Electromagnetic Transition of Hyperons

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Electromagnetic Structure of Strange Baryons

at GSI, Darmstadt

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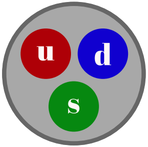
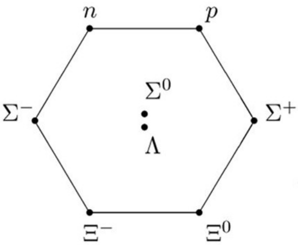
Focus on:

Σ^0 and Λ hyperon

$$\Sigma^0: \quad I(J^P) = 1(\frac{1}{2}^+)$$

$$\Lambda: \quad I(J^P) = 0(\frac{1}{2}^+)$$





1. The electromagnetic Sigma-to-Lambda hyperon transition form factors at low energies

   Granados/Leupold/Perotti, Eur.Phys.J. A53, 117 (2017)

2. Constraining P and CP violation in the main decay of the neutral Sigma hyperon

   Nair/Perotti/Leupold, accepted for Phys.Lett.B

Outline

Motivation – part I

To further investigate the structure of matter one can try the phenomenal combo:

Electromagnetism
Form Factors (FF)

+

Strangeness
Hyperons (Y)

Hyperons are not stable!

Experimental information about hyperon form factors is rather limited

- Hyperon FFs are more easily accessible in the time-like region ($q^2 > 0$) for high and low energies via:
  - $e^+e^- \rightarrow Y_1 \bar{Y}_2$ reactions (BESIII)
  - $Y_1 \rightarrow Y_2\, e^+e^-$ Dalitz decays (PANDA, HADES)

Focus on:

Electric and magnetic transition form factor Σ^0 to Λ

- accessible by high-precision measurement of the decay $\Sigma^0 \to \Lambda e^+ e^-$ (possible @FAIR)

Experimental feasibility:

- FFs are functions of dilepton invariant mass q^2

  $\to$ not very large range available, $q^2 < (m_{\Sigma^0} - m_\Lambda)^2 \approx (77\,\mathrm{MeV})^2$

  $\to$ high experimental precision required

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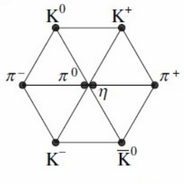
Ingredients

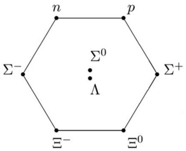
- Chiral Perturbation Theory (EFT) $\rightarrow$ pseudo-Goldstone bosons



$\rightarrow$ systematically improvable, reliable uncertainty estimate

Ingredients

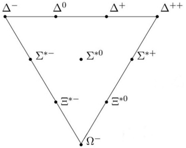
- Chiral Perturbation Theory (EFT) $\rightarrow$ pseudo-Goldstone bosons
  - Include baryon octet



Kubis/Meißner, Eur. Phys. J. C 18, 747 (2001)

Ingredients

- Chiral Perturbation Theory (EFT) $\rightarrow$ pseudo-Goldstone bosons
  - Include baryon octet
  - Include baryon decuplet



Jenkins/Manohar, Phys.Lett. B259, 353 (1991)

Pascalutsa/Vanderhaeghen/Yang, Phys.Rept. 437, 125 (2007)

Ledwig/Camalich/Geng/Vacas, Phys.Rev. D 90, 054502 (2014)

Ingredients

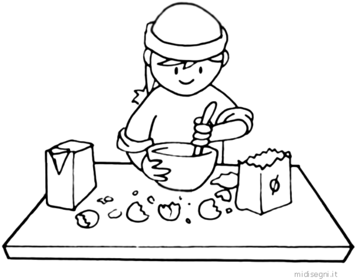
- Chiral Perturbation Theory (EFT) $\rightarrow$ pseudo-Goldstone bosons
  - Include baryon octet
  - Include baryon decuplet

Q: What about other hadronic states, e.g. vector mesons?

A: It's unclear how to treat them in a systematic, model-independent way.

- ρ meson experimentally shows up in pion form factor and p-wave pion phase shift (well-known quantities)

Dispersion theory allows to combine these ingredients:



EFT + data

i.e. ChPT and extension thereof + F_π^V

A few words about dispersion relations

Consider the S-matrix $S = \mathbb{1} + iT$

Unitarity requires

$$SS^\dagger = \mathbb{1} + i(T - T^\dagger) + |T|^2 = \mathbb{1}$$

which implies that

$$2\,\mathrm{Im}\,T = |T|^2 \quad \longrightarrow \quad \boxed{\mathrm{Im}\,T_{A\to B} = \frac{1}{2}\sum_X T_{A\to X}\,T^\dagger_{X\to B}}$$ Optical theorem

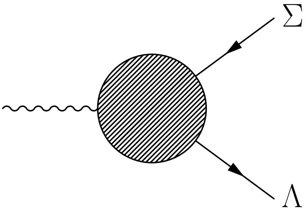
$\rightarrow$ consider only most relevant intermediate states X

Analyticity requires

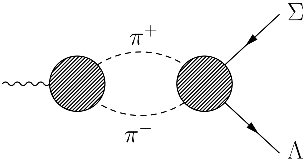
$$\boxed{T(q^2) = T(0) + \frac{q^2}{\pi}\int_{-\infty}^{+\infty} \mathrm{d}s\, \frac{\mathrm{Im}\,T(s)}{s\,(s - q^2 - i\epsilon)}}$$

$\rightarrow$ get the whole amplitude T from its imaginary part

Our paper in figures...



$\Sigma - \Lambda$ transition form factor



Two-pion exchange: dominant contribution at low energies

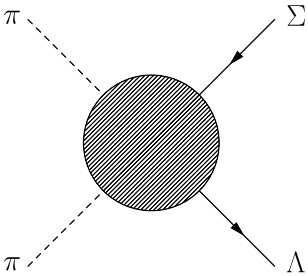
Need:

- pion vector form factor F_π^V ✓
- $\Sigma\Lambda\pi\pi$ scattering amplitude $A_{\Sigma\Lambda\pi\pi}$

Granados/Leupold/Perotti, Eur.Phys.J. A53, 117 (2017)

Alarcon/Blin/Vacas/Weiss, Nucl.Phys. A964, 18 (2017)

Let's take a closer look at $A_{\Sigma\Lambda\pi\pi}$



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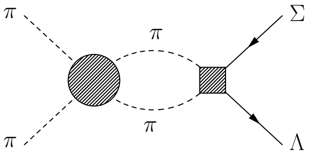
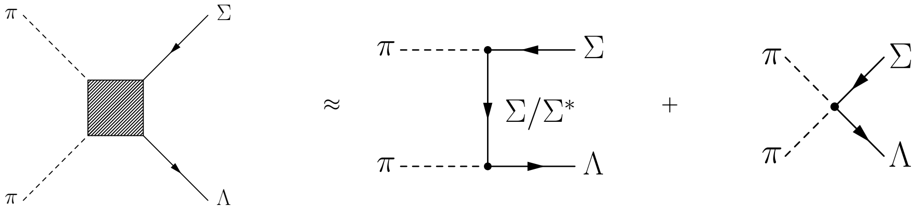


pion rescattering (circle) + a part containing left-hand cuts and polynomials (box)



- no pion-hyperon scattering data available :(

  → use three-flavor baryon ChPT at LO and NLO

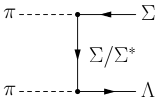
  → include decuplet states

Parameters

Baryon exchange diagrams from LO BChPT:

- octet baryon → Born diagrams

  vertices: $\Sigma\Lambda\pi$ and $\Sigma\Sigma\pi$ (F and D parameters)



- decuplet baryon

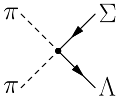
  vertices: $\Sigma^*\Lambda\pi$ and $\Sigma^*\Sigma\pi$ (h_A parameter, $2.2 < h_A < 2.4$)

Four-point diagram from NLO BChPT:

- vertex $\Sigma\Lambda\pi\pi$ (b_{10} parameter, $0.85 < b_{10} < 1.35\,\mathrm{GeV}^{-1}$)

  → b_{10} is not very well known!

  → b_{10} is directly related to magnetic transition radius of $\Sigma - \Lambda$



Results: TFF at photon point

Electric charge, magnetic moment and electric(magnetic) radius of $\Sigma - \Lambda$ transition

Λ [GeV]	quantity	Born	NLO	NLO+res	χPT
1	$G_M(0)$	-0.438	5.55	2.58	1.98 (exp.)
2		-0.65	5.98	2.66	
1	$\langle r_M^2 \rangle$ [GeV^{-2}]	0.453	33.7	17.9	18.6
2		0.613	35.2	18.8	
1	$G_E(0)$	-0.432	-	0.0026	0
2		-0.562	-	-0.031	
1	$\langle r_E^2 \rangle$ [GeV^{-2}]	-3.13	-	0.866	0.773
2		-2.91	-	1.044	

Comparison to χPT (Kubis, Meißner 2001), using $h_A = 2.3$, $b_{10} = 1.1\ \text{GeV}^{-1}$

- Born terms alone are insufficient to produce reasonable results

  $\rightarrow$ need NLO and decuplet-resonance exchange

- varying the cut off Λ has rather small impact (10% at most)

However...

- uncertainty related to h_A moderate

quantity	$h_A = 2.2$	$h_A = 2.4$	χPT
$G_M(0)$	2.94	2.36	1.98 (exp.)
$\langle r_M^2 \rangle$ [GeV^{-2}]	20.2	17.3	18.6
$G_E(0)$	-0.076	0.016	0
$\langle r_E^2 \rangle$ [GeV^{-2}]	0.708	1.40	0.773

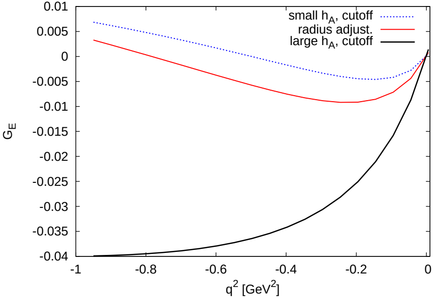
Comparison to χPT using $\Lambda = 2$ GeV and $b_{10} = 1.1$ GeV^{-1}

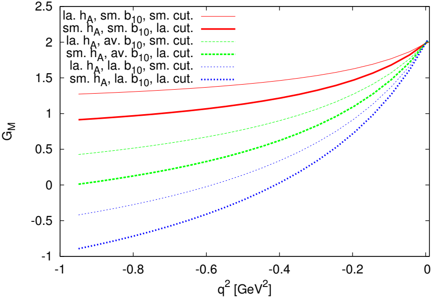
- uncertainty related to b_{10} sizable

b_{10}	quantity	NLO	NLO+res	χPT
0.85	$G_M(0)$	4.47	1.15	1.98 (exp.)
1.35		7.49	4.17	
0.85	$\langle r_M^2 \rangle$ [GeV^{-2}]	27.4	10.9	18.6
1.35		43.1	26.7	

Comparison to χPT using $\Lambda = 2$ GeV and $h_A = 2.3$

Results: TFF





- G_E close to zero at low energies
- G_M is very sensitive to variations of b_{10}
  $\rightarrow$ need input from experiment

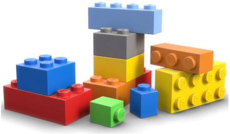
Summary - part I

So...What have we learned so far?

- Dispersion theory relates the low-energy electromagnetic $\Sigma\Lambda$ TFF with F_π^V
- Relativistic NLO BChPT determines $A_{\Sigma\Lambda\pi\pi}$
  - Inclusion of decuplet baryons essential to obtain reasonable results!
- Electric TFF very small in the whole low-energy region
- Magnetic TFF depends strongly on a poorly known LEC of the NLO Lagrangian (b_{10})

  $\rightarrow$ can be determined from measurement of the magnetic transition radius (@FAIR)

  $\rightarrow$ obtain predictive power



THE END

THE END

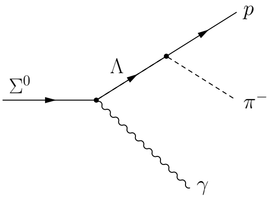
... of part I

Motivation – part II

- Look for **baryonic** CP violation

  $\rightarrow$ Might give us some insight into the origin of:

  - baryon-antibaryon asymmetry
  - strong CP problem



Strong CP problem in a nutshell

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \bar{q}(i\not{D} - \mathcal{M})q + \theta \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

- QCD doesn't seem to break CP symmetry (in contrast to the electroweak theory)
  $\rightarrow$ *i.e.*, no experimental evidence for strong CP violation

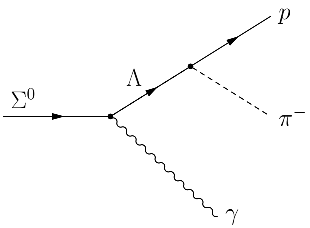
Our conservative approach:

1. allows strong CP violation via θ-term
2. stays faithful to the Standard Model (SM)

Focus on:

$\Sigma^0 \rightarrow \Lambda\gamma$ BR: 100%

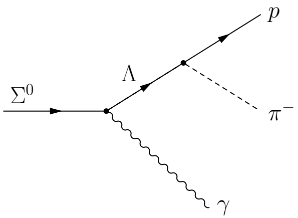
$\Lambda \rightarrow p\pi$ BR: 64%



Two-step decay chain

Our starting point:

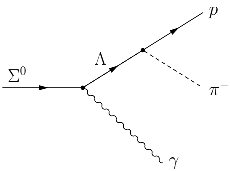
1. $\Sigma^0 \to \Lambda\gamma$
   - electromagnetic decay
   - parity conserving magnetic dipole transition moment
   - parity violating electric dipole transition moment
2. $\Lambda \to p\pi$
   - weak decay
   - self-analyzing



Asymmetry in the angular distribution!

Decay asymmetries

1. $\Sigma^0 \to \Lambda\gamma$
   - $\mathcal{M}_1 = \bar{u}_\Lambda \left(a\,\sigma_{\mu\nu} - b\,\sigma_{\mu\nu}\gamma_5\right) u_{\Sigma^0}\,(-i)q^\nu\,\epsilon^{\mu *}$
   - a and b related to transition moments κ_M and $d_{\Sigma\Lambda}$
   - final-state interaction leads to additional phase shift δ_F
   - $\alpha_{\Sigma^0} := \frac{2\mathrm{Re}(a^* b)}{|a|^2+|b|^2}$
2. $\Lambda \to \mathrm{p}\pi$
   - $\mathcal{M}_2 = \bar{u}_p(\mathcal{A} - \mathcal{B}\gamma_5)\,u_\Lambda$
   - $\mathcal{A}$ and $\mathcal{B}$ related to s- and p-wave
   - $\alpha_\Lambda := \frac{2\mathrm{Re}(s^* p)}{|s|^2+|p|^2}$



To reveal CP violation compare particle and antiparticle decays

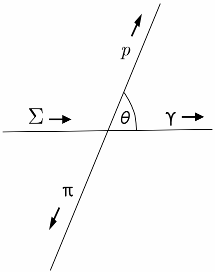
Three-body decay

$$\mathcal{M}_3 = \bar{u}_p\,(\mathcal{A} - \mathcal{B}\gamma_5)\,(\not{p}_\Lambda + m_\Lambda)\,(a\,\sigma_{\mu\nu} - b\,\sigma_{\mu\nu}\gamma_5)\,u_{\Sigma^0}\,(-i)p_\gamma^\nu\,\epsilon^{\mu *}\,D_\Lambda(m_{12}^2)$$

- Λ resonance propagator

  $D_\Lambda(s) := (s - m_\Lambda^2 + i m_\Lambda \Gamma_\Lambda)^{-1}$

- Λ is long-lived $\rightarrow$ displaced vertex!



in the Λ rest frame

$$\frac{\mathrm{d}\Gamma_{\Sigma^0\to\gamma p\pi^-}}{\mathrm{d}\cos\theta} = \frac{1}{2}\Gamma_{\Sigma^0\to\gamma\Lambda}\mathrm{Br}_{\Lambda\to p\pi^-}\,(1 - \alpha_\Lambda\alpha_{\Sigma^0}\cos\theta)$$

Constraining CP violation in $\Sigma^0 \to \Lambda\gamma$

How?

- Compare decay distributions of Σ^0 and $\bar{\Sigma}^0$

$$\frac{\mathrm{d}N}{\mathrm{d}\cos\theta} = \frac{N}{2}(1 - \alpha_\Lambda \alpha_{\Sigma^0} \cos\theta) \qquad \text{Vs} \qquad \frac{\mathrm{d}\bar{N}}{\mathrm{d}\cos\theta} = \frac{\bar{N}}{2}(1 - \bar{\alpha}_\Lambda \bar{\alpha}_{\Sigma^0} \cos\theta)$$

- Exploit $SU(3)_F$ symmetry:

  from upper limit on neutron EDM $\to$ get upper limit for the angular asymmetry

$$\boxed{|\alpha_{\Sigma^0}| \leq 3.0 \cdot 10^{-12} |\sin\delta_F| \to 3.0 \cdot 10^{-14}}$$

- Search for CP violation $\mathcal{O}_{\mathrm{CP}} := \alpha_{\Sigma^0} + \bar{\alpha}_{\Sigma^0}$

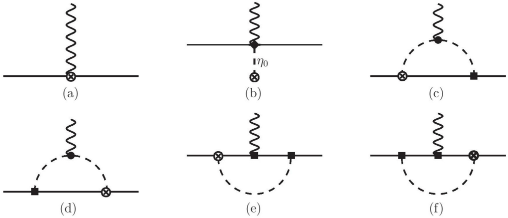
$$\boxed{|\mathcal{O}_{\mathrm{CP}}| \leq 6.0 \cdot 10^{-14}}$$

Nair/Perotti/Leupold, accepted for Phys.Lett.B

Theory approach

Σ^0-Λ EDM is related to nEDM via $SU(3)_F$ symmetry

- use baryon chiral perturbation theory (incl. θ-term) to connect them
- calculate Σ^0-Λ EDM at LO
  - one-loop diagrams contribute!
  - meson-baryon couples: $\pi^+\Sigma^-$, $\pi^-\Sigma^+$, $K^+\Xi^-$, K^-p



Guo/Meißer, JHEP 12 (2012) 097

Tree-level contribution to EDM

NEUTRON

$$d_n^{\text{tree}} = \frac{8}{3}\, e\bar{\theta}_0\,(\alpha w_{13} + w'_{13})$$

Σ-Λ

$$d_{\Sigma\Lambda}^{\text{tree}} = -\frac{4}{\sqrt{3}}\, e\bar{\theta}_0\,(\alpha w_{13} + w'_{13})$$

where:

- w_{13} and w'_{13} are low-energy constants from baryon NLO Lagrangian
- $\alpha := 144\, V_0^{(2)} V_3^{(1)} / (F_0 F_\pi M_{\eta_0})^2$
- $\bar{\theta}_0 = \left[1 + \frac{4V_0^{(2)}}{F_\pi^2} \frac{4M_K^2 - M_\pi^2}{M_\pi^2 (2M_K^2 - M_\pi^2)}\right]^{-1} \theta_0$

Loop contribution to EDM

NEUTRON

$$d_n^{\text{loop}} = -\frac{8e\bar{\theta}_0 V_0^{(2)}}{F_\pi^4} \sum_{\{M,B\}} C_{\text{cd}}\, J_{MM}(0)$$

loops	C_{cd}
$\{\pi^-, p\}$	$2(D+F)(b_D+b_F)$
$\{K^+, \Sigma^-\}$	$-2(D-F)(b_D-b_F)$

$\Sigma \to \Lambda\gamma$

$$d_{\Sigma\Lambda}^{\text{loop}} = -\frac{4e\bar{\theta}_0 V_0^{(2)}}{\sqrt{3}F_\pi^4} \sum_{\{M,B\}} (C_{\text{ce}} - C_{\text{df}})\, J_{MM}(0)$$

loops	C_{ce}	C_{df}
$\{\pi^+, \Sigma^-\}$	$-4Db_F$	$4Fb_D$
$\{\pi^-, \Sigma^+\}$	$-4Db_F$	$4Fb_D$
$\{K^+, \Xi^-\}$	$-(3F-D)(b_D+b_F)$	$(D+F)(3b_F-b_D)$
$\{K^-, p\}$	$-(D+3F)(b_D-b_F)$	$(D-F)(3b_F+b_D)$

where:

$$J_{MM}(q^2) = i\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - M^2 + i\epsilon)((k+q)^2 - M^2 + i\epsilon)}$$
$$= 2L + \frac{1}{16\pi^2}\left(\ln\frac{M^2}{\mu^2} - 1 - \sigma \ln\frac{\sigma-1}{\sigma+1}\right)$$

- L contains a divergence, absorbed into the renormalization of w'_{13}

Parameter estimates

Our calculations give:

$$\frac{d_{\Sigma\Lambda}}{d_n} = \frac{d_{\Sigma\Lambda}^{\text{tree}} + d_{\Sigma\Lambda}^{\text{loop}}}{d_n^{\text{tree}} + d_n^{\text{loop}}} \approx -0.88$$

Use the experimental upper limit for the nEDM:

$$|d_n^{\text{exp}}| \leq 2.9 \times 10^{-26} e\,\text{cm}$$

$$\downarrow\downarrow$$

to get an upper limit for the Σ^0-Λ EDM:

$$|d_{\Sigma\Lambda}| \leq 2.5 \times 10^{-26} e\,\text{cm}$$

Summary - part II

Goal: Constrain CP violation in $\Sigma^0 \to \Lambda\gamma$

Our result:

$$\alpha_{\Sigma^0} \approx -\frac{2 d_{\Sigma\Lambda} \sin\delta_F}{a}$$

$$\boxed{|\alpha_{\Sigma^0}| \leq 3.0 \cdot 10^{-14}} \quad \to \quad \boxed{|\mathcal{O}_{\rm CP}| \leq 6.0 \cdot 10^{-14}}$$

- far below any experimental resolution!
  - observation of CP violating angular asymmetry implies physics beyond the Standard Model (BSM)
- suitable for experimental check @PANDA

Back-up slides

Relevant interaction part of the LO chiral Lagrangian:

- including only octect baryons

$$\mathcal{L}_8^{(1)} = i\langle\bar{B}\gamma_\mu D^\mu B\rangle + \frac{D}{2}\langle\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\rangle + \frac{F}{2}\langle\bar{B}\gamma^\mu\gamma_5[u_\mu, B]\rangle$$

- including also decuplet resonances

$$\mathcal{L}_{8+10}^{(1)} = \mathcal{L}_8^{(1)} + \frac{1}{2\sqrt{2}} h_A\,\epsilon_{ade}\,g_{\mu\nu}\,(\bar{T}^\mu_{abc}\,u^\nu_{bd}\,B_{ce} + \bar{B}_{ec}\,u^\nu_{db}\,T^\mu_{abc})$$

Relevant interaction part of the NLO chiral Lagrangian:

- including only octect baryons

$$\begin{aligned}\mathcal{L}_8^{(2)} \quad = \quad & b_D\langle\bar{B}\{\chi_+, B\}\rangle + b_3\langle\bar{B}\{u^\mu, [u_\mu, B]\}\rangle + ib_6\big(\langle\bar{B}[u^\mu, \{u^\nu, \gamma_\mu D_\nu B\}]\rangle \\ & - \langle\bar{B}\overleftarrow{D}_\nu\{u^\nu, [u^\mu, \gamma_\mu B]\}\rangle\big) + \frac{i}{2}\, b_{10}\,\langle\bar{B}\{[u^\mu, u^\nu], \sigma_{\mu\nu}B\}\rangle\end{aligned}$$

Form Factors

$$\langle 0|j^\mu|\Sigma^0\bar{\Lambda}\rangle = e\,\bar{v}_\Lambda\left(\left(\gamma^\mu + \frac{m_\Lambda - m_\Sigma}{q^2}q^\mu\right)F_1(q^2) - \frac{i\,\sigma^{\mu\nu}q_\nu}{m_\Sigma + m_\Lambda}F_2(q^2)\right)\,u_\Sigma$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{(m_\Sigma + m_\Lambda)^2}F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$\langle r_E^2\rangle = 6\,\frac{dG_E(q^2)}{dq^2} \qquad \text{at } q^2 = 0$$

$$\langle r_M^2\rangle = \frac{6}{G_M(0)}\,\frac{dG_M(q^2)}{dq^2} \qquad \text{at } q^2 = 0$$

Dispersion Relations

We use the subtracted dispersion relations:

$$G_{M/E}(q^2) = G_{M/E}(0) + \frac{q^2}{12\pi} \int_{4m_\pi^2}^{\infty} \frac{ds}{\pi} \frac{T_{M/E}(s) p_{c.m.}^3(s) F_\pi^{V*}(s)}{s^{3/2}(s - q^2 - i\epsilon)}$$

with $T_{M/E}$ electric and magnetic scattering amplitudes:

$$T(s) = K(s) + \Omega(s) P_{n-1}(s) + \Omega(s) s^n \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\sin \delta(s') K(s')}{|\Omega(s')|(s' - s - i\epsilon) s'^n}$$

- K contains left-hand cuts from pole terms in u- and t-channel exchange diagrams
- P_{n-1} polynomial coming from contact terms

Consider pion rescattering encoded in the Omnes function:

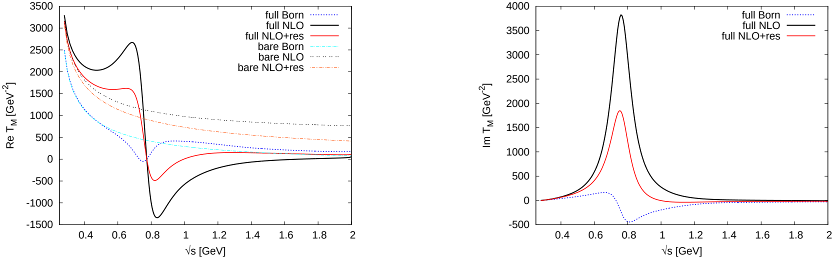
$$\Omega(s) = exp\left(s \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\delta(s')}{s'(s' - s - i\epsilon)} \right) \approx F_\pi^V(s)$$

with δ pion p-wave phase shift

Results: Helicity Amplitudes

Real and Imaginary part of $\Sigma\bar{\Lambda} \to \pi^+\pi^-$ helicity amplitudes

- Magnetic part

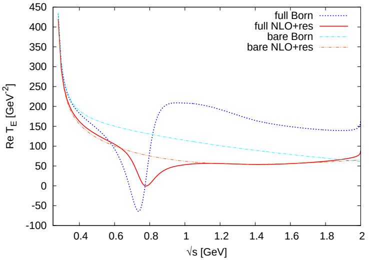


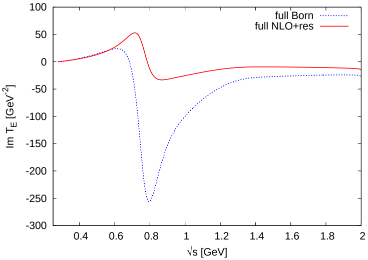
- full amplitudes have both real and imaginary part
- full amplitudes include ρ meson (via π-π phase shift)
- decuplet resonance exchange modifies considerably the amplitudes

Results: Helicity Amplitudes

Real and Imaginary part of $\Sigma\bar{\Lambda} \to \pi^+\pi^-$ helicity amplitudes

- Electric part





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Electric and magnetic dipole moment

Baryon coupling to the EM current J^μ:

$$\langle B'(p')|J^\mu|B(p)\rangle = e\,\bar{u}_{B'}(p')\,\Gamma^\mu(q)\,u_B(p), \qquad \text{with} \quad q := p - p'$$

and

$$\Gamma^\mu(q) = -\frac{i}{m_B + m_{B'}}\,\sigma^{\mu\nu} q_\nu\, F_2(q^2) - \frac{1}{m_B + m_{B'}}\,\sigma^{\mu\nu} q_\nu \gamma_5\, F_3(q^2)\; + \ldots$$

NEUTRON

$$F_{2,n}(0) = \kappa_n \approx -1.91$$

$$d_n = \frac{e}{2m_n}\,F_{3,n}(0)$$

Σ-Λ

$$\kappa_M := F_{2,\Sigma\Lambda}(0) \approx 1.98$$

$$d_{\Sigma\Lambda} := \frac{e}{m_{\Sigma^0} + m_\Lambda}\,F_{3,\Sigma\Lambda}(0)$$

Decay parameters

The two decay parameters a and b are related to the transition moments via:

$$a = \frac{e}{m_{\Sigma^0} + m_\Lambda} \kappa_M \,, \qquad b = i\, d_{\Sigma\Lambda}$$

For the second decay we have:

$$s := \mathcal{A} \qquad \text{and} \qquad p := \eta\mathcal{B}$$

with $\eta := |\vec{p}_p|/(m_p + E_p)$