Nucleon and resonance imaging from experiment and lattice QCD

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FAIR Lattice QCD Days @ GSI

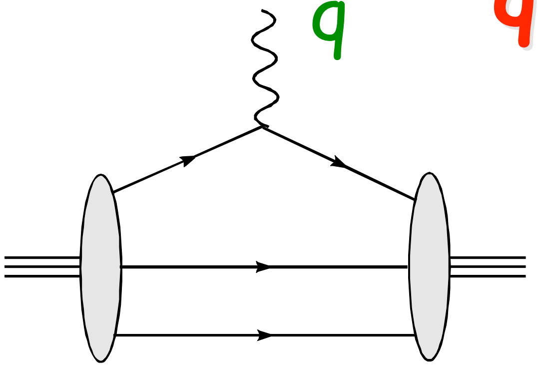
Darmstadt, November 23-24, 2009

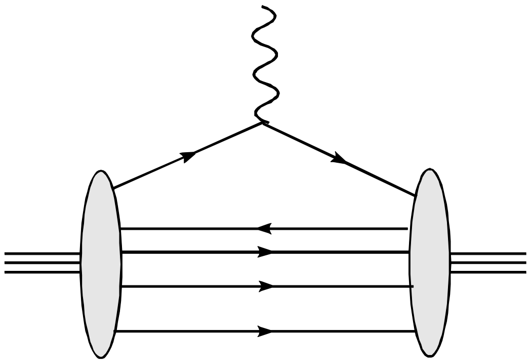
Outline

What do we know about the transverse structure (imaging) of hadrons from phenomenology and lattice QCD ?

- Light-front charge densities <-> elastic nucleon Form Factors
- Shape of hadrons <-> higher e.m. moments of transverse charge densities (systems of spin 1 or higher)
- Deformation of light-front charge densities in external e.m. field (polarization) <-> nucleon Generalized Polarizabilities
- Resonance structure / transition charge densities <-> $N \to N^*$ Transition Form Factors
- Quark densities in both transverse position and longitudinal momentum <-> nucleon Generalized Parton Distributions

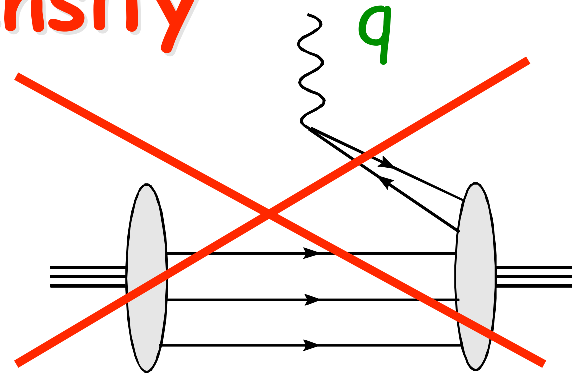
interpretation of Form Factor as quark density





overlap of wave function Fock components with same number of quarks

interpretation as probability/charge density



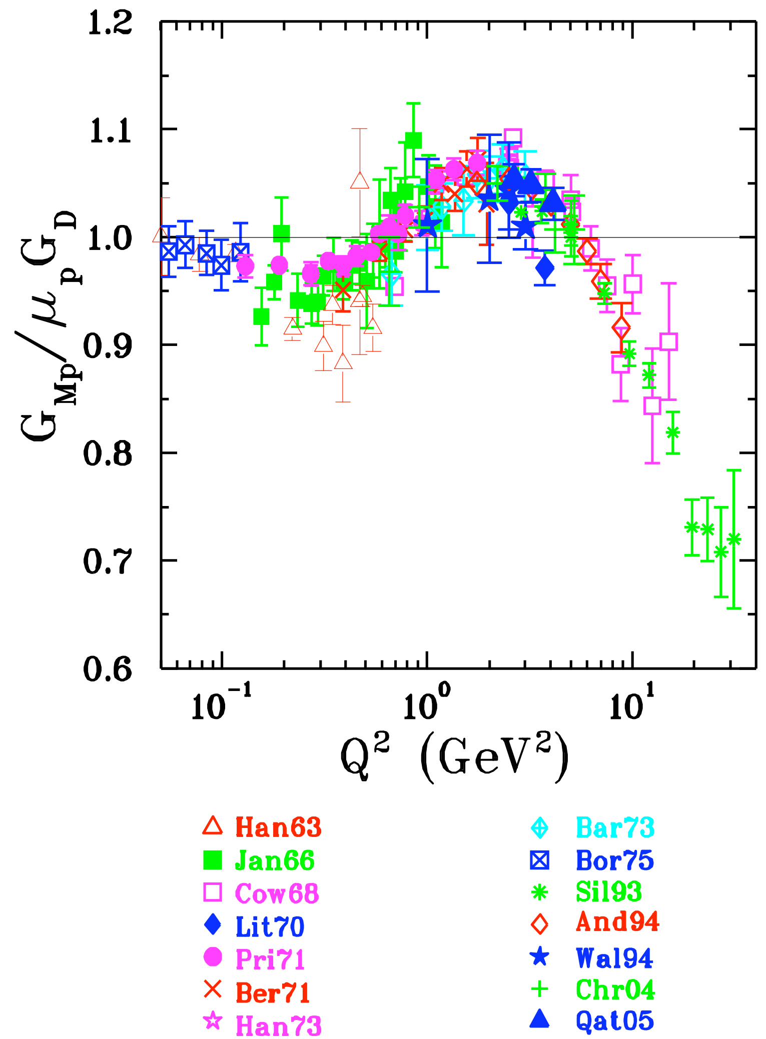
overlap of wave function Fock components with different number of constituents

NO probability/charge density interpretation

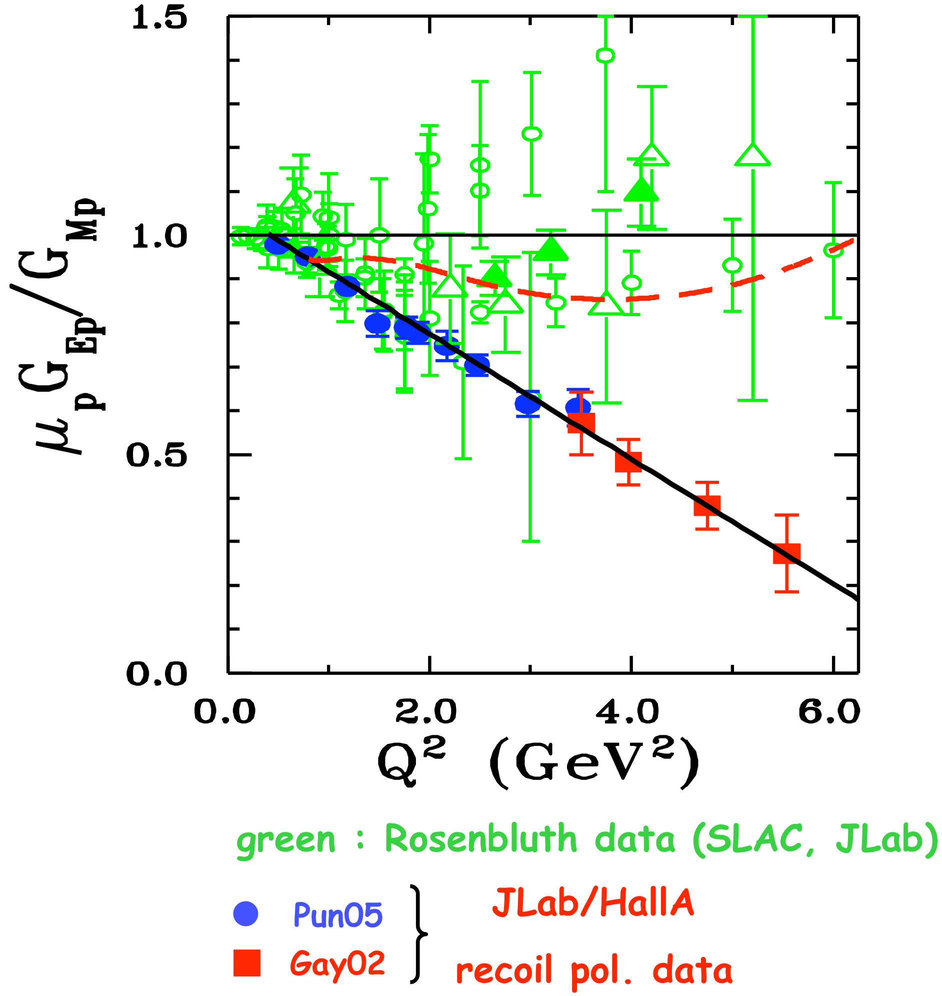
absent in a LIGHT-FRONT frame !

$$q^+ = q^0 + q^3 = 0$$

proton e.m. form factor : status



new MAMI/A1 data up to $Q^2 \approx 0.7$ GeV^2



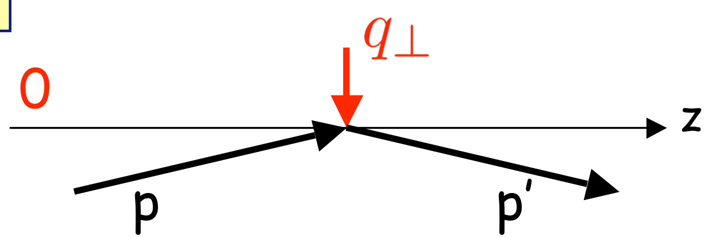
new JLab/HallC recoil pol. exp. (spring 2008) :
extension up to $Q^2 \approx 8.5$ GeV^2

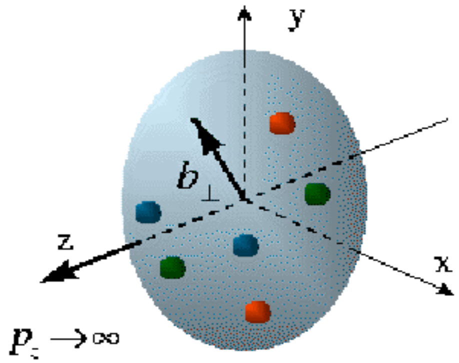
quark transverse charge densities in nucleon (I)

light-front

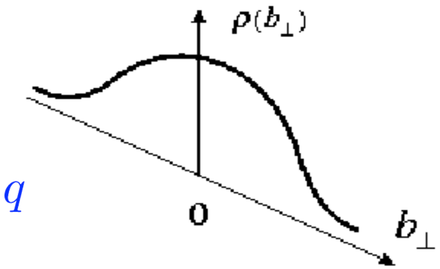
- $q^+ = q^0 + q^3 = 0$

  $Q^2 \equiv \vec{q}_\perp^{\,2}$





photon only couples to forward moving quarks



- quark charge density operator

$$J^+ \equiv J^0 + J^3 = \bar{q}\gamma^+ q = 2\, q_+^\dagger\, q_+, \quad \text{with} \quad q_+ \equiv \frac{1}{4}\gamma^-\gamma^+ q$$

☆ longitudinally polarized nucleon

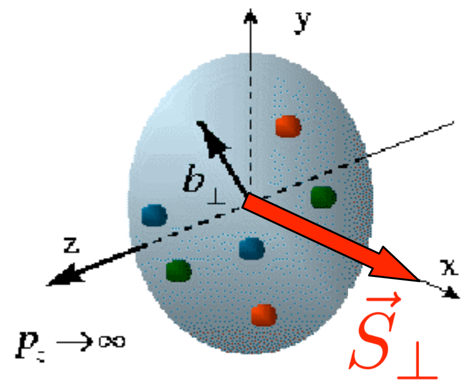
$$\rho_0^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\, \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda\,|\, J^+(0)\,|\, P^+, -\frac{\vec{q}_\perp}{2}, \lambda\rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(b\,Q) F_1(Q^2)$$

Miller (2007)

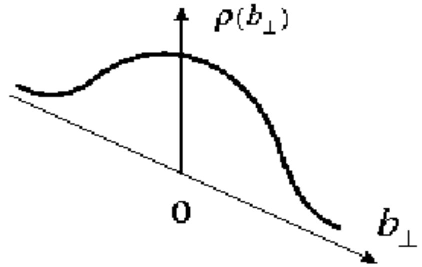
quark transverse charge densities in nucleon (II)

☆ transversely polarized nucleon



transverse spin $\vec{S}_\perp = \cos\phi_S\,\hat{e}_x + \sin\phi_S\,\hat{e}_y$

e.g. along x-axis : $\phi_S = 0$



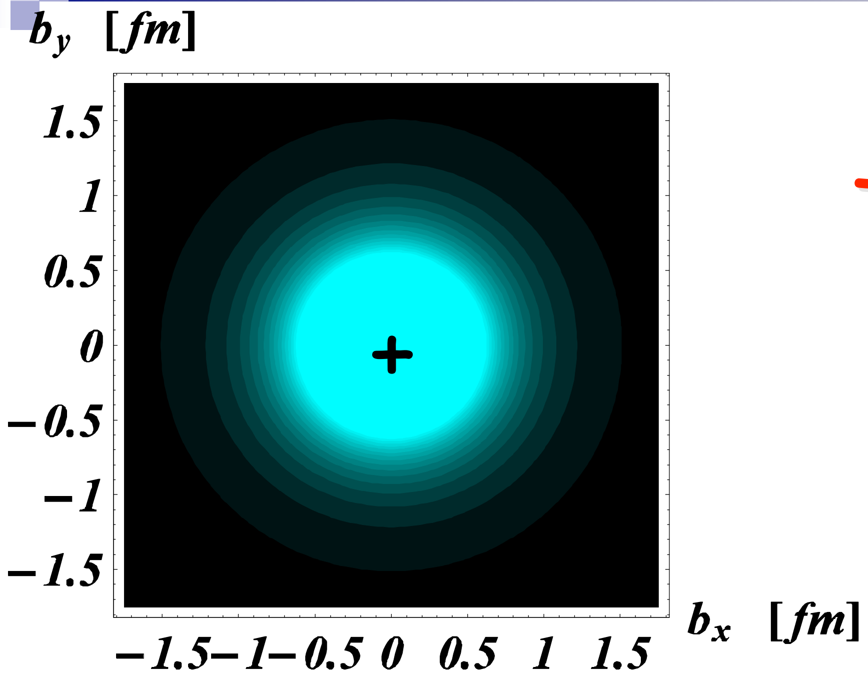
$$\vec{b} = b\,(\cos\phi_b\,\hat{e}_x + \sin\phi_b\,\hat{e}_y)$$

$$\rho_T^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\, \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2}\,|\,J^+(0)\,|\,P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2}\rangle$$

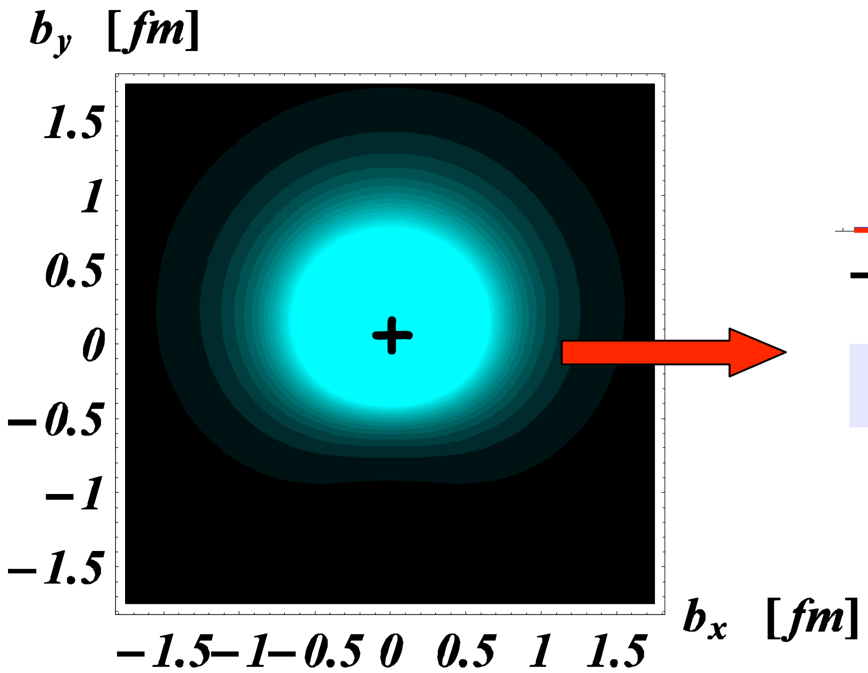
$$= \rho_0^N(b) + \sin(\phi_b - \phi_S)\int_0^\infty \frac{dQ}{2\pi}\,\frac{Q^2}{2M_N}\,J_1(b\,Q)\,F_2(Q^2)$$

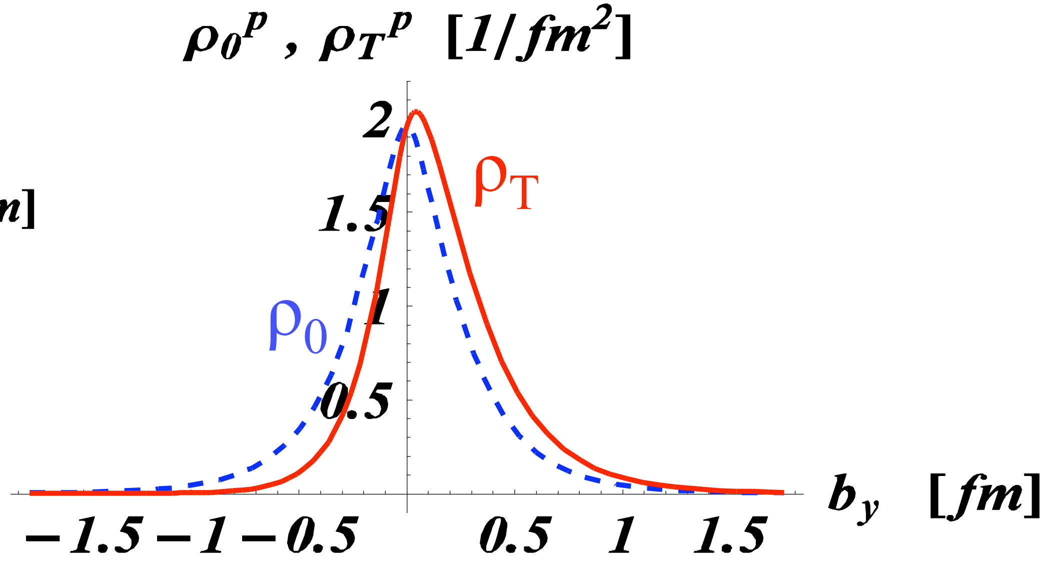
dipole field pattern

Carlson, Vdh (2007)



empirical quark transverse densities in proton



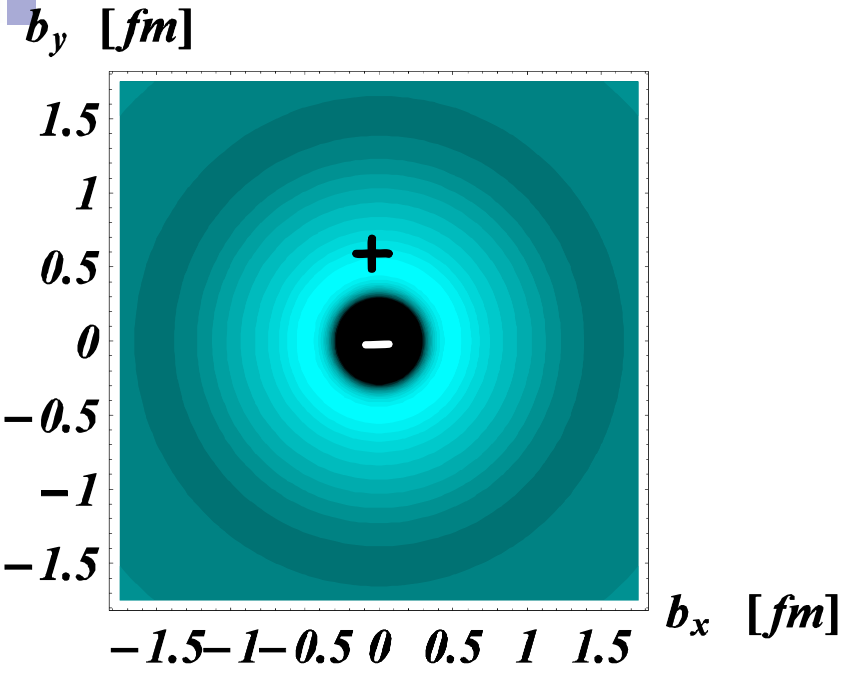


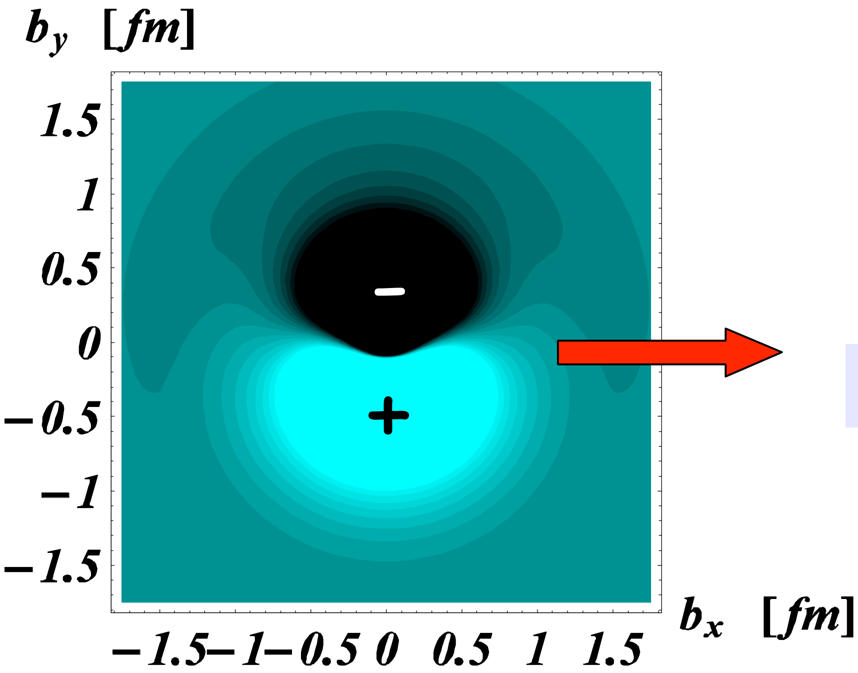
induced EDM : $d_y = F_{2p}(0) . e / (2 M_N)$

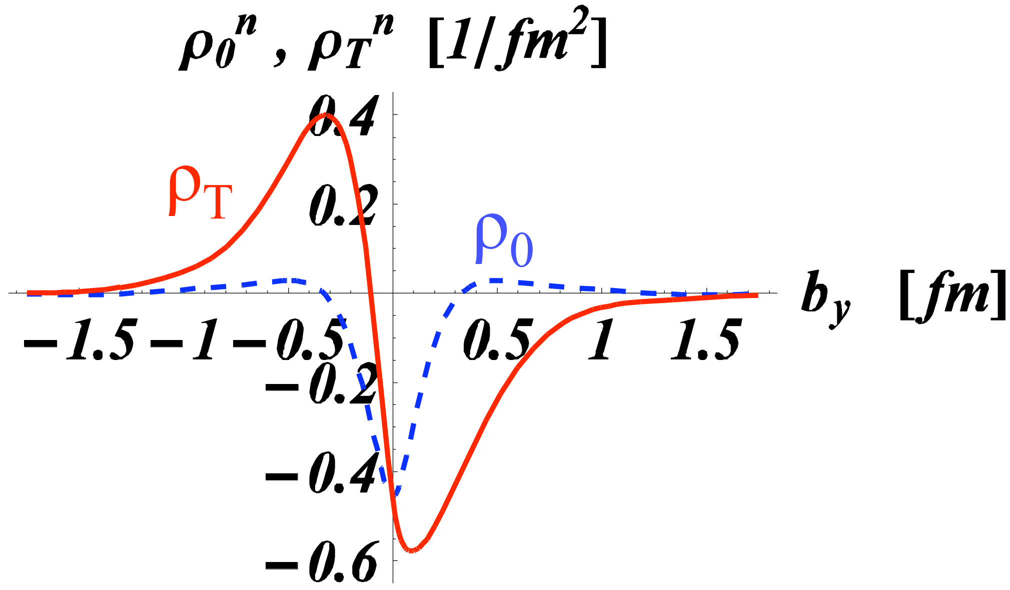
data : Arrington, Melnitchouk, Tjon (2007)

densities : Miller (2007); Carlson, Vdh (2007)

empirical quark transverse densities in neutron







induced EDM : $d_y = F_{2n}(0) . e / (2 M_N)$

data: Bradford, Bodek, Budd, Arrington (2006)

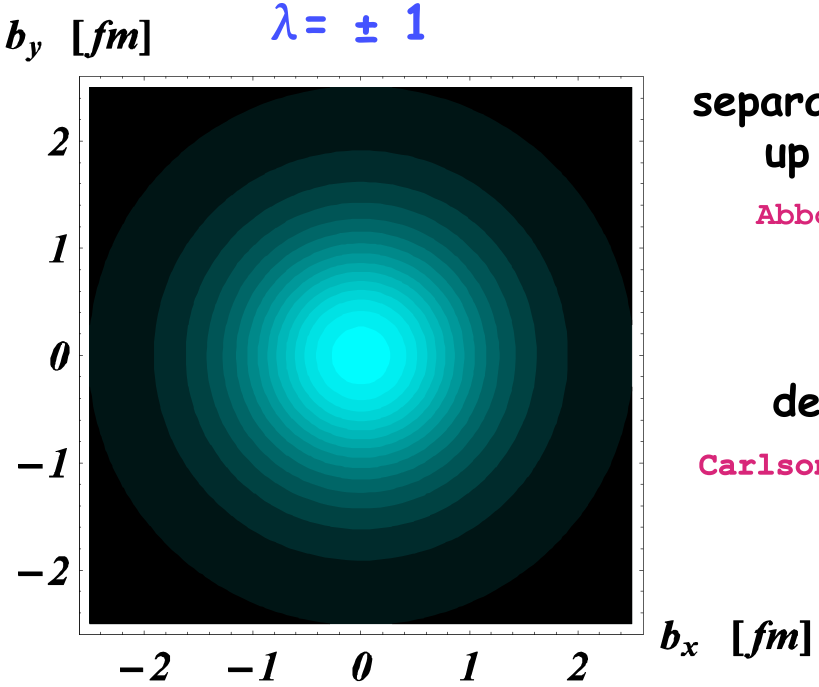
densities : Miller (2007); Carlson, Vdh (2007)

empirical quark transverse densities in deuteron

$$\rho^d_\lambda(b) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\,\vec{q}_\perp\cdot\vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+ | P^+, \frac{-\vec{q}_\perp}{2}, \lambda \rangle$$
$$= \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(b\,Q)\, G^+_{\lambda\lambda}(Q^2)$$

$$G^+_{11} = \frac{1}{1+\eta}\left\{G_C + \eta\, G_M + \frac{\eta}{3} G_Q\right\}$$
$$G^+_{00} = \frac{1}{1+\eta}\left\{(1-\eta)\, G_C + 2\eta\, G_M - \frac{2\eta}{3}(1+2\eta) G_Q\right\}$$

$\lambda = \pm 1$



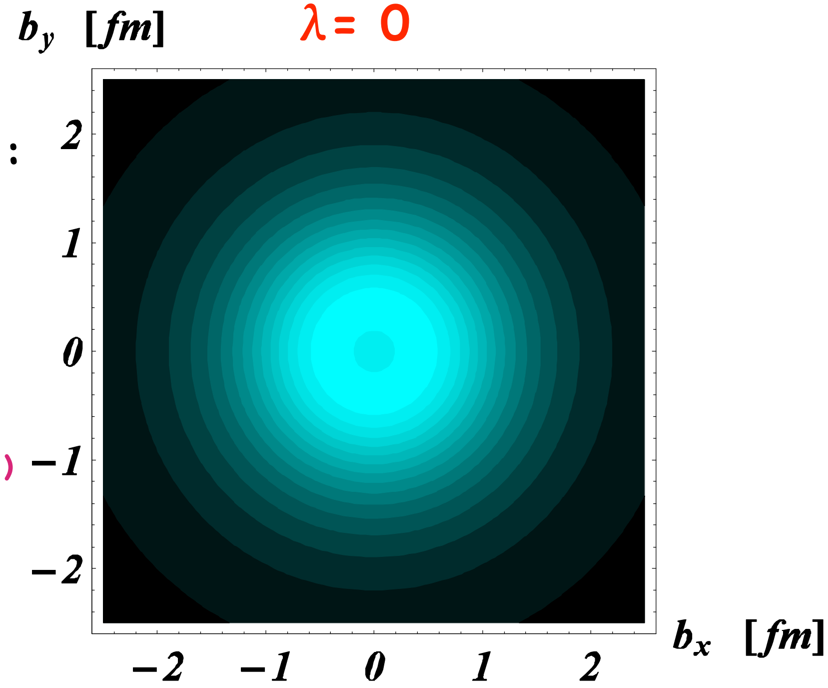
separated data up to 2 GeV^2 :

Abbott et al. (2000)

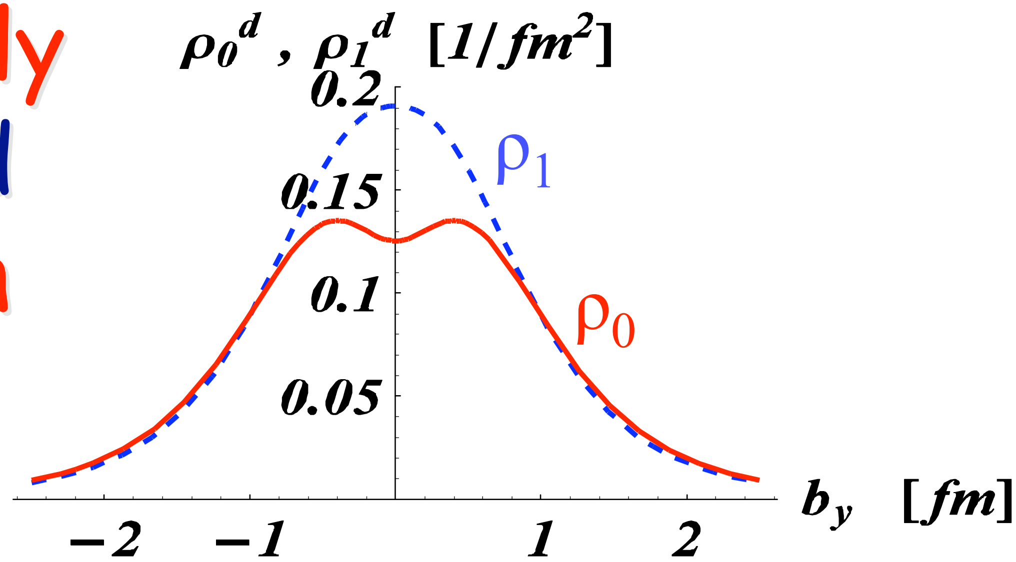
densities :

Carlson, Vdh (2008)

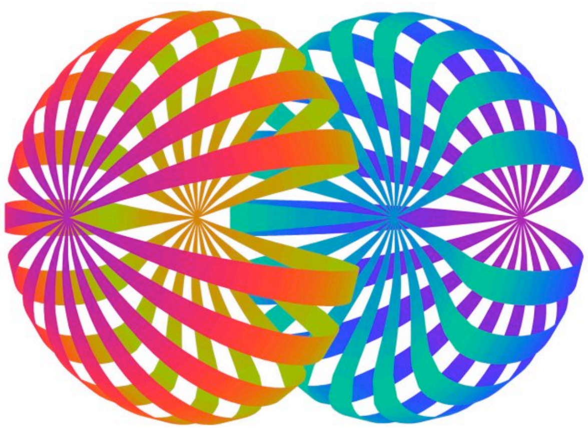
$\lambda = 0$



longitudinally polarized deuteron



λ= ± 1



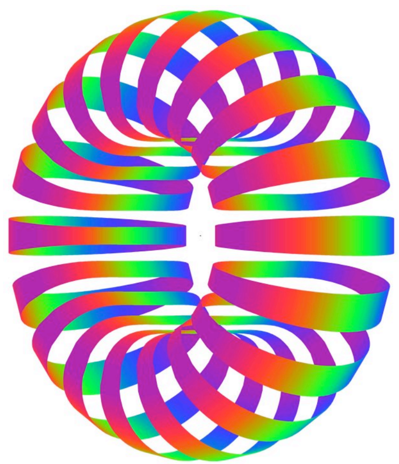
deuteron equidensity surfaces

(ρ_d = 0.24 fm^{-3})

from Argonne v_{18} :

Forest et al. (1996)

λ= 0



transversely polarized deuteron

$$Q^d_{s_\perp} \equiv e \int d^2\vec{b}\,(b_x^2 - b_y^2)\,\rho^d_{T\,s_\perp}(\vec{b})$$

$$Q^d_1 = -\frac{1}{2} Q^d_0 = \frac{1}{2}\{[G_M(0) - 2] + [G_Q(0) + 1]\}\left(\frac{e}{M_d^2}\right)$$

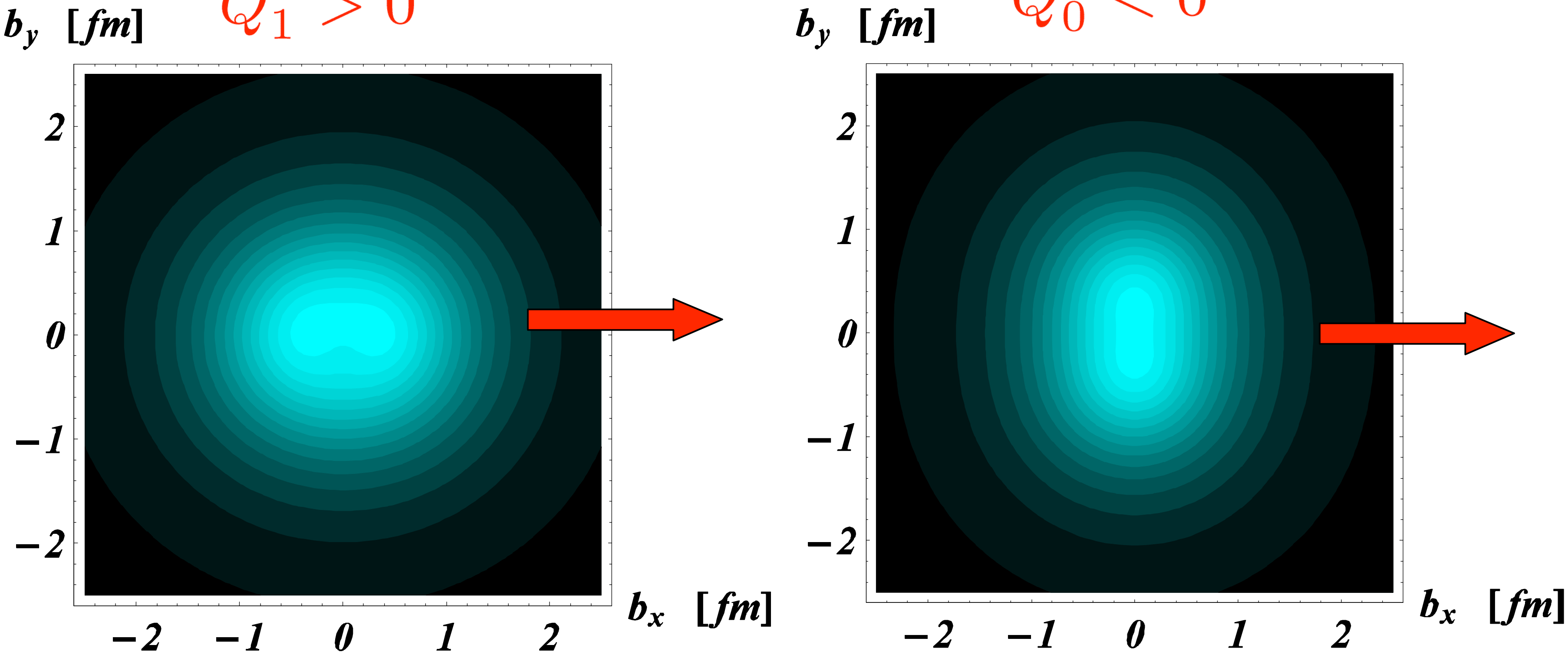
experiment :

$$G_M(0) = 1.71$$

$$G_Q(0) = 25.84(3)$$

$s_\perp = +1$ $\qquad$ $s_\perp = 0$

$Q^d_1 > 0$ $\qquad$ $Q^d_0 < 0$



E.M. moments of W bosons

for spin-1 point particle

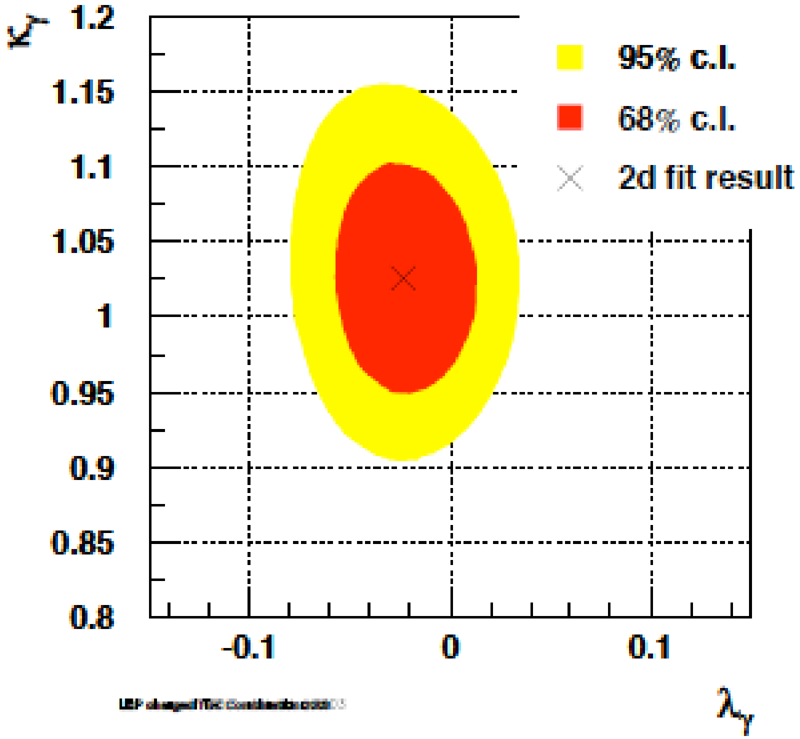
$G_M(0) = 2$ and $G_Q(0) = -1$

$$\mu = \frac{e}{2M_W}\{2 + (\kappa - 1) + \lambda\}$$

$$Q = -\frac{e}{M_W^2}\{1 + (\kappa - 1) - \lambda\}$$

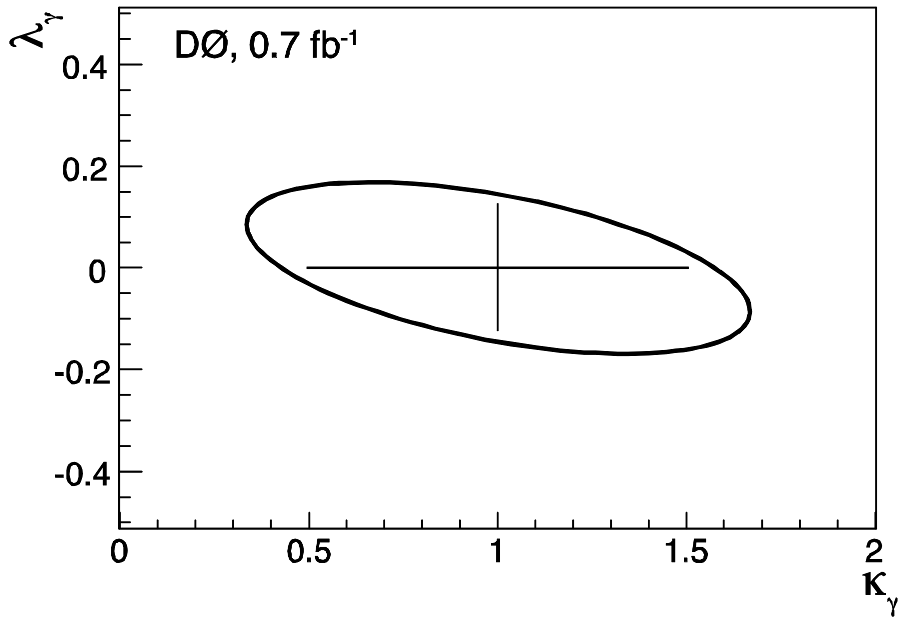
LEP Electroweak working group

hep-ex/0612034



DO Collaboration

PRL100, 241805 (2008)



natural values for e.m. moments of point particle with spin j

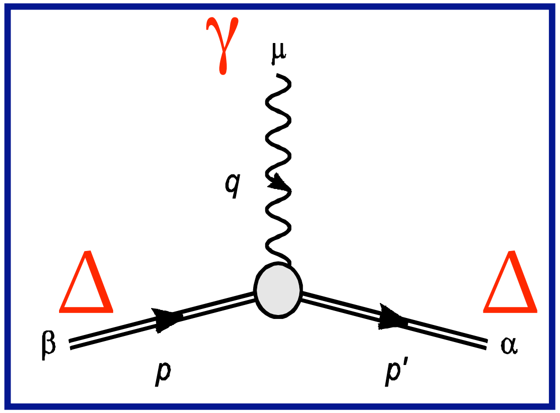
Lorcé (2008)

$$
\begin{aligned}
G_{E0}(0) &= 1 \\
G_{M1}(0) &= 2j \\
G_{E2}(0) &= -j(2j-1) \\
G_{M3}(0) &= -\frac{1}{3}j(2j-1)(2j-2)
\end{aligned}
$$

j	$G_{E0}(0)$	$G_{M1}(0)$	$G_{E2}(0)$	$G_{M3}(0)$	$G_{E4}(0)$	$G_{M5}(0)$	$G_{E6}(0)$
0	1	0	0	0	0	0	0
1/2	1	1	0	0	0	0	0
1	1	2	-1	0	0	0	0
3/2	1	3	-3	-1	0	0	0
2	1	4	-6	-4	1	0	0
5/2	1	5	-10	-10	5	1	0
3	1	6	-15	-20	15	6	-1
...							

→ transverse charge densities depend only on anomalous values of e.m. moments → reflect internal structure

$\gamma^* \Delta\Delta$ vertex



$$\langle \Delta(p',\lambda') \,|\, J^\mu(0) \,|\, \Delta(p,\lambda) \rangle$$
$$= -\bar{u}_\alpha(p',\lambda') \left\{ \left[F_1^*(Q^2) g^{\alpha\beta} + F_3^*(Q^2) \frac{q^\alpha q^\beta}{(2M_\Delta)^2} \right] \gamma^\mu + \left[F_2^*(Q^2) g^{\alpha\beta} + F_4^*(Q^2) \frac{q^\alpha q^\beta}{(2M_\Delta)^2} \right] \frac{i\sigma^{\mu\nu} q_\nu}{2M_\Delta} \right\} u_\beta(p,\lambda)$$

mass M_Δ

multipole transitions

$$G_{E0} = (F_1^* - \tau F_2^*) + \frac{2}{3}\tau G_{E2}$$
$$G_{E2} = (F_1^* - \tau F_2^*) - \frac{1}{2}(1+\tau)\,(F_3^* - \tau F_4^*)$$
$$G_{M1} = (F_1^* + F_2^*) + \frac{4}{5}\tau G_{M3}$$
$$G_{M3} = (F_1^* + F_2^*) - \frac{1}{2}(1+\tau)\,(F_3^* + F_4^*)$$

electric charge $e_\Delta = G_{E0}(0)$

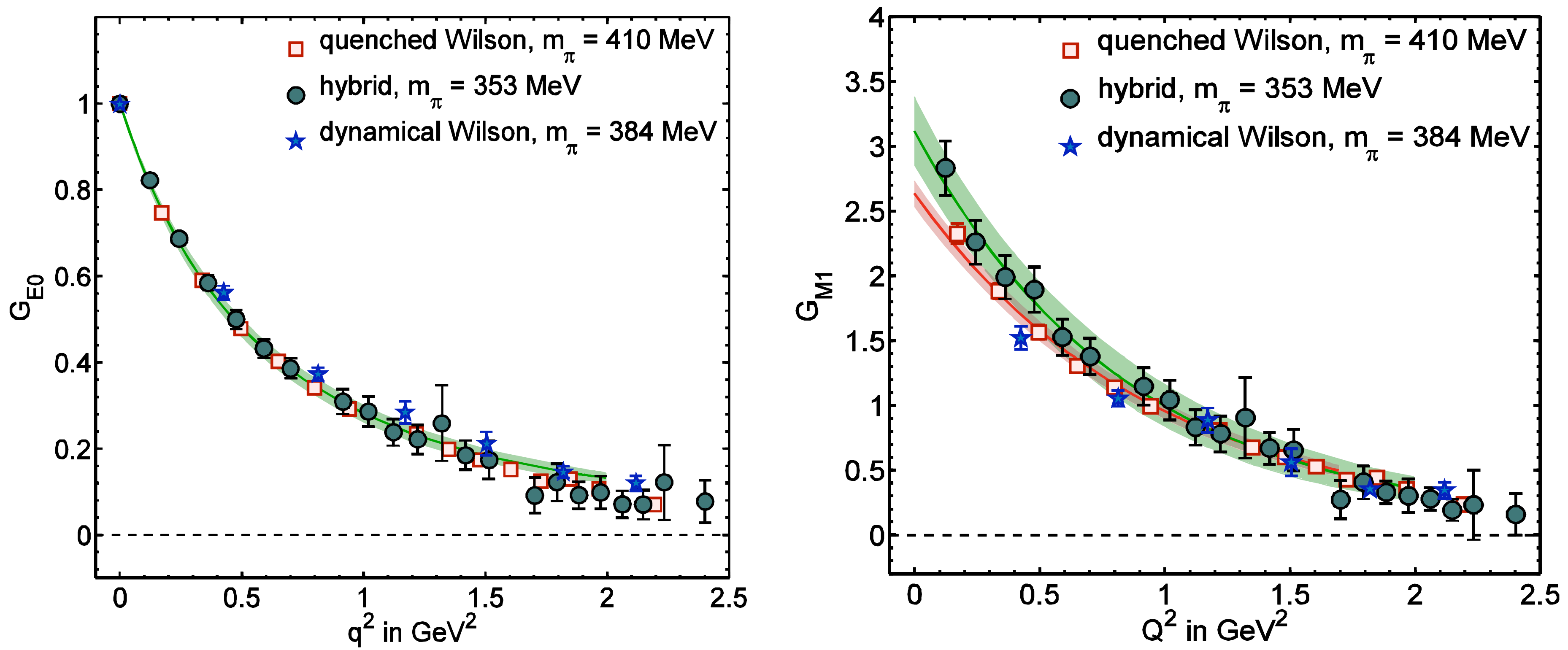
charge quadrupole $Q_\Delta = \frac{e}{M_\Delta^2} G_{E2}(0)$

magnetic dipole $\mu_\Delta = \frac{e}{2M_\Delta} G_{M1}(0)$

magnetic octupole $O_\Delta = \frac{e}{2M_\Delta^3} G_{M3}(0)$

$$\tau \equiv Q^2/(4M_\Delta^2)$$

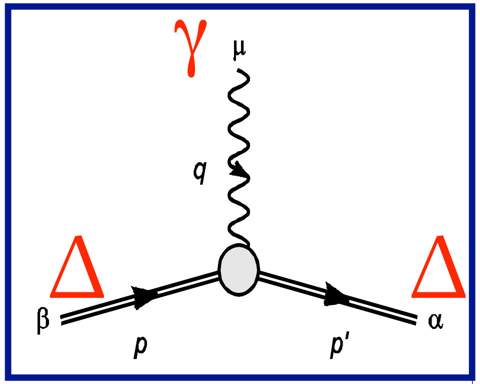
e.m. Δ to Δ transition : lattice results



lattice analysis :

Alexandrou, Korzec, Koutsou, Leontiou, Lorcé, Negele, Pascalutsa, Tsapalis, Vdh (2008)

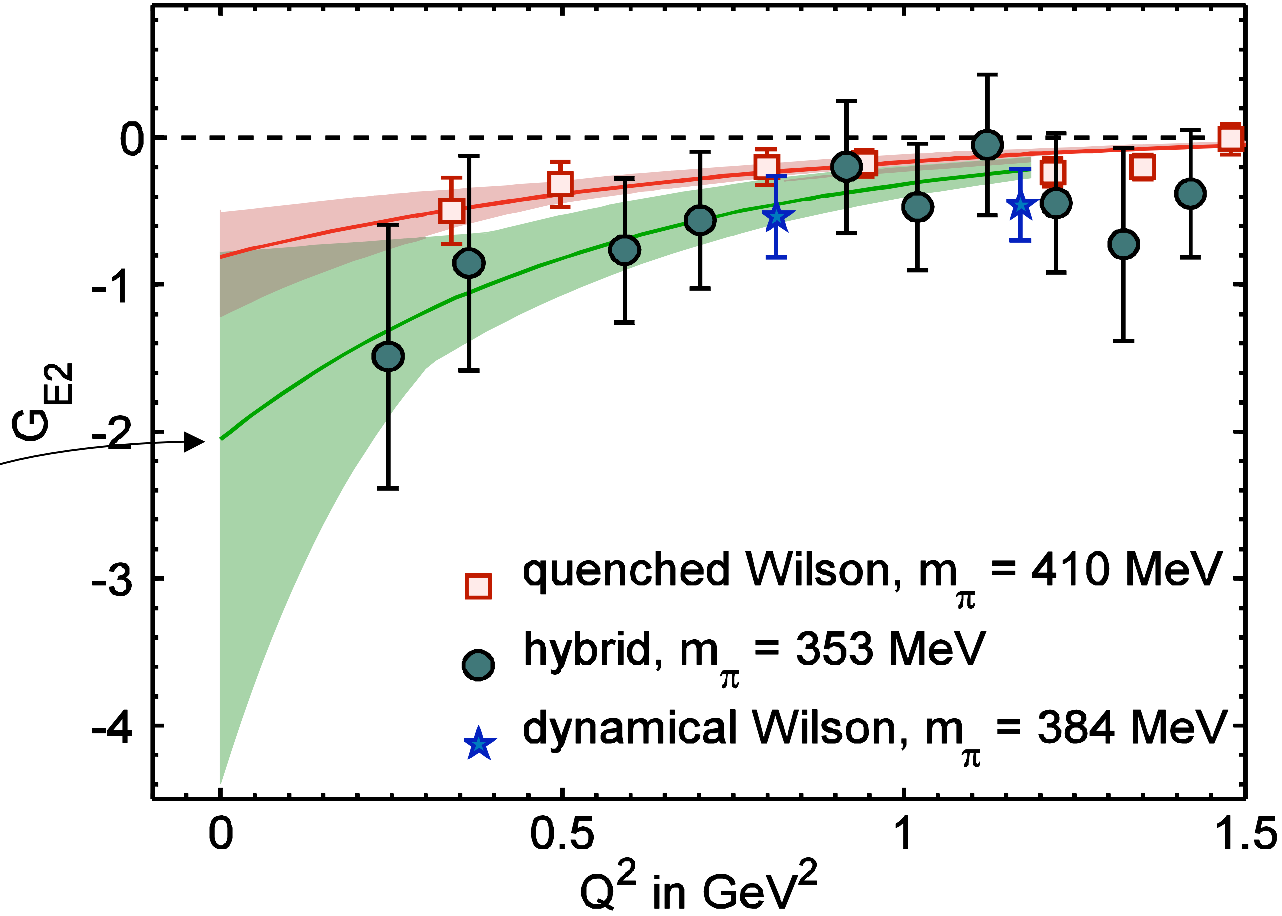
Hadron shape : e.m. Δ to Δ transition



C0 , M1,

C2 , M3

transitions



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transversely polarized $\Delta(1232)$

$$\rho^{\Delta}_{T\, s_\perp}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp \,|\, J^+(0)\,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp \rangle$$

$$\begin{aligned}\rho^{\Delta}_{T\,\frac{3}{2}}(\vec{b}) = \int_0^{+\infty} \frac{dQ}{2\pi}\, Q \quad \Big[\quad & J_0(Qb)\, \frac{1}{4}\left(A_{\frac{3}{2}\frac{3}{2}} + 3A_{\frac{1}{2}\frac{1}{2}}\right) \\ - \quad & \sin(\phi_b - \phi_S)\, J_1(Qb)\, \frac{1}{4}\left(2\sqrt{3}A_{\frac{3}{2}\frac{1}{2}} + 3A_{\frac{1}{2}-\frac{1}{2}}\right) \\ - \quad & \cos[2(\phi_b - \phi_S)]\, J_2(Qb)\, \frac{\sqrt{3}}{2} A_{\frac{3}{2}-\frac{1}{2}} \\ + \quad & \sin[3(\phi_b - \phi_S)]\, J_3(Qb)\, \frac{1}{4} A_{\frac{3}{2}-\frac{3}{2}} \Big]\end{aligned}$$

monopole

dipole

quadrupole

octupole

$$\begin{aligned}\rho^{\Delta}_{T\,\frac{1}{2}}(\vec{b}) = \int_0^{+\infty} \frac{dQ}{2\pi}\, Q \quad \Big[\quad & J_0(Qb)\, \frac{1}{4}\left(3A_{\frac{3}{2}\frac{3}{2}} + A_{\frac{1}{2}\frac{1}{2}}\right) \\ - \quad & \sin(\phi_b - \phi_S)\, J_1(Qb)\, \frac{1}{4}\left(2\sqrt{3}A_{\frac{3}{2}\frac{1}{2}} - A_{\frac{1}{2}-\frac{1}{2}}\right) \\ + \quad & \cos[2(\phi_b - \phi_S)]\, J_2(Qb)\, \frac{\sqrt{3}}{2} A_{\frac{3}{2}-\frac{1}{2}} \\ - \quad & \sin[3(\phi_b - \phi_S)]\, J_3(Qb)\, \frac{3}{4} A_{\frac{3}{2}-\frac{3}{2}} \Big]\end{aligned}$$

quark transverse charge densities in $\Delta(1232)$

$$\rho^{\Delta}_{T\,s_\perp}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp \,|\, J^+(0) \,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp \rangle$$

$$Q^{\Delta}_{s_\perp} \equiv e \int d^2\vec{b}\, (b_x^2 - b_y^2)\, \rho^{\Delta}_{T\,s_\perp}(\vec{b})$$

$$Q^{\Delta}_{\frac{3}{2}} = -Q^{\Delta}_{\frac{1}{2}} = \frac{1}{2}\left\{2\left[G_{M1}(0) - 3e_\Delta\right] + \left[G_{E2}(0) + 3e_\Delta\right]\right\}\left(\frac{e}{M_\Delta^2}\right)$$

$s_\perp = +3/2$

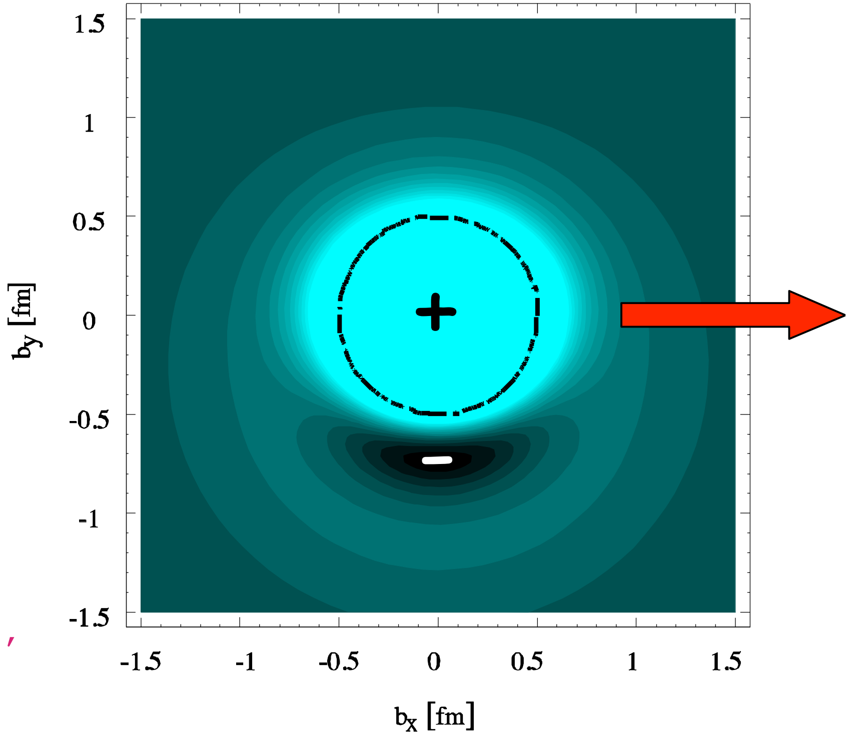
for spin-3/2 point particle

$G_{M1}(0) = 3e_\Delta$ and $G_{E2}(0) = -3e_\Delta$

transverse charge densities depend only on anomalous values of e.m. moments -> reflect internal structure

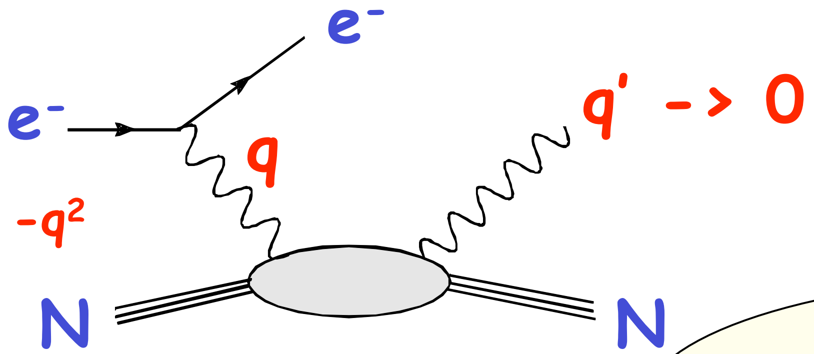
lattice analysis :

Alexandrou, Korzec, Koutsou, Leontiou, Lorcé, Negele, Pascalutsa, Tsapalis, Vdh (2008)



Induced polarization in proton

VCS :



low energy photon plays role of applied e.m. dipole field

nucleon response :
Generalized Polarizabilities (GP)
α(Q²), β(Q²), and 4 spin GPs

$$H^{\mu\nu} = -i \int d^4x\, e^{-iq\cdot x} \langle p', \lambda'_N | T\left[J^\mu(x), J^\nu(0)\right] | p, \lambda_N \rangle$$

light-front + component

linear response in outgoing photon energy ~ $\nu = q \cdot P/M$

$$i\,\vec{\varepsilon}_\perp^{\,\prime *} \cdot \vec{P}_0 \equiv \varepsilon_\nu^{\prime *} \frac{(1+\tau)}{(2P+)} \frac{\partial H^{+\nu}}{\partial \nu}\Big|_{\nu=0}$$

induced electric dipole moment

$$\vec{q}_\perp^{\,2} = Q^2$$

$$\vec{P}_0(\vec{q}_\perp) = i\,\hat{q}_\perp\, A(Q^2)$$

function of GPs

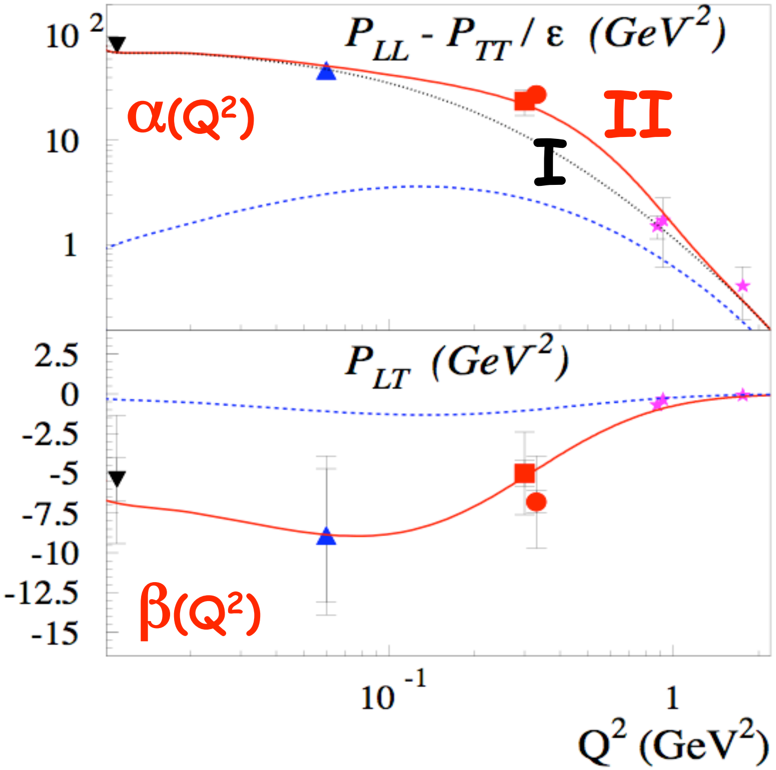
induced polarization in proton

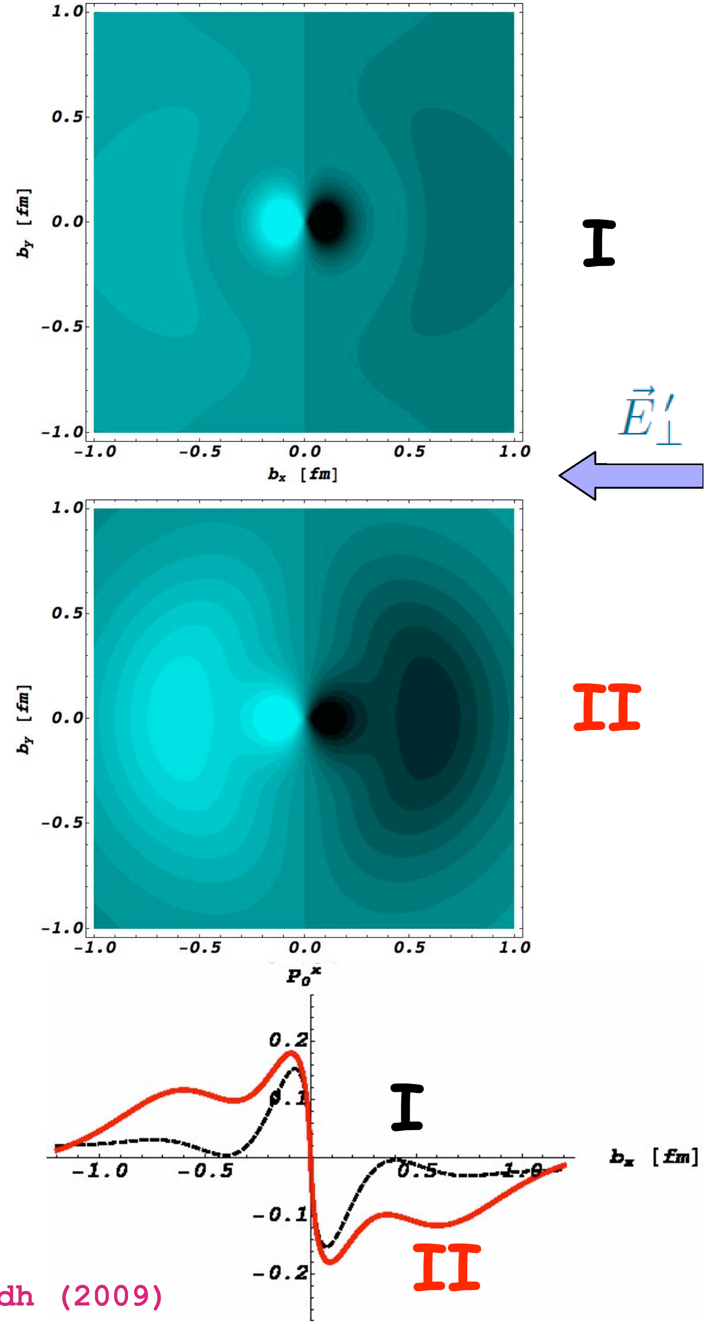
$$\vec{P}_0(\vec{b}) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\vec{q}_\perp\cdot\vec{b}}\, \vec{P}_0(\vec{q}_\perp)$$

$$= \hat{b} \int_0^\infty \frac{dQ}{(2\pi)}\, Q\, J_1(b\,Q)\, A(Q^2)$$

data :

RCS
MIT-Bates
MAMI
JLab





Gorchtein, Lorcé, Pasquini, Vdh (2009)

Transverse transition charge densities

for

N -> Δ(1232), P_{11}(1440), S_{11}(1535), D_{13}(1520)

N -> Δ (1232) transition

- **experiment measures multipoles**

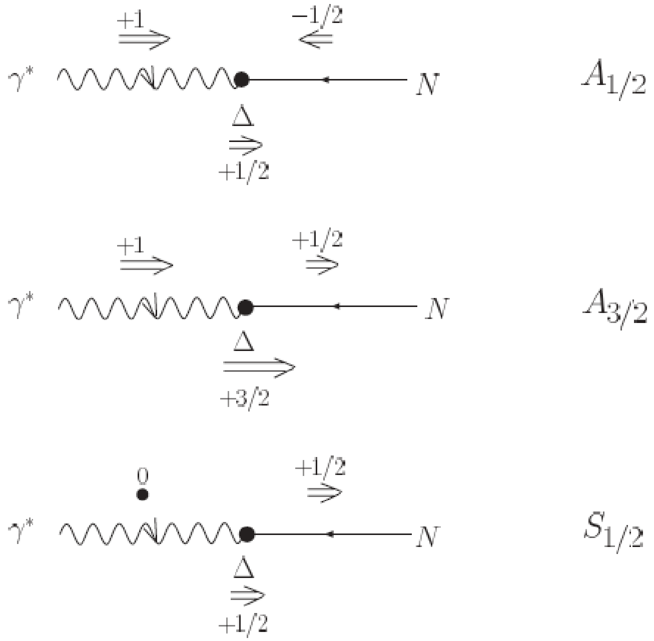
$$\bar{M}_{1+}^{(3/2)}(Q^2) \equiv \sqrt{\frac{2}{3}}\, a_\Delta \, \mathrm{Im} M_{1+}^{(3/2)}(Q^2, W = M_\Delta)$$

- **theory calculates helicity form factors**

$$A_{3/2} \equiv -\frac{e}{\sqrt{2 q_\Delta}} \frac{1}{(4 M_N M_\Delta)^{1/2}} \langle\, \Delta(\vec{0}, +3/2)\, |\, \mathbf{J} \cdot \epsilon_{\lambda=+1}\, |\, N(-\vec{q}, +1/2)\, \rangle$$

$$A_{1/2} \equiv -\frac{e}{\sqrt{2 q_\Delta}} \frac{1}{(4 M_N M_\Delta)^{1/2}} \langle\, \Delta(\vec{0}, +1/2)\, |\, \mathbf{J} \cdot \epsilon_{\lambda=+1}\, |\, N(-\vec{q}, -1/2)\, \rangle$$

$$S_{1/2} \equiv \frac{e}{\sqrt{2 q_\Delta}} \frac{1}{(4 M_N M_\Delta)^{1/2}} \langle\, \Delta(\vec{0}, +1/2)\, |\, J^0\, |\, N(-\vec{q}, +1/2)\, \rangle,$$



- **define resonance properties**

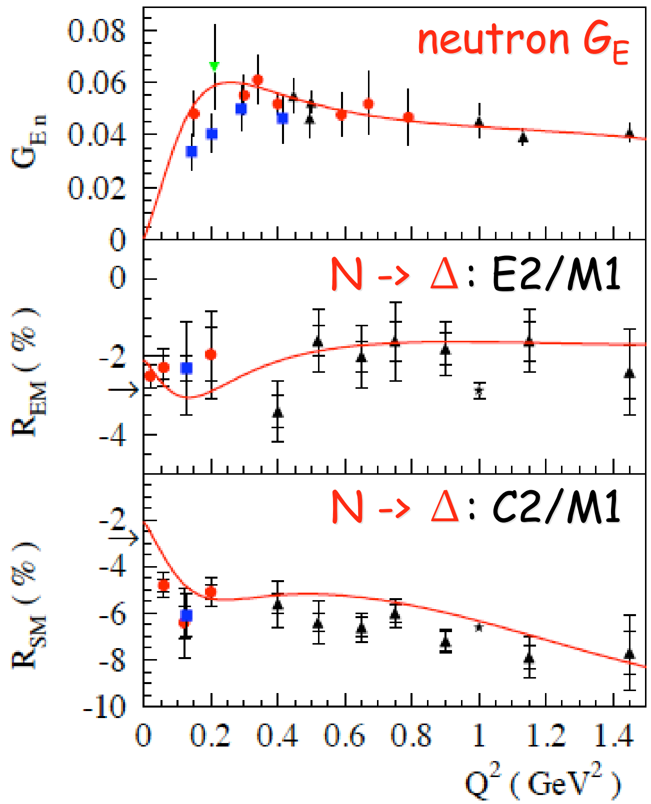
$$A_{3/2} = -\frac{\sqrt{3}}{2} \left\{ \bar{M}_{1+}^{(3/2)} - \bar{E}_{1+}^{(3/2)} \right\}$$

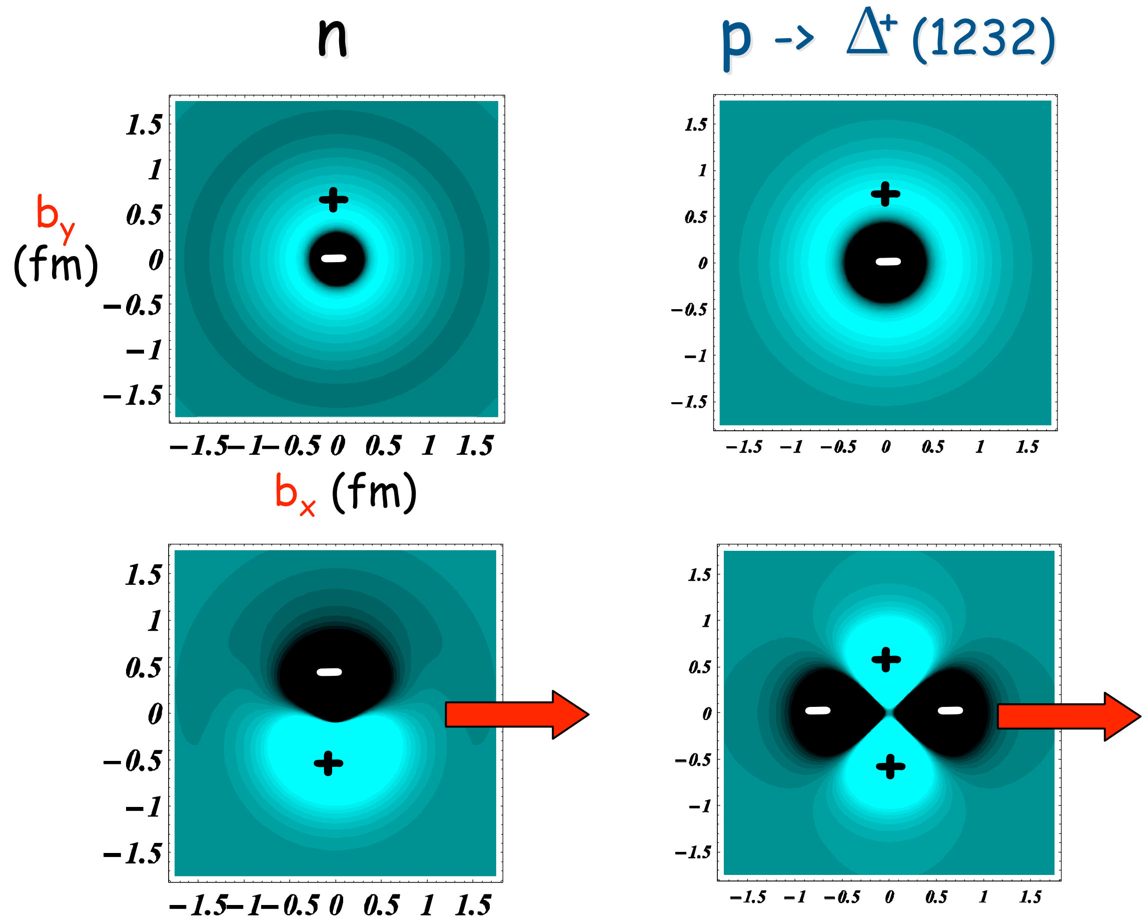
$$A_{1/2} = -\frac{1}{2} \left\{ \bar{M}_{1+}^{(3/2)} + 3\, \bar{E}_{1+}^{(3/2)} \right\}$$

$$S_{1/2} = -\sqrt{2}\, \bar{S}_{1+}^{(3/2)},$$

empirical transverse transition densities for N -> Δ(1232) excitation

data : MAMI, NIKHEF, MIT-Bates, JLab

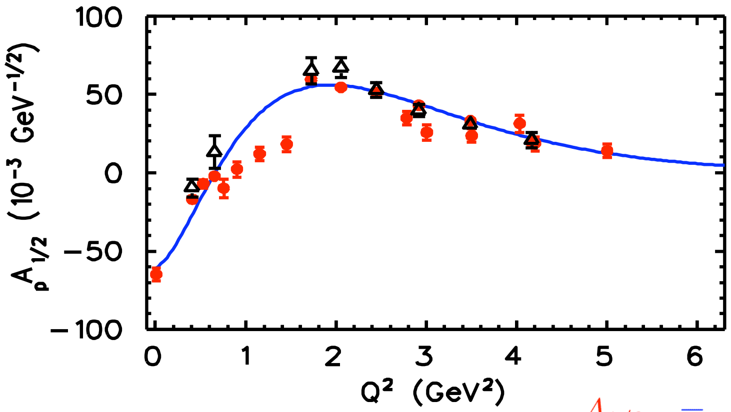




Miller (2007),
Carlson, Vdh (2007)

quadrupole pattern

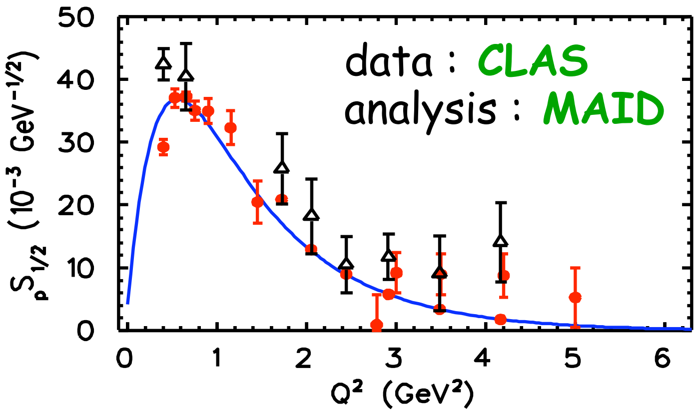
empirical e.m. transition FFs for p -> N*(1440) excitation

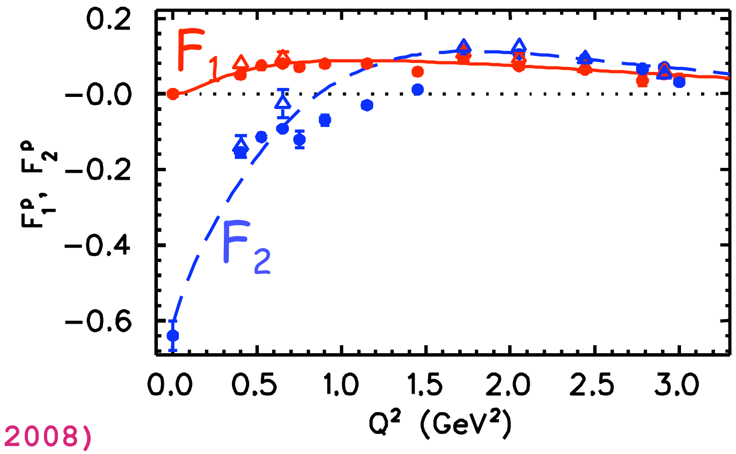


$$\langle N^*(p',\lambda')\,|\,J^\mu(0)\,|\,N(p,\lambda)\rangle$$
$$= \bar{u}(p',\lambda')\left\{F_1^{NN^*}(Q^2)\left(\gamma^\mu - \gamma\cdot q\,\frac{q^\mu}{q^2}\right) + F_2^{NN^*}(Q^2)\,\frac{i\sigma^{\mu\nu}q_\nu}{(M^*+M_N)}\right\}u(p,\lambda)$$

$$A_{1/2} = e\,\frac{Q_-}{\sqrt{K}\,(4M_N M^*)^{1/2}}\left\{F_1^{NN^*} + F_2^{NN^*}\right\}$$

$$S_{1/2} = e\,\frac{Q_-}{\sqrt{2K}\,(4M_N M^*)^{1/2}}\left(\frac{Q_+Q_-}{2M^*}\right)\frac{(M^*+M_N)}{Q^2}\left\{F_1^{NN^*} - \frac{Q^2}{(M^*+M_N)^2}F_2^{NN^*}\right\}$$

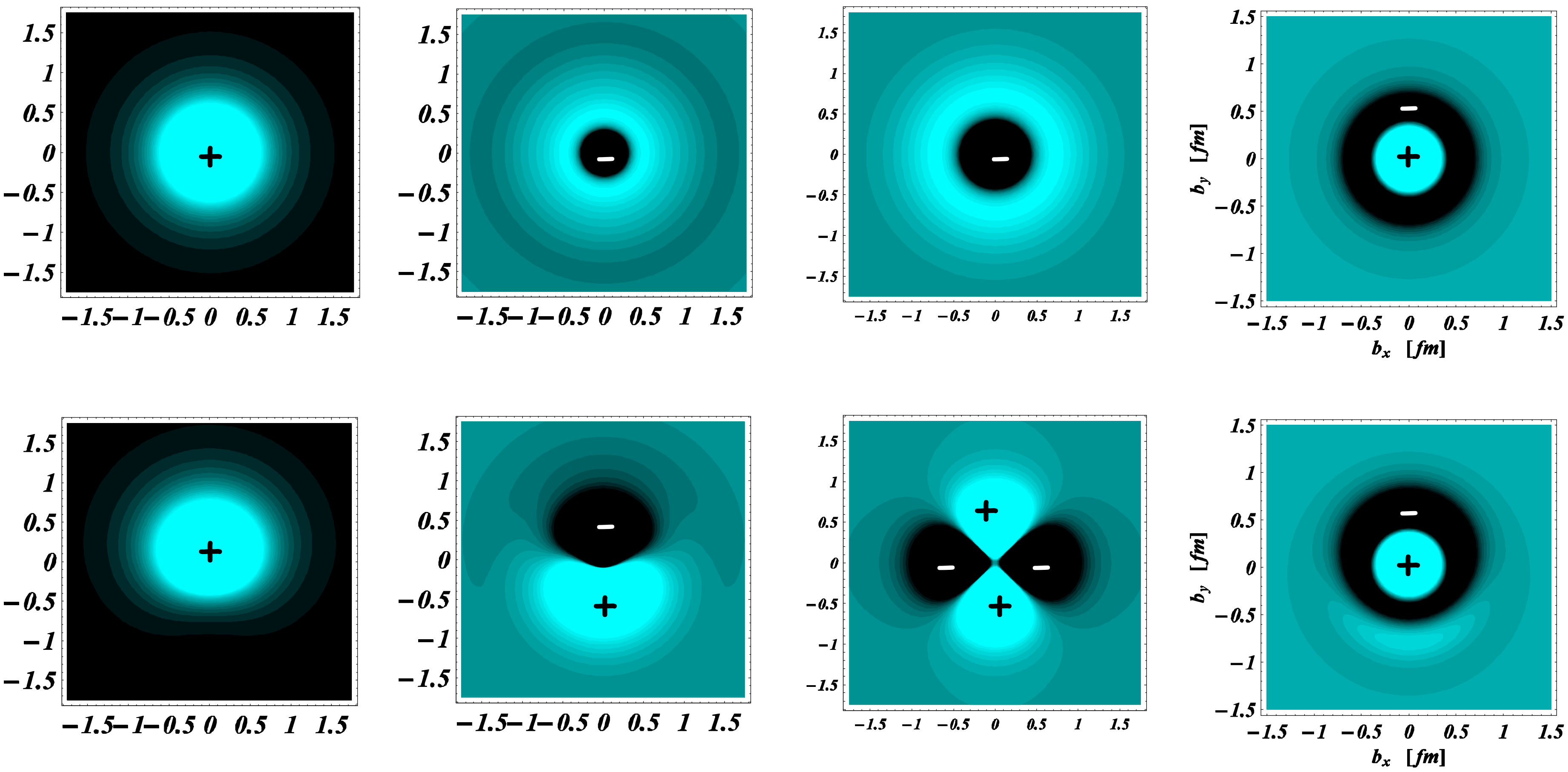




Tiator, Vdh (2008)

empirical transverse transition densities

p | n | p -> Δ^+ (1232) | p -> N^* (1440)



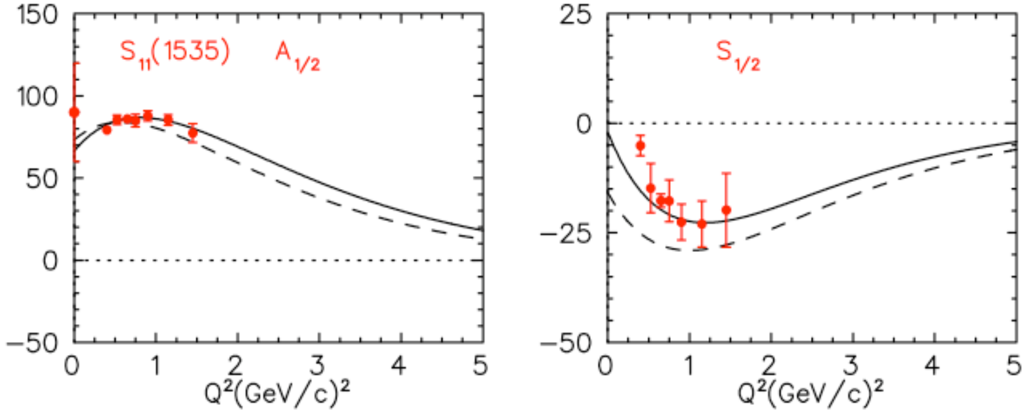
Carlson, Vdh (2007)

quadrupole pattern

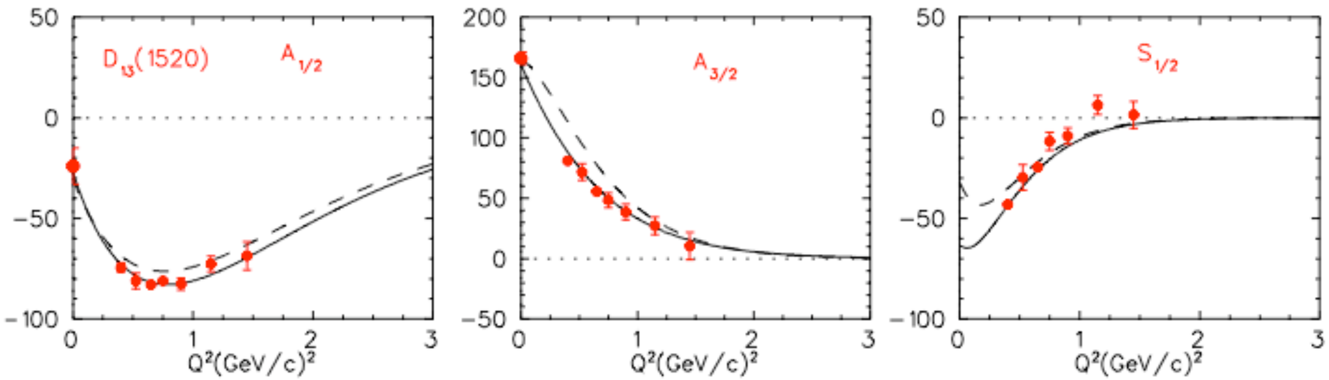
Tiator, Vdh (2008)

empirical transition FFs for

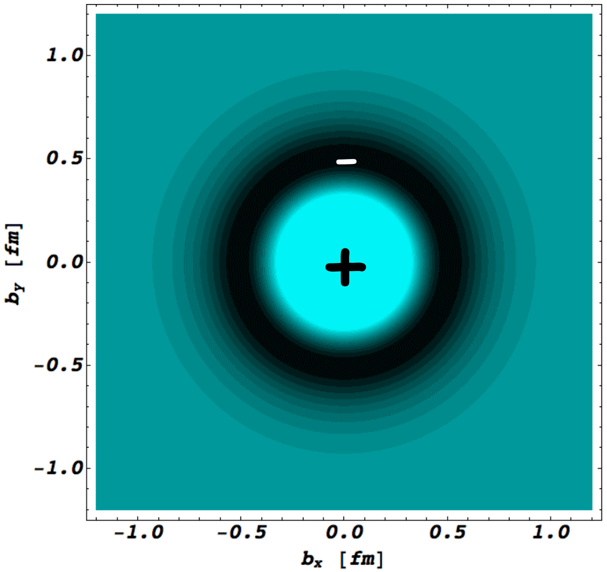
p -> $S_{11}(1535)$, $D_{13}(1520)$ excitations

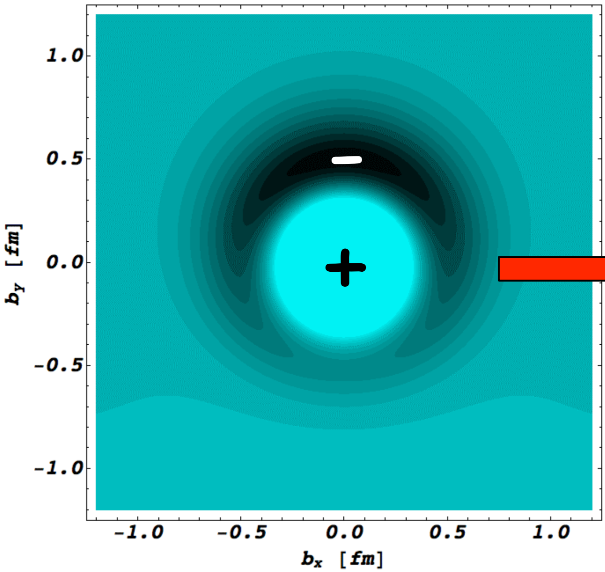


data : **CLAS** analysis : **MAID** —— 2007 - - - 2003

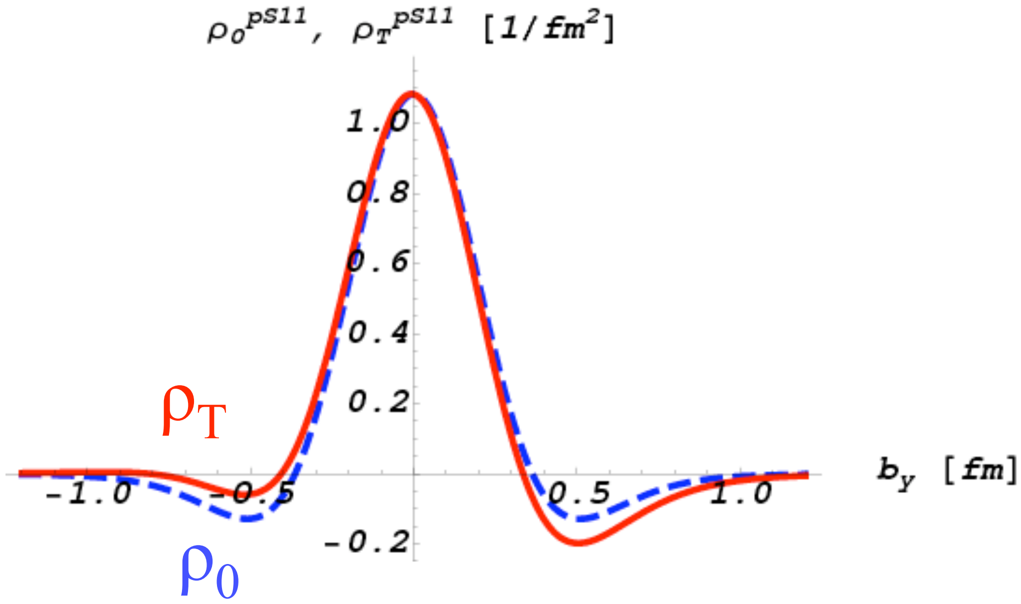


empirical transverse transition densities for $p \to S_{11}(1535)$ excitation



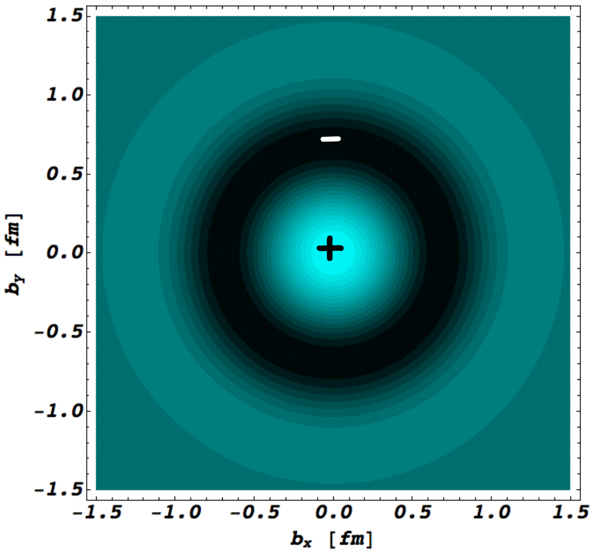


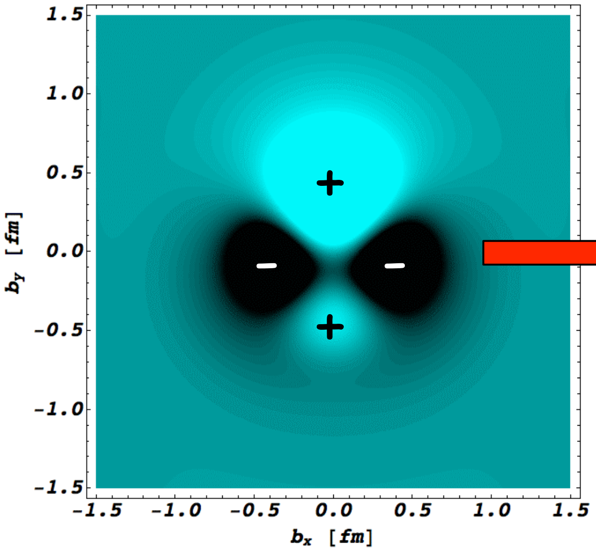
transition $s_\perp = +1/2 \to s_\perp = -1/2$



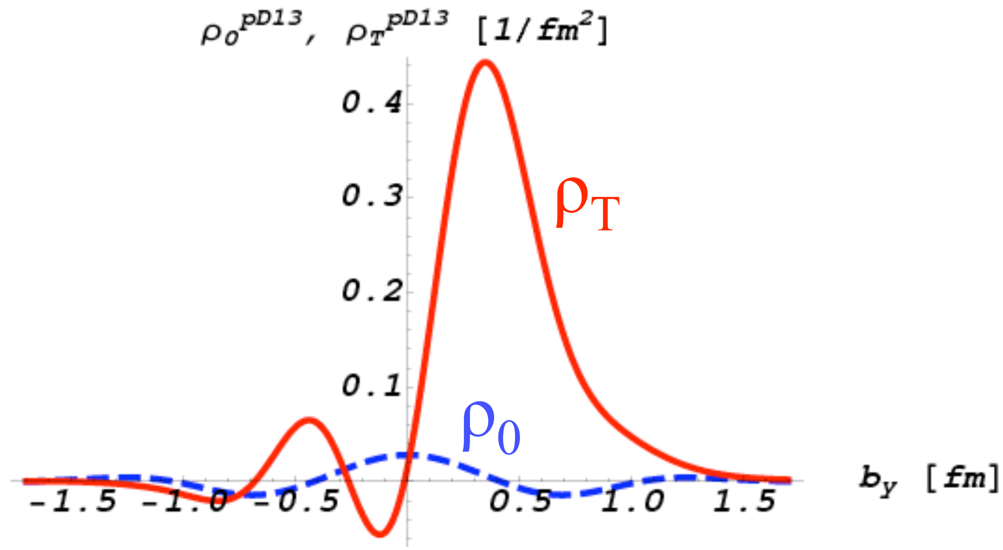
Tiator, Vdh (2009)

empirical transverse transition densities for $p \rightarrow D_{13}(1520)$ excitation





transition $s_\perp = +1/2 \rightarrow s_\perp = -1/2$



Tiator, Vdh (2009)

Generalized Parton Distributions (GPDs) : 3D picture of nucleon

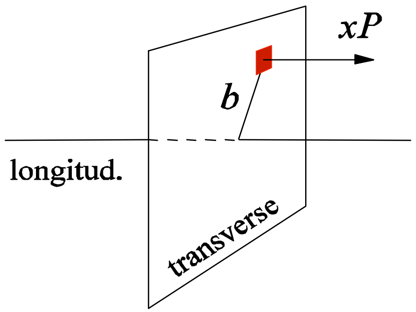
DIS
longitudinal
quark distribution
in momentum space

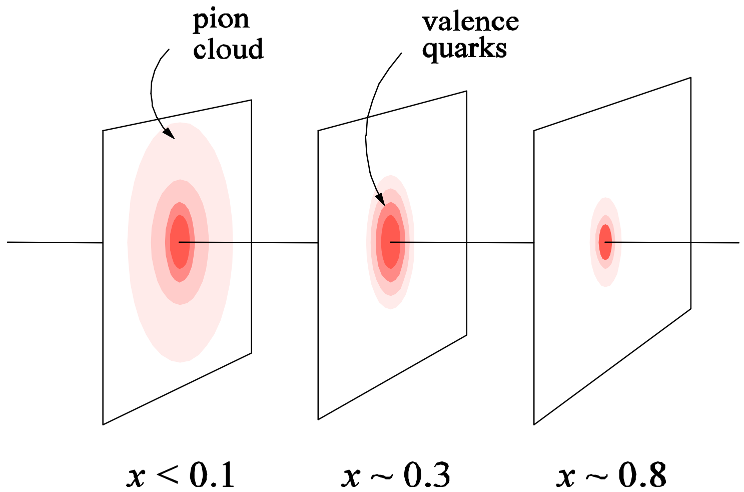
Unifying concept

GPDs

Elastic Scattering
transverse quark
distribution in
coordinate space

fully-correlated
quark distributions in **both**
coordinate and momentum space

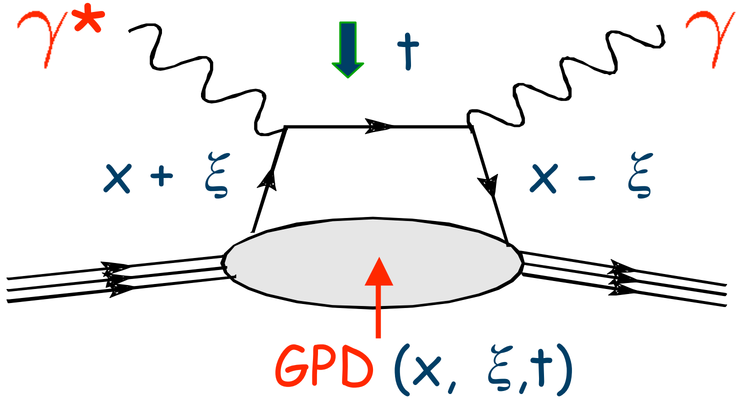




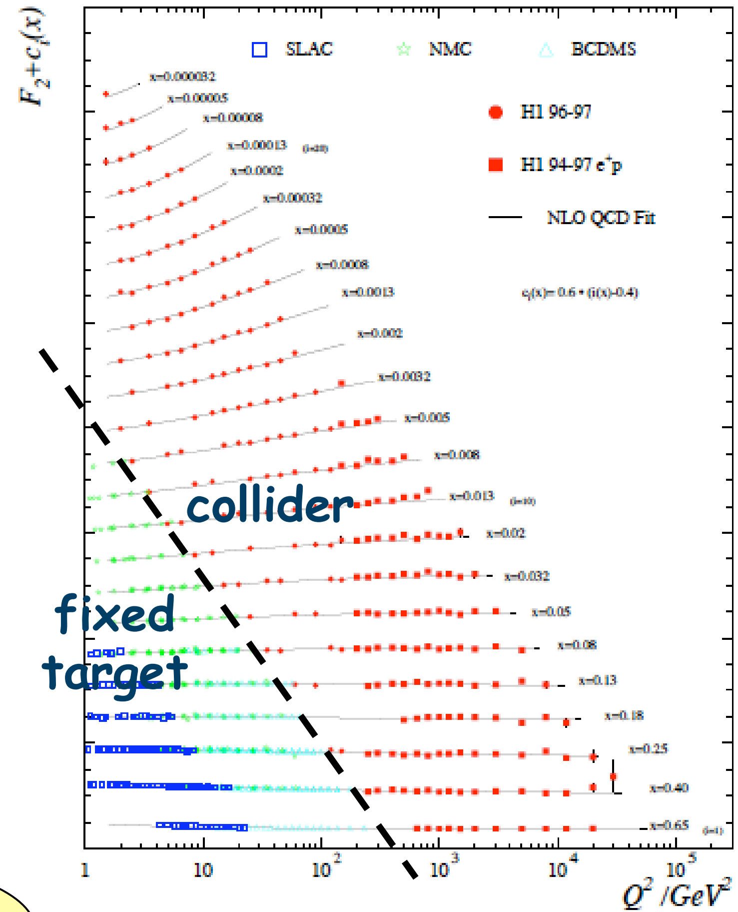
Burkardt (2000, 2003),
Belitsky, Ji, Yuan (2004)

QCD factorization : tool to access GPDs

$Q^2 >> 1\ GeV^2$



world data on proton F_2



➡ at large Q^2 : **QCD factorization theorem** :

hard exclusive process described by GPDs
model independent !

Müller et al. (1994),
Ji (1995), Radyushkin (1995),
Collins, Frankfurt, Strikman (1996)

➡ KEY Q^2 leverage required to test

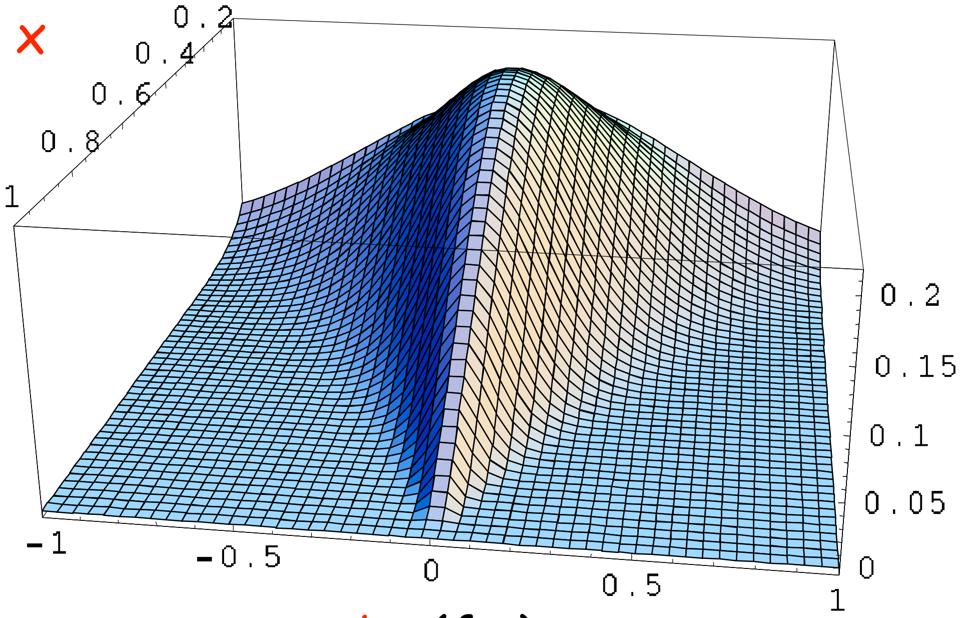
QCD scaling ➡ e N collider

GPDs : transverse image of nucleon

GPDs : quark distributions w.r.t. longitudinal momentum x and transverse position $b_\perp$

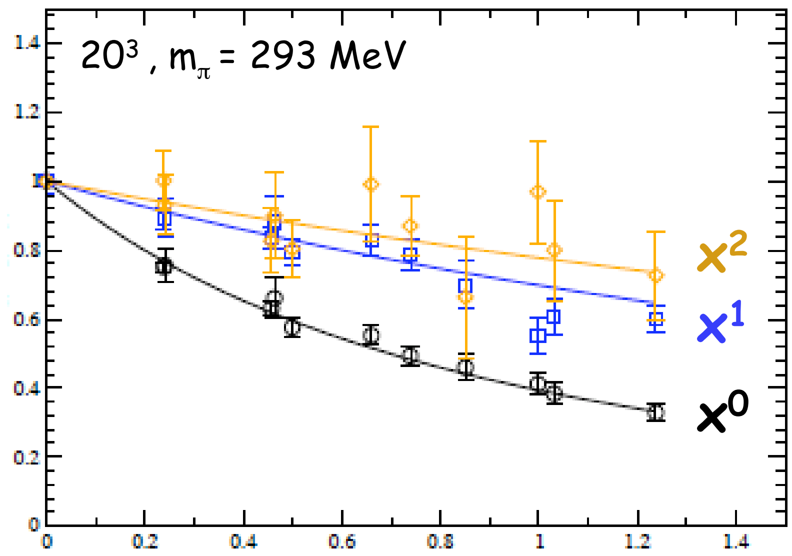
lattice QCD : moments of GPDs

$H^u(x, b_\perp)$



$b_\perp$ (fm)

x^n moment of H^{u-d}



-t (GeV^2)

Fourier transform

Guidal, Polyakov, Radyushkin, Vdh (2005),

Diehl, Feldmann, Jakob, Kroll (2005)

LHPC Coll.