω^- baryon is very well suited for this purpose:

- the Ω^- is stable in QCD
- it contains only strange quarks in the valence sector

  $\Rightarrow$ a long chiral extrapolation in the valence quark mass can be avoided

Our simulations are at this moment not yet advanced enough for a reliable determination of the mass of the Ω^-

For the time being we use m_K as a reference scale, and need also to determine the mass of the $K^\star$-meson (procedure of the CERN group)

$K^\star$-meson: a vector particle with one s antiquark and one u or d quark (here: degenerate)

This method seems to work reasonably well

Setting the scale

SHORT DESCRIPTION:

For a fixed m_u, compute $m_{K^\star}/m_K$ for a few values of m_s

Interpolate $(m_{K^\star}/m_K)^2$ as a function of $(am_K)^2$ to its physical value $m_{K^\star}/m_K = 0.554$

Repeat similar determinations of m_s for various values of m_u

Unfortunately it is not practical to fix m_u by extrapolating to the physical value of m_π/m_K

It would require a long extrapolation in m_u, and the $K^\star$ would become unstable ($\rightarrow$ kinematical threshold)

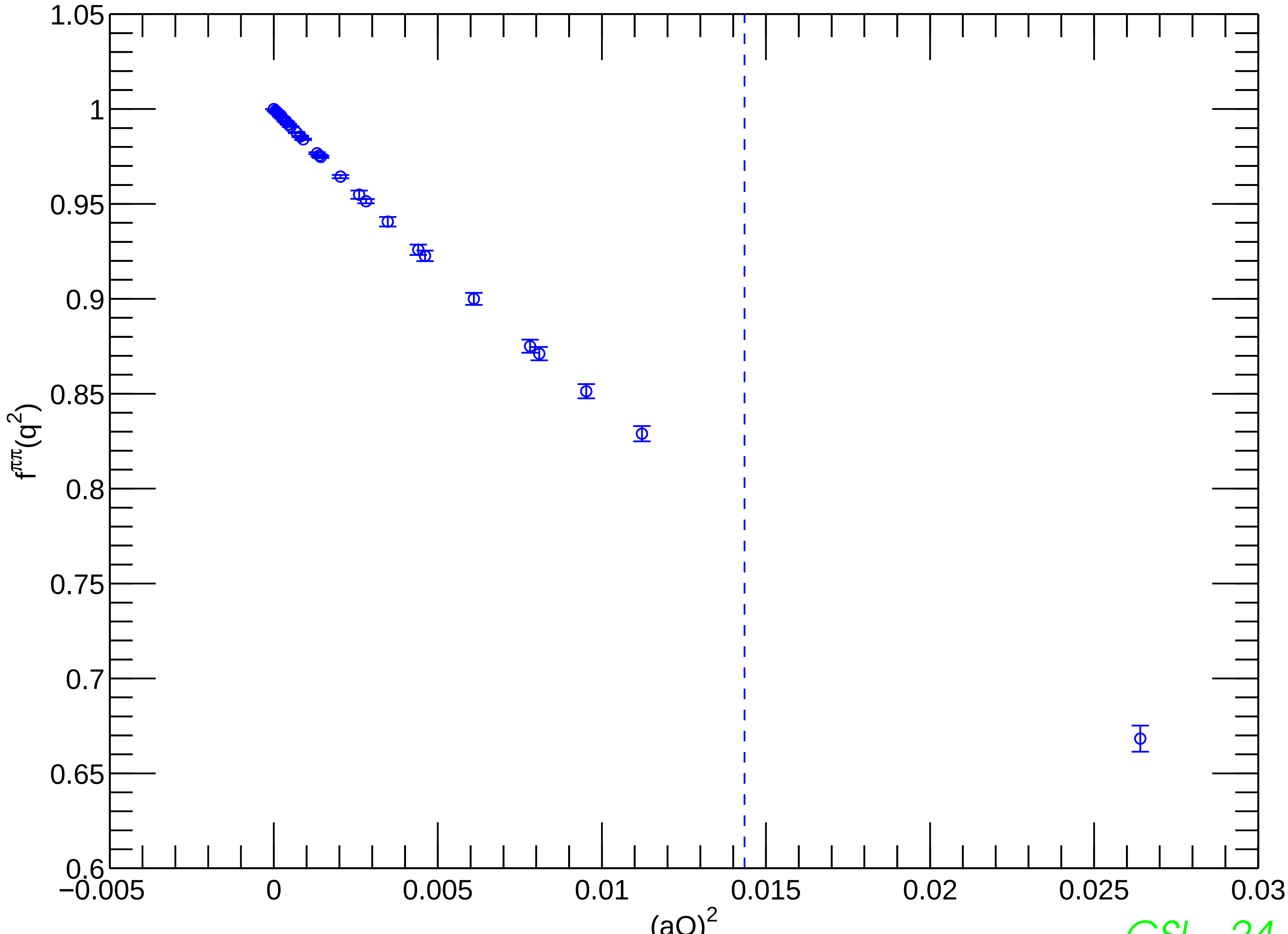
But, noticing that am_K is very weakly dependent on m_u, we interpolate am_K in m_u to the reference point $m_\pi/m_K = 0.85$

Comparing this am_K^{ref} in lattice unit with the physical value $m_K = 495\ MeV$, we obtain the value of a

In our present lattice: $am_K = 0.1512(38)$ $\Rightarrow$ $a = 0.0603(15)\ fm$

Pion form factor

Andreas Jüttner (N4, absolutely preliminary) :



Conclusions

Large lattices at fine resolution

They can be simulated efficiently on commodity clusters

In spite of a sharp increase in the autocorrelation time of the topological charge observed at even smaller lattice spacings: the distributions for this quantity obtained in our runs are not pathological

We plan to compute several two-and three-point correlation functions for mesonic and baryonic states

$\Rightarrow$ determine a variety of observables

$\longrightarrow$ use of twisted boundary conditions

$\longrightarrow$ look for better interpolating operators

$\longrightarrow$ in general: improve techniques

Perspectives

In the longer term:

- LOWER THE QUARK MASS

  So far our minimum pion mass is about $360\ MeV$, and $m_\pi L \geq 5$

  We think that maintaining $m_\pi L > 3$ is a necessary condition to obtain significant results

- STUDY LARGE VOLUMES

  Further lowering the quark mass, in order to access pion masses of less than $300\ MeV$ would necessitate going to larger lattice sizes, if one wants to maintain the condition $m_\pi L > 3$

  Planned: $L = 3.8\ fm$

- POSTPONE STUDY OF $a \approx 0.04\ fm$ UNTIL TOPOLOGY ISSUE IS BETTER UNDERSTOOD

  With the currently available algorithms, i.e. while a satisfactory solution to the problem of critical slowing down is still under investigation, it is not worth investing more effort into the generation of ensembles with smaller lattice spacings