Knowing What You Don't Know: Nuclear Reactions, Effective Field Theory & Uncertainty Quantification

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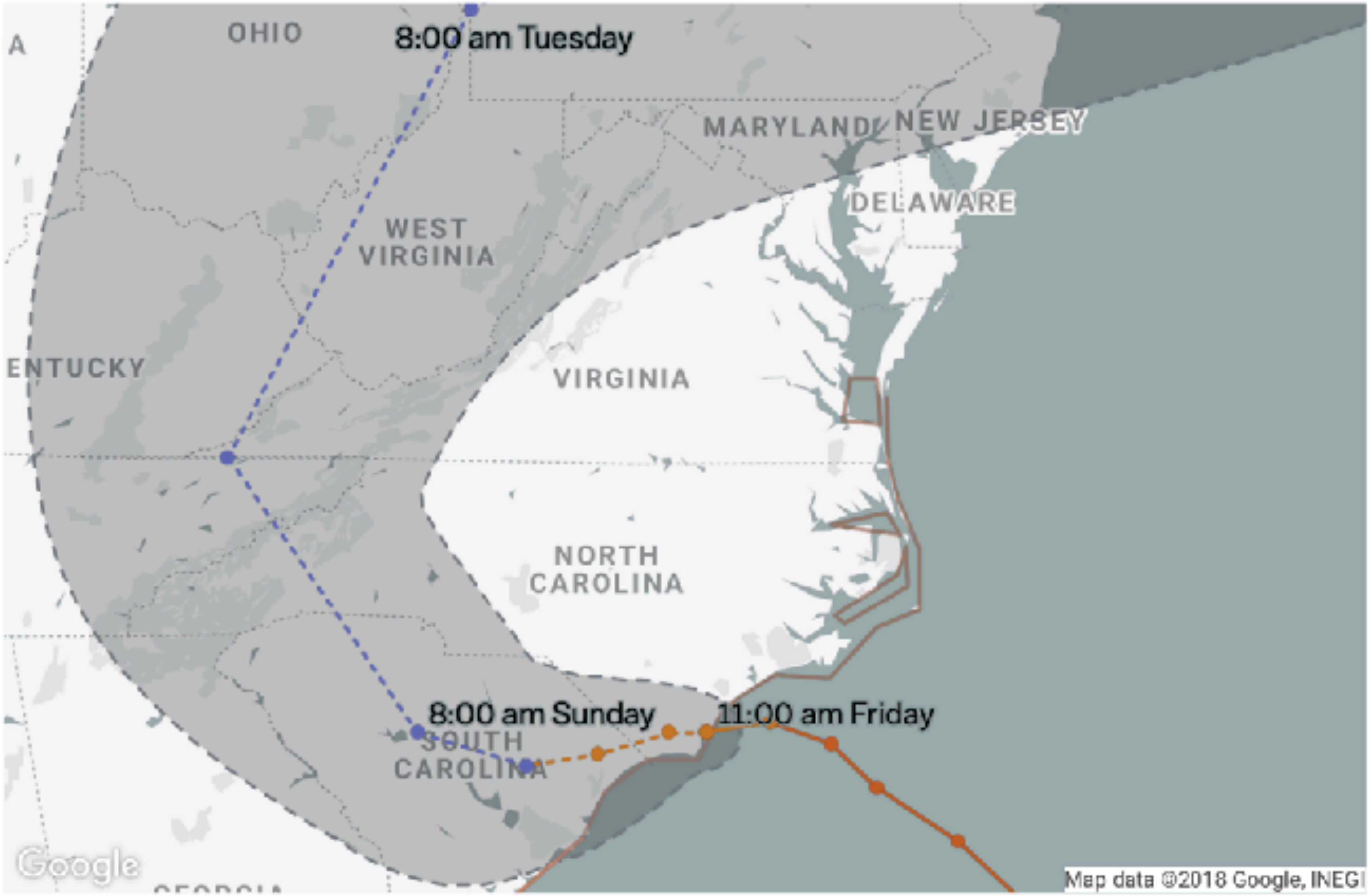
Hurricane forecasting



Data: **National Hurricane Center**, NOAA. Updated: Sep 14, 2018, 2:00 pm

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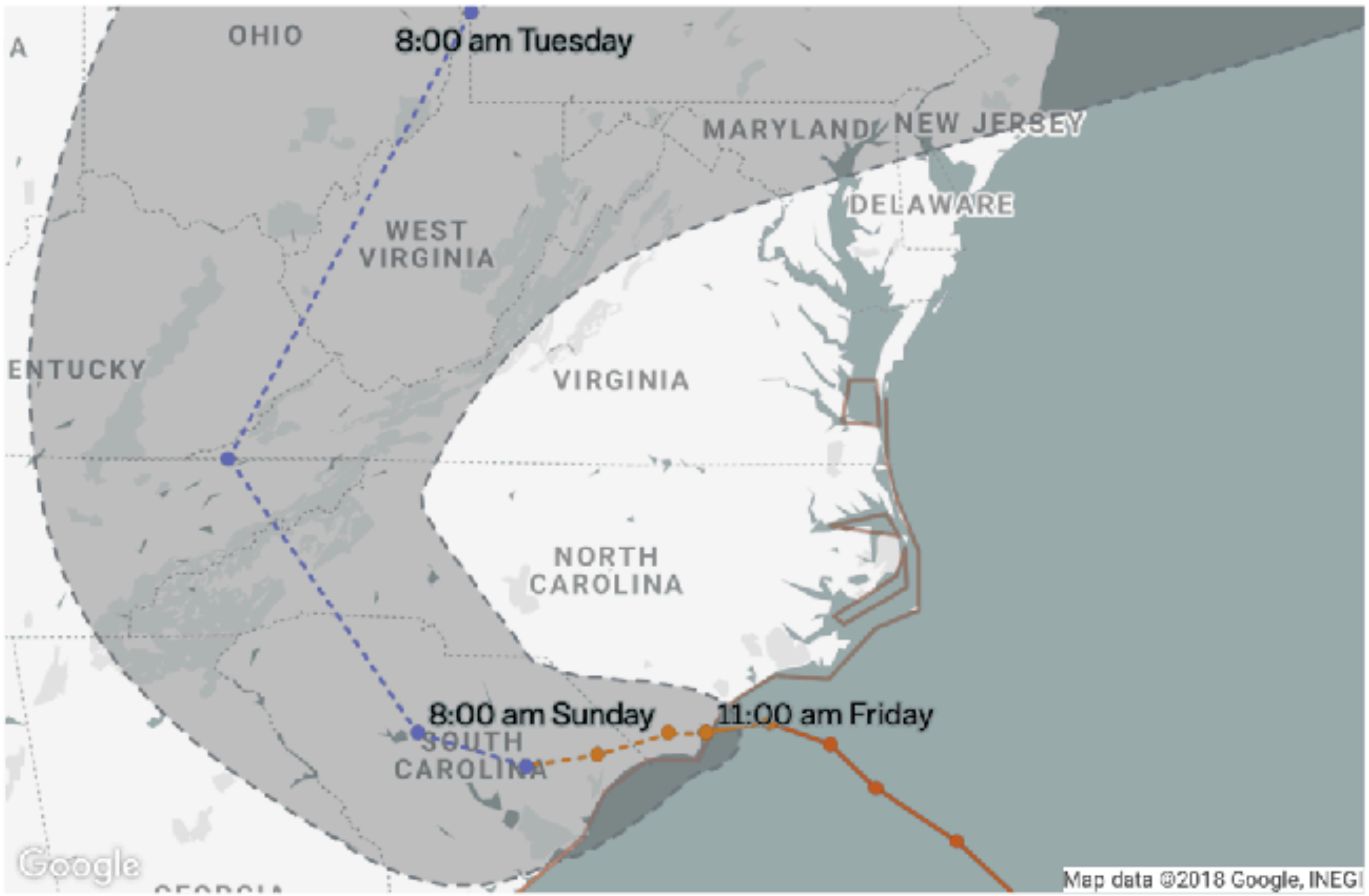

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- Conservation laws
- Parameterizations

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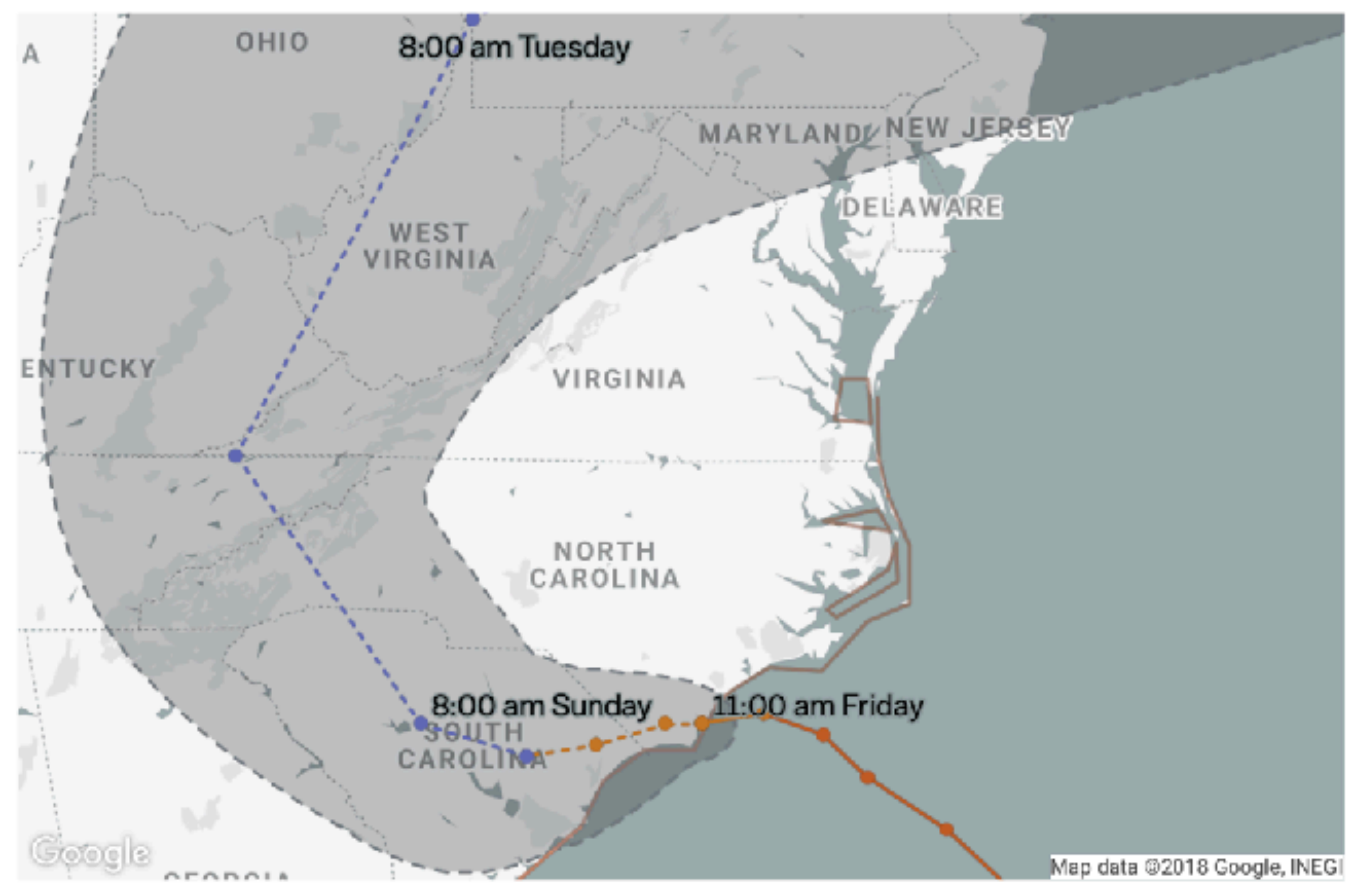

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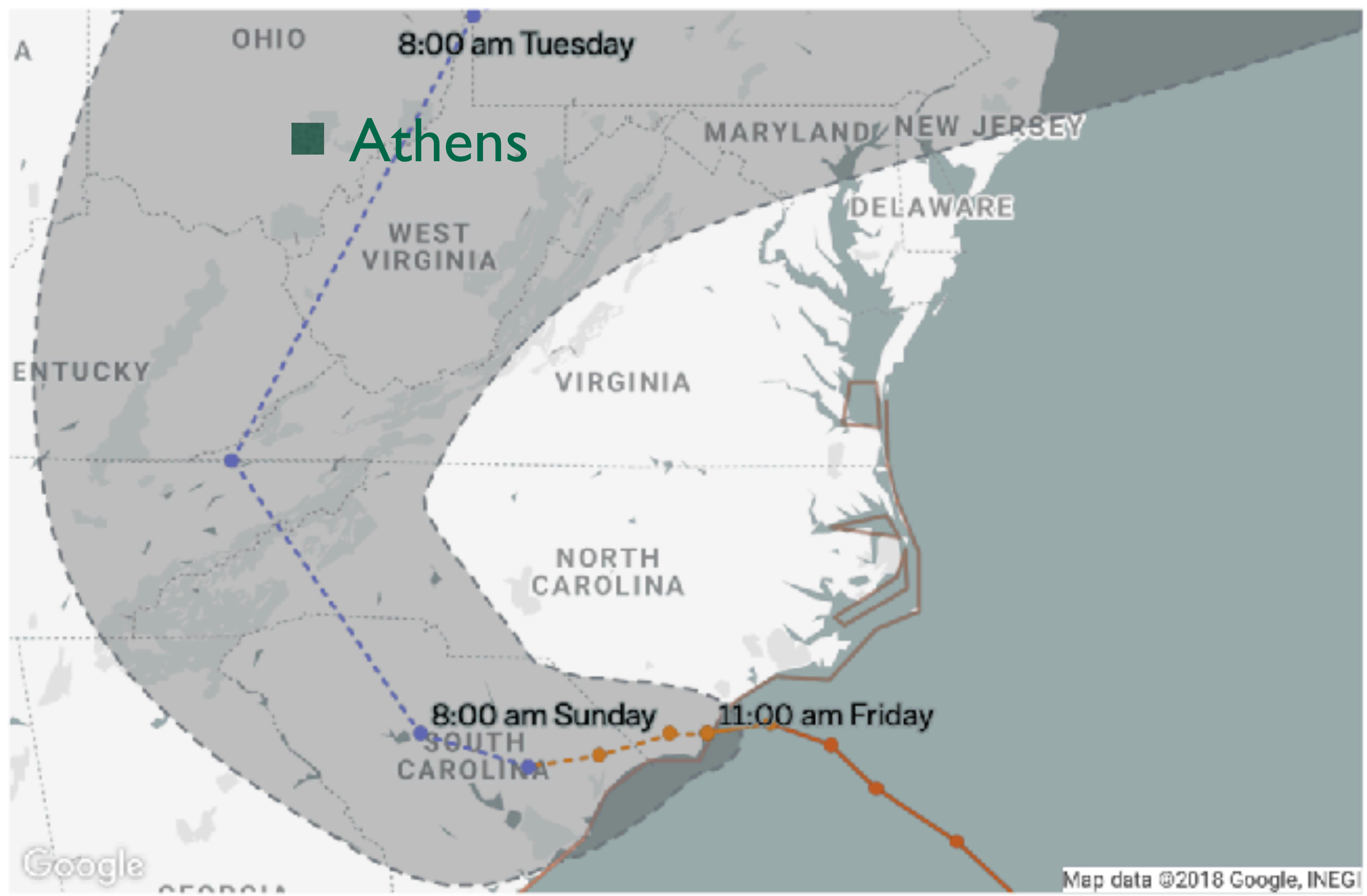

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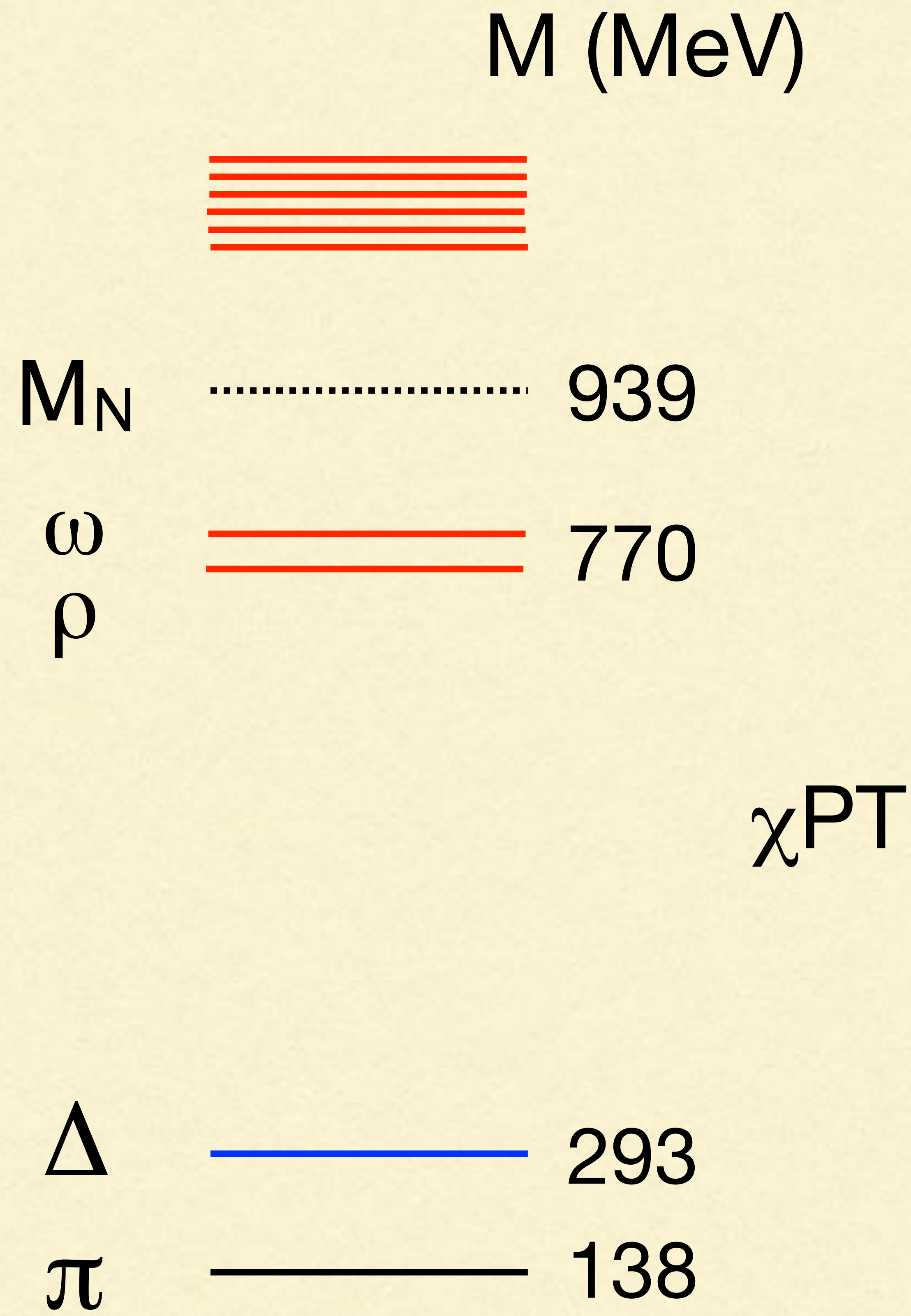
- Forces: electromagnetic, strong nuclear
- Conservation laws, e.g., probability, energy, momentum
- Some parameterizations
- Accurate knowledge of initial state (nuclear structure)
- Computing to evolve state forward in time
- Uncertainty quantification

Outline

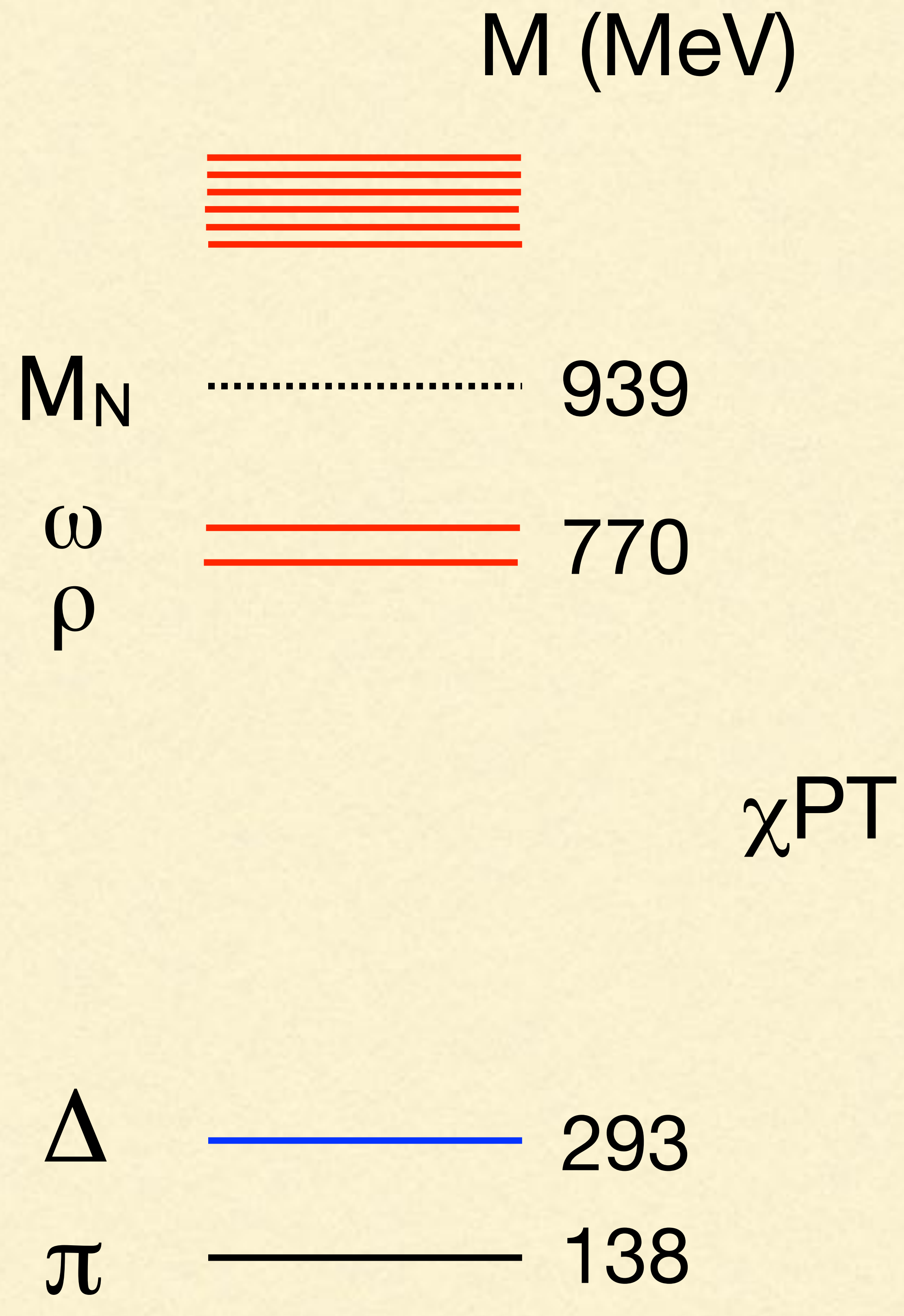
- What we do and don't know about the strong nuclear force
- EFT: organizing what we know, constraining what we don't
- EFT truncation errors from a Bayesian analysis: NN scattering
- EFT for halo nuclei: universal formula for $\gamma + {}^{A}Z \rightarrow {}^{A-1}Z + n$
- Uncertainty quantification for fusion: ^{7}Be(p,γ) at solar energies
- Conclusion

The long and the short of hadron physics



The long and the short of hadron physics

- Spectrum of QCD bound states



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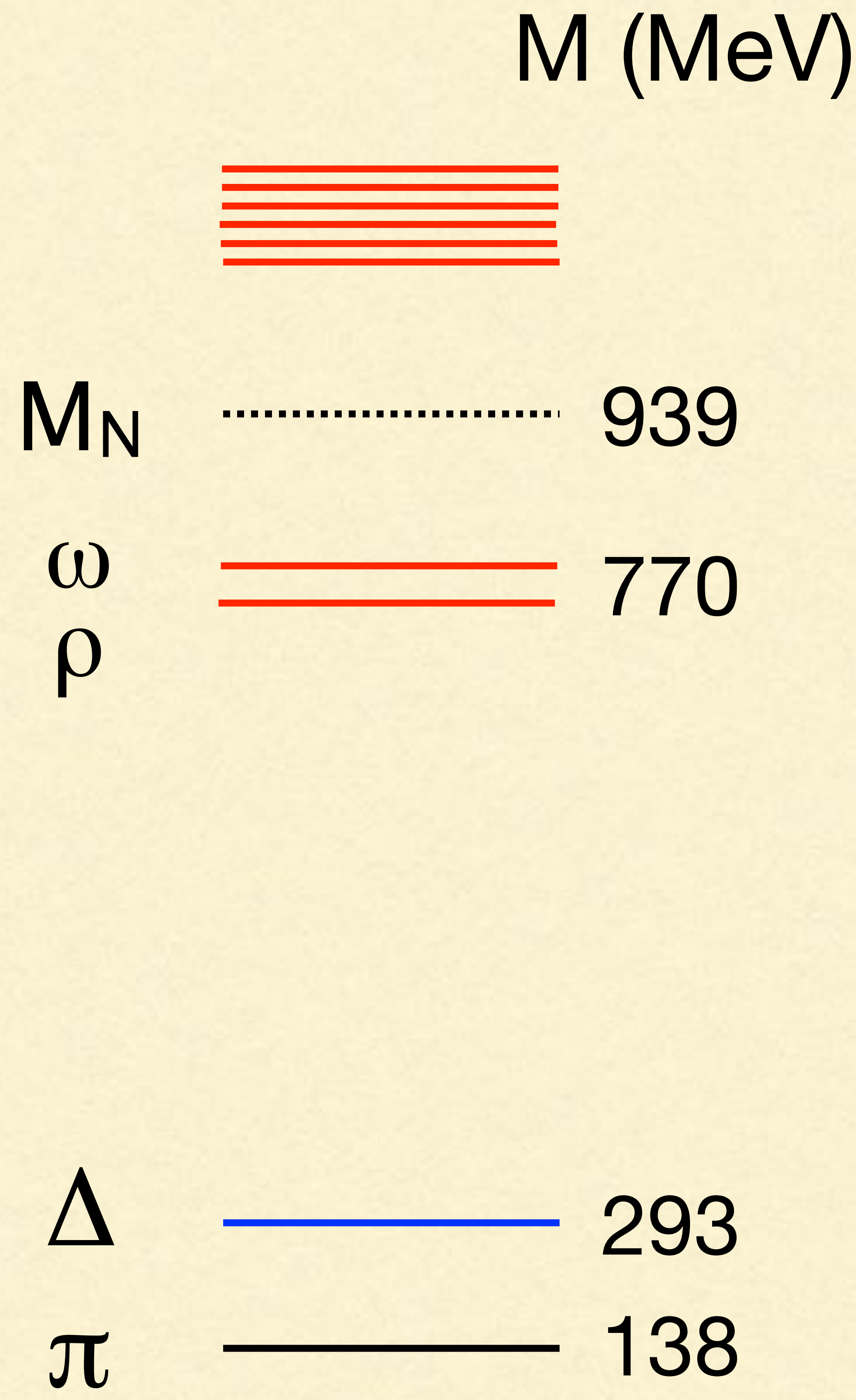

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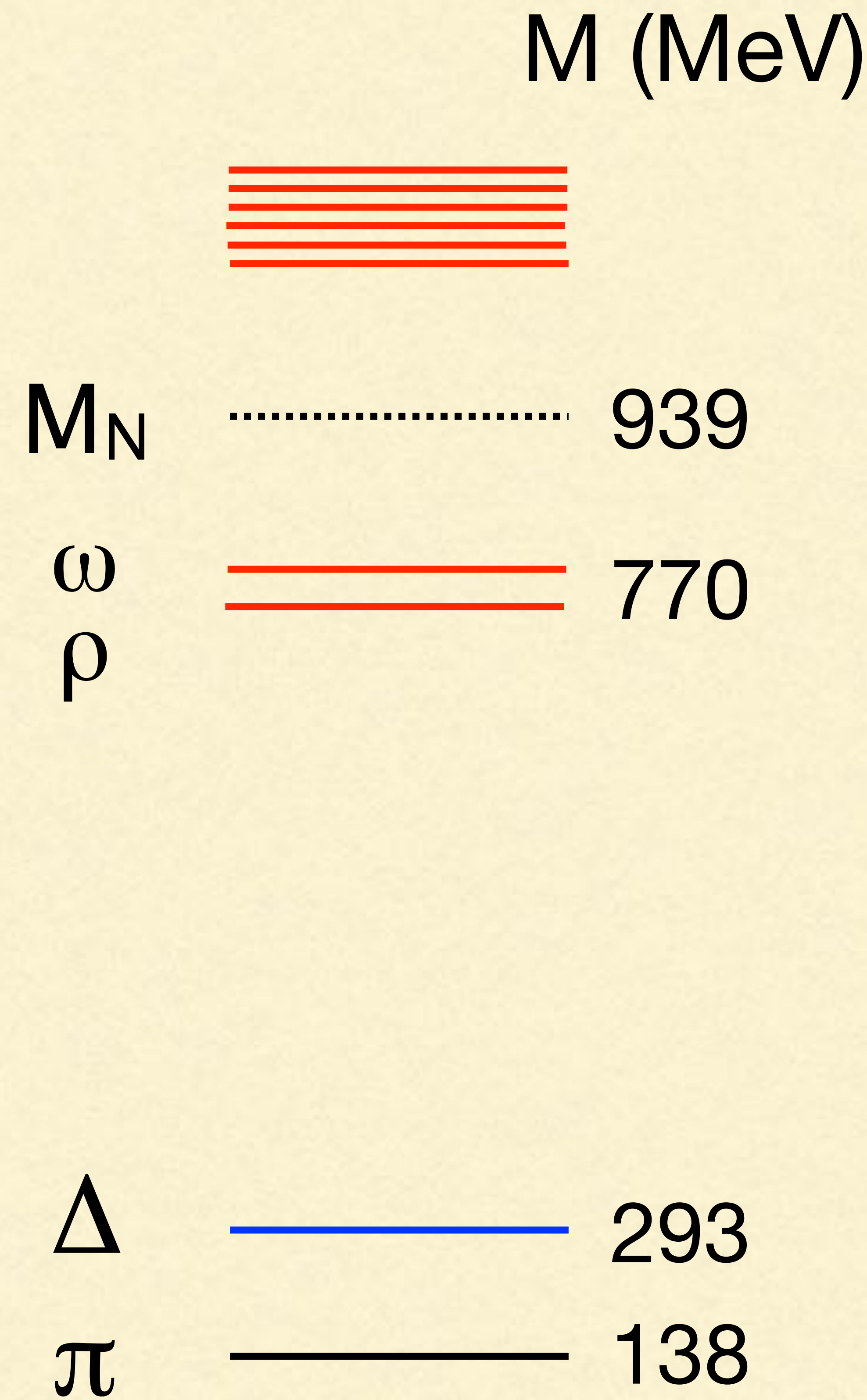
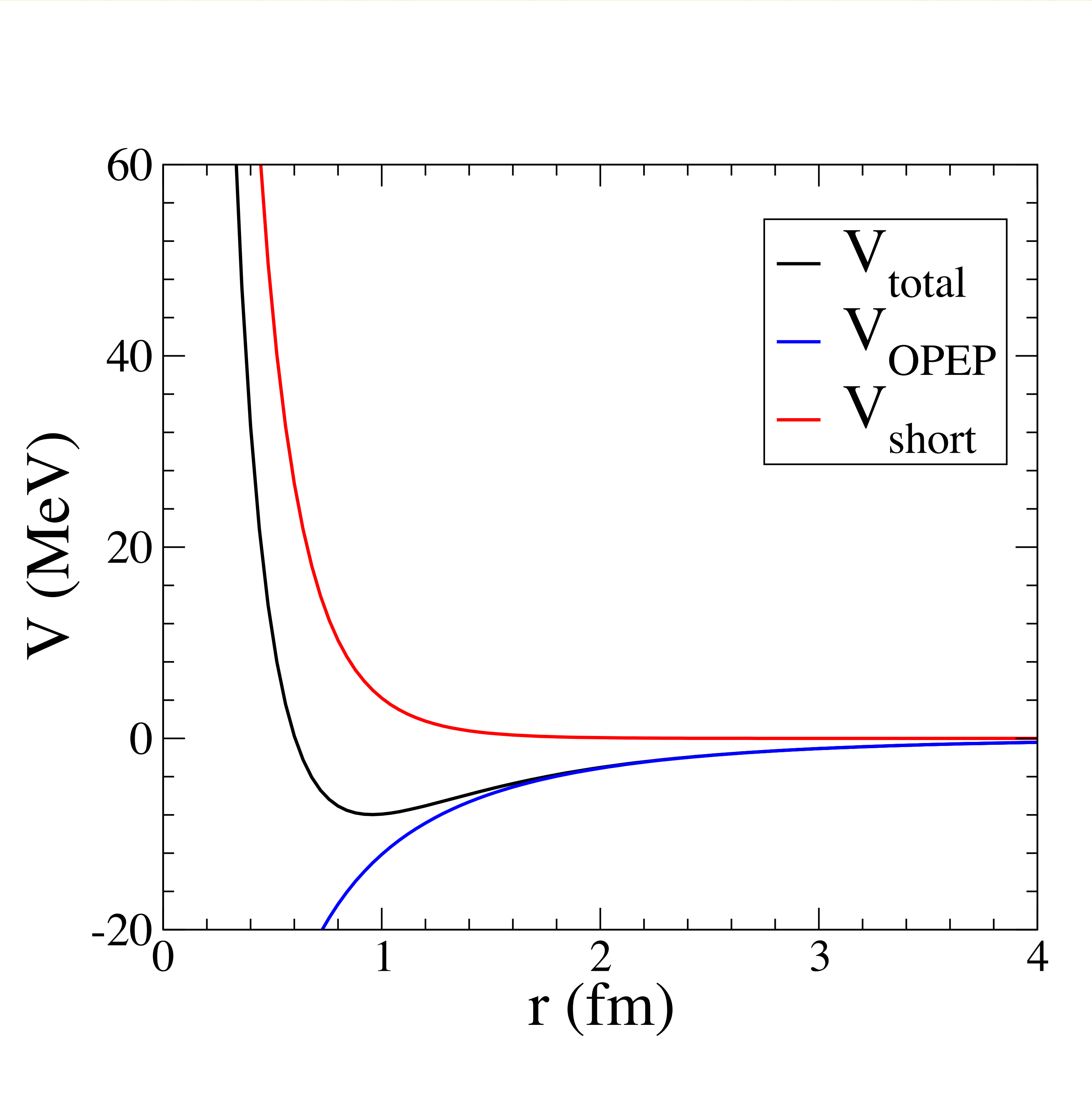
- For probe energies ~a hundred MeV, simplifications of the rich QCD dynamics emerge: processes dominated by πs (and Δs)
- Pion exchange generates longest-range part of NN force
- But short-distance dynamics too



The NN potential: a cartoon



- Long-range part generated by one-pion exchange
- Intermediate ranges: multiple pion exchange
- Short ranges: "other stuff" exchange
- Needs to be parameterized, then fit to NN scattering data

Effective Field Theory

- Simpler theory that reproduces results of full theory at long distances
- Short-distance details irrelevant for long-distance (low-momentum) physics, e.g. multipole expansion
- Expansion in ratio of physical scales: $p/\Lambda_b=\lambda_b/r$
- Symmetries of underlying theory limit possibilities: all possible terms up to a given order present in EFT
- Short distances: unknown coefficients at a given order in the expansion need to be determined. Symmetry relates their impact on different processes
- Examples: standard model, chiral perturbation theory, Halo EFT

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Error grows as first omitted term in expansion

χEFT for nuclear forces

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- χPT⇒pion interactions are weak at low energy.

  Weinberg (1990), apply χPT to V, i.e. expand it in $x=p/\Lambda_b$

$$(E - H_0)|\psi\rangle = V|\psi\rangle$$

$$V = V^{(0)} + V^{(2)} + V^{(3)} + \ldots$$

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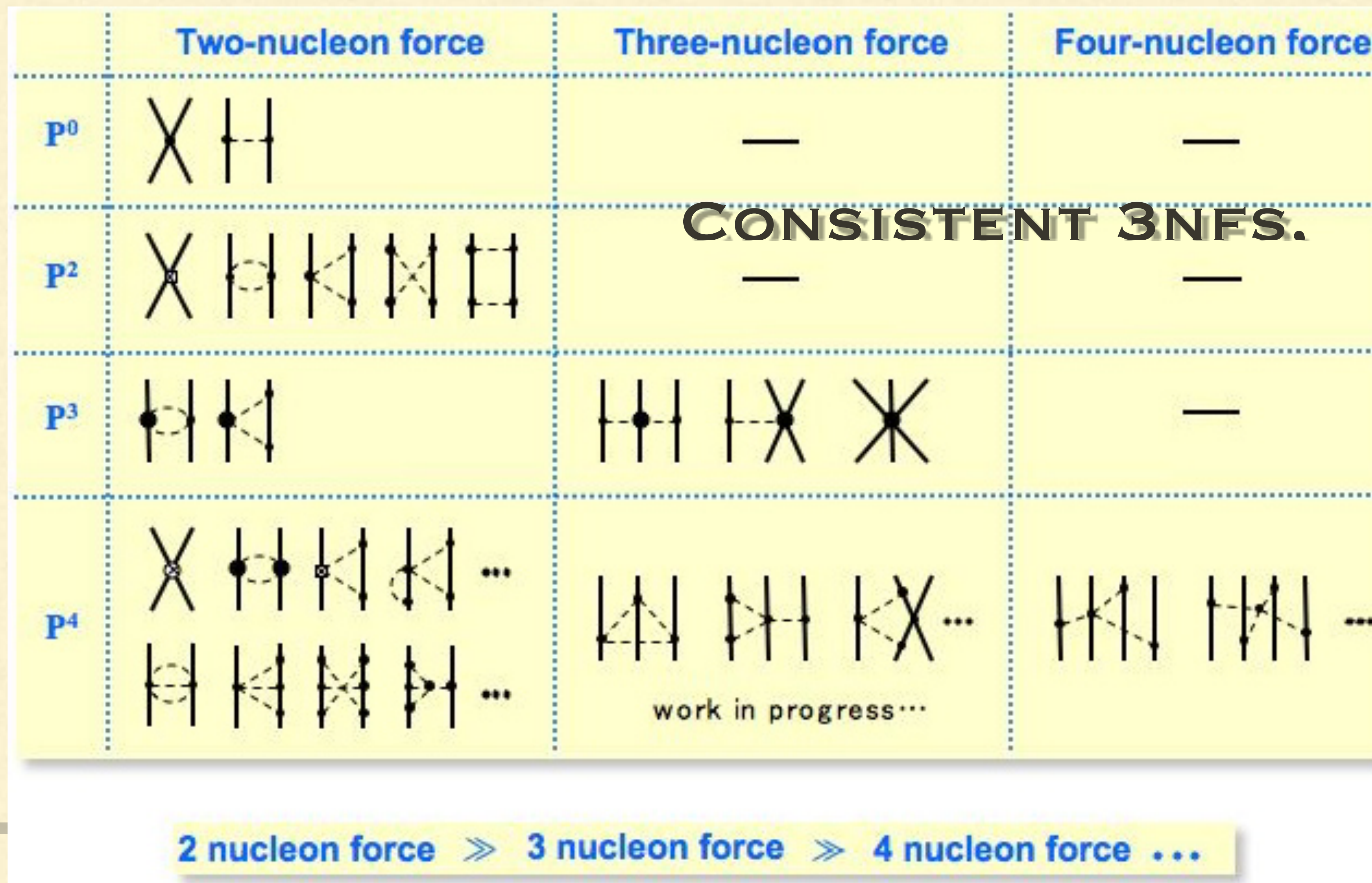

Behavior of a χEFT series

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- Expansion in $m_\pi/(M_\Delta-M_N)\simeq 0.4$
- For proton electric polarizability, χPT$\Rightarrow$ $\alpha_{\rm E1}^{(p)} = 12.5 - 2.3 + 1.5 = 11.7$
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- Rewrite as $\alpha_{\mathrm{E1}}^{(p)} = \alpha_{\mathrm{LO}}[1 + c_1(0.4) + c_2(0.4)^2 + c_3(0.4)^3]$
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- Two questions:
  - What is expectation for c_3 before we know c_0, c_1, c_2?
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Updating

- One possibility: $c_3=\max\{c_0,c_1,c_2\}$

Epelbaum, Krebs, Meissner (2014)
cf. McGovern, Griesshammer, Phillips (2013); many others.

Bayesian tools

Thomas Bayes (1701?-1761)

http://www.bayesian-inference.com

$$\mathrm{pr}(A|B,I) = \frac{\mathrm{pr}(B|A,I)\mathrm{pr}(A|I)}{\mathrm{pr}(B|I)}$$

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Likelihood

Prior

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Posterior

Normalization

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Likelihood   Prior

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Posterior   Normalization

Marginalization: $\mathrm{pr}(x|\mathrm{data},I) = \int dy\, \mathrm{pr}(x,y|\mathrm{data},I)$

Allows us to integrate out "nuisance" (e.g. higher-order) parameters

Probability for EFT coefficients

Furnstahl, Klco, DP, Wesolowski, PRC,2015 after Cacciari and Houdeau, JHEP, 2011

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- Compute conditional probability distribution: pr(c_{k+1}|c_0,…,c_k,I)
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Result: $\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) \propto \begin{cases} 1 & \text{if } c_{k+1} < c_{\max} \\ \left(\frac{c_{\max}}{c_{k+1}}\right)^{k+2} & \text{if } c_{k+1} > c_{\max} \end{cases}$

$[-c_{max}X_0x^{k+1}, c_{max}X_0x^{k+1}]$ is a $\frac{k+1}{k+2} * 100\,\%$ DoB interval

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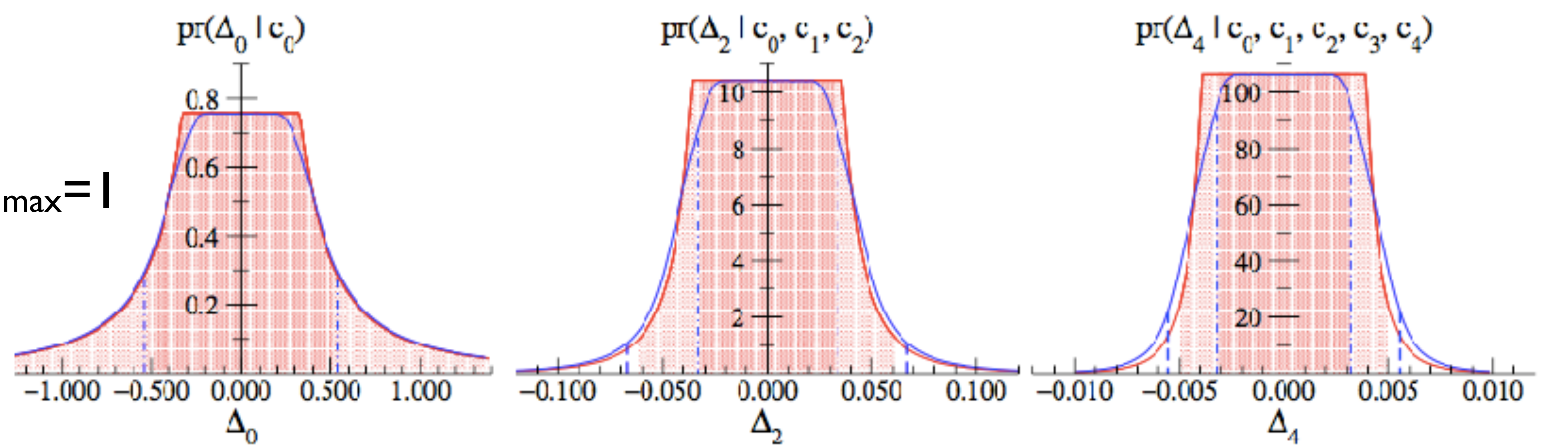
x=0.33; c_{max}=1



NN scattering cross sections

- NN cross section at T_{lab}=50, 96, 143, 200 MeV
- Potential regulated by local function, parameterized by R. Here: R=0.9 fm data
- Results at LO, NLO, N^2LO, N^3LO, N^4LO (k=0, 2, 3, 4, 5)

Epelbaum, Krebs, Meissner, PRC, 2015

$$\sigma_{np}(E_{\rm lab}) = \sigma_{\rm LO} \sum_{n=0}^{k} c_n(p_{\rm rel}) \left(\frac{p_{\rm rel}}{\Lambda_b}\right)^n$$

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EKM state

Λ_b=600 MeV

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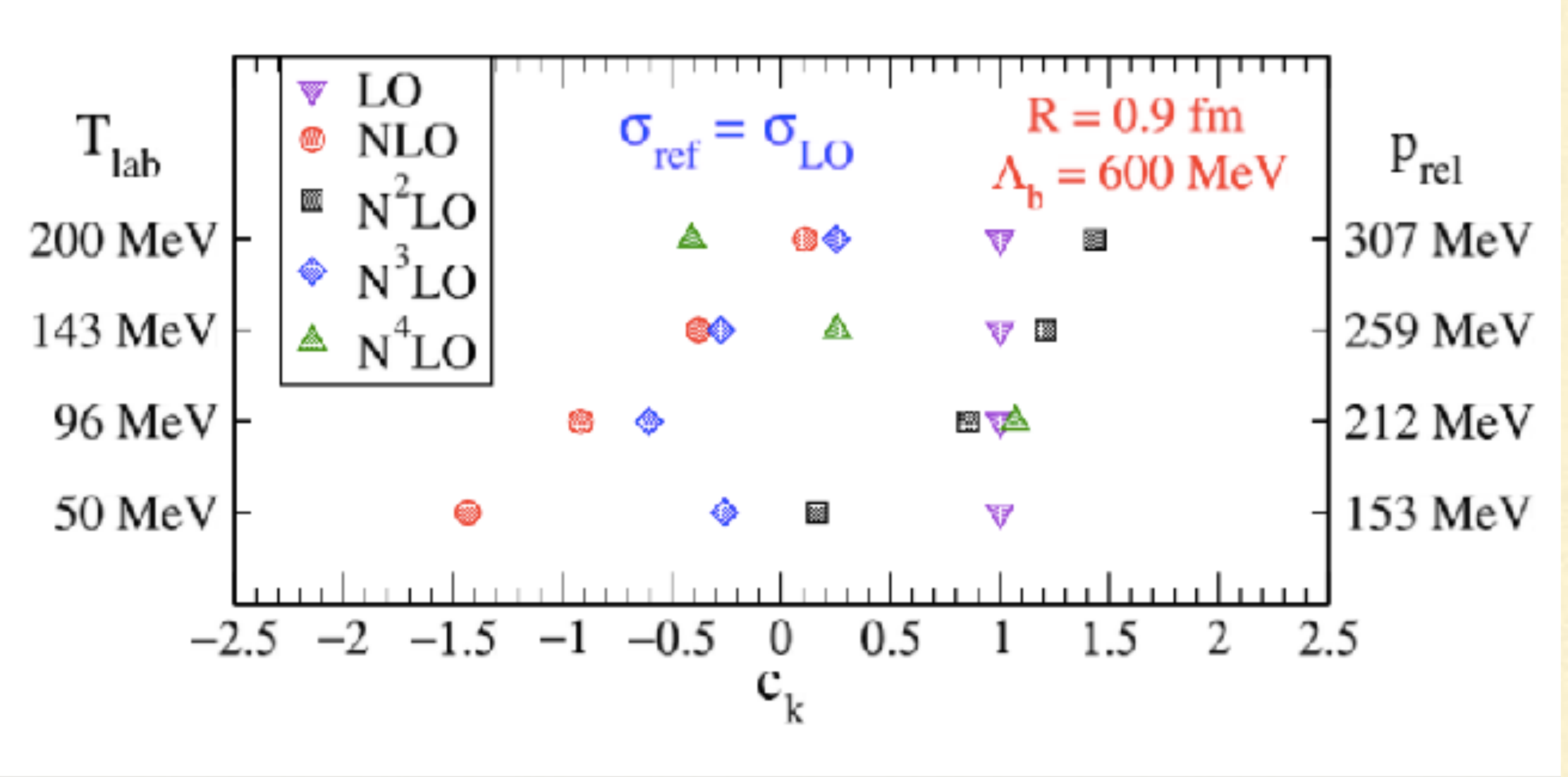
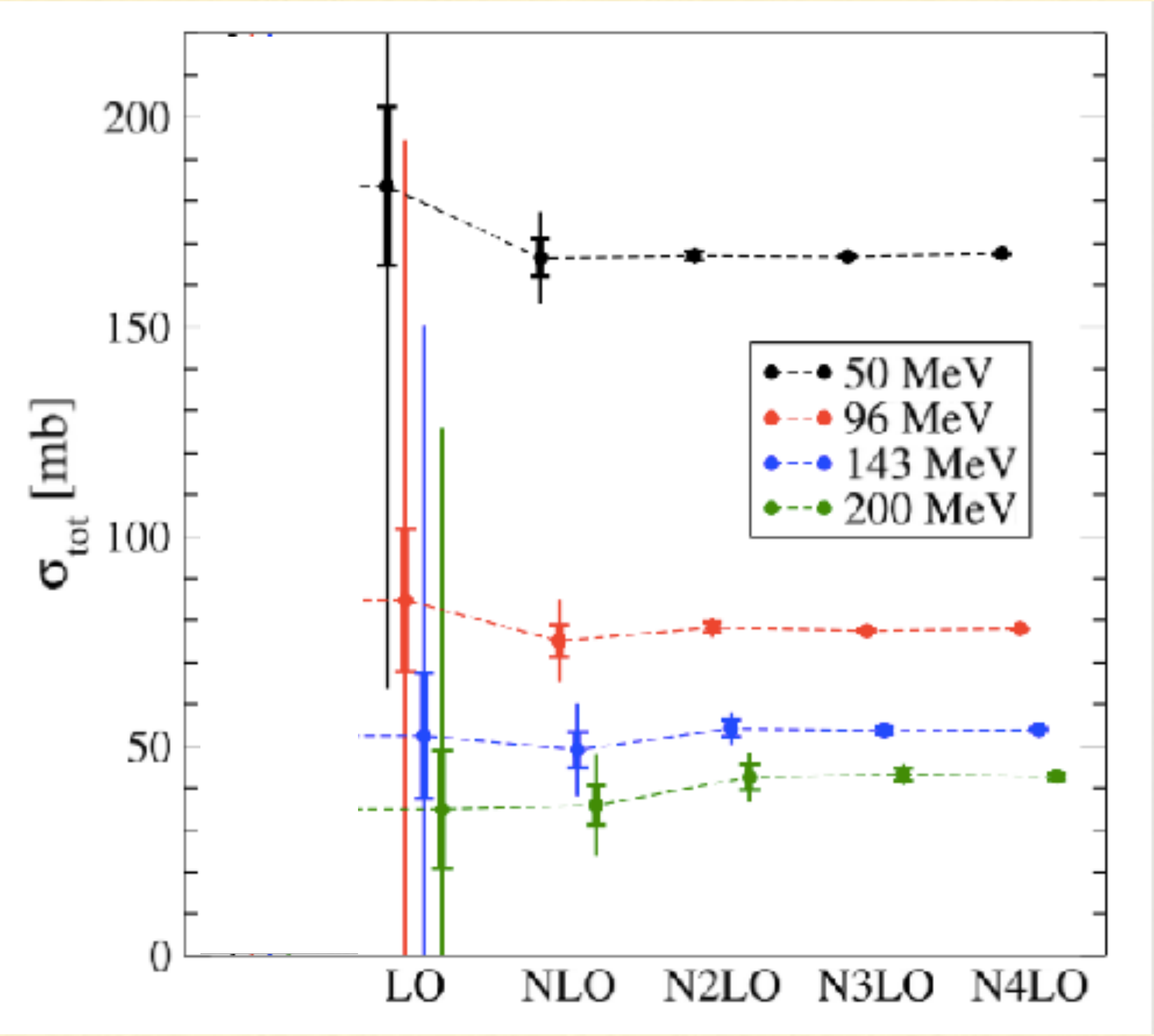


Results



200
150
100
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σ_{tot} [mb]
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NLO
N2LO
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N4LO
50 MeV
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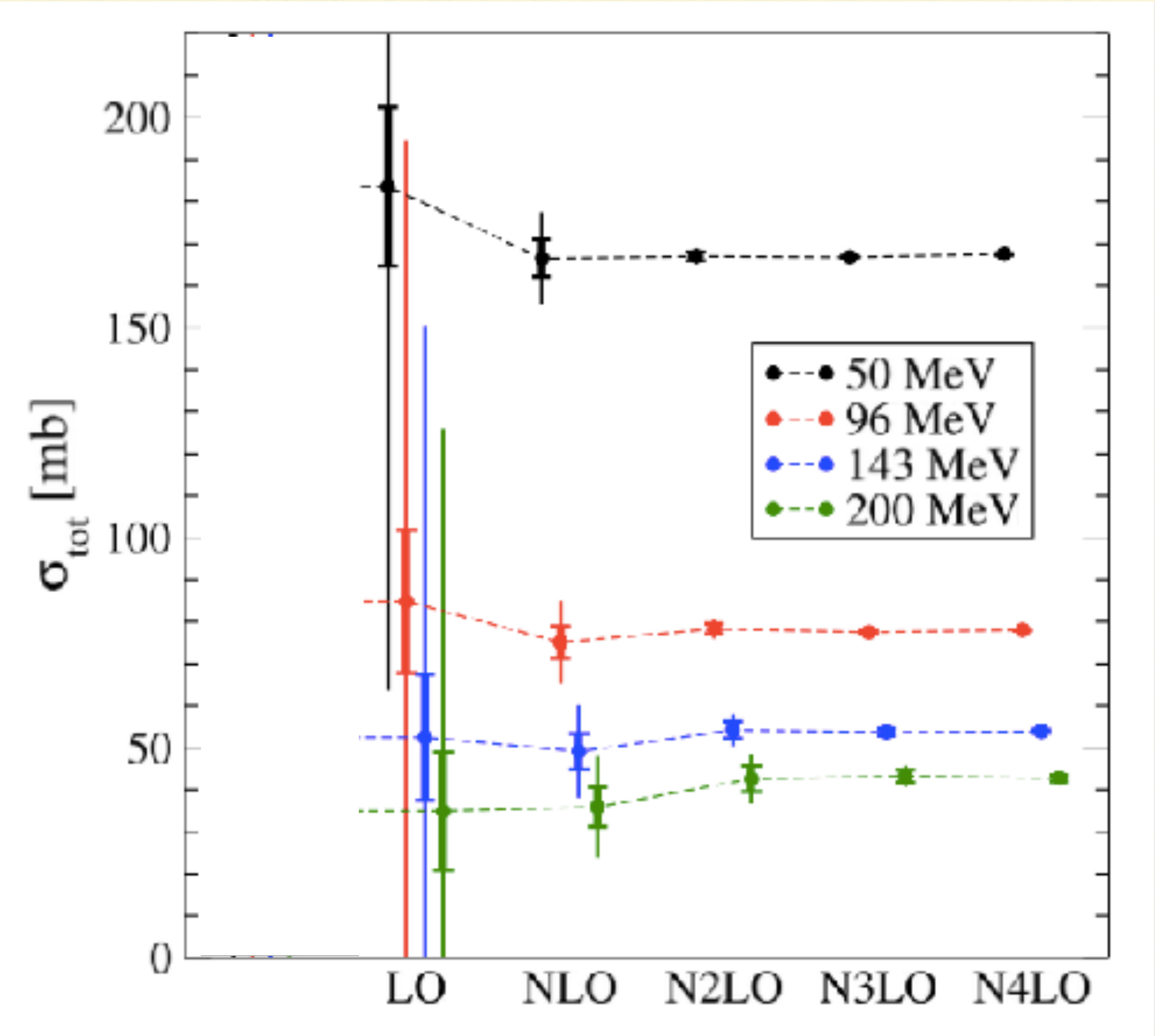

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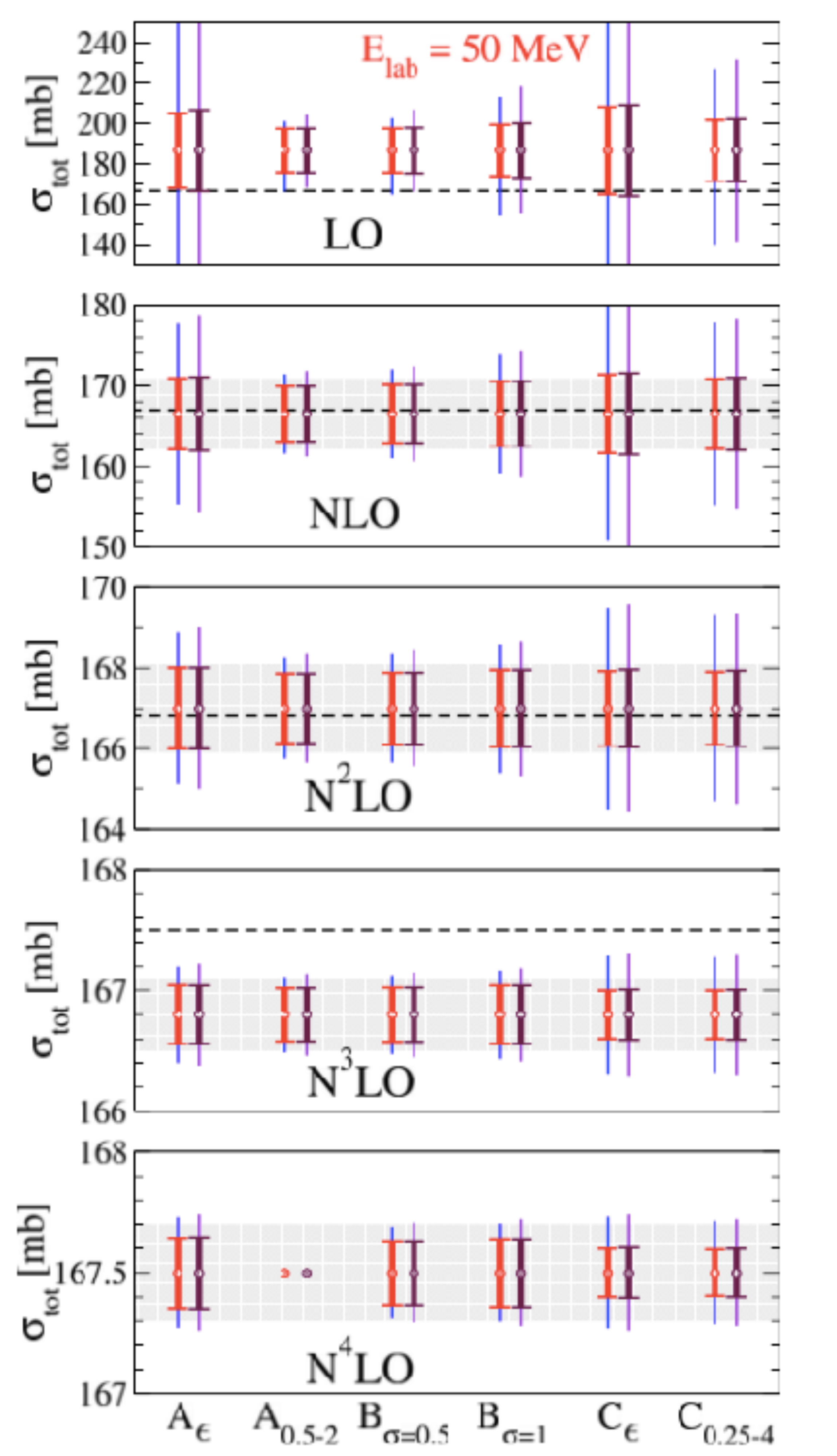

E_lab = 50 MeV
σ_tot [mb]
LO
NLO
N²LO
N³LO
N⁴LO
A_ε
A_0.5-2
B_σ=0.5
B_σ=1
C_ε
C_0.25-4

The well-calibrated EFTist

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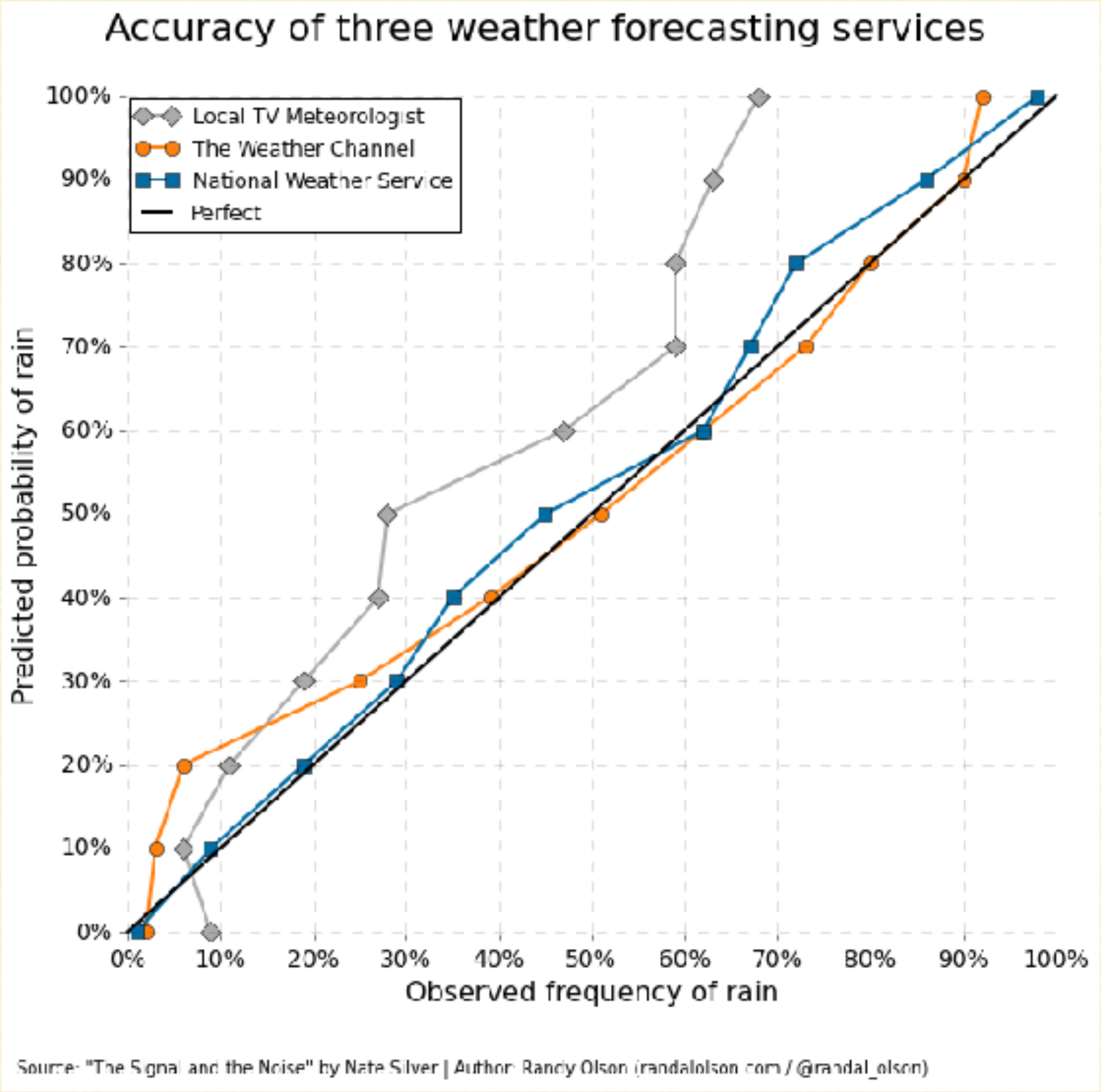

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Furnstahl, Klco, DP, Wesolowski, PRC, 2015

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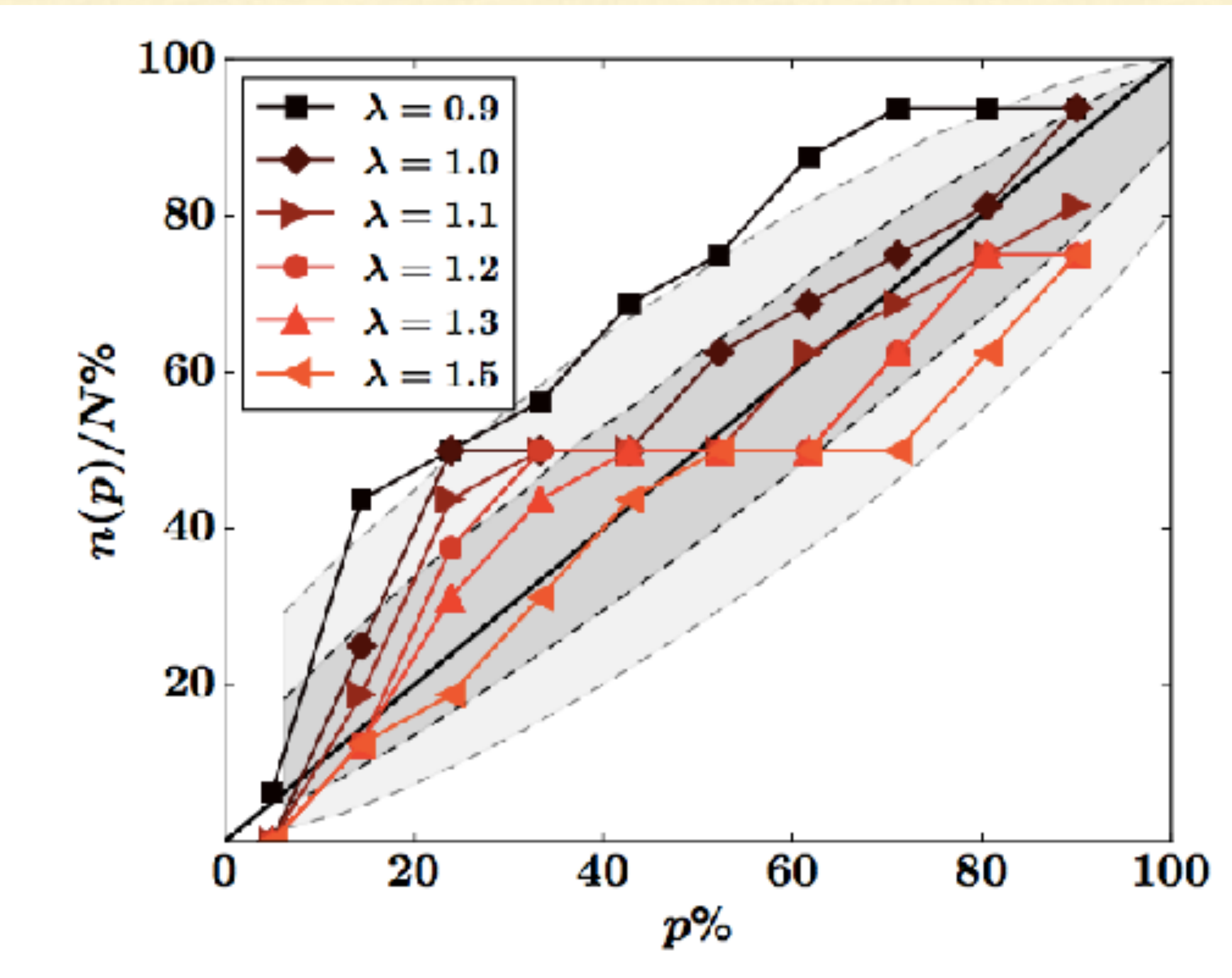

- Now we consider predictions at each order, with their error bars, as data and test them to see if the procedure is consistent
- Fix a given DOB interval, compute actual success ratio and compare
- Look at this over EKM predictions at four different orders and four different energies
- Interpret in terms of rescaling of Λ_b by a factor λ

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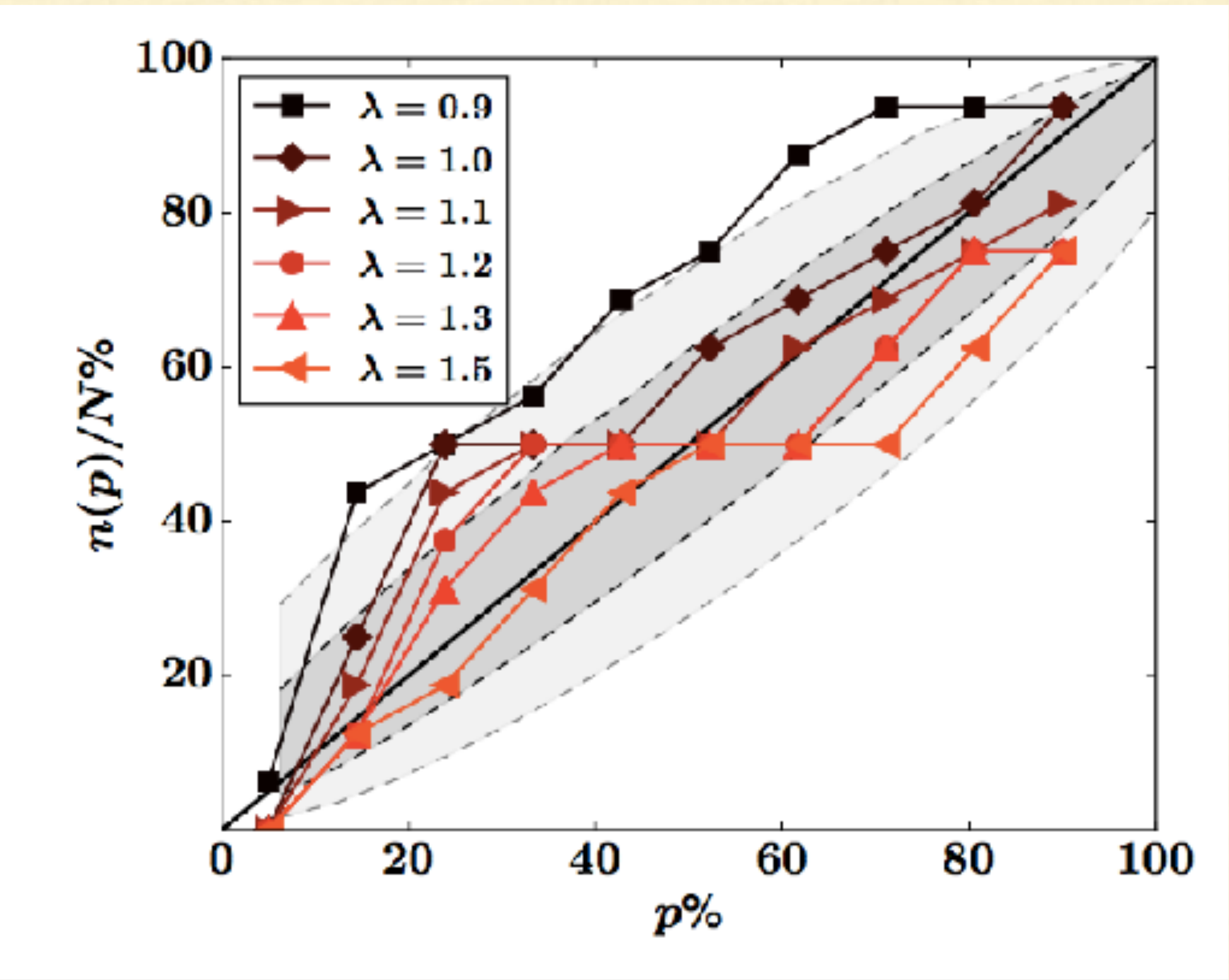

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No evidence for significant rescaling of Λ_b

Outline

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Ordinary vs. halo nuclei

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- In nuclei, each nucleon moves in the potential generated by the others
- The nuclear size grows as $A^{1/3}$; cross sections like $A^{2/3}$
- Nuclear binding energies are on the order of 8 MeV/nucleon

http://alternativephysics.org

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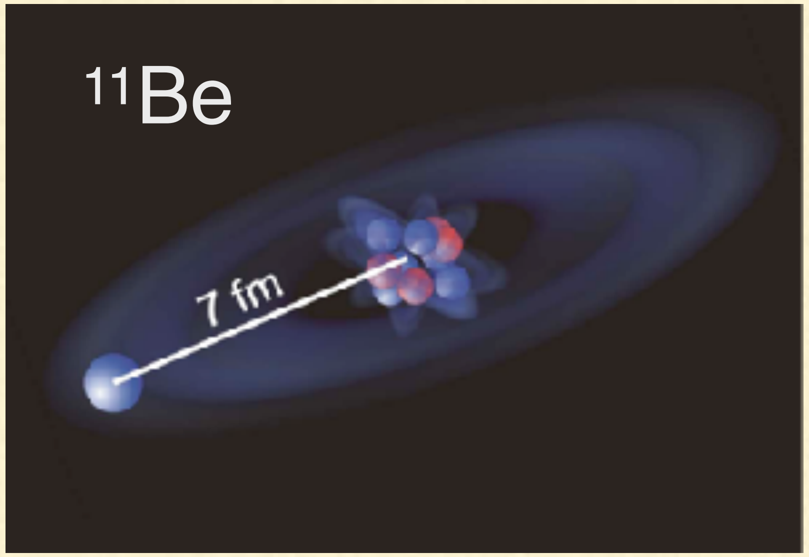

http://www.uni-mainz.de

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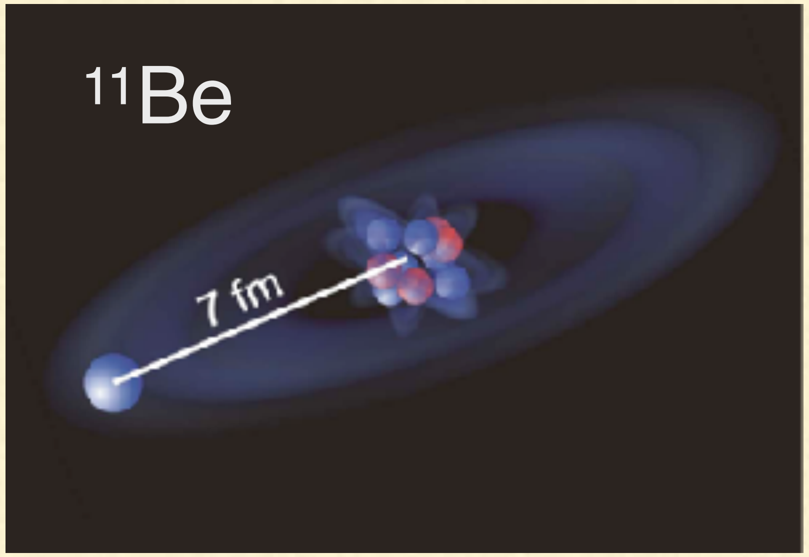

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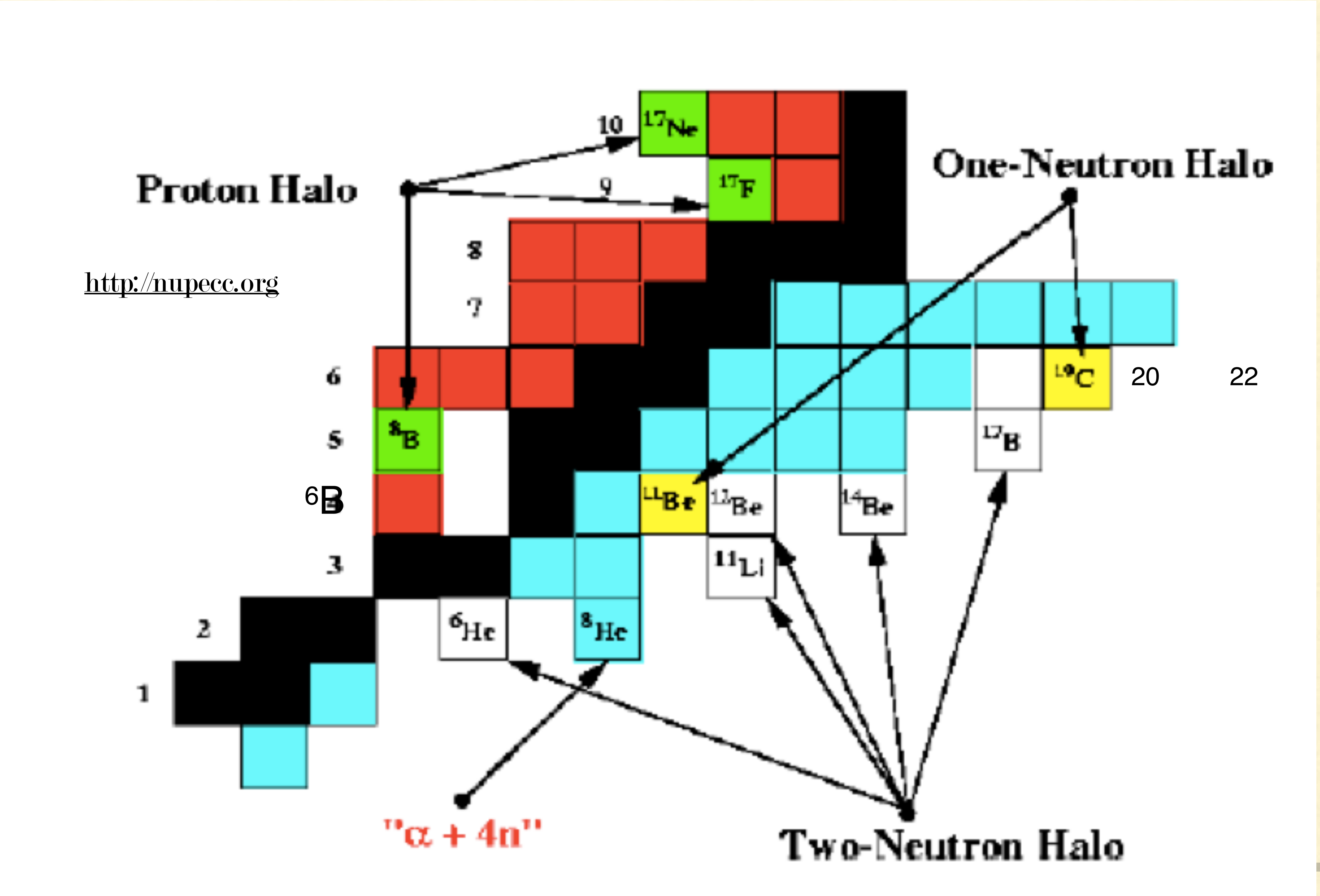
- Halo nuclei: the last few nucleons "orbit" far from the nuclear "core"

- Characterized by small nucleon binding energies, large radii, large interaction cross sections, large E1 transition strengths.

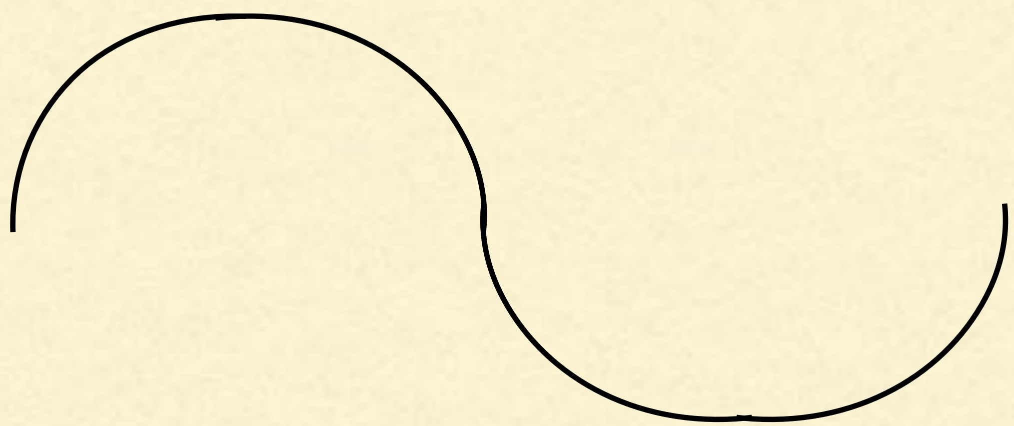
Halo nuclei: examples

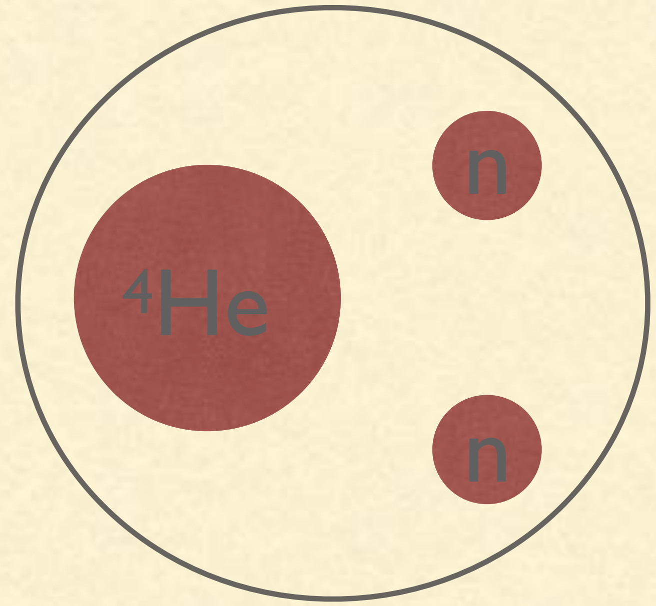
Halo nuclei: examples



Halo EFT

$\lambda \gg R_{core}; \ \lambda \lesssim R_{halo}$





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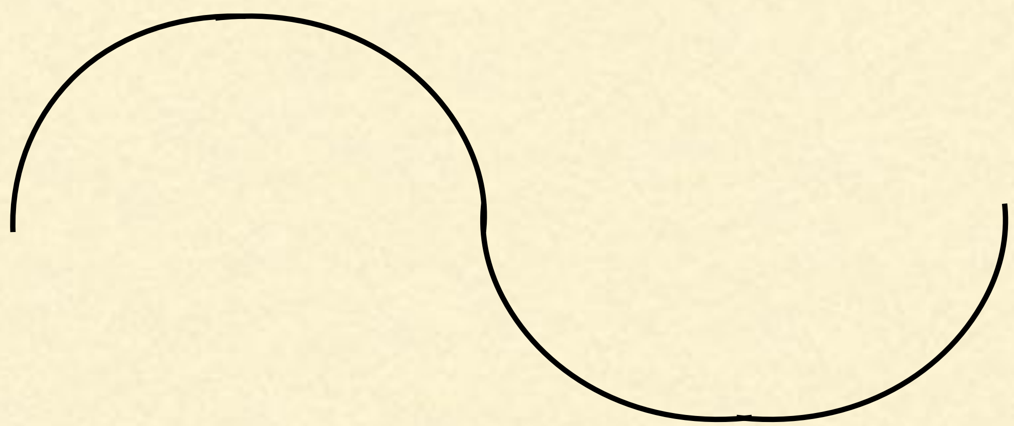
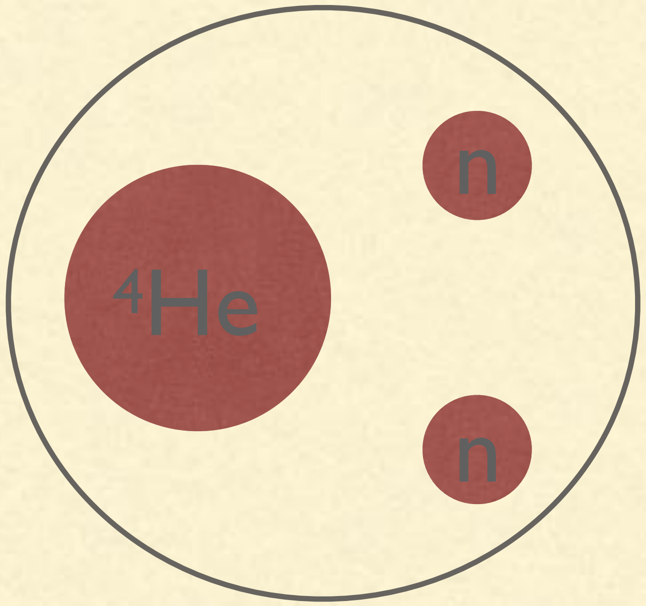




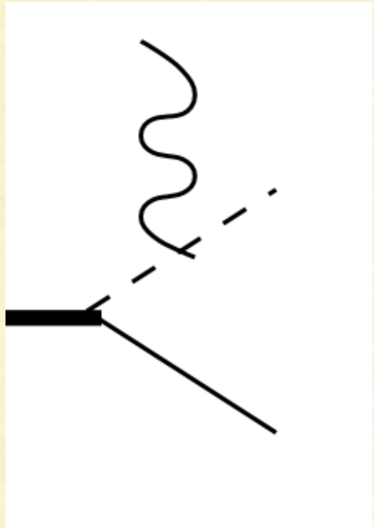
- Define $R_{halo}=\langle r^2\rangle^{1/2}$. Seek EFT expansion in R_{core}/R_{halo}. Valid for $\lambda \lesssim R_{halo}$
- Typically $R \equiv R_{core} \sim 2$ fm. And since $\langle r^2\rangle$ is related to the neutron separation energy we are looking for systems with neutron separation energies of order 1 MeV or less
- By this definition the deuteron is the lightest halo nucleus, and the pionless EFT for few-nucleon systems is a specific case of Halo EFT

Predicting dissociation

$$\mathcal{M} = \frac{eZg_0 2m_R}{\gamma_0^2 + \left(\mathbf{p} - \frac{\mathbf{k}}{A}\right)^2} \qquad \gamma_0 = \sqrt{2m_R S_{1n}}$$

$$p = \sqrt{2m_R E}$$

$$\mathrm{E1} \propto \int_0^\infty dr\, j_1(pr) r u_0(r); \quad u_0(r) = A_0 e^{-\gamma_0 r}$$



Chen, Savage (1999)

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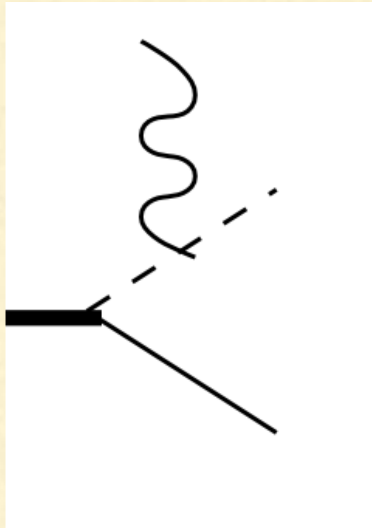

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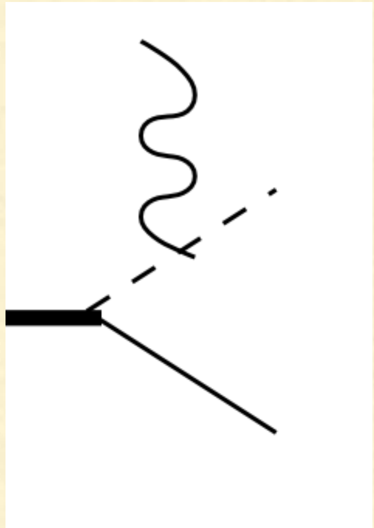
Predicting dissociation

- Leading order: no FSI ⇒ γ_0 is only free parameter=0.16 fm^{-1} for ^{19}C

$$\mathcal{M} = \frac{eZg_0 2m_R}{\gamma_0^2 + \left(\mathbf{p} - \frac{\mathbf{k}}{A}\right)^2} \qquad \gamma_0 = \sqrt{2m_R S_{1n}}$$

$$p = \sqrt{2m_R E}$$

$$\mathrm{E1} \propto \int_0^\infty dr\, j_1(pr) r u_0(r); \qquad u_0(r) = A_0 e^{-\gamma_0 r}$$



Chen, Savage (1999)

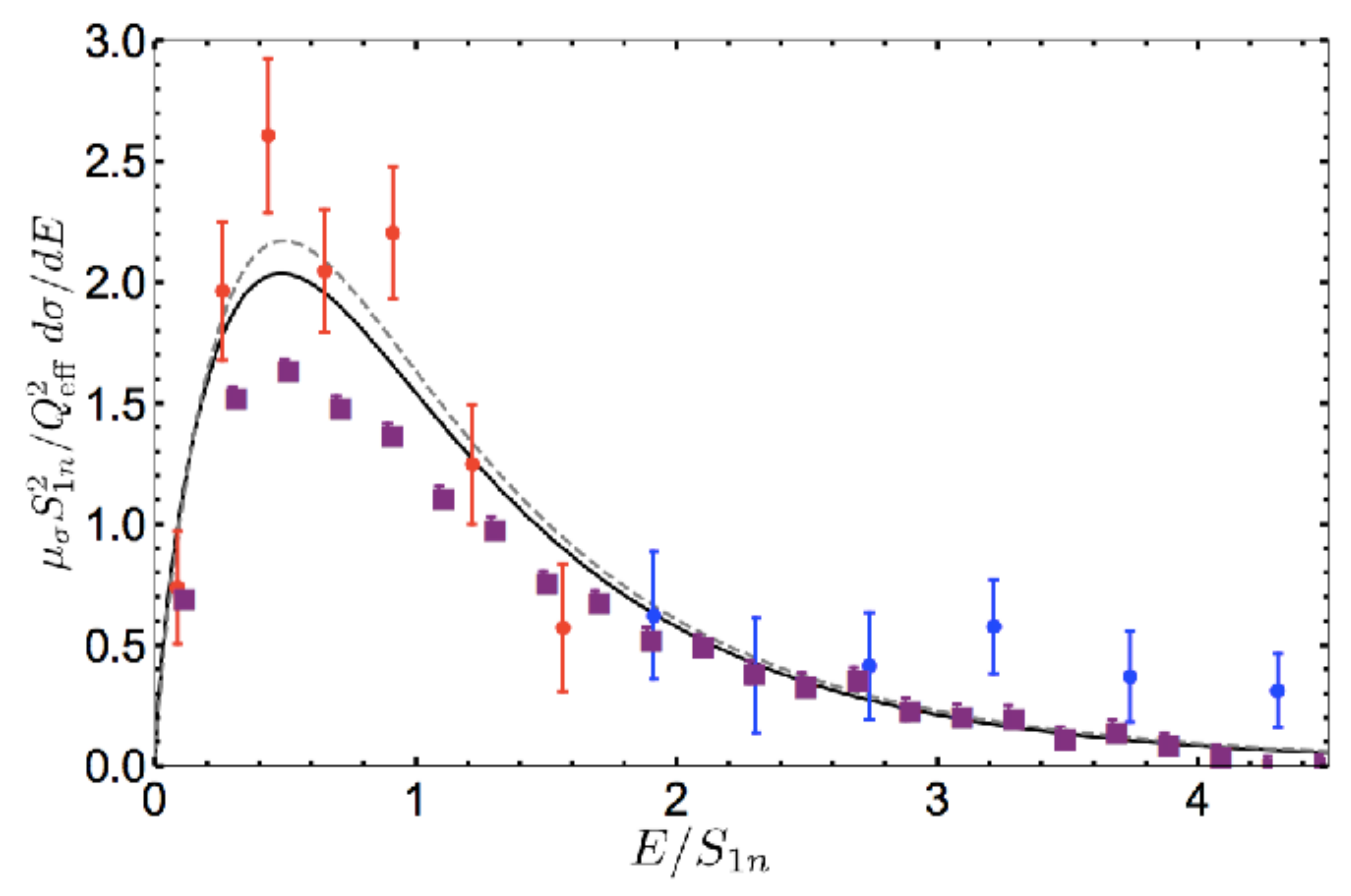
$$\frac{dB(E1)}{e^2 dE} = \frac{6m_R}{\pi^2} \frac{Z^2}{A^2} A_0^2 \frac{p^3}{(\gamma_0^2 + p^2)^2}$$

Universal E1 strength formula for S-wave halos

- Final-state interactions suppressed by $(R_{core}/R_{halo})^3$
- Short-distance piece of E1 m.e.: $L_{E1}\sigma^\dagger \mathbf{E} \cdot (n \overleftrightarrow{\nabla} c) + \mathrm{h.c.} \sim \left(\frac{R_{\mathrm{core}}}{R_{\mathrm{halo}}}\right)^4$

Results

Results



Data: Nakamura et al., 1999, 2003;
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Analysis: Acharya, Phillips, 2013;
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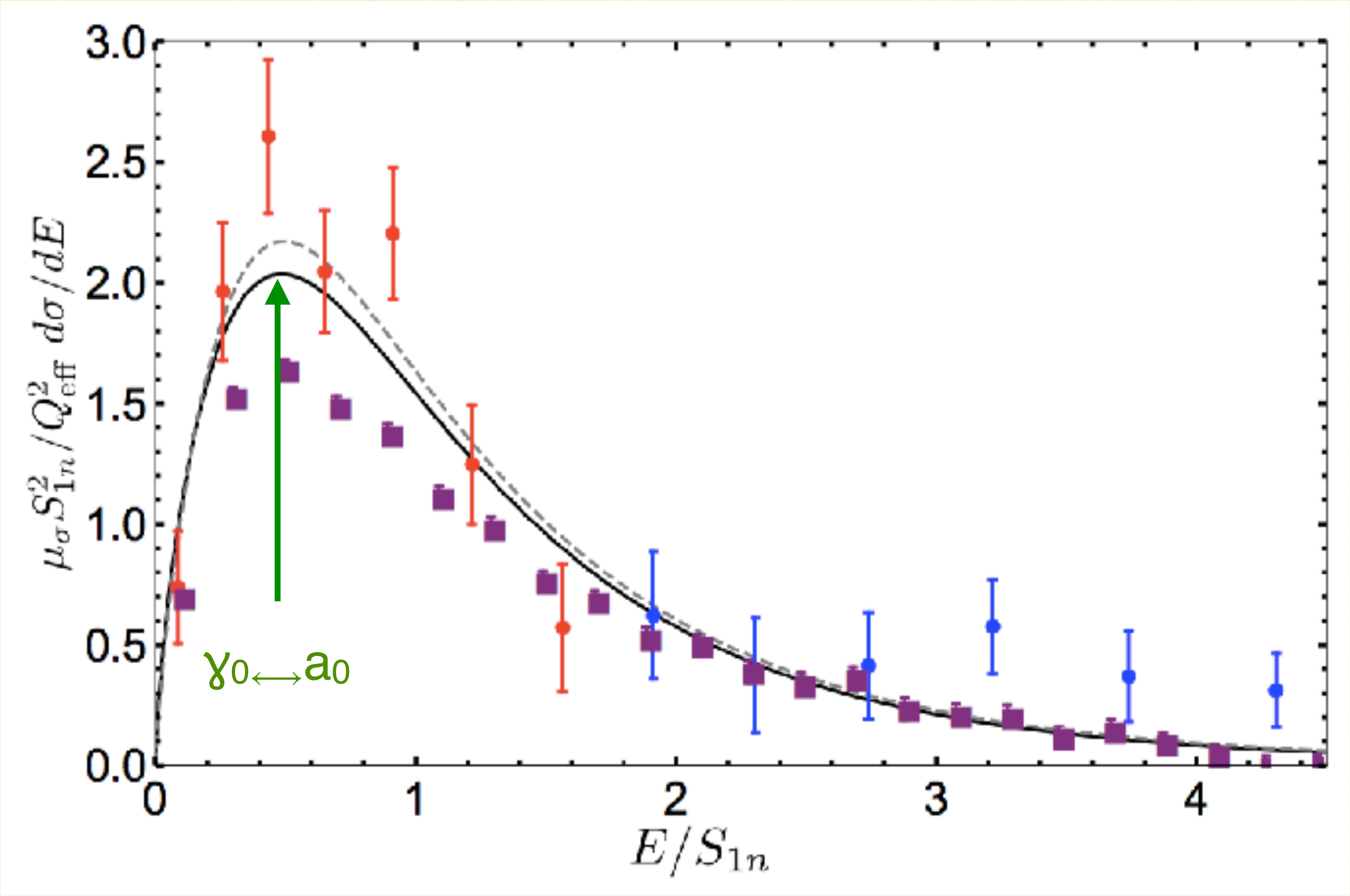
Results



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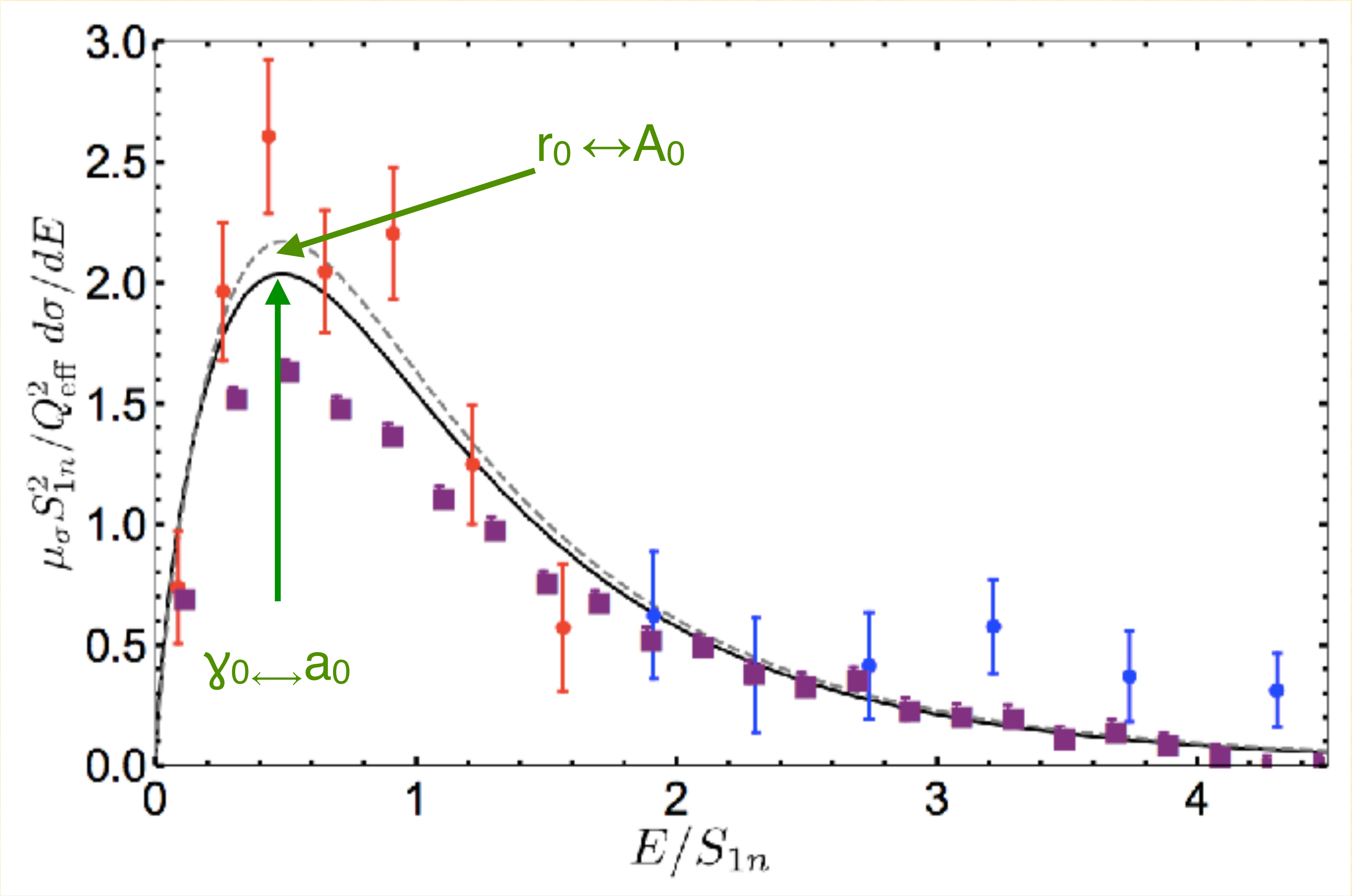

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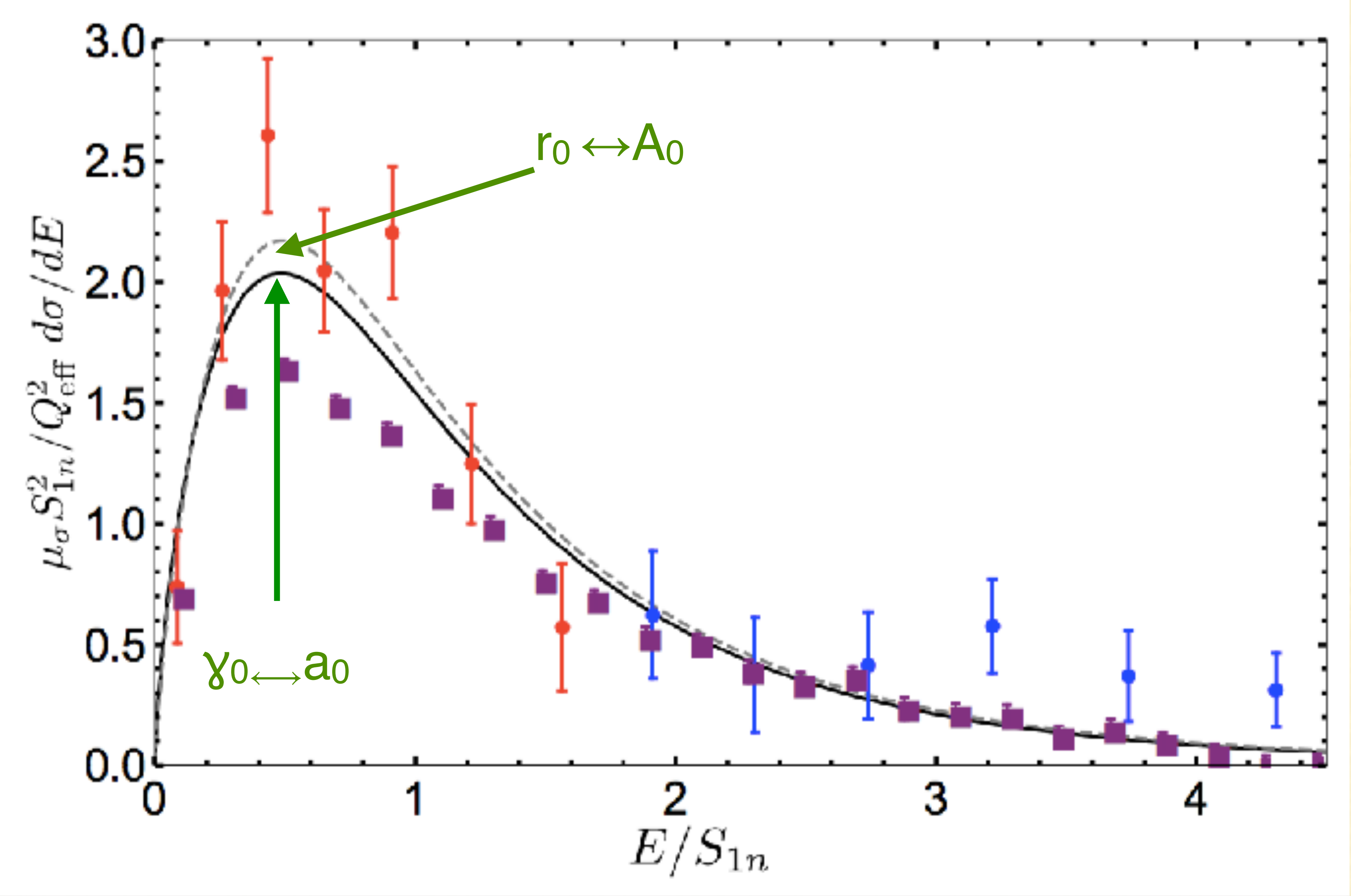

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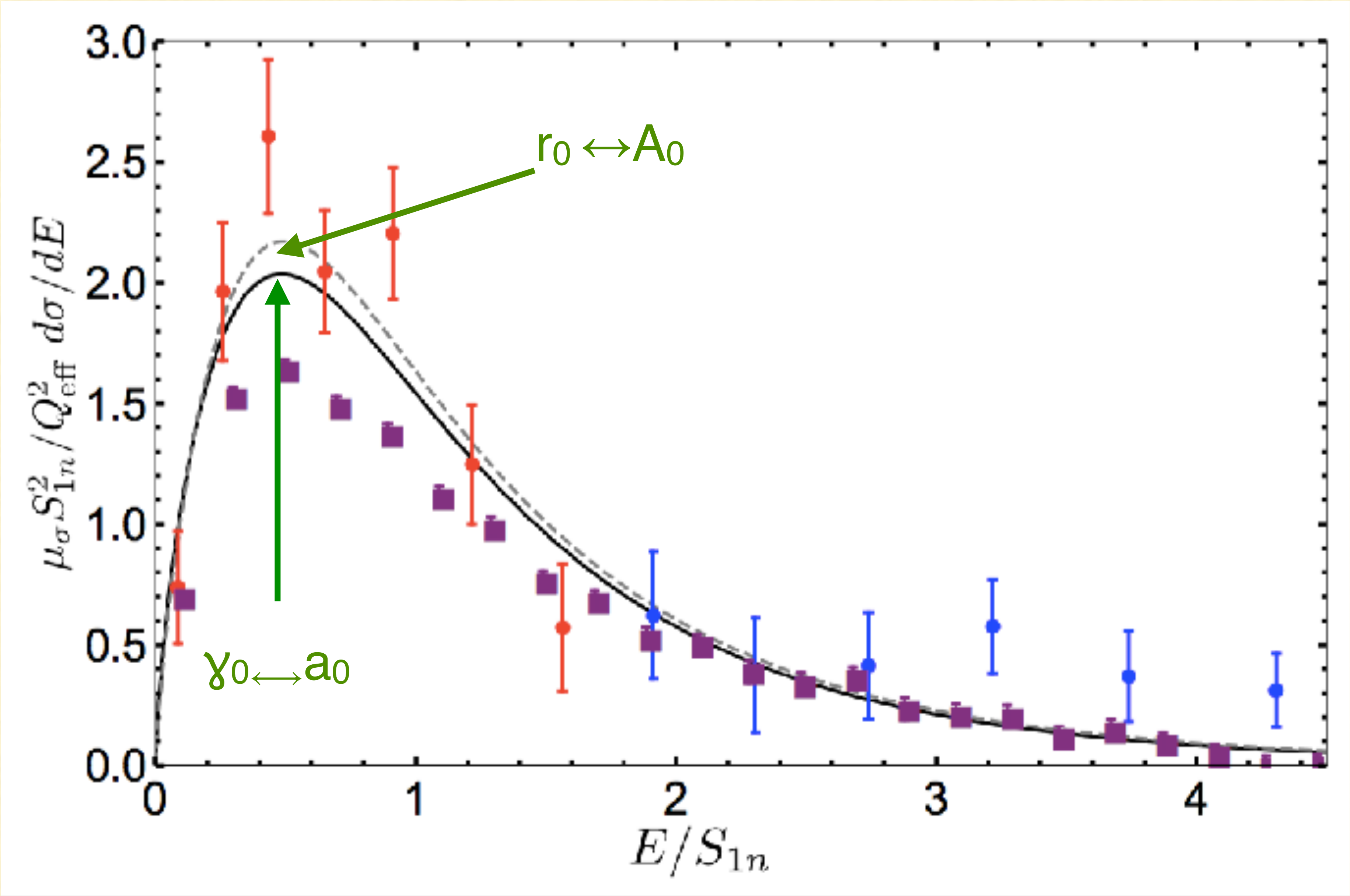
Determine S-wave ^{18}C-n scattering parameters⇔ ^{19}C ANC from dissociation data.

Results



Data: Nakamura et al., 1999, 2003;
Fukuda et al., 2004
Analysis: Acharya, Phillips, 2013;
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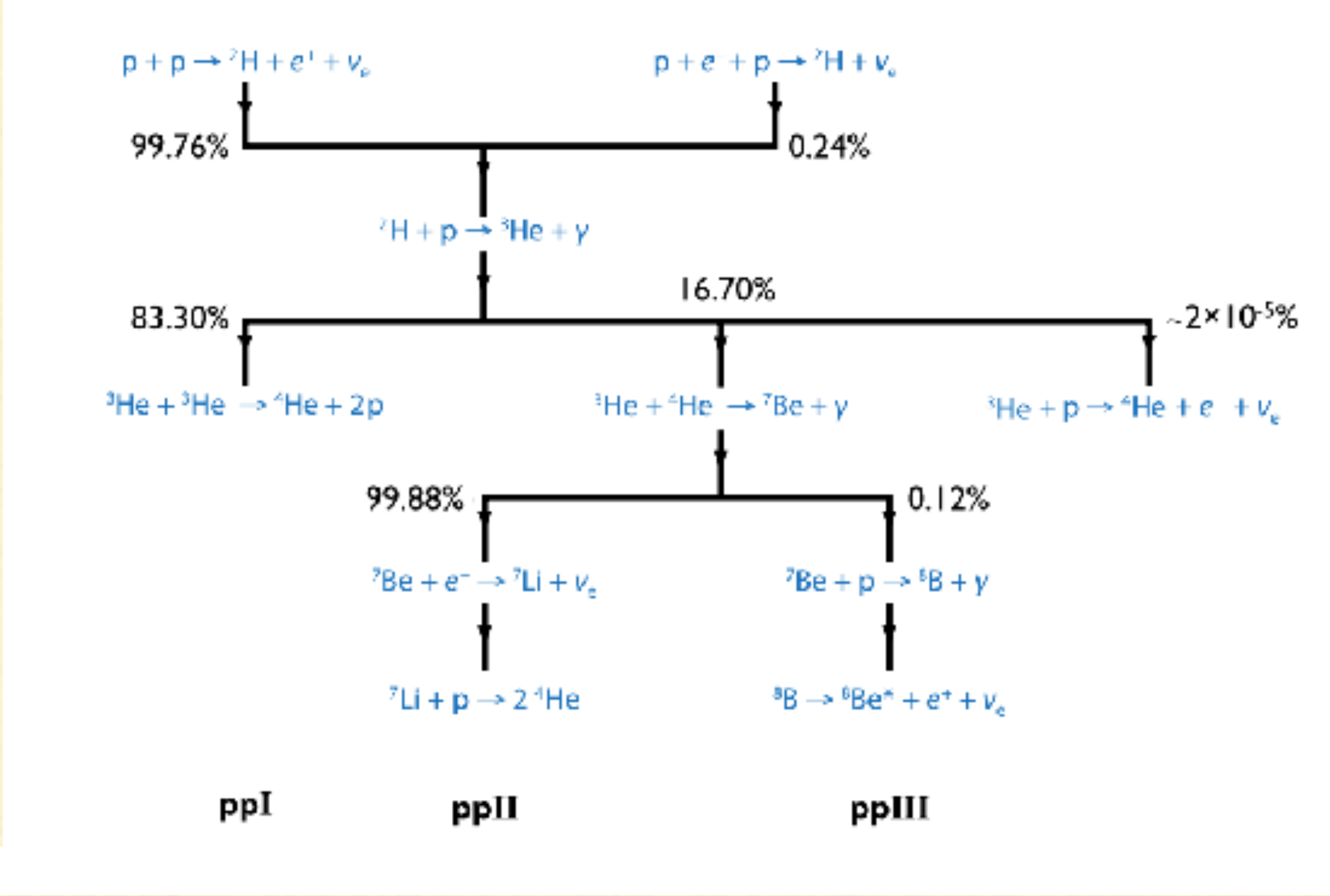
- For ^{19}C:

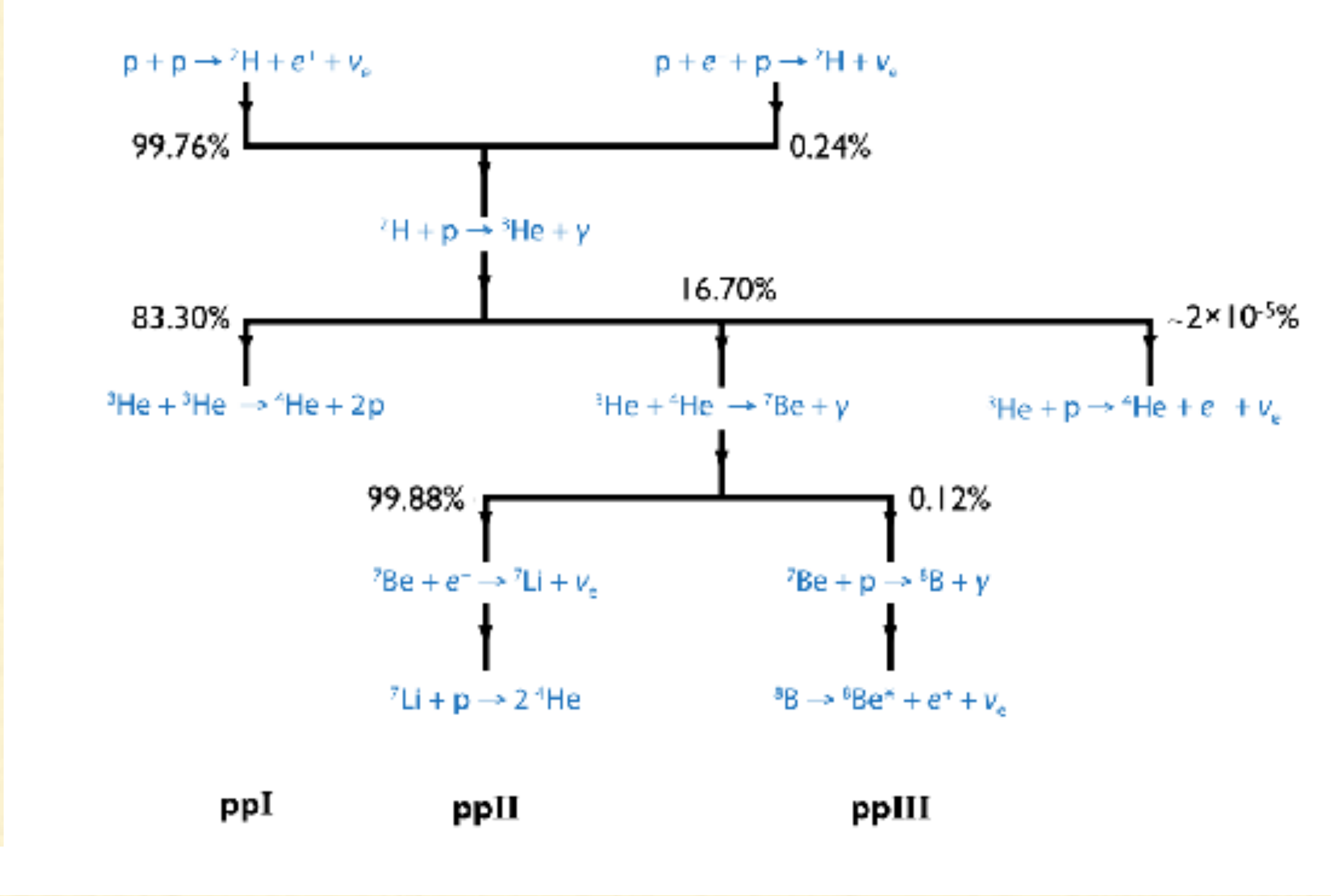
$$\begin{aligned} a &= (7.75 \pm 0.35(\text{stat.}) \pm 0.3(\text{EFT}))\ \text{fm}; \\ r_0 &= (2.6^{+0.6}_{-0.9}(\text{stat.}) \pm 0.1(\text{EFT}))\ \text{fm}. \end{aligned}$$

Why is $^7Be(p,\gamma)$ important?



Why is $^7Be(p,\gamma)$ important?

- Part of pp chain (ppIII)
- Key for predictions flux of solar neutrinos, especially high-energy (^8B) neutrinos
- Accurate knowledge of $^7Be(p,\gamma)$ needed for inferences from solar-neutrino flux regarding chemical composition of Sun→solar-system formation history
- S(0)=20.8 ± 0.7 ± 1.4 eV b



"SFII": Adelberger et al. (2010)

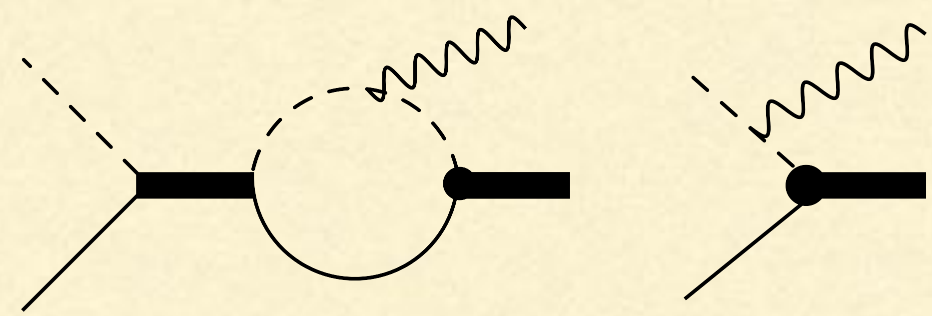
Capture to p-wave halo in EFT

Hammer & DP, NPA (2011)

- At LO: p-wave 1n halo described solely by its ANC and binding energy

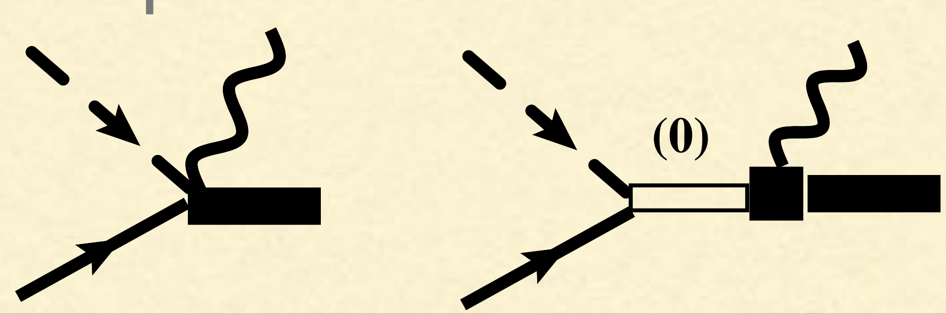
$$u_1(r) = A_1 \exp(-\gamma_1 r)\left(1 + \frac{1}{\gamma_1 r}\right)$$

- Capture to the p-wave state proceeds via the one-body E1 operator: "external direct capture"



$$\mathrm{E1} \propto \int_0^\infty dr\, u_0(r) r u_1(r); \quad u_0(r) = 1 - \frac{r}{a}$$

- NLO: piece of the amplitude representing capture at short distances, represented by a contact operator⇒there is an LEC that must be fit



NLO for ^{7}Be(p,γ)

Zhang, Nollett, Phillips, PRC (2014)
cf. Ryberg, Forssen, Hammer, Platter, EPJA (2014)
Zhang, Nollett, Phillips, PLB (2015); PRC (2018)

- LO calculation: ISI in S=2 & S=1 into p-wave bound state. Scattering wave functions are linear combinations of Coulomb wave functions F_0 and G_0. Bound state wave function=the appropriate Whittaker function

- We also incorporate a low-lying excited state ($1/2^-$) in ^{7}Be

- NLO: piece of the amplitude representing capture at short distances, represented by a contact operator⇒there is an LEC that must be fit

$$S(E) = \quad f(E) \sum_s C_s^2 \left[\left| \mathcal{S}_{\rm EC}\left(E;\delta_s(E)\right) + \overline{L}_s \mathcal{S}_{\rm SD}\left(E;\delta_s(E)\right) + \epsilon_s \mathcal{S}_{\rm CX}\left(E;\delta_s(E)\right) \right|^2 + |\mathcal{D}(E)|^2 \right]$$

Four parameters at LO; five more at NLO

- ANCs in $^5P_2$ and $^3P_2$: A_{5P2} and A_{3P2}
- Scattering lengths and effective ranges in both $^5S_2$ and $^3S_1$: a_2, r_2 and a_1, r_1
- Core excitation: determined by ratio of ^{8}B couplings of ^{7}Be*p and ^{7}Be-p states: ϵ_1
- LECs associated with contact interaction, one each for S=1 and S=2: L_1 and L_2

Extrapolation to zero energy

Zhang, Nollett, DP, PLB, 2015; arXiv:1708.04017

$$\mathrm{pr}\left(\bar{F}|D;T;I\right) = \int \mathrm{pr}\left(\vec{g},\{\xi_i\}|D;T;I\right)\delta(\bar{F}-F(\vec{g}))d\xi_1\ldots d\xi_5 d\vec{g}$$

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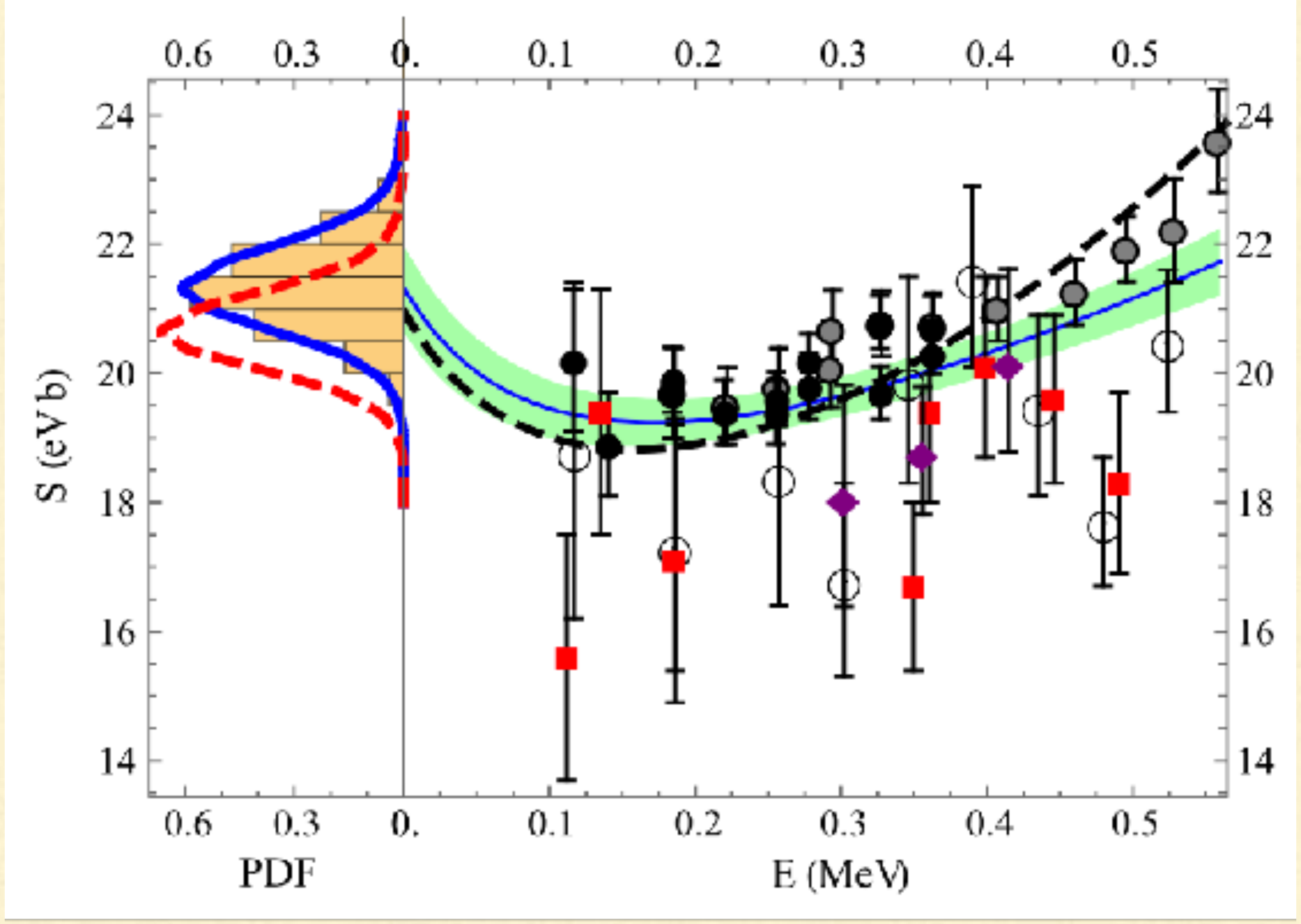

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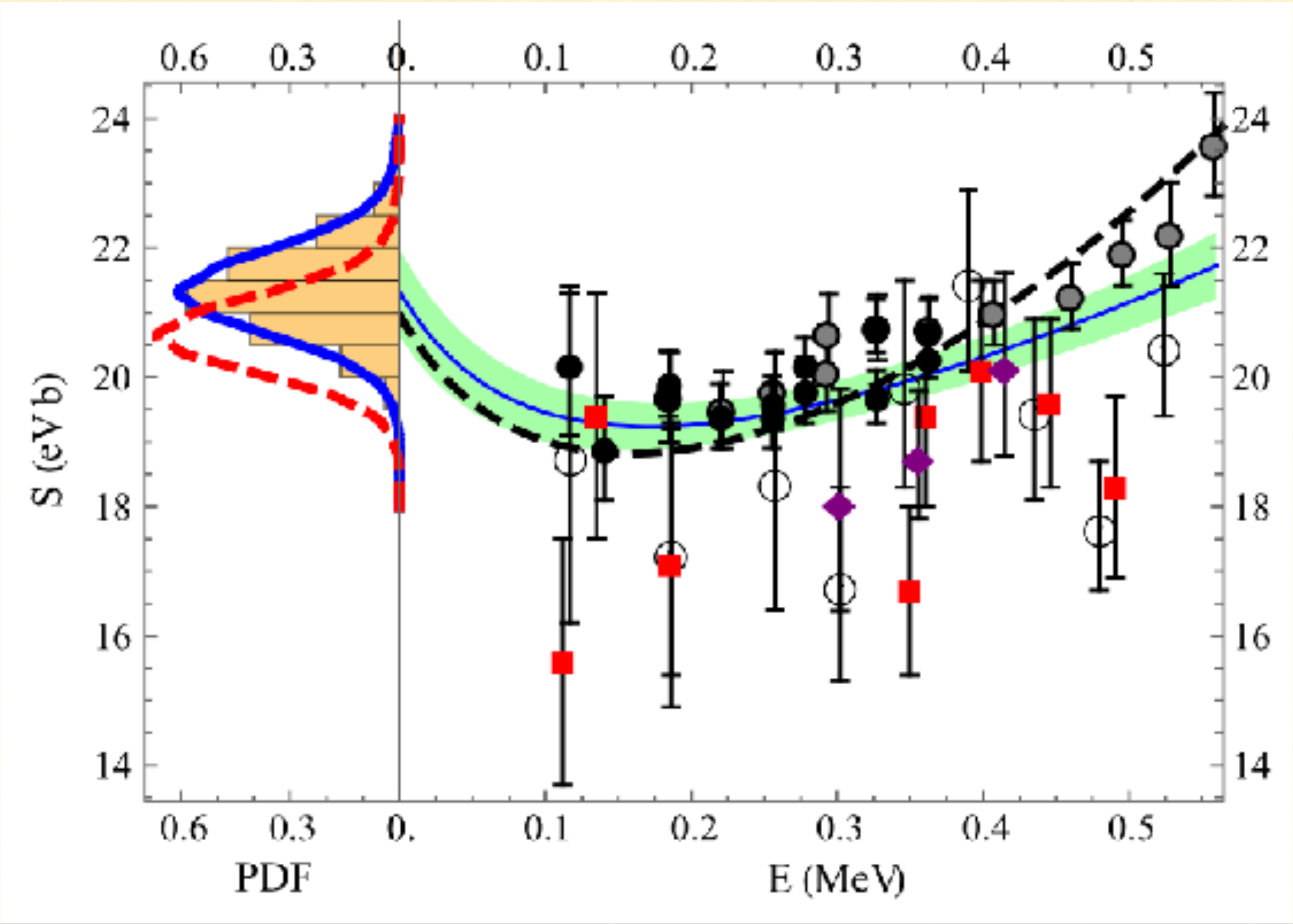

$$S(0) = 21.33^{+0.66}_{-0.69}\ \mathrm{eV\ b}$$

No N^{2}LO corrections

Also assessed impact of N^{3}LO contact operator

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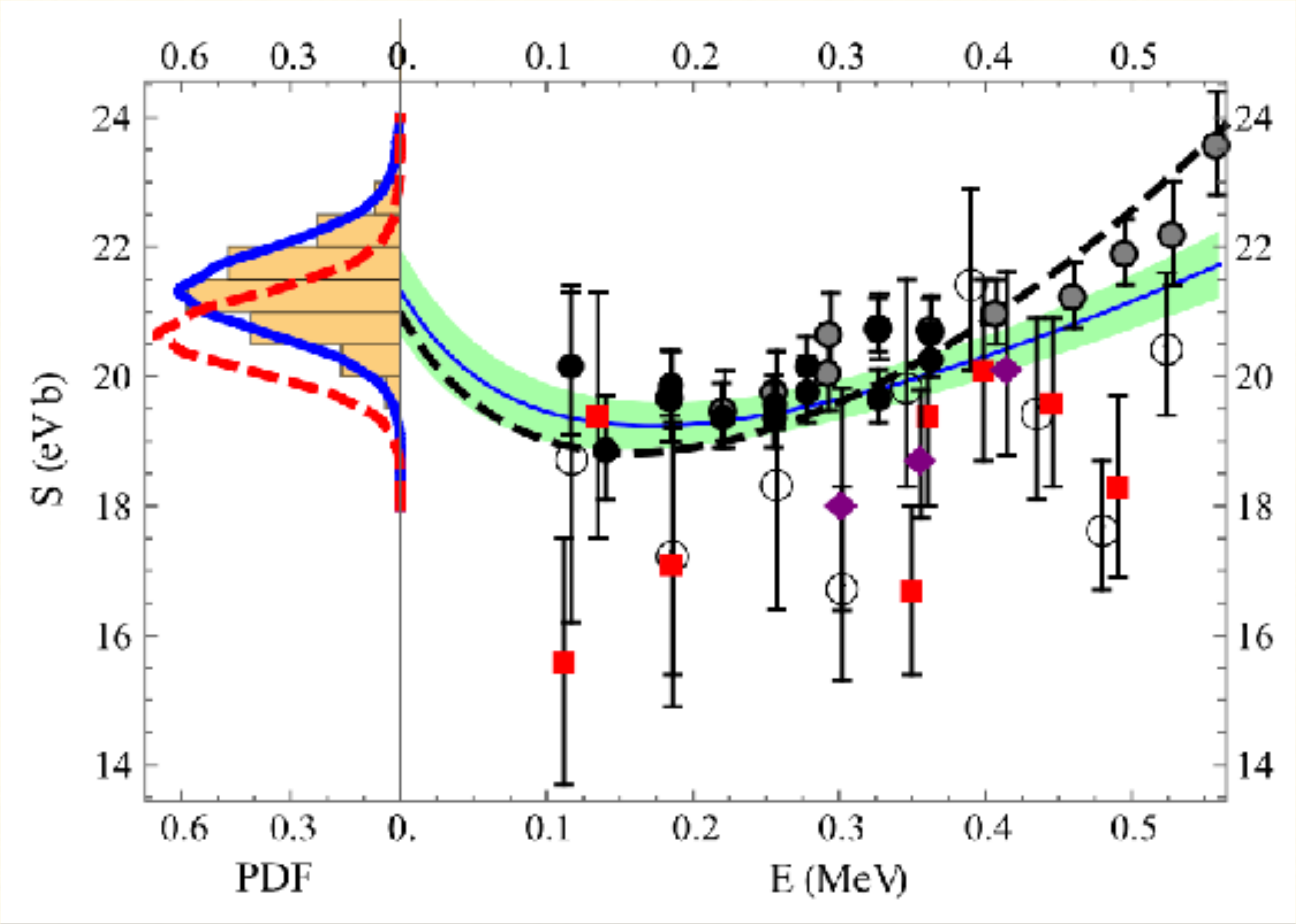

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Some remaining uncertainty due to ^{8}B S_{1p}

Uncertainty reduced by factor of two: model selection

Ongoing work along these lines

- Simultaneous fit to ^{7}Be+p scattering data: requires inclusion of resonances (TRIUMF experiment)

  Brown, Hale, Paris
  Poudel, Zhang, DP

- Same techniques applied to ^{3}He(^{4}He,γ)

  Vaghani, Higa, Rupak
  Zhang, Nollett, DP

- Coulomb dissociation: better reaction theory and connection to *ab initio* structure

  Capel, Hammer, DP

- Rotational states as explicit degrees of freedom

  Coello Pérez, Papenbrock
  Alnamlah, Coello Perez, DP

- Gaussian process models for ChiEFT truncation errors

  Melendez, Furnstahl, DP, Wesolowski

- ChiEFT truncation errors in nuclear & neutron matter

  Drischler, Melendez, Furnstahl, DP

- Parameter estimation for 3NFs

One thing is certain….

The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities.

It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication in *Physical Review A* without a detailed discussion of the uncertainties involved in the measurements….

The question is to what extent can the same high standards be applied to papers reporting the results of theoretical calculations…..There are many cases where it is indeed not practical to give a meaningful error estimate for a theoretical calculation….However, there is a broad class of papers where estimates of theoretical uncertainties can and should be made.

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1. If the authors claim high accuracy, or improvements on the accuracy of previous work.
2. If the primary motivation for the paper is to make comparisons with present or future high precision experimental measurements.
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Physical Review A Editorial, 29 April 2011

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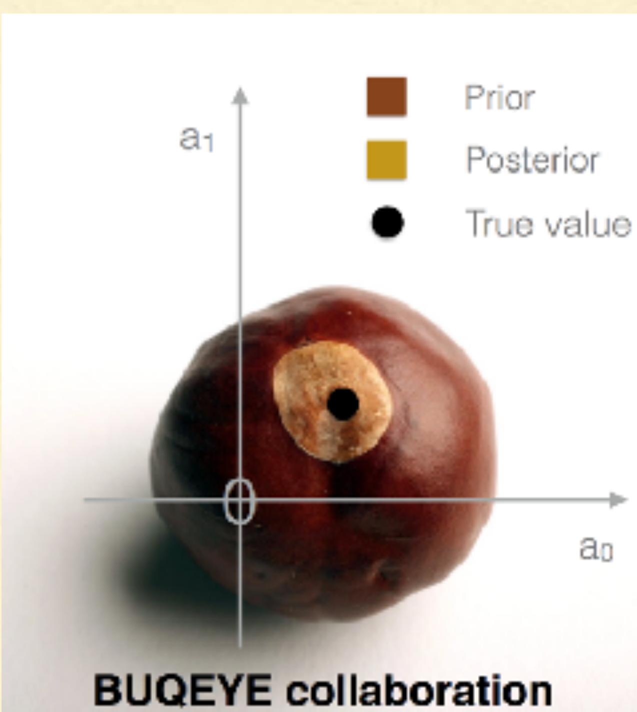

Bayesian Uncertainty Quantification:
Errors for Your EFT

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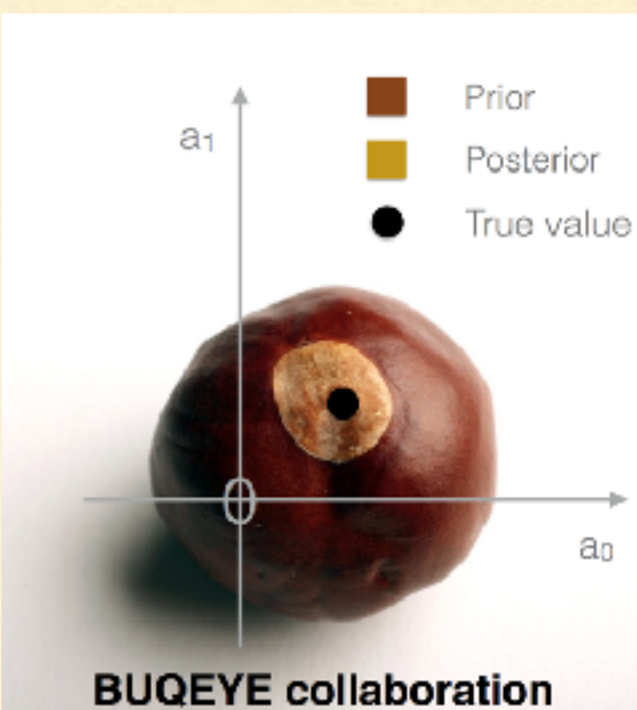

Bayesian Uncertainty Quantification:
Errors for Your EFT

EMMI workshop: ISNET-6, Uncertainty Quantification at the Extremes

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- “Halo effective field theory constrains the solar $^{7}Be + p \rightarrow {}^{8}B + \gamma$ rate”, X. Zhang, K. Nollett, D. R. Phillips, Phys. Lett. **B751**, 535 (2015); “Models, measurement and EFT: proton capture on ^{7}Be at NLO”, Phys. Rev. C **98**, 034616 (2018).
- “Effective field theory for halo nuclei”, H.-W. Hammer, C. Ji, D. R. Phillips, Topical Review for J. Phys. G **44**, 103002 (2017).

Backup Slides

A Generic EFT

$$g(x) = \sum_{i=0}^{k} \mathcal{A}_i(x) x^i$$

$$x = \frac{p}{\Lambda_b}$$

A Generic EFT

- Suppose we are interested in a quantity as a function of a momentum, p, that is small compared to some high scale, Λ_b.

- EFT expansion for quantity is $$g(x) = \sum_{i=0}^{k} \mathcal{A}_i(x) x^i$$

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$$\mathcal{A}_i(x) = a_i(\mu) + f_i(x, \mu) \qquad a_i, f_i = \mathcal{O}(1) \text{ for } \mu \sim \Lambda_b, x \sim 1$$

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- $f_i(x, \mu)$ is a calculable function, that encodes IR physics at order i
- a_i is a low-energy constant (LEC): encodes UV physics at order i. Must be fit to data
- Complications: multiple light scales, multiple functions at a given order, skipped orders, ….

Bayes→Result

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- Bayes theorem: $\mathrm{pr}(\bar{c}|c_0, c_1, \ldots, c_k) = \dfrac{\mathrm{pr}(c_0, c_1, \ldots, c_k|\bar{c})\mathrm{pr}(\bar{c})}{\mathrm{pr}(c_0, c_1, \ldots, c_k)}$

$$= \mathcal{N}\mathrm{pr}(\bar{c})\Pi_{n=0}^{k}\mathrm{pr}(c_n|\bar{c})$$

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- Marginalization:

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) = \int_0^\infty d\bar{c}\,\mathrm{pr}(c_{k+1}|\bar{c})\mathrm{pr}(\bar{c}|c_0, c_1, \ldots c_k)$$

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- This is generic, but the integrals are simple in the case of "Prior A"

$$\mathrm{pr}(\bar{c}|c_0, c_1, \ldots, c_k) \propto \begin{cases} 0 & \text{if } \bar{c} < \max\{c_0, \ldots, c_k\} \\ 1/\bar{c}^{k+2} & \text{if } \bar{c} > \max\{c_0, \ldots, c_k\} \end{cases}$$

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) \propto \begin{cases} 1 & \text{if } c_{k+1} < c_{\max} \\ \left(\frac{c_{\max}}{c_{k+1}}\right)^{k+2} & \text{if } c_{k+1} > c_{\max} \end{cases}$$

I don't like THAT prior!

- Modify Set A to restrict cbar to a finite range, e.g. $A_{[0.25,4]}$
- Set B: give cbar a log-normal prior: $\mathrm{pr}(\bar{c}) = \frac{1}{\sqrt{2\pi}\bar{c}\sigma} e^{-(\log \bar{c})^2/2\sigma^2}$
- Set C: $\mathrm{pr}(c_n|\bar{c}) = \frac{1}{\sqrt{2\pi}\bar{c}} e^{-c_n^2/2\bar{c}^2}$; $\mathrm{pr}(\bar{c}) \propto \frac{1}{\bar{c}}\theta(\bar{c} - \bar{c}_<)\theta(\bar{c}_> - \bar{c})$
- Same formulas as before can be invoked. Now numerical.

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) = \int_0^\infty d\bar{c}\, \mathrm{pr}(c_{k+1}|\bar{c})\mathrm{pr}(\bar{c}|c_0, c_1, \ldots c_k)$$

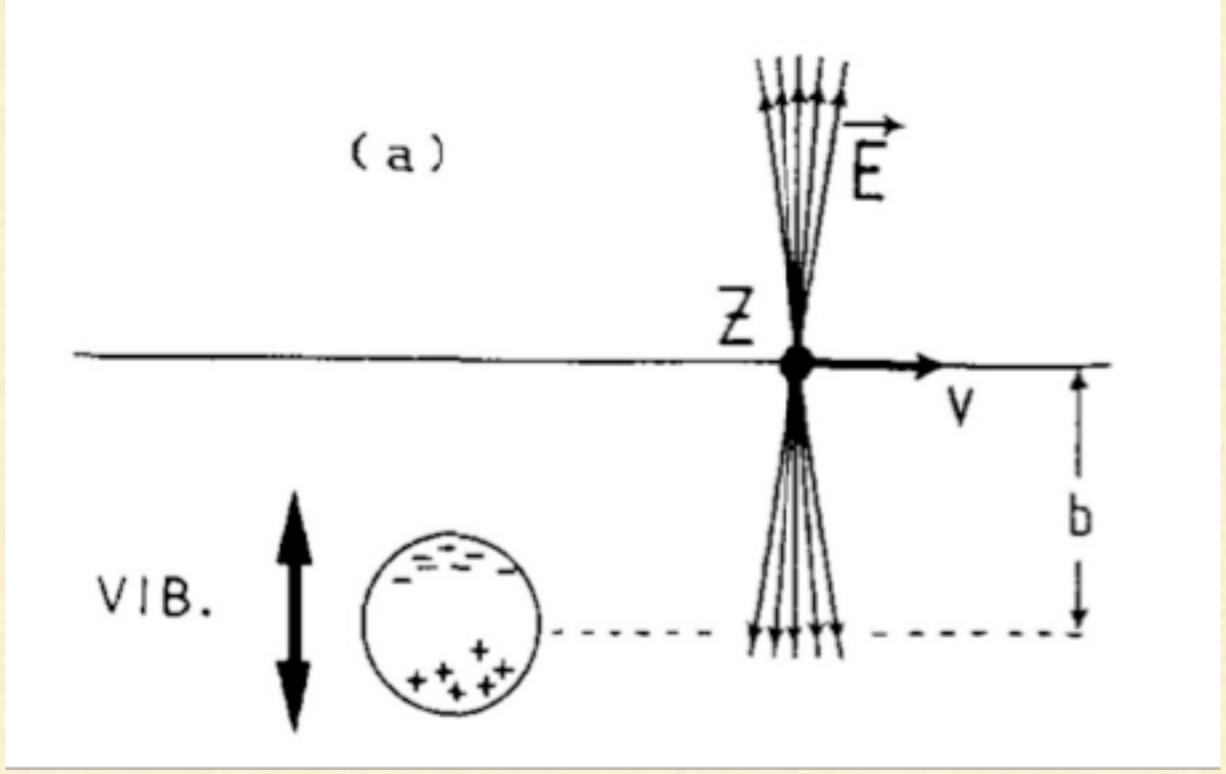
$$\mathrm{pr}(\bar{c}|c_0, c_1, \ldots, c_k) = \mathcal{N}\mathrm{pr}(\bar{c})\Pi_{n=0}^{k}\mathrm{pr}(c_n|\bar{c})$$

- You don't like these? Pick your own and follow the rules…
- First omitted term approximation

Coulomb dissociation of halo nuclei

Bertulani, arXiv:0908.4307

- Coulomb dissociation: collide halo nucleus (we hope peripherally) with a high-Z nucleus
- Do with different Z, different nuclear sizes, different energies to test systematics



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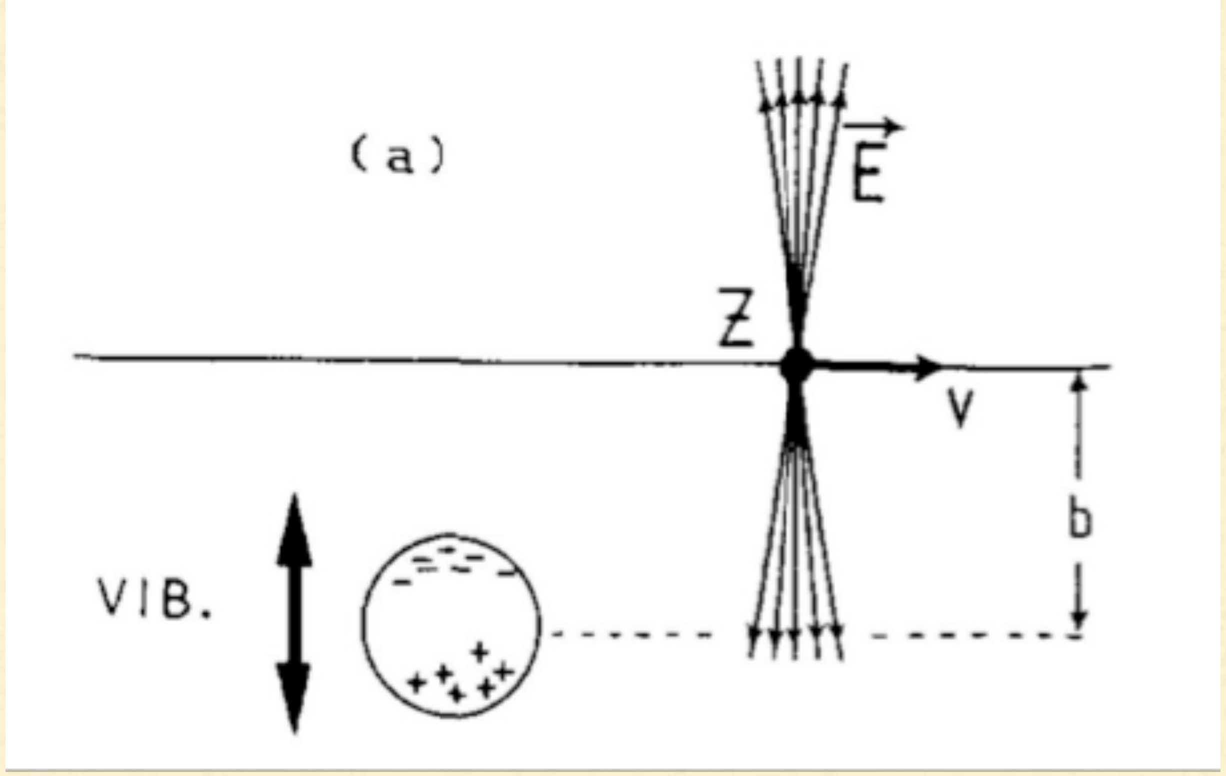

- Do with different Z, different nuclear sizes, different energies to test systematics

- Coulomb excitation dissociation cross section (p.v. b≫R_{target})

$$\frac{d\sigma_C}{2\pi b db} = \sum_{\pi L} \int \frac{dE_\gamma}{E_\gamma} n_{\pi L}(E_\gamma, b) \sigma_\gamma^{\pi L}(E_\gamma)$$

- $n_{\pi L}(E_\gamma, b)$ virtual photon numbers, dependent only on kinematic factors. Number of equivalent (virtual) photons that strike the halo nucleus.

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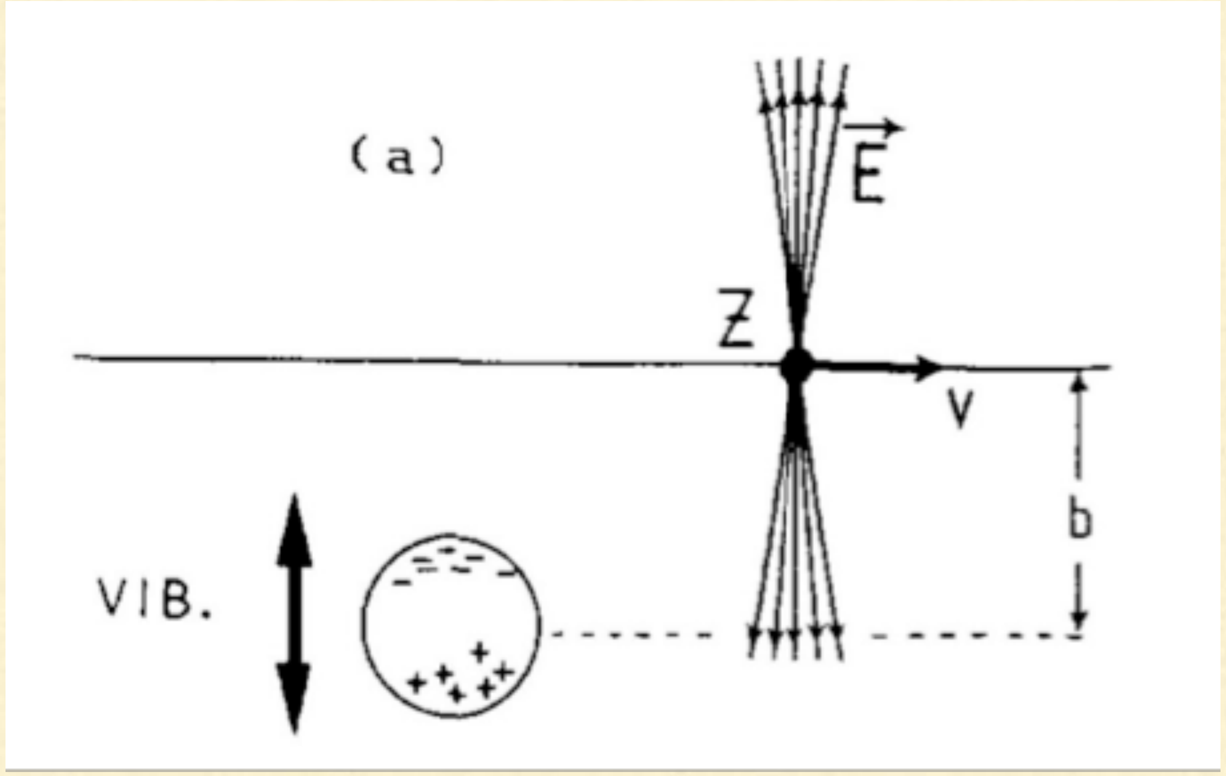

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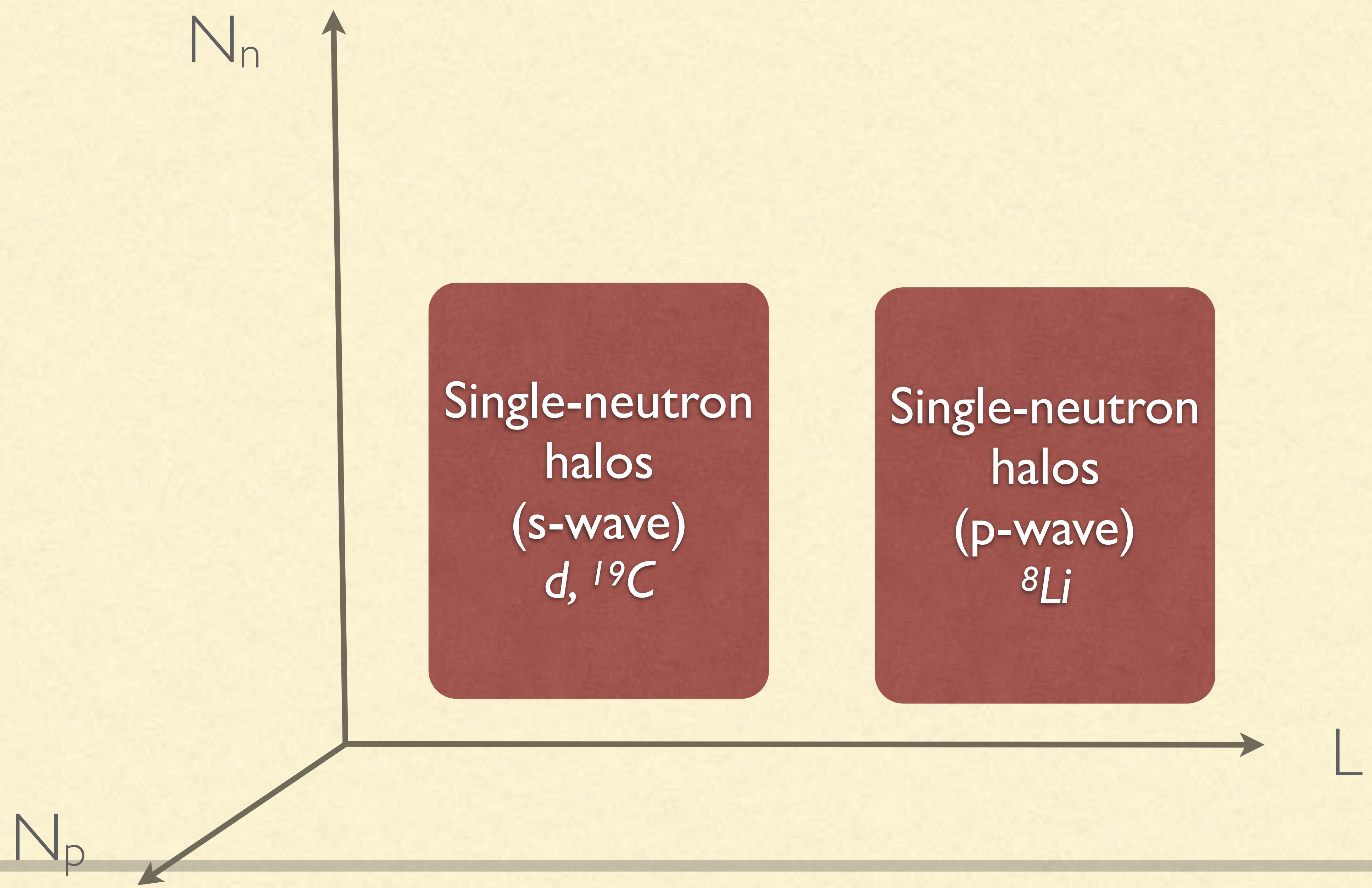
- $n_{\pi L}(E_\gamma, b)$ virtual photon numbers, dependent only on kinematic factors. Number of equivalent (virtual) photons that strike the halo nucleus.
- $\sigma_\gamma^{\pi L}(E_\gamma)$ can then be extracted: it's the (total) cross section for dissociation of the nucleus due to the impact of photons of multipolarity πL.

The multi-dimensional Halo EFT space



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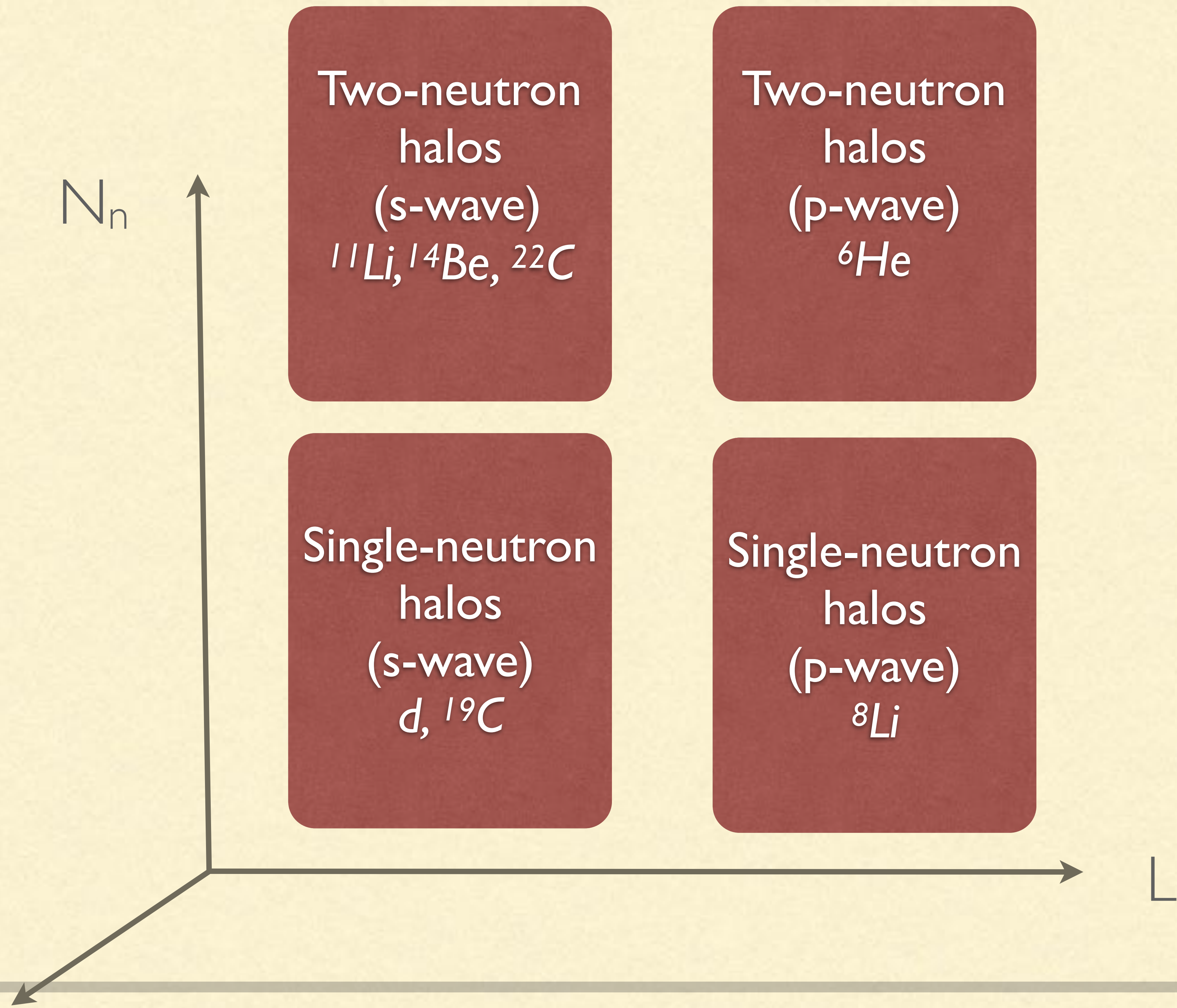

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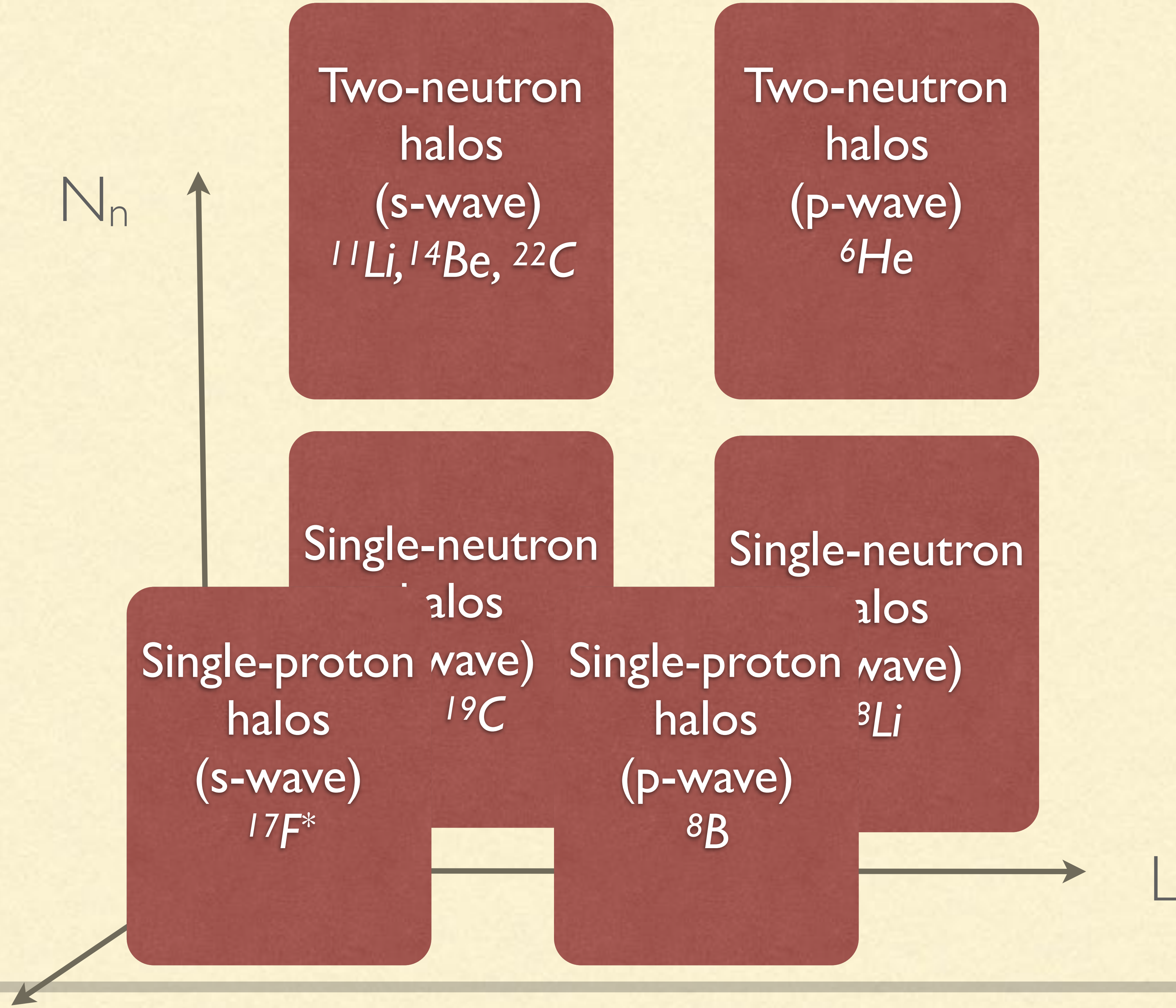

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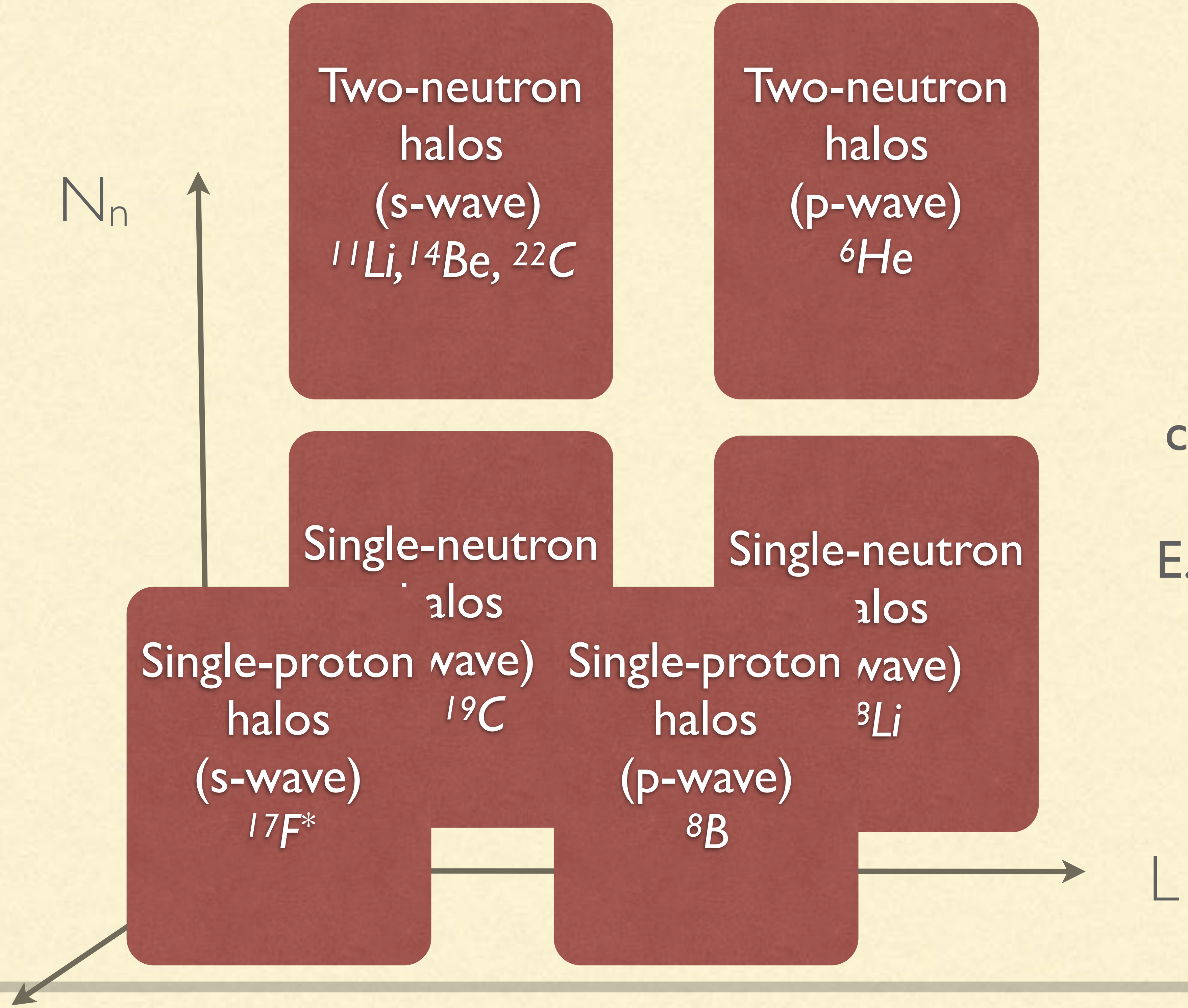

Plus complementary direction: N_α
E.g. ^{9}Be, $^{12}C^{*}$, etc.

Lagrangian for s- and p-wave states

s-wave: Kaplan, Savage, Wise (1998); van Kolck (1999); Birse, Richardosn, McGovern 1999)

p-wave: Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)

$$
\begin{aligned}
\mathcal{L} \quad = \quad & c^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) c + n^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) n \\
& + \sigma^\dagger \left[\eta_0 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_0 \right] \sigma + \pi_j^\dagger \left[\eta_1 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_1 \right] \pi_j \\
& - g_0 \left[\sigma n^\dagger c^\dagger + \sigma^\dagger n c \right] - \frac{g_1}{2} \left[\pi_j^\dagger (n \, i\overleftrightarrow{\nabla}_j \, c) + (c^\dagger \, i\overleftrightarrow{\nabla}_j \, n^\dagger) \pi_j \right] \\
& - \frac{g_1}{2} \frac{M - m}{M_{nc}} \left[\pi_j^\dagger \, i\overrightarrow{\nabla}_j \, (nc) - i\overleftrightarrow{\nabla}_j \, (n^\dagger c^\dagger) \pi_j \right] + \ldots ,
\end{aligned}
$$

- c, n: “core”, “neutron” fields. c: boson, n: fermion
- σ, π_j: S-wave and P-wave fields
- Minimal substitution generates leading EM couplings

One-slide p-wave review

$$\langle \mathbf{k} | t_1 | \mathbf{k}' \rangle = -\frac{6\pi}{m_R} \frac{\mathbf{k} \cdot \mathbf{k}'}{-\frac{1}{a_1} + \frac{1}{2} r_1 k^2 - i k^3}$$

Bethe (1949)

One-slide p-wave review

- For a short-ranged potential, if kR≲1:

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Wigner (1955); Hammer & Lee (2009); Nishida (2012)

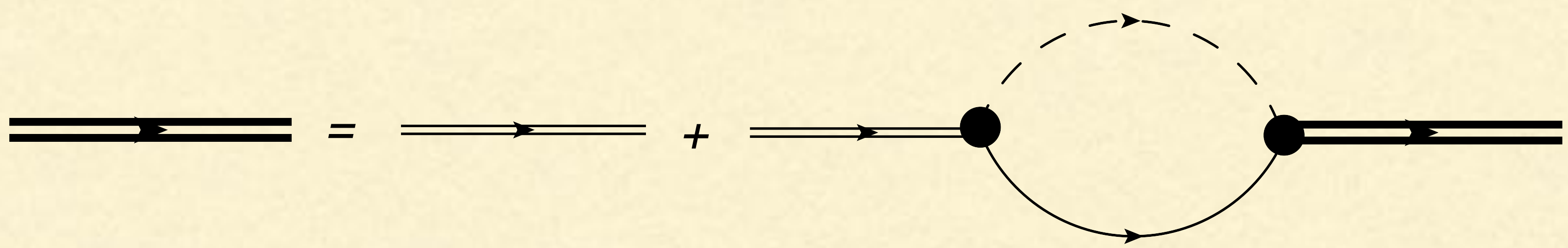
- So low-energy resonance/bound state would seem to have to arise due to cancellation between $-1/a_1$ and $1/2\ r_1 k^2$ terms.
- $a_1 \sim R/M_{lo}^2$ gives $k_R \sim M_{lo}$

Bedaque, Hammer, van Kolck (2003)

Dressing the p-wave state

Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)

- Proceed similarly for p-wave state as for s-wave state



$$D_\pi(p) = \frac{1}{\Delta_1 + \eta_1[p_0 - \mathbf{p}^2/(2M_{nc})] - \Sigma_\pi(p)}$$

- Here both Δ_1 and g_1 are mandatory for renormalization at LO

$$\Sigma_\pi(p) = -\frac{m_R g_1^2 k^2}{6\pi}\left[\frac{3}{2}\mu + ik\right]$$

- Reproduces ERE. But here (cf. s waves) cannot take $r_1=0$ at LO

A narrow p-wave resonance/bound state

Bertulani, Hammer, van Kolck (2002)

Bedaque, Hammer, van Kolck (2003)

A narrow p-wave resonance/bound state

- First EFT paper to do this assigned $a_1 \sim 1/M_{lo}^3$; $r_1 \sim M_{lo}$ Bertulani, Hammer, van Kolck (2002)
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- So, off resonance, $\mathrm{Re}[t^{-1}] > \mathrm{Im}[t^{-1}]$: phase shifts are $O(M_{lo}R)$ and scattering is perturbative away from resonance cf. Pascalutsa, DP (2003)

$$\langle \mathbf{k} | t_1 | \mathbf{k}' \rangle = -\frac{12\pi}{m_R r_1} \frac{\mathbf{k} \cdot \mathbf{k}'}{k^2 - k_R^2} \qquad k_R^2 = \frac{2}{a_1 r_1}$$

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- Resonance width is $\sim E_R \, k_R/r_1$, so it is parametrically narrow. Need to resum width if $k^2 - k_R^2$ gets small

P-wave FSI in γ_{E1} + ^{11}Be→^{10}Be + n

Typel & Baur, Phys. Rev. Lett. 93, 142502 (2004); Nucl. Phys. A759, 247 (2005); Eur. Phys. J. A 38, 355 (2008)

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$$k^3 \cot \delta_1 = -1/2\, r_1 (k^2 + \gamma_1^2) \Rightarrow \delta_1 \sim R_{core}/R_{halo} \text{ if } k \sim 1/R_{halo} \sim \gamma_1.$$

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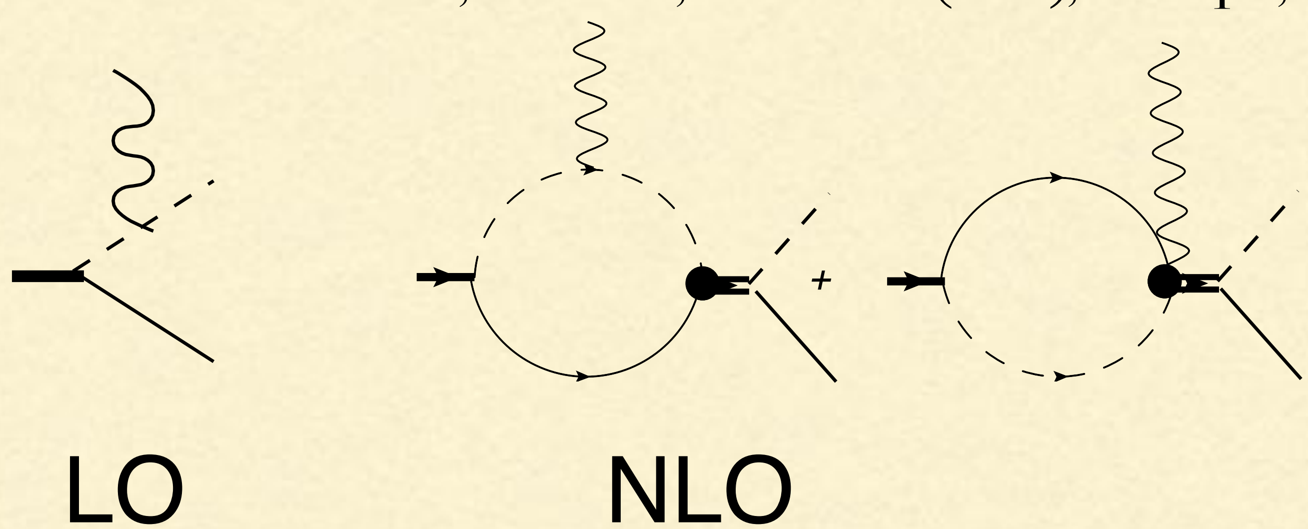

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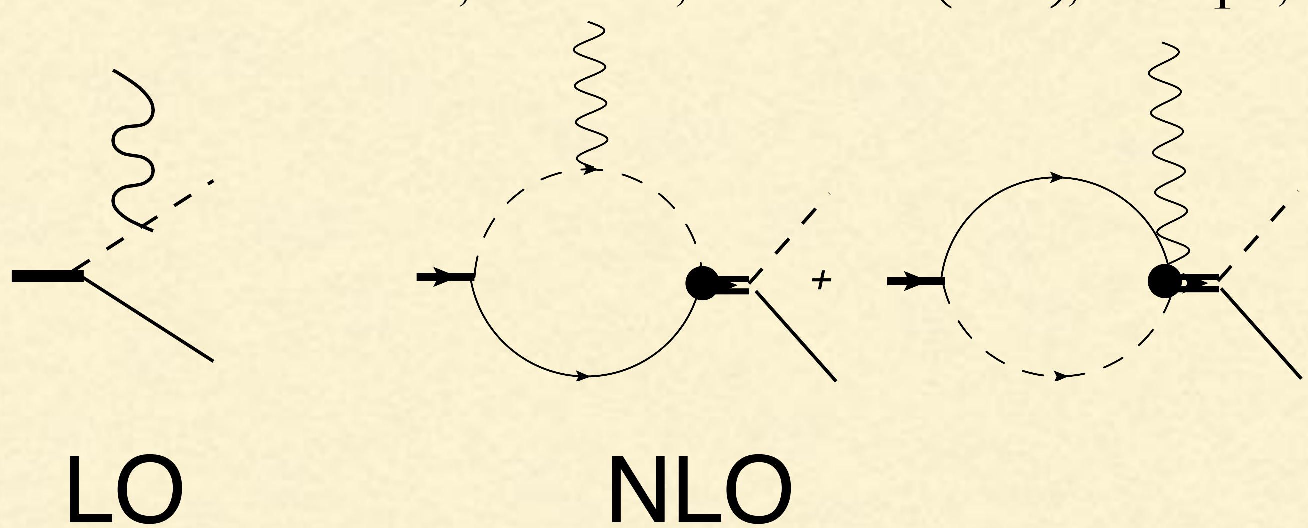
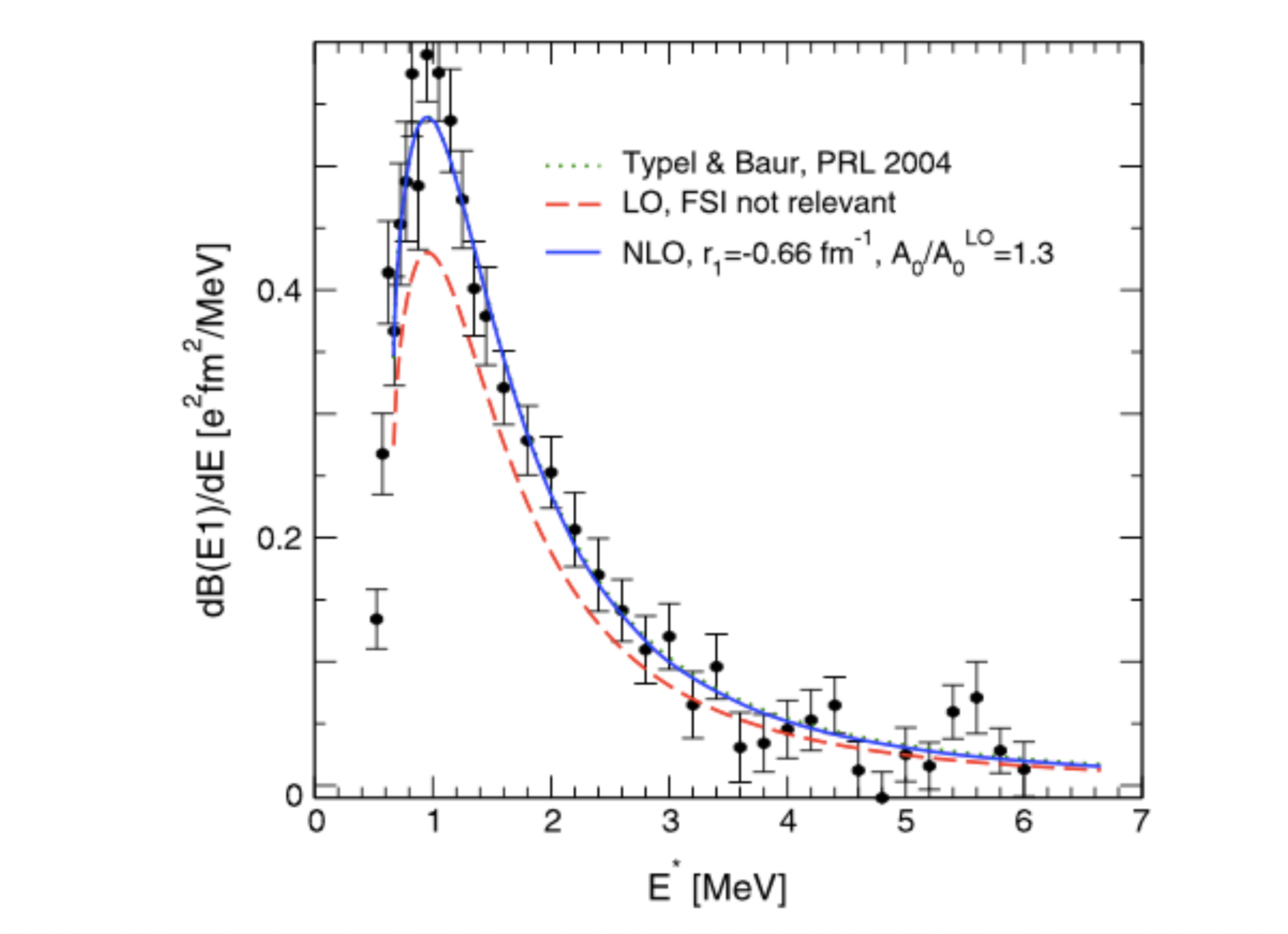


- Need both γ_1 and $r_1 \equiv A_1$ at NLO in this observable. A_0 also becomes a free parameter at NLO: fit it to Coulomb dissociation data

Coulomb dissociation of ^{11}Be: result



Data: Palit et al., 2003
Analysis: Hammer, Phillips. NPA, 2011

- Reasonable convergence
- Information on value of r_0 through fitting of A_0:

r_0=2.7 fm

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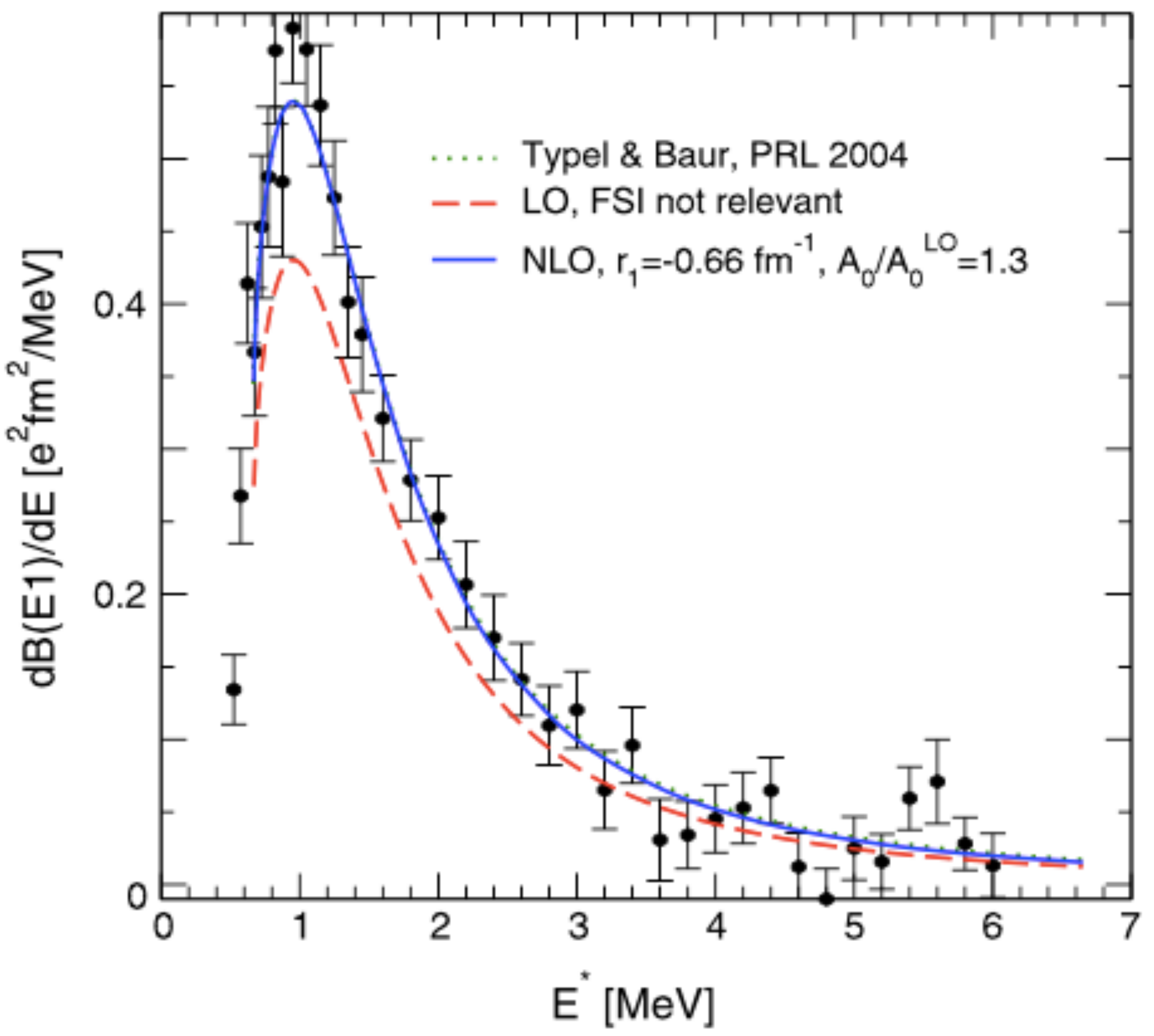

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- Here value of r_1 used to fit B(E1:1/2$^+$→1/2$^-$) works.

  r_1=-0.66 fm^{-1}

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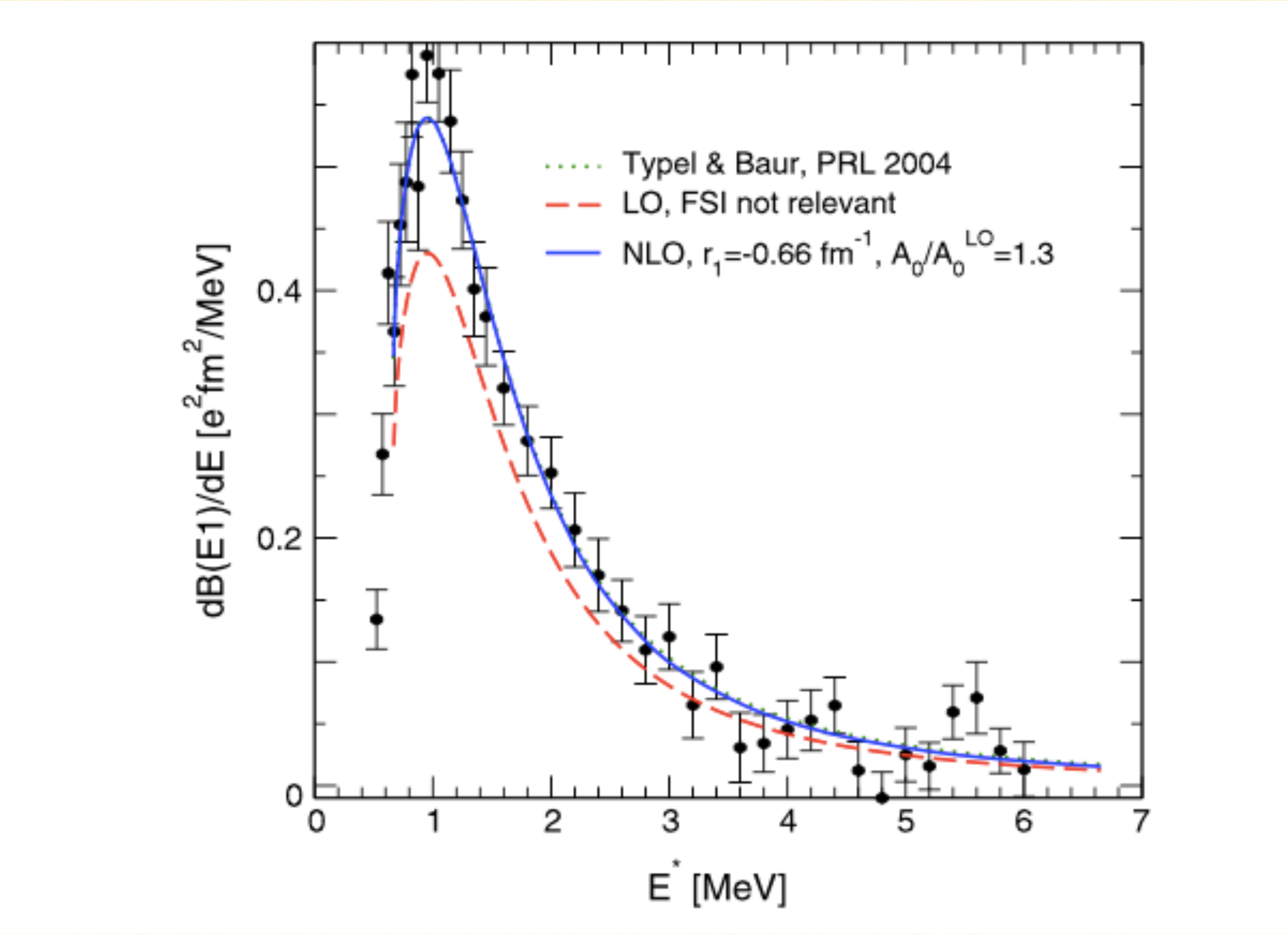

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$$\text{NLO: } (\langle r_c^2\rangle+\langle r_{Be}^2\rangle)^{1/2}=2.44 \text{ fm}$$

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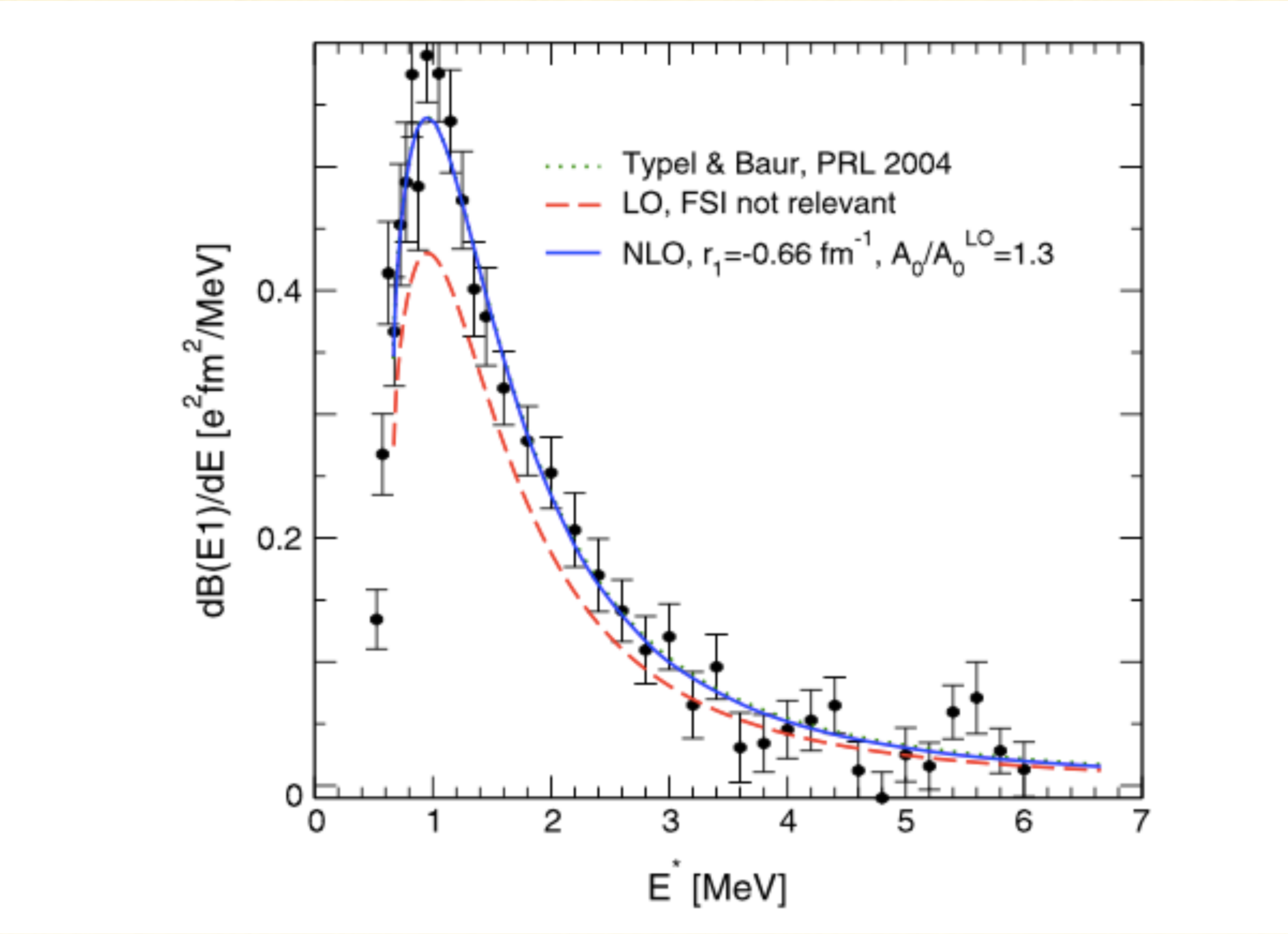

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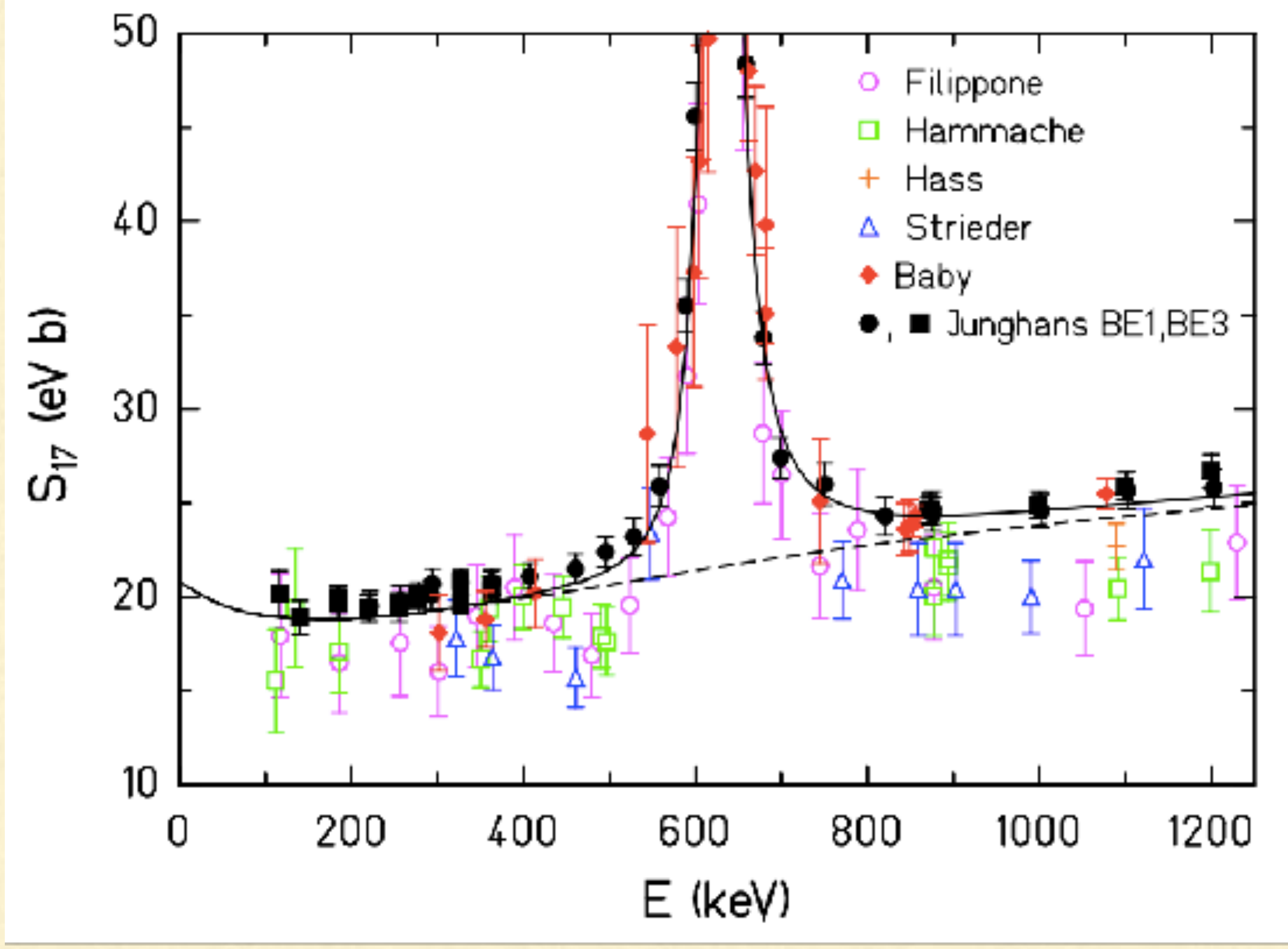
Use of ab initio input, e.g. for ANC?

Status as in "Solar Fusion II"

- Energies of relevance≈20 keV
- There dominated by ^{7}Be-p separations ~10s of fm
- Below narrow 1^+ resonance proceeds via s- and d-wave direct capture
- Energy dependence due to interplay of bound-state properties, Coulomb, strong ISI



- SF II central value used energy-dependence from Descouvemont's ab initio eight-body calculation. Errors from consideration of energy-dependence in a variety of "reasonable models"

Data situation

- 42 data points for 100 keV < $E_{c.m.}$ < 500 keV
  - Junghans (BE1 and BE3)
  - Fillipone
  - Baby
  - Hammache (1998 and 2001)
- CMEs
  - 2.7% and 2.3%
  - 11.25%
  - 5%
  - 2.2% (1998)
- Subtract M1 resonance: negligible impact at 500 keV and below
- Deal with CMEs by introducing five additional parameters, ξ_j

Building the pdf

- Bayes:

$$\mathrm{pr}\left(\vec{g}, \{\xi_i\}|D;T;I\right) = \mathrm{pr}\left(D|\vec{g}, \{\xi_i\};T;I\right)\mathrm{pr}\left(\vec{g}, \{\xi_i\}|I\right),$$

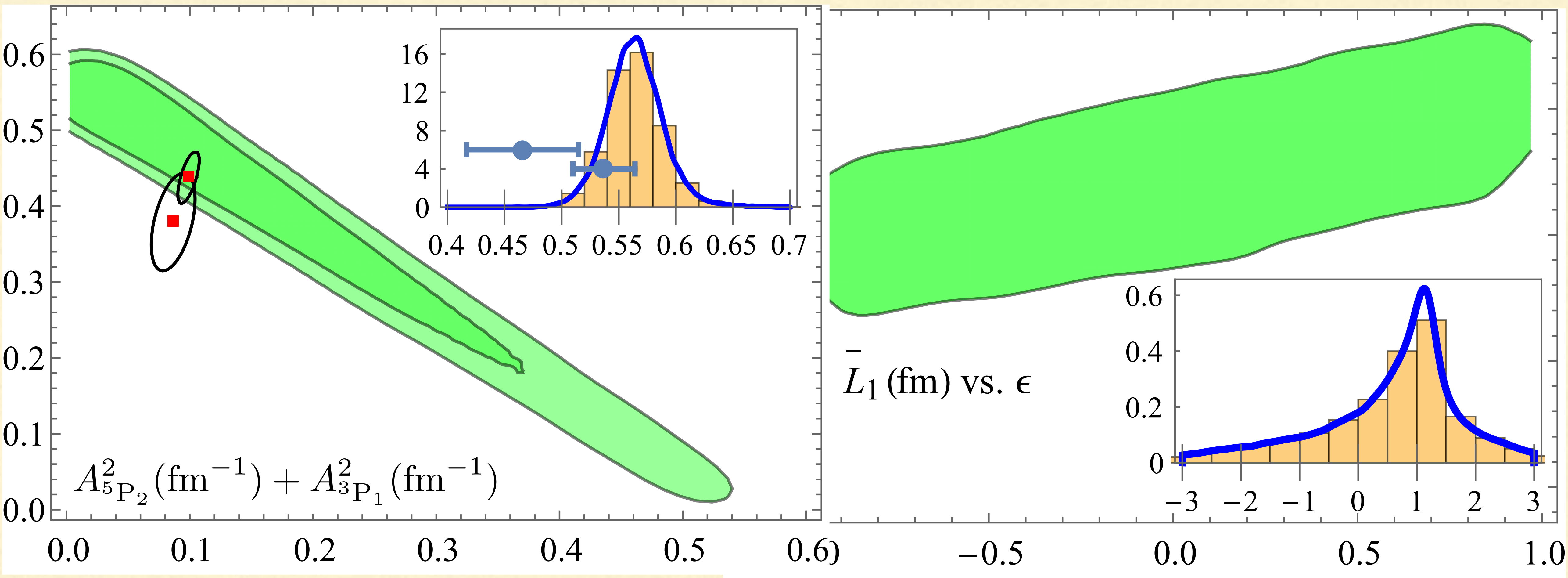
- First factor: likelihood

$$\ln \mathrm{pr}\left(D|\vec{g}, \{\xi_i\};T;I\right) = c - \sum_{j=1}^{N} \frac{\left[(1-\xi_j)S(\vec{g};E_j) - D_j\right]^2}{2\sigma_j^2},$$

- Second factor: priors
  - Independent gaussian priors for ξ_j, centered at zero and with width=CME
  - Gaussian priors for $a_{S=1}$ and $a_{S=2}$, based on Angulo et al. measurement
  - All other EFT parameters assigned flat priors, corresponding to natural ranges
- No s-wave resonance below 600 keV

Marginalizing→pdfs

$$\text{pr}\left(g_1, g_2 | D; T; I\right) = \int \text{pr}\left(\vec{g}, \{\xi_i\} | D; T; I\right)\, d\xi_1 \ldots d\xi_5 dg_3 \ldots dg_9$$



- ANCs are highly correlated but sum of squares strongly constrained
- One spin-1 short-distance parameter: $0.33\ \bar{L}_1/(\mathrm{fm}^{-1}) - \epsilon_1$